A Mixed Industrial Structure Magnifies the Importance of Menu Costs*

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Abstract

New Keynesian literature assumes symmetric industrial structure when analysing explanations of money non-neutrality. This paper analyses the impact of modifying this assumption by allowing for a mixed industrial structure; some industries are characterized by monopolistic competition, others by perfect competition. The mixed industrial structure implies misallocation of labour between the different industries which may contribute to explanations of non-neutrality of money. Following a 5% money increase, the menu costs needed for non-neutrality may be 40 times smaller and ratio of welfare gain over private loss more than 100 times larger than in the corresponding model with a symmetric structure.

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1. Introduction

"*....industrial economists have been aware that the responsiveness of prices to changes in demand differs sharply across industries.*" (Gordon 1990, pp. 1126).

"*The scope for small menu costs to lead to large output effects in our model depends critically on the elasticity of labour supply with respect to the real wage being large enough. Evidence on individual labour supply suggests, however, a small elasticity.*" (Blanchard & Kiyotaki 1987, pp. 369).

In this paper we look at the effects of nominal disturbances in the context of menu costs and the resultant nominal rigidity of prices. The existing macroeconomic literature on this topic has adopted the representative sector approach: the whole economy consists of symmetric monopolistically competitive firms (see e.g. Mankiw 1985, Blanchard and Kiyotaki 1987, Rotemberg 1987, Ball and Romer 1990, 1991). However, whilst this feature might be convenient, it is not realistic, as indicated by the quote above from Gordon's survey of the New Keynesian literature. The structure of industries is diverse: concentration ratios vary greatly between industries, as do measures of the degree of product differentiation, returns to scale and so on.[1] To what extent is the variety of industrial structure important for the macroeconomic behaviour of the economy?[2]

We adopt the theoretical framework of the existing menu-cost literature, but extend it to allow for some variety amongst output markets: proportion β are monopolistically competitive, and proportion $1-\beta$ are perfectly competitive (where $\beta \in [0,1]$). The representative sector model is thus the special cases of $\beta = 1$ (monopolistic) and $\beta = 0$ (Walrasian). Introducing two sectors leads to some very different conclusions: there is an additional dimension of causality in terms of the reallocation of labour between sectors. The possibility of labour reallocation is important in presence of a mixed industrial structure for two reasons; (i) menu costs can only prevent price changes in the monopolistic competitive sector; (ii) the equilibrium allocation of aggregate employment between the two sectors is inefficient, because employment is too low in the monopolistic sector relative to the competitive sector. The implication of (i) is that a monetary expansion increases labour demand in the monopolistic sector relative to the competitive sector.[3] This changes the allocation of labour in favour of the monopolistic sec-

[1] See e.g. Scherer (1980). For difference in price cost margins see also Bresnahan (1989).

[2] See Dixon (1994) which explores this issue in a context without menu-costs, but finds significant structural effects.

[3] This happens because prices are totally rigid in the monopolistically competitive sector whereas they respond at least partly in the perfect competitive sector. Evidence of such an relationship goes back to Gardiner C. Means who observed price decreases in competitive industries

tor which increases welfare according to (*ii*). The menu costs required for price rigidity may be lower because it is cheaper to expand production by attracting labour from the other sector than to expand aggregate employment. Furthermore, the labour reallocation effect increases real GNP for a given employment level. Thus, the model provides a new explanation for demand driven procyclicality of productivity. The most closely related explanation is found in Basu (1995) where procyclical productivity arises in a menu costs model because of a mixed *production* structure; some firms produce intermediate goods which is used by other firms to produce final goods.[4]

In representative sector models with a competitive labour market small menu costs matter only if labour supply is highly elastic. An inelastic labour supply implies that an increase in aggregate employment is costly in terms of lost utility to the households and increased marginal costs to the firms. This is a serious problem, as the evidence points towards a small wage elasticity of labour supply (cf. the opening quote). Our model provides an example of how different market structures may interact to improve the performance of these models for realistic labour supply elasticities and menu-costs. We consider a bench-mark case with a wage elasticity of labour supply equal to 0.2 (a reasonable value according to evidence in Killingsworth 1983 and Pencavel 1986) and a "macro" degree of imperfect competition equal to an average Lerner index of 1/4 (not too high a value according to evidence reported in Hall 1986, 1988, Domowitz, Hubbard & Petersen 1988, and Bresnahan 1989). Comparing the consequences of a 5% money increase for the mixed industrial structure $\beta = 1/2$ and the symmetric structure $\beta = 1$ reveals that the menu costs needed for non-neutrality are 40 times smaller and ratio of welfare gain over private loss more than 100 times larger in the former case. The mixed industrial structure case requires menu costs of about 0.2% of revenue, which is within limits of Levy *et.al.* (1996) who estimated menu costs ranging from 0.61% to 0.76% within large U.S. supermarket chains. Increased productivity accounts for 61% of the increase in GNP which is close to evidence in Rotemberg & Summers (1990) from U.S. in the period 1962-84 ; a demand shock yielding a one per cent increase in output is associated with a 0.59% increase in total factor productivity. Whilst the welfare response is still modest, i.e. only

such as grain farming and crude oil extraction and insignificant price changes in monopolized industries such as nickel production during the Great Depression (see Scherer 1980 ch. 10). The strong correlation between industry concentration and rigid prices observed in Carlton (1986) may also indicate such an relationship. However, less conclusive evidence also exists (see Martin 1993 ch. 15).

[4]Other explanations include Rotermberg & Summers (1990) who attribute their empirical evidence of procyclical productivity to labour hoarding. Caballero & Lyons (1992) provide empirical evidence in favour of productive externalities. However, this evidence has been challenged by Basu & Fernald (1995).

1.2% of GNP, it is still significant in terms of the annual variation in GNP.

The results are obtained within a framework that resembles existing work. However, we have excluded all imperfections other than monopolistic competition in output markets in order to simplify the exposition and focus our attention solely on the effects of a mixed industrial structure. For example, we assume perfect competition in the labour market instead of a unionized labour market (as in Blanchard & Kiyotaki 1987) or a labour market influenced by efficiency wage considerations (see Akerlof and Yellen 1985). We also abstract from other imperfections in the goods market such as imperfect information and customer markets (e.g. see Ball & Romer 1990 and Nishimura 1992 ch. 3 & 8).

The paper is organized as follows. In the second section we describe the model and derive the equilibrium without any nominal rigidities. This section also discusses the additional equilibrium welfare loss due to misallocation of inputs caused by the mixed industrial structure. The third section derives the consequences of nominal disturbances when firms face small menu costs of price changes. It looks at three issues and their relationship to industrial structure: (*i*) how monetary disturbances affect the two different industries; (*ii*) the size of menu costs needed to ensure nominal rigidity; (*iii*) welfare consequences of monetary disturbances. We conclude with a wider discussion of alternative models and the limitations of our results.

2. The model

The model is based on a simplified Blanchard and Kiyotaki model, with a perfectly competitive sector added. Indeed, we assume that the economy can be divided into a monopolistic sector and a competitive sector. The monopolistic sector consists of an infinite number of firms each producing a differentiated good and engaged in monopolistic competition, whereas the competitive sector is represented by a composite good traded in a perfectly competitive market by price taking firms. In order to isolate the effect of different market structure, it is assumed that all firms have an identical constant returns technology. The only input in production is labour which is bought in a common perfectly competitive market. Households earn income by supplying labour to the common labour market and by receiving dividend (as a lump sum) on firm shares. Household utility is increasing in consumption of the different goods and decreasing in number of working hours. To simplify the exposition we exclude real money balance from the utility function and assume like Ball & Romer (1989, 1990) that a simple transactions technology determines the relation between aggregate spending and real money balances.

2.1. Households

There is a continuum of households $i \in [0,1]$. Each household i has the following utility function where superscripts M and C represent variables corresponding to the monopolistic sector and the competitive sector, respectively[5]

$$U\left(C_i^M, C_i^C, \ell_i\right) = \frac{\left(C_i^M\right)^{\beta}\left(C_i^C\right)^{1-\beta}}{\beta^{\beta}\left(1-\beta\right)^{1-\beta}} - \tfrac{\gamma}{\gamma+1}\ell_i^{\frac{\gamma+1}{\gamma}} \qquad \forall i \in [0,1], \tag{2.1}$$

where

$$C_i^M \equiv \left(\int_{j=0}^{1} c_{ij}^{1-\mu} dj\right)^{\frac{1}{1-\mu}} \quad , \quad \mu \in (0,1)\,.$$

The first term in the utility function represents utility from consumption of the two composite goods C_i^M and C_i^C, the relative importance of the two goods being determined by the size of $\beta \in [0,1]$. The composite good C_i^M representing the monopolistic sector consists of a continuum of differentiated goods c_{ij} with an elasticity of substitution between any two goods equal to $\frac{1}{\mu}$. The second term in the utility function represent disutility from working where the elasticity of marginal disutility of work $\frac{1}{\gamma}$ is assumed positive. As is well-known, this kind of utility function excludes wealth effects in the labour supply. Thus, our result will not depend on the ownership structure which is more than usually convenient in an asymmetric setting where firms may earn different profits.

The corresponding consumer price index equals

$$P\left(P^M, P^C\right) = \left(P^M\right)^{\beta}\left(P^C\right)^{1-\beta}, \tag{2.2}$$

where

$$P^M = \left(\int_{j=0}^{1} p_j^{1-\frac{1}{\mu}} dj\right)^{\frac{\mu}{\mu-1}}.$$

The budget constraint of household i equals

$$P^M C_i^M + P^C C_i^C \leq W\ell_i + \pi_i \equiv I_i \qquad \forall\, i, \tag{2.3}$$

[5]The Cobb-Douglas sub-utility function over consumption goods is chosen for tractability reasons. Furthermore, it is practical because the β parameter becomes a natural measure of the relative size of the two sectors in equilibrium. Allowing for a more general homothetic utility function would allow the budget shares to respond to relative prices, rather than being constant (see Dixon 1994). However, this is a secondary effect that complicates the analysis but would be quantitatively unimportant unless budget shares are highly elastic with respect to relative prices. Note, that a change in weighting of the two terms in (2.1) wont change the results in Section 3. E.g. a constant multiplied on one of the terms wont enter into the formulas used to derive Table 2.

where W is the nominal hourly wage obtained in the common labour market. The LHS is nominal spending which most be less than or equal to nominal income consisting of wage income and dividends. Maximization of (2.1) subject to (2.3) yields the following first order conditions

$$C_{ij}^{M} = \left(\frac{p_j}{P^M}\right)^{-1/\mu} \frac{\beta I_i}{P^M}, \tag{2.4}$$

$$C_i^C = \frac{(1-\beta)\, I_i}{P^C}, \tag{2.5}$$

$$\ell_i = \left(\frac{W}{P}\right)^{\gamma}. \tag{2.6}$$

Equation (2.4) states that each household spends a fraction β of income on consumption of monopolistic goods with the distribution on the different brands depending on the relative prices and the degree of substitution between the goods, (2.5) states that remaining income is spent on competitive goods, and (2.6) states that labour supply depends on the real wage only. The wage-elasticity of labour supply γ is going to be an important parameter in what follows.

Finally, we assume that some transactions technology (e.g. a cash-in advance constraint) determines the relation between aggregate spending of the households and money balances

$$\int_{i=0}^{1} I_i di = M. \tag{2.7}$$

2.2. Firms

Each firm produces either one of the monopolistic goods included in C^M or one of the competitive goods included in C^C. Firms in both the monopolistic and competitive sectors are infinitesimal, and the monopolistic firms treat both the aggregate price in their own sector P^M and for the whole economy P as given. However, whilst firms in the monopolistic sector face non-infinitely elastic demand and conversely act as price-setters, firms in the competitive sector act as price-takers. It is assumed that an identical constant returns technology is available to all firms in the economy as we are not interested in technological asymmetry. We normalize the input-output coefficient to unity. Thus, production of differentiated good j equals the employment of labour L_j^M. The firm producing this good maximizes profits subject to the aggregate demand for this particular brand derived from (2.4) taking other prices including the market wage W as given. The first order condition equals

$$\frac{W}{p_j} = 1 - \mu \qquad \forall\, j \in [0,1], \tag{2.8}$$

where μ is the degree of imperfect competition measured as the Lerner Index. The aggregate production of competitive goods equals L^C giving the corresponding marginal condition

$$\frac{W}{P^C} = 1. \tag{2.9}$$

2.3. Symmetric equilibrium

The symmetric structure of the model implies that prices of all monopolistic goods are equal to P^M. The aggregate labour demand of monopolistic firms is obtained by using this relationship as well as (2.4), (2.7), (2.8), and clearing in the output market, yielding

$$L^M = \beta \frac{M}{W} (1 - \mu). \tag{2.10}$$

The corresponding equation for the competitive goods is obtained from (2.5), (2.7), and (2.9)

$$L^C = (1 - \beta) \frac{M}{W}. \tag{2.11}$$

By dividing (2.10) with (2.11), we get

$$\frac{L^M}{L^C} = \frac{\beta}{1 - \beta} (1 - \mu) < \frac{\beta}{1 - \beta}, \tag{2.12}$$

which shows that monopolistic competition implies misallocation of labour between industries producing competitive goods and industries producing differentiated goods. The market allocates too much of the aggregate employment to production of the competitive goods and too little to production of the monopolistic goods.

The equilibrium level of aggregate employment is now obtained by assuming equilibrium in the labour market and by combining (2.2), (2.6), (2.8), and (2.9), yielding (see Appendix)

$$L^* = (1 - \mu)^{\beta\gamma} \leq 1. \tag{2.13}$$

Equilibrium employment increases if the differentiated goods become closer substitutes (corresponding to a lower value of μ) since this reduces the market power in the monopolistic sector implying larger labour demand and real wage. The socially optimal level of employment is 1, corresponding to the Walrasian equilibrium. This can be derived for any value of μ by assuming that the monopolistic competitors behave as price-takers, and so (2.8) becomes $\frac{W}{p_j} = 1$. There are 3 conditions under which the equilibrium level of employment equals 1: *i*) the differentiated goods are perfect substitutes ($\mu \to 0$), *ii*) the economy consists of only the competitive goods ($\beta = 0$), or *iii*) the labour supply is perfectly inelastic ($\gamma \to 0$).

Only the first two possibilities correspond, however, to a social optimum. This is clear if we derive the equilibrium value of aggregate consumption/real GNP.[6] It is defined as and equal to (see Appendix):

$$C^* \equiv \frac{\int_{j=0}^{1} p_j c_j dj + P^C C^C}{P} = (1-\mu)^{\beta(\gamma+1)} \frac{1}{1-\beta\mu} \leq 1. \tag{2.14}$$

Aggregate consumption increases if the differentiated goods become better substitutes through the same channel that increased aggregate employment. However, the increase in consumption is larger than the increase in aggregate employment because of an additional effect to be discussed below. The socially optimal level of aggregate consumption is 1. This level is only obtained in the decentralized economy if: *i*) the differentiated goods are perfect substitutes, or *ii*) the economy consists of only the competitive goods. Taking the limit of (2.14) gives

$$C^*|_{\gamma\to 0} = (1-\mu)^{\beta} \frac{1}{1-\beta\mu} \leq 1,$$

which shows that real GNP is inefficiently low even with perfectly inelastic labour supply (implying that aggregate employment is at its efficient level). This may at first seem a little odd because the total number of physical output units produced in the two sectors is identical to that produced in the social optimum. The reason is that monopolistic competition leads to misallocation of labour across the sectors: employment and output are too high in the competitive sector relative to the monopolistic. The suboptimal output mix causes a fall in real GNP as prices on the competitive goods become relative low compared to prices on the monopolistic goods. The equilibrium welfare loss is illustrated in Figure 1 for the general case with a finite wage elasticity of labour supply. The equilibrium is a point like **A**, whereas the social optimum is point **C**. The total effect of imperfect competition can now be divided into two: *i*) The distance $|\mathbf{AB}|$ which is due to the decline in aggregate employment because of a lower equilibrium wage in the presence of imperfect competition. *ii*) The distance $|\mathbf{BC}|$ which is due to the misallocation of labour between the industries. In the case of a perfectly inelastic labour supply the first effect disappears (**A** and **B** will be identical) and the equilibrium **B** lies on the same iso-employment curve as the social optimum but not on the same iso-GNP curve. Thus, any change in the allocation of labour towards point **C** will increase GNP and welfare.

< Figure 1 here >

[6]We are dealing with income in its Hicksian sense since our numeraire P is the cost-of-living index. (2.14) is the indirect utility function for consumption goods.

Some examples are provided in Table 1. The first two rows consider the case of a completely inelastic labour supply. The first row reveals that aggregate employment, consumption, and utility equal their socially optimal values when we consider a symmetric industrial structure whereas the second row gives an example of a mixed industrial structure where only aggregate employment equals the socially optimal level.[7]

Table 1. Examples of equilibrium

γ	β	μ	$\xi = \beta\mu$	L^*	C^*	U^*
0	1.0	μ	μ	1	1	1
	0.5	0.50	0.250	1	0.943	0.943
0.2	1.0	0.25	0.250	**0.944**	**0.944**	**0.826**
	0.5		0.125	0.972	0.962	0.821
		0.5	0.250	**0.933**	**0.880**	**0.770**
2	1.0	0.25	0.250	0.563	0.563	0.281
	0.5		0.125	0.750	0.742	0.309
		0.5	0.250	0.500	0.471	0.236

The next three rows consider the bench mark value of γ equal to 20%. The first of them looks at a Lerner index μ equal to 0.25, and a share of income spent on monopolistic goods β equal to 1. In this case aggregate employment and consumption are approximately 6% lower than the socially optimal levels. The only thing changed in the next row is the households share of income spent on monopolistic goods; it is now assumed that households spend half the income on monopolistic goods and half the income on competitive goods. The consequence is larger values of both employment and consumption but a lower level of utility. The introduction of the competitive goods increases labour demand and market wage thereby increasing employment and production. In isolation this effect increases utility as employment was inefficiently low initially. The total effect is, however, a decrease in utility because of the misallocation of labour between the industries which decreases GNP (cet.par.). The introduction of the perfect competition sector increases employment by 3% but GNP only by 2% making the increase in disutility of work larger than the increase in utility from consumption. In this example we changed β without changing μ implying that the average mark-up in the economy decreased as well. From a macro point of view, it seems more interesting to look at consequences of different industrial structure for a given average value of the Lerner index $\left(\xi \equiv \beta \frac{P^M - W}{P^M} + (1-\beta)\frac{P^C - W}{P^C} = \beta\mu\right)$ which is what we have done in the next row. It shows that a mixed industrial structure

[7] We use the word "symmetric" here to mean symmetry *across* sectors. Even in the asymmetric case of a mixed structure, there is symmetry *within* sectors.

has only small consequences on employment but large consequences on real GNP and utility compared to the corresponding symmetric model for a given macro value of the Lerner index. The same experiment with an elastic labour supply is provided in the last three rows.

3. Menu costs and non-neutrality of money

The previous section derived equilibrium levels of aggregate employment, consumption, and utility in the presence of a mixed industrial structure. These solutions are completely independent of nominal variables, since objective functions are defined on real variables and trades are frictionless. However, monopolistic competition and small nominal frictions (e.g. menu costs) may interact to make money non-neutral and causing nominal disturbances to imply welfare fluctuations of a much larger magnitude than the frictions causing them. Previous results (see e.g. Mankiw 1985, Akerlof & Yellen 1985, Blanchard & Kiyotaki 1987, Rotemberg 1987, Ball & Romer 1989, 1990, 1991) have been derived in the special case of $\beta = 1$, in which there is no competitive sector.

