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# Bank Lending, Housing and Spreads* 

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#### Abstract

The framework presented in this paper takes its cue from recent financial events and attempts to develop a tractable framework for policy analysis of macro-linkages, in particular a first attempt at the integration of an independent profit-maximising banking sector that lends to and borrows from agents in the economy, and through which changes in the monetary policy rate by the central bank are transmitted. The inter-linkages between housing and the role of the banking sector in the transmission of monetary policy is emphasized. Two competing effects are highlighted: (i) a financial accelerator channel, due to the presence of collateralized borrowers, and (ii) a banking attenuator effect, which crucially arises from the spread in interest rates caused by the introduction of monopolistically competitive financial intermediaries. We show how the classical amplification mechanism explored in models of private borrowing between collaterally-constrained 'impatient' households and unconstrained 'patient' households, such as those put forward by Kiyotaki and Moore (1997) and Iacoviello (2005), is counteracted by the banking attenuator effect, given an endogenous steady state spread between loan and savings rates. Attenuation occurs therefore even under the assumption of flexible interest rates. This effect is further magnified when sluggishness in the interest rate-setting mechanism is introduced.


JEL: E32; E43; E44; E51; E52; E58
Keywords: Bank Lending, Housing, Liquidity, Credit, Staggered Interest RateSetting, Collateral Constraints.

[^0]
## 1 Introduction

Recent years have borne witness to increasingly frequent episodes of financial turmoil, characterized by contracted liquidity in the global banking system. The recent subprime mortgage crisis has highlighted the detrimental effects on credit and spending induced by the deterioration of bank balance sheets and those of other leveraged lenders. Central bankers are particularly sensitive to the obvious possibility of negative feedback from economic activity to the financial system, with the growing possibility of even more severe second-round effects on the macroeconomy through reduced credit availability.

The framework presented in this paper takes its cue from recent financial events and attempts to develop a tractable framework for policy analysis of macro-linkages, in particular a first attempt at the integration of an independent profit-maximizing banking sector that lends to and borrows from agents in the economy, and through which changes in monetary policy are transmitted. Two competing effects are highlighted: (i) the standard financial accelerator channel, due to the presence of collateralized borrowing (against durable goods), and (ii) a banking attenuator effect, which crucially arises from the spread in interest rates caused by the introduction of monopolistically competitive financial intermediaries. We show analytically and numerically, via impulse response analysis, how the classical amplification mechanism explored in models of private borrowing between collaterally-constrained 'impatient' households and unconstrained 'patient' households, such as those put forward by Kiyotaki and Moore (1997), Iacoviello (2005) and Monacelli (2008), is counteracted by the banking attenuator effect, given an endogenous steady state spread between loan and savings rates. Attenuation occurs therefore even under the assumption of flexible interest rates. The impact of a monetary policy tightening in a model with banking depresses the down payment necessary to increase borrowers' consumption of durable services by more than what is observed in models with direct credit flows between private agents. This effect is reinforced even further when sluggishness in the interest rate-setting mechanism is introduced.

The recent crisis is remarkable not just for its propagation across credit markets and its potency, but also because it has brought to the forefront numerous issues which have been highlighted and commented on by economists for a number of years, but have never been fully resolved. For example, understanding whether monetary authorities should react to asset prices bubbles, particularly in the housing market, as well as the role of monetary aggregates in informing and influencing central bank responses are both features of the current crisis - yet they are old issues on which work has been steadily progressing and developing. Most notably, the recent crisis has thrown the role of banks as financial intermediaries into sharp relief. The breakdown of the link between monetary policy instruments and the interbank lending rate has highlighted that any model which attempts to analyze the traditionally healthy monetary transmission, now needs to include
explicitly a financial sector that actively intermediates funds and provides vital structural loans to all participants in the economy, be they households, firms or even other financial institutions. ${ }^{1}$

There is therefore a rich vein of topics waiting to be tapped by monetary economists, and despite this plethora of issues, the ultimate aim of this paper is to provide a simple upgrade to the existing New Neoclassical Synthesis (NNS) framework ${ }^{2}$ as dubbed by Goodfriend and King (1997). Recent events in particular provide a useful list of ingredients for this upgrade, such that any micro-founded DSGE framework for the macroeconomy should ideally integrate the following at the simplest level: liquidity-constrained households, durable goods (specifically housing), and interest rate spreads generated by a profit-maximizing interest rate-setting banking sector with balance sheet considerations. ${ }^{3}$ The standard NNS is ultimately restricted since the transmission mechanism of monetary policy remains limited to a single real interest rate channel on aggregate demand. This means that a number of interesting issues are automatically excluded, namely interest rate spreads, as well as the potential impact of such spreads in the face of other modelling assumptions such as credit market imperfections, collateral and wealth effects linked to the evolution of asset (house) prices and household heterogeneity in saving rates.

Constrained households and durable goods have already been introduced and explored in previous papers. Specifically, Bernanke et al. (1996) developed the concept of the financial accelerator, which emphasizes the role of collateral requirements in affecting aggregate fluctuations. Kiyotaki and Moore (1997) built a general equilibrium model with two types of agents (borrowers and savers) and introduced heterogeneity via differential patience rates. Agents trade private debt intertemporally and collateral requirements are motivated by the presence of limited enforcement. And building on Kiyotaki and Moore's work, Iacoviello (2005) actively linked their framework with the NNS, showing how the role of nominal debt and asset prices are central for propagation of monetary policy shocks. The model set out in this paper builds on the DSGE framework in Iacoviello (2005) but differs importantly in one respect - household debt no longer reflects intertemporal trading between an impatient borrower and a patient saver. Instead there is a role for banks as explicit profit-maximizing interest rate-setting agents that intermediate funds, balancing loans and deposits via their balance sheet. The modelling of durable goods used here follows from Barsky et al. (2007) and Monacelli (2008).

With regards the banking sector and interest rate spreads, some attempts have been made to analyze the implications of broad money, banking and interest rate spreads in a

[^1]DSGE setting, most notably Goodfriend (2005) and Goodfriend and McCallum (2007) . However, these papers have resorted to RBC-style loan production functions and pricetaking banks, which render loan rates as residual to the system of endogenous variables. In particular, the banking sector, captured via the loan production function, enters via a cash-in-advance constraint, without which the model reduces to the standard NNS framework. Curdia and Woodford (2008) also use an intermediation technology to introduce a spread between the lending and saving rate available to a continuum of households who are indexed by the utility and disutility obtained from current expenditure and labor respectively. However, the spread itself varies exogenously over time - despite different interpretations for the source of the spread. They also show that optimal spread-adjusted monetary policy should typically target a weighted average of the different interest rates without much consideration for the credit spread itself.

Here we attempt to model a fully endogenous, monopolistically competitive banking sector with a distinctly New Keynesian flavor, where policy rate changes feed through the financial intermediation sector to the rest of the economy via the rate of interest that banks use to lend to one another. ${ }^{4}$ In the absence of borrowing and lending by the goods sector, ${ }^{5}$ we can trace the impact of bank-determined interest rate spreads on monetary transmission. Furthermore, these particular features should help to address some of the questions and issues which the current crisis has forced into the open. For instance, what are the implications for durable and non-durable consumption in the presence of banking? How will the real economy react to changing spreads when interest rate adjustment by banks is sluggish? What does the introduction of monopolistically competitive banks add to the transmission mechanism highlighted by the literature on the financial accelerator?

The remainder of the paper is laid out as follows: section two provides a brief background to some of the key issues and characteristics addressed in the model and examines why they are important to integrate into the framework; section three introduces the core theoretical setting; section four discusses the differences between the bank and non-bank settings of the model, and section five looks at the dynamics of the model economy under different shocks, with continued emphasis on the comparison between banking and non-banking economies. The final section concludes.

[^2]
## 2 Bank Lending, Housing and Liquidity

The recent financial storm has highlighted the crucial interactions between house prices, collateralized debt and bank lending, as the main ingredients. This section reviews some of the findings on these factors and ends by proposing a modelling strategy for tackling the related policy debate.

### 2.1 Housing

It is well documented that the value of mortgage debt represents a significant portion of outstanding household debt (e.g. Calza et al. (2007)), such that in OECD countries the share has increased from $60 \%$ to $75 \%$ between 1952 and 2005. Co-movement between consumption and house prices is also a well documented finding (Aoki et al. (2004)). It is self-evident in the most recent housing market boom that households were consciously raising consumption on the back of drastic house price increases. Moreover, the sizeable increases in house prices combined with an unprecedented rise in household debt are typically mutually reinforcing.

A large part of the observed increase in consumer borrowing has primarily been in the form of collateralized debt. The rise in house prices has induced households to increasingly extract equity from their accumulated assets, encouraging further borrowing against realized capital gains. Dynamics of this sort have been considered important in sustaining the level of private spending in several countries, especially during the business cycle downturn of 2001. However, the implications of a housing bust, coinciding with a complicated deterioration of credit markets (either exacerbated by or causing them), are still to be understood. In light of this, one focus of this paper is to unravel how changes in house prices in response to various shocks may directly impact on consumption via credit market effects, specifically bank lending.

Goodfriend and McCallum (2007) have highlighted the role of collateralized and uncollateralized external finance premia as significant factors in driving business cycle fluctuations. In this paper, we are interested in housing (durable goods) as collateral as opposed both collateralized and uncollateralized spreads in borrowing rates - all borrowing is collateralized in the framework presented here, and there can be no uncollateralized borrowing by constrained households. When the value of housing acts as collateral, an increase in house prices makes more collateral available to homeowners, which in turn encourages them to borrow more to finance their desired level of consumption. The increase in house prices may itself be caused by a variety of factors, including an unanticipated reduction in real interest rates, which lowers the rate at which future housing services are discounted. Taylor (2007) suggests that a sustained period of exceptionally low short-term interest rates between 2003 and 2004 may have played an important role in the rise of the housing market bubble. As noted by Jarocinski and Smets (2008),
the recent financial turmoil, triggered by the exceptionally high sequence of defaults in the sub-prime mortgage market in the United States, has fostered an increasing interest in the role of the housing market in the economy and on how monetary policy should respond to misalignments in house prices from their equilibrium level so as to prevent the occurrence of non-fundamental dynamics.

Given the uncertainty over wealth effects ${ }^{6}$ and the importance of housing, it is very clear that monetary policymakers must consider housing market developments, despite the traditional caveats about responding to asset prices. ${ }^{7}$

### 2.2 Bank Lending and Interest Rate Spreads

Given our focus on integrating banking into the NNS framework, it is important to review what has come before on banking and financial intermediation within economics. The economic analysis of the banking industry has traditionally placed strong emphasis on the importance of financial intermediaries in the provision of credit and the special nature of bank loans. In macroeconomics the role of credit has been widely explored within the strand of the monetary economics that deals with the credit channel of the transmission mechanism (see Blinder and Stiglitz (1983) and Bernanke and Blinder (1988)). According to this credit view, in conditions of asymmetric information between borrowers and lenders, monetary policy actions affect the external finance premium through the balance sheet channel and the bank lending channel. While the former emphasizes the relevance of borrowers' balance sheets, ${ }^{8}$ the bank lending channel highlights the importance of liabilities on the banks' balance sheet in channeling macroeconomic fluctuations.

Specifically, the bank lending channel suggests that monetary policy affects the real economy not only through the impact of the interest rate changes on aggregate demand, but also through impacts on bank liabilities (reservable deposits in Kashyap and Stein (1995); bank equity in Van den Heuvel (2002)) which can lead to shifts in the supply of credit and bank loans, particularly loans from the commercial banking sector. An important assumption is that banks are assumed to play a special role in the financial system as primary lender to a wide range of individuals and sectors (referred to as 'bankdependent borrowers'). A second assumption is that banks typically fund a fraction of

[^3]their loans with liabilities (such as deposits) that carry reserve requirements. By reducing bank reserves, a monetary policy tightening lowers the level of 'reservable' deposits banks can take on when reserve requirements are binding. The decrease in reservable liabilities will in turn lead banks to reduce lending if they cannot easily switch to alternative forms of finance or liquidate assets other than loans. Therefore, as Van den Heuvel (2007) notes, the two necessary conditions for a bank lending channel to be operative is that reserve requirements are binding and that the market for non-reservable bank liabilities is not frictionless. Otherwise, banks could simply replace funding from reservable liabilities (e.g. deposits) with liabilities that have no reserve requirements, thereby allowing the bank to continue to exploit any profitable lending opportunities that might arise even if it faced a binding reserve requirement.

Another important issue regarding monetary transmission and banks is inertia in interest rate changes, which are often found to be sticky, such that they do not respond immediately or fully to changes in the corresponding reference market rates against which they are priced. Berger and Udell (1992) focus on the response of credit rates, and show that a $1 \%$ increase in the T-Bill rate only induces a $0.5 \%$ response in credit rates. This has important monetary policy implications, as changes in the monetary policy rate may not be fully reflected in the interest rates banks offer their customers. Moreover, Hannan and Berger (1991), and Borio and Fritz (1995) show that stickiness increases with market concentration. This evidence shifts our attention to a class of models that stresses the role of the competitive regime in the banking industry and its implications for the interest rate setting mechanism.

Goodfriend and McCallum (2007) provide a first attempt to embed a fully-specified (perfectly competitive) banking sector into a dynamic stochastic general equilibrium framework. The aim of their paper, as of this one, was to quantify the effects of banking in general equilibrium, in terms of both its steady state implications and dynamic properties. In particular, the model characterizes steady-state interest rate spreads given key assumptions regarding the loan production function and the degree of substitutability between collateral and monitoring effort. Moreover, they show that the effects of exogenous shocks can be amplified or attenuated due to changes in the external finance premium (EFP). Two competing forces are at work. A positive monetary shock raises asset prices, which leads to a rise in the collateral value and subsequently the supply of loans - this reduces the EFP. The initial impulse also raises aggregate demand and the demand for deposits, and hence the amount of banking services that are provided - this raises the EFP. The latter effect dominates leading to the EFP rising, and so the effects of the shock are attenuated.

From the perspective of introducing a fully fledged profit maximizing banking sector into a DSGE model for the analysis of the interdependence between housing, credit and banks' balance sheet, we find it key to introduce an element of imperfect competition in
the banking industry. Huelsewig et al. (2006) analyze effects of bank behavior on the supply side of the economy in a general equilibrium setting. In that paper, banks supply loans to firms that must pay their wage bills before they receive revenues from selling their output, leading to the so-called cost channel of monetary policy transmission. We exploit an analogous mechanism in a model featuring credit market frictions in the form of collateralized borrowing on the demand side of the economy. Moreover, compared to the steady state evidence provided by Goodfriend and McCallum (2007), where interest rate differentials are affected by the marginal value of collateral, the interest rate spread generated by our model depends upon the degree of competitive pressure in the banking sector (through the elasticity of substitution between bank loans), as well as by the reserve requirement.

