Endogenous Growth:
A Knife-Edge or the Razor's Edge?*

Carl-Johan Dalgaard
University of Copenhagen and EPRU

Claus Thustrup Kreiner
University of Copenhagen, EPRU and CESifo

Forthcoming: *Scandinavian Journal of Economics*

Abstract

According to much of the recent growth literature the dramatic worldwide fertility decline currently taking place should ultimately lead to global economic stagnation. This pessimistic prediction is not shared by the original innovation-based growth literature. However, this strand of literature has in recent year been criticized for resting on implausible knife-edge assumptions and for being inconsistent with available evidence. In this paper, we argue that this conclusion is unwarranted.

Keywords: Endogenous Growth Theory
JEL Classification: O40

I. Introduction

Recent population estimates (United Nations, 1998) emphasize the need for establishing whether long-run economic growth is feasible in the absence of population growth. On all continents of the world, and in particular in the regions where the bulk of the global R&D effort is taking place (i.e. US, Japan and Western Europe), fertility is rapidly declining. In fact, even if immigration is taken into account the total population in the G7 area in 2050 is expected to be roughly the same as in 1998. These demographic changes can be expected to entail the ultimate demise of global growth in living standards if an expanding population is deemed necessary for perpetual (exponential) growth. Indeed, according to one strand of the literature on innovation-based growth, perpetual increases in living standards is only possible through continual population growth (e.g. Jones, 1995b; Segerstrom, 1998). In contrast to these "semi-endogenous" growth models, the original innovation-based endogenous growth literature (Romer, 1990; Grossman and Helpman, 1991; Aghion and Howitt, 1992) delivers less gloomy outlooks because growth in income per capita is possible without increases in the size of population.[1] But which theory should one rely on when assessing the prospects for future income growth?

Due to two distinct arguments against the endogenous growth theory, the received perception seems to be that semi-endogenous growth theory is superior. The main point of this paper is to demonstrate that these two arguments rely on specific functional forms, which are chosen only for analytical convenience.

The first argument is that endogenous growth constitutes a 'knife-edge case' since it requires

---

[1]The term "semi-endogenous growth" was originally coined by Jones (1995). A model is said to exhibit semi-endogenous growth if the growth rate in per capita income is determined by an (some) exogenous – non-technological – growth rate(s). However, for all practical purposes, this implies that permanent growth hinges entirely on population growth since it is hard to think of any non-producible factor of production, other than the number of individuals in the population, that can keep on growing.

exactly constant returns to producible (and essential) factors of production. A feature that seems quite unrealistic as the standard replication argument cannot be applied to the production of factors such as scientific knowledge and human capital. Moreover, it is well known that small deviations from constant returns dramatically alter the nature of the growth equilibrium. If slightly increasing returns to producible factors are allowed, growth will 'explode'; if slightly decreasing returns to producible factors are imposed, growth will come to a halt in the absence of population growth. Hence, from this perspective, endogenous growth theory seems to be a rather special case. Indeed, in the words of Solow (1994, p.51):

> "The knife-edge character of the constant-returns model cannot be evaded [...]. The conclusion has to be that this version of the endogenous-growth model is very un-robust. It can not survive without *exactly* constant returns to capital. But you would have to believe in the tooth fairy to expect that kind of luck".

While Solow's criticism was directed primarily towards the first wave of endogenous growth models, i.e. the AK-model (Frankel, 1962; Romer, 1986), other authors (e.g. Jones, 1999 and Li, 2000; 2002) have recently argued that the innovation-based endogenous growth theory initiated by Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992) rests on a similar knife-edge assumption.

In this paper we argue that the innovation-based endogenous growth theory is more general than it appears. The argument is simple. In general, the knowledge production function may exhibit decreasing returns, increasing returns, or perhaps change over time between decreasing and increasing returns. The crucial condition for endogenous growth is that the marginal product of knowledge in producing new ideas converges towards some positive constant in the long run.

From a technical perspective this argument builds on Jones and Manuelli (1990) who demonstrated that perpetual growth may arise in the standard neoclassical growth model when the marginal product of capital is bounded from below by a positive constant, thus violating one of the Inada-conditions.[2] In contrast, we focus on the marginal productivity of *knowledge* in creating new ideas. This difference in scope is more than semantic. Whereas the Inada-condition stating that the marginal product of physical capital tends to zero as the capital stock tends to infinity (given other factors of production such as labor or land) might seem reasonable, it seems much less intuitive when imposed on the marginal productivity of the stock of knowledge in producing new ideas. The central question is this: Why would a new piece of information be completely unproductive in producing new ideas even if there did in fact exist infinitely many *other* pieces of information?[3]

The second argument against the theory of endogenous growth is that it is inconsistent with available evidence. For example, the models predict that economic policy, and incentives in general, can affect the growth rate in the long run. But as pointed out by Jones (1995a), this proposition seems inconsistent with the empirical evidence; time-series data for several OECD countries covering the last few decades shows that the growth rate in income per capita has remained remarkably stable even though tax structures have changed substantially.[4] In addition, endogenous growth models have been criticized for being at variance with observed convergence patterns (e.g. Eicher and Turnovsky, 1999b). By way of contrast we argue that endogenous growth theory may be consistent with these observations.

---

[2] Actually, this point was made early on by Pitchford (1960).

[3] In addition, assuming that the marginal productivity of knowledge in producing new ideas is bounded away from zero, asymptotically, does not lead to the unwelcome implication that the capital share of total income tends to one in the long run as is the case in Jones and Manuelli (1990).

[4] In addition, Jones notes that the number of scientists has increased rapidly in this period. This is inconsistent with endogenous growth models, which feature a scale effect on the growth rate from the number of scientists. However, by now several papers (e.g., Young, 1998; Howitt, 1999; Dalgaard and Kreiner; 2001) have shown that this prediction may be avoided in endogenous growth theory.

II. On the Generality of Perpetual Growth in the Absence of Population Growth

We consider a simple reduced form framework. Final goods, Y, are produced according to

$$Y = F(A, L; \alpha_Y), \quad 0 \le \alpha_Y \le \bar{\alpha}_Y, \tag{1}$$

where $F(\cdot)$ is a standard neoclassical production function, A is the stock of (scientific) knowledge, and L is labor input. Since our focus is on growth driven by R&D we exclude other reproducible factors of production – such as physical capital and human capital - but such inputs could be added without altering the arguments that follow in any essential way.[5] Similarily, one could add other non-producible inputs aside from labour. However, this would add nothing, since labor is the only non-producible input which can keep on growing.

For given stocks of A and L, production may change due to the shift parameter α_Y. For example, α_Y could represent the fraction of labour used in final goods production, or it may more generally represent economic institutions that are important for the effectiveness of inputs.

The R&D equation is given by

$$\dot{A} = H(A, L; \alpha_A), \quad 0 \le \alpha_A \le \bar{\alpha}_A, \tag{2}$$

where $H > 0$, $H_1 \gtreqless 0$, $H_2 \ge 0$, and α_A is a shift parameter.[6] Note, that the two shift parameters may be constrained by each other. If, for instance, the two parameters denote the fraction of labour input used in the two types of production, respectively, then the two parameters are constrained according to $\alpha_Y + \alpha_A = 1$. However, our results are independent on whether such constrains exist or not.

---

[5] In fact, one could think of A as a composite input of different producible factors of production including scientific knowledge.

[6] The case $H_1 < 0$ corresponds to the 'fishing-out' scenario (see e.g. Segerstrom, 1998).

The literature on innovation-based growth usually adopts special functional forms. For instance, it is common to assume $F(A, L; \alpha_Y) = A^\gamma (\alpha_Y L)$ and $H(A, L; \alpha_A) = \delta A^\varepsilon (\alpha_A L)^\eta$, where $\alpha_Y + \alpha_A = 1$. The parameters γ, δ, and η are all positive, while $\varepsilon \gtreqless 0$.