In this section we analyze the consequences of having a mixed industrial structure by looking at a monetary expansion when there are menu costs within the monopolistic sector. The main questions addressed are: (*i*) what is the impact of a monetary expansion on the different industries if menu costs are sufficiently large; (*ii*) how large menu costs need to be (as a proportion of firm revenue) for monopolistic firms to chose not to change prices; (*iii*) how large menu costs need to be relative to the change in social welfare generated by a monetary expansion. What will be crucial here is the structure of the economy in terms of the proportion β, and the nature of preferences in terms of the two parameters μ and γ. We will address the three questions in turn, and throughout compare our results to the corresponding symmetric model ($\beta = 1$) that resembles models used in the literature.

3.1. Price, quantity and productivity responses with menu costs

Suppose that there is a monetary expansion. If the menu costs are sufficiently large, then the monopolistic price is fixed, so that all of the demand increase will feed through into an output response in that sector (cf. eq. (2.4) and (2.7)). In the competitive sector, however, nominal prices can increase in response to higher wages (note that eq. (2.8) is violated because of the menu costs whereas (2.9) still holds), so that one would expect the price of competitive goods to rise relative to monopolistic goods. Gross substitution then implies that the proportional increase in the quantity of monopolistic goods will be higher than that of competitive. If the initial equilibrium is point **A** in Figure 1, it implies that a monetary expansion will move the economy to the left of the ray going through the origin and **A**, thereby reducing the inefficiency stemming from misallocation of labour. In addition there will be an increase in aggregate employment because of increasing labour demand moving the economy to the north-east of the iso-employment curve $\bar{L}_1$. This well-known effect increases welfare by reducing the amount of underemployment caused by imperfect competition. The two effects are illustrated by the arrows in Figure 1 from which it is clear that the total output response in the monopolistic sector is positive whereas the response in the competitive sector is indeterminate.

In order to establish the size of these effects, it is important to know the response of the nominal market wage to a monetary expansion *given that prices in the monopolistic sector do not change*. This is derived from (2.2), (2.4), (2.7), (2.9), (2.11), and (2.12) and given in elasticity form by (see Appendix)

$$\zeta_{WM} = \frac{dW/W}{dM/M} = \frac{1-\beta\mu}{\gamma(1-\beta\mu)\beta + 1 - \beta} \geq 0. \tag{3.1}$$

Clearly, $\zeta_{WM} \geq 0$ for all possible parameter values, so that nominal wages must increase with the money supply. There are two reasons for this: firstly, the output expansion in the monopolistic sector increases the labour demand thereby increasing wages, secondly the price increase in the competitive sector increases the cost-of-living index thereby increasing the nominal wage claim of the households for a given amount of work. For latter reference if might be useful to look at the two polar cases $\beta = 0$ and $\beta = 1$. The first extreme amounts to removing the monopolistic sector and implies money neutrality and $\zeta_{WM} = 1$. The second extreme results in the symmetric market structure normally assumed in models explaining money non-neutrality, namely an economy characterized only by monopolistic competition. In this case $\zeta_{WM} = 1/\gamma$ making the wage elasticity of labour supply of crucial importance for the wage response of a monetary expansion. Note, for example that the wage response goes towards infinity as labour supply becomes perfectly inelastic, whereas it will converge towards a constant for a mixed economy $\beta \in (0, 1)$. This feature is going to be important in the next section when we look at the monopolistic firms costs of not changing prices because the nominal wage equals the firms marginal costs.

Returning to output and price responses, it is useful to define the elasticity of the competitive price wrt M as well $\zeta_{P^C M}$. We obtain the following,

Proposition 1. *Consider $dM > 0$ small enough to prevent price changes in the monopolistic sector then:*

(a) The output of the monopolistic sector is increasing in M.

(b) The output response of a monetary expansion is always larger in the monopolistic sector than in the competitive sector.

(c) Aggregate productivity C/L is increasing in M.

(d) If $\gamma \leq \frac{1-\mu}{1-\beta\mu}$ $\left(\gamma > \frac{1-\mu}{1-\beta\mu}\right)$ then $\zeta_{WM} \geq 1$ $(\zeta_{WM} < 1)$, $\zeta_{P^C M} \geq 1$ $(\zeta_{P^C M} < 1)$, output of the competitive sector is decreasing (increasing) in M.

Proof. (a) follows from (2.4) and (2.7). (b) follows by comparison of (2.4) and (2.5). (c) Consider some distribution α of total input such that $L^M = \alpha L$. It follows from the definition of GNP in (2.14) that $C/L = (\alpha/\beta)^{\beta}\left((1-\alpha)/(1-\beta)\right)^{1-\beta}$. C/L is increasing in α when $\alpha < \beta$ which is fulfilled in the initial equilibrium, cf. (2.12). Finally, (b) implies that α increases when M increases. (d) The relationship between γ and ζ_{WM} follows from (3.1) whereas the next two statements are obtained from (2.9) and (2.11). ■

We have discussed (a) and (b). (c) is an implication of (b) as we have previously concluded that movements from **B** towards **C** in Figure 1 increase GNP for given aggregate employment. This feature is not present in the representative sector framework as movements from **A** towards **B** leave productivity unchanged. (d) captures the spillover effect from the monopolistic sector to the competitive: the nominal wage and price in the competitive sector rise more than the money increase if the labour supply is relatively inelastic- a sufficient condition for this to happen is $\gamma < 1 - \mu$, which is likely to be satisfied for empirically relevant magnitudes. Thus, the increase in monopolistic output is likely to crowd out competitive output. The initial expansion in the monopolistic sector is not magnified by an increase in activity in the competitive sector: rather, the fact that output in the competitive sector falls allows the labour to be reallocated to the monopolistic sector, thus facilitating its expansion. If the labour supply is relative elastic then the competitive output may also increase (a sufficient condition is $\gamma > 1$), but always by less than the monopolistic sector (from (b)).

Prop. 1 states that one should expect asymmetric responses on different markets following a monetary expansion, a conclusion assumed away in symmetric models. Another difference is the positive correlation between the cost-of-living index and money not present in symmetric models when menu costs are sufficiently large. Finally, productivity becomes procyclical following demand shocks which is in fact observed empirically (e.g. Rotemberg & Summers 1990). We will return to this issue in Section 3.3.

