LIMITS TO GROWTH?

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INTRODUCTION

Classical writers in economics, like Thomas Malthus, saw *land* as an *essential* and *fixed* factor of production. He also argued the size of population rises, if income goes up.

As consequence of these two premisses he argued growth must come to a halt; population would over the long-run be kept at a level of subsistence

Basic logic of the Malthusian "trap".

Suppose $Y_t = X^\beta L_t^{1-\beta}$, where X is the fixed supply. Observe

$$y_t \equiv \frac{Y_t}{L_t} = \left(\frac{X}{L_t}\right)^\beta \tag{1}$$

for which reason $L \uparrow \Rightarrow y \downarrow$.

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Moreover, Malthus argued that population size (fertility) was *endogenous.* If $y \uparrow \Rightarrow n \uparrow$. To capture this is a simple way, suppose

$$n_t = s_n \cdot y_t, \; s_n < 1. \tag{2}$$

Remainig part $(1 - s_n)$ is consumed. To equations (1) and (2) we add

$$L_{t+1} = n_t L_t, \; n > 0. \tag{3}$$

To analyse the model (equation 1-3) substitute for n and y into (3)

$$L_{t+1} = s_n y_t L_t = s_n Y_t = s_n X^{\beta} L_t^{1-\beta} \equiv G\left(L_t\right),$$

which is the law of motion for population, aggregate output, and output per capita. [Insert phase diagram]

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The steady state level of population

$$L_{t+1} = L_t = L^* = s_n^{1/\beta} X.$$

Thus more land would sustain greater numbers of individuals. But, they would not be more "wealthy":

$$y^* = \left(\frac{X}{s_n^{1/\beta} X} \right)^{\beta} = s_n^{-1}.$$

Hence, "accumulation" of land would not *permanently* be able to improve living standards. The reason is (1) diminishing returns to labor input (conseqence of land entering into the production function), and (2) n increases with income

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It was not long after Malthus completed his thesis that (2) ($n_t = s_n \cdot y_t$) started breaking down. Today, in rich places anyway, rising income does *not* lead to population growth. Rather it is other way around.

BUT, if land indeed is important (i.e, present) in the production function it *may* modify our results from the basic Solow model. Note that.

$$Y_t = AK^{\alpha} L_t^{\beta} X^{\kappa},\ \alpha + \beta + \kappa \equiv 1$$

thus

$$y_t \equiv \frac{Y_t}{L_t} = A \left(\frac{K_t}{L_t} \right)^{\alpha} \left(\frac{X}{L_t} \right)^{\kappa}$$

As L rises the last term declines, pushing in the direction of *lower* standards of living.

ISSUE 1: Under what circumstances can growth be sustained?

ISSUE 1: LIMITS TO GROWTH?

Consider a standard Solow model with technological change, augmented to include land. Slightly augemented replication argument ...:

$$y_t = A^{\beta}\left(\frac{K_t}{L_t}\right)^{\alpha}\left(\frac{X}{L_t}\right)^{\kappa} \Leftrightarrow y_t = A_t^{\frac{\beta}{1-\alpha}}\left(\frac{K_t}{Y_t}\right)^{\frac{\alpha}{1-\alpha}}\left(\frac{X}{L_t}\right)^{\frac{\kappa}{1-\alpha}}$$

Otherwise the model is standard. That is, we have

$$k_{t+1} = \frac{sy_t + (1-\delta)\,k_t}{(1+n)} \Rightarrow \frac{k_{t+1}}{k_t} = \frac{sy_t/k_t + (1-\delta)}{(1+n)}$$

To solve the model in a simple way, define

$$z_t \equiv \frac{K_t}{Y_t} \Rightarrow \frac{z_{t+1}}{z_t} = \frac{\frac{k_{t+1}}{k_t}}{\frac{y_{t+1}}{y_t}} = \frac{\frac{s/z_t+(1-\delta)}{(1+n)}}{\left(\frac{A_{t+1}}{A_t}\right)^{\frac{\beta}{1-\alpha}}\left(\frac{z_{t+1}}{z_t}\right)^{\frac{\alpha}{1-\alpha}}\left(\frac{L_t}{L_{t+1}}\right)^{\frac{\kappa}{1-\alpha}}}$$

LIMITS TO GROWTH? DYNAMICAL ANALYSIS

After some rearrangements

$$\frac{z_{t+1}}{z_t} = \left(\frac{s/z_t + (1-\delta)}{(1+g)^{\frac{\beta}{1-\alpha}} (1+n)^{\frac{1-\alpha-\kappa}{1-\alpha}}} \right)^{1-\alpha} \equiv \Phi\left(z_t\right)$$

Which is the law of motion for capital intensity in the model.