Endogenous growth arises only when $\varepsilon = 1$, in which case the growth rate in income, $g_Y \equiv \dot{Y}/Y$, becomes

$$g_Y = \gamma \delta (\alpha_A L)^\eta, \tag{3}$$

where L is assumed fixed. Accordingly, perpetual growth does not hinge on an expanding population. However, it may now be argued that this theory of endogenous growth is a rather special case, i.e. a knife-edge case, since it requires exactly constant returns to knowledge in producing new ideas. A feature that might seem unrealistic since the standard replication argument cannot be applied to the production of knowledge. Moreover, if ε is just slightly above one growth explodes. On the other hand, if ε is just slightly below one the nature of the growth process changes in a fundamental way. In particular, the steady state growth rate equals

$$g_Y = \left(\frac{\gamma \eta}{1 - \varepsilon} + 1 \right) n, \tag{4}$$

where n is the population growth rate, which is assumed exogenous and constant. Thus, population growth is required in order to keep economic growth in momentum. This so-called semi-endogenous growth theory arises for all values of ε below one and has therefore been argued to be more plausible than the theory of endogenous growth. However, this argument is misleading.

Going back to equation (1) and differentiating with respect to time, we have

$$g_Y = \frac{F_1 A}{Y} \frac{\dot{A}}{A} + \frac{F_2 L}{Y} \frac{\dot{L}}{L}.$$

In order to sustain innovation-based perpetual growth, it is nessesary to assume that $\frac{F_1 A}{Y} > 0$.

Except from that the production function for final goods is not crucial. From equation (2), we obtain the rate of growth in knowledge

$$g_A = \frac{H\left(A, L; \alpha_A\right)}{A} \equiv h\left(A, L; \alpha_A\right).$$

With the specific functional forms considered above $h\left(A, L; \alpha_A\right)$ is independent of the first argument in the case of endogenous growth and decreasing in the case of semi-endogenous growth. Thus, this feature of the $h\left(\cdot\right)$ function seems crucial in determining the type of growth. But this is not so. If we exclude the possibility of explosive growth, then the growth rate of A will, for a given L, converge towards some constant, i.e.

$$\lim_{A \to \infty} h\left(A, L; \alpha_A\right) = \tilde{h}\left(L; \alpha_A\right), \tag{5}$$

where $\tilde{h}$ is a constant that may depend on the parameter α_A. This yields

Proposition 1 *Perpetual growth arises in the absence of population growth if $\tilde{h}\left(L; \alpha_A\right) > 0$. Perpetual growth requires population growth if $\tilde{h}\left(L; \alpha_A\right) = 0$.*

Hence, endogenous growth is compatible with both decreasing, constant and increasing returns to knowledge in creating new ideas. The only restriction is that $h\left(A, L; \alpha_A\right)$ asymptotically tends to a strictly positive constant.[7] This is illustrated in Figure 1 where curve 1 corresponds to exactly constant returns, curve 2 to decreasing returns, curve 3 to increasing returns, whereas curve 4 shows a case where the returns to scale change as a function of the stock of knowledge. All of these curves converge towards $\bar{h}$ and feature therefore endogenous

---

[7] Weitzman (1998) provides some microfoundations for the R&D process that points toward an ever expanding number of ideas, i.e. for the condition $\lim_{A \to \infty} H_1 > 0$. The argument builds on the presumption that scientific progress often takes on the shape of combining existing pieces of knowledge in new ways. On this basis the number of ideas will tend to increase through time by combinatorial expansion. Eventually, however, growth will (or rather might) tend to a constant due to man's limited capacity to process an ever increasing abundance of new ideas.

growth. Semi-endogenous growth, on the other hand, occurs only when $h(A, L; \alpha_A)$ tends asymptotically to zero as illustrated by curve 5 in Figure 1.[8]

Note, that in the case of endogenous growth $\lim_{A \to \infty} h(A, L; \alpha_A) = \lim_{A \to \infty} H_1 > 0$, where the equality follows from L'Hopitals rule. Hence, what endogenous growth requires is that an incremental increase in the stock of knowledge, when this stock is already 'large', leads to some new ideas. Semi-endogenous growth theory, on the other hand, arises when an incremental increase in the stock of knowledge asymptotically leads to no new ideas, i.e. $\lim_{A \to \infty} H_1 \leq 0$.