3.2. Private loss and menu costs

In the previous section it was assumed that menu costs were sufficiently large to prevent price changes in the monopolistic sector. This section asks how large these menu costs need to be. In the representative sector literature, this can be done by looking at the ratio of private losses from not adjusting prices relative to revenue. However, in an economy with $\beta < 1$, the relevant private losses are only incurred by the monopolistic part of the economy. This raises the question whether the private losses are to be measured relative to revenue in the monopolistic sector or relative to aggregate revenue in the whole economy. We use the first approach in this section since we are considering the individual firm's decision, and the second approach later when dealing with the size of private losses relative to gains in social welfare. The important thing to explore is how the likelihood of price rigidity varies with the size of the monopolistic sector β. This is captured by the loss/revenue ratio for an individual monopolist $\frac{d\pi}{R}$ *given that it expects the other monopolistic firms not to adjust.*[8] Taking a second order Taylor expansion around the initial equilibrium gives (see Appendix)

$$\frac{d\pi}{R} \approx \Phi(\mu, \beta, \gamma, m) \equiv \frac{1}{2}\left(\frac{1-\mu}{\mu}\right)(\zeta_{WM})^2 m^2, \tag{3.2}$$

where $m = \frac{dM}{M}$. Clearly, all of the parameters (μ, γ, β) influence this ratio for a given m. However, it should be apparent that factors which reduce ζ_{WM} tend to reduce the loss (recall that ζ_{WM} is non-negative) as it reduces the increase in marginal costs following a monetary expansion. From (3.1) and (3.2), we obtain the following

Proposition 2. *Consider a sufficiently small monetary expansion* $m > 0$, *then*

$$\Phi'_\beta(\mu, \beta, \gamma, m) \geq 0 \ \left(\Phi'_\beta(\mu, \beta, \gamma, m) < 0\right) \text{ if } \gamma \leq \tfrac{1-\mu}{(1-\beta\mu)^2} \ \left(\gamma > \tfrac{1-\mu}{(1-\beta\mu)^2}\right).$$

Proof. Take the derivative of (3.1) with respect to β. ■

The result states that the menu-cost needed for price-rigidity is increasing (decreasing) in the size of the monopolistic sector if labour supply is relative inelastic (elastic). The effects of changes in β works entirely through ζ_{WM}, but through two effects of opposite sign. First, an increase in β makes it is more difficult for the monopolistic firms to capture workers from the competitive sector and thus

[8] We restrict the analysis to this approach adopted from Blanchard & Kiyotaki (1987). However, flexibility may also be an equilibrium unless menu costs are sufficiently large, as noted in Ball & Romer (1991).

wage increases are needed to expand aggregate employment. This positive effect is large if labour supply is relatively inelastic. Second, an increase in β increases the size of the sector where prices are rigid implying less increase in the cost-of-living index following a monetary expansion. This tends to reduce nominal wage claims of households for a given amount of work. The first effect dominates if labour supply is relative inelastic, and the second if it is relative elastic. A sufficient condition for the first effect to dominate is $\gamma < 1 - \mu$; a condition that seems to be fulfilled for all realistic values of the two parameters (a sufficient condition for the other case is that the labour supply is elastic $\gamma > 1$).

It is thus quite possible that the smaller is the monopolistic sector relative to the whole economy, the smaller are the menu costs required for monopolistic firms not to change their price. This is interesting, since it shows that from the perspective of the whole economy, menu costs in a small monopolistic sector may be smaller per firm, and hence certainly smaller relative to the aggregate.

3.3. Consequences of a monetary expansion: Numerical examples

In this section we provide some numerical examples in order to compare the welfare consequences of a monetary expansion with the (menu) costs causing them. Taking a second order Taylor expansion on the utility function around the initial equilibrium, the deviation relative to GNP yields (see Appendix)

$$\begin{aligned}\frac{dU}{C} &\approx \beta m + (1-\beta)(1-\zeta_{WM})\, m - (1-\beta\mu)\,\gamma\beta\,(\zeta_{WM})\, m \\ &\quad -\frac{1}{2}\beta(1-\beta)(\zeta_{WM})^2 m^2 - (1-\beta\mu)\,\gamma\frac{\beta^2}{2}(\zeta_{WM})^2 m^2. \qquad (3.3)\end{aligned}$$

The first row contains the first order terms. The first and second term reflect the change in utility due to changes in consumption of the two goods; the third term reflects the reduced utility due to an increase in time spent on work. The second order terms in the second row reflect the non-linear responses of the utility of the composite goods and the disutility of work. Clearly, the overall welfare consequences of a money increase will tend to be larger the smaller is ζ_{WM}, since this implies a larger increase (smaller decrease) in the production of competitive goods (cf. prop. 1 (d)) and a smaller total increase in working time following a monetary expansion. Thus, the responsiveness of wages to money changes is crucial for the welfare consequences of a monetary expansion. After inserting (3.1), (3.3) simplifies to (see Appendix)

$$\frac{dU}{C} \approx \Omega(\mu,\beta,\gamma,m) \equiv \beta\mu m - \frac{\beta}{2}\frac{(1-\beta\mu)^2}{\gamma\beta(1-\beta\mu)+1-\beta}m^2. \qquad (3.4)$$

To compare the welfare response with the costs causing them, we define the ratio

$$\Psi(\mu, \beta, \gamma, m) \equiv \frac{\Omega(\mu, \beta, \gamma, m)}{\beta\Phi(\mu, \beta, \gamma, m)}, \tag{3.5}$$

which measures welfare gain relative to private loss where the private loss is measured relative to GNP by multiplying by β. We will take this as a measure of the possible macroeconomic impact of menu costs. Possible private losses of changing prices in the competitive sector are excluded in this definition as such cost will be paid whether money is neutral or not; they do not contribute to money non-neutrality. It is now possible to derive numerical examples by inserting different parameter constellations into equation (3.2), (3.4), and (3.5). This is done in Table 2 where we consider the consequences of a 5% money increase. The last column $\Upsilon(\mu, \beta, \gamma, m)$ in Table 2 contains the increase in productivity following a monetary expansion (cf. Prop. 1) measured as the percentage change in productivity C/L relative to the percentage change in GNP (see Appendix).

Table 2. Private loss and social gain of $m = 5\%$[9]

γ	β	μ	$\xi = \beta\mu$	$\Phi(.,.,.,.)$	$\Omega(.,.,.,.)$	$\Psi(.,.,.,.)$	$\Upsilon(.,.,.,.)$
0	1.0	μ	μ	∞	0	0	–
	0.5	0.50	0.250	0.28	1.18	8.39	1
0.2	1.0	0.25	0.250	**9.38**	**0.78**	**0.08**	**0**
	0.5		0.125	0.83	0.54	1.30	0.38
		0.50	0.250	**0.21**	**1.19**	**11.18**	**0.61**
2	1.0	0.25	0.250	0.09	1.20	12.83	0
	0.5		0.125	0.15	0.59	7.77	0.06
		0.50	0.250	0.05	1.22	54.31	0.14

The first two rows give an example of a symmetric industrial structure and a mixed industrial structure when labour supply is completely inelastic. In the symmetric case menu costs of any size cannot prevent price changes, and if (for some reason) prices were fixed firms would not chose to satisfy the extra demand (cf. footnote 9) and so the welfare response is zero. In the asymmetric case menu

[9] It is possible to derive the new equilibrium after the monetary expansion and then calculate the welfare response etc. by subtracting welfare in the initial equilibrium from welfare in the new equilibrium. This procedure does, however, involve simulation of the new equilibrium wage and does not give significantly different results. Thus, we have chosen to report the more tractable approximations instead. Another issue is whether firms will satisfy the extra demand or choose to ration. In the first row of Table 2 firms would always chose to ration if (for some reasons) they could not change the price; a marginal increase in the production would make the wage and thus the firms marginal costs go towards infinity. In the other cases it is possible to show that firms always satisfy demand. These calculations are available on request from the authors.

costs equal to 0.3% of the monopolistic firms' revenue prevent price changes, and a monetary expansion increases welfare by 1.2% of GNP. The menu costs equal 0.15% of GNP giving a welfare gain relative to private loss equal to 8.39. The increase in GNP is driven solely by an increase in aggregate productivity arising from the reallocation of input.

