DOCUMENTATION OF MATLAB OLG PROGRAM

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1. Introduction

In the baseline Diamond OLG model the equilibrium path of the economy can be described by the fundamental difference equation,

$$k_{t+1} = \frac{s(w(k_t), r(k_{t+1}))}{1+n}. \tag{1}$$

As seen, k_{t+1} appears on both sides of the equation. Under some specifications of the model, k_{t+1} will not be uniquely determined by k_t. The reaction curve may be S-shaped such that k_{t+1} is not a single-valued function of k_t. The "Matlab OLG Program" described here provides a method for visualizing the reaction curve even in such cases, by plotting pairs of k_t and k_{t+1} that satisfy equation (1).

Define the function G by

$$G(k_t, k_{t+1}) = \frac{s(w(k_t), r(k_{t+1}))}{1+n} - k_{t+1},$$

and note that equation (1) is satisfied when $G(k_t, k_{t+1}) = 0$.

Assume the setup to be well-defined, i.e. the value of $G(k_t, k_{t+1})$ to be unique for any $k_t, k_{t+1} \geq 0$. For a discrete set of non-negative values of k_t and k_{t+1} placed in a grid, we calculate the value of $G(k_t, k_{t+1})$ and interpolate linearly between the points in the grid. We can then approximate the reaction curve of the economy by the zero contour line of the resulting surface, and that is exactly what the program does.

2. Instructions

The program is run by executing the file `main.m` in Matlab. To change parameters and options, open `main.m` in a text editor. The code is heavily commented to minimize the need for cross referencing with this document.

2.1. **Options.** The user can specify the period utility function, the production function, and a range of program parameters.

2.2. **Specification.** The production function and the utility function can be specified by altering the program parameters `Prod_type` and `Util_type` corresponding to the values in the leftmost columns in the tables in Section 2.2.1 and 2.2.2.

2.2.1. *The production function.* At present three types of production functions are implemented. To add more, modify the file `f.m`.

Table 1. Production functions implemented.

Type	Name	Function
1	CES	$f(k) = a(bk^p + (1-b))^{(1/p)}, \quad a > 0,\ 0 < b < 1,\ p \leq 1$
2	Cobb-Douglas	$f(k) = ak^b, \quad a > 0,\ 0 < b \leq 1$
3	Unnamed	$f(k) = b + \frac{ak}{1+k}, \quad a > 0,\ b > 0$

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2.2.2. *The utility function.* At present four types of period utility functions are implemented (excluding the case of distinctive period utility, see below). To add more, modify the file `v.m`. In addition to the parameter restrictions in Table 2, it is also required that $m > 0$.

Table 2. Utility functions implemented.

Type	Name	Function
1	CRRA	$u(c) = \begin{cases} \ln(c) & \text{when } m = 1, \\ \frac{c^{1-m}}{1-m} & \text{otherwise} \end{cases}$
2	Subsistence cons.	$u(c) = \begin{cases} \ln(c-h), \ h \text{ free} & \text{when } m = 1, \\ \frac{(c-h)^{1-m}}{1-m}, \ h \geq 0 & \text{otherwise} \end{cases}$
3	CARA	$u(c) = -\exp(-mc)$
4	CARA-like	$u(c) = \begin{cases} -c^{-1} & \text{when } c < 0.703... \\ -\exp(-mc), & \text{otherwise} \end{cases}$

The user can specify the lifetime utility function to have distinctive period utility functions, by changing the parameter `Dist_util` from 0 to 1. In this case, the first period utility is given by $\ln(Dd + De * c)$ and the second period utility is given by $\ln(Df + Dg * c)$.

2.3. **Parameters.** The interpretation of the model parameters m, h, Dd, De, Df, Dg, a, b, and p depends on the choice of utility and production functions (see Section 2.2.2 and 2.2.1). The parameters m and h are used in the utility function when the period utility is the same in both periods. The parameter m is the (absolute) elasticity of marginal utility of consumption in the CRRA case, a general parameter in the "Subsistence cons." case, and the absolute risk aversion in the CARA and the CARA-like case. And the parameter h is the subsistence consumption in the "Subsistence cons." case. The parameters Dd, De, Df, and Dg are used in the utility function when the utility in the two periods are distinct. They are all general parameters. The parameters a, b, and p are used in the production functions where a is total factor productivity, b is the output elasticity wrt. capital in the Cobb-Douglas case and a minimal production level in the "Unnamed" case, and p is a parameter determining the elasticity of substitution between capital and labour in the CES case as $1/(1-p) \lesseqgtr 1$ as $p \lesseqgtr 0$.

The parameter d is the capital depreciation rate ($0 \leq d \leq 1$), which $r(k)$ in (1) depends on in the following way: $r(k) = f'(k) - d$. Finally, rho is the utility discount rate ($rho > -1$), and n is the population growth rate ($n > -1$).

When choosing values for the parameters d, rho, and n, one should keep in mind that the period length in the model is 25-30 years.

Adding technical progress: In the case where the period utility function is of type 1 (i.e., CRRA), Harrod-neutral technical progress at the rate γ can easily be incorporated in the simulation by replacing the parameter value for n by the value $n' \equiv (1+\gamma)(1+n) - 1$, where n is the population growth rate.

2.4. **Program options.** The output and precision of the numerical calculations can be manipulated with the following parameters,

`Dist_util`	Set to 1 to use distinct period utility functions.
`Util_type`	Set to the value of the instantaneous utility function.
`Prod_type`	Set to the value of to the production function.
`Line_f`	Set to 1 to plot $f(k_t)/(1+n)$. This line is dashed.
`Line_w`	Set to 1 to plot $w(k_t)/(1+n)$. This line is dash/dot'ed.
`Line_45_d`	Set to 1 to draw the 45-degree line in the phase diagram.
`Show_para`	Set to 1 to show the parameter values on the reaction curve.
`grid`	The number of evaluations of G on each axis, such that the total number of grid points are `grid`^{2}. A higher number is better, but the expense is increased calculation time. A value of 200 is recommended to give an idea of how the phase diagram looks. To gain more precision, increase the number to somewhere in the range 200–500.
`xmin`	The minimal value of k_t in the grid.
`ymin`	The minimal value of k_{t+1} in the grid.
`ymin`	The maximal value of k_t in the grid.
`ymax`	The maximal value of k_{t+1} in the grid.

2.5. **Files.** When modifying the program, please note the following. To add or change production functions, modify the files `f.m`, `R.m` and `W.m`. To add or change period utility functions, modify the file `v.m`. To change the intertemporal utility function, modify the file `U.m`. In Table 3 below, the content of each file of the program is described.

Table 3.

`f.m`	The production function on intensive form.
`G.m`	The function G described above.
`main.m`	The main program containing the parameter values and code to draw the reaction curve.
`pp.m`	Prints the parameters when the program is run.
`pw.m`	Prints a warning when parameter makes no economic meaning or can cause problems.
`R.m`	The interest rate.
`S.m`	The utility maximizing saving.
`U.m`	The intertemporal utility function.
`v.m`	The period utility function.
`W.m`	The wage rate.

2.6. **Notes.** One should always be cautious when interpreting numerical results. In the present case, the user should be aware that some specifications of the model lead to numerical imprecision, which roughly translates to jagged plots. To avoid this, increase the `grid` parameter at the expense of increased calculation time. Also note that it is assumed that the model is well-defined i.e. that the value of $s(w,r)$ is unique for any w and r. The program will not be able to tell if this is not the case.

If needed, calculation in Matlab can be cancelled by pressing CTRL+C.