>Figure 1 about here<

A simple example that illustrates some of the issues involved can be constructed using a CES function for knowledge production, i.e.,

$$H(A, L; \alpha_A) = \left[(\phi A)^{\frac{\sigma-1}{\sigma}} + ((1-\phi)\alpha_A L)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \quad \dot{L}/L = n, \tag{6}$$

where the parameters fulfill $0 \leq n < \phi < 1$, $\sigma > 0$, and $\sigma \neq 1$. Due to strict concavity of the specified H-function, the growth rate of knowledge is always a decreasing function of the stock of knowledge. Thus, the example only illustrates cases like "curve 2" or "curve 5" in Figure 1. Whether the knowledge function is described by curve 2 or curve 5 depends on whether the elasticity of substitution, σ, is above one or below one. Accordingly, the evolution of the economy depends crucially on whether $\sigma > 1$ or $\sigma < 1$. From equation (6), we obtain the following equation governing the growth rate of knowledge

$$\dot{g}_A = \left[1 - (\phi/g_A)^{\frac{\sigma-1}{\sigma}} \right] [n - g_A] g_A. \tag{7}$$

This equation gives rise to the two possible phase diagrams displayed in Figure 2a and Figure 2b. Technically speaking, there are three steady states corresponding to $g_A = 0$, $g_A = n$, and $g_A = \phi$.

---

[8] Of course, curve 5 need not be monotonically decreasing toward zero.

However, in the case $\sigma > 1$, the average product of knowledge, or equivalently g_A, is bounded from below by ϕ, implying that the economy can never venture into the shaded area. Thus, the growth rate of knowledge will gradually decline and ultimately approach ϕ. Accordingly, the long-run growth rate depends neither on the size nor the growth rate of population when the elasticity of substitution is above one. On the other hand, if the elasticity of substitution is below one, the average product of knowledge lies in the range zero to ϕ, and the growth rate of knowledge will therefore converge towards n. Hence, in this case, exponential growth in knowledge will eventually cease unless the population continues to expand.

>Figure 2a and Figure 2b about here<

Normally in the literature, a strictly concave R&D production function, like equation (6), gives rise to semi-endogenous growth. However, this is due to the focus on Cobb-Douglas functions, i.e. on an elasticity of substitution exactly equal to one.[9] Unfortunately, our knowledge of the R&D production function is limited. Specifically, we do not know whether the elasticity of substitution is above or below one.[10] Certainly, there is no particular reason to believe in exactly unity. Thus, to conclude, it is impossible to say which theory, endogenous growth or semi-endogenous growth, is the more realistic, even when confining ourselves to strictly concave R&D production function.

---

[9] Notice from equation (6) that $\lim_{\sigma \to 1} \left[(\phi A)^{\frac{\sigma-1}{\sigma}} + ((1-\phi) \alpha_A L)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} = A^\phi (\alpha_A L)^{1-\phi}$, in which case the model features semi-endogenous growth.

[10] In fact, there is little hope that such an elasticity can ever be recovered econometrically even if all inputs in the R&D production function could be quantified (i.e. including "knowledge"), cf. the non-identification theorem by Diamond, MacFadden, and Rodriguz (1978). Furthermore, even if these empirical complications are resolved and the evidence point to an elasticity of substitution below one, this does exclude that the elasticity converges to a value above one, which may be sufficient for endogenous growth.

9

III. Empirical Issues

A second line of critique raised against endogenous growth theory is empirically founded. Specifically, it has been argued that endogenous growth theory involves unrealistic convergence speeds and is inconsistent with evidence on the effect of policy on growth. We will revisit these two issues in turn.

