THE OPEN ECONOMY SOLOW MODEL: CAPITAL MOBILITY

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OUTLINE

Part I: Assessing international capital mobility empirically.

– The Feldstein-Horioka Puzzle (S&W-J, Ch. 4.1.)

– The Lucas Paradox (Lucas, 1990)

– Resolving the Lucas paradox? (Caselli and Feyrer, 2005)

Part II: Open economy Solow model - Capital mobility

– The basic model

– Empirical issues

PART I

- THE FELDSTEIN-HORIOKA PUZZLE

BACKGROUND

In a closed economy setting we know the following must hold

$$Y = C + I \Leftrightarrow I = S.$$

Hence, total investments (or the investment share of GDP, I/Y) must vary 1:1 with total savings (or the savings rate S/Y).

Thus, a simple regression

$$\left(\frac{I}{Y}\right)_i = \alpha + \beta \left(\frac{S}{Y}\right)_i + \epsilon_i$$

should return $\beta_{OLS} = 1$ (and $\alpha_{OLS} = 0$ in the absence of national accounts mistakes).

BACKGROUND

In an open economy, however, things should work differently. In particular, the following must be true:

$$S - I = \Delta F$$

If savings exceed domestic investments, the country is building up net foreign assets. That is, on net the country is *investing abroad*.

As a result

$$S - \Delta F = I. \tag{*}$$

which says domestic investments equal savings minus what we (on net) invest abroad.

BACKGROUND

Reconsider the regression model from before

$$\left(\frac{I}{Y}\right)_i = \alpha + \beta \left(\frac{S}{Y}\right)_i + \epsilon_i$$

In light of equation (*)

$$\epsilon_i \equiv -\Delta F/Y - \alpha.$$

Estimating the above by OLS we get

$$\beta_{OLS} = \beta - \frac{COV\left(\frac{S}{Y}, \Delta F/Y\right)}{var\left(\frac{S}{Y}\right)}$$

as $COV\left(\left(\frac{S}{Y}\right)_i, \epsilon_i\right) = -COV\left(\frac{S}{Y}, \Delta F/Y\right)$. Thus β_{OLS} is expected to be (much) smaller than 1.

THE PUZZLE

The startling finding was, however, this

Table 2

The Relation between Domestic Saving Ratios and Domestic Investment Ratios

	Gross saving and investment			Net saving and investment		
Sample period	Constant	S/Y	R^2	Constant	S/Y	R^2
1960–74	0·035 (0·018)	0·887 (0·074)	0·91	0·017 (0·014)	0·938 (0·091)	0·87
1960–64	0·029 (0·015)	0·909 (0·060)	0·94	0·017 (0·011)	0·936 (0·072)	0·91
1965–69	0·039 (0·025)	0·872 (0·101)	0·83	0·022 (0·020)	0·908 (0·133)	0·75
1970–74	0·039 (0·024)	0·871 (0·092)	0·85	0·018 (0·018)	0·932 (0·107)	0·83

Parameter estimates refer to equation (1) in the text. All equations are based on observations for 16 countries, with the variables averaged for the sample period indicated. Standard errors are shown in parentheses.

Figure 1: Source: Feldstein and Horioka, 1980.

Suggests *limited* capital mobility. In striking contrast to e.g. evidence on very similar interest rates on similar assets dispite being located in different countries (thus "a puzzle").

THE PUZZLE

This finding remains something of a puzzle, and is robut to more recent periods (albeit the size of the coefficient shrinks)

Table 2 FELDSTEIN–HORIOKA REGRESSIONS, $I/Y = \alpha + \beta NS/Y + \epsilon$, 1990–1997[a]

	No. of obs.	α	β	R^2
All countries[b]	56	0.15 (0.02)	0.41 (0.08)	0.33
Countries with GNP/cap. > 1000	48	0.13 (0.02)	0.48 (0.09)	0.39
Countries with GNP/cap. > 2000	41	0.07 (0.02)	0.70 (0.09)	0.62
OECD countries[c]	24	0.08 (0.02)	0.60 (0.09)	0.68

[a]OLS regressions. Standard errors in parentheses.
[b]Israel is excluded from all regressions in this table. If Israel is added to the samples of size (56, 48, 41), the estimates of β are (0.39, 0.45, 0.63).
[c]If one adds Korea to the OECD sample, the estimate for β rises to 0.76. Korea is included in the larger samples.

Figure 2: Source: Obstfeld and Rogoff (2000).

