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Incomplete financial markets and jumps in asset prices

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Abstract

A dynamic pure-exchange general equilibrium model with uncertainty is studied. Fundamentals are supposed to depend continuously on states of nature. It is shown that: 1. if financial markets are complete, then asset prices vary continuously with states of nature, and; 2. if financial markets are incomplete, jumps in asset prices may be unavoidable. Consequently incomplete financial markets may increase volatility in asset prices significantly.

Keywords: General equilibrium, financial markets, jumps in asset prices.

JEL-classification: D52, D53, G12.

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1 Introduction

In the present paper we provide an explanation of jumps in asset prices based on the interaction of real markets and financial markets.

An empirical characteristic of asset prices is that distributions of price changes have thick tails, i.e., large changes in asset prices are overly represented in observed data. Indeed in Merton (1976), motivated by the observation that stock prices tend to show far too many outliers, the study of option prices in case of jumps in the underlying security prices was initiated. Thick tails are not consistent with the standard assumption of Gaussian processes widely used in the finance literature. Therefore jump processes such as Poisson processes seem to be necessary to account for the thick tails (see e.g. Andersen, Benzoni & Lund (2002)). In Bansal & Shaliastovich (2008) it is mentioned that the frequency of jumps is 1-1.5 per year and that around 10 percent of the volatility in asset prices is explained by jumps. The consequences of jumps in asset prices are potentially significant as jumps in asset prices increase uncertainty: fundamentals are uncertain and small changes in fundamentals can result in dramatic changes of prices.

Several contributions aim at explaining jumps in asset prices. In Calvet & Fisher (2008) an optimal growth model where endowments and dividends are uncertain is considered and it is shown that jumps in the drift and/or volatility of endowments and dividends generate jumps in asset prices even though sample paths of endowments and dividends are continuous. In Balduzzi, Foresi & Hait (1997) and Lim, Martin & Teo (1998) partial equilibrium models with ad hoc behaviour of some investors are considered and this behaviour causes supply curves to be non-monotonic leading to jumps in asset prices. In Bansal & Shaliastovich (2008) an optimal growth model with a representative consumer, where dividends are uncertain, information is incomplete and the consumer can buy a precise signal, is considered and it is shown that from time to time the representative consumer buys the precise signal in which case asset prices jump.

According to the market efficiency hypothesis changes in asset prices must be due to changes in dividends or conditional expectations because, as shown in Huang (1985), if both dividends and conditional expectations vary continuously, then asset prices vary continuously too. In Calvet & Fisher (2008) and Bansal & Shaliastovich (2008) jumps in asset prices are caused by jumps in conditional expectations.

Some contributions aim at exploring a possible link between incomplete financial markets and volatility of asset prices. In Geanakoplos (1997) the use of collateral in contracts is shown to induce an excess volatility in the prices of the durable goods that are used as collateral, excess volatility in the sense that the variance is larger with the use of collateral in contracts than with complete markets. In Citanna & Schmedders (2001) financial innovation is shown to induce excess volatility. In Calvet (2001) incomplete financial markets are shown to lead to excess volatility. The difference between "jumps in asset prices" and "volatility of asset prices" should be noted. Indeed volatility of asset prices does not necessarily involve jumps, but merely changes of asset prices.

In the present paper a dynamic, finite horizon, pure-exchange general equilibrium model with uncertainty is studied. Fundamentals are assumed to be continuous functions of states of nature. We show that: 1. if financial markets are complete, then prices (including asset prices), consumption bundles and portfolios are continuous functions of the states of nature, and; 2. if financial markets are incomplete, then neither prices, consumption bundles nor portfolios need to be continuous functions of states of nature. Therefore incompleteness of financial markets may increase volatility in asset prices significantly.

The paper proceeds as follows: In Section 2 the set-up, the equilibrium concepts and our maintained assumptions are introduced. In Section 3, respective Section 4, complete financial markets, respective incomplete financial markets, are considered. Finally in Section 5 some final remarks are provided.

2 The model

Set-up

There is a finite number $T+1$ of dates with $t \in \{0, \ldots, T\}$. There is uncertainty, the set of states at date $t \geq 1$ is $S = [0,1]$ with $s \in S$ and $\pi : S^T \to \mathbb{R}_+$ is the density on the set of states S^T. There is a finite number of goods ℓ at every state with $j \in \{1, \ldots, \ell\}$. A collection of maps $p = (p_t)$, where $p_t : S^t \to \mathbb{R}^\ell_{++}$, is a price system for goods.

