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(Un)anticipated Technological Change in an Endogenous Growth Model

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(Un)anticipated Technological Change in an Endogenous Growth Model

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Abstract

This paper examines numerically the impact of a *negative* exogenous shock to marginal productivity (such as ecological government regulation that becomes effective at some point in time) in an endogenous finite-time growth model with sluggish reallocation of human capital. The policy can be anticipated or unanticipated by firms, and it can also be announced but not implemented. It turns out that these frictions have a very strong long-run effect on output, consumption and on the optimal allocation of capital and labor in particular. The qualitative properties relate to homogenous labor models with *positive* productivity shocks. The problem is thus to maximize a function of a continuous system, where the system is subject to frictions and stepwise changes; for such a problem the application of calculus of variations necessary conditions is problematic. A numerical optimization method, which has had much success on qualitatively similar problems in engineering, has been employed.

JEL-Classification: C61, E32

Keywords: two-sector endogenous growth model, unanticipated and anticipated technological change, frictions in reallocation of human capital, Runge-Kutta parallel shooting algorithm.

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1 Introduction

Our goal in this paper is to study the announcement and implementation effects of a *negative* exogenous change to consumption good productivity when the allocation of labor is subject to frictions. Physical capital however is completely mobile and may partly offset the effect of this friction. To this end we analyze numerically the optimal policy in a standard endogenous growth model with production and education sectors. The analysis focuses on a technology with a finite lifetime. The current physical production technology (and with it all specialized human capital) becomes obsolete at a certain point in time, for instance, due to a structural break.

The exogenous change to the physical capital sector can be anticipated or unanticipated by firms. Moreover, we allow for the possibility that it is announced but not implemented. One can think of a change in productivity caused by government regulation such as anti-pollution laws, or any other measure that lowers marginal productivity of firms.

The main feature of the model considered here is that frictions in the reallocation of labor between the two sectors rule out an instantaneous adjustment, for instance in response to an unanticipated (or non-enacted) change. This may lead to an imbalance between the levels of physical and human capital in both sectors. On the optimal path, allocation of physical capital is thus indirectly affected by the friction. We show that the sluggish reallocation of labor has severe long-run effects on output and labor- and capital-allocation. None of these qualitative properties (except for the interest rate in the pre-change time when the constraint is not binding) are observed for the balanced growth path of the corresponding infinite-horizon model without frictions.

Our findings are related to insights from homogenous labor models in which the impact of a *positive* production shock on employment is analyzed. For a brief summary see e.g. Trehan [10]. For instance, Phelps and Zoega [8] argue that the news of a productivity jump that will materialize in some point in the future leads to an expansion (and to more employment in particular) before the change actually occurs; the effect will dissipate once the change happens. We find that the news of a *negative* productivity shock (and its materialization) leads to the same pattern of employment in the sector in which technology changes.

The presence of frictions and exogenous changes (when they are anticipated) to the technology makes the application of calculus of variations necessary conditions problematic. The optimal path is instead found numerically, using a method that has had much success on qualitatively similar problems in engineering. In this method the continuous problem is discretized and converted into a nonlinear programming problem. The system governing equations are enforced through the use of nonlinear constraint equations that are implicit integration rules. The method is referred to as direct in that the system Lagrange multipliers (i.e. adjoint variables) are not required.

The next section introduces the model. Its numerical study is presented in Section 3.

2 Model

The model is a finite time-horizon version of the two-sector model of endogenous growth with different technologies for production and education, c.f. Barro and Sala-i-Martin [1, Sec. 5.2.1] and Rebelo [9]. Education is labor-augmenting in both sectors. There are additional constraints on labor mobility and the availability of information with respect to exogenous changes to the marginal productivity. The technology's lifetime is finite and the human skills associated with this technology eventually become worthless. Only the terminal stock of consumption good is assigned a value as consumption good.

Production functions are Cobb-Douglas in both sectors. The output of physical goods is given by

$$Y_t = C_t + \dot{K}_t + \delta_K \, K_t = A_t \, (\phi_t \, K_t)^{\alpha} \, (\psi_t \, H_t)^{1-\alpha} \tag{1}$$

and human capital growth is given by

$$\dot{H}_t + \delta_H \, H_t = B \, ([1 - \phi_t] \, K_t)^{\eta} \, ([1 - \psi_t] \, H_t)^{1-\eta} \tag{2}$$

The exogenous technology parameter $A_t > 0$ is time-dependent. Changes in A_t are Hicks neutral, i.e. marginal productivity of both inputs is affected to the same extent.

The control variables $0 \leq \phi_t \leq 1$ and $0 \leq \psi_t \leq 1$ are the fractions of physical and human capital respectively used in production of physical capital, the remainder being employed in education. The physical good can be used for consumption or investment. δ_K and δ_H denote the depreciation rate of physical and human capital respectively. We assume a constant population size, normalized to one.

In contrast to the standard assumption of frictionless reallocation of labor and capital between the two sectors, the model assumes that only physical capital is completely mobile while adjustment in human capital is subject to frictions. These frictions originate e.g. from the requirement of different skills in each sector. The friction in the reallocation of labor between the two sectors is modeled here in the simplest way possible:

$$-b_\psi \leq \frac{d\psi_t}{dt} \leq +b_\psi \tag{3}$$

Human capital is a perfectly mobile for $b_\psi = \infty$.

The central planner's maximization problem is given by

$$\max \int_0^T e^{-\rho t} \frac{C_t^{1-\sigma} - 1}{1-\sigma} \, dt + e^{-\rho T} \frac{1}{\rho} \frac{(\rho\, K_T)^{1-\sigma} - 1}{1-\sigma} \tag{4}$$

$\rho > 0$, $0 < \sigma \neq 1$ subject to (1), (2), (3), and the no-borrowing condition $K_t \geq 0$. The term on the far right is the present value of the remaining stock of consumption good K_T when production ceases.

A change in the technology parameter A_t (which determines the return on inputs in the production of the consumption good) can be anticipated or unanticipated by firms. A policy can also be announced but not enacted. Four cases are studied in this paper. They are delineated in terms of the optimization problem faced by the central planner in response to the revelation of information over time:

No Action [**N**] The optimization problem is solved subject to $A_t = 1$ for all $t \in [0, T]$.

Anticipated [**A**] The optimization problem is solved subject to $A_t = 1$ for all $t \in [0, T/2]$, and $A_t = 1/2$ for all $t \in (T/2, T]$.

Not Anticipated [**NA**] The optimization problem is solved subject to $A_t = 1$ for all $t \in [0, T]$, but at time $t = T/2$ it is revealed that $A_t = 1/2$ for all $t > T/2$. At time $t = T/2$ the optimization problem is newly solved for the remaining time-horizon subject to $A_t = 1/2$ for all $t > T/2$.

Not Enacted [**NE**] The optimization problem is solved subject to $A_t = 1$ for all $t \leq T/2$, and $A_t = 1/2$ for all $t > T/2$. At time $t = T/2$ it is revealed that A_t will remain equal to 1 for all $t > T/2$. The optimization problem is newly solved for the remaining time-horizon at time $t = T/2$ subject to $A_t = 1$ for all $t > T/2$.

3 Analysis

3.1 Numerical Method

There are essentially two approaches to solving optimal control problems. The indirect approach uses the calculus of variations to derive the necessary conditions of optimality, i.e., the Euler-Lagrange equations. This results in a two-point boundary-value-problem (TPBVP), which may then be solved numerically, although this becomes a very difficult task for significant problems. The direct approach is to discretize the original problem, transforming it into a parameter optimization problem, which is then solved using mathematical programming, Enright and Conway [4].

Numerous methods have been used to solve the TPBVP that results from the indirect approach to the optimal control problem. One approach to solving this problem is the initial value "shooting" method. The shooting method requires a guess of either the initial or terminal boundary conditions, from which the Euler-Lagrange system is then integrated forward or backward, iteratively adjusting the guess in an attempt to satisfy the boundary conditions at the other end. Common difficulties with shooting methods are the requirement of an initial guess for the adjoint variables of the TPBVP, the sensitivity of the adjoint equations to variations in the initial values of the adjoint variables, and possible occurrences of discontinuities in the optimal control, Enright and Conway [4, 5].

Another approach to solving the TPBVP avoids explicitly integrating the system differential equations. The system history is first divided into a large number of segments. The algebraic relationships approximate the differential equations locally; i.e. within each time segment. These algebraic relationships, along with the boundary conditions, form a system of nonlinear simultaneous equations. This "direct" approach ignores the calculus of variations first-order necessary conditions and approximates the original optimal control problem with a nonlinear programming (NLP) problem which often consists of a large number (many thousands) of variables and constraints. The most common implementation of this method is known as "direct collocation with nonlinear programming;" how the method is implemented varies, particularly with regard to how the states are approximated in time (the most common approach is to use a cubic polynomial to represent a given state in a given segment) and what implicit integration rule is used to satisfy the system differential equations ([2], [3], [4], [5], [6]).

Another direct approach, similar to collocation, is used to find the optimal path for the current problem. This transcription method is essentially an adaptation of the parallel-shooting method for boundary-value problems to the discretization of the equation of motion constraints in the direct approach to the optimal control problem (Enright and Conway [5]). An explicit four-stage Runge-Kutta integration rule is used to propagate the governing equations across the discretized segments of the trajectory. Requiring continuity at the nodes of the segment boundaries generates a set of discrete nonlinear algebraic constraints involving the states and controls on the boundaries of the segment. These nonlinear constraints replace the constraints from a collocation method.

In the Runge-Kutta parallel shooting algorithm the previously defined optimal control problem is first discretized into a sequence of stages. The partition $[t_0, t_1, ..., t_N]$ is introduced, with $t_0 = 0$, $t_N = t_f$, where $t_0 < t_1 < ... < t_N$, and let $h_i = t_i - t_{i-1}$ for $i = 1, 2, ..., N$. The h_i's may or may not be uniform. The mesh points t_i are referred to as *nodes*, whereas the intervals $[t_{i-1}, t_i]$ are referred to as *segments*. The state variables $x_i = x(t_i)$ are approximated by values at the nodes, for $i = 0, 1, ..., N$. Control variables are provided at the nodes t_i as well as the segment center points, $t_i + h/2$, by $u_i = u(t_i)$ for $i = 0, 1, ..., N$ and $v_i = u(t_{i-1} + h/2)$ for $i = 1, 2, ..., N$. From a given node, t_{i-1}, the equations of motion are integrated forward from initial condition x_{i-1} to the next node t_i using the controls u_{i-1}, v_i, and u_i by a step of a four-stage Runge-Kutta formula (Enright [3]):

$$y_i^1 = x_{i-1} + \frac{h}{2} f(x_{i-1}, u_{i-1}) \tag{5}$$

$$y_i^2 = x_{i-1} + \frac{h}{2} f(y_i^1, v_i) \tag{6}$$

$$y_i^3 = x_{i-1} + h\, f(y_i^2, v_i) \tag{7}$$

$$y_i^4 = x_{i-1} + \frac{h}{6} f(x_{i-1}, u_{i-1}) + 2f(y_i^1, v_i) + 2f(y_i^2, v_i) + f(y_i^3, u_i) \tag{8}$$

The state at the next node is estimated by y_i^4, thus, for continuity it is necessary that the "Runge-Kutta defects"

$$\Delta_i = y_i^4 - x_i = 0 \tag{9}$$

for $i = 1, 2, ..., N$. The Runge-Kutta procedure has order h^5 local truncation errors.

Perhaps the biggest advantage of this method is that since it is explicit, it can be incorporated into a parallel-shooting approach, as illustrated in Figure 1. The single step of the Runge-Kutta procedure previously described is replaced with multiple steps. This allows the use of larger intervals, resulting in smaller NLP problems. However, in each segment additional control variables must be introduced to accommodate the multiple integration steps.

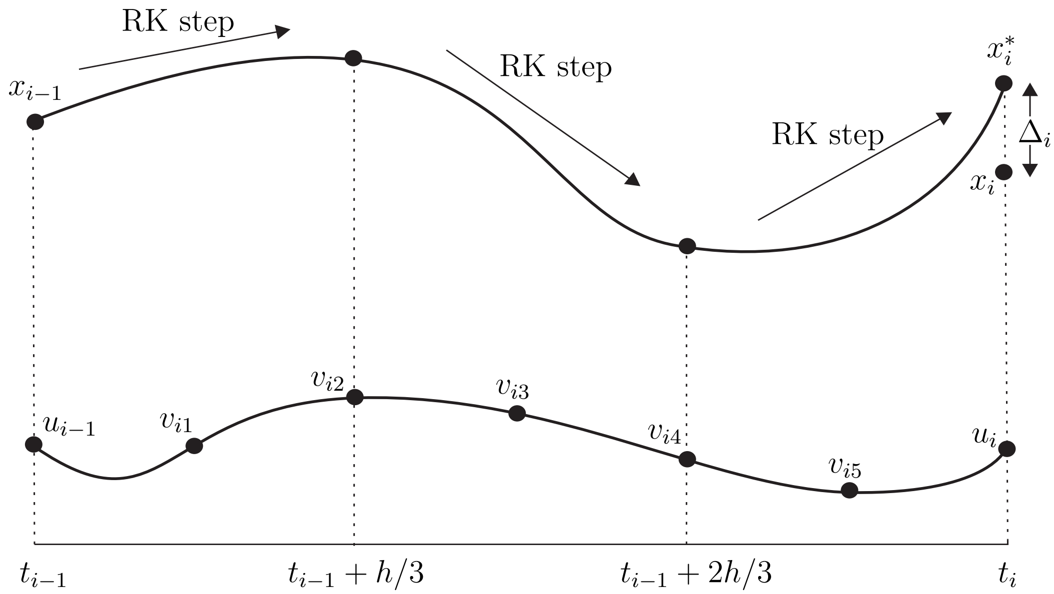


Figure 1: Illustrating the Runge-Kutta parallel-shooting discretization of the continuous optimal control problem. Only one of many segments is shown.

Let p be the number of integration steps per segment, $[t_{i-1}, t_i]$. In the usual manner, states and controls are provided at the nodes, $x_i = x(t_i)$ and $u_i = u(t_i)$. Now, the "center" controls $v_{ij} = u(t_{i-1} + jh/2p)$ for $j = 1, 2, ..., 2p - 1$ and for $i = 1, 2, ..., N$ must be provided, as shown in Figure 1 for $p = 3$. Then, using equations (5) through (8) the states are integrated from t_{i-1} forward one step to $t_{i-1} + h/p$ with controls u_{i-1}, v_{i1}, and v_{i2}. Using the resulting estimate of the state at $t_{i-1} + h/p$, the state integration is continued forward using the four-stage Runge-Kutta procedure and the controls yielding the estimate of the state at $t_{i-1} + 2h/p$. The process is repeated once more using controls resulting in an estimate of the state, x_i^*, at node t_i which then replaces y_i^4 in the defect formula, equation (9). Note that the $p-1$ intermediate estimates of the state vector do not appear explicitly in the NLP problem.

The parallel-shooting method allows for the use of larger intervals, and results in smaller NLP problems than collocation methods. Although additional control variables must be introduced in each segment to accommodate the multiple integration steps, the intermediate state variables (resulting from the forward propagation) are used to reinitialize values for the next step, and never appear explicitly in the NLP problem. Since most optimal control problems have many more state variables than control variables, the tradeoff of introducing more control variables and reducing the number of state variables is a favorable one.

The Runge-Kutta defects constitute a set of nonlinear "defect" equations, i.e. nonlinear equality constraints. These defect equations become the nonlinear constraints in the NLP problem. Collecting all the independent variables into a single vector P defined as:

$$P^T = (x_0, u_0, x_1, u_1, ..., x_N, u_N, t_f) \tag{10}$$

where, in this problem, the cost function is the final time t_f (the last parameter in the vector P^T), and similarly collecting all the nonlinear constraint equations into a vector C^T, the optimal control problem can then be restated as an NLP problem of the form:

$$\text{minimize } \varphi(P)$$

subject to:

$$b_L \leq \left\{ \begin{array}{c} P \\ AP \\ C(P) \end{array} \right\} \leq b_U \tag{11}$$

where AP contains all the linear relationships of the stated problem, and b_L and b_U are the lower and upper bounds on the variables and constraints, [6]. The vast majority of the nonlinear constraint equations comprising vector C^T are the defect equations (9) for which upper and lower bounds will both be zero. Another attractive feature of this method, in comparison to the Euler-Lagrange necessary conditions, is that known discontinuities in either the state or control variables are easily accommodated, for example through the linear equations AP in (11) or directly using the upper and lower bounds of the parameter vector P containing the state and control variables.

3.2 Simulation Results

We now report the optimal solution to the model for the four cases detailed in Section 2 using the numerical procedure described in the preceding section.

Parameter values are fixed throughout the numerical analysis as follows.

$$\rho = 0.02,\ \sigma = 3,\ \alpha = 0.4,\ \eta = 0.2,\ B = 0.136, \text{ and } \delta_K = \delta_H = 0.05. \tag{12}$$

In this model the education sector is relatively intensive in human capital ($\eta < \alpha$). We further set

$$b_\psi = 0.05,\ T = 50,\ K_0 = 1,\ H_0 = 1, \text{ and } \psi_0 = 0.5 \tag{13}$$

The friction in the reallocation of labor is significant, that is, the constraint is frequently effective as observed in the simulations. With the discount rate set to $\rho = 0.02$, one unit of time can be interpreted as a year; the lifetime of the technology thus being $T = 50$ years.

The results from the optimal paths are shown in the figures of Appendix A. We depict the optimal time paths of consumption, physical and human capital in production, net interest rate of physical capital in production, fraction of human capital in production as well as its change, physical capital–human capital ratio, and consumption–physical capital ratio.

The total utility, equation (4), denoted by $U[\cdot]$, is ranked as follows: $U[N] = -13.49 > U[NE] = -23.58 > U[A] = -62.72 > U[NA] = -94.39$. This result is perfectly in line with intuition: (a) no decrease in productivity is best, (b) if a change is anticipated it is better if it does not occur, (c) an unanticipated change is worst.

The optimal consumption path goes through two phases: a pre-change and a post-change regime. Since households prefer to smooth consumption over time, the consumption path does not change significantly when the change is anticipated, [A], or does not happen at all, [N]. In both cases the consumption path is increasing at roughly constant rate. Consumption growth slows down only in the last periods. An unanticipated change implies that the consumption path is equal to the [N] case before the change and that consumption falls considerably after marginal productivity is decreased. A non-enacted change has the effect that consumption is equal to [A] before $T/2$ but rises after $T/2$.

In cases [A] and [NE] a "build up" of physical capital takes place in the pre-change regime as a response to the expected change in productivity.

After the change occurs in case [A], most of the capital is used to smooth consumption. This "build up" is accompanied by an accumulation of human capital that peaks around $t = 20$ due to the friction in reallocation of labor. In case [NE] human capital is further reallocated to the physical capital sector as the no-change policy is revealed.

The optimal path of the change in the flow of human capital, $d\psi/dt$, depicted in Appendix A, allows several episodes to be distinguished. Recall that the friction in labor reallocation becomes binding when the rate of readjustment in either direction reaches $b_\psi = 5\%$.

In case [N] human capital is first allocated at maximum rate to physical production and then back to education. From period 11 to 33 the constraint is not binding. In the last few periods the same pattern is observed as in the first 10 periods. When the change of productivity occurs as a surprise, case [NA], human capital is first reallocated at maximum rate to education to make up for the decline in productivity and at period 30 is moved back to production. However, consumption declines dramatically.

In case [A], in which the change is anticipated, the time-path of $d\psi/dt$ is governed by the conflicting goals of increasing productivity to ease the future transition to a low productivity regime and the need to produce more consumption good to smooth consumption over time. This causes a reallocation of labor to the production sector before the change to build up a stock of consumption good. In period 20, human capital flows back to the education sector to improve future production of the consumption good.

In case [NE], where productivity does not change, human capital is reallocated to the consumption good sector immediately after the information is released. The surge in consumption is a response to too little consumption and too much production of human capital before period 25 when the productivity cut was expected to occur.

The first 10 periods are an adjustment phase in all four cases. The interest rate falls to about 8%, the interest rate that would prevail along the balanced growth path of the corresponding unconstrained infinite-horizon economy. As the build-up of consumption good takes place, the interest rate falls to about 5% with a trough at the expected productivity cut date. After the policy is enacted the interest rate increases. This is observed even in the [NE] case. After period 32, the pattern of the interest rate is caused by the finite time-horizon of the technology. When productivity does not change, cases [N] and [NE], it takes about 7 periods for the interest rate to converge. With a change, [A] and [NA], convergence takes 14 periods. The changes in the interest rate cannot be explained by the friction in labor allocation alone. For instance the flow of human capital away from the education sector in periods 16-20, which happens at maximum rate, is not followed by any interest rate change.

The most remarkable effect of the friction is its impact on the human capital in production of physical capital. Due to the capital allocation before the date at which the change is expected, case [A] and [NE], the amount of human capital in production is almost identical over the entire remaining time after period 25. This is so despite the fact that productivity does not change in the [NE] case.

A similar pattern can be observed when comparing cases [N] and [NA] (though at a lower level of human capital in the physical production sector). In case [NA] the optimal allocation in the pre-change time, $t \leq 25$, has a long-run impact that does not permit achievement of the optimal level of human capital one would have when the change is anticipated.

3.3 Interpretation

How does the qualitative behavior of the optimal solution relate to that of the balanced growth path of the corresponding unconstrained infinite-horizon economy? In that economy, one finds the following long-run characteristics. For $A = 1.0$, $\psi^\star = 0.167$, $(K/H)^\star = 3.11$, $(C/K)^\star = 0.0433$, and the net interest rate is 8.0%. For $A = 0.5$, $\psi^\star = 0.318$, $(K/H)^\star = 1.57$, and $(C/K)^\star = 0.0886$, and the interest rate is 5.9%.

It is apparent that only the interest rate gives some guidance for the finite-horizon model with frictions. In particular, the ratios of physical to human capital and consumption to physical capital bear no relation.

All other characteristics do not carry over. The share of labor in the consumption good sector is almost always higher than in the infinite-horizon model. Only around the period in which productivity is cut by half does this fraction fall below the optimal share in the infinite-horizon model. What appears to be counterintuitive at the first glance is actually an optimal reaction to the fall in productivity: The high initial share of human capital in production contributes to the build up of physical capital to smooth consumption. While this "saved" good is used for consumption, human capital is moved to education to increase the total amount of human capital, which partly offsets the drop in productivity for the remaining period of time.

In the case of an anticipated *negative* productivity shock, case [A], the time-pattern of employment in the consumption goods sector (which is directly hit by the shock) exhibits a dynamics that closely resembles the impact of a *positive* production shock on employment in Phelps and Zoega [8], see also Phelps [7]. Phelps and Zoega argue that the news of a productivity jump that will materialize at some point in the future leads to an expansion (and to more employment in particular) before the change actually occurs; the effect will dissipate once the change happens. Our numerical results show that the news of a *negative* productivity shock (and its materialization) leads to the same pattern of employment in the sector in which technology changes. Of course, it is important to emphasize that our model only allows for structural unemployment, i.e. the gap between supply and demand of labor in the consumption good or education sector caused by the friction in the reallocation of human capital.

While the positive content of Phelps and Zoega's results are not challenged by our findings, they shed doubts on the validity of any argument that attempts to exploit an inverse causality.

References

[1] R. J. Barro and X. Sala-i-Martin. *Economic Growth*. McGraw-Hill, New York, 1995.

[2] E. Dickmanns and K. Well, editors. *Approximate Solution of Optimal Control Problems Using Third-Order Hermite Polynomial Functions*, volume Proceedings for the 6th Technical Conference on Optimization Techniques of *IFIP-TC7*, New York, 1975. Springer-Verlag.

[3] P. Enright. *Optimal Finite-Thrust Spacecraft Trajectories Using Direct Transcription and Nonlinear Programming*. PhD thesis, University of Illinois, 1991.

[4] P. Enright and B. Conway. Optimal finite-thrust spacecraft trajectories using collocation and nonlinear programming. *Journal of Guidance, Control, and Dynamics*, 14:981–985, 1991.

[5] P. Enright and B. Conway. Discrete approximations to optimal trajectories using direct transcription and nonlinear programming. *Journal of Guidance, Control, and Dynamics*, 15:994–1002, 1992.

[6] C. Hargraves and S. Paris. Direct trajectory optimization using nonlinear programming and collocation. *Journal of Guidance, Control, and Dynamics*, 1:338–342, 1987.

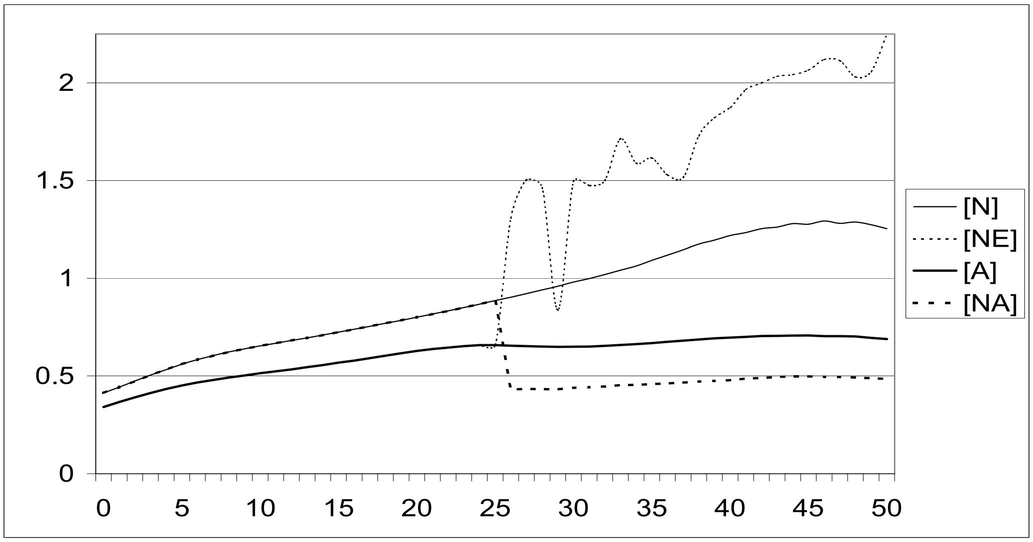
[7] E. S. Phelps. *Structural Slumps: The Modern Equilibrium Theory of Unemployment, Interest, and Assets.* Harvard University Press, Cambridge, MA, 1994.

