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# The Endogenous Grid Method without Analytical Inverse Marginal Utility*

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**Abstract**

The computational time required to solve and estimate dynamic economic models is one of the main constraints in empirical research. The Endogenous Grid Method (EGM) proposed by Carroll (2006) is known to offer impressive speed gains over more traditional stochastic dynamic programming methods, such as Value Function Iterations (VFI). However, existing EGM implementations implicitly require an analytical expression for the inverse marginal utility, which is not known in many interesting cases. We propose a simple and fast approach, which we refer to as the interpolated EGM (iEGM), that can be applied even when the inverse marginal utility is not known analytically. We show through two applications that the iEGM inherits the speed and accuracy of the EGM and that our approach is an order of magnitude faster than traditional approaches.

**JEL-codes:** D13, D15, C61, C63, C78.

**Keywords:** Endogenous grid method, dynamic programming, numerical methods, limited commitment.

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# 1 Introduction

The Endogenous Grid Method (EGM) proposed by Carroll (2006) is an example of a computational advancement that offers impressive speed gains over more traditional stochastic dynamic programming methods, such as value function iterations, VFI (see e.g. White, 2015 and Fella, 2014). The EGM alleviates the need for numerical solvers by inverting first order conditions, but relies on the existence of an inverse marginal utility function. All existing work employing the EGM has been restricted to models with an analytical inverse marginal utility.

In this paper, we propose an approach for using the EGM even when the inverse marginal utility is not known analytically. Concretely, we suggest constructing an interpolator of the inverse marginal utility before solving the model. Precomputing the interpolator is computationally costless and simple to implement. We refer to the EGM with interpolated inverse marginal utility as the interpolated EGM (iEGM).

The iEGM expands the class of models that are practically solvable with the EGM. Examples without an analytical inverse marginal utility function include many models of consumption and housing where consumers have CES-type utility over consumption and housing services (see e.g. Yang, 2009) and dynamic bargaining models with limited commitment (see eg. Mazzocco, 2007; Hallengreen, Jørgensen and Olesen, 2024). Until now, such popular models could only be solved using potentially time consuming numerical solvers. The iEGM alleviates this computational burden, allowing for more realistic economic models. Importantly, the iEGM can be applied in combination with previous extensions of the EGM to models with multiple dimensions, multiple constraints, and mixed discrete-continuous choices.[1]

We demonstrate the performance of the iEGM through two examples. For expositional simplicity, we illustrate the approach by solving the canonical buffer-stock model (see e.g. Carroll, 1992; Deaton, 1991; and Gourinchas and Parker, 2002) with four methods: VFI, EGM, EGM with a numerical inverse, and iEGM. In this example, the inverse marginal utility is known analytically, enabling speed and accuracy comparisons across the four approaches. We show that the iEGM inherits the speed and accuracy benefits of the EGM. Next, we turn to a much richer model of dynamic household bargaining with limited commitment where the original EGM cannot be used, but the iEGM can. We illustrate through this example that the iEGM is 10 times faster than the EGM using a numerical

[1] See e.g. Barillas and Fernández-Villaverde (2007); Hintermaier and Koeniger (2010); Ludwig and Schön (2014); Fella (2014); White (2015); Iskhakov, Jørgensen, Rust and Schjerning (2017); and Druedahl and Jørgensen (2017).

solver to invert the marginal utility and 50 times faster than VFI.[2]

The remainder of this paper is organized as follows. In the next section, we illustrate the idea of the basic EGM before presenting our iEGM approach in Section 3. In Section 4, we report timing and accuracy results for the two models across solution approaches before concluding in Section 5.

## 2 The Endogenous Grid Method (EGM)

To fix ideas, we first illustrate the EGM in the canonical buffer-stock model. The Bellman equation of this model is

$$\begin{aligned}
V_t(M_t, P_t) &= \max_{C_t} U(C_t) + \beta \mathbb{E}_t \left[ V_{t+1}(M_{t+1}, P_{t+1}) \right] \\
&\text{s.t.} \\
A_t &= M_t - C_t && \text{(assets)} \\
M_{t+1} &= RA_t + Y_{t+1} && \text{(resources/cash-on-hand)} \\
Y_{t+1} &= P_{t+1}\xi_{t+1} && \text{(income)} \\
P_{t+1} &= GP_t\psi_{t+1} && \text{(perm. income)} \\
\log \xi_{t+1} &\sim \mathcal{N}(-0.5\sigma_\xi, \sigma_\xi^2) && \text{(trans. income shock)} \\
\log \psi_{t+1} &\sim \mathcal{N}(-0.5\sigma_\psi, \sigma_\psi^2) && \text{(perm. income shock)} \\
A_t &\geq 0, \forall t && \text{(no borrowing)}
\end{aligned} \tag{1}$$

The first order condition for an interior solution is

$$U'(C_t) = \beta R \mathbb{E}_t[V_{t+1}^1(M_{t+1}, P_{t+1})] \tag{2}$$

where $V_{t+1}^1(M_{t+1}, P_{t+1})$ is the partial derivative w.r.t. resources, $M_{t+1}$ and $U'(C_t)$ is the marginal utility of consumption. The envelope theorem then gives the standard Euler equation. The main idea of the EGM is to realize that given a post-decision level of assets after consumption, $A_t = a$, the expected discounted marginal utility,

$$W_t = \beta R \mathbb{E}_t[V_{t+1}^1(Ra + Y_{t+1}, P_{t+1})]$$

on the right-hand-side of (2) is known up to random shocks. We can then invert the first

[2] All code is available at https://github.com/ThomasHJorgensen/iEGM.

order condition to get consumption in closed form without having to rely on numerical solvers[3]

$$C_t^{\star} = U'^{-1}(W_t) \tag{3}$$

where $U'^{-1}(W)$ is the *inverse marginal utility function* taking marginal utility as input and returns consumption. The associated endogenous level of resources is then $M_t = C_t^{\star} + a$.

All existing research, employing the EGM, has employed functional forms that give analytical expressions for this key function. For example, if utility is constant relative risk aversion (CRRA), the inverse marginal utility function is $U'^{-1}(W) = W^{-1/\rho}$ where $\rho$ is the CRRA coefficient.