There is a finite number m of consumers with $i \in \{1, \ldots, m\}$. Consumers are described by their identical consumption sets $X = (\mathbb{R}^\ell_{++})^{T+1}$, their endowments $\omega_i = (\omega_i^t)_t$, where endowments at date t is described by a map $\omega_i^t : S^t \to X$, and their state utility functions $u_i : X \to \mathbb{R}$. A consumption bundle is a collection of maps $x_i = (x_i^t)_t$, where $x_i^t : S^t \to \mathbb{R}^\ell_{++}$. An allocation of goods $x = (x_i)_i$ is a list of individual consumption bundles.

Walrasian equilibrium

Let $s^t = (s_1, \ldots, s_t)$ denote the history of states up to and including date t, then the problem of consumer i is:

$$\max_{x_i} \int_{S^T} u(x_i^0, \ldots, x_i^T(s^T))\ \pi(s^T)\ ds^T$$

$$\text{s.t.} \int_{S^T} \sum_t p_t(s^t) \cdot x_i^t(s^t)\ ds^T \ \leq \int_{S^T} \sum_t p_t(s^t) \cdot \omega_i^t(s^t)\ ds^T$$

Formally integrability assumptions are needed.

In a Walrasian equilibrium consumers choose consumption bundles that solve their problems and markets clear.

Definition 1 *A* **Walrasian equilibrium** *is a price system for goods and an allocation of goods $(\bar{p}, \bar{x})$ such that:*

- *$\bar{x}_i$ is a solution to the problem of consumer i for all i, and;*
- *markets clear, $\sum_i \bar{x}_i^t(s^t) = \sum_i \omega_i^t(s^t)$ for all t and s^t.*

Financial market equilibrium

There is a finite number n of assets with $k \in \{1, \ldots, n\}$ where the dividend of asset k at date t is described by a map $a_k^t : S^t \to \mathbb{R}^\ell$. An asset structure is a collection of assets $a = (a_k)_k$, where $a_k = (a_k^t)_t$. A price system for an asset structure is a collection of maps $q = (q_t)$, where $q_t : S^t \to \mathbb{R}^n$. A portfolio plan is a collection of maps $z_i = (z_i^t)_t$, where $z_i^t : S^t \to \mathbb{R}^n$. An allocation of assets $z = (z_i)_i$ is a list of portfolio plans.

A price system (p, q) is a price system for goods and a price system for assets. An allocation (x, z) is an allocation of goods and an allocation of assets.

Let $a_t(s^t)$ be the $\ell \times n$-matrix of dividends $(a_t^1(s^t) \ldots a_t^n(s^t))$ at date t in state s^t, then the problem of consumer i is:

$$\max_{(x_i, z_i)} \int_{S^T} u(x_i^0, \ldots, x_i^T(s^T)) \; \pi(s^T) \; ds^T$$

$$\text{s.t.} \begin{cases} p_0 \cdot x_i^0 + q_0 \cdot z_i^0 \; \leq \; p_0 \cdot \omega_i^0 \\ p_t(s^t) \cdot x_i^t(s^t) + q_t(s^t) \cdot z_i^t(s^t) \\ \qquad \leq \; p_t(s^t) \cdot \omega_i^t(s^t) + (q_t(s^t) + p_t(s^t) a^t(s^t)) \cdot z_i^{t-1}(s^{t-1}) \\ \qquad \text{for all } t \in \{1, \ldots, T-1\} \\ p_T(s^T) \cdot x_i^T(s^T) \\ \qquad \leq \; p_T(s^T) \cdot \omega_i^T(s^T) + (p_T(s^T) a^T(s^T)) \cdot z_i^{T-1}(s^{T-1}) \end{cases}$$

In a financial market equilibrium consumers choose consumption bundles and portfolio plans that solves their problems and markets clear.

Definition 2 *A* **financial market equilibrium** *is a price system and an allocation* $((\bar{p}, \bar{q}), (\bar{x}, \bar{z}))$ *such that:*

- $(\bar{x}_i, \bar{z}_i)$ *is a solution to the problem of consumer* i *for all* i*, and;*
- *markets clear,* $\sum_i \bar{x}_i^t(s^t) = \sum_i \omega_i^t(s^t)$ *and* $\sum_i \bar{z}_i^t(s^t) = 0$ *for all* t *and* s^t*.*

Assumptions

The consumers are supposed to satisfy the following assumptions:

(A.1) $\omega_i^t \in \mathrm{C}^1(S^t, X)$.