Observe that $1-\alpha-\kappa=\beta$ by constant returns to capital, labor and land.

$[$*Insert phasediagram*$]$

LIMITS TO GROWTH? DYNAMICAL ANALYSIS

Hence, *contingent on the condition* $[(1+n)(1+g)]^{\frac{\beta}{\beta+\kappa}} > (1-\delta)$ a steady state exists, and it is stable.

$$z^* = \frac{s}{[(1+n)(1+g)]^{\frac{\beta}{\beta+\kappa}} - (1-\delta)} > 0.$$

What about *growth* in GDP per capita?

$$\left(\frac{y_{t+1}}{y_t}\right)^* = \left(\frac{A_{t+1}}{A_t}\right)^{\frac{\beta}{1-\alpha}} \left(\frac{z^*}{z^*}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{L_t}{L_{t+1}}\right)^{\frac{\kappa}{1-\alpha}} = \frac{(1+g)^{\frac{\beta}{1-\alpha}}}{(1+n)^{\frac{\kappa}{1-\alpha}}}$$

Note: $\left(\frac{y_{t+1}}{y_t}\right)^* > 1$ requires

$$(1+g)^{\frac{\beta}{1-\alpha}} > (1+n)^{\frac{\kappa}{1-\alpha}}$$

That is, *only* if the rate of technological change is *sufficiently rapid* is growth sustainable!(note: $1-\alpha = \beta + \kappa$ by CRTS)

LIMITS TO GROWTH?

With land entering the production function we have reached the following conclusion: **Even with technological change growth in GDP per capita is not nessesarily sustainable, if the population expands expotentially**

One may view this as a "Neo-Malthusian" result: There are limits to growth. At some level perhaps uninteresting: it is obvious that population growth itself cannot go on indefinitely.

Still, worth noting that "technological change" is not nessesarily enough to ensure growth, with limited resources and a rising population. Either y or n will have to "give in".

Comparative economic growth?

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"New" predictions (compared to standard Solow):

(1) The *steady state* growth rate of GDP per capita is negatively *affected* by population growth. Intuition: familiar capital dillution mechanism "on steroids"

$$\left(\frac{y_{t+1}}{y_t}\right)^* = \left[\frac{(1+g)^\beta}{(1+n)^\kappa}\right]^{\frac{1}{\beta+\kappa}}$$

(2) More land *increases* GDP per capita in the long-run

$$y^* = A_t^{\frac{\beta}{1-\alpha}} (z^*)^{\frac{\alpha}{1-\alpha}} \left(\frac{X}{L_t}\right)^{\frac{\kappa}{1-\alpha}} = A_0^{\frac{\beta}{1-\alpha}} \left(\frac{X}{L_0}\right)^{\frac{\kappa}{\beta+\kappa}} (z^*)^{\frac{\alpha}{1-\alpha}} \left[\frac{(1+g)^\beta}{(1+n)^\kappa}\right]^{\frac{1}{\beta+\kappa}}$$

with $z^* = \left\{ s/\left[(1+n)(1+g)\right]^{\frac{\beta}{\beta+\kappa}} - (1-\delta) \right\}$. Both predictions are *consistent* with cross-country data (cf textbook).

NATURAL RESOURCES...A CURSE?

We just saw that land is “good” for long-run living standards

But there are other forms of natural resources which relate directly to production: Oil and mineral extraction in particular.

Both are (for practical purposes) as *nonrenewable* natural resources. Hence, as the resource is used the stock of it declines.