The next three rows consider the bench mark value of γ equal to 0.2. The first of them looks at a uniform industrial structure $\beta = 1$ and a Lerner index equal to 1/4 which results in menu costs equal to 9.4% of revenue, welfare increase equal to 0.8% of GNP, and a ratio of welfare gain relative to private loss equal to only 0.08. As in all cases with a symmetric structure there is no increase in productivity. The only change in the next row is $\beta = 1/2$ which also implies a halving of the degree of imperfect competition on the aggregate level ξ. Despite this we observe that menu costs needed are more than 10 times smaller than before, whereas there is only a small decrease in the welfare response implying a large increase in the ratio of welfare gain over private loss. In this case, increased productivity account for 40% of the increase in GNP. The next row looks at the same asymmetric structure but with a degree of imperfect competition on the aggregate level that corresponds to the symmetric model. Menu costs needed are now around 40 times smaller, the welfare gain 1.5 times larger, so that the welfare gain over private loss is more than 100 times larger than in the symmetric case. Now, increased productivity accounts for 61% of the increase in GNP which is in line with empirical evidence in Rotemberg & Summers (1990). The last three rows consider examples of elastic labour supply. They reveal the same direction of change as in the inelastic case (but smaller magnitudes) when we vary β.

3.4. Limitations and alternatives

In this paper, we have assumed a particular structure for different sectors of the economy which generalizes the existing approach. However, there are alternative possibilities, some of which we will briefly consider. Firstly, we could consider an economy in which labour was differentiated, so that the there was a separate labour market for the two sectors, with different wages. There are two differences with the current model here: (i) the reallocation effect cannot operate in the same manner as our model: to increase the labour supply in the monopolistic sector requires the relative wage to rise: (ii) a given rise in the nominal wage in the monopolistic sector will represent a higher real wage increase the larger is the competitive sector (since with menu costs the monopolistic price is fixed, the aggregate price level can only increase insofar as competitive sector wages increase). The presence of menu costs will still give rise to an increase in welfare in that the output of the monopolistic sector will rise and that of the competitive sector fall as the relative price of the monopolistic output falls. Unlike the case of mobile labour, however, a perfectly inelastic labour supply still requires infinite menu costs, unlike the case in Table 2. Secondly, we can consider an economy with immobile labour which is unionized in the monopolistic sector (this is similar to the setup in Blanchard and Kiyotaki (1987) but with a competitive sector). Of course, in this case it is natural to interpret the "menu-cost" idea as applying to the nominal wage (due to re-negotiation costs, nominal contracts etc.). In this case, the representative sector model means that output can increase and that "nominal prices and nominal wages do not adjust to the change in nominal money" (Blanchard and Kiyotaki p. 655). The presence of a competitive sector will tend to *reduce* the effect of menu costs: as output increases, so will the competitive sector wages and prices, which will put pressure on the unionized wages to change. This negative effect will tend to be larger the less elastic is demand. Thirdly, we have also assumed that there are no frictions in the labour market, and rely on the reallocation of labour between the two sectors. Real frictions and costs connected with such a transfer will tend to reduce the increase in welfare and increase the menu cost requirement. Whilst the importance of heterogenous markets depends on the nature of the labour market, we have shown that in an important class of models the introduction of a competitive sector in addition to the monopolistic can lead to menu costs being important even with low labour supply elasticities.

4. Concluding remarks

New Keynesian literature adopts a representative sector approach when analyzing explanations of money non-neutrality. However, this restricts non-neutrality to implausible values of the elasticity of labour supply. Allowing for a mixed industrial structure, menu costs can be important even with low elasticities of labour supply: a monetary expansion increases employment in the monopolistically competitive sector relative to the perfectly competitive sector thereby decreasing the inefficiency due to misallocation of inputs in the initial equilibrium. While this magnifies the importance of menu costs, it should be noted that the employment responses and welfare responses involved are still modest when labour supply is inelastic. Thus, a mixed industrial structure does not explain away 'the employment variability puzzle' (Prescott 1986). In Table 2 the increase in welfare resulting from a 5% increase in the money supply is no more than 1.2%. Whilst this is modest, it is not insignificant in relation to the usual magnitude of annual GNP growth (in the US and UK this is 2-3%). It is also perhaps more realistic, and in addition requires a much smaller (and empirically plausible) level of menu costs relative to GNP than the conventional story when labour supply is inelastic.

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A. Appendix

A.1. Derivation of (2.13)

The aggregate labour supply is obtained from (2.6) giving the following equilibrium condition

$$L = \ell = \left(\frac{W}{P}\right)^{\gamma}.$$

Insert (2.9) and (2.2), to get

$$L = \left(\frac{P^C}{P^M}\right)^{\beta\gamma}.$$

The price ratio is found using (2.8) and (2.9) and inserted into the above equation giving (2.13)

$$L^* = (1-\mu)^{\beta\gamma} \leq 1.$$

A.2. Derivation of (2.14)

Aggregate consumption/GNP is defined as

$$C \equiv \frac{\int\limits_{j=0}^{1} p_j c_j dj + P^C C^C}{P} = \frac{\left(C^M\right)^{\beta}\left(C^C\right)^{1-\beta}}{\beta^{\beta}\left(1-\beta\right)^{1-\beta}} = \frac{\left(L^M\right)^{\beta}\left(L^C\right)^{1-\beta}}{\beta^{\beta}\left(1-\beta\right)^{1-\beta}},$$

where the last equality follows from the production functions and clearing. L^C is isolated from (2.12) and inserted into the above equation yielding

$$C = \frac{1}{\beta}\left(\frac{1}{1-\mu}\right)^{1-\beta} L^M.$$

Combining (2.12) and the identity $L = L^M + L^C$ give $L^M = L\left(1+\frac{1-\beta}{\beta}\frac{1}{1-\mu}\right)^{-1}$ which is inserted into the above equation. This yields

$$C = \frac{1}{\beta}\left(\frac{1}{1-\mu}\right)^{1-\beta}\left(1+\frac{1-\beta}{\beta}\frac{1}{1-\mu}\right)^{-1} L.$$