[8] E. S. Phelps and G. Zoega. Structural booms. *Economic Policy*, 16:83–126, 2001.

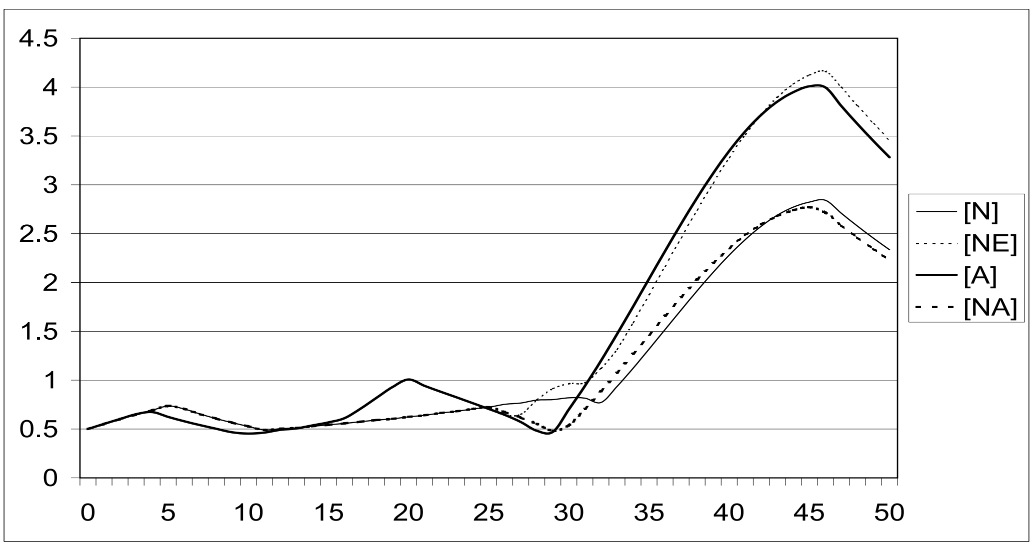
[9] S. Rebelo. Long-run policy analysis and long-run growth. *Journal of Political Economy*, 99:500–521, 1991.

[10] B. Trehan. Productivity shocks and the unemployment rate. In *Economic Review 2003.* Federal Reserve Bank of San Francisco, Economic Research Department, 2003.