(A.2) $u_i \in \mathrm{C}^2(X, \mathbb{R})$ with $Du_i(x_i) \in \mathbb{R}^{\ell T}_{++}$ for all x_i and $v' D^2 u_i(x_i) v < 0$ for all x_i and $v \neq 0$.

The economy is supposed to satisfy the following assumptions:

(A.3) $\pi \in \mathrm{C}^1(S^T, \mathbb{R}_{++})$.

(A.4) $a_k^t \in \mathrm{C}^1(S^t, \mathbb{R}^\ell)$ for all k and t.

Existence of equilibrium

The focus of the present paper is on properties of equilibria rather than existence. However a short discussion of existence of Walrasian equilibrium and of financial market equilibrium for economies with infinite dimensional commodity spaces is provided below.

In Bewley (1972) the existence of a Walrasian equilibrium is shown for consumption bundles in L^∞ and price systems in L^1. In the proof it is crucial that consumption sets have non-empty interior. In Mas-Colell (1986, 1991) existence of Walrasian equilibrium in more general vector lattices, where the consumption set does not necessarily have a non-empty interior, is considered. The assumption of uniform properness of preferences, which implies the existence of supporting prices, replaces the assumption that consumption sets have non-empty interior.

The problem with changes in the dimension of set of income transfers spanned by assets carries over from economies with finitely many states. Moreover as shown in Mas-Colell & Monteiro (1996) and Mas-Colell & Zame (1996) there is a problem with feasibility of consumption bundles. The assumption that every feasible portfolio results in a feasible consumption bundle appears to be needed to ensure existence of equilibrium. However the assumption is very strong, especially for economies with at least two goods per state.

3 Complete financial markets

In the present paper functions that are identical except for a set of measure zero are considered to be identical.

Definition 3 *A measurable function $f : S^T \to \mathbb{R}$ is continuous at $\hat{s}^T$ if and only if there exist a neighborhood A of $\hat{s}^T$ and a function $g : S^T \to \mathbb{R}$, where $g^{-1}(B)$ is open for B open, such that*

$$\int_A 1_{\{s^T | f(s^T) \neq g(s^T)\}} \; \pi(s^T) \; ds^T \;=\; 0.$$

A function is continuous if and only if it is continuous at all points.

Walrasian equilibrium

At Walrasian equilibria, prices and consumption bundles are differentiable functions of states of nature. The proof consists of two steps: in Lemma 1 it is shown that prices and consumption bundles are continuous functions, and; in Theorem 1 it is shown that if they are continuous functions of states, then they are differentiable functions.

Lemma 1 *Suppose that $(\bar{p}, \bar{x})$ is a Walrasian equilibrium. Then $(\bar{p}, \bar{x})$ is continuous in s^T.*

Proof: Suppose that $(\bar{p}, \bar{x})$ is a Walrasian equilibrium, then there exists $\lambda_1, \ldots, \lambda_m > 0$ such that $\bar{x}$ is the solution to the following problem

$$\begin{aligned} \max_x \;\; & \sum_i \lambda_i \int u_i(x_i^0, \ldots, x_i^T(s^T)) \; \pi(s^T) \; ds^T \\ \text{s.t.} \;\; & \sum_i x_i^t(s^t) \;=\; \sum_i \omega_i^t(s^t) \text{ for all } t \text{ and } s^t \end{aligned} \tag{1}$$

The proof that $\bar{x}$ is continuous in s^T is by backward induction on t.

"$t = T$" Suppose that $\hat{c}^{T-1} = (\hat{x}^0, \ldots, \hat{x}^{T-1})$ and $\hat{s}^T$ are fixed and consider the following maximization problem

$$\begin{aligned} \max_{x^T} \quad & \sum_i \lambda_i u_i(\hat{c}_i^{T-1}, x_i^T) \\ \text{s.t.} \quad & \sum_i x_i^T \;=\; \sum_i \omega_i^T(\hat{s}^T). \end{aligned}$$