### 2.3 A Modelling Perspective

We now briefly preview how we hope to integrate the disparate factors related to banking, borrowing and housing in a DSGE setting. Durable goods and collateral constraints following Kiyotaki and Moore (1997), Iacoviello (2005) and Monacelli (2008) are an obvious starting point. Their 'multi-representative agent' frameworks with collaterallyconstrained consumers can explain positive elasticities of consumption to housing wealth. Constrained (impatient) households value current consumption more than unconstrained (patient) households and can therefore raise borrowing and consumption more than proportionally when housing prices increase, leading to positive effects on aggregate demand from asset price rises.

For the banking sector, we introduce independent profit-maximizing and interest-rate setting banks, as mentioned above. By modelling loans and deposits as the outcome of optimizing behavior, we can introduce interest rates beyond the single bond rate prevalent in NNS models, as well as interest rates which are determined separately and independently of the instrument rate. The use of an aggregate bank balance sheet allows for the possibility of liquidity injections into the financial sector, with obvious spillovers into lending and credit for households.

Banks will be modelled along the lines of the simple Industrial Organization (IO) models of banking as set out in Freixas and Rochet (1997). The problem is one of constrained optimization, where the profits for the $j$ th bank are calculated as interest income earned from loans net of interest paid on deposits. In the simplest setting, profits, $\Psi_{j, t}^{B}$, are defined as:

$$
\begin{equation*}
\Psi_{j, t}^{B} \equiv R_{j, t}^{L} L_{j, t}+R_{t}^{I B} B_{j, t}^{I B}-R_{t}^{D} D_{j, t}, \tag{1}
\end{equation*}
$$

where $L_{j, t}$ represent mortgage loans, $B_{j, t}^{I B}$ captures interbank (net) lending to other banks, and $D_{j, t}$ are deposits made at the $j$ th bank by households. $R_{t}^{I B}$ is the interbank lending
rate and will be the policy rate in the model, while $R_{j, t}^{L}$ is the bank-determined loan rate facing borrowers, and $R_{t}^{D}$ is the deposit rate paid by savers. Maximization is subject to the constraint that the bank balances it assets and liabilities. The former comprises the various loans made to households as well as to other banks, while the latter includes deposits. This constraint is formally referred to as the bank balance sheet and is written as follows:

$$
\begin{equation*}
L_{j, t}+B_{j, t}^{I B}=D_{j, t} . \tag{2}
\end{equation*}
$$

Within this profit-maximizing setting, we specifically implement a monopolistically competitive banking framework with staggered interest rate adjustment. For the purposes of this paper, and in order to track more easily the changes in transmission that come with adding banks to the system, we only allow banks the power to set the loan rate, and not the deposit rate. Control of the latter rate could be introduced with little additional complication. In the case of flexible price-setting (zero stickiness), the loan rate will be a mark-up over the deposit rate, and this gives us the crucial spread between interest rates faced by households. Once stickiness is introduced, this spread can vary over time.

It is clear that our model features deposits as the only source of funding for the lending activities of the whole banking sector. Therefore, a monetary policy tightening necessarily acts through a quasi-bank lending channel (which we later re-christen the bank balance sheet channel), by determining an equal reduction in deposits and household debt in the form of loans (mortgages).

## 3 The Model

This section sets out the model to be discussed in this paper. As to the demand side, there are two types of consumer: patient households and impatient households, which are differentiated by the discount factor with which they discount their future stream of instantaneous utility. ${ }^{9}$ The supply side is populated by monopolistically competitive firms producing intermediate goods, as well as a perfectly competitive final goods production sector. Moreover, there is a financial intermediation sector, specializing in extending loans and mortgages to collaterally-constrained impatient households, and a monopolistically competitive banking sector, which provides lendable resources to the intermediaries. Money is also included in this model via money-in-utility (MIU) as justified by Kiyotaki and Wright (1989).

[^4]

Diagram summarising various relations in basic 2-agent version of the
banking model

### 3.1 Consumers

There are two households in the economy:

1. Patient households, who are financially unconstrained and consume as standard permanent income households would. They choose how much to deposit with banks (save) and receive all residual profits from the monopolistically competitive intermediate goods producers and monopolistically competitive banks;
2. Impatient households, who are credit-constrained and borrow from financial intermediaries using their holdings of durable goods as collateral. They also earn wages from renting their labor to the intermediate goods sector;

Household debt no longer represents intertemporal trading between the different types of household. In this model, the important financial (credit) choice variables of the household, deposits, $d_{t}$, and the amount of borrowing (or saving) by impatient households, $l_{t}$, are linked through the aggregate balance sheet of the banking sector. This crucial link provides an explicit role for monopolistically competitive interest-rate setting banks to intermediate funds across the economy. ${ }^{10}$

[^5]
### 3.1.1 Patient Households

The preferences of the representative patient household are defined over a nondurable consumption good $\left(C_{t}^{P}\right)$, housing services $\left(H_{t}^{P}\right)$ and labor $\left(N_{t}^{P}\right)$. These households maximize the present discounted value of their expected utility:

$$
\begin{equation*}
E_{0} \sum_{t=0}^{\infty}\left(\beta^{P}\right)^{t} U^{P}\left(C_{t}^{P}, H_{t}^{P}, \frac{M_{t}^{P}}{P_{t}}, N_{t}^{P}\right) \tag{3}
\end{equation*}
$$

where $U^{P}\left(C_{t}^{P}, H_{t}^{P}, \frac{M_{t}^{P}}{P_{t}}, N_{t}^{P}\right)=\log C_{t}^{P}+\varepsilon_{t}^{P} \log H_{t}^{P}+\varrho^{P} \log \left(\frac{M_{t}^{P}}{P_{t}}\right)-\nu^{P} \frac{\left(N_{t}^{P}\right)^{1+\varphi^{P}}}{1+\varphi^{P}}$ and $\beta^{P}$ is the discount factor. Parameter $\varphi^{P}$ denotes the inverse of the elasticity of substitution between work and leisure. Maximization is subject to the following sequence of (nominal) budget constraints:

$$
\begin{equation*}
P_{t} C_{t}^{P}+Q_{t}^{H} X_{t}^{P}+D_{t}+M_{t}^{P}=R_{t-1}^{D} D_{t-1}+M_{t-1}^{P}+W_{t}^{P} N_{t}^{P}+\Psi_{t-1}^{B}+\Psi_{t}^{G}+T_{t}^{P} \tag{4}
\end{equation*}
$$

where $P_{t}$ is the price of nondurable goods, $D_{t}$ indicates end-of-period deposits which earn an interest $R_{t}^{D}$, payable at the start of period $t+1 ; Q_{t}^{H}$ is the nominal price of durables (housing), and $\Psi_{t}^{B}$ and $\Psi_{t}^{G}$ are dividends (profits) remitted to patient households by the banking and goods sectors:

$$
\Psi_{t}^{B}=\int_{0}^{1} \Psi_{i t}^{B} d i, \quad \Psi_{t}^{G}=\int_{0}^{1} \Psi_{i t}^{G} d i
$$

Patient households enter each period with $M_{t-1}^{P}$ nominal money balances, receive transfers from the central government, $T_{t}^{P}=M_{t}^{P}-M_{t-1}^{P}$, and earn $W_{t}^{P}$ for their labor input to the goods sector. Note that housing does not depreciate across time and so we have assumed perfect durability of housing. Furthermore, $X_{t}^{P}$ denotes the purchase of new durables and the accumulation equation for the stock of durables is:

$$
H_{t}^{P}=X_{t}^{P}+H_{t-1}^{P}
$$

where we assume that there is perfect durability, i.e. durable goods do not depreciate.
In real terms this constraint becomes:

$$
\begin{equation*}
C_{t}^{P}+q_{t}^{H} X_{t}^{P}+m_{t}^{P}+d_{t}=R_{t-1}^{D} \frac{d_{t-1}}{\Pi_{t}}+\frac{m_{t-1}^{P}}{\Pi_{t}}+w_{t}^{P} N_{t}^{P}+\psi_{t}^{G}+\frac{\psi_{t-1}^{B}}{\Pi_{t}}+\tau_{t}^{P} \tag{5}
\end{equation*}
$$

where $d_{t} \equiv \frac{D_{t}}{P_{t}}, q_{t}^{H} \equiv \frac{Q_{t}^{H}}{P_{t}}, \psi_{t}^{B} \equiv \frac{\Psi_{t}^{B}}{P_{t}}, \psi_{t}^{G} \equiv \frac{\Psi_{t}^{G}}{P_{t}}, w_{t}^{P} \equiv \frac{W_{t}^{P}}{P_{t}}, \tau_{t}^{P} \equiv \frac{T_{t}^{P}}{P_{t}}$ and $\Pi_{t} \equiv \frac{P_{t}}{P_{t-1}}$ is the gross inflation rate. The first order conditions for this agent are as follows: ${ }^{11}$

[^6]\[

$$
\begin{gather*}
U_{H, t}^{P}=\frac{\varepsilon^{P}}{H_{t}^{P}} \text { and } U_{C, t}^{P}=\frac{1}{C_{t}^{P}}  \tag{6}\\
\frac{1}{C_{t}^{P}}=\lambda_{t}^{P}  \tag{7}\\
\beta^{P} E_{t}\left[\lambda_{t+1}^{P} \frac{R_{t}^{D}}{\Pi_{t+1}}\right]=\lambda_{t}^{P},  \tag{8}\\
\nu^{P}\left(N_{t}^{P}\right)^{\varphi^{P}}=\lambda_{t}^{P} w_{t}^{P}  \tag{9}\\
\frac{\varepsilon_{t}^{P}}{H_{t}^{P}}+\beta^{P} E_{t}\left[\lambda_{t+1}^{P} q_{t+1}^{H}\right]=\lambda_{t}^{P} q_{t}^{H},  \tag{10}\\
\frac{\varrho^{P}}{m_{t}^{P}}+\beta^{P} E_{t}\left[\frac{\lambda_{t+1}^{P}}{\Pi_{t+1}}\right]=\lambda_{t}^{P}, \tag{11}
\end{gather*}
$$
\]

where $\lambda_{t}^{P}$ is the multiplier associated with the budget constraint. As in the standard NNS framework, we recover dynamic Euler equations, which together with the budget constraint and the appropriate transversality conditions, characterize equilibrium dynamics for the patient household. ${ }^{12}$

Patient savers therefore operate in perfect financial markets, not suffering from any collateral constraints with respect to their borrowing. We can show analytically that patient consumption will co-move with durable goods prices, which shows up later in all impulse responses. This feature, however, will not be present with impatient borrowers, for whom this mechanism is de-linked due to the presence of the collateral constraint. We can see this perfect co-movement between asset prices and non-durable consumption by analyzing (10), which after repeated forward substitution gives:

$$
\begin{equation*}
U_{C, t}^{P} q_{t}^{H}=\sum_{j=0}^{\infty}\left(\beta^{P}\right)^{j} U_{H, t+j}^{P} \tag{12}
\end{equation*}
$$

Barsky et al. (2007) examine the interplay between intertemporal substitution and the purchases of durable and non-durable goods. They note, as recalled later by Calza et al. (2007), that in the case of durables with low depreciation rates, ${ }^{13}$ the right hand-side of (12) is heavily influenced by the marginal utilities of durable service flows in the distant future. When shocks hitting the economy (and their effects) are temporary, the forwardlooking terms in (12) do not deviate from their steady-state values, and so even significant variation in the first few terms only have a small impact on the present value. As Barsky

[^7]et al. (2007) point out, this means the present value remains constant (or invariant) even in the face of substantial temporary movements in $U_{H, t+j .}^{P} .{ }^{14}$ And so given that the right hand side of (12) remain fairly constant following any temporary shock, any variation in asset prices immediately impacts on the marginal utility of consumption. Specifically, as asset prices rise, $U_{C, t}^{P}$ must fall, which means that patient non-durable consumption will rise.

### 3.1.2 Impatient Households

Preferences of the representative impatient household are defined over consumption of the final non-durable good $\left(C_{t}^{I}\right)$, housing services $\left(H_{t}^{I}\right)$ and labor $\left(N_{t}^{I}\right)$. Impatient households maximize the expected present discounted value of utility:

$$
\begin{equation*}
E_{0} \sum_{t=0}^{\infty}\left(\beta^{I}\right)^{t} U^{I}\left(C_{t}^{I}, H_{t}^{I}, \frac{M_{t}^{I}}{P_{t}}, N_{t}^{I}\right), \tag{13}
\end{equation*}
$$

where $U^{I}\left(C_{t}^{I}, H_{t}^{I}, \frac{M_{t}^{I}}{P_{t}}, N_{t}^{I}\right)=\log C_{t}^{I}+\varepsilon_{t}^{I} \log H_{t}^{I}+\varrho^{I} \log \left(\frac{M_{t}^{I}}{P_{t}}\right)-\nu^{I} \frac{\left(N_{t}^{I}\right)^{1+\varphi^{I}}}{1+\varphi^{I}}$, and $\beta^{I}$ $\left(\beta^{P}>\beta^{I}\right)$ is the intertemporal discount factor. Parameter $\varphi^{I}$ is as for the patient agents. Maximization is subject to the following sequence of (nominal) budget constraints:

$$
\begin{equation*}
P_{t} C_{t}^{I}+Q_{t}^{H} \Delta H_{t}^{I}+R_{t-1}^{L} L_{t-1}+M_{t}^{I}=L_{t}+W_{t}^{I} N_{t}^{I}+M_{t-1}^{I}+T_{t}^{I} . \tag{14}
\end{equation*}
$$

Impatient households enter period $t$ with $L_{t}$ of nominal debt (or mortgage), which has been borrowed from the bank, and will pay $R_{t}^{L}$ gross nominal interest on this debt at the start of $t+1$. They increase their housing holdings by $\Delta H_{t}^{I}$ at the nominal price $Q_{t}^{H}$, and earn $W_{t}^{I} N_{t}^{I}$ from working in the goods sector. Once again these consumers receives transfers from the government, $T_{t}^{I}=M_{t}^{I}-M_{t-1}^{I}$ and we use $X_{t}^{I}$ to denote the purchase of new durables, where durables accumulate as follows: $H_{t}^{I}=X_{t}^{I}+H_{t-1}^{I}$. In real terms this constraint is:

$$
\begin{equation*}
C_{t}^{I}+q_{t}^{H} \Delta H_{t}^{I}+R_{t-1}^{L} \frac{l_{t-1}}{\Pi_{t}}+m_{t}^{I}=l_{t}+w_{t}^{I} N_{t}^{I}+\frac{m_{t-1}^{I}}{\Pi_{t}}+\tau_{t}^{I} \tag{15}
\end{equation*}
$$

where $l_{t} \equiv \frac{L_{t}}{P_{t}}, w_{t}^{I} \equiv \frac{W_{t}^{I}}{P_{t}}$ and $m_{t}^{I} \equiv \frac{M_{t}^{I}}{P_{t}}$.
Borrowing is restricted by the additional collateral constraint:

$$
R_{t}^{L} L_{t} \leq \chi^{I} E_{t} Q_{t+1}^{H} H_{t}^{I},
$$

which in real terms becomes

[^8]$$
l_{t} \leq \chi^{I} E_{t}\left[\frac{\Pi_{t+1} q_{t+1}^{H}}{R_{t}^{L}} H_{t}^{I}\right]
$$
that is, the maximum amount of borrowable resources is equal to a fraction of the value of housing holdings available at the start of period $t$. Parameter $\chi^{I}$ denotes the margin requirement or loan-to-value ratio.