## 3 EGM without Analytical Inverse Marginal Utility (iEGM)

Many interesting economic models do not, however, have an analytic inverse marginal utility function.[4] An example of such a model is the so-called limited commitment model in which couples bargain dynamically over economic resources to be allocated to private consumption, $c_{j,t}$ for $j \in \{1,2\}$ and public consumption, $c_t$ (see e.g. Mazzocco, 2007). Assuming that the consumption allocation between private and public consumption is a purely intra-temporal problem conditional on total consumption, $C$, the household objective function when remaining a couple can be thought of as a function of total consumption,

$$\begin{aligned} U(C) &\equiv \max_{c_{1,t},c_{2,t}} \mu U_1(c_{1,t}, c_t) + (1-\mu)U_2(c_{2,t}, c_t) \\ &\text{s. t.} \\ c_t &= C - c_{1,t} - c_{2,t} \end{aligned}$$

where $\mu$ is the bargaining weight on the utility of household member 1. The inverse of the marginal household utility, $U'^{-1}(W)$, cannot be found analytically in this relevant case. In fact, even in cases where the inverse marginal utility can be found analytically, it might be easier and less error-prone to implement our approach.

In cases where the inverse marginal utility, $U'^{-1}(W)$, is not known analytically, it could

[3]This trick often provides great speed gains, see e.g. Jørgensen (2013) and references in the Introduction.

[4]The conditions for applicability of the EGM, discussed in Druedahl and Jørgensen (2017), should still be fulfilled.

be found using a numerical solver such that

$$C_t^\star = \{C : U'(C) - W_t = 0\}. \tag{4}$$

Since the bulk of the computational cost lies in computing $W_t$, such a numerical inverse could be a viable approach, if the computational cost of evaluating $U'(C)$ is not too large. However, in our limited commitment example, evaluating the marginal utility using finite differences would potentially require solving the intra-temporal problem several times. In such cases, evaluating the inverse marginal utility numerically is rather costly.

We instead propose to replace the inverse with a precomputed interpolator, $\check{C}(W)$, that typically can be constructed without a significant change in the computation speed and accuracy of the numerical solution. In turn, the only modification to the EGM implementation is replacing equation (3) with

$$C_t^\star = \check{C}(W_t), \tag{5}$$

where a wide variety of interpolation methods such as B-splines or projection methods can be used to construct the interpolator.

We construct the interpolator through two simple steps.[5] First, we construct a grid over consumption, $\overrightarrow{C}$, with $\#_C$ number of points. Evaluating the marginal utility for all points in this grid gives a grid of associated marginal utilities, $\overrightarrow{W} = U'(\overrightarrow{C})$.[6] Panel a) of Figure 1 shows this relationship with the constructed consumption grid, $\overrightarrow{C}$, on the x-axis and the associated marginal utility on the y-axis under the assumption that preferences are CRRA with $\rho = 1.5$ such that $U'(C) = C^{-1.5}$.

Second, we use these associated grids to construct an interpolator for $C$ as a function of $W$. We then simply "flip the axes", such that we can use the calculated marginal utility grid points in $\overrightarrow{W}$ together with the associated consumption grid points in $\overrightarrow{C}$ to construct an interpolator, $\check{C}(W)$, of optimal consumption for values of marginal utility. We show this in panel b) of Figure 1. The solid line shows the actual inverse marginal utility in the CRRA example from above, and the dashed lines connecting the known node points illustrate linear interpolation between these points.

In our example, we have abstracted from the potential dependency on other choices and/or state variables since the interpolation can simply be constructed for all relevant

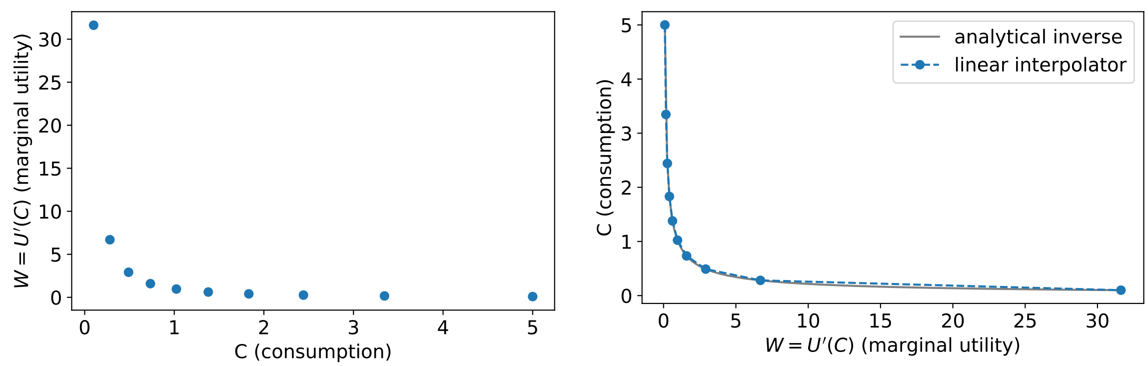
[5]Chebyshev interpolation can be employed following an alternative strategy. Instead of the two steps, use a numerical solver, as discussed above, to get the inverse marginal utility in the precomputation step in order to control exactly where the grid points in $\overrightarrow{W}$ are placed.

[6]If the marginal utility is not known analytically, it can e.g. be approximated by finite differences of the utility function, $U(C_t)$. This is the approach we employ in the limited commitment application below.

conditioning variables that enter in the marginal utility. In the buffer-stock example, the interpolator is independent of both state variables.

Figure 1: Construction of Interpolator of Optimal Consumption, $\check{C}(W)$.

(a) Grids over $\overrightarrow{C}$ and $\overrightarrow{W} = U'(\overrightarrow{C})$. (b) Flipped axis and interpolator, $\check{C}(W)$.




*Notes:* The figure illustrates the two steps of the proposed approach in panel a) and b). Panel a) shows how constructing a grid of $\#_C = 10$ points over values of consumption $\overrightarrow{C}$ can be used to evaluate the marginal utility, $\overrightarrow{W} = U'(\overrightarrow{C})$. Panel b) shows how flipping the axes provides nodes which can be used to construct an interpolator for the object of interest, namely $\check{C}(W)$ using the known points in $(\overrightarrow{W}, \overrightarrow{C})$. The solid line shows the actual inverse marginal utility with CRRA utility with CRRA coefficient of $\rho = 1.5$, and the dashed lines connecting the known node points illustrate linear interpolation between these points.

# 4 Accuracy and Speed

Here we show accuracy and computation time for VFI, the analytical EGM (when available), the EGM with numerical inverse as in eq. (4) and the iEGM as in eq. (5) for an increasing number of precomputation nodes, $\#_C$. The computation time is relative to that of VFI. We show results from a linear interpolator and a modified linear interpolator, where we interpolate (linearly) the reciprocal of consumption, $1/C$, since that function has less curvature and is bounded at zero for low values of marginal utility (see panel b in Figure 1).

To measure the accuracy of the solution across implementations, we first solve a very accurate benchmark model with VFI and very dense solution grids. We then compare the average sum of discounted simulated utility of each of our implementations with that of the "true" model. We report the percentage difference from the "true" discounted lifetime utility as a measure of the solution accuracy, following Druedahl (2021).

We report results for the buffer-stock model and the richer limited commitment model in Table 1. The Supplemental Material contains a description of the numerical solution and simulation of both models. See also Hallengreen, Jørgensen and Olesen (2024) for a detailed description of the limited commitment.[7] All results are based on 200 Monte Carlo simulations of $N = 10,000$ consumers for $T = 20$ periods.

Table 1: Accuracy and Computation Time across Methods and Models.

| | Buffer-stock model | | Limited commitment | |
|---|---|---|---|---|
| | Accuracy (%) | Time (rel. to VFI) | Accuracy (%) | Time (rel. to VFI) |
| VFI | 0.000 | 1.000 | 0.163 | 1.000 |
| EGM, analytical | 0.000 | 0.063 | N.A. | N.A. |
| EGM, numerical | 0.000 | 0.111 | 0.163 | 0.290 |
| iEGM, linear | | | | |
| $\#_C = 20$ | 0.791 | 0.064 | 0.179 | 0.021 |
| $\#_C = 50$ | 0.081 | 0.064 | 0.163 | 0.021 |
| $\#_C = 100$ | 0.015 | 0.064 | 0.163 | 0.021 |
| $\#_C = 200$ | 0.003 | 0.064 | 0.163 | 0.021 |
| iEGM, linear reciprocal | | | | |
| $\#_C = 20$ | 0.197 | 0.064 | 0.163 | 0.020 |
| $\#_C = 50$ | 0.014 | 0.064 | 0.163 | 0.021 |
| $\#_C = 100$ | 0.003 | 0.064 | 0.163 | 0.021 |
| $\#_C = 200$ | 0.000 | 0.064 | 0.163 | 0.021 |

*Notes:* The table reports accuracy and computation time for the buffer-stock model in columns 2-3 and the limited commitment model in columns 4-5. Accuracy is measured as percent deviation in avg. discounted sum of simulated life time utility from the "true" model and computation time is relative to that of VFI. The "true" model is a version of the model solved with VFI with significantly denser grids over all states. See the Supplementary Material for details. We show results for the analytical endogenous grid method (EGM) proposed by Carroll (2006) when available and a numerical inverse EGM, using eq. 4. For all models, we report results for our iEGM method with varying number of precomputation grid points, $\#_C$, using linear interpolation of consumption and linear interpolation of the reciprocal of consumption. See the Supplementary Material and Hallengreen, Jørgensen and Olesen (2024) for implementation details.

The results are very encouraging. From the buffer-stock model in columns 2-3, in line with the existing literature, we see that the EGM produces a more accurate solution in only around 7% of the time it takes to solve the model with VFI (White, 2015; Fella, 2014). We also see that our iEGM using linear interpolation is just as fast as the baseline EGM. Even with relatively few points in the precomputed interpolator, the deviations from the "true" model are less than 1%. Increasing the number of points improves accuracy to be almost on par with VFI without notably increasing computation time. Linear interpolation of the reciprocal consumption further improves accuracy.

Turning to the limited commitment model in columns 3-4, we see a similar picture. The iEGM is 50 times faster than the VFI without a significant reduction in the accuracy

[7] All results are generated on a Dell PowerEdge R640 Server with 36 dual-core Intel Xeon Gold 6154 3.0GhZ processors and 768GB RAM.

of the solution. Additionally, the iEGM solves the model ten times faster than the EGM using a numerical inverse with no reductions in accuracy. Such large speed gains are rare and could lead to CPU times associated with estimation of, say, a month rather than a year.

## 5 Concluding Discussion

We propose a simple and yet fast and accurate approach to implement the EGM even when a key object, namely the inverse marginal utility, is not known analytically. The approach, which we refer to as the interpolated EGM (iEGM), builds an interpolator of the inverse marginal utility and uses that to solve for continuous choices without having to rely on numerical solvers.

We illustrate the effectiveness of the iEGM through two applications; the buffer-stock model and the limited commitment model of dynamic household bargaining. While the former can be solved with the standard EGM, the latter cannot. The iEGM delivers accurate model solutions. In the buffer-stock model example, we can compare performance with the analytical EGM and show that the iEGM can attain similar accuracy without a significant increase in the computation time. In our second example with a model of dynamic bargaining with limited commitment, where marginal utility is costly to evaluate, the iEGM is an order of magnitude faster than EGM with numerical inverse while preserving accuracy. The iEGM is up to 50 times faster than VFI in this example.

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# Supplementary Material

## A Numerical Details related to the Buffer-Stock Model

We solve the buffer-stock model in ratio form, where all small-letter variables denote upper-case variables normalized by permanent income, $P_t$, $x_T = X/P_t$, similarly to e.g. Gourinchas and Parker (2002). We approximate the two-dimensional integral over future transitory and permanent shocks using Gauss-Hermite quadrature with $Q = 5$ nodes in each dimension (25 nodes in total). We solve and simulate the models for $N = 10,000$ consumers for $T = 20$ periods in each of the 200 Monte Carlo runs. The parameters of the model are $\beta = 0.99$, $\rho = 1.5$, $G = 1.02$, $R = 1.03$, $\sigma_\psi = 0.1$, $\sigma_\xi = 0.1$.

**VFI** is implemented as follows: We construct a grid over beginning-of-period (normalized) resources, $\overrightarrow{m}$ with $\#_m = 100$ grid points with a maximum level of resources of 5. We assume that all resources are consumed in the final period of life such that $\overrightarrow{c}_T = \overrightarrow{m}$ and $\overrightarrow{v}_T = U(\overrightarrow{c}_T)$. In previous periods, we loop through points in $\overrightarrow{m}$ and for the $k$th point in the grid, we solve using a numerical solver,

$$c_t[k] = \arg\max_c U(c) + \beta \sum_{j=1}^{Q} \sum_{l=1}^{Q} \omega_j \omega_l (G_{t+1}\psi^j)^{1-\rho} \breve{v}_{t+1}(\underbrace{R(\overrightarrow{m}[k] - c) \cdot (G_{t+1}\psi^j)^{-1} + \xi^l}_{=m_{t+1}^{jl}})$$

where $(\omega_j, \omega_l)$ are Guass-Hermite quadrature nodes associated with the permanent and transitory income shocks, respectively, $(\psi^j, \xi^l)$, and $\breve{v}_{t+1}(m_{t+1})$ is a linear interpolator of the next-period value function. The value function is then
$v_t[k] = U(c_t[k]) + \beta \sum_{j=1}^{Q} \sum_{l=1}^{Q} \omega_j \omega_l (G_{t+1}\psi^j)^{1-\rho} \breve{v}(R(\overrightarrow{m}[k] - c_t[k]) \cdot (G_{t+1}\psi^j)^{-1} + \xi^l)$.

**EGM** is implemented as follows: We construct a grid over end-of-period (normalized) savings, $\overrightarrow{a}$ with $\#_a = 100$ grid points with a minimum level of zero and a maximum of 10. We assume that all resources are consumed in the final period of life such that $\overrightarrow{c}_T = \overrightarrow{m}_T$, where $\overrightarrow{m}_T = \overrightarrow{m}$ from above. In previous periods, we loop through points in $\overrightarrow{a}$ and for the $k$th point in the grid, we invert the Euler equation, using that $U'^{-1}(W) = W^{-1/\rho}$ in this simple example,

$$c_t[k] = (W_t[k])^{-1/\rho}$$

where

$$W_t[k] = \beta R \sum_{j=1}^{Q} \sum_{l=1}^{Q} \omega_j \omega_l (G_{t+1}\psi^j)^{-\rho} \left[ \check{c}_{t+1}(\underbrace{R\overrightarrow{a}[k] \cdot (G_{t+1}\psi^j)^{-1} + \xi^l}_{=m_{t+1}^{jl}}) \right]^{-\rho}$$

is the expected discounted marginal utility of consumption next period and $\check{c}_{t+1}(m_{t+1})$ is a linear interpolator of the next-period consumption. The endogenous level of resources is then $\overrightarrow{m}_t = \overrightarrow{c}_t + \overrightarrow{a}$ and the value function is not stored since it is not needed.

We also implement a numerical inverse for comparison. In this case, we use a root finding algorithm to find the consumption level that solves

$$c_t[k] = \{c : c^{-\rho} = W_t[k]\}.$$

**iEGM** is implemented as the EGM above but with the only change that

$$c_t[k] = \check{C}(W_t[k])$$

where $\check{C}(W)$ is an interpolator constructed as described in the main text. We vary the number of points, $\#_C$ in the the grid over consumption, $\overrightarrow{C}$ across specifications but fix the maximum amount in the grid to 10. All other grids are like above.

**The "true"** model is solved using VFI as above but with $\#_m = 500$ points in the state variable.

**The initial** normalized wealth, $a_{i,0}$, in simulations are drawn from a uniform distribution between zero and 2.5 (50% of maximum level of resources in the in the VFI solution).

# B Numerical Details related to the Limited Commitment Model

Here we first give a brief overview of how the model is solved before for a detailed description of the model and solution algorithm. The states in this problem for a couple remaining together is wealth, $A_{t-1}$, match quality, $\psi_t$, and the bargaining power, $\mu_{t-1}$. We use $\#_A = 50$, $\#_\psi = 21$, and $\#_\mu = 21$. The choices of a couple are all related to the consumption allocation between public consumption, $c_t$, and private consumption of each household member, $c_{j,t}$ for $j \in \{1,2\}$. The household utility can thus be thought of as a function of total consumption, $C$,

$$\begin{aligned} U(C;\mu_{t-1}) &= \max_{c_{1,t},c_{2,t}} \mu_{t-1}U_1(c_{1,t},c_t) + (1-\mu_{t-1})U_2(c_{2,t},c_t) \\ &\text{s. t.} \\ c_t &= C - c_{1,t} - c_{2,t} \end{aligned} \tag{B.1}$$

Conditional on $C$, we can then solve for optimal allocation between $c_t$, $c_{1,t}$ and $c_{2,t}$ using a numerical solver. All code is implemented in c++ and parallelized wrt. $\mu_{t-1}$ using 20 threads. We use NLopt's BOBYQA algorithm in all numerical solvers.

**VFI** is implemented similarly to the buffer-stock model above. We solve for *total consumption*, $C_t$, for a couple conditional on remaining together, at the $k$th point in the wealth grid, the $l$th point in the match quality grid and the $m$th grid point in the power-grid, as

$$C_t^\star[k,l,m] = \arg\max_{C_t} U(C_t; \overrightarrow{\mu}[m]) + \beta \sum_{q=1}^{Q} \omega_q \check{V}_{t+1}^m(A_t, \psi_{t+1}, \overrightarrow{\mu}[m]) \tag{B.2}$$

where $A_t = R\overrightarrow{A}[k] + Y_{t+1}$ and $\psi_{t+1} = \overrightarrow{\psi}[l] + \varepsilon_q$. We use $Q = 5$ Gauss-Hermite nodes to approximate the expectation wrt. future match quality. $\check{V}_{t+1}^m(\bullet)$ is a bilinear interpolator, wrt. wealth and match quality, of the next-period value function conditional entering the next period as married. The household utility function is found by solving eq. B.3.

**EGM** can only be solved using a numerical solver for the inverse of the marginal household utility. We solve this for the $k$th point in the wealth grid, the $l$th point in the match quality grid and the $m$th grid point in the power-grid, using a numerical solver that minimizes

$$C_t^\star[k,l,m] = \arg\min_{C} (U'(C; \overrightarrow{\mu}[m]) - W_t[k,l,m])^2 \tag{B.3}$$

wrt. consumption, $C$, given the discounted expected marginal utility

$$W_t[k,l,m] = \beta \sum_{q}^{Q} \omega_{q=1} d\check{V}_{t+1}^m(A_t, \psi_{t+1}, \overrightarrow{\mu}[m])$$

where $A_t = R\overrightarrow{A}[k] + Y_{t+1}$ and $\psi_{t+1} = \overrightarrow{\psi}[l] + \varepsilon_q$. We use $Q = 5$ Gauss-Hermite nodes to approximate the expectation wrt. future match quality. $d\check{V}_{t+1}^m$ is a linear interpolator of the marginal utility of wealth. This object is very complicated, since it depends on, among other elements, the marginal effect of wealth on the future bargaining power. We thus approximate this object using the found value function *level* and calculating finite differences a a given grid point, $k$, wrt. wealth,

$$dV_t^m(\overrightarrow{A}[k],\bullet) \approx (V_t^m(\overrightarrow{A}[k+1],\bullet) - V_t^m(\overrightarrow{A}[k-1],\bullet))/(\overrightarrow{A}[k+1] - \overrightarrow{A}[k-1]).$$

Since $U'(C)$ is not known analytically in B.3, we find it using forward finite differences of eq. B.1. We use as starting values for $C$ (and $c_{1,t}, c_{2,t}$, and $c_t$) the previously found solution (across end-of-period wealth).

