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The Allocation of Talent Under Perfect and Imperfect Information*

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Abstract

We develop a general sorting model that incorporates both self-selection by applicants, who assess their own skills, and evaluation by admissions agents, who rely on noisy signals of skills. Building upon the theoretical framework of the Roy model, our analysis examines how the presence of noise in signals of skills influences admission behaviour and the wage distribution. Our analysis reveals two novel behavioural effects on admission procedures. First, institutions may optimally assign positive weight to skill signals that are uncorrelated and unproductive in the relevant sector—a phenomenon we call *talent hoarding*. This talent-hoarding effect disrupts comparative advantages, reallocates talent towards restricted sectors, and diminishes overall efficiency. With the presence of noise in signals, the admissions agent is further incentivised to increase the number of admitted applicants, which lowers wages. This, in turn, reduces reliance on admission rules and promotes more informative self-selection—a behavioural effect we label *talent separation*. Under relatively lenient assumptions, talent separation improves efficiency. Evidence from Danish administrative data reveals empirical patterns consistent with the predicted talent-hoarding effect, and a structural model of Denmark's education system highlights that the two behavioural effects can have a substantial impact on the distribution of wages.

Keywords: talent allocation, asymmetric information, noisy signals, self-selection, two-sided selection, college admissions, recruitment.

JEL classifications: D82, I23, J24, D83.

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1 Introduction

The allocation of talent is central to economic performance, individual outcomes, and the distribution of earnings and inequality. In many economic models, and most notably in the Roy (1951)-model, the sorting of individuals and the allocation of skills into sectors, regions, or educational tracks is based purely on individual self-selection.[1] In many real-life settings, the sorting of individuals is rarely based on self-selection alone. Instead, it is often the result of a double-sided selection: On one side, applicants self-select into different choices based on private information about their skills and comparative advantages. On the other side, admissions agents representing colleges, immigration authorities, or hiring committees typically play an important role in governing access to restricted choices. They seek to admit the absolute best subset of applicants, often based on noisy signals of the applicants' underlying skills.

In recent years, trends have made it harder to identify the true skills of applicants. Consider the example of college admissions: grade inflation, the widespread use of artificial intelligence to write papers and applications, and even outright cheating in applications have increased noise in the signals of applicants' underlying skills, making it harder for college admissions agents to assess the skills of college applicants.[2] We do not know how the presence of noise in the signals of skills affects the equilibrium allocation of talent, as we currently lack a unified framework for understanding these double-sided selection dynamics.

In this paper, we therefore build a sorting model with double-sided selection to analyse how the presence of noise in signals of skills affects admission behaviour and the resulting distribution of wages. Our model reveals two novel behavioural effects that arise when admission is based on noisy signals. The first reflects an incentive for the admissions agent to optimally consider signals of irrelevant and even uncorrelated skills of applