Long Run Impact of Increased Wage Pressure*

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Abstract

An unanticipated permanent increase in wage pressure is analysed in a dynamic general equilibrium model combining standard theory of capital accumulation and monopolistic wage setting. The long run (steady state) implications are identical percentage reduction in employment, consumption, and capital stock whereas wages and the real interest rate are unchanged. The reduction in employment on impact is larger than the steady state reduction whereas wages rise and the real interest rate declines on impact.

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1 Introduction

Changes in wage pressure are normally analysed in static general equilibrium frameworks that exclude capital accumulation (e.g., Layard, Nickell & Jackman 1991 ch. 1 and 8 or Booth 1995 ch. 8). The results show that increases in wage pressure either (a) reduce employment and leave real wages unchanged if production functions are linear in labour input or (b) reduce employment to a *smaller* extent and increase real wages if the technology exhibits diminishing returns to scale in labour input. One may argue that diminishing returns to labour is a good assumption in the short run where capital is fixed whereas constant returns to scale is reasonable in the long run where all inputs can vary. Hence, it may be natural to think of result (a) as the long run outcome and result (b) as the short run outcome of an unanticipated permanent increase in wage pressure.[1] However, it is not clear whether such interpretation is warranted: What happens after the short run reduction in employment that reduces the returns on capital? Does a production function that is linear in labour and exclude capital yield the same long run consequences as one with constant returns to both labour and capital? Wont the long run effects have an impact in the short run if agents anticipate the long run consequences? To address these questions and to improve the predictions concerning changes in wage pressure it is necessary to include capital accumulation in the analysis. Therefore, this paper embeds a standard static analysis of monopolistic wage setting into a Ramsey model.

The results confirm the above assertion concerning wages but not concerning employment: higher wage pressure increases wages on impact and afterwards wages converge back to the same steady state level but the impact reduction on employment is *larger* than the steady state reduction. Thus, employment overshoots the new permanent level. To understand this, it is necessary to look at the macroeconomic implications.

There are many ways of modelling the interaction between unions and investment in capital equipment even in partial static models. The pioneering

[1] Layard *et.al.* (1991) does in fact make such interpretation on p. 107.

paper on the topic (Grout 1984) shows that unions decrease firm investments because of the 'hold-up' problem; i.e., the incentive to make an irreversible investment is low in a firm with organized workers because the firm knows that the union increases the wage claim after the investment is made.[2] I abstract from this effect by assuming that capital goods are perfectly mobile between industry sectors and that households can convert capital goods into consumption goods at no costs. Hence, the incorporation of capital does not add any new 'frictions' to the standard static general equilibrium analysis. Despite this, unions still reduce the capital stock due to a 'hold-up' problem at the aggregate level: an unanticipated permanent increase in wage pressure increases wage claims and reduces employment; this reduces the lifetime income of households who reduce savings (and consumption) and thus capital accumulation.

The percentage fall in consumption and capital stock from the initial steady state to the new one equals the percentage fall in employment (which approximately equals the percentage point increase in unemployment) whereas the real interest rate is unchanged. The economy would jump immediately to this new steady state if households consumed the difference between the capital stock at the initial steady state and the new steady state. However, this does not occur because households prefer to smooth consumption. Instead, consumption falls on impact and both consumption and capital decrease over time towards their new steady state levels. The development of the capital stock implies that the real interest rate is increasing over time which is the reason for the overshooting of employment: It is, ceteris paribus, better to work at times where the interest rate is high which implies that unions adjust wages over time such that employment is increasing over time.

[2] The under-investment result in Grout (1984) is unambiguous because firm and workers bargain over both employment and wages after the investment is made. Hoel (1990) shows that there may be over-investment if the firms has the 'right-to-manage' employment and there is only bargaining over the wage. Anderson & Devereux (1988) obtains similar results in a non-cooperative setting. Devereux & Lockwood (1991) shows that Grout's negative effect on capital may be reversed in an OLG-model through a positive impact on household savings. However, in a similar setting de la Croix & Licandro (1995) shows that union power in general is less favorable to physical capital in the presence of irreversibilities.