To date: No complete resolution.[1]

[1]Perhaps in part because no-one seem to know how small β is supposed to be in order to be consistent with capital mobility.

THE LUCAS PARADOX

SET-UP

Another contribution striking a similar cord is Lucas (1990).

Lucas' focus is on rich and poor countries; not just "within the group of rich"

Basic point of departure is a one good economy, featuring competitive market.

Firms use a Cobb-Douglas production function. They maximize profits

$$\max_{K,L} \underbrace{K^{\alpha}L^{1-\alpha}}_{=Y} - wL - \underbrace{(r+\delta)}_{\text{user cost of capital}} K.$$

Focusing on FOC wrt K

$$r+\delta = \alpha K^{\alpha-1}L^{1-\alpha} = \alpha k^{\alpha-1}.$$

Suppose this condition holds in *any country.*

THE PARADOX

In particular, suppose we consider India and the US. Then (ignoring δ) $r_{INDIA} = \alpha k_{INDIA}^{\alpha-1}$ and $r_{US} = \alpha k_{US}^{\alpha-1}$.Implying

$$\frac{r_{INDIA}}{r_{US}} = \left(\frac{k_{INDIA}}{k_{US}}\right)^{\alpha-1}$$

Capital is hard to measure. But note: $y = k^{\alpha}$ (cf production function). SO

$$\frac{r_{INDIA}}{r_{US}} = \left(\frac{y_{INDIA}}{y_{US}}\right)^{\frac{\alpha-1}{\alpha}}$$

Since $y_{US}/y_{IND} \approx 15$ and $\alpha = .4$, this implies

$$\frac{r_{INDIA}}{r_{US}} = \left(\frac{1}{15}\right)^{\frac{.4-1}{.4}} \approx 58$$

Why doesn't capital flow to poor countries???

A SOLUTION TO THE PARADOX?

Maybe we are getting it wrong because we are missing something. Consider the modified production function

$$Y = XK^{\alpha}L^{1-\alpha}$$

where X could be human capital (Lucas' favorit), or something else (technology). Observe that we now get

$$y = k^{\alpha}X \Leftrightarrow k = (y/X)^{1/\alpha}$$

Hence the first order condition from profit maximization is (still ignoring δ)

$$r = MP_K = \alpha k^{\alpha-1}X = \alpha y^{\frac{\alpha-1}{\alpha}}X^{\frac{1}{\alpha}}$$

A SOLUTION TO THE PARADOX?

Now, if

$$\frac{r_{IND}}{r_{US}} \approx 1$$

then we need

$$\frac{r_{IND}}{r_{US}} = \left(\frac{y_{IND}}{y_{US}}\right)^{\frac{\alpha-1}{\alpha}} \left(\frac{X_{IND}}{X_{US}}\right)^{\frac{1}{\alpha}} \approx 1$$

or

$$\frac{X_{us}}{X_{ind}} = \left(\frac{y_{us}}{y_{ind}}\right)^{1-\alpha} = 15^{0.6} \approx 5.$$

Lucas manages to motivate "X" almost *entirely* by human capital; $h = X$.

WHY IT MAY NOT BE A RESOLUTION

1. Evidence for external effects of human capital is not strong.

2. Lucas' calculation is, under reasonable assumptions, not entirely internally consistent.

To see this, suppose we rewrite the production function slightly

$$y = k^{\alpha} X \Leftrightarrow y = \left(\frac{K}{Y}\right)^{\frac{\alpha}{1-\alpha}} X^{\frac{1}{1-\alpha}}$$

If $\frac{X_{us}}{X_{ind}} = 5$, then $\left(\frac{X_{us}}{X_{ind}}\right)^{\frac{1}{1-\alpha}} = 5^{\frac{1}{1-.4}} \approx 15$! If X is human capital, this implies that we can account for the *entire* observed difference in labor productivity by this variable *alone* (growth "multiplier effect"). Not plausible.

Of course, things like "A" (TFP) could be included in X. But that violates the calibration. Another look is warranted.

A RESOLUTION TO THE LUCAS PARADOX?

SET-UP

We begin with a set of basic assumptions

A1 $Y = F(K, XL)$, X= "efficiency" (human capital, productivity).
CRTS: $F_K \cdot K + F_L L = F = Y$

A2 Competitive markets, and multi-good economy ($p_Y \neq p_I$)

Implication 1 $R \cdot K + w \cdot L = p_Y Y$, where p_Y is the GDP deflator, and R is the rental rate of capital (sometimes called: "usercost of capital", Hall and Jorgenson, 1963)

$$R = p_I \cdot (r + \delta),$$

p_I = price of investment good.