The first order conditions for this agent are as follows:

$$
\begin{gather*}
U_{H, t}^{I}=\frac{\varepsilon^{I}}{H_{t}^{I}} \text { and } U_{C, t}^{I}=\frac{1}{C_{t}^{I}},  \tag{16}\\
\frac{1}{C_{t}^{I}}=\lambda_{t}^{I},  \tag{17}\\
\beta^{I} E_{t}\left[\lambda_{t+1}^{I} \frac{R_{t}^{L}}{\Pi_{t+1}}\right]+R_{t}^{L} \mu_{t}^{I}=\lambda_{t}^{I},  \tag{18}\\
\nu^{I}\left(N_{t}^{I}\right)^{\varphi^{I}}=\lambda_{t}^{I} w_{t}^{I},  \tag{19}\\
\frac{\varepsilon^{I}}{H_{t}^{I}}+\beta^{I} E_{t}\left[\lambda_{t+1}^{I} q_{t+1}^{H}\right]+\mu_{t}^{I} \chi^{I} \Pi_{t+1} q_{t+1}^{H}=\lambda_{t}^{I} q_{t}^{H},  \tag{20}\\
\frac{\varrho^{I}}{m_{t}^{I}}+\beta^{I} E_{t}\left[\frac{\lambda_{t+1}^{I}}{\Pi_{t+1}}\right]=\lambda_{t}^{I}, \tag{21}
\end{gather*}
$$

where $\lambda_{t}^{I}$ is the multiplier associated with the budget constraint. In Section 4, we will analyze how the collateral constraint affects the choice between nondurable and durable consumption, and how this in turn is affected by the introduction of banks into the model. When linearized, (8), (10), (18) and (20) together with the aggregate resource constraint (and the fixed supply of housing), will correspond to the demand-side or "IS" portion of the standard three equation "IS-PC-MR" NNS model.

### 3.2 Financial Intermediation and Banking

We now set out the specifics of the banking sector and the system of financial intermediation used. The set-up is deliberately New Keynesian in nature and exploits the existing approach traditionally applied to the goods sector. The key difference emerges in the 'intermediate' banking sector, which is made up of a continuum of monopolistically competitive banks who maximize profits subject to a balance sheet constraint. Instead of being a passive agent of monetary transmission, these intermediate banks are independent bodies which react optimally to the economic environment. We assume furthermore that they are subject to Calvo-type staggered interest rate adjustment, which yields a 'New Keynesian' interest rate-setting curve, where the change in current loan rates depend on the change in expected future loan rates as well as the spread between the current loan rates and the interbank rate (which is taken in this framework as a proxy for the policy
rate as in Goodfriend and McCallum (2007)).

### 3.2.1 Loan Demand

The final bank sector can be viewed as the ultimate mortgage provider, e.g. a retail bank in the UK, or Countrywide Financial in the USA. This sector effectively bundles loans from the intermediate banking sector and issues them directly to the collaterallyconstrained the impatient households. Ultimately, this sector allows us, as in the case of the goods sector, to derive algebraically from microfounded behavior the impatient consumers' demand for loans, rather than postulate it.

Loan providers are taken to be perfectly competitive and act to aggregate mortgages to impatient households using a standard CES aggregator:

$$
L_{t}=\left(\int_{0}^{1}\left(L_{j, t}\right)^{\frac{\eta^{H}-1}{\eta^{H}}} d j\right)^{\frac{\eta^{H}}{\eta^{H}-1}} .
$$

Profit maximization leads to a familiar loan demand function:

$$
\begin{equation*}
L_{j, t}=\left(\frac{R_{j, t}^{L}}{R_{t}^{L}}\right)^{-\eta^{H}} L_{t} \quad \forall j, \tag{22}
\end{equation*}
$$

where $R_{t}^{L}=\left(\int_{0}^{1}\left(R_{j, t}^{L}\right)^{1-\eta^{H}} d j\right)^{\frac{1}{1-\eta^{H}}}$.

### 3.2.2 Banking Sector

We next derive the optimal relations for loan supply by profit-maximizing banks that choose interest rates subject to a binding balance sheet constraint. We show how loan rate stickiness is introduced by assuming that banks operating in a customer market face nominal frictions as in Calvo (1983). Each bank resets its loan rate only with a probability $1-\xi$ each period, independently of the time elapsed since the last adjustment. Thus, each period a fraction $1-\xi$ of banks reset their loan rates, while a fraction $\xi$ keep their rates unchanged. The aggregate loan rates are given by:

$$
\begin{equation*}
R_{t}^{L}=\left[\xi\left(R_{t-1}^{L}\right)^{1-\eta^{H}}+(1-\xi)\left(R_{j, t}^{L^{*}}\right)^{1-\eta^{H}}\right]^{\frac{1}{1-\eta^{H}}} \tag{23}
\end{equation*}
$$

where $R_{j, t}^{L *}$ is the loan rate chosen by banks who are able to adjust.
As before, the $j$ th bank maximizes profits by choosing a sequence $\left\{R_{j, t}^{L}, D_{j, t}, B_{j, t}^{I B}\right\}$, taking nominal profits to be:

$$
\begin{equation*}
E_{t} \sum_{k=0}^{\infty}(\xi)^{m} \Lambda_{k, t+k} \Psi_{j, t+k}^{B}=E_{t}\left\{\sum_{k=0}^{\infty}(\xi)^{k} \Lambda_{k, t+k}\left[R_{j, t}^{L} L_{j, t+k}+R_{t+k}^{I B} B_{j, t+k}^{I B}-R_{t+k}^{D} D_{j, t+k}\right]\right\} . \tag{24}
\end{equation*}
$$

Maximization is subject to the bank balance sheet constraint:

$$
\begin{equation*}
L_{j, t}+B_{j, t}^{I B}+H_{j, t}=X_{j, t}^{C B}+D_{j, t}, \tag{25}
\end{equation*}
$$

where $H_{j, t}$ is high-powered money, itself defined as:

$$
H_{j, t}=\theta_{j, t},
$$

where $\theta_{j, t}$ are cash reserves transferred by bank $j$ on its account at the Central Bank, i.e. the bank-specific amount of fractional reserves. $\theta_{j, t}$ typically bears no interest ${ }^{15}$ and is therefore optimally chosen at its minimal level defined by the regulator:

$$
\theta_{j, t}=\kappa D_{j, t}, \quad 0<\kappa<1,
$$

where $\kappa$ is the reserve requirement. ${ }^{16}$ Therefore we can rewrite the balance sheet constraint as:

$$
L_{j, t}+B_{j, t}^{I B}=X_{j, t}^{C B}+(1-\kappa) D_{j, t} .
$$

The bank's balance sheet therefore tells us how the $j^{\text {th }}$ bank can obtain funding in three ways: (i) it receives cash injections $\left(X_{j, t}^{C B}\right)$ from the monetary authority, where $X_{j, t}^{C B}=\kappa D_{j, t}$; (ii) it can obtain funds on the interbank market, where the net position of the $j^{\text {th }}$ bank on the interbank market is denoted by $B_{j, t}^{I B}$, and finally (iii) banks receive deposits from patient households. Changes in the first source of funding is closest in spirit to the bank lending channel, in which central banks very bank reserves to affect loan supply.

For the dynamic analysis later, we add an iid shock, $\varepsilon_{t}^{x}$, to the bank balance sheet. This is to capture the impact of exogenous changes in balance sheet liquidity and could be useful when attempting to unravel the implications from the recent contraction in

[^9]balance sheets. The loan demand curve (22) from the final banking sector also serves as an additional constraint for maximization and the interbank rate $R_{t}^{I B}$ and deposit rate, $R_{t}^{D}$, are both taken as given by the banking sector.

After denoting with $\mu_{t}^{B}$ the Lagrange multiplier associated to the balance sheet identity (the nominal marginal cost in this case), we retrieve the following first order conditions:

$$
\begin{gather*}
R_{t}^{D}=\mu_{t}^{B}(1-\kappa)  \tag{26}\\
R_{t}^{I B}=\mu_{t}^{B}  \tag{27}\\
E_{t} \sum_{m=0}^{\infty}(\xi)^{m} \Lambda_{k, t+k}\left\{\left(\frac{R_{j, t}^{L *}}{\mu_{t+k}^{B}}\right)-\frac{\eta^{H}}{\left(\eta^{H}-1\right)}\right\} L_{j, t+k}=0 \tag{28}
\end{gather*}
$$

We find that the interbank rate is the adjusted deposit rate, where the factor of adjustment is the reciprocal of the fraction of non-reservable deposits:

$$
R_{t}^{I B}=\frac{R_{t}^{D}}{(1-\kappa)}
$$

Therefore when linearized, price-taking behavior by banks with respect to the deposit rate, means that the deposit and interbank rates are equal $\left(\hat{R}_{t}^{I B}=\hat{R}_{t}^{D}\right)$, and so changes in the policy rate translate one-to-one to changes in the deposit rate.

Linearizing the first order condition with respect to $R_{t}^{L}$ and combining it with the aggregate loan rate (23) we obtain the previously-mentioned 'New Keynesian' interest rate-setting curve for the mortgage rate:

$$
\begin{equation*}
\Delta \hat{R}_{t}^{L}=\beta^{P} E_{t} \Delta \hat{R}_{t+1}^{L}-\frac{\left(1-\beta^{P} \xi\right)(1-\xi)}{\xi}\left(\hat{R}_{t}^{L}-\hat{R}_{t}^{I B}\right) . \tag{29}
\end{equation*}
$$

while under flexible interest rates, we find that $\hat{R}_{t}^{L}=\hat{R}_{t}^{I B}=\hat{R}_{t}^{D}$, as expected.

### 3.2.3 Loan Supply and Demand

Given the collateral constraints and the staggered adjustment of interest rates, we have linearized relations which define loan supply and loan demand functions in $\left[\hat{l}_{t}, \hat{R}_{t}^{L}\right]$-space. In the case of mortgage demand we obtain a downward-sloping relation between $\hat{R}_{t}^{L}$ and $\hat{l}_{t}$ :

$$
\begin{equation*}
\hat{l}_{t}=E_{t}\left[\hat{q}_{t+1}^{H}+\hat{H}_{t}^{I}+\hat{\Pi}_{t+1}-\hat{R}_{t}^{L}\right] . \tag{30}
\end{equation*}
$$

Mortgage supply, as derived from the Banking sector, is however perfectly elastic, and differs depending upon the assumption of flexible or sticky interest rates:

$$
\begin{equation*}
\Delta \hat{R}_{t}^{L}=\beta^{P} E_{t} \Delta \hat{R}_{t+1}^{L}-\frac{\left(1-\beta^{P} \xi\right)(1-\xi)}{\xi}\left(\hat{R}_{t}^{L}-\hat{R}_{t}^{I B}\right) . \tag{31}
\end{equation*}
$$

### 3.3 Goods Sector

### 3.3.1 Final Good Producers

The aggregate non-durable good is produced by perfectly competitive firms and requires the assembly of a continuum of intermediate goods, indexed by $i \in[0,1]$, via the following technology: $Y_{t}=\left(\int_{0}^{1}\left(Y_{i, t}\right)^{1-\frac{1}{\theta}} d i\right)^{\frac{\theta}{\theta-1}}$. Profit maximization leads to the typical demand function:

$$
\begin{equation*}
Y_{i, t}=\left(\frac{P_{i, t}}{P_{t}}\right)^{-\theta} Y_{t} \quad \forall i, \tag{32}
\end{equation*}
$$

where $P_{t}=\left(\int_{0}^{1}\left(P_{i, t}\right)^{1-\theta} d i\right)^{\frac{1}{1-\theta}}$ is the price index consistent with the final good producer earning null profits.

### 3.3.2 Intermediate Goods Production

A continuum of firms produces intermediate goods. Shares of these firms are owned by patient households. Each firm $i \in[0,1]$ employs labor (supplied by patient and impatient households) in a constant-return-to scale production function:

$$
\begin{equation*}
Y_{i, t}=A_{t}\left(N_{i, t}^{P}\right)^{\gamma}\left(N_{i, t}^{I}\right)^{1-\gamma}, \tag{33}
\end{equation*}
$$

where $A_{t}$ is a total factor productivity shifter, $N_{i, t}^{P}$ is the firm-specific total demand for patient household labor, and $N_{i, t}^{I}$ is the firm-specific total demand for impatient household labor. Price setting behavior in this sector is detailed below.

### 3.3.3 Pricing of Intermediate Goods

Each intermediate good firm has monopolistic power in the production of its own variety and therefore has leverage in setting the price. In so doing it faces a quadratic cost equal to $\frac{\phi}{2}\left(\frac{P_{i, t}}{P_{i, t-1}}-\Pi\right)^{2} Y_{t}$, first proposed by Rotemberg (1982), where $\Pi$ is the steady state inflation rate and where the parameter $\phi$ measures the degree of nominal price rigidity.