Then for every $\hat{c}^{T-1}$ and $\hat{s}^T$ there exists a unique continuous solution to the maximization problem according to assumptions (A.2) and (A.3). Let $f^T : (X^m)^T \times S^T \to X^m$ be the solution, then it is continuous according to Berge's maximum theorem and if $c^{T-1} = (\bar{x}^0(s^0), \ldots, \bar{x}^{T-1}(s^{T-1}))$, then $f^T(c^{T-1}, s^T) = \bar{x}^T(s^T)$. Moreover the function $v_i^T : (X^m)^T \times S^{T-1} \to \mathbb{R}$ defined by

$$v_i^T(c^{T-1}, s^{T-1}) \;=\; \int u_i(c_i^{T-1}, f_i^T(c^{T-1}, s^T)) \; \pi(s_T | s^{T-1}) \; ds_T$$

is strictly concave in $x^0, \ldots, x^{T-1}$.

"$t = T - 1$" Suppose that $\hat{c}^{T-2} = (\hat{x}^0, \ldots, \hat{x}^{T-2})$ and $\hat{s}^{T-1}$ are fixed and consider the following maximization problem

$$\begin{aligned} \max_{x^{T-1}} \quad & \sum_i \lambda_i v_i(\hat{c}^{T-2}, x^{T-1}, \hat{s}^{T-1}) \\ \text{s.t.} \quad & \sum_i x_i^{T-1} \;=\; \sum_i \omega_i^{T-1}(\hat{s}^{T-1}). \end{aligned}$$

Then for every $\hat{c}^{T-2}$ and $\hat{s}^{T-1}$ there exists a unique continuous solution to the maximization problem according to assumptions (A.2) and (A.3). Let $f^{T-1} : (X^m)^{T-1} \times S^{T-1} \to X^m$ be the solution, then it is continuous according to Berge's maximum theorem and if $c^{T-2} = (\bar{x}^0(s^0), \ldots, \bar{x}^{T-2}(s^{T-2}))$, then $f^{T-1}(c^{T-2}, s^{T-1}) = \bar{x}^{T-1}(s^{T-1})$. Moreover the function $v_i^{T-1} : (X^m)^{T-1} \times S^{T-2} \to \mathbb{R}$ defined by

$$\begin{aligned} & v_i^{T-1}(c^{T-2}, s^{T-2}) \\ & \quad = \int v_i^T(c^{T-2}, f^{T-1}(c^{T-2}, s^{T-1}), s^{T-1}) \; \pi(s_{T-1} | s^{T-2}) \; ds_{T-1} \end{aligned}$$

is strictly concave in c^{T-2}.

The steps for $t = T-2, \ldots, 0$ are similar to the step for $t = T-1$. The solution $(\bar{x}^t)_t$, where $\bar{x}_t : S^t \to X^m$, to problem (1) is defined as follows

$$\begin{aligned}
\bar{x}^0 &= f^0 \\
\bar{x}^1(s^1) &= f^1(\bar{x}^0, s^1) \\
&\vdots \\
\bar{x}^{T-1}(s^{T-1}) &= f^{T-1}(\bar{x}^0, \bar{x}^1(s^1), \ldots, \bar{x}^{T-2}(s^{T-2}), s^{T-1}) \\
\bar{x}^T(s^T) &= f^T(\bar{x}^0, \bar{x}^1(s^1), \ldots, \bar{x}^{T-1}(s^{T-1}), s^T).
\end{aligned}$$

The price system $\bar{p}$ is collinear with the gradients of the consumers, so the price system is continuous in s^T too. Indeed there exists $\tau > 0$ such that

$$\bar{p}_t(s^t) = \tau\lambda_i \int D_{x^t} u_i(\bar{x}_i(s^T))\ \pi(s_{t+1}, \ldots, s_T | s^t)\ d(s_{t+1}, \ldots, s_T).$$

for all i, t and s^t.

$\square$

Remark: In the proof of Lemma 1 it is only used that utility functions are once differentiable and strictly concave, but it is not used that utility functions are twice differentiable with negative definite Hessian matrices.