This paper is not the first one to introduce monopolistic wage setting in a Ramsey model. In fact, the framework is almost a continous-time version of Bénassy (1996). The main difference is that Bénassy (1996) treats capital as an intermediate good by assuming that capital depreciates fully in one period. This is done to get analytical solutions outside steady state but it also implies that there is no transitional dynamics following an unanticipated permanent increase in wage pressure: the economy jumps instantly to a new steady state where employment is lower and the wage level is unchanged. Thus, when analysing changes in wage pressure full depreciation is not only an unrealistic assumption it also yields qualitatively different results.

The next section describes the model. Section 3 solves the model for aggregate variables and describes the dynamics. The main results, established in Section 4, are finally discussed in Section 5.

2 The Model

The economy consists of m sectors each producing an intermediate good and one sector using the intermediate goods to produce a final good. The final good is chosen as numeraire. Households buy the final good which may be used either for consumption or savings in new capital equipment. Each intermediate good is produced using capital and sector specific labour. All markets are perfectly competitive except the labour markets where each type of labour is controlled by one household/union. The household uses this power to set a wage per unit of labour that is above the opportunity costs of employment. Employment enters directly into the utility function due to increasing disutility from work and into the budget constraint due to increased wealth from working. All agents have perfect foresight.

2.1 The Representative Household

Each household i consists of $\bar{L}_i$ members. The household maximizes discounted lifetime utility defined as (time subscripts are omitted to ease notation)

$$U_s = \int_{t=s}^{\infty} \left(\ln C_i - \beta L_i^{\gamma}\right) e^{\rho(s-t)} dt \quad , \quad \gamma > 1, \tag{1}$$

where C_i is household consumption, L_i is the number of employed household members which depends on the wage claim of the household, ρ is the rate of time preference, and γ is a parameter that determines the change in marginal disutility from increased work. Because of analytical tractability the analysis is confined to this utility function which is a special case of a more general class of utility functions that have reasonable properties (cf. Barro & Sala-i-Martin 1995 ch. 9).[3]

The flow budget constraint is

$$\dot{A}_i = W_i L_i + R A_i - C_i + \kappa W \left(\bar{L}_i - L_i\right) + \tau, \tag{2}$$

where A_i is holdings of capital by the household, R is the rate of return on capital, W_i is the wage claim of the household, κW is unemployment benefits which are linked to the overall wage level, and τ is a lump-sum tax that finances unemployment benefits. All households have the same initial holding of capital. In what follows, it is assumed that the upper limit on household employment, $\bar{L}_i$, is non binding and that each household has negligible influence on the overall wage level. Maximization of (1) subject to (2), a labour demand relationship $L_i(W_i)$, a non-negative consumption condition, a non-negative employment condition, and a no-Ponzi game condition yields the usual Keynes-Ramsey rule for consumption

$$\frac{\dot{C}_i}{C_i} = R - \rho, \tag{3}$$

and the wage equation

$$W_i = \frac{C_i \beta \gamma \left[L_i(W_i)\right]^{\gamma-1} + \kappa W}{1 - 1/\eta_i}, \tag{4}$$

where $\eta_i \equiv -L_i'(W_i) W_i / L_i(W_i)$ is the (numerical) wage elasticity of labour demand. The last equation states that the wage is set as a mark-up on the

[3] E.g., employment does not converge towards zero or infinity but is constant along the balanced growth path.

(marginal) opportunity costs of employment equal to the forgone utility of leisure measured in money terms and the forgone unemployment benefits. Equation (4) may be interpreted as standard wage-curve changing over time because of capital accumulation. To derive η_i, it is necessary to look at the behaviour of firms.

2.2 The Final Good Sector

Final goods, Y, are produced using m intermediate goods according to the constant elasticity production function

$$Y = m^{\frac{1}{1-\varepsilon}} \left(\sum_{i=1}^{m} X_i^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}},$$

where ε is the elasticity of substitution between inputs. Cost minimization implies that the demand for intermediate good i equals

$$X_i = \left(\frac{P_i}{P}\right)^{-\varepsilon} \frac{Y}{m} = \left(\frac{P_i}{P}\right)^{-\varepsilon} \frac{C+\dot{A}}{m}, \quad C \equiv \sum_{i=1}^{m} C_i, \; A \equiv \sum_{i=1}^{m} A_i, \tag{5}$$

where $C + \dot{A}$ equals the aggregate demand for final goods and where

$$P = \left(\frac{1}{m} \sum_{i=1}^{m} P_i^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}} = 1, \tag{6}$$

is the marginal (and average) costs of producing Y. The last equality in (6) follows from the assumption of perfectly competitive output markets.