Finally, insert aggregate employment from equation (2.13) and rearrange to obtain equation (2.14)

$$C^* = \frac{1}{1-\beta\mu}\left(1-\mu\right)^{\beta(\gamma+1)} \leq 1.$$

A.3. Derivation of (3.1)

The aggregate labour demand of monopolistic firms is obtained by aggregating (2.4) and using symmetry (note that we cannot use (2.10) because the marginal condition (2.8) is violated in the presence of menu costs)

$$L^M = \beta \frac{M}{P^M}. \tag{A.1}$$

Equilibrium in the labour market implies

$$\left(\frac{W}{P}\right)^{\gamma} = \beta \frac{M}{P^M} + \frac{M}{W}(1-\beta),$$

where the aggregate labour demand is obtained from (A.1) and (2.11). Inserting (2.2) and (2.9) in the above equation gives

$$\left(\frac{W}{(P^M)^{\beta}(W)^{1-\beta}}\right)^{\gamma} = \beta \frac{M}{P^M} + \frac{M}{W}(1-\beta).$$

This implies

$$\gamma \left(\frac{W}{(P^M)^{\beta}(W)^{1-\beta}}\right)^{\gamma}\left(\frac{dW}{W} - (1-\beta)\frac{dW}{W}\right) = \beta \frac{M}{P^M}\left(\frac{dM}{M}\right) + \frac{M}{W}(1-\beta)\left(\frac{dM}{M} - \frac{dW}{W}\right)$$

$\Leftrightarrow$

$$\beta\gamma \frac{dW}{W} = \frac{L^M}{L}\frac{dM}{M} + \frac{L^C}{L}\left(\frac{dM}{M} - \frac{dW}{W}\right).$$

Using (2.12) and the indentity $L = L^M + L^C$, we obtain

$$\zeta_{WM} = \frac{dW/W}{dM/M} = \frac{1-\beta\mu}{\gamma(1-\beta\mu)\beta + 1 - \beta}.$$

A.4. Derivation of (3.2)

The loss of not adjusting the price when the monopolistic firm expects that non of the other monopolistic firms change prices is approximated by making a second order Taylor expansion around the initial equilibrium. If the firm chose not to adjust the price p_j profits equal

$$\pi_1 \approx \pi_0 + \frac{\partial \pi(p_j, M)}{\partial M} dM + \frac{1}{2}\frac{\partial^2 \pi(p_j, M)}{(\partial M)^2}(dM)^2,$$

where π_0 is profits before the monetary expansion. If the firm chose to adjust the price profits equal

$$\pi_2 \approx \pi_0 + \frac{\partial \pi (p_j, M)}{\partial p_j} dp_j + \frac{\partial \pi (p_j, M)}{\partial M} dM + \frac{1}{2} \frac{\partial^2 \pi (p_j, M)}{(\partial p_j)^2} (dp_j)^2 + \frac{1}{2} \frac{\partial^2 \pi (p_j, M)}{(\partial M)^2} (dM)^2 + \frac{\partial^2 \pi (p_j, M)}{\partial p_j \partial M} dp_j dM.$$

The loss from not adjusting the price p_j is found by subtracting π_1 from π_2 and using the Envelope Theorem (i.e. $\partial \pi (p_j, M) / \partial p_j = 0$):

$$d\pi = \pi_2 - \pi_1 \approx \frac{1}{2} \frac{\partial^2 \pi (p_j, M)}{(\partial p_j)^2} (dp_j)^2 + \frac{\partial^2 \pi (p_j, M)}{\partial p_j \partial M} dp_j dM. \tag{A.2}$$

The profit function equals

$$\pi (p_j, M) = p_j \left(\frac{p_j}{P^M} \right)^{-\frac{1}{\mu}} \frac{\beta M}{P^M} - W \left(\frac{p_j}{P^M} \right)^{-\frac{1}{\mu}} \frac{\beta M}{P^M}. \tag{A.3}$$

The first order condition equals

$$\frac{\partial \pi_j}{\partial p_j} = \left(\frac{p_j}{P^M} \right)^{-\frac{1}{\mu}} \left(1 - \frac{1}{\mu} \right) \frac{\beta M}{P^M} + \frac{W}{P^M} \left(\frac{p_j}{P^M} \right)^{-\frac{1}{\mu}-1} \frac{1}{\mu} \frac{\beta M}{P^M},$$

which is equal to zero in the initial equilibrium. The second order derivatives evaluated in the initial equilibrium equal

$$\frac{\partial^2 \pi_j}{(\partial p_j)^2} = \frac{1 - \frac{1}{\mu}}{p_j} \left(\frac{p_j}{P^M} \right)^{-\frac{1}{\mu}} \frac{\beta M}{P^M}$$

$$\frac{\partial^2 \pi_j}{\partial p_j \partial M} = \left(-\frac{dW/W}{dM/M} \right) \left(\frac{p_j}{P^M} \right)^{-\frac{1}{\mu}} \left(1 - \frac{1}{\mu} \right) \frac{\beta}{P^M}.$$

Now, we derive $\frac{dp_j}{dM}$ and $\left(\frac{dp_j}{dM} \right)^2$. If the firm adjust the price, it will adjust according to the above first order condition or equivalent (2.8). This implies that $dp_j / p_j = dW/W$ giving

$$\frac{dp_j}{dM} = \frac{p_j}{M} \frac{dW/W}{dM/M}$$

$\Rightarrow$

$$\left(\frac{dp_j}{dM} \right)^2 = \left(\frac{dW/W}{dM/M} \right)^2 \left(\frac{p_j}{M} \right)^2.$$

Insert it all into (A.2) and collect the terms

$$\begin{aligned} d\pi &\approx \frac{1}{2}p_j\left(\frac{p_j}{P^M}\right)^{-\frac{1}{\mu}}\left(1-\frac{1}{\mu}\right)\frac{\beta M}{P^M}\left(\frac{dW/W}{dM/M}\right)^2\left(\frac{dM}{M}\right)^2 + \\ & \left(-\frac{dW/W}{dM/M}\right)p_j\left(\frac{p_j}{P^M}\right)^{-\frac{1}{\mu}}\left(\frac{1}{\mu}-1\right)\frac{\beta M}{P^M}\left(\frac{dW/W}{dM/M}\right)\left(\frac{dM}{M}\right)^2 \end{aligned}$$

$\Leftrightarrow$

$$d\pi \approx \frac{1}{2}p_j\left(\frac{p_j}{P^M}\right)^{-\frac{1}{\mu}}\left(\frac{1}{\mu}-1\right)\frac{\beta M}{P^M}\left(-\frac{dW/W}{dM/M}\right)^2\left(\frac{dM}{M}\right)^2.$$