The speed of convergence is defined as $\Phi = [\dot{g}_a(t) - \dot{g}_a^*(t)] / [g(t) - g^*(t)]$, that is the rate at which the growth rate of knowledge approaches its long-run value.[11] It follows immediately that $\lim_{g_a \to g^*} \Phi = \frac{\partial \dot{g}_a}{\partial g_a} \big|_{g_a = g_a^*}$. Hence, the rate of convergence and the condition for local stability are identical expressions. Now, consider again the commonly used functional form: $H(A, L; \alpha_A) = \delta A^\varepsilon (\alpha_A L)^\eta$. In the endogenous growth case, $\varepsilon = 1$, convergence to the steady state growth rate is instantaneous. In contrast, the convergence speed in the semi-endogenous growth model is given by ηn, reflecting the fact that the rate of knowledge growth only gradually adjusts to its long-run rate. However, with this specialized production function for knowledge the elasticity of substitution is restricted to equal one. If we instead consider the more general production function (6) then the case of endogenous growth, $\sigma > 1$, also displays a gradual adjustment towards the steady state growth rate. More specifically, the rate of convergence is $\frac{\sigma-1}{\sigma}(\phi - n)$ in the endogenous growth case, $\sigma > 1$, compared with $\left[1 - (\phi/n)^{\frac{\sigma-1}{\sigma}}\right] n$ in the case of semi-endogenous growth, $\sigma < 1$. In this example the speed of convergence under endogenous growth lies in the range zero to infinity, while it lies in the range zero to n in the semi-endogenous growth case. Without additional information about the parameters, ϕ and σ, matters cannot

---

[11] Often, the rate of convergence is formulated in levels, rather than in terms of growth rates. However, in comparing the predictions of the model with the empirical evidence, these two formulations are equivalent. To see this, note that a standard cross-county regression would be $g_y^t \equiv \ln y_t - \ln y_{t-1} = \lambda \ln y_{t-1} + \gamma x^t$, where the estimate for λ can be translated into a rate of convergence – x is a vector of additional controls. Now, take the difference between this equation and the lagged version. Then, one obtains $\Delta g_y \equiv g_y^t - g_y^{t-1} = \lambda g_y^{t-1} + \gamma \left(x^t - x^{t-1}\right)$, which corresponds to the approach taken here. Accordingly, empirical estimates for λ can equally well be capturing the convergence speed in levels as in growth rates.

be pushed much further without becoming very speculative in nature. But this illustration does suggest that, a priori, the convergence speed in the endogenous growth case is as flexible as that of the semi-endogenous growth model. This impression mirrors the conclusion emerging from recent work on implied rates of convergence within multi-sector general equilibrium endogenous and semi-endogenous growth models. Eicher and Turnowsky (1999b) show that the semi-endogenous growth model allows for convergence speeds for output per capita in line with empirical findings, i.e. in the vicinity of three percent. A similar result is, however, derived for a base line endogenous growth model augmented with capital installation costs in Ortigueria and Santos (1997). In a similar vein, Howitt (2000) demonstrates within a multi-country endogenous growth model that the implied rate of convergence towards steady state is strictly below the rate implied by the neoclassical growth model. Hence, both frameworks appear to be capable of quantitatively matching observed patters of convergence reasonably well, even when the growth engine is Cobb-Douglas.[12]

With the special functional forms considered above it is clear from equations (3) and (4) that economic incentives and policy can affect steady state growth in per capita income only in the case of endogenous growth. This feature has been criticized by Jones (1995a) on the grounds that growth in several OECD countries has been fairly constant for decades, while e.g. tax policies have changed quite extensively. Jones (1995b) argues that the semi-endogenous growth model, on the other hand, is capable of matching the facts. First, note that the condition for semi-endogenous growth, $\tilde{h}(L; \alpha_A) = 0$, implies that the importance of the shift parameter, α_A, vanishes asymptotically. Or to put it differently, economic incentives, policy, and institutions

---

[12]A separate issues raised by Eicher and Turnowsky (1999b) is whether theories of growth are consistent with differential rates of convergence in output and technology, respectively. They show that two-sector semi-endogenous growth models can have this property. While most endogenous growth models constrain output and technology to accumulate in proportion, the model developed by Howitt (2000) is also capable of generating such asymmetries.