2.3 The Intermediate Good Sectors

Each intermediate good sector i is characterized by many identical firms each optimizing an intertemporal profit function. The production of good i is given by Cobb-Douglas technology

$$X_i = F(K_i, ZL_i) = \alpha K_i^{1-\nu} (ZL_i)^{\nu}, \tag{7}$$

where K_i and L_i are input of capital and labour, respectively. Z represents the current state of knowledge which grows at the exogenous rate g. There is no adjustment costs associated with changing inputs, and so firms use inputs according to the marginal conditions

$$F_{L_i}(K_i, L_i) = \nu\alpha K_i^{1-\nu} Z^{\nu} L_i^{\nu-1} = \frac{W_i}{P_i}, \tag{8}$$

$$F_{K_i}(K_i, L_i) = (1-\nu)\,\alpha K_i^{-\nu} Z^{\nu} L_i^{\nu} = \frac{R}{P_i}, \tag{9}$$

where the final good price, P_i, and the nominal wage, W_i, are sector specific while the rental rate on capital goods, R, is identical in all sectors. The labour demand in sector i is obtained from (5), (7), (8), and (9)

$$L_i = W_i^{-(\nu\varepsilon-\nu+1)} R^{(1-\nu)(1-\varepsilon)} Z^{\nu(\varepsilon-1)} \left[\frac{C+\dot{A}}{m}\right] \nu^{1+\nu(\varepsilon-1)} \alpha^{\varepsilon-1} (1-\nu)^{(1-\nu)(\varepsilon-1)}, \tag{10}$$

which states that labour demand is decreasing in the wage and increasing in aggregate demand for final goods (consumption and investment) treated as exogenous by each household and firm, both of which are assumed too small to influence the overall economy. Since the capital stock in sector i is endogenous, the labour demand also depends on the rental price R; an increase in R reduces the capital stock which reduce the marginal product of labour. The assumption of perfect capital mobility across sectors influences the wage responsiveness of labour demand; the households must take into consideration that a wage rise reduces employment, which reduces the marginal product of capital and thereby the amount of capital, which decreases employment further. Equation (10) yields

$$\eta_i = \nu\varepsilon - \nu + 1 \equiv \eta, \tag{11}$$

which relates the (numerical) wage elasticities of labour demand to the degree of substitution between intermediate goods in the production of final goods. The elasticity of substitution between intermediate goods, ε, is also an indirect measure of the degree of competition in the wage setting. Equation (4), (10), and (11) determine employment and wage in each sector as function of the aggregate variables C, R, and X.

3 General Equilibrium and the Dynamic System

Symmetric households and firms implies that sector related variables (with subscript i) are identical across sectors, e.g., $P_i = P \ (= 1)$. We are interested in the development of aggregate employment, consumption, and capital stock equal to $L = mL_i, C = mC_i$, and $K = mK_i$, respectively. In order to express the dynamics in a two dimensional system, it is useful to define $k \equiv K/\left(ZL\right) = K_i/\left(ZL_i\right)$ and $c \equiv C/\left(ZL\right) = C_i/\left(ZL_i\right)$. Aggregate employment is derived as a function of k and c using (4), (8), and (11):

$$L = m\left(\frac{\nu\alpha k^{1-\nu}\left(1 - 1/\eta - \kappa\right)}{\beta\gamma c/m}\right)^{\frac{1}{\gamma}}. \tag{12}$$

Equations (3), (5), (7), (9), and (12) yield (see Appendix)

$$\hat{k} \equiv \frac{\dot{k}}{k} = \alpha k^{-\nu} - \frac{\left(\gamma - 1\right)c/k + \rho + \gamma g}{\gamma - \nu}, \tag{13}$$

$$\hat{c} \equiv \frac{\dot{c}}{c} = \left(1 - \nu\right)\alpha k^{-\nu} + \frac{\left(1 - \nu\right)c/k - \left(\gamma - \nu + 1\right)\rho - \gamma g}{\gamma - \nu}. \tag{14}$$