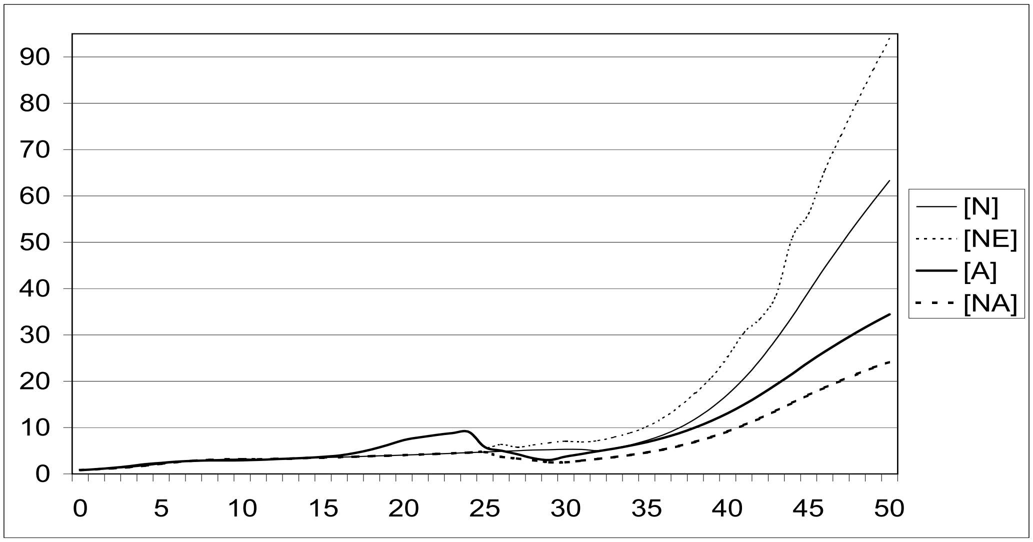
A Optimal solution and controls



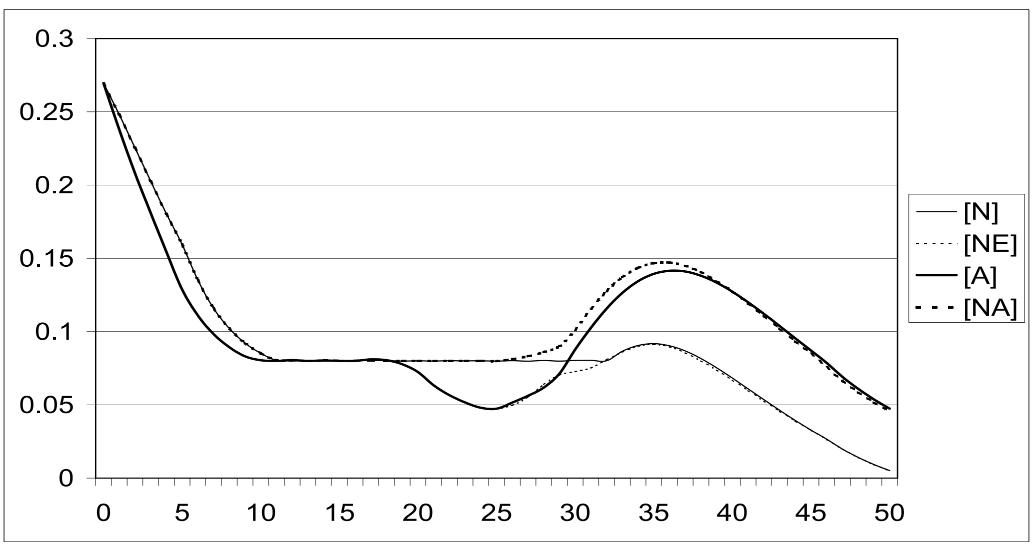
Consumption C_t



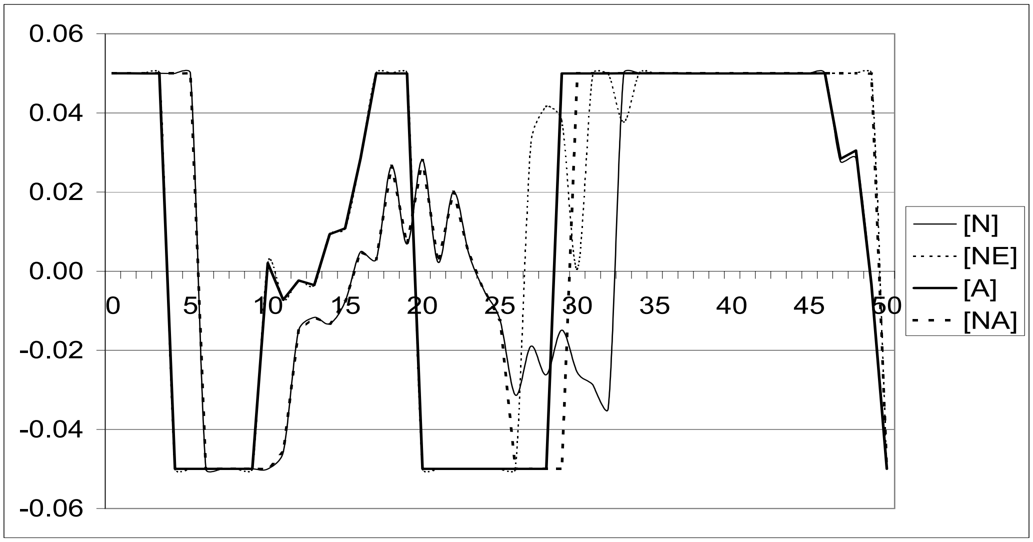
Human capital in production $\psi_t \, H_t$



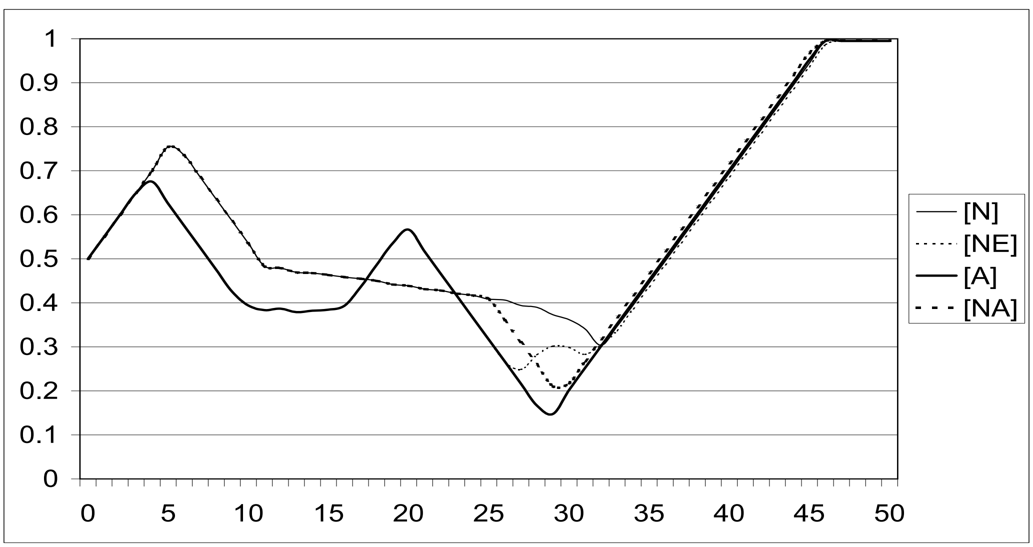
Physical capital in production $\phi_t \, K_t$