**iEGM** replaces eq. B.3 with a precomputed interpolator,

$$C_t^\star[k,l,m] = \check{C}(W_t[k,l,m]; \overrightarrow{\mu}[m]) \tag{B.4}$$

as described in the main text. Since this depends on the bargaining power, we construct this interpolator for each point of the $\#_\mu$ points in the power-grid, $\overrightarrow{\mu}$. We construct this interpolator by constructing a grid over total consumption with $\#_C$ points in it (for each point in $\overrightarrow{\mu}$). We, as discussed above, use forward finite differences when evaluating the marginal utility in each consumption grid point to get the associated marginal utility grid points.

**The "true"** model is solved using VFI and $\#_A = 250$, $\#_\psi = 51$, and $\#_\mu = 51$ grid points in each of the states.

**Initial states** when simulating are given as follows. All individuals are initialized in a couple with a match quality of zero, $\psi_{i,0} = 0\,\forall i$, with a uniformly distributed distributed level of wealth, $A_{i,0}$ between zero and 7.5 (50% of the maximum point in the grid of wealth, used when solving the model with VFI).

## B.1 Limited Commitment Details

In this section, we present a model of dynamic consumption allocations with limited commitment bargaining within a household. The household consists of a woman and a man, indexed $w$ and $m$, respectively. The couple bargains according to the algorithm described in Hallengreen, Jørgensen and Olesen (2024) and split up if an agreement cannot be reached. Single individuals participate in the marriage market and can remarry if they meet a suitable partner. The full dynamics of the couples' and singles' problems, respectively, are described in the following.

In this example, we set up a model where couples choose individual consumption, $c_{j,t}$ to $j \in \{w, m\}$, and public consumption, $c_t$. We have three state variables: beginning of period $t$ wealth, $A_{t-1}$, match quality, $\psi_t$ and the bargaining power coming into the period, $\mu_{t-1}$. From the previous notation, this corresponds to $\mathcal{S}_t = (\psi_t, A_{t-1})$.

Individual preferences are of the CES type,

$$U_j(c_{j,t}, c_t) = \frac{1}{1-\rho_j} \left( \alpha_{1,j} c_{j,t}^{\phi_j} + \alpha_{2,j} c_t^{\phi_j} \right)^{1-\rho_j} \tag{B.5}$$

and the budget constraint for a couple is

$$A_t + c_t + c_{w,t} + c_{m,t} = RA_{t-1} + Y_{w,t} + Y_{m,t}, \quad A_t \geq 0$$

where $R$ is the gross interest rate and $Y_{j,t}$ is exogenous income of member $j$. The household utility function is a weighted sum of individual utilities with the weight $\mu$ on the woman's utility. Couples also receive utility from match quality, $\psi_t$, which enters additively in the value function. Match quality follows a unit root process:

$$\psi_{t+1} = \psi_t + \varepsilon_{t+1}$$

where $\varepsilon \sim iid\mathcal{N}(0, \sigma_\psi^2)$. This "love shock" is the only source of uncertainty for couples.

Single individuals also choose individual consumption, $c_{j,t}$ and public consumption, $c_t$. The state variable for singles is $\mathcal{S}_{j,t} = (A_{j,t-1})$, since singles do not engage in bargaining or have a match quality. Individual preferences are still described by (B.5). Singles face the budget constraint

$$A_{j,t} + c_t + c_{j,t} = RA_{j,t-1} + Y_{j,t}, \quad A_{j,t} > 0$$

**The value of remaining single** is

$$V_{j,t}^{s\to s}(A_{j,t-1}) = \max_{c_{j,t}, c_t} U_j(c_{j,t}, c_t) + \beta \mathbb{E}_t[V_{j,t+1}^s(A_{j,t}, A_{j,t}^p, \psi_{t+1})]$$

where $\mathbb{E}_t[V_{j,t+1}^s(A_{j,t}, A_{j,t}^p, \psi_{t+1})]$ denotes the expected value of entering period $t+1$ as single (described below).

**The expected value of entering a period as single** is comprised of the value of meeting a partner, which happens with probability $p_t$, and the value of staying single:

$$\mathbb{E}_t[V^s_{j,t+1}(A_{j,t}, A^p_{j,t}, \psi_{t+1})] = p_t \mathbb{E}_t\left[\tilde{V}_{j,t+1}(A_{j,t}, A^p_{j,t}, \psi_{t+1})\right] + (1-p_t)V^{s\to s}_{j,t+1}(A_{j,t})$$

where $\tilde{V}_{j,t}(A_{j,t-1}, A^p_{j,t-1}, \psi_t)$ denotes the value of meeting a partner with assets $A^p_{j,t-1}$ and initial match quality $\psi_t$:

$$\tilde{V}^s_{j,t}(A_{j,t-1}, A^p_{t-1}, \psi_t) = M^\star_t V^{s\to m}_{j,t}(\psi_t, A_{t-1}) + (1-M^\star_t)V^{s\to s}_{j,t}(A_{j,t-1})$$
$$\text{s. t.}$$
$$A_{t-1} = A_{j,t-1} + A^p_{j,t-1},$$

where $V^{s\to m}_{j,t}(\psi_t, A_{t-1})$ is the value of transitioning from singlehood to marriage, $V^{s\to s}_{j,t}(A_{j,t-1})$ is the value of remaining single, and $M^\star_t$ is the optimal choice to marry or not (defined later).

When taking the expectation of $\tilde{V}_{j,t}(\bullet)$ with respect to the characteristics of the partner, we let the partner's wealth conditional on own wealth, and initial match quality follow the independent distributions $\Gamma_{A^p_j}(a|A_{j,t})$ and $\Gamma_\psi(\psi)$. In turn, the expected value is

$$\mathbb{E}_t[\tilde{V}^s_{j,t}(A_{j,t-1}, A^p_{j,t-1}, \psi_t)] = \int_0^\infty \int_{-\infty}^\infty \tilde{V}^s_{j,t}(A_{j,t-1}, a, \psi)\Gamma_\psi(\psi)\Gamma_{A^p_j}(a|A_{j,t-1})d\psi da.$$

**The value of transitioning from couple to single** is similar to remaining single but couples incur a divorce cost of $\chi$

$$V^{m\to s}_{j,t}(A_{j,t-1}) = V^{s\to s}_{j,t}(A_{j,t-1}) - \chi$$

where all choices thus are identical as those of someone remaining single.