The path of the economy is characterized by (13), (14), and a conventional transversality condition. Figure 1 illustrates the phase-diagram corresponding to (13) and (14). The economy follows the saddle path towards the unique non-trivial steady state (k^*, c^*); point **C** in Figure 1. At the balanced growth path, the growth rates of aggregate consumption, $\hat{C}$, and capital, $\hat{K}$, equal the growth rate of knowledge, g, whereas aggregate employment is constant (cf. (12)).

< Figure 1 >

Both (13) and (14) are independent of the two unions related parameters ε and κ. Thus, the dynamics of k and c are identical for two economies having identical initial value of k but different values of ε and κ. Note though, that k depends both on the state variables K and Z and the control variable L

which implies that k jumps if L do so (in response to some unanticipated change). Hence, to use the phase diagram, it is important to recognize that the state variable $\tilde{K} \equiv K/Z$ is below its steady state value, $\tilde{K}^*$, whenever k is below its steady state value, k^*, and vice versa.[4]

4 The Impact of Monopolistic Wage Setting

This section studies the implications of monopolistic wage setting in two ways. First, we compare the wage-setting economy and the corresponding competitive economy with identical capital intensities. In the competitive economy households take wages as given and unemployment benefits do not exist.[5] Second, we analyse the consequences of changes in wage pressure caused by changes in the (indirect) degree of competition among wage setters, ε, or in the replacement ratio, κ.

Let u denote the steady state unemployment rate defined as the relative difference in employment between the wage-setting economy and the corresponding competitive economy in steady state.[6] Then (11) and (12) yield

$$u = 1 - \left(1 - \frac{1}{\nu\varepsilon - \nu + 1} - \kappa\right)^{\frac{1}{\gamma}}. \tag{15}$$

As usual the unemployment rate is an increasing function of the replacement ratio, κ, and a decreasing function of the degree of competition among wage setters, ε.

Proposition 1 *Compare a wage-setting economy and a competitive economy that have the same capital intensity k and the same values of all parameters except ε and κ.* ***(i)*** *The employment, L, consumption, C, and capital stock,*

[4] This is an implication of Lemma 2 in the proof of Proposition 2.

[5] The aggregate variables in the competitive economy are independent of ε. However, the competitive solution may be found from the previous equations by letting ε converge towards infinity and setting κ equal to zero.

[6] This is in accordance with the standard definition of unemployment equal to the difference between employment and the number of persons willing to work at the going wage rate as equation (8) shows that the steady state wage level is identical in the two economies.

K, are $u\%$ smaller in the wage-setting economy. ***(ii)*** *The two economies have identical growth rates in employment, $\hat{L}$, consumption, $\hat{C}$, and capital stock, $\hat{K}$.* ***(iii)*** *The real wage, W, and real interest rate, R, are identical.*

Proof. (ii) Both (13) and (14) are independent of ε and κ. $\hat{L}$ is also independent of ε and κ for a given value of k according to (12). This implies that the two economies have identical values of $\hat{K}$ and $\hat{C}$. (i) The two economies have identical steady state values of k and c. It then follows from (15) that K and L are $u\%$ smaller in the steady state of the wage-setting economy. It follows from the identical growth rates that this also holds outside steady state. (iii) follows directly from (8) and (9). ■

In general, it is not likely that a wage-setting economy has the same capital-labour ratio as a corresponding competitive economy since parameter changes that influence the rate of unemployment (i.e., $\varepsilon, \kappa, \nu, \gamma$) have different effects on the capital-labour ratio in the two types of economies. Proposition 1 does though describe the long run consequences of monopolistic wage setting as both economies converge to the same long run (steady state) capital intensity. This is clear from the phase diagram in Figure 1 which applies both to the wage-setting economy and the competitive economy. Outside the steady state, the main difference between the wage-setting economy and the competitive economy is not the position of the saddle path itself but the position *on* the path. This position change in the wage-setting economy when ε or κ changes.