Oil, for instance, is used in production for its value as an *energy input.*

What are the implications of admitting exhaustible natural ressources into the model?

NATURAL RESOURCES...A CURSE?

The simplest version of the model has the following production function

$$Y_t = \min\left(K_t^{\alpha} L_t^{1-\alpha}, A_t E_t\right)$$

where E is energy. (textbook asumes Cobb-Douglas. (1) more complicated, (2) substitution of E for K with A given ... ultimately not meaningful from a thermodynamical perspective). Hence, we require

$$A_t e_t \equiv \frac{E_t}{L_t} = k_t^{\alpha} \equiv \left(\frac{K_t}{L_t}\right)^{\alpha} \text{ for all } t.$$

Now, suppose

$$E_t = s_E R_t, \; s_E < 1.$$

where s_E is the *extraction rate.* Finally, suppose $R_{t+1} = (1 - s_E) R_t$, where R is the stock of the resource (i.e., oil).

NATURAL RESOURCES...A CURSE?

Studying the dynamics

$$\frac{k_{t+1}}{k_t} = \frac{s_K \frac{y_t}{k_t} + (1-\delta)}{1+n}$$

$$R_{t+1} = (1 - s_E)\, R_t$$

define

$$z_t \equiv \frac{K_t}{Y_t} \Rightarrow \frac{z_{t+1}}{z_t} = \frac{\frac{k_{t+1}}{k_t}}{\frac{y_{t+1}}{y_t}} = \frac{\frac{s_K/z_t + (1-\delta)}{(1+n)}}{\frac{A_{t+1}}{A_t}\frac{R_{t+1}/L_{t+1}}{R_t/L_t}}$$

where it has been used that $k_t^{\alpha} = A_t e_t = A_t s_E R_t / L_t$ at all points in time.

$$\frac{z_{t+1}}{z_t} = \frac{s_K/z_t + (1-\delta)}{(1+g)(1-s_E)} \equiv \Phi(z_t)$$

NATURAL RESOURCES...A CURSE?

After some rearrangements, the law of motion reads

$$\frac{z_{t+1}}{z_t} = \frac{s_K/z_t + (1-\delta)}{(1+g)(1-s_E)} \equiv \Phi(z_t)$$

$\Phi(0) = \infty$ and $\Phi(\infty) = (1-\delta)/[(1+g)(1-s_E)] < 1$ if $\delta > s_E$.

With this condition, the phasediagram looks very much like the one for the model with land in the production function

Steady state capital-output ratio:

$$z^* = \frac{s_K}{(1+g)(1-s_E) - (1-\delta)} > 0.$$

NATURAL RESOURCES...A CURSE?

What about long-run growth? At any point in time

$$k_t^{\alpha} = A_t e_t$$

We have shown, $k/y = z^*$. So rewriting the above

$$(z^* y_t^*)^{\alpha} = A_t e_t \Rightarrow y_t^* = z^{*-1} (A_t e_t)^{1/\alpha}$$

$$\left(\frac{y_{t+1}}{y_t}\right)^* = \left[(1+g)\frac{s_E R_{t+1}/L_{t+1}}{s_E R_t/L_t}\right]^{1/\alpha} = \left(\frac{(1+g)(1-s_E)}{1+n}\right)^{1/\alpha}$$

Once again we have that technological change is not *sufficient* for economic growth to last. Without (energy saving) technological change, growth must come to a halt. **Note**: This is true *even if population growth is absent* $(n = 0)$.

NATURAL RESOURCES...A CURSE?

$$\left(\frac{y_{t+1}}{y_t}\right)^* = \left(\frac{(1+g)(1-s_E)}{1+n}\right)^{1/\alpha}$$

The intuition is as with land, only the rate at which the resource becomes dilluted is increasing in s_E. This is why the resource extraction rate *lowers* long-run growth.

The more “general” specification where substitution is possible softens this conclusion slightly; changes in s_E and n does not map into changes in growth on a 1:1 basis (see textbook).

The way in which we have introduced technology into the model makes clear that energy saving technological change are needed, if we do not rely on “substitution”. Is this process possible forever?