**The value of remaining a couple** with bargaining power $\mu$ is:

$$
\begin{aligned}
V_{j,t}^{m\to m}(\psi_t, A_{t-1}, \mu) = U_j(\tilde{c}_{j,t}, \tilde{c}_t) + \psi_t + \beta \mathbb{E}_t[V_{j,t+1}^m(\psi_{t+1}, A_t, \mu)] \\
\text{s.t.} \\
A_t = RA_{t-1} + Y_{w,t} + Y_{m,t} - (\tilde{c}_t + \tilde{c}_{w,t} + \tilde{c}_{m,t}) \\
\psi_{t+1} = \psi_t + \varepsilon_{t+1}
\end{aligned}
$$

where $(\tilde{c}_{w,t}, \tilde{c}_{m,t}, \tilde{c}_t)$ is the optimal consumption allocation conditional on $\mu$. This is determined by solving the couple's optimization problem conditional on remaining together with the level of bargaining power being $\mu$:

$$
\begin{aligned}
\tilde{c}_{w,t}(\mu), \tilde{c}_{m,t}(\mu), \tilde{c}_t(\mu) = \arg \max_{c_{w,t}, c_{m,t}, c_t} \mu v_{w,t}(\psi_t, A_{t-1}, c_{w,t}, c_{m,t}, c_t, \mu) \\
+ (1-\mu) v_{m,t}(\psi_t, A_{t-1}, c_{w,t}, c_{m,t}, c_t, \mu) \\
\text{s.t.} \\
A_t = RA_{t-1} + Y_{w,t} + Y_{m,t} - (c_t + c_{w,t} + c_{m,t}) \\
\psi_{t+1} = \psi_t + \varepsilon_{t+1},\ \varepsilon_t \sim iid\mathcal{N}(0, \sigma_\psi^2)
\end{aligned}
\tag{B.6}
$$

where the value-of-choice given some $\mu$ is

$$
v_{j,t}(\psi_t, A_{t-1}, \mu, c_{w,t}, c_{m,t}, c_t) = U_j(c_{j,t}, c_t) + \psi_t + \beta \mathbb{E}_t[V_{j,t+1}^m(\psi_{t+1}, A_t, \mu)] \tag{B.7}
$$

where $V_{j,t+1}^m(\bullet)$ denotes the value of entering period $t+1$ as married.

**The value of entering a period as a couple** is

$$
V_{j,t}^m(\psi_t, A_{t-1}, \mu_{t-1}) = D_t^\star V_{j,t}^{m\to s}(\kappa_j A_{t-1}) + (1 - D_t^\star) V_{j,t}^{m\to m}(\psi_t, A_{t-1}, \mu_t^\star)
$$

where $\kappa_j$ is the share of household wealth member $j$ gets in case of divorce ($\kappa_w + \kappa_m = 1$). The determination of the bargaining weight $\mu_t^\star$ and the choice of divorce, $D_t^\star$, are determined as follows.

The bargaining power is updated according to the algorithm described in Hallengreen, Jørgensen and Olesen, 2024. For this purpose let

$$
S_{j,t}(\psi_t, A_{t-1}, \mu) = V_{j,t}^{m\to m}(\psi_t, A_{t-1}, \mu) - V_{j,t}^{m\to s}(\kappa_j A_{t-1})
$$

denote the marital surplus of household member $j$.

This gives the updating rule

$$\mu_t^\star = \begin{cases} \mu_{t-1} & \text{if} \quad S_{j,t}(\psi_t, A_{t-1}, \mu_{t-1}) \geq 0 \quad \text{for} \quad j \in \{w, m\} \\ \tilde{\mu}_w & \text{if} \quad S_{w,t}(\psi_t, A_{t-1}, \mu_{t-1}) < 0 \quad \text{and} \quad S_{m,t}(\psi_T, A_{t-1}, \tilde{\mu}_w) \geq 0 \\ \tilde{\mu}_m & \text{if} \quad S_{m,t}(\psi_t, A_{t-1}, \mu_{t-1}) < 0 \quad \text{and} \quad S_{w,T}(\psi_T, A_{t-1}, \tilde{\mu}_m) \geq 0 \\ \emptyset & \text{else} \end{cases}$$

where

$$\tilde{\mu}_j = \{\mu : S_{j,t}(\psi_t, A_{t-1}, \mu) = 0\}$$

The divorce indicator $D^\star$ takes the value 1 if a cooperative bargaining outcome cannot be reached and 0 otherwise, that is

$$D^\star = \begin{cases} 1 & \text{if} \quad \mu^\star = \emptyset \\ 0 & \text{else} \end{cases}$$

**The value of transitioning from single to couple** is

$$V_{j,t}^{s \to m}(\psi_t, A_{t-1}) = V_{j,t}^{m \to m}(\psi_t, A_{t-1}, \mu^0)$$

where $V_{j,t}^{m \to m}(\psi_t, A_{t-1}, \mu^0)$ is the value of remaining married (described above) with initial bargaining weight determined through Nash bargaining,

$$\mu^0 = \arg\max_{\mu} (V_{w,t}^{m \to m}(\psi_t, A_{t-1}, \mu) - V_{w,t}^{s \to s}(A_{w,t-1})) \\ \times (V_{m,t}^{m \to m}(\psi_t, A_{t-1}, \mu) - V_{m,t}^{s \to s}(A_{m,t-1}))$$

If $V_{j,t}^{m \to m}(\psi_t, A_{t-1}, \mu^0) - V_{j,t}^{s \to s}(A_{j,t-1}) > 0$ for $j \in \{m, w\}$ they form a couple and $M_t^\star = 1$, otherwise they do not and $M_t^\star = 0$.

## B.2 Numerical solution

In this section, we describe how we solve the model and highlight some specific tricks used. For a detailed description of the implementation, we refer to the GitHub repository with the accompanying code used to generate the results here.

### B.2.1 Precomputations

Before solving the model, we precompute an interpolater for total consumption. We construct a grid over total consumption, $C$, and for each grid point, we compute the marginal utility by taking the numerical derivative of the couple's utility function:

$$\begin{aligned} U(C) &= \max_{c_j, c_m, c} \mu U_w(c_w, c) + (1-\mu) U_m(c_m, c) \\ st. \quad C &= c_w + c_m + c \end{aligned}$$

We save this on a grid of marginal utility, $U'$, and then construct an interpolator of $C$ over marginal utility as described in the main text, $\check{C}(U')$.