Proposition 2 *Consider an unanticipated permanent reduction in ε or rise in κ.* ***(i)*** *On impact employment, L, and consumption, C, fall whereas the capital stock, K, is unchanged.* ***(ii)*** *Present and future growth rates of consumption, $\hat{C}$, and capital, $\hat{K}$, decrease whereas growth rates of employment, $\hat{L}$, increase.* ***(iii)*** *Present and future levels of real wages, W, increase whereas the levels of the real interest rate, R, decrease.*[7]

[7] Note, that the proposition holds for any initial value (not only the steady state value) of the state variable $\tilde{K} \equiv K/Z$ and for all finite changes (not only marginal changes) in ε and κ.

Proof. See Appendix. ■

Both a reduction in the degree of competition among wage setters, ε, and a rise in the replacement ratio, κ, increase the wage claims of the households and reduce employment.[8] This causes an immediate rise in the capital-labour ratio (as capital cannot jump). Two examples of that is illustrated in Figure 1: if the economy is in the steady state initially then it may move from **C** to **D** whereas the movement from **A** to **B** illustrates a case where the economy is below its steady initially. It is clear from Figure 1 that c always increases. This is due to the reduction in employment as aggregate consumption C declines as explained below. The increase in the capital-labour ratio raises the overall wage level and reduces the return on capital.

The wage rise of one household increases the aggregate real income of the household members but has negative externalities on the real income of other households as in static models.[9] In total the negative externalities dominate and therefore all households expect lower real income in the future. The households respond by reducing both present and future consumption and savings. The reduction in savings reduces the capital stock over time which makes real wages a decreasing function of time and real interest rates an increasing function of time. This implies that the opportunity costs of consumption are an increasing function of time making the reduction of consumption an increasing function of time, i.e., the growth rates of consumption decrease. The development of real interest rates increases the marginal gain of working making the wage claims of the households a decreasing function

[8] In fact, this is not as straightforward as in static models since (4) depends on consumption. The fall in consumption increases the marginal utility of employment which counteract the direct effect. The direct effect does, however, dominate as shown in the proof of Proposition 2.

[9] There are 3 negative externalities: (i) The costs of producing final goods increase which reduces real wages by increasing the consumer price index. (ii) The costs of producing capital goods increase which reduces capital demand and labour demand. (iii) The reduction in aggregate employment reduces the household's return on their capital stock. However, there are also 2 positive externalities: (i) The reduction in the employment of one sector increases the price of that particular intermediate good. Final goods producers substitute towards the other intermediate goods which increase labour demand in other sectors. (ii) Capital tends to flow to the other sectors which also increases the demand for labour in these sectors.

of time and employment an increasing function of time, and so employment overshoots the new steady state level.

The economy converges towards a new steady state characterized by larger unemployment, lower capital stock, and lower consumption but unchanged wages and real interest rate (cf. Proposition 1). The differences between the long run (steady state) consequences in Proposition 1 and the short- and medium run consequences in Proposition 2 are caused by households desire for consumption smoothing. After the parameter change it is in principle possible for the households to instantly consume the difference between the two steady state capital stocks after which the economy would be in the new steady state with a permanently lower consumption level. However, such a consumption path conflicts with the households desire for a smooth consumption path. Instead, households reduce consumption at all points in time after the parameter change and run down the capital stock smoothly.[10]

5 Concluding Remarks

In static general equilibrium analysis of changes in wage pressure, it is common to illustrate the macroeconomic outcome in a wage-price-setting diagram similar to Figure 2 (e.g., Layard, Nickell & Jackman 1991 ch. 1 and 8 or Booth 1995 ch. 8).[11] The equilibrium is determined by the intersection of an upward-sloping wage-setting curve (WS) and a downward sloping or horizontal price-setting curve (PS) in a wage-employment diagram. The WS curve describes the aggregate wage-setting behaviour of unions and is a mark-up on the labour supply whereas the PS curve describes the aggregate labour demand of firms. The slope of the PS curve is determined by the production function which only contains labour input; the horizontal PS curve arises when the production function is linear in labour whereas the downward

[10] Mathematically, the jump down to the new permanent level after the increase on impact would violate (3).