The revenue of the firm equals

$$R = \left(\frac{p_j}{P^M}\right)^{1-\frac{1}{\mu}}\beta M.$$

The loss relative to the revenue of the firm equals

$$\frac{d\pi}{R} \approx \Phi\left(\mu,\beta,\gamma,m\right) \equiv \frac{1}{2}\left(\frac{1}{\mu}-1\right)\left(\zeta_{WM}\right)^2 m^2,$$

where $m = \frac{dM}{M}$. The monopolistic sector's loss relative to the total revenue in the economy (GNP) equals

$$\frac{d\pi}{GNP} \approx \frac{\beta}{2}\left(\frac{1}{\mu}-1\right)\left(\zeta_{WM}\right)^2 m^2.$$

A.5. Derivation of (3.3)

The second order approximation of the response of aggregate utility relative to GNP equals

$$\frac{dU}{C} \approx \frac{dC}{C} - \frac{\ell^{\frac{\gamma+1}{\gamma}}}{C}\left(\frac{d\ell}{\ell}+\frac{1}{2\gamma}\left(\frac{d\ell}{\ell}\right)^2\right).$$

From (2.13) and (2.14), we get

$$\frac{\ell^{\frac{\gamma+1}{\gamma}}}{C} = \frac{(1-\mu)^{\beta(\gamma+1)}}{(1-\mu)^{\beta(\gamma+1)}\frac{1}{1-\beta\mu}} = 1-\beta\mu,$$

which implies

$$\frac{dU}{C} \approx \frac{dC}{C} - (1-\beta\mu)\left(\frac{d\ell}{\ell}+\frac{1}{2\gamma}\left(\frac{d\ell}{\ell}\right)^2\right). \tag{A.4}$$

The second order approximation of the consumption/real GNP response equals

$$\begin{aligned}\frac{dC}{C} &\approx \beta\frac{dC^M}{C^M}+(1-\beta)\frac{dC^C}{C^C}+\frac{1}{2}(\beta-1)\beta\left(\frac{dC^M}{C^M}\right)^2+\\ &\quad \frac{1}{2}\beta(\beta-1)\left(\frac{dC^C}{C^C}\right)^2+\beta(1-\beta)\frac{dC^M}{C^M}\frac{dC^C}{C^C}\end{aligned}$$

From (2.4), (2.7), and (2.11), we get $\frac{dC^M}{C^M}=\frac{dM}{M}$ and $\frac{dC^C}{C^C}=\frac{dM}{M}-\frac{dW}{W}$ which are inserted into the above equation. This gives

$$\begin{aligned}\frac{dC}{C} &\approx \left(\beta+(1-\beta)\left(1-\frac{dW/W}{dM/M}\right)\right)\frac{dM}{M}\\ &\quad +\frac{1}{2}\beta(\beta-1)\left(1+\left(1-\frac{dW/W}{dM/M}\right)^2-2\left(1-\frac{dW/W}{dM/M}\right)\right)\left(\frac{dM}{M}\right)^2\end{aligned}$$

$\Leftrightarrow$

$$\frac{dC}{C}\approx(\beta+(1-\beta)(1-\zeta_{WM}))\,m-\frac{1}{2}\beta(1-\beta)(\zeta_{WM})^2m^2,$$

where $m=\frac{dM}{M}$ and $\zeta_{WM}=\frac{dW/W}{dM/M}$. Now, we elaborate on the the second order approximation of the disutility of work in (A.4). From (2.6), (2.2), and (2.9), we get $\frac{d\ell}{\ell}=\beta\gamma\zeta_{WM}m$

$$\begin{aligned}\left(\frac{d\ell}{\ell}+\frac{1}{2\gamma}\left(\frac{d\ell}{\ell}\right)^2\right) &= \beta\gamma\zeta_{WM}m+\frac{1}{2\gamma}(\beta\gamma\zeta_{WM}m)^2\\ &= \beta\gamma\left(\zeta_{WM}m+\frac{\beta}{2}(\zeta_{WM}m)^2\right).\end{aligned}$$

Inserting this together with $\frac{dC}{C}$ into (A.4), we obtain

$$\begin{aligned}\frac{dU}{C} &\approx (\beta+(1-\beta)(1-\zeta_{WM}))\,m-\frac{1}{2}\beta(1-\beta)(\zeta_{WM})^2m^2\\ &\quad -(1-\beta\mu)\beta\gamma\left(\zeta_{WM}m+\frac{\beta}{2}(\zeta_{WM})^2m^2\right),\end{aligned}\tag{A.5}$$

which is identical to (3.3).

A.6. Derivation of (3.4)

We start by deriving the first order terms by inserting ζ_{WM} from (3.1) into (3.3). This gives

$$\frac{dU}{C}\approx\beta m+(1-\beta)\left(1-\frac{1-\beta\mu}{\gamma\beta(1-\beta\mu)+1-\beta}\right)m-\left(\frac{\gamma\beta(1-\beta\mu)^2}{\gamma\beta(1-\beta\mu)+1-\beta}\right)m$$

$\Leftrightarrow$
$$\frac{dU}{C} \approx \beta\mu m.$$
The second order terms are derived by inserting ζ_{WM} from (3.1) into (A.5) and added to the above equation. This gives the second order approximation:
$$\frac{dU}{C} = \beta\mu m - \frac{\beta}{2}\left(1-\beta+\gamma\beta\left(1-\beta\mu\right)\right)\left(\zeta_{WM}\right)^2 m^2$$
$\Rightarrow$
$$\frac{dU}{C} \approx \beta\mu m - \frac{\beta}{2}\left(1-\beta+\gamma\beta\left(1-\beta\mu\right)\right)\left(\frac{1-\beta\mu}{\gamma\beta\left(1-\beta\mu\right)+1-\beta}\right)^2 m^2$$
$\Leftrightarrow$
$$\frac{dU}{C} \approx \Omega\left(\mu,\beta,\gamma,m\right) \equiv \beta\mu m - \frac{\beta}{2}\frac{\left(1-\beta\mu\right)^2}{\gamma\beta\left(1-\beta\mu\right)+1-\beta}m^2.$$
This is (3.4).

A.7. Derivation of $\Upsilon\left(.,.,.,.\right)$ in Table 2

Productivity in this setting is defined as $Z \equiv C/L$. The relative change in productivity equals
$$\frac{dZ}{Z} = \frac{dC}{C} - \frac{dL}{L} = \frac{dC}{C} - \frac{d\ell}{\ell}.$$
The last column in Table 2 states the relative change in productivity relative to the relative change in GNP
$$\Upsilon\left(.,.,.,.\right) \equiv \frac{dZ/Z}{dC/C} = 1 - \frac{d\ell/\ell}{dC/C}.$$
Inserting $d\ell/\ell$ and dC/C derived in Section A.5 and plugging in the different parameter constellations yield the figures in Table 2.

Figure 1. Initial equilibrium and monetary expansion