As is standard in the New Keynesian literature, intermediate goods producers solve a two-stages budgeting problem. Given $W_{t}^{P}, W_{t}^{I}$ they rent $N_{i, t}^{P}, N_{i, t}^{I}$ in perfectly competitive factor markets in order to minimize real cost to get:

$$
\begin{equation*}
\frac{W_{t}^{P}}{P_{i, t}}=m c_{t}^{G} A_{t} \gamma\left(N_{i, t}^{P}\right)^{\gamma-1}\left(N_{i, t}^{I}\right)^{1-\gamma}=m c_{t}^{G} \gamma \frac{Y_{i, t}}{N_{i, t}^{P}}, \tag{34}
\end{equation*}
$$

$$
\begin{equation*}
\frac{W_{t}^{I}}{P_{i, t}}=m c_{t}^{G} A_{t}(1-\gamma)\left(N_{i, t}^{I}\right)^{-\gamma}\left(N_{i, t}^{P}\right)^{\gamma}=m c_{t}^{G}(1-\gamma) \frac{Y_{i, t}}{N_{i, t}^{I}} \tag{35}
\end{equation*}
$$

Each monopolistic firm chooses its optimal price $\left\{P_{i, t}\right\}$ taking nominal profits:

$$
\begin{equation*}
\max _{P_{i t}} E_{t} \sum_{s=t}^{\infty} \Lambda_{s, t} \Psi_{i, t}^{G}=\max _{P_{i, t}} E_{0}\left\{\sum_{t=s}^{\infty} \Lambda_{s, t}\left[P_{i, t} Y_{i, t}-\left(T C_{i, t}-\frac{\phi P_{t}}{2}\left(\frac{P_{i, t}}{P_{i, t-1}}-\Pi\right)^{2} Y_{t}\right)\right]\right\}, \tag{36}
\end{equation*}
$$

where $\Psi_{i, t}^{G}$ are the nominal profits of firm $i$ which will be transferred to shareholders in the form of dividends, $T C_{i, t}=W_{t}^{P} N_{i, t}^{P}+W_{t}^{I} N_{i, t}^{I}$ are the total cost of production, and $\Pi=1$ is the steady state of gross inflation; and where the stochastic discount factor (SDF) is defined as: $\Lambda_{s, t}=\left(\beta^{P}\right)^{s-t} E_{t}\left[\frac{\lambda_{s+1}^{P}}{\lambda_{s}^{P}}\right]$. The terms on the right-hand side are, respectively, revenue from sales to final goods firms, the wage bill to patient and impatient households, and the quadratic cost of changing prices.

Maximization is subject to the constraints $Y_{i, t} \geq A_{t} F\left(N_{i, t}^{P}, N_{i, t}^{I}\right)$ and to (32). The optimum firm-specific price is given by:
$0=(1-\theta) \frac{Y_{i, t}}{P_{t}}+\theta \frac{Y_{i, t}}{P_{t}} m c_{t}^{G}-\phi\left(\frac{P_{i, t}}{P_{i, t-1}}-1\right) \frac{Y_{t}}{P_{i, t-1}}+\phi E_{t}\left[\left(\frac{P_{i, t+1}}{P_{i, t}}-1\right) \frac{Y_{t+1} P_{i, t+1}}{\left(P_{i, t}\right)^{2}} \frac{\Lambda_{t+1}}{\Lambda_{t}}\right]$,
where $m c_{t}^{G}$ is the Lagrange multiplier on the demand constraint and $\Pi_{t}$ is the gross aggregate inflation rate. Notice that all firms employ an identical capital/labor ratio in equilibrium. The Lagrange multiplier $m c_{t}^{G}$ plays the role of the real marginal cost of production and in a symmetric equilibrium it must hold that $P_{i, t}=P_{t}$. The linearized form of this first order condition therefore gives us a New Keynesian Phillips Curve:

$$
\begin{equation*}
\hat{\Pi}_{t}=\beta E_{t} \hat{\Pi}_{t+1}+\frac{\theta-1}{\phi} \widehat{m c}_{t}^{G} \tag{38}
\end{equation*}
$$

### 3.4 The Government-Monetary Authority

The government sets the nominal interbank interest rate according to a standard Taylor rule:

$$
\begin{equation*}
\frac{R_{t}^{I B}}{R^{I B}}=\left(\frac{R_{t-1}^{I B}}{R^{I B}}\right)^{r_{R}}\left(\left(\frac{\Pi_{t}}{\Pi}\right)^{r_{\Pi}}\left(\frac{Y_{t}}{Y}\right)^{r_{Y}}\right)^{1-r_{R}} \varepsilon_{t}^{R} \tag{39}
\end{equation*}
$$

As in the standard New Keynesian literature, the government achieves such rules via open market operations (OMOs). These operations are financed by lump-sum cash transfers, $X_{t}^{C B}$, to the banking sector as well as money transfers to the two households, $T_{t}^{P}$ and $T_{t}^{I}$, such that any deficits are equal to zero.

### 3.5 Market Clearing and Aggregation

This section lists the market-clearing conditions. The aggregate resource constraint covers the goods market clearing:

$$
\begin{equation*}
Y_{t}=C_{t}+\frac{\phi}{2}\left(\frac{P_{t}}{P_{t-1}}-\Pi\right)^{2} Y_{t} \tag{40}
\end{equation*}
$$

where $C_{t}=C_{t}^{I}+C_{t}^{P}$. The supply of durable goods is held fixed for the analysis that follows. In particular, the total supply of durable goods, $H$, is set equal to 1 , such that

$$
\begin{equation*}
H_{t}^{P}+H_{t}^{I}=H=1 . \tag{41}
\end{equation*}
$$

There is also equilibrium in the labor markets for patient and impatient labor:

$$
\begin{equation*}
\int_{0}^{1} N_{i, t}^{I} d i \equiv N_{t}^{I}, \quad \int_{0}^{1} N_{i, t}^{P} d i \equiv N_{t}^{P} \tag{42}
\end{equation*}
$$

For the banking sector, we have the crucial money market clearing condition which replaces the traditional inter-household lending-borrowing clearing condition. This balance sheet embodies the idea of the banking sector intermediating funds within the economy between agents. It states that the supply of deposits and loans must be equal in every period, and this constraint must be satisfied by banks in their profit maximization:

$$
\begin{gather*}
L_{t}=D_{t}  \tag{43}\\
\int_{0}^{1} B_{j, t}^{I B} d j \equiv B_{t}^{I B}=0 . \tag{44}
\end{gather*}
$$

## 4 The Model With and Without Banks

The preceding section set out a model of two durable good-accumulating households (patient, impatient), a standard New Keynesian non-durable goods sector with sticky goods prices, and a banking sector with staggered interest rate-setting. In addition, we have introduced broad money via money-in-utility following. The full linearized system therefore constitutes 23 equations with 23 variables. When examining the effects of monetary policy, we can decompose the model into simpler variants which can be used to analyze how the introduction of additional sectors and assumptions alter the traditional responses of major macroeconomic variables. The natural progression therefore will be: (1) a model of collateralized borrowing without banks, (2) the same model with the introduction of banks, and finally (3) the model with banks in the presence of flexible and sticky interest rate-setting.

The first step of introducing collaterally-constrained agents was explored by Kiyotaki and Moore (1997) and nested in the New Keynesian framework by Iacoviello (2005). The basic two-agent framework set up here differs from the two-agent problem in Iacoviello (2005) which consisted of patient households and entrepreneurs - the latter borrowed for consumption and engaged in production using durable goods as well as labor in production. The output form these entrepreneurs was sold to retailers who marked up prices in a staggered way, thereby introducing the New Keynesian element of price stickiness. Here our agents have more in common with Monacelli (2008), where the constrained agent does not engage in production, and instead merely provides elastic labor to a separate goods sector, the profits of which are remitted to the patient households (and owners of these firms). Impatient households are constrained in their borrowing, which is undertaken to supplement their labor income in order to fund current consumption of both durables and non-durables.

In the absence of banks, households return to the familiar pattern of trading their debt intertemporally between one another, and there is no divergence in interest rates such that the deposit rate for patient savers equals the loan rate for impatient borrowers, $R_{t}^{D}=R_{t}^{L}=R_{t}$. Both households borrow and save at the same rate of interest, $R_{t}$, which is set by the monetary authority via OMOs. ${ }^{17}$ This is the same as the single aggregate interest rate specification of the standard NNS. Equilibrium borrowing and lending between both types of household is now simply $b_{t}^{P}=b_{t}^{I}$, where $b_{t}^{P}$ denotes patient saving and $b_{t}^{I}$ denotes impatient borrowing. This is used instead of the equilibrium (real) bank balance sheet: $l_{t}=d_{t}$.

### 4.1 Deterministic Steady State

The steady states of the system can be solved for analytically, and are laid out in Appendix B. In particular, we are able to recover steady state expressions for the various interest rates in the system. We find that the mortgage rate is a mark-up over the interbank rate in the steady state:

$$
\begin{equation*}
R^{L}=\frac{\eta^{H}}{\left(\eta^{H}-1\right)} R^{I B}=\frac{\eta^{H}}{\left(\eta^{H}-1\right)} \frac{R^{D}}{(1-\kappa)}=\frac{\eta^{H}}{\beta^{P}(1-\kappa)\left(\eta^{H}-1\right)}, \tag{45}
\end{equation*}
$$

since $\eta^{H}>1$, such that $\frac{\eta^{H}}{\left(\eta^{H}-1\right)}$ is greater than 1 . It is also decreasing in $\eta^{H}$, since $\frac{\partial R^{L}}{\partial \eta^{H}}=$ $-\frac{1}{\left(\eta^{H}-1\right)^{2}} R^{I B}<0$, reflecting the fact that the market power of the $j$ th bank diminishes as the elasticity of substitution increases. We also find that as the reserve requirement

[^10]increases from zero, the mark-up increases $\frac{\partial R^{L}}{\partial \kappa}=\frac{\eta^{H}}{\left(\eta^{H}-1\right)} \frac{(1-\kappa)^{2}}{\beta^{P}}>0$. Therefore the spread between the interbank rate and the loan rate is simply a function of the parameters that characterize the system - this mark-up is parallel in the non-durable goods sector where price is a mark-up over marginal cost.

In addition, these gross nominal interest rates are equal to the gross real interest rates, since inflation is assumed to equal zero in the steady state: $\Pi=1+\pi=1$, where $\pi$ is the steady state (net) rate of inflation and $\Pi$ is steady state gross inflation.

An important point to turn to in this section is how our setup guarantees positive consumption by both agents as well as a unique, determinate level of debt held by the impatient borrower in the steady state. It turns out the combination of differential discount rates, where the borrower is more impatient than the saver, and the presence of binding collateral constraints ensures that both these characteristics are present.

Becker (1980) and Becker and Foias (1987) initially set out how the long-run steady state of the income distribution in a heterogeneous agents model, where heterogeneity is characterized by different discount rates across households, is determined by the household with the lowest discount rate. Furthermore, if discount rates are equal then the steady state income distribution would be indeterminate. This notion also appeared as the 'unit root' problem in the literature for small open economies with incomplete international asset markets, which has been addressed by Schmitt-Grohe and Uribe (2003) among others. ${ }^{18}$

In addition, as Campbell and Hercowitz (2005) explain, in a model of heterogeneous discount rates but no constraints to borrowing, the impatient household's debt to the lender, which in our case is the banking sector (or alternatively the patient household in a model without banks) increases over time to the maximum level the borrower can service via labor income. The introduction of binding collateral constraints restricts the level of debt such that the economy possesses a unique steady state with positive consumption by both households. Therefore a combination of determinate debt under different discount rates, enhanced by positive steady state consumption when collateral constraints are deployed, serve to return a unique, determinate and stable steady state path for debt and consumption by both households.

This shopping list for determinacy therefore imposes certain restrictions for the parameters in our model. From the first order condition (18), the collateral constraint will be binding in the steady state if and only if $\beta^{I}<\frac{1}{R^{L}}$ since:

[^11]\[

$$
\begin{equation*}
\mu^{I}=\frac{\lambda^{I}}{R^{L}}\left(\frac{1}{R^{L}}-\beta^{I}\right) \tag{46}
\end{equation*}
$$

\]

If this restriction is not satisfied and instead $\beta^{I} R^{L}=1$, then $\mu^{I}=0$ meaning that the constraint would not bind and the level of borrowing by impatient households would be indeterminate and not always positive in the steady state. ${ }^{19}$

Therefore, the model is calibrated such that the multiplier on the collateral constraint, $\mu^{I}$, is positive in the steady state. In fact, the relative size of the discount factors, as well as the elasticity of substitution, $\eta^{H}$, are all important when determining the collateral constraint multiplier since we have already seen that the loan rate is a function of $\eta^{H}$ :

$$
\begin{equation*}
\beta^{I}<\frac{1}{R^{L}}=\frac{\left(\eta^{H}-1\right)(1-\kappa)}{\eta^{H}} \beta^{P}<\beta^{P} . \tag{47}
\end{equation*}
$$

Therefore (47) shows how the right hand-side inequality is the result of bank-determined interst rate spreads, and the left hand inequaility arises due to the binding collateral constraint.

We also carry out simulations to explore the sign of the multiplier and as in Iacoviello (2005), we recover a broad range of parameters for which the multiplier is positive in the steady state and we use this result to confirm that locally the collateral constraint is binding. In effect, the collateral constraint may bind only occasionally. However, it always binds if the economy remains close to its nonstochastic steady state; so following Campbell and Hercowitz (2005), Iacoviello (2005) and Monacelli (2008), standard first-order approximation techniques, specifically log-linearization, can characterize the equilibrium of the model in the presence of small disturbances and 'low' uncertainty.

We can also try and impute (or calibrate) the value of $\eta^{H}$ from the empirical mean of the spread between the loan rate and the deposit rate. If we let $s^{L} \equiv R^{L}-R^{D}$ denote the spread (margin) between the two interest rates faced by the household. Using the analytical formula for the steady state loan rate, we can derive the following condition:

$$
\begin{equation*}
\frac{s^{L}}{R^{D}}=\frac{1-\left(\eta^{H}-1\right) \kappa}{\left(\eta^{H}-1\right)(1-\kappa)} \tag{48}
\end{equation*}
$$

Assuming $\kappa \rightarrow 0$ (i.e. negligible or zero reserve requirement), then we can find the appropriate calibration for $\eta^{H}$ :

$$
\begin{equation*}
\eta^{H}=\frac{R^{D}}{s^{L}}+1 \tag{49}
\end{equation*}
$$

Given the endogenous interest rate spread generated by the profit-maximising banking sector, we can find out the range of values that $\kappa$ must satisfy: $\kappa<\left(1-\frac{\eta^{H}-1}{\eta^{H}}\right)$. This

[^12]has implications for the calibration of $\kappa$, which we will discuss later.

### 4.2 Collateral Constraint Effects with Banking Attenuation

Given that this framework builds on the foundations of Monacelli (2008) it is useful to highlight the key mechanisms at work given the presence of banks: ${ }^{20}$ The collateral constraint channel can potentially act to accelerate the effects from policy changes. For example, looser monetary policy that stimulates asset prices (in our case house prices) can loosen the collateral constraint. Constrained households who value current consumption more than future consumption might therefore be capable of raising borrowing and consumption more than proportionally when asset prices rise - and so increases in asset prices could have positive effects on aggregate demand. However, this effect is modified in the presence of a banking sector which drives a wedge between interest rates charged to each household.