[11] This is only possible because of symmetry assumptions on both sides of the market.

sloping PC curve occurs when there is decreasing returns to labour.[12] An increase in wage pressure, say because of less elastic labour demands or higher replacement ratios, moves the wage setting curve upwards (WS_1 to WS_2). Depending on the production function the consequences are then either (a) an increase in wages and a relatively moderate reduction in employment (**A** to **B**) or (b) unchanged wages and a large reduction in employment (**A** to **C**).[13]

< Figure 2 >

This paper assumes instead constant returns to both labour and capital and add standard theory of capital accumulation to the analysis. In this setting, an unanticipated permanent increase in wage pressure yields an increase in wages and a relatively *large* reduction in employment on impact (**A** to **B**), gradually decreasing wages and increasing employment afterwards (**B** to **D**), and unchanged wages and a relatively *small* reduction in employment in the long run (**A** to **D**). Thus, the result is different from both (a) and (b) but share some elements of both.

The analysis is confined to a closed economy. In a small open economy with free capital movements and no capital adjustment costs the economy would jump instantly from **A** to **D** in Figure 2; the wage at the macro level would then be determined uniquely by the exogenous international real interest rate.

The model uses specific functional forms but does have reasonable properties; e.g., the economy converges towards a balanced growth path where the employment and the labour share of income are untrended. In a more general setting many things are possible. For instance, multiple equilibria and sunspots may arise if mark-ups are not constant (see Gali 1994).

[12] It is presumed that all agents have correct expectations. It is also possible to have a downward sloping PS curve with constant returns if the expectations of the agents are not fulfilled (see Layard *et.al.* 1991).

[13] In both cases, the equilibrium is Pareto inoptimal due to a 'coordination failure' among the unions. E.g., because each union does not take into consideration that a larger wage claim increases the price level which reduces the real wages of the other unions.

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1 Appendix

1.1 Dynamic equations for $\hat{k}$ and $\hat{c}$

Equations (7) and (5) and the assumption of clearing in the capital market yield

$$\dot{K} = \alpha K^{1-\nu} (ZL)^{\nu} - C \tag{16}$$

$\Rightarrow$

$$\frac{\dot{k}}{k} = \frac{\dot{K}}{K} - \frac{\dot{L}}{L} - \frac{\dot{Z}}{Z} = \alpha k^{-\nu} - c/k - \frac{\dot{L}}{L} - g, \tag{17}$$

and (3) and (9) give

$$\frac{\dot{C}}{C} = (1-\nu)\,\alpha K^{-\nu} (ZL)^{\nu} - \rho \qquad \Rightarrow$$

$$\frac{\dot{c}}{c} = \frac{\dot{C}}{C} - \frac{\dot{L}}{L} - \frac{\dot{Z}}{Z} = (1-\nu)\,\alpha k^{-\nu} - \rho - \frac{\dot{L}}{L} - g. \tag{18}$$

The growth rate of employment is derived by differentiating (12) and inserting $\dot{k}/k$ and $\dot{c}/c$:

$$\frac{\dot{L}}{L} = \frac{1}{\gamma}\left((1-\nu)\frac{\dot{k}}{k} - \frac{\dot{c}}{c}\right) \qquad \Rightarrow$$

$$\frac{\dot{L}}{L} = -\frac{(1-\nu)\,c/k}{\gamma-\nu} + \frac{\rho+\nu g}{\gamma-\nu}. \tag{19}$$

Inserting this into (17) and (18) gives (13) and (14).

1.2 Proving Proposition 2

Let a "hat" over a variable denote the growth rate of the variable. Before proving the proposition, we need to establish 4 lemmas:

Lemma 1 *If $\chi(k) \equiv c/k$ then $\chi'(k) < 0$.*

Proof. From (17) and (18), we get

$$\hat{\chi} \equiv \frac{\dot{\chi}}{\chi} = \frac{\dot{c}}{c} - \frac{\dot{k}}{k} = -\nu\alpha k^{-\nu} - \rho + \chi \qquad \Rightarrow$$

$$\frac{d\hat{\chi}}{dt} = \nu^2 \alpha k^{-\nu} \hat{k} + \dot{\chi}.$$

The term $\nu^2 \alpha k^{-\nu} \hat{k}$ is positive when $k < k^*$. The above equation implies that $\dot{\chi}$ continues to increase if $\dot{\chi} > 0$ contradicting the fact that the economy will ever approach a steady state. Thus, $\hat{\chi} < 0$ if $k < k^*$ or as k is increasing $\chi'(k) < 0$. By the opposite reasoning it follows that $\hat{\chi} > 0$ if $k > k^*$ or as k is decreasing $\chi'(k) < 0$. ■

Lemma 2 *If* $\hat{K}(k) \equiv \dot{K}/K$ *then* $\hat{K}'(k) < 0$.

Proof. From (16), we have $\dot{K} = \alpha K^{1-\nu} (ZL)^{\nu} - C \;\Rightarrow$

$$\hat{K} = \frac{\dot{K}}{K} = \alpha k^{-\nu} - c/k \qquad \Rightarrow$$

$$\frac{d\hat{K}}{dt} = -\nu \alpha k^{-\nu} \frac{\dot{k}}{k} - \dot{\chi} \qquad \Rightarrow$$

$$\frac{d\hat{K}}{dt} = -\nu \alpha k^{-\nu} \left(\alpha k^{-\nu} - \frac{(\gamma - 1)\chi + \rho + \gamma g}{\gamma - \nu} \right) - \left(-\nu \alpha k^{-\nu} - \rho + \chi \right) \chi.$$

The first term is negative whereas the second term is positive. Lemma 1 implies that $\hat{\chi} = -\nu \alpha k^{-\nu} - \rho + \chi < 0 \Leftrightarrow \nu \alpha k^{-\nu} > \chi - \rho$. Using this inequality to substitute the term outside the first bracket yields

$$\frac{d\hat{K}}{dt} < -(\chi - \rho) \left(\alpha k^{-\nu} - \frac{(\gamma - 1)\chi + \rho + \gamma g}{\gamma - \nu} \right) - \left(-\nu \alpha k^{-\nu} - \rho + \chi \right) \chi \qquad \Leftrightarrow$$

$$\frac{d\hat{K}}{dt} < \frac{(\chi + (\gamma - \nu)\alpha k^{-\nu} - \rho)(\rho + \nu g - \chi(1 - \nu)) + (\gamma - \nu) g (\chi - \nu \alpha k^{-\nu} - \rho)}{\gamma - \nu}.$$

Lemma 1 implies that $\hat{\chi} = \chi - \nu \alpha k^{-\nu} - \rho < 0$ when $k < k^*$. Lemma 1 and $\chi^* \equiv c^*/k^* = (\rho + \nu g)/(1 - \nu)$ imply that $\rho + \nu g - \chi(1 - \nu) < 0$. Thus, $d\hat{K}/dt < 0$ if $\chi > \rho - (\gamma - \nu)\alpha k^{-\nu}$. This is fulfilled as

$$\chi > \chi^* = \frac{\rho + \nu g}{1 - \nu} > \frac{\rho + \nu g - \gamma(\rho + g)}{1 - \nu} = \rho - (\gamma - \nu)\alpha (k^*)^{-\nu} > \rho - (\gamma - \nu)\alpha k^{-\nu},$$

holds whenever $k < k^*$. Thus, $d\hat{K}/dt < 0$ if $k < k^*$ or as k is increasing $\hat{K}'(k) < 0$. Applying the same method for the case $k > k^*$ gives a corresponding inequality that has to be fulfilled:

$$\frac{d\hat{K}}{dt} > \frac{(\chi + (\gamma - \nu)\alpha k^{-\nu} - \rho)(\rho + \nu g - \chi(1 - \nu)) + (\gamma - \nu) g (\chi - \nu \alpha k^{-\nu} - \rho)}{\gamma - \nu}.$$

Lemma 1 implies that $\hat{\chi} = \chi - \nu\alpha k^{-\nu} - \rho > 0$ when $k > k^*$. Lemma 1 and $\chi^* \equiv c^*/k^* = (\rho + \nu g) / (1 - \nu)$ imply that $\rho + \nu g - \chi (1 - \nu) > 0$. Thus, $d\hat{K}/dt > 0$ if

$$\chi > \rho - (\gamma - \nu)\, \alpha k^{-\nu}.$$

Lemma 1 implies that $\hat{\chi} = \chi - \nu\alpha k^{-\nu} - \rho > 0 > -\gamma\alpha k^{-\nu}$ when $k > k^*$. ■