in general do not have any systematic impact in the long run on the marginal product of knowledge in producing new knowledge, and therefore no effects on long-run income growth.[13] Although, this is consistent with the above facts the question is whether observing the lack of co-movements between some economic variables is sufficient to exclude altogether that some economic factors may matter for long-run growth.[14] Second, in the endogenous growth variant α_A influences the long-run growth rate only insofar as $\tilde{h}$ depends on this parameter. If $\tilde{h}$ and also the elasticities $F_1 A/Y$ and $F_2 L/Y$ are independent of α_A, endogenous growth occurs but policy cannot affect growth permanently. Indeed, this proporty arises in our CES example where the long-run growth rate equals ϕ when $\sigma > 1$; in this case, policy can only temporarily affect the growth rate, like in the semi-endogenous growth model. Finally, one may ask whether observations of changes in different policy variables during a period with fairly constant growth rates imply that these policy variables do not affect growth. It is easy to see that this assertion – at least in principle – is wrong. Suppose, for example, that the knowledge production function exhibits diminishing returns to knowledge, as in the case of curve 2 in Figure 1, then growth enhancing policies may simply work to counteract a tendency for declining growth rates that would otherwise have prevailed. In addition, one may argue that regressions showing that the growth rate is independent of the tax rate may be suffering from omitted variable bias. It may well be the case that a higher tax rate has significant negative effect on the growth rate, but that this is roughly offset by a significant positive growth effect of the productive government expenditure that the higher tax rate is financing, thus resulting in a small over-all net effect. In

---

[13] Note though that growth may be influenced indirectly by e.g. economic policies if it changes the elasticities in the production functions as first pointed out by Eicher and Turnovsky (1999a). Formally this arises in our model if the elasticities $F_1 A/Y$, $F_2 L/Y$, $g_1 A/g$, and $g_2 L/g$ depend on the α parameters. However, this does not lead to any obvious policy prescriptions since there is no theoretical relationship between the elasticities and the parameters.

[14] In fact, recent empirical studies, Kocherlakota and Yi (1997) and Kneller et al. (1999), do find that permanent changes in economic policy have permanent effects on income growth rates.

fact, the welfare maximizing tax rate in a simple Barro (1990) model of productive government expenditure coincides with the growth maximizing tax rate, so that if the tax rate is close to being optimal, there should be little effect on the growth rate; precisely as the observation by Jones (1995b) seems to suggest.[15]

IV.   Concluding Remarks

Is it unrealistic to assume constant returns to knowledge in producing new ideas? While it is tempting to conclude so when focusing on specific functional forms, as is normally done in the endogenous growth literature, we have argued that this conclusion is unwarrented. In fact, such production technologies arise naturally in the long run if one is willing to subscribe to the presumption that new pieces of knowledge always lead to the formation of some new ideas. In addition, we have argued that endogenous growth is consistent with available evidence on rates of convergence and policy effectiveness. Thus, endogenous growth theory should not be regarded as an implausible knife-edge case.

References

[1] Aghion, P. and P. Howitt (1992), A Model of Growth Through Creative Destruction, *Econometrica 60*, 323-51.

[2] Barro, R. (1990), Government Spending in a Simple Model of Endogenous Growth, *Journal of Political Economy 98*, S103-26.

[3] Dalgaard, C-J. and C. T. Kreiner (2001), Is Declining Productivity Inevitable?, *Journal of Economic Growth 6*, 187-203.

---

[15]We thank an anonymous referee for suggesting this argument. In this context, it is worth noting that the study by Kocherlakota and Yi (1997) demonstrates a negative effect from taxes on growth only after the expenditure side has been controlled for.

[4] Diamond, P., D. McFadden, and M. Rodriguz (1978), Measurement of the elasticity of factor substitution and bias of technical change, in Melvyn Fuss and Daniel McFadden (eds.), *Production Economics: A Dual Approach to Theory and Applications*, Vol. II, North-Holland, Amsterdam.

[5] Eicher, T. S. and S. J. Turnovsky (1999a), Non-scale Models of Economic Growth, *Economic Journal 109*, 394-415.

[6] Eicher, T.S. and S. J. Turnovsky (1999b), Convergence in a Two-Sector Nonscale Growth model, *Journal of Economic Growth 4*, 413-28

[7] Frankel, M (1962), The Production Function in Allocation and Growth: A Synthesis, *American Economic Review 52*, 995-1022.

[8] Grossman, G. M. and E. Helpman (1991), *Innovation and growth in the global economy.* MIT Press, Cambridge, MA.

[9] Howitt, P. (1999), Steady Endogenous Growth with Population and R&D Inputs Growing, *Journal of Political Economy 107*, 715-730.