In order to understand how the collateral constraint channel is modified in the presence of banks, we examine the dynamic system of equations facing impatient households. First we look at the intertemporal effects on consumption of non-durable goods. From the linearized impatient consumption Euler:

$$
\begin{equation*}
\hat{C}_{t}^{I}=-\hat{R}_{t}^{L}+\underbrace{\beta^{I} R^{L}}_{<1} E_{t}\left[\hat{\Pi}_{t+1}\right]+\underbrace{\beta^{I} R^{L}}_{<1} E_{t}\left[\hat{C}_{t+1}^{I}\right]-\underbrace{R^{L} \mu_{B}^{I} C_{B}^{I} \hat{\mu}_{t}^{I} . . ~ . . ~}_{>0} \tag{50}
\end{equation*}
$$

We can see that as the multiplier increases and the collateral constraint tightens $\left(\hat{\mu}_{t}^{I} \uparrow\right)$, impatient household demand for non-durable goods falls. This linearized Euler contrasts to that under the model without banks:

$$
\begin{equation*}
\hat{C}_{t}^{I}=-\hat{R}_{t}+\underbrace{\frac{\beta^{I}}{\beta^{P}}}_{<1} E_{t}\left[\hat{\Pi}_{t+1}\right]+\underbrace{\frac{\beta^{I}}{\beta^{P}}}_{<1} E_{t}\left[\hat{C}_{t+1}^{I}\right]-\underbrace{\frac{\mu_{N B}^{I} C_{N B}^{I}}{\beta^{P}}}_{>0} \hat{\mu}_{t}^{I} . \tag{51}
\end{equation*}
$$

A tightening of the constraint (an increase in the multiplier) will lead to a larger fall in consumption in the presence of banks if $R^{L}>\frac{1}{\beta^{P}}$, which is always satisfied when the collateral constraint is binding in the steady state.

Next we can uncover how the marginal rate of substitution (MRS), $\frac{U_{H, t}^{I}}{U_{C, t}^{I}}$, varies in the presence of banks. Starting with the impatient household demand for housing (20), we can assume that the multiplier can be decomposed into the multiplier on the budget constraint and a second multiplier, $\omega_{t}^{I}: \mu_{t}^{I}=\lambda_{t}^{I} \omega_{t}^{I}$. In addition we can rewrite some

[^13]terms in the Euler in terms of asset price (durable good) inflation, $\Pi_{t}^{H}=\frac{Q_{t+1}^{H}}{Q_{t}^{H}}$ since $\Pi_{t+1} q_{t+1}^{H}=q_{t}^{H} \Pi_{t+1}^{H}$. This gives:
\[

$$
\begin{equation*}
\frac{U_{H, t}^{I}}{U_{C, t}^{I}}=q_{t}^{H}\left[1-\chi^{I} \omega_{t}^{I} \Pi_{t+1}^{H}\right]-\beta^{I} E_{t}\left[\frac{U_{C, t+1}^{I}}{U_{C, t}^{I}} q_{t+1}^{H}\right] \tag{52}
\end{equation*}
$$

\]

We can see how the presence of the multiplier increases the MRS and 'de-links' it from asset price movements. In the absence of the collateral constraint we would have a one-to-one relationship between the relative price of durables, $q_{t}^{H}$, and the MRS.

We can also obtain a richer expression which shows how the MRS (and user cost) also depends on the ex ante real loan rate, $\frac{R_{t}^{L}}{E_{t} \Pi_{t+1}}$ by combining (20) and (18). This gives us an expression which shows how the MRS can be equated with the user cost of durables, $\Lambda_{B A N K S, t}^{I}$ :

$$
\begin{equation*}
\frac{U_{H, t}^{I}}{U_{C, t}^{I}}=\Lambda_{B A N K S, t}^{I} \equiv q_{t}^{H}\left[1-\chi^{I} \omega_{t}^{I} \Pi_{t+1}^{H}\right]-E_{t}\left[\left(1-R_{t}^{L} \omega_{t}^{I}\right) \frac{\Pi_{t+1}}{R_{t}^{L}} q_{t+1}^{H}\right] \tag{53}
\end{equation*}
$$

As in Kiyotaki and Moore (1997), we can now see how the user cost captures the 'down payment' required to purchase a unit of durable (services), which is the difference between the 'collateral-adjusted' price of the land and the effective 'collateral-adjusted' amount that can be borrowed against the additional unit of durables. Linearizing (53) gives us an expression for the MRS (user cost):

$$
\hat{\Lambda}_{B A N K S, t}^{I}=\frac{1}{\Phi_{B A N K S}^{I}}\left\{\begin{array}{c}
{\left[1-\chi^{I} \omega^{I}\right] \hat{q}_{t}^{H}+\frac{1}{R^{L}} E_{t}\left[\hat{R}_{t}^{L}-\hat{\Pi}_{t+1}\right]-\left[\frac{1}{R^{L}}-\omega^{I}\right] \hat{\mathrm{a}}_{t+1}^{H}}  \tag{54}\\
+\omega^{I}\left\{\left[1-\chi^{I}\right] \hat{\omega}_{t}^{I}-\left(\chi^{I} \hat{\Pi}_{t+1}^{H}-E_{t}\left[\hat{\Pi}_{t+1}\right]\right)\right\}
\end{array}\right\}
$$

where $\Phi_{B A N K S}^{I}=\left\{1-\chi^{I} \omega^{I}-\frac{1}{R^{L}}+\omega^{I}\right\}$.
From the nondurable Euler, we know that $\omega^{I}=\frac{1}{R^{L}}-\beta^{I}$. Therefore, in the model with banks the constraint binds when $\frac{1}{R^{L}}>\beta^{I}$ (see equation (46) above). Equation (54) therefore becomes:

$$
\hat{\Lambda}_{B A N K S, t}^{I}=\frac{1}{\Phi_{B A N K S}^{I}}\left\{\begin{array}{c}
{\left[1-\chi^{I}\left(\frac{1}{R^{L}}-\beta^{I}\right)\right] \hat{q}_{t}^{H}+\frac{1}{R^{L}} E_{t}\left[\hat{R}_{t}^{L}-\hat{\Pi}_{t+1}\right]-\beta^{I} \hat{q}_{t+1}^{H}}  \tag{55}\\
+\left(\frac{1}{R^{L}}-\beta^{I}\right)\left[\left[1-\chi^{I}\right] \hat{\omega}_{t}^{I}-\left(\chi^{I} \hat{\Pi}_{t+1}^{H}-E_{t}\left[\hat{\Pi}_{t+1}\right]\right)\right]
\end{array}\right\},
$$

where: ${ }^{21}$

[^14]$$
\Phi_{B A N K S}^{I}=\left\{\left(1-\frac{1}{R^{L}}\right)+\left(1-\chi^{I}\right)\left(\frac{1}{R^{L}}-\beta^{I}\right)\right\}>0 .
$$

Therefore, a tightening of the collateral constraint, captured via an increase in the multiplier, $\hat{\omega}_{t}^{I}$, increases the marginal rate of substitution, $\frac{U_{H, t}^{I}}{U_{C, t}^{I}}$, since

$$
\frac{1}{\Phi_{\text {BANKS }}^{I}}\left(\frac{1}{R^{L}}-\beta^{I}\right)\left[1-\chi^{I}\right]>0
$$

and so we see a substitution away from durables to non-durables.
In the absence of banks $\frac{1}{R^{L}}=\beta^{P}$ since there is there is only one interest rate in the economy ( $R^{L}=R^{D}=R$ ), and so the multiplier, $\omega^{I}$, equals $\beta^{P}-\beta^{I}$ in the steady state. Thus:

$$
\Phi_{\text {NO BANKS }}^{I}=\left\{\left(1-\beta^{P}\right)+\left(1-\chi^{I}\right)\left(\beta^{P}-\beta^{I}\right)\right\}>0,
$$

and it is always the case that $\Phi_{N O B A N K S}^{I}<\Phi_{B A N K S}^{I}$. Therefore, we can see that the introduction of banks actually attenuates the increase in the MRS which arises from a tightening in the collateral constraint, since

$$
\frac{1}{\Phi_{N O B A N K S}^{I}}\left(\beta^{P}-\beta^{I}\right)\left[1-\chi^{I}\right]>\frac{1}{\Phi_{B A N K S}^{I}}\left(\frac{1}{R^{L}}-\beta^{I}\right)\left[1-\chi^{I}\right]>0
$$

given that $\beta^{I}<\frac{1}{R^{L}}<\beta^{P}$ from (46).
Given expression (55) for the impatient household's MRS, we can also show how the effect of a rise in the policy rate (a monetary policy tightening) is attenuated in the presence of a banking sector with flexible interest rate setting ( $\hat{R}_{t}^{L}=\hat{R}_{t}^{I B}$ ). From (55), the marginal effect of $\hat{R}_{t}^{L}$ on $\hat{\Lambda}_{t}$ is equal to $\frac{1}{R^{L}}\left(\Phi_{B A N K S}^{I}\right)^{-1}$. This gives:

$$
\begin{equation*}
\frac{1}{R^{L}\left\{\left(1-\frac{1}{R^{L}}\right)+\left(1-\chi^{I}\right)\left(\frac{1}{R^{L}}-\beta^{I}\right)\right\}}=\frac{1}{\left\{\left(R^{L}-1\right)+\left(1-\chi^{I}\right)\left(1-\beta^{I} R^{L}\right)\right\}}>0 \tag{56}
\end{equation*}
$$

This is always greater than zero $\left(R^{L}>1\right.$ and $\left.1>\beta^{I} R^{L}\right)$. For analogous arguments, a rise in $\hat{\omega}_{t}^{I}$ (a tightening of the collateral constraint) means that the user cost rises. Comparing the relative impact of the policy rate in the model with and without banking, we find that following responses can be computed in each case (in the bank model, $\hat{R}_{t}^{I B} \equiv \hat{R}_{t}$ ):

$$
\begin{gather*}
\left.\quad \frac{\partial \hat{\Lambda}_{t}}{\partial \hat{R}_{t}}\right|_{N O B A N K S}=\frac{1}{\frac{1}{\beta^{P}}-1+\left(1-\chi^{I}\right)\left(1-\frac{\beta^{I}}{\beta^{P}}\right)}>0,  \tag{57}\\
\left.\frac{\partial \hat{\Lambda}_{t}}{\partial \hat{R}_{t}^{I B}}\right|_{B A N K S}=\frac{1}{\frac{1}{\beta^{P}} \frac{\eta^{H}}{\left(\eta^{H}-1\right)}-1+\left(1-\chi^{I}\right)\left(1-\beta^{I} \frac{\eta^{H}}{\beta^{P}\left(\eta^{H}-1\right)}\right)}>0 . \tag{58}
\end{gather*}
$$

We can compare the two responses such that $\left.\frac{\partial \hat{\Lambda}_{t}}{\partial \hat{R}_{t}}\right|_{N O B A N K S}>\left.\frac{\partial \hat{\Lambda}_{t}}{\partial \hat{\hat{t}}_{t}^{I B}}\right|_{B A N K S}$. The inequality can be reduced to a condition

$$
\left(1-\frac{\eta^{H}}{\eta^{H}-1}\right)\left(1-\left(1-\chi^{I}\right) \beta^{I}\right)<0
$$

which is always satisfied. Therefore, we can see how the presence of banks in a model of constrained and unconstrained households can cause attenuation of the effects of a rise in the policy rate on the marginal rate of substitution between housing services and consumption. The presence of banks means that the substitution away from durables to non-durables is relatively lower when monetary policy is contractionary. As $\frac{\eta^{H}}{\eta^{H}-1}$ is the markup of the loan rate over the deposit rate, we can also see that as substitutability increases between the banks in the intermediate banking sector ( $\eta^{H} \rightarrow \infty$ ), it tends to 1 , the wedge between $\left.\frac{\partial \hat{\Lambda}_{t}}{\partial \hat{R}_{t}}\right|_{N O \text { BANKS }}$ and $\left.\frac{\partial \hat{\Lambda}_{t}}{\partial \hat{R}_{t}^{I B}}\right|_{B A N K S}$ vanishes, and therefore the attenuation effects of financial intermediation themselves abate.

## 5 Dynamic Analysis

In this section, we explore the dynamic implications for monetary policy and the economy in the presence of bank lending, against a backdrop of collaterally-constrained households. To this end, we investigate two shocks in the context of the dual-household framework with and without banks. The first is a standard monetary policy shock, while the second is a collateral constraint shock. The latter is meant to capture the effects of financial distress (or gain) by constrained households. Then we consider two shocks emanating from the banking sector itself (which have no counterpart in the benchmark non-bank model): a shock to the New Keynesian interest rate-setting relation, and a shock to liquidity in the bank balance sheet, which is treated as a liquidity shock. First, we discuss the calibration of the model.

### 5.1 Calibration

As is necessary with such large-scale models, several parameters were calibrated before the model was solved. The calibrations chosen matched typical values chosen across the New Keynesian literature, e.g. elasticities of substitution, labor shares in utility. In the goods sector, the factor share of patient labor, $\gamma$, was chosen to be 0.4. The shares of housing in impatient and patient household utility were chosen to equal 0.1 as in Iacoviello (2005). Given the uncertainty surrounding calibration, various ranges were chosen around some of the more traditional New Keynesian parameters, given the complexity of the model. In the Appendix, the specific calibration parameters used to generate the impulse responses are recorded, but in this section, we note and discuss the ranges chosen for the key
parameters in the system. On reason was that simple sensitivity analysis revealed that certain parameters exerted little effect at the extrema of each range, and so it was safe for them to vary more than others.

The patient savers' discount rate was varied between 0.99 and 0.998 , and those of the impatient borrowers between 0.95 and 0.98 . The latter discount rates were chosen to ensure that the collateral constraints bind in the steady state, namely the Lagrange multipliers associated with the collateral constraints were greater than zero. These discount rates suggest annual average real rates for patient households of between $0.8 \%$ and $4 \%$. Given the current calibration, the spread between the deposit rate and the loan rate in the steady state is approximately $2.69 \%$.

For the bank elasticity of substitution parameter $\eta^{H}$, there was little guidance in the literature. This was therefore reverse-engineered from historical averages of the USD LIBOR, GBP LIBOR and EUR LIBOR rates, all obtained from the British Bankers' Association (BBA). The overnight indexed swap rates (Euronia and Sonia) were also considered. Therefore ranges were also obtained for these parameters, $90<\eta^{H}<170$. There was similarly little guidance for the Calvo parameter for the case of sticky interest rates, $\xi$. As a starting point we used the empirical estimates from Huelsewig et al. (2006), whereby $\xi$ was calibrated to 0.36 .