Implication 2 $p_Y \cdot F_K = R$ and $p_Y \cdot F_L = w$.

SET-UP

Under these assumptions, we can now obtain an estimate for F_K. Let $\alpha_K \equiv RK/p_Y Y$ (capital's share). Then

$$F_K = \alpha_K \frac{Y}{K} = \frac{p_I}{p_Y}\left(r + \delta\right).$$

Using data on capital's share in national accounts, we can *calculate* F_K for a number of countries.

The question is whether marginal products are equalized ...

RESULT 1: MARGINAL PRODUCTS ARE NOT EQUALIZED

They are not ...

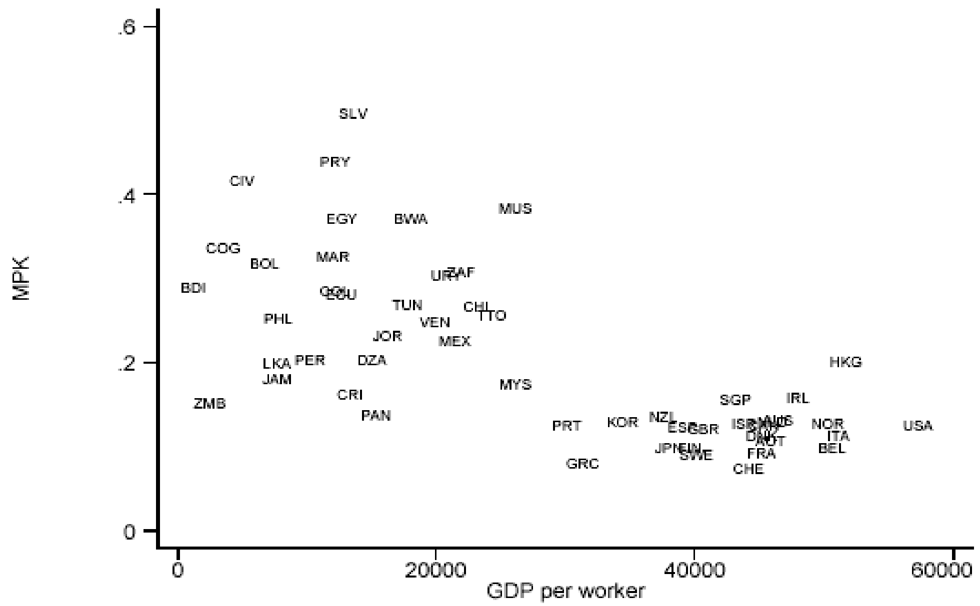

Figure 1: Implied MPKs

Figure 3:

REFLECTING ON THE RESULT

Investors probably do not care about the marginal product *per se.* They care about the *return* to their investment, r

Perfect capital mobility would require the equalization of the r's

Fairly easy to calculate the implied r, given the above "view of the world":

$$\alpha_K \frac{Y}{K} = MP_K = \frac{p_I}{p_Y}(r+\delta) \Leftrightarrow r+\delta = \frac{p_Y}{p_I} MP_K,$$

it is assumed that δ is about the same in all countries ...