### B.2.2 Solving the model

We solve the model by iterating backwards, starting in the terminal period $T$. When doing so using the iEGM algorithm, we need to know the expected marginal value of entering period $t$ as single,

$$w_{j,t}(A_{j,t}) = \beta \mathbb{E}_t \left[ \frac{\partial V^s_{j,t+1}(A_{j,t})}{\partial A_{j,t}} \right]$$

and for couples,

$$w_t(A_t, \mu) = \beta \mathbb{E}_t \left[ \mu \frac{\partial V^m_{w,t+1}(\psi, A_t, \mu)}{\partial A_t} + (1-\mu) \frac{\partial V^m_{m,t+1}(\psi, A_t, \mu)}{\partial A_t} \right].$$

We describe the construction of these objects below.

### B.2.3 Terminal period

**The value of remaining single** in the terminal period $T$ is:

$$V^{s \to s}_{j,T}(A_{j,T-1}) = U_j \left( c_j^{single}(C_T), C_T - c_j^{single}(C_T) \right)$$

where total consumption is $C_T = RA_{j,t-1} + Y_{j,t}$, i.e. all resources.

**The value of transitioning from marriage to singlehood** is identical to the above apart from a divorce cost,

$$V_{j,T}^{m\to s}(A_{j,t-1}) = V_{j,T}^{s\to s}(A_{j,T-1}) - \chi$$

**The value of remaining a couple** in the terminal period is

$$V_{j,T}^{m\to m}(\psi_T, A_{T-1}, \mu) = U_j(\check{c}_j(C_T, \mu), \check{c}(C_T, \mu)) + \psi_T$$

where total consumption again amounts to all resources, $C_T = RA_{T-1} + Y_{w,T} + Y_{m,T}$. Note that $V_{j,t}^{m\to m}$ is defined for an arbitrary bargaining power $\mu$.

The marital surplus as a function of $\mu$ is then

$$S_{j,T}(\psi_T, A_{T-1}, \mu) = V_{j,T}^{m\to m}(\psi_T, A_{T-1}, \mu) - V_{j,T}^{m\to s}(\kappa_j A_{T-1})$$

where $\kappa_j$ denotes the share of marital assets received by spouse $j$ in the event of divorce.

**The value of entering a period as a couple** includes both the possibility of remaining married and divorcing, such that

$$V_{j,T}^{m}(\psi_T, A_{T-1}, \mu_{T-1}) = D_T^{\star} V_{j,T}^{m\to s}(\kappa_j A_{T-1}) + (1 - D_T^{\star}) V_{j,T}^{m\to m}(\psi_T, A_{T-1}, \mu_T^{\star})$$

This value depends on the outcome of any potential bargaining, $\mu_T^{\star}$ and divorce $D_T^{\star}$ which is updated according to the algorithm described in section **??**.

Knowing this value, we can precompute the expected marginal value of entering period $T$. First, we compute the household value of entering period $T$ as married over a grid of post-decision assets, $\overrightarrow{A}$:

$$V_T^m(\psi_T, \overrightarrow{A}, \mu_{T-1}) = \mu_{T-1} V_{w,T}^m(\psi_T, \overrightarrow{A}, \mu_{T-1}) + (1 - \mu_{T-1}) V_{m,T}^m(\psi_T, \overrightarrow{A}, \mu_{T-1})$$

Next, we approximate the marginal value by centered finite differences on the grid $\overrightarrow{A}$. Letting $\overrightarrow{A}[i]$ denote index $i$ on $\overrightarrow{A}$:

$$\frac{\partial V_{j,T}^m(\psi_T, \overrightarrow{A}[i], \mu_{T-1})}{\partial A} = \frac{V_{j,T}^m(\psi_T, \overrightarrow{A}[i+1], \mu_{T-1}) - V_{j,T}^m(\psi_T, \overrightarrow{A}[i-1], \mu_{T-1})}{\overrightarrow{A}[i+1] - \overrightarrow{A}[i-1]}$$

where we extrapolate the slope at the first and last grid points.

Finally, we compute the expected marginal value over a grid of post decision assets,

$\overrightarrow{A}$ and post-decision bargaining $\overrightarrow{\mu}$:

$$w_{T-1}(\overrightarrow{A}, \overrightarrow{\mu}) = \beta \sum_{q=1}^{Q} \omega^q \left[ \overrightarrow{\mu} \frac{\partial V^m_{w,T}(\psi^q, \overrightarrow{A}, \overrightarrow{\mu})}{\partial A} + (1 - \overrightarrow{\mu}) \frac{\partial V^m_{m,T}(\psi^q, \overrightarrow{A}, \overrightarrow{\mu})}{\partial A} \right]$$

where we use $Q$ Gauss Hermite quadrature nodes to take expectations over future values of match quality $\psi_T$ and interpolate $V'_{j,T}$ using linear interpolation. This allows us to construct an interpolator for the expected marginal value, $\breve{w}_{T-1}(A_{T-1}, \mu_{T-1})$.

**The value of starting as single** conditional on meeting a partner with assets $A^p_{T-1}$ and initial match quality $\psi_T$ is

$$\begin{aligned} \tilde{V}^s_{j,t}(A_{j,t-1}, A^p_{t-1}, \psi_t) &= M^\star_t V^{s\to m}_{j,t}(\psi_t, A_{t-1}) + (1 - M^\star_t) V^{s\to s}_{j,t}(A_{j,t-1}) \\ &\text{s. t.} \\ A_{t-1} &= A_{j,t-1} + A^p_{t-1}, \end{aligned}$$

We precompute initial bargaining power for each combination of own assets and partner's assets, $(A_{j,t}, A_{i,t})$ by first computing repartnering surplus over a grid of bargaining power $\overrightarrow{\mu}$:

$$\begin{aligned} S^{s\to m}_{j,T}(\psi_T, A_{j,T-1}, A_{i,T-1}, \overrightarrow{\mu}) &= V^{m\to m}_{j,T}(\psi_t, A_{T-1}, \overrightarrow{\mu}) - V^{s\to s}_{j,T}(A_{j,T-1}) \\ S^{s\to m}_{i,T}(\psi_T, A_{i,T-1}, A_{j,T-1}, \overrightarrow{\mu}) &= V^{m\to m}_{i,T}(\psi_T, A_{T-1}, \overrightarrow{\mu}) - V^{s\to s}_{i,T}(A_{i,T-1}) \\ &\text{st. } A_{T-1} = A_{i,T-1} + A_{j,T-1} \end{aligned}$$

We interpolate the values of $V^{m\to m}_{j,T}$ using linear interpolation.