Given the calibration of $\eta^{H}$, this suggests a range of values for $\kappa$, which must satisfy the restriction obtained above: $\kappa<\left(1-\frac{\eta^{H}-1}{\eta^{H}}\right)$. In this case $\kappa<[0.006,0.01]$. These values for $\kappa$ are extremely low and according to the Bank of International Settlements, the average reserve requirement across OECD is approximately $10 \%$. Therefore for the purposes of our impulse response analysis, we set $\kappa=0$.

The loan-to-value ratios for impatient households, $\chi^{I}$ was chosen to lie between 0.5 and 0.8 , which captured the numbers reported in Iacoviello (2005) and Calza et al. (2007). Given the presence on money and its inclusion in the system, it was necessary also to calibrate the share of real money balances in utility. Once again, the literature was scarce in suggested values, and so a value was chosen which was greater than or equal to the share of housing in utility. Given that housing is not as convertible as money as a medium of exchange, it seemed plausible to presume that households preferred to hold more money, the most liquid asset, relative to other assets in their portfolios, i.e. $\frac{\varrho^{P}}{\varepsilon^{P}} \geq 1$.

### 5.2 What to Expect from the Model

As discussed earlier, it is important to get an idea of the key mechanisms that are at work in the framework set out in the preceding sections. We recover two familiar effects, which have been detailed in previous works: the collateral constraint effect, which has the potential to amplify the impact of monetary policy actions, and the debt deflation channel, which deals with the transfer of wealth from lenders to borrowers when inflation
rises, such that borrowers, who have higher propensities to consume, will be able to consume more when inflation rises.

However, as described in section 4.2, our model incorporates additional effects and channels. We recover an attenuation of shocks for certain variables, due to the presence of a spread between the loan and savings rates. The presence of the balance sheet as a binding financial resource constraint on banking activity also leads to a unique transmission of shocks between households and the rest of the economy.

Banking Attenuator and Interest Rate Spreads Should we expect to recover an attenuation of shocks, or an even greater acceleration of the effects of monetary policy changes, given the way we have introduced profit-maximizing interest rate-setting banks? As shown in section 4.2, we showed analytically the existence of a banking attenuator effect that limits the monetary policy innovations on durables accumulation. Analogously, a tightening of the collateral constraint, which reflects an increase in the user cost of durables, is dampened by the presence of a monopolistically competitive banking sector. The introduction of price-setting in the banking sector is crucial to these results, as it allows us to introduce a spread between the loan rate $\left(R_{t}^{L}\right)$ and the deposit rate $\left(R_{t}^{D}\right)$. On the other hand, the impact of a monetary policy innovation on non-durables is magnified by the presence of a banking sector, given the choice of parameter values. And staggered interest rate adjustment increases the magnitude of these competing effects on durables and non-durables.

Goodfriend and McCallum (2007) identify a banking "attenuator" ${ }^{22}$ as a mechanism which tends to reduce the impact of monetary policy actions. The way this mechanism operates is rather different from the one presented in this paper. This attenuator effect recognizes that any monetary stimulus to spending increases the demand for bank deposits, thereby raising the external finance premium for a given value of collateral-eligible assets in the economy - this is unique to the cash-in-advance (or transaction) constraint that the paper uses, and works differently in our model. In the framework presented here, there is no such constraint that nests a loan production function.

As noted before, the major contrast with the standard NNS models and those of Iacoviello (2005) and Monacelli (2008) is that the introduction of monopolistically competitive banks leads to a divergence in the interest rates faced by households in the economy. Therefore, it is no longer the case that monetary policy, which traditionally sets a single interest rate via a Taylor rule in the NNS, directly influences the 'IS' or demand-side of the economy. This signals a move away from the 'money view' of monetary transmission

[^15]where this unique interest rate is the summary statistic for all credit conditions in the economy, simultaneously capturing the return on all assets present. There are now three distinct interest rates in the model: interest earned on deposits by patient households, $R_{t}^{D}$; interest earned by banks on loans to impatient households, $R_{t}^{L}$, and an interbank rate set by the monetary authority - usually charged by banks to one another for lending - which serves as a proxy for the policy rate, $R_{t}^{I B}$. In this situation it is necessary to track the impact of monetary policy shocks via the loan supply functions, which characterize how banks adjust credit and loan rates in equilibrium.

In our model, adjustments of the interbank (policy) rate by the central monetary authority mean that both the loan rate and deposit rate move in the same direction. The policy rate equals the deposit rate (due to our assumption of price-taking by banks in the market for deposits), and the loan rate is marked-up over the deposit rate. Therefore the only source for changes in the interest rate spread over time is via staggered interest rate-setting by banks. Nevertheless, the mark-up between the loan rate and the deposit rate is crucial in determining the strength of the attenuation effect from the presence of banks, as shown in section 4.2.

Bank Balance Sheet Channel As discussed in the introduction, the bank lending channel and traditional credit view note that banks have access to more than one type of asset - at least one security as well as loans. An assumption of imperfect substitutability of loans for other forms of securities (assets) in bank portfolios is then required such that when a central bank reduces the volume of reserves, loans are also reduced.

In our model we do not have any form of substitutable assets for banks. Banks can only make loans to impatient households and cannot buy any other assets. As such therefore the traditional bank lending channel which relies on central banks engaging in reserve management to affect the supply of loans via changing the liabilities side of the balance sheet is not apparent. ${ }^{23}$ Instead, our model appears to include a bank balance sheet channel, where changes in the liabilities side of the bank balance sheet have a direct effect on the assets of the bank, and through this on to the amount of credit available in the economy. We deliberately choose to refer to this channel as a bank balance sheet channel as opposed to a bank lending channel, since a change in monetary policy cannot directly affect the composition of bank balance sheet (via interest rates), ${ }^{24}$ as banks have only one asset and one liability. The bank balance sheet is ultimately, in the context of the simple banking framework we develop in this paper, the market clearing condition for loans and deposits between households.

By considering a liquidity shock to the balance sheet of the bank, we attempt to

[^16]explore this channel in more detail. For a given level of loans and deposits, we will see that an exogenous increase on the liabilities side of the equation, will lead to a fall in deposits and, depending on the persistence of the shock, a rise or fall in loans. ${ }^{25}$

### 5.3 The Effects of adding a Banking Sector

In this section we compare the output from the model for the benchmark model with banks to one without a banking sector. The Figures displaying the impulse responses for each of the shocks we discuss are reported in Appendix C.

### 5.3.1 Monetary Policy Shocks

The Figures in Appendix C. 1 shows the effects of a $1 \%$ shock to the monetary policy rule. As in the standard NNS framework, an increase in the policy rate leads to an immediate increase in ex ante real interest rates. In the case of flexible interest rate-setting by banks, this rise in the nominal rate also translates into a proportionate rise in the ex ante real loan rate $\left(\frac{R_{t}^{L}}{E_{t}\left[\Pi_{t+1}\right]} \uparrow\right)$, as well as an equal increase in the ex ante real deposit rate $\left(\frac{R_{t}^{D}}{E_{t}\left[\Pi_{t+1}\right]} \uparrow\right)$. This rise would typically induce both sets of household to postpone current consumption of non-durable goods, which follows from the two Euler equations (8) and (18). The responses show that despite their impatience to shift consumption forward, impatient households reduce consumption relatively more than patient households, since their ability to borrow falls with the increase in the ex ante real interest rate:

$$
\begin{equation*}
l_{t}=\chi^{I} \frac{E_{t}\left[q_{t+1}^{H}\right] H_{t}^{I}}{R_{t}^{L} / E_{t}\left[\Pi_{t+1}\right]} . \tag{59}
\end{equation*}
$$

Furthermore, this tightening of borrowing conditions for impatient households from the contraction in the discounted value of their collateral is exacerbated by the fall in house prices which follows the monetary policy contraction. Real loans, $l_{t}$, therefore fall and demand for both durables and non-durables falls, i.e. the effects on the collateral constraint lead to complementary effects on impatient households' consumption of both types of good, which is captured in the responses. ${ }^{26}$ Therefore impatient borrowers see a fall in nondurable consumption greater than that of patient savers.

As we noted earlier, the collateral constraint effect is such that as the relative price of durables falls, the de-linking of asset prices from the user cost, means that the user cost (MRS) in fact rises, leading to a substitution away from housing to non-durable consumption. This reinforces the asset price effects on loan demand above and explains

[^17]why we see a larger effect on durables in the impulse responses. We also recover the debt deflation effect, as inflation falls. There is a transfer of wealth from impatient households (borrowers) to the patient households (savers) as the real cost of current borrowing increases and the real return to current deposits increases.

How does the presence of banks affect these various channels? We find increasing attenuation of most effects once banks are introduced, as set out earlier in section 4.2, and an even stronger attenuation once the stickiness of interest rate-setting increases. Therefore by simply comparing the model without banks to the model with flexible interest rate-setting banks, we find an attenuation of the drop in borrowers' consumption of durables. This latter response is also related to the attenuation of the collateral constraint effect mentioned earlier, whereby the presence of banks means that the rise in the MRS is smaller when banks set loan rates differently to savings rates. Therefore the substitution from durables to non-durables is not as severe, and so instead impatient households cut back relatively more on non-durable consumption, given their overall diminished capacity to borrow due to lower collateral values.

When we compare the flexible interest rate-setting bank model to the one with sticky interest rates, the collateral constraint tightening, captured by the increase in $\hat{\mu}_{t}^{I}$, has a stronger effect on non-durable consumption. With sticky interest rates and given the calibration, we see an amplification of the fall in non-durable consumption compared to the case of flexible interest rate-setting.

The responses also show that the presence of banks has no real effect on the longevity of the shock. The rate of the decay of the impulse to each variable is not significantly altered when monetary policy changes are intermediated through banks, and therefore banks no not appear to affect the propagation of the shock beyond attenuating (or accelerating in the case of non-durable consumption) the existing effects.

### 5.3.2 Collateral Constraint Shocks

In the figures displayed in Appendix C.2, we show the effects of a shock to the collateral constraint:

$$
\begin{equation*}
\hat{l}_{t}=E_{t}\left[\hat{q}_{t+1}^{H}\right]+\hat{H}_{t}^{I}-\left\{\hat{R}_{t}^{L}-E_{t}\left[\hat{\Pi}_{t+1}\right]\right\}+\varepsilon_{t}^{L} \tag{60}
\end{equation*}
$$

where

$$
\begin{equation*}
\varepsilon_{t}^{L}=\rho^{L} \varepsilon_{t-1}^{L}+e_{t}^{L} \tag{61}
\end{equation*}
$$

represents the $\mathrm{AR}(1)$ loan demand shock and $e_{t}^{L}$ is an iid disturbance. In this case we allow for some persistence of the shock by setting $\rho^{L}=0.6$. This positive shock is akin to an outward shift in the loan demand schedule, which leans to an increase in loans and the loan rate. However, in order to fund this increase in loans, banks must attract deposits
which require an increase in the deposit rate. In the case of the non-bank model and the bank model with flexible prices, both the deposit rate and loan rate rise by the same proportion. And given the mark-up of the loan rate over the deposit rate in the model with banks, bank profits increase following the increase in loan demand.

The increase in loans means that impatient households experience a loosening of their collateral constraint. Subsequently, their consumption of both durables and nondurables can increase. These responses once again reflect the complementarity between the consumption of both types of good that a relaxation or tightening of the collateral constraint induces.

The rise in deposits (savings) by patient households means that they postpone their consumption of durables and non-durables, which both fall (the former changes in response to adjustment by impatient households given the fixed supply of durables). The responses also show that the fall in $C_{t}^{P}$ and $H_{t}^{P}$ is less than the rise in $C_{t}^{I}$ and $H_{t}^{I}$ respectively, given the loosening of the collateral constraint and the higher marginal propensity to consume of impatient households. Since the rise of impatient nondurable consumption is greater than the fall of patient non-durable consumption, we see an increase in aggregate demand, which leads to an increase in output. And to produce this increased output, firms must hire more labor and raise wages to induce greater labor supply.

Furthermore, in response to an increase in interest rates, we see asset prices, $\hat{q}_{t}^{H}$, fall. And as rates begin to fall following the initial impulse, we see asset prices rise once again. There is an element of overshooting as prices adjust to ensure that the housing market clears. Inflation also spikes as aggregate demand is boosted by the positive shift in loan demand.

When we look at the responses as banks gradually adjust the loan rate, we find the increase in aggregate demand has been mildly attenuated and therefore there is a smaller increase in output relative to the flexible price and non-bank cases. Consequently inflation also rises by a smaller amount and the slower change in loan rates also means that bank profits rise by less.

This positive loan demand shock can be compared to a situation where the borrowing opportunities of impatient households have increased. Parallels between such a relaxation in constraints could be compared to a situation with an incipient bubble in house prices, since an increase in expected future house prices (as could occur in a rational bubble) can cause a similar outward shift in loan demand.

### 5.4 Bank Model Shocks

In this section we compare shocks, displayed in Appendices D.1, D. 2 and D.3, which are specific to the banking model and look at the effects within the context of sticky and flexible interest rate setting by banks. The issue of attenuation is no longer relevant here
and so

### 5.4.1 Balance Sheet Shocks

We also investigate the effects of a $1 \%$ liquidity shock to the model in Appendices D. 1 and D.2, whereby we shock the bank balance sheet as follows:

$$
\begin{equation*}
\hat{l}_{t}=\hat{d}_{t}+\varepsilon_{t}^{x} \tag{62}
\end{equation*}
$$

where

$$
\begin{equation*}
\varepsilon_{t}^{x}=\rho^{x} \varepsilon_{t-1}^{x}+e_{t}^{x} \tag{63}
\end{equation*}
$$

and $e_{t}^{x}$ is an iid innovation, and we look at the effects of the shock under zero and mild persistence, $\rho^{x}=0$ and $\rho^{x}=0.6$ respectively. According to Bernanke (1983) banking and financial crises, lead to different types of large unplanned changes in the channels of credit flow: (1) a fear of runs leading to large withdrawals of deposits; (2) precautionary increases in reserve-deposit ratios, and (3) an increased desire by banks for very liquid or rediscountable assets. These factors are ideally captured by this shock to the balance sheet and can lead to a forced contraction of the banking system's role in the intermediation of credit and therefore to real effects on the economy.