End of remark

Theorem 1 *Suppose that $(\bar{p}, \bar{x})$ is a Walrasian equilibrium. Then $(\bar{p}, \bar{x})$ is differentiable in s^T.*

Proof: Suppose that $(\bar{p}, \bar{x})$ is a Walrasian equilibrium, then according to Lemma 1 it is continuous in s^T and there exists $\lambda_1, \ldots, \lambda_m > 0$ such that $\bar{x}$ is the solution to the following problem

$$\begin{aligned}
\max_x \quad & \sum_i \lambda_i \int u_i(x_i^0, \ldots, x_i^T(s^T))\ \pi(s^T)\ ds^T \\
\text{s.t.} \quad & \sum_i x_i^t(s^t) = \sum_i \omega_i^t(s^t) \text{ for all } t \text{ and } s^t.
\end{aligned}$$

The proof that $\bar{x}$ is differentiable in s^T is by induction on t. At step t it is assumed that x^0 is differentiable in s^0,…, x^{t-1} is differentiable in s^{t-1}.

"$t = 0$" The first-order conditions with respect to x^0 at s^0 are

$$\begin{aligned} \lambda_i \int D_{x^0} u_i(x_i^0, \ldots, x_i^T(s^T))\ \pi(s^T)\ ds^T - \alpha^0 &= 0 \text{ for all } i \\ \sum_i x_i^0 - \sum_i \omega_i^0 &= 0 \end{aligned}$$

The $\ell(m+1) \times \ell(m+1)$-matrix H^0 of derivatives with respect to x^0 and α^0 of the first-order conditions is

$$\begin{pmatrix} D_1^0 & & & -I \\ & \ddots & & \vdots \\ & & D_m^0 & -I \\ I & \cdots & I & \end{pmatrix}$$

where D_i^0 is a $\ell \times \ell$-matrix defined by

$$D_i^0 \ = \ \lambda_i \int D^2_{x^0 x^0} u_i(x_i^0, \ldots, x_i^T(s^T))\ \pi(s^T)\ ds^T$$

and I is a the $\ell \times \ell$-identity matrix. The matrix H^0 has full rank. Therefore according to the Implicit Function Theorem x^0 is a differentiable function of s^0, because x^1 is a continuous function of s^1,…, x^T is a continuous function of s^T.

"$t = T$" The first-order conditions with respect to x^T at s^T are

$$\begin{aligned} \lambda_i D_{x^T} u_i(x_i^0, \ldots, x_i^T(s^T)) - \alpha^T &= 0 \text{ for all } i \\ \sum_i x_i^T(s^T) - \sum_i \omega_i^T(s^T) &= 0 \end{aligned}$$

The $\ell(m+1) \times \ell(m+1)$-matrix H^T of derivatives with respect to x^T and α^T of the first-order conditions is

$$\begin{pmatrix} D_1^T & & & -I \\ & \ddots & & \vdots \\ & & D_m^T & -I \\ I & \cdots & I & \end{pmatrix}$$

where D_i^T is a $\ell \times \ell$-matrix defined by

$$D_i^T \;=\; \lambda_i D^2_{x^T x^T} u_i(x_i^0, \ldots, x_i^T(s^T)).$$

The matrix H^T has full rank. Therefore according to the Implicit Function Theorem x^T is a differentiable function of s^T, because x^0 is a differentiable function of s^0,..., x^{T-1} is a differentiable function of s^{T-1}.

The fact that $\bar{p}$ is differentiable in s^T follows from the proof that $\bar{p}$ is continuous in s^T in the proof of Lemma 1 and that $\bar{x}$ is differentiable in s^T.

$\square$

Financial market equilibrium: complete markets

At financial market equilibria, where the allocation is Pareto optimal, prices of goods and assets, consumption bundles and portfolios are differentiable functions of states.

Corollary 1 *Suppose that $(\bar{p}, \bar{x})$ is a Walrasian equilibrium and that $a = (a_k)_k$ is an asset structure such that $((\bar{p}, \bar{q}), (\bar{x}, \bar{z}))$ is a financial market equilibrium. Then $\bar{q}$ is differentiable in s^T.*

Proof: The proof that $\bar{q}$ is differentiable in s^T is by backward induction on t.

"$t = T-1$" The price of asset k at date $T-1$ in state s^{T-1} is

$$\bar{q}^k_{T-1}(s^{T-1}) \;=\; \int \bar{p}_T(s^{T-1}, s_T) \cdot a_k^T(s^{T-1}, s_T)\; ds_T$$

where $\bar{p}_T$ is continuous in s^T according to Lemma 1 and a_k^T is continuous in s^T according to assumption (A.5). Therefore $\bar{q}^k_{T-1}$ is continuous in s^{T-1}.

"$t = 0$" Trivial because $\bar{q}^k_0$ is a number rather than a function. However the asset price of asset k at date 0 is

$$\bar{q}^k_0 \;=\; \int (\bar{p}_1(s_1) \cdot a_k^1(s_1) + q_1^k(s_1))\; ds_1$$

where $\bar{p}_1$ is continuous in s^1 according to Lemma 1 and a_k^1 is continuous in s^1 according to assumption (A.5). Therefore $\bar{q}^k_0$ is continuous.

$\square$

Remark: In the proof of Corollary 1 it is only used that $(\bar{p}, \bar{x})$ is continuous and that a is continuous, but it is not used that a is differentiable.

End of remark

4 Incomplete financial markets

Financial market equilibrium: incomplete markets

At financial market equilibria, where financial markets are incomplete, there may be jumps in prices including asset prices, consumption bundles and portfolios. The proof is based on an example.

Theorem 2 *There exists an economy such that if* $((\bar{p}, \bar{q}), (\bar{x}, \bar{z}))$ *is a financial market equilibrium, then* $\bar{q}$ *is discontinuous in* s^T*.*

Proof: Consider an economy with three dates $T = 2$, one good per state $\ell = 1$, two consumers $m = 2$ and one asset $n = 1$. The dividend of the asset is supposed to be one unit of the good at the last date. Endowments and asset dividends are supposed to independent of the state at the last date. For the density $\pi : S \to \mathbb{R}_{++}$ suppose that $\pi(s) = 1$ for all $s \in S$.

Endowments at the first date are supposed to be identical $\omega_2^0 = \omega_1^0$ and endowments at the last two dates are supposed to be reverse in the sense that $\omega_2^1(s) = \omega_1^2(1-s)$ and $\omega_2^2(s) = \omega_1^1(1-s)$. Similarly utility functions are supposed to be identical for the first date and reverse for the last two dates in the sense that $u_2(x^0, x^1, x^2) = u_1(x^0, x^2, x^1)$.

For c_i^0 let $f_i(\cdot; c_i^0) : \mathbb{R}^2_{++} \times \mathbb{R}_{++} \to \mathbb{R}^2_{++}$ denote the demand function for the consumer with endowments $e_i(s) = (\omega_i^1(s), \omega_i^2(s))$ and utility function $v_i(\cdot; c_i^0) : \mathbb{R}^2_{++} \to \mathbb{R}$ defined by $v_i(x_i^1, x_i^2; c_i^0) = u_i(c_i^0, x_i^1, x_i^2)$. Then $(p, s) \in \times \mathbb{R}^2_{++} \times S$ is an equilibrium for the Edgeworth box economy $\mathcal{E}(s; (c_i^0)_i) = (e_i(s), v_i(\cdot, ; c_i^0))_i$ if and only if

$$f_1(p, p \cdot e_1(s); c_1^0) + f_2(p, p \cdot e_2(s); c_2^0) \;=\; e_1(s) + e_2(s).$$

Clearly (p_1, p_2, s) is an equilibrium for $\mathcal{E}(s; (c_i^0)_i)$ if and only if $(p_2, p_1, 1-s)$ is an equilibrium for $\mathcal{E}(1-s; (d_i^0)_i)$, where $d_1^0 = c_2^0$ and $d_2^0 = c_1^0$.

Suppose that equilibrium prices are normalized such that the sum of the prices is equal to one and let $E \subset \mathbb{R}^2_{++} \times S$ be the equilibrium set for the collection of Edgeworth economies $(\mathcal{E}(s; (c_i^0)_i)_s$, where $c_i^0 = \omega_i^0$, so

$$E \;=\; \{\, (p, s) \,|\, (p, s; (\omega_i^0)_i) \text{ is an equilibrium for } \mathcal{E}(s; (\omega_i^0))\,\}.$$

Suppose that E is S-shaped as shown in Figure 1 and let $r : S \to \mathbb{R}^2_{++}$ be a selection from $\mathcal{E}$ such that $r_1(s)$ is the lowest equilibrium price for $s < 1/2$, $r_1(s) = (1/2, 1/2)$ for $s = 1/2$ and $r_1(s)$ is the highest equilibrium price for $s > 1/2$. In order to construct a financial market equilibrium: let the

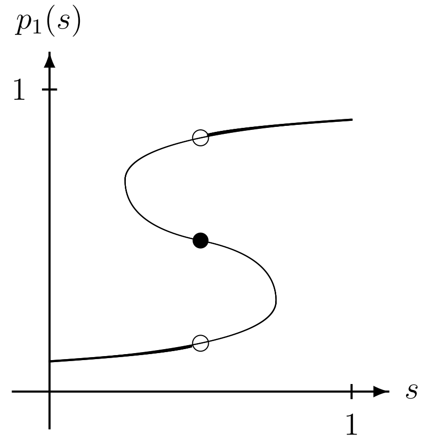

Figure 1: *The equilibrium set E and the selection r.*

allocation x be defined by $x_i^0 = \omega_i^0$, $x_i^j(s) = f_i^j(r(s), e_i(s); \omega_i^0)$ for $j \in \{1, 2\}$; let the portfolio plan z be defined by $z_i^0 = 0$ and $z_i^1(s) = (r_1(s)/r_2(s))(\omega_i^1(s) - f_i^1(r(s), e_i(s); \omega_i^0)) \;=\; f_i^2(r(s), e_i(s); \omega_i^0) - \omega_i^2(s)$; let the price system p be defined by $p_2(s) = p_1(s) = p_0 = 1$, and; let the price system for assets q be defined by $q_1(s^1) = r_2(s^1)/r_1(s^1)$ and $q_0 > 0$ such that

$$\int \left(-q_0 \frac{\partial u_i(x_i(s))}{\partial x_i^0} + q_1(s) \frac{\partial u_i(x_i(s))}{\partial x_i^1} \right) \, ds \;=\; 0.$$

Then $((p,q),(x,z))$ is a financial market equilibrium and the asset price at date 1 is discontinuous at $s = 1/2$.

Finally the portfolio z_1^0 of consumer 1 at date 0 is bounded from below by $-\min_s(\omega_1^1(s) + q_1(s)\omega_1^2(s))/q_1(s)$ and from above by $\min_s(\omega_2^1(s) + q_1(s)\omega_2^2(s))/q_1(s)$. Therefore suppose that $\|(\omega_1^1(s), \omega_1^2(s))\|$ is bounded from above by $\varepsilon > 0$ for $s \in \{0,1\}$ and that the marginal rates of substitution at the Pareto optimal allocations in the Edgeworth box economies for $s \in \{0,1\}$ are bounded away from zero and infinity. Then for ε sufficiently small the set of equilibria for the collection of Edgeworth box economies is S-shaped for all feasible portfolios so there is a discontinuity in prices.

□

Remark: The proof of Theorem 2 reveals that any measurable selection $r : S \to \mathbb{R}^2_{++}$ such that $r_1(s) = 1 - r_1(1-s)$ and $r_2(s) = 1 - r_2(1-s)$ is part of a financial market equilibrium. Therefore as shown in Mas-Colell (1991) there is a continuum of financial market equilibria.

End of remark

On the example in the proof of Theorem 2

Let us try, informally, to argue that the example in the proof of Theorem 2 is robust. In order to consider pertubations of fundamentals suppose that the set of fundamentals is endowed with the Whitney topology, endowments and dividends with the C^1-topology and utility functions with the C^2-topology.

The S-shape of the equilibrium set E is robust to perturbations in fundamentals and small changes in portfolios. Therefore every selection from the equilibrium set is discontinuous. Hence assets prices are discontinuous. The robustness of the example in the proof of Theorem 2 shows that the symmetry in the example is not essential, but merely convenient.

5 Final remarks

In the present paper we have shown that jumps in asset prices may be unavoidable in case of incomplete financial markets. Moreover we have shown that jumps are impossible in case of complete financial markets. Therefore our results implies that incompleteness of financial markets is a possible explanation of jumps in asset prices. Hence incompleteness of financial markets may increase uncertainty significantly compared to complete financial markets.

In the example, where asset prices jump, endowments vary continuously with states of nature, while dividends are constant across states of nature. Thus it should be pointed out that jumps in asset prices have to be seen as the outcome of the interaction of real markets and financial markets.

From a finance perspective it would be interesting to calibrate a parametric model such as an optimal growth model or an overlapping generations model in order to study whether jumps in asset prices are compatible with data. From a general equilibrium perspective a partial answer to the question of the appropriate commodity space for economies with infinite dimensional commodity spaces has been provided. Indeed we have shown that for Walrasian equilibria restricting attention to continuous maps on the underlying state space as in Chichilnisky & Zhou (1998) is no real restriction.

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