[10] Howitt, P. (2000), Endogenous Growth and Cross-Country Income Differences, *American Economic Review 90*, 829-46.

[11] Jones, C. I. (1995a), Time Series Tests of Endogenous Growth Models, *Quarterly Journal of Economics 110*, 495-525.

[12] Jones, C. I. (1995b), R&D-based Models of Economic Growth, *Journal of Political Economy 103*, 759-83.

[13] Jones, C. I., (1999), Growth: With or Without Scale Effects?, *American Economic Review Papers and Proceedings 89*, 139-144.

[14] Jones, L. E. and Manuelli, R. E. (1990), A Convex Model of Equilibrium Growth: Theory and Policy Implications, *Journal of Political Economy 98*, 1008-1038.

[15] Kneller, R., M. Bleaney, and N. Gemmell (1999), Fiscal Policy and Growth: Evidence from OECD countries, *Journal of Public Economics 74*, 171-190.

[16] Kocherlakota, N. R. and Kei-Mu Yi (1997), Is there Endogenous Long-Run Growth? Evidence from the United States and the United Kingdom, *Journal of Money, Credit, and Banking 29*, 235-262.

[17] Li, Chol-Won (2000), Endogenous vs. Semi-Endogenous Growth in a Two-R&D-Sector Model, *Economic Journal 110*, C109-C122.

[18] Li, Chol-Won (2002), Growth and Scale Effects: The Role of Knowledge Spillover, *Economics Letters 74*, 177-85.

[19] Ortigueria, S. and M. Santos (1997), On the Speed of Convergence in Endogenous Growth Models, *American Economic Review 87*, 383-99.

[20] Pitchford, J. D. (1960), Growth and the Elasticity of Factor Substitution, *Economic Record 36*, 491-504.

[21] Romer, P. (1986), Increasing Returns and Long-run Growth, *Journal of Political Economy 94*, 1002-37.

[22] Romer, P. (1990), Endogenous Technical Change, *Journal of Political Economy 98*, 71-102.

[23] Segerstrom, P. (1998), Endogenous Growth without Scale Effect, *American Economic Review 88*, 1290-1310.

[24] Solow, R. M. (1994), Perspectives on Growth Theory, *Journal of Economic Perspectives 8*, 45-54.

[25] United Nations (1998), *World Population Projections to 2150*, Department of Economic and Social Affairs, Population Division, New York.

[26] Weitzman, M.L. (1998), Recombinant Growth, *Quarterly Journal of Economics 113*, 331-60.

[27] Young, A. (1998), Growth without Scale Effects, *Journal of Political Economy 106*, 41-63.