Lemma 3 *If $\hat{C}(k) \equiv \dot{C}/C$ and $\hat{L}(k) \equiv \dot{L}/L$ then $\hat{C}'(k) < 0$ and $\hat{L}'(k) > 0$.*

Proof. Equation (3) and (9) yield $\hat{C}(k) = (1 - \nu)\, \alpha k^{-\nu} - \rho \;\Rightarrow$

$$\hat{C}'(k) = -\nu (1 - \nu)\, \alpha k^{-\nu - 1} < 0.$$

Equation (19) yields $\hat{L}(k) = -\frac{(1-\nu)\chi(k)}{\gamma - \nu} + \frac{\rho + \nu g}{\gamma - \nu} \;\Rightarrow$

$$\hat{L}'(k) = -\frac{(1 - \nu)\, \chi'(k)}{\gamma - \nu} > 0,$$

according to Lemma 1. ■

Lemma 4 *A reduction in ε or rise in κ increase present and future values of k.*

Proof. Let $\tilde{K} \equiv K/Z$ and denote by $\tilde{K}_1^*$ the steady state value of $\tilde{K}$ before the parameter change and $\tilde{K}_2^*$ the steady state value after the parameter change. Note, that the growth rates of $\tilde{K}$ and k are independent of ε and κ for a given value of k. Thus, a change in ε or κ that increases (decreases) k on impact also increases (decreases) future values of k and according to Lemma 2 this decreases (increases) present and future growth rates of $\tilde{K}$. From Proposition 1 it follows that $\tilde{K}_2^* < \tilde{K}_1^*$ after a reduction in ε or rise in κ. This is only possible if the growth rates of $\tilde{K}$ decrease as $\tilde{K}$ cannot change on impact. Thus, present and future values of k have to increase after a reduction in ε or rise in κ. ■

Proof of Proposition 2. *Part (i).* K is the state variable and is therefore unchanged. $k = \frac{K}{ZL}$ increases on impact according to Lemma 4 which is only possible if L decreases as Z is also a state variable. Lemma 1 implies that $\chi = C/K$ decreases when k increases which is only possible if C decreases. *Part (ii).* Lemma 2 and 3 imply that $\hat{C}$ and $\hat{K}$ is a decreasing function of k whereas $\hat{L}$ is an increasing function of k. It then follows from Lemma 4 that present and future values of $\hat{C}$ and $\hat{K}$ decrease whereas the values of $\hat{L}$ increase. *Part (iii).* Equation (8) and (9) yield

$$W = \nu\alpha k^{1-\nu} Z \quad , \quad R = (1-\nu)\,\alpha k^{-\nu},$$

showing that W is an increasing function of k whereas R is a decreasing function of k. It then follows from Lemma 4 that present and future values W increase whereas the values of R decrease. ■

Figure 1. Phase diagram.

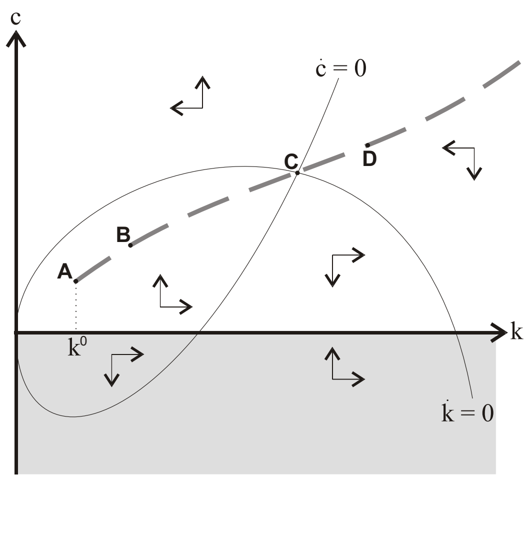

Figure 2. Macroeconomic outcome.