Bernanke (1992) also set out empirically how a reduction in the volume of deposits held by depository institutions, triggered by tight monetary policy, will affect a banks' balance sheet. The effect of the reduction in deposits starts immediately, grows gradually, reaches its maximum after approximately three quarters, and appears to be permanent, which means that bank assets fall along with bank liabilities. For the first six months after the policy shock, the fall in assets is concentrated almost entirely in securities, with loans hardly moving. However, shortly after, security holdings begin gradually to be rebuilt, while loans start to fall and, after two years, security holdings have almost returned to their original value, and the entire decline in deposits is reflected in loans. The simple bank balance sheet we introduce contains only one asset and one liability, and therefore instead we capture an instantaneous impact on loans rather than a gradual effect.

When there is some persistence in the liquidity shock (Appendix D.2), i.e. $\rho^{x}>0$, we find that loans increase initially following the positive impulse to the balance sheet, before falling to match deposits, which fall on impact (but by less than when the shock to the balance sheet is $i i d$ ). The reasoning for the behavior of deposits and loans following the shock is as follows: in order for (62) to hold it must be the case that either loans increase or deposits fall, or both. Loans increase initially since $\Delta \hat{d}_{1}<\Delta \varepsilon_{1}^{x}$, but from approximately the 7 th or 8 th quarter the shock has mostly dissipated and the perfect co-movement between deposits and loans due to the balance sheet is almost completely restored.

However, when there is no persistence in the liquidity shock ( $\rho^{x}=0$ ) in Appendix D.1, we see a concurrent fall in loans and deposits occurs,.despite a seemingly 'positive' shock to the liabilities side of the balance sheet. This happens because an injection of funds into the bank balance sheet means that less funding needs to be undertaken immediately from deposits for the given amount of loans. Therefore we see deposits fall. Given the balance sheet constraint, however, loans must also fall with deposits and so the supply of loans falls relative to its steady state level. In particular, we can see that deposits fall by more than the upward impulse from the liquidity shock, $\varepsilon_{t}^{x}$, to ensure that loans fall in the period that the shock hits the economy, i.e. $\Delta \hat{d}_{1}>\Delta \varepsilon_{1}^{x}$. From period 2 there is no further shock, and the balance sheet constraint holds perfectly, such that $\hat{l}_{t}=\hat{d}_{t}$, for $t \geq 2$.

We also find that inflation and output fall following this liquidity shock as in the case of the negative monetary policy shock, whereas we see an opposite movement in house prices. The reason for the fall in output is that this shock to the balance sheet shows up negatively in the aggregate resource constraint: ${ }^{27}$

$$
\begin{equation*}
\hat{Y}_{t}=\hat{C}_{t}^{P}+\hat{C}_{t}^{I}-\varepsilon_{t}^{x}, \tag{64}
\end{equation*}
$$

and so represents a contraction in output.
Since there is no issue about banking attenuation in this case, we can look instead at the effects on the spread between the loan and deposit rates. The increase in the spread under mild persistence is greater than that with zero persistence, and we can see that consumption of both durables and non-durables is accelerated by the presence of the increasing changing interest rate spread when loan rates are sticky, as well as sharper changes to real loans and deposits.

### 5.4.2 Interest Rate Setting Shocks

Finally, we turn to a shock to the New Keynesian interest rate setting relation which was derived in section 2, the responses of which are reported in Appendix D.5. As with the preceding liquidity shock, we can only examine this type of impulse in the case of the bank model with flexible vs. sticky loan rates. The shock will be to the loan supply relation (31) - a 'credit supply'-side shock. This contrast to the 'credit demand'-side collateral constraint shock. We treat the shock as a change in bank competitiveness on the system, following the findings of Hannan and Berger (1991) and Borio and Fritz (1995) reported earlier, and simulate a change to the degree of substitutability in bank loans. Following the modelling strategy pursued by Steinsson (2003) and Smets and Wouters (2003) to obtain a cost push shock in the NK Phillips curve, we assume that the elasticity

[^18]of substitution in the demand for loans $\left(\eta_{t}^{H}\right)$ follows a log-stationary stochastic process. Therefore, the loan demand function faced by each intermediary now reads as:
\[

$$
\begin{equation*}
L_{j, t}=\left(\frac{R_{j, t}^{L}}{R_{t}^{L}}\right)^{-\eta_{t}^{H}} L_{t} \quad \forall j, \tag{65}
\end{equation*}
$$

\]

In this case, we obtain the following interest rate-setting equation under the assumption of Calvo-type interest rate setting:

$$
\begin{equation*}
\Delta \hat{R}_{t}^{L}=\beta^{P} E_{t} \Delta \hat{R}_{t+1}^{L}-\frac{\left(1-\beta^{P} \xi\right)(1-\xi)}{\xi}\left(\hat{R}_{t}^{L}-\hat{R}_{t}^{I B}\right)+\varepsilon_{t}^{R L} \tag{66}
\end{equation*}
$$

where

$$
\begin{equation*}
\varepsilon_{t}^{R L}=\underbrace{\left(1-\frac{\eta^{H}}{\eta^{H}-1}\right)}_{<0} \frac{\left(1-\beta^{P} \xi\right)(1-\xi)}{\xi} \hat{\eta}_{t}^{H} . \tag{67}
\end{equation*}
$$

where $\hat{\eta}_{t}^{H}=\ln \left(\eta_{t}^{H} / \eta^{H}\right)$. Notice that the structural shock $\varepsilon_{t}^{R L}$ will enter with a negative sign into (66), implying that an increase in competitive pressure in the banking sector $\left(\eta_{t}^{H}>\eta^{H}\right)$ - more elastic demand and lower mark-up of the loan rate over the deposit rate - will increase the rate of change of the loan rate.

A cost-push shock in the banking sector translates into an increase in the full set of interest rates in our model economy and in a drop in the supply of loans. Simultaneously, a fall in loans means that banks require less deposits from patient savers to fund their lending activities. This effect is determined by the fact that the loan supply schedule is perfectly elastic, since the interest rate charged on mortgages is independent of the amount of loans being issued. Therefore, a positive shock to the interest rate-setting relation leads to an upward shift of the perfectly elastic loan supply schedule. And given that loan demand is a negatively-sloped function in $\left[\hat{l}, \hat{R}^{L}\right]$-space, this translates into an increase in the interest rate and fall in the amount of loans provided to the impatient borrowers. This has effects on the household sectors in two ways.

First, impatient households shift from housing to non-durable consumption given the fall in their funding opportunities. This fall in durables by impatient borrowers is greater than the increase in the demand for housing by patient savers. This drives down the asset price. As for patient consumers, as shown in equation (12), durables price dynamics are perfectly reflected in their consumption of non-durable goods. Therefore, the two impulse responses will follow the same dynamic pattern following the shock. Patient consumers will therefore initially increase their householdings, given the contracted price of durables and the decrease in their deposits. Expected bank profits will increase, and the overall effect on production and inflation is positive.

### 5.5 Conclusion

The aim of this paper was to graft a profit-maximizing interest rate-setting banking sector onto a model with collaterally-constrained agents to produce a framework that can track the effects of monetary aggregates, such as loans and deposits, and interest rates on the real economy. Introducing banks in this way allows us to embed interest rate spreads within the economy and see how they alter the traditional monetary transmission mechanism in DSGE models. Exploring the role of banks and financial intermediaries is particularly relevant given how the financial crisis had its genesis in the banking sector. We uncover, both analytically and via impulse response analysis, that the banking sector attenuates the effects from a number of shocks on most variables, due to the differential loan and savings rates introduced.

Some light can therefore be shed on how the monetary transmission mechanism is altered in the presence of a profit-maximizing interest-rate setting banking sector. However, this is only the very first step, as there are a number of other issues embedded within the most recent and preceding literature on banking which can be incorporated, and indeed new puzzles and issues have been thrown up by recent events.

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## A Log-linearized Equilibrium Conditions

## A. 1 Patient Households

$$
\begin{gather*}
0=\hat{C}_{t}^{P}+\hat{\lambda}_{t}^{P}  \tag{68}\\
E_{t} \hat{C}_{t+1}^{P}-\hat{C}_{t}^{P}=\hat{R}_{t}^{D}-E_{t} \hat{\Pi}_{t+1}  \tag{69}\\
\hat{q}_{t}^{H}=\hat{C}_{t}^{P}+\left(1-\beta^{P}\right)\left(\hat{\varepsilon}_{t}^{P}-H_{t}^{P}\right)+\beta^{P}\left(E_{t} \hat{q}_{t+1}^{H}-E_{t} \hat{C}_{t+1}^{P}\right)  \tag{70}\\
\frac{\varrho^{P}}{m^{P}} \hat{m}_{t}^{P}+\frac{\beta^{P}}{C^{P}} E_{t}\left[\hat{C}_{t+1}^{P}+\hat{\Pi}_{t+1}\right]=\frac{1}{C^{P}} \hat{C}_{t}^{P}  \tag{71}\\
=C^{P} \hat{C}_{t}^{P}+q^{H} H^{P}\left(\hat{H}_{t}^{P}-\hat{H}_{t-1}^{P}\right)+m^{P} \hat{m}_{t}^{P}+d \hat{d}_{t} \\
m^{P}\left(\hat{m}_{t-1}^{P}-\hat{\Pi}_{t}\right)+Y \hat{Y}_{t}-w^{I} N^{I}\left(\hat{w}_{t}^{I}+\hat{N}_{t}^{I}\right) \\
+R^{L} l\left(\hat{R}_{t-1}^{L}+\hat{l}_{t-1}-\hat{\Pi}_{t}\right) \tag{72}
\end{gather*}
$$

## A. 2 Impatient Households

$$
\begin{gather*}
\hat{l}_{t}=E_{t}\left[\hat{q}_{t+1}^{H}\right]+\hat{H}_{t}^{I}-\left\{\hat{R}_{t}^{L}-E_{t}\left[\hat{\Pi}_{t+1}\right]\right\}+\varepsilon_{t}^{L}  \tag{73}\\
0=\hat{C}_{t}^{I}+\hat{\lambda}_{t}^{I} \\
\beta^{I} R^{L} E_{t}\left[\hat{\Pi}_{t+1}\right]+\beta^{I} R^{L} E_{t}\left[\hat{C}_{t+1}^{I}\right]=R^{L} \mu^{I} C^{I} \hat{\mu}_{t}^{I}+\left(\hat{R}_{t}^{L}+\hat{C}_{t}^{I}\right)  \tag{74}\\
\varphi^{I} \hat{N}_{t}^{I}=\hat{w}_{t}^{I}-\hat{C}_{t}^{I}  \tag{75}\\
\left.=\left(\beta^{I} \frac{q^{H}}{C^{I}}+\chi^{I} \mu^{I} q^{H}\right) E_{t}\left[\hat{q}_{t+1}^{H}\right]-\beta^{I} \frac{q^{H}}{C^{I}} \hat{C}_{t}^{I}-\frac{\varepsilon^{I}}{H^{I}}\left(\hat{\varepsilon}_{t}^{I}-\hat{H}_{t}^{I}\right)-\chi^{I} \mu^{I} q^{H} \hat{\mu}_{t+1}^{I}\right]+\chi^{I} \mu^{I} q^{H} E_{t}\left[\hat{\Pi}_{t+1}\right] \\
\frac{\varrho^{I}}{m^{I}} \hat{m}_{t}^{I}+\frac{\beta^{I}}{C^{I}} E_{t}\left[\hat{C}_{t+1}^{I}+\hat{\Pi}_{t+1}\right]=\frac{1}{C^{I}} \hat{C}_{t}^{I}  \tag{76}\\
=l
\end{gather*}
$$

## A. 3 Goods Sector

$$
\begin{equation*}
\hat{Y}_{t}=\hat{A}_{t}+\gamma \hat{N}_{t}^{I} \tag{78}
\end{equation*}
$$

$$
\begin{gather*}
\widehat{w}_{t}^{I}=\widehat{m c}_{t}^{G}+\hat{Y}_{t}-\hat{N}_{t}^{I}  \tag{79}\\
\widehat{m c}_{t}^{G}=-\frac{1}{\gamma} \hat{A}_{t}+\widehat{w}_{t}^{I}  \tag{80}\\
\hat{\Pi}_{t}=\beta E_{t} \hat{\Pi}_{t+1}+\frac{\theta-1}{\phi} \widehat{m c}_{t}^{G}+u_{t} \tag{81}
\end{gather*}
$$

## A. 4 Banking Sector

$$
\begin{gather*}
\hat{R}_{t}^{D}=\widehat{\mu}_{t}^{B}  \tag{82}\\
\hat{R}_{t}^{I B}=\widehat{\mu}_{t}^{B}  \tag{83}\\
\Delta \hat{R}_{t}^{L}=\beta^{P} E_{t} \Delta \hat{R}_{t+1}^{L}-\frac{\left(1-\beta^{P} \xi_{H}\right)\left(1-\xi_{H}\right)}{\xi_{H}}\left(\hat{R}_{t}^{L}-\hat{R}_{t}^{I B}\right)+\varepsilon_{t}^{R L}  \tag{84}\\
l \hat{l}_{t}=d \hat{d}_{t}+\varepsilon_{t}^{x} \tag{85}
\end{gather*}
$$

## A. 5 Central Bank Interest Rate Rule

$$
\begin{equation*}
\hat{R}_{t}^{I B}=r_{R} \hat{R}_{t-1}^{I B}+\left(1-r_{R}\right) r_{\pi} \hat{\Pi}_{t}+\left(1-r_{R}\right) r_{Y} \hat{Y}_{t}+\varepsilon_{t}^{R} \tag{86}
\end{equation*}
$$

## A. 6 Market-Clearing Conditions

$$
\begin{gather*}
H^{P} \hat{H}_{t}^{P}+H^{I} \hat{H}_{t}^{I}=0  \tag{87}\\
Y \hat{Y}_{t}=C^{I} \hat{C}_{t}^{I}+C^{P} \hat{C}_{t}^{P} \tag{88}
\end{gather*}
$$

## B Deterministic Steady States

This section sets out the analytical formulae for steady states:

- We assume that $\Pi=(1+\pi)=1$, i.e. inflation is zero in the steady state;
- Housing supply is fixed: $H=1$;
- List of coefficients:

$$
\begin{gather*}
\zeta_{1}=\frac{\varepsilon^{I}}{\left\{\left(1-\beta^{I}\right)-\frac{\left(1-\beta^{I} R^{L}\right)}{R^{L}} \chi^{I}\right\}}  \tag{89}\\
\zeta_{3}=\left\{1+\left(R^{L}-1\right) \frac{\chi^{I} \zeta_{1}}{R^{L}}\right\}  \tag{90}\\
\zeta_{5}=\frac{\chi^{I} \zeta_{1}}{R^{L}} m c^{G} \frac{(1-\gamma)}{\zeta_{3}}  \tag{91}\\
\zeta_{7}=\left[\left(1-(1-\gamma) m c^{G}\right)+\left(R^{L}-1\right) \zeta_{5}\right]  \tag{92}\\
\zeta_{8}=\frac{\varepsilon^{P}}{\left(1-\beta^{P}\right)} \tag{93}
\end{gather*}
$$

- Interest Rates:

$$
\begin{array}{ll}
\text { Deposit Rate: } & R^{D}=\frac{1}{\beta} \\
\text { Interbank Rate: } & R^{I B}=\frac{R^{D}}{1-\kappa} \\
\text { Mortgage (Loan) Rate: } & R^{L}=\frac{\eta^{H}}{\left(\eta^{H}-1\right)} R^{I B}
\end{array}
$$

- Remaining Steady States

$$
\begin{array}{r}
m c^{G}=\frac{\theta-1}{\theta} \\
\psi^{G}=\left(1-m c^{G}\right) Y \\
N^{P}=\left(\frac{m c^{G} \gamma}{\nu^{P} \zeta_{7}}\right)^{\frac{1}{\varphi^{P}+1}} \\
N^{I}=\left(\frac{\zeta_{3}}{\nu^{I}}\right)^{\frac{1}{\varphi^{I}+1}} \\
Y=\left(N^{P}\right)^{\gamma}\left(N^{I}\right)^{1-\gamma} \\
\frac{H^{I}}{H}=\frac{\zeta_{1} m c^{G} \frac{(1-\gamma)}{\zeta_{3}}}{\zeta_{8} \zeta_{7}+\zeta_{1} m c^{G} \frac{(1-\gamma)}{\zeta_{3}}} \tag{99}
\end{array}
$$

$$
\begin{gather*}
\frac{H^{P}}{H}=1-\frac{H^{I}}{H}=\frac{\zeta_{8} \zeta_{7}}{\zeta_{8} \zeta_{7}+\zeta_{1} m c^{G} \frac{(1-\gamma)}{\zeta_{3}}}  \tag{100}\\
q^{H}=\zeta_{8} \zeta_{7} \frac{Y}{H^{P}}  \tag{101}\\
l=\zeta_{5} Y  \tag{102}\\
C^{P}=\zeta_{7} Y  \tag{103}\\
C^{I}=Y-C^{P}  \tag{104}\\
w^{P}=m c^{G} \gamma \frac{Y}{N^{P}}  \tag{105}\\
w^{I}=\frac{C^{I} \zeta_{3}}{N^{I}}=m c^{G}(1-\gamma) \frac{Y}{N^{I}}  \tag{106}\\
m^{P}=\frac{\varrho^{P}}{\left(1-\beta^{P}\right)} C^{P}  \tag{107}\\
m^{I}=\frac{\varrho^{I}}{\left(1-\beta^{I}\right)} C^{I}  \tag{108}\\
\mu^{I}=\frac{\left(1-\beta^{I} R^{L}\right)}{R^{L} C^{I}}  \tag{109}\\
d=l  \tag{110}\\
\psi^{B}=R^{L} l-R^{D} d \tag{111}
\end{gather*}
$$

## B. 1 Description of System

1. System Variables:

Specifically, the key log-linearized variables ${ }^{28}$ of the system which are determined endogenously are:

- Prices:

$$
\left\{\hat{q}_{t}^{H}, \hat{w}_{t}^{P}, \hat{w}_{t}^{I}, \hat{\Pi}_{t}\right\}
$$

- Household Variables (including multipliers):

$$
\left\{\hat{C}_{t}^{P}, \hat{C}_{t}^{I}, \hat{C}_{t}, \hat{H}_{t}^{P}, \hat{H}_{t}^{I}, \hat{l}_{t}, \hat{d}_{t}, \hat{\mu}_{t}^{I}, \hat{m}_{t}^{P}, \hat{m}_{t}^{I}\right\}
$$

- Goods Sector Variables:

$$
\left\{\hat{Y}_{t}, \widehat{m c}_{t}^{G}, \hat{N}_{t}^{P}, \hat{N}_{t}^{I}\right\}
$$

[^19]- Banking Variables:

$$
\left\{\widehat{\mu}_{t}^{B}, \hat{R}_{t}^{L}, \hat{R}_{t}^{D}, \hat{R}_{t}^{I B}, \hat{m}_{t}\right\}
$$

2. Example Calibration:

| Parameters | Values | Parameters | Values |
| :--- | :--- | :--- | :--- |
| $\beta^{P}$ | 0.99 | $\eta^{H}$ | 150 |
| $\beta^{I}$ | 0.97 | $\kappa$ | 0 |
| $\varepsilon^{P}$ | 0.1 | $\xi$ | 0.36 |
| $\varepsilon^{I}$ | 0.1 | $\gamma$ | 0.4 |
| $\chi^{I}$ | 0.75 | $r_{R}$ | 0.8 |
| $\varphi^{P}$ | 1.01 | $r_{\Pi}$ | 1.5 |
| $\varphi^{I}$ | 1.01 | $r_{Y}$ | 0.1 |
| $\nu^{P}$ | 1 | $\rho^{A}$ | 0.5 |
| $\nu^{I}$ | 1 | $\rho^{x}$ | $0,0.5$ |
| $\theta$ | 3.5 | $\rho^{L}$ | 0.5 |
| $\phi$ | $83 \frac{1}{3}$ |  |  |

## C Bank vs. Non-Bank Impulse Responses

## C. 1 Monetary Policy Shock

C.1.1 Sticky Bank (Solid) vs. Non-bank (Dashed) vs. Flex Bank (Dotted)






## C. 2 Collateral Constraint Shock

C.2.1 Sticky Bank (Solid) vs. Non-bank (Dashed) vs. Flex Bank (Dotted)


## D Bank Impulse Responses

## D. 1 Balance Sheet/Liquidity Shock (Bank Model only)

D.1.1 Sticky Bank (Solid) vs. Flex Bank (Dotted), Zero persistence of Shock


## D. 2 Balance Sheet/Liquidity Shock (Bank Model only)

D.2.1 Sticky Bank (Solid) vs. Flex Bank (Dotted), Mild persistence of Shock ( $\rho^{x}=0.5$ )




## D. 3 New Keynesian Interest Rate-Setting Shock (Bank Model only)

D.3.1 Sticky Bank (Solid) vs. Flex Bank (Dotted)




[^0]:    *We would like to thank David Aikman, Piergiorgio Alessandri and Chryssi Giannitsarou for their invaluable discussions, and Alexis Anagnostopoulos, Sean Holly, Ivan Petrella, James Cloyne and others at the Bank of England for their suggestions, as well as participants at the 14th International Computing in Economics and Finance Conference in Paris, 26th June 2008; the Macroeconomics Workshop, University of Cambridge and the Macroeconomics Seminar, University of Trento. All errors are our own.
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[^1]:    ${ }^{1}$ In addition, it would appear that in the recent turmoil, liquidity management, typically treated by central banks as a marginal operation, is now proving vital given the current urgent need for injections of short-term funds into beleaguered interbank markets.
    ${ }^{2}$ See Woodford (2003) and Galí (2008) for detailed expositions on the construction and mechanics of New Keynesian DSGE models.
    ${ }^{3}$ The balance sheet would appear to be a crucial part of any specification of the banking sector, as it is an important route for both understanding and introducing liquidity considerations.

[^2]:    ${ }^{4}$ There is an important assumption underlying this particular formulation of the Taylor rule using the interbank rate: changes to the policy rate have a one-to-one effect on the interbank lending rate. As is very clear at the moment, there is an obvious divergence between central bank policy rates (e.g. the Fed Funds rate in the US and the repo rate in the UK) and interbank rates (e.g. the London Interbank Offer Rate, LIBOR). However, as this paper is concerned with the effects of introducing an interest rate-setting banking sector into the NNS framework, this issue is temporarily side-stepped, and it is assumed that monetary authorities can intervene directly in interbank markets to soak up excess supply and demand using OMOs and subsequently set the interbank rate, $R_{t}^{I B}$.
    ${ }^{5}$ An important next step is the modelling of the impact of spreads on production and output. For the time being this paper abstracts from financially-constrained firms and focuses on the impact of banking on transmission and feedback in a simple DSGE model of household borrowing and lending.

[^3]:    ${ }^{6}$ Wealth effects arising from changes in the value of housing are still contested within central banks and the academic literature.
    ${ }^{7}$ The Federal Reserve has typically advocated against 'leaning into the wind' when considering its response to asset price bubbles. The argument is that it is difficult to assess which asset prices to respond to and what their equilibrium or fundamental level is. The preference has traditionally been to use monetary policy to engage in 'cleaning-up' once the bubbles have burst, as in 1987, 1990-91, 1997 and 2001.
    ${ }^{8}$ This is notionally different to the bank balance sheet channel we discuss later in the paper. The most prominent line of work in this strand of the literature is the one developed by Bernanke and Gertler (1989, 1995), Greenwald and Stiglitz (1993), Bernanke, Gertler, and Gilchrist (1999), Kiyotaki and Moore (1997), Carlstrom and Fuerst (1997), Kocherlakota (2000), Cooley et al. (2004).

[^4]:    ${ }^{9}$ Ramsey (1928) postulates that if two dynasties have different discount rates and a loan market is in operation, then "equilibrium would be attained by a division of society into two classes, the thrift enjoying bliss and the improvident at the subsistence level". Becker (1980) and Becker and Foias (1987) confirm this conjecture, by designing dynamic general equilibrium frameworks where two classes of agents coexist and differ with respect to their discount factor.

[^5]:    ${ }^{10}$ An obvious extension to this framework, would be the introduction of working capital loans to the goods sector, as set out in the cost channel literature (see Christiano and Eichenbaum (1992), Ravenna and Walsh (2006)).

[^6]:    ${ }^{11} U_{x, t}^{P}$ denotes the partial derivative of patient utility with respect to variable $x^{P}$ at time $t$.

[^7]:    ${ }^{12}$ It is useful to point out that we also include a private bond, $B_{t}$ and the corresponding bond rate, $R_{t}$, in the system. This allows us to compare the responses of the single interest rate in the non-bank formulation with the deposit rate in the model with banks, since this bond rate is identical to the deposit rate, delivering an identical Euler and steady state.
    ${ }^{13}$ Technically, we assume a zero depreciation rate for durable goods (perfect durability), which implies an infinite stock-flow ratio.

[^8]:    ${ }^{14}$ This approximation is equivalent to saying that the demand for durable goods displays an almost infinite elasticity of intertemporal substitution. Even a small rise in the price of the durable today relative to tomorrow would cause people to delay their purchases.

[^9]:    ${ }^{15}$ As part of the emergency measures granted to the Federal Reserve by the US Treasury Department and Congress, was the ability to pay interest on reserves that banks maintain at the central bank. This enables the Fed to pump almost unlimited cash into the money market without fear of interest rates falling to zero, as they did in Japan during its lost decade. This is new instrument for monetary policy which we can partly capture within our framework by altering the profits that the bank earns. The balance sheet of the bank remains unchanged, however, there will be an effect on the equilibrium determination of the deposit rate with implications for bank profits.
    ${ }^{16}$ In the literature, $\kappa$ is a regularly-commented on, but little-used, tool of monetary policy. As the specification stands, $\kappa$ has not been endogenised. However, to do so would require little complication and its endogenisation is saved for future work.

[^10]:    ${ }^{17}$ The rate, $R_{t}$, is plotted in the impulse responses as the bond rate and is used to compare the deposit rate in the bank model with the single interest rate of the non-bank model, since $\hat{R}_{t}=\hat{R}_{t}^{D}$ in the bank model (from the first order conditions of the patient household when a private bond is also included).

[^11]:    ${ }^{18}$ Lubik (2007), in particular, gives an example of the canonical small open economy model, where a steady state exists only if the product of the discount factor and the exogenous foreign interest rate equal one, $\beta R^{*}=1$. Furthermore, the steady-state level of one of the control variables, namely net foreign assets (debt, $l$, in our model), is not pinned down by the model's optimality conditions. and so there are a multiplicity of steady states indexed by the initial condition for debt.

[^12]:    ${ }^{19}$ This is equivalent to requiring that $R^{L}<\frac{1}{\beta^{I}}$ where $\frac{1}{\beta^{I}}$ would have been the steady state loan rate in the absence of the collateral constraint.

[^13]:    ${ }^{20}$ Monacelli (2008) uses his two-person framework without banks to understand the co-movement problem that occurs in the presence of durable goods' price stickiness and perfect financial markets without constrained borrowing: namely, durable consumption is found to contract (as expected) following a monetary policy tightening and subsequently does not co-move with nondurable consumption, which rises. This is at odds with the empirics. Once collateral constraints are introduced, positive co-movement between durable and nondurable consumption is recovered.

[^14]:    ${ }^{21}$ We can see that $\Phi_{B A N K S}^{I}$ is always greater than zero, since $R^{L}>1, \chi^{I}<1$ and $\frac{1}{R^{L}}>\beta^{I}$.

[^15]:    ${ }^{22}$ Gerali et al. (2008) develop a DSGE model with banking along the lines of Iacoviello (2005). They discuss a banking attenuator effect due to the sole assumption of sticky interest rate-setting on deposits and loans. We have shown that an attenuator effect on durables is present even under the assumption of flexible interest rates. As explained in section 4.2, this is a direct consequence of having a monopolistically competitive banking sector, which produces an endogenous spread between the loan and deposit rates in the steady state. This fundamental attenuator mechanism is further magnified by the assumption of interest rate stickiness.

[^16]:    ${ }^{23}$ The parameter $\kappa$ captures the reserve requirement, however, the maximum possible value of $\kappa$ that would permit the collateral constraint to bind in the steady state (i.e. for $\mu^{I}>0$ ) is just under $1 \%$. As already noted, this is much lower than the average reserve requirement of $10 \%$ used by most central banks and so we set $\kappa=0$.
    ${ }^{24}$ However, the monetary authority could affect the balance sheet via the reserve requirement.

[^17]:    ${ }^{25}$ It is possible to interpret the shock to the balance sheet as a shock to the reserve requirement of the banking sector.
    ${ }^{26}$ Given a fixed supply of durables (housing), patient durable consumption is always directly opposite to that of impatient households. As they are unconstrained they smooth intertemporally and 'take up the slack' in the housing market, so to speak.

[^18]:    ${ }^{27}$ This expression can be derived by combining the two household budget constraints with the housing supply condition and bank balance sheet (62).

[^19]:    ${ }^{28}$ The complete set of log-linerized equilibrium conditions is reported in Appendix A.