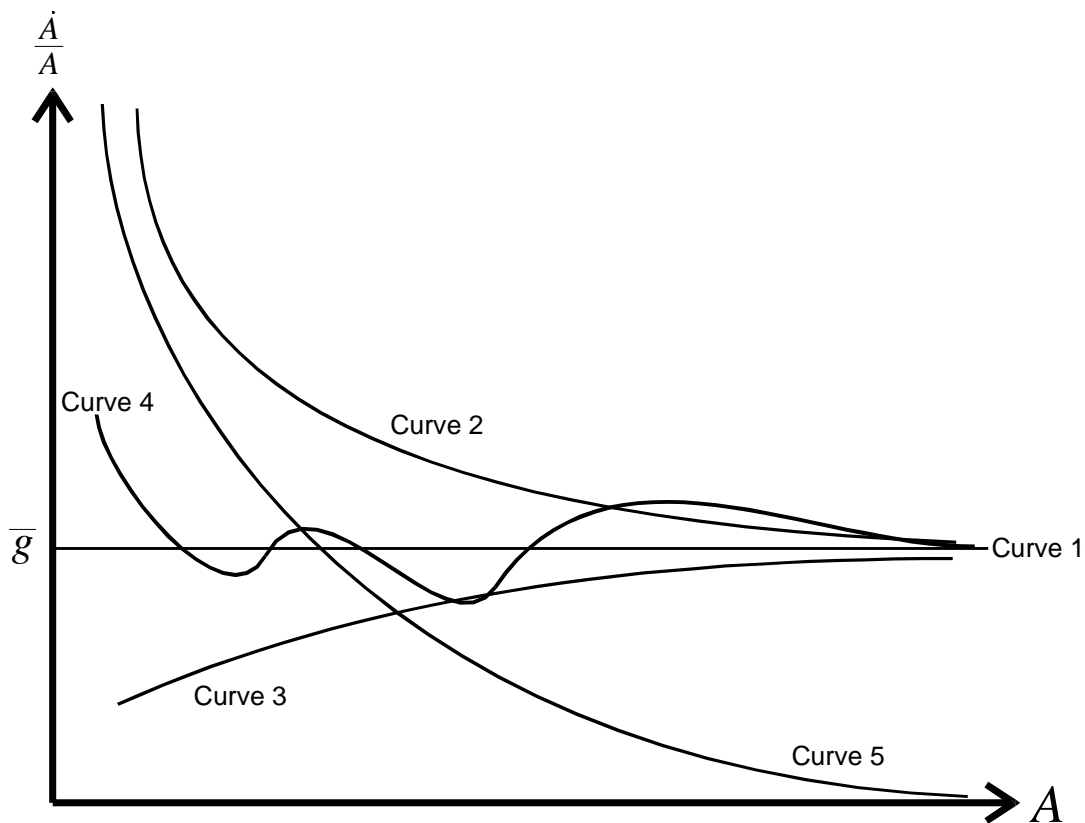**Figure 1. Knowledge growth as function of the stock of knowledge**
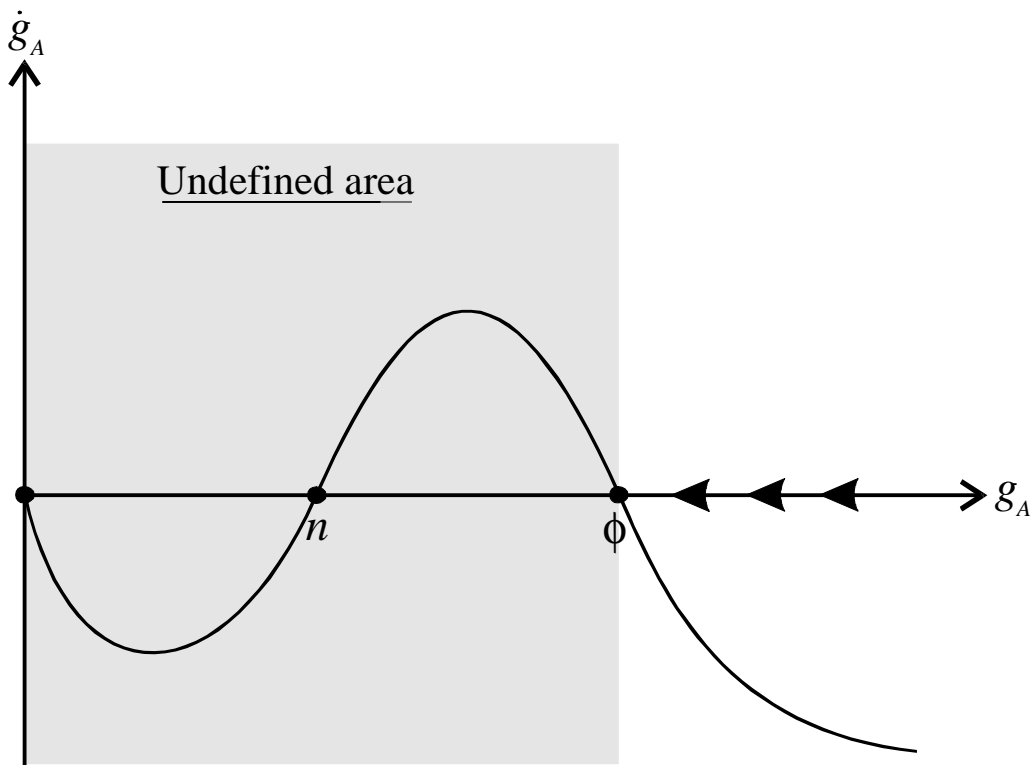
Figure 2a. Dynamics when σ > 1

Figure 2b. Dynamics when σ < 1