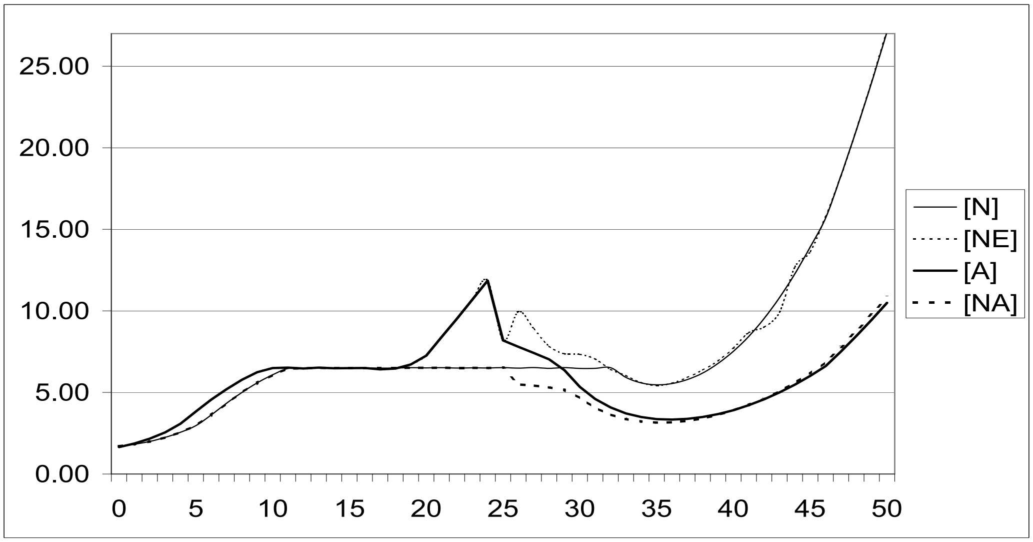
Net interest rate of physical capital in production



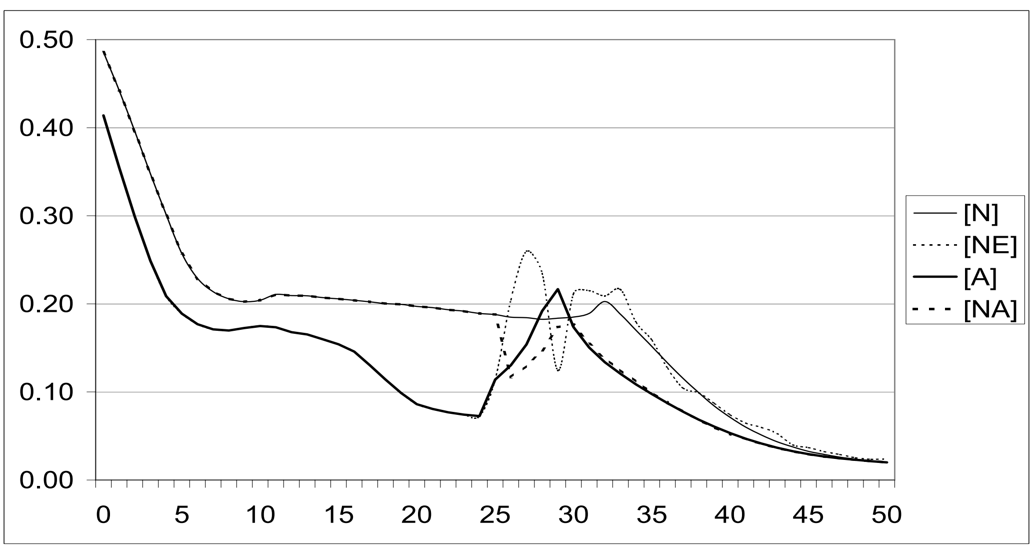
Change of human capital share in production $d\psi_t/dt$



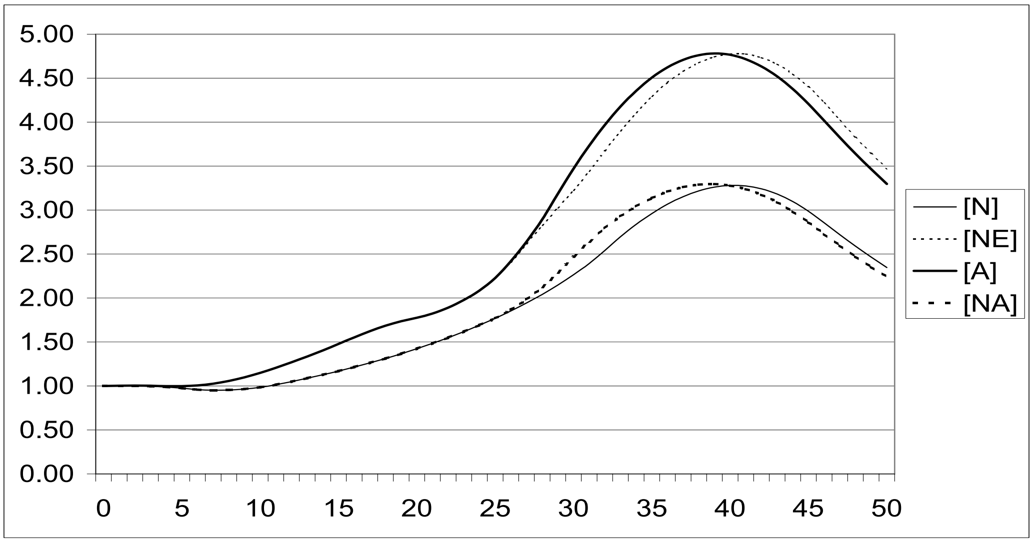
Human capital share in production ψ_t



Capital-labor ratio K_t/H_t



Consumption-capital ratio C_t/K_t



Human capital H_t