We then determine initial bargaining power:

$$\mu_0(\psi_T, A_{j,T}, A_{i,T}) = \arg\max_{\mu} S^{s\to m}_{j,T}(\psi_T, A_{j,T-1}, A_{i,T-1}, \mu) S^{s\to m}_{i,T}(\psi_T, A_{j,T-1}, A_{i,T-1}, \mu)$$

**The expected value of starting as single** is computed as

$$\mathbb{E}_{T-1}\left[V^s_{j,T}(A_{j,T-1})\right] = p_T \left[\sum_{k}^{K_\psi} \sum_{n}^{N_A} p^\psi_k p^A_n \tilde{V}^s_{j,T}(A_{j,T-1}, A^p_n, \psi_k)\right] + (1 - p_T) V^{s\to s}_{j,T}(A_{j,T-1})$$

where $p^\psi_k$ denotes the probability of drawing initial match quality $\psi_i$ and $p^A_n = p^A_n(A_{j,T-1})$ denotes the probability of drawing partner's assets $A^p_j$. These probabilities are specified on a grid of $(A_j, A_j^p)$ to represent discretized approximations of $\Gamma_\psi$ and $\Gamma_{A_j^p}$.

Finally, much like in the case of couples, we compute the expected marginal value of entering period $T$ as single over a grid of post-decision assets, $\overrightarrow{A}$ using centered finite differences, with:

$$w_{j,T}(\overrightarrow{A}[i]) = \frac{\mathbb{E}_{T-1}\left[V^s_{j,T}(\overrightarrow{A}[i+1])\right] - \mathbb{E}_{T-1}\left[V^s_{j,T}(\overrightarrow{A}[i-1])\right]}{\overrightarrow{A}[i+1] - \overrightarrow{A}[i-1]} \tag{B.8}$$

We use this to construct an interpolator for the marginal expected value of entering a period as single, $\breve{w}_{j,T-1}(A_{T-1})$.

### B.2.4 Earlier periods

Solving earlier periods follows almost the same approach as the terminal period, except that we use EGM to determine consumption.

**The value of remaining single** is computed using standard EGM. We construct a grid over post-decision assets, $\overrightarrow{A}$. We can then interpolate the expected marginal value of entering period $t+1$ using the interpolator $\breve{w}_{j,t}(A_{j,t})$. We make use of the fact that the marginal utility for singles is analytically invertible to find the total consumption, $C_{j,t}$:

$$C_{j,t}(\overrightarrow{A}_t) = U_j'^{-1}(\breve{w}_{j,t}(\overrightarrow{A}_t))$$

With this, we construct an endogenous grid over resources:

$$\overrightarrow{A}_{j,t-1} = R(C_{j,t}(\overrightarrow{A}_t) + \overrightarrow{A}_t) + Y_{j,t}$$

from which we can now interpolate optimal consumption given beginning of period assets $A_{j,t-1}$. We enforce the credit constraint by setting consumption equal to total resources for all asset values below the first point in the endogenous grid. Consumption is now computed over an endogenous grid. It can be helpful to interpolate consumption back onto a common grid for $A$ used throughout all periods.

This allows us to compute the value of remaining single:

$$\begin{aligned} V^{s\to s}_{j,t}(A_{j,t-1}) &= U_j(c_j^{single}(C_{j,t}), C_{j,t} - c_j^{single}(C_{j,t})) + \beta\mathbb{E}_t\left[V^s_{j,t+1}(A_{j,t})\right] \\ \text{st.} \quad A_{j,t} &= RA_{j,t-1} - C_{j,t} + Y_{j,t} \end{aligned}$$

Due to the discrete choice of whether to remarry, the value function for singles may have non-concave regions. We deal with this by taking an upper envelope over decision-specific value functions to determine optimal consumption (see Iskhakov, Jørgensen, Rust and Schjerning, 2017).

Consequently, the computation of the value of transitioning from marriage to singlehood is identical to that of the terminal period.

**The value of remaining a couple** is similarly computed by interpolating the expected marginal value over a grid of post-decision states, $\overrightarrow{A}$. However, this time, we cannot analytically invert the couple's marginal utility function. Instead, we compute total consumption using the precomputed interpolater:

$$C_t = \check{C}(w_{t+1})$$

From here, we follow the same EGM approach as described above to compute consumption as a function of beginning of period assets. Again, we take an upper envelope over decision-specific valuefunctions to deal with potential non-concave regions stemming from the possibility of divorce. We can then compute the value of remaining a couple with bargaining power $\mu$:

$$V_{j,t}^{m\to m}(\psi_t, A_{t-1}, \mu) = U_j(\check{c}_j(C_t), \check{c}(C_t)) + \beta \mathbb{E}_t \left[ V_{j,t+1}^{m}(\psi_{t+1}, A_t, \mu) \right]$$

The consecutive steps to compute the expected marginal values of entering period $t$ as a couple $w_t$, and as single, $w_{j,t}$ follow the approach of the terminal period, and the steps can be iteratively repeated until the initial time period.

The parameters on the model are given in Table B.1

Table B.1: Parameter Values.

| | |
|---|---|
| **Income** | |
| $R$ | 1.03 |
| $Y_w$ | 1.0 |
| $Y_m$ | 1.0 |
| **Preferences** | |
| $\beta$ | $1/R$ |
| $\rho_w$ | 2.0 |
| $\rho_m$ | 2.0 |
| $\alpha_{1,w}$ | 1.0 |
| $\alpha_{1,m}$ | 1.0 |
| $\alpha_{2,w}$ | 1.0 |
| $\alpha_{2,m}$ | 1.0 |
| $\phi_w$ | 0.2 |
| $\phi_m$ | 0.2 |
| **Household bargaining** | |
| $\kappa_w$ | 0.5 |
| $\kappa_m$ | 0.5 |
| $\sigma_\psi$ | 0.1 |
| $\chi$ | 0.0 |
| **Repartnering** | |
| $p_t$ | 0.1 |
| $\Gamma_\psi(\psi)$ | $\mathcal{N}(0, \sigma_\psi)$ |
| $\Gamma_A(A_j^p \mid A_j)$ | Deterministic* |

*In the example, we set $A_j^p$ as deterministic conditional on $A_j$, such that $A_j^p = A_j$ for all values of $A_j$.