RESULT 2: RENTAL RATES ARE INDEED NOT THAT DIFFERENT

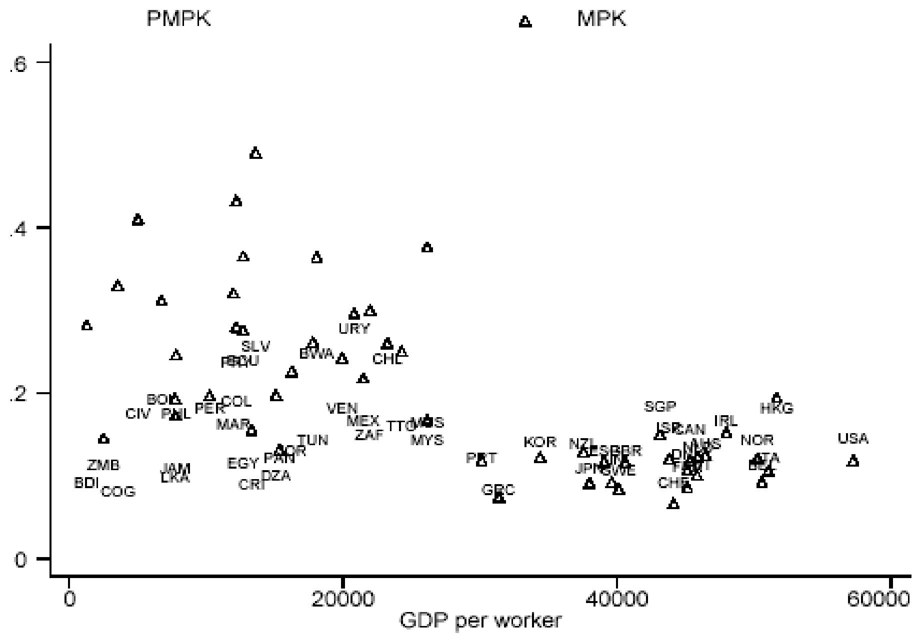

Figure 4: Implied PMPKs

Figure 4:

SUMMING UP

Lucas: There are no differences in real rates. Human capital is solely responsible. Shortcoming: Overestimates productivity differences. Ignores relative price differences

C&F: marginal products are not equalized. But rental rates are not that different.

– Rich places have a lot of capital (even in a fully integrated world) because: "X" is large (not only human capital though), and because the relative price of investment is low (p_I/p_y).

– Hence if international capital markets do allocate capital reasonably efficiently, then we better think about how it affects our understanding of the growth process

Part II: OPEN ECONOMY SOLOW MODEL
- CAPITAL MOBILITY

THE BASIC MODEL: SET UP

We are considering an open economy, where *capital* is fully mobile. Labor, however, is not. All markets are competitive.

Two new basic relationsships:

1. Savings $\neq$ Domestic total investment
2. Production and Income are not longer identical

The national accounts identity is

$$Y = C + I + NX \Longleftrightarrow Y + rF = C + I + NX + rF$$

where NX represents net exports, and F is holdnings of foreign capital; rF=income inflow from foreign capital holdnings.

THE BASIC MODEL: SET UP

Gross National Income (GNI) is therefore $Y+rF$, cf 2. Y is production, or, Gross Domestic Product.

Note that if $NX>0$ the economy must be building up assets abroad (exports > imports). Hence, in general (r is assumed constant):

$$NX_t + rF = F_{t+1} - F_t$$

Finally, by definition

$$S_t = Y_t + rF_t - C_t$$

Combining

$$S_t = I_t + F_{t+1} - F_t$$

Hence, savings can be used to accumulate domestic capital (I), or, foreign assets ($F_{t+1} - F_t$), cf. 1.

THE BASIC MODEL: SET UP

As "usual" we have that

$$K_{t+1} = I_t + K_t$$

(i.e., here we assume $\delta = 0$, for simplicity)

But, observe that we now have

$$K_{t+1} = S_t - (F_{t+1} - F_t) + K_t \Leftrightarrow K_{t+1} + F_{t+1} = S_t + K_t + F_t$$

If we define total wealth (domestically owned local (K) and Foreign (F) capital)

$$V_t = K_t + F_t$$

Leaving us with

$$V_{t+1} = S_t + V_t. \tag{1}$$

THE BASIC MODEL: SET UP

The fundamental assumption about *savings* behavior is the same: People save a constant fraction of total *income*. In the open economy

$$S_t = s \cdot (Y_t + rF_t), \; 0 < s < 1. \tag{2}$$

We will also maintain our basic assumption about *production*

$$Y_t = F(K_t, L_t; A) = AK_t^{\alpha} L_t^{1-a}$$

From (A) competitive market, and (B) constant returns to scale follows

$$Y_t = w_t L_t + rK_t \tag{3}$$

Since

$$w_t = \frac{\partial F[\cdot]}{\partial L_t}, r = \frac{\partial F[\cdot]}{\partial K_t}$$

THE BASIC MODEL: SET UP

The fact that capital is fully mobile has an important implication. Denote by r^w the world real rate of interest. Then *at all points in time*

$$r^w = r = \frac{\partial F[\cdot]}{\partial K_t}$$

Substituting for $\frac{\partial F[\cdot]}{\partial K_t}$ we find

$$r^w = \alpha A k^{\alpha - 1} \Leftrightarrow \bar{k} = \left(\frac{\alpha A}{r^w} \right)^{\frac{1}{1-\alpha}}.$$

Hence the capital-labor ratio is constant, *absent* changes in A. Suppose A rises ...

Note also, that this implies a constant wage rate

$$w_t = \bar{w} = \frac{\partial F[\cdot]}{\partial L_t} = (1 - \alpha) A \bar{k}^{\alpha} \tag{4}$$

SOLVING THE BASIC MODEL

Starting with eq (1):

$$
\begin{aligned}
V_{t+1} &= S_t + V_t \overset{\text{eq (2)}}{=} s \cdot (Y_t + r^w F_t) + V_t \\
&\overset{\text{eq (3)}}{=} s \cdot (w L_t + r^w K_t + r^w F_t) + V_t \\
&\overset{\text{def of } V \text{ and eq (4)}}{=} s \cdot \bar{w} L_t + (1 + s r^w)\, V_t
\end{aligned}
$$

As a final step: $L_{t+1} = (1+n)\, L_t$, $n > -1$. Let $v_t \equiv V_t / L_t$. Then

$$
v_{t+1} = \frac{s\bar{w}}{1+n} + \frac{1 + s r^w}{1+n} v_t \equiv \Phi\,(v_t)
$$

is the fundamental law of motion for wealth in the open economy setting.

[Insert Phasediagram]

SOLVING THE BASIC MODEL

Definition A steady state of the model is a $v_{t+1} = v_t = v^*$ such that
$v^* = \Phi(v)$

For existence of a steady state, we require the following *stability condition*

$$\Phi'(v) = \frac{1 + sr^w}{1+n} < 1 \Leftrightarrow sr^w < n$$

Plausible? r is the world market interest rate. The "world" is a *closed* economy. In the steady state of a closed economy the real rate is given by (when $\delta = g = 0$)

$$r^* = MPK^* = \alpha \left(\frac{Y}{K}\right)^* = \alpha \frac{n}{s}.$$

For the world, define $r^w = \alpha n^w / s^w$. Inserted

$$s\left(\frac{\alpha n^w}{s^w}\right) < n \Leftrightarrow \alpha \frac{s}{n} < \frac{s^w}{n^w}.$$

SOME OBSERVATIONS ABOUT THE STEADY STATE

Unique (non-trivial) steady state, where

$$v^* = \frac{s\bar{w}}{n - sr^w},$$
$$\bar{w} = (1-\alpha)\,\bar{y} = (1-\alpha)\,A^{1/(1-\alpha)}\,(\alpha/r^w)^{\frac{\alpha}{1-\alpha}}$$

Globally stable. For any $v_0 > 0$ $\lim_{t\to\infty} v_t \to v^*$

v^* determined by local structural charactaristics: s, A, n.

Specifically: $\partial v^*/\partial s > 0$, $\partial v^*/\partial n < 0$ and $\partial v^*/\partial A > 0$.

Qualitatively, just as in a closed economy Solow model (with k exchanged for v).

SOME OBSERVATIONS ABOUT THE STEADY STATE

Net foreign position?

Rercall that $v^* = \bar{k} + f^*$. If we use that $r^w = \alpha A k^{\alpha-1}$ and that $\bar{w} = (1-\alpha)\, A\bar{k}^{\alpha}$, we obtain that

$$\frac{r^w}{\bar{w}} = \frac{\alpha}{(1-\alpha)} \frac{\bar{k}^{\alpha-1}}{\bar{k}^{\alpha}} \Leftrightarrow \bar{k} = \frac{\alpha}{1-\alpha} \frac{\bar{w}}{r^w}.$$

As a result

$$f^* = v^* - \bar{k} = \frac{s\bar{w}}{n - sr} - \frac{\alpha}{1-\alpha} \frac{\bar{w}}{r^w} = \frac{1}{1-\alpha} \frac{s}{n} \frac{1}{r^w} \left[\frac{r^w - \alpha n/s}{1 - sr^w/n} \right] \bar{w}.$$

Recall, the steady state autarky real rate of return $r^* = \alpha n/s$. Hence, if $r^w > r^* \Rightarrow$ creditor, and debitor otherwise $(r^w < r^*)$.

Of course, a "low" r^* implies high savings, and vice versa.

EMPIRICS: LONG-RUN GROWTH

Observe that GNI per capita

$$y_t^n = \bar{y} + r f_t = \bar{w} + r^w v_t$$

We can therefore convert the law of motion for v, into one for y^n

$$\frac{1}{r}\left(y_{t+1}^n - \bar{w}\right) = \frac{s\bar{w}}{1+n} + \frac{1+sr}{1+n}\left(\frac{1}{r}\left(y_t^n - \bar{w}\right)\right)$$

$$\Downarrow$$

$$y_{t+1}^n = \frac{n}{1+n}\bar{w} + \frac{1+sr}{1+n} y_t^n \Rightarrow (y^n)^* = \frac{n}{n-sr}\bar{w}.$$

As in the closed economy Solow model: Growth in income per capita will come to a halt (provided $\frac{1+sr}{1+n} < 1$).

You will still see growth *in transition* however. Note, GDP per capita, y (as well as w, r and k), is *constant at all points in time.* Changes require human capital accumulation, or, technological change.

EMPIRICS: CONVERGENCE PROCESS

Unique steady state $\Rightarrow$ Model predicts *conditional convergence.*

Since n, r^w and s are constants, we can solve for the entire path for y^n

$$y_{t+1}^n = \frac{n}{1+n}\bar{w} + \frac{1+sr^w}{1+n}y_t^n \Leftrightarrow y_t^n = \left(\frac{1+sr^w}{1+n}\right)^t \left(y_0^n - (y^n)^*\right) + (y^n)^*$$

with $(y^n)^* \equiv \frac{n\bar{w}}{n-sr^w}$.

Speed of convergence?

$$\frac{y_t^n - (y^n)^*}{y_0^n - (y^n)^*} = \frac{1}{2} = \left(\frac{1+sr^w}{1+n}\right)^{t_{1/2}} \Rightarrow t_{1/2} = \frac{-\log 2}{\log\left(\frac{1+sr^w}{1+n}\right)}.$$

.

EMPIRICS: CONVERGENCE PROCESS

In the *closed economy* (if $\delta = g = 0$)

$$t_{1/2}^{c} = \frac{-\ln(2)}{\ln\left(\frac{1+\alpha n}{1+n}\right)}$$

Hence by comparison

$$t_{1/2}^{c} < t_{1/2}^{open} \Leftrightarrow \frac{-\ln(2)}{\ln\left(\frac{1+\alpha n}{1+n}\right)} < \frac{-\log 2}{\log\left(\frac{1+sr^{w}}{1+n}\right)} \Leftrightarrow r^{w} > \alpha\frac{n}{s} = r^{*}$$

Hence slower if a *creditor* ($r^{w} > r^{*}$), and vice versa for a debitor.

Intuition...

EMPIRICS: INCOME DIFFERENCES

If we compare two countries who only differ in terms of savings (1 and 2, respectively)

$$\frac{y_1^n}{y_2^n} = \frac{\frac{n}{n-s_1 r}\bar{w}}{\frac{n}{n-s_2 r}\bar{w}} = \frac{n - s_2 r^w}{n - s_1 r^w}$$

Using reasonable parameter values ($r^w = 0.03$ and population growth in the world of about 2 percent), maximum variation in savings rates translate into:

$$\frac{n - s_2 r^w}{n - s_1 r^w} = \frac{0.02 - 0.1 \cdot 0.03}{0.02 - 0.4 \cdot 0.03} \approx 2,$$

which is about the same income difference that we could generate in the closed economy Solow model, with similar (1:4) variation in s.

SUMMING UP

The open economy model does not radically change our priors *viz* standard model.

Countries that save more, have slower population growth and higher levels of technological sophistication are still predicted to the more prosperious.

No growth in the long-run (absent...). Lengthy transitions can be motivated

The model does about as well as Solow model in accounting for per capita income differences

But we can ask new questions: Does liberalising capital mobility increase income per capita? How will a world wide credit-crunch (higher r^w) impact living standards?

ADVANTAGES OF FREE CAPITAL MOBILITY?

Compare levels of national income per capita in the two settings (open vs closed). Closed: attain a steady state associated with r^*.

RESULT: We find national income *always* rises, unless $r^w = r^*$. Formal proof p. 113-14

Suppose $r^w > r^*$. In stead of being forced to invest at home, at the rate r^*, the country can now *invest abrod*, and reap the gains $r^w > r$. This will increase national income. Impact on wages?

Suppose $r^w < r^*$. In the absence of capital mobility the economy would have ended up in a low steady state (note: r^* is high). Opening up to capital flows therefore leads to *capital imports* which increases GDP per capita and therefore GNI. Impact on wages?

CHANGES IN THE REAL RATE OF INTEREST

Permanent increase in r^w.

When $r^w > r \Rightarrow k \downarrow \Rightarrow y \downarrow \rightarrow y^n \downarrow$

In addition: $r^w \uparrow \Rightarrow r^w f \uparrow$ $(if\ \ f > 0) \Rightarrow y^n \uparrow$

Creditor: Stands to win in terms of income; due to the second mechanism.

Debitor: Stands to loose in terms of income.

In either case, capital flows out, and depresses wages

An example of how international capital market can lead to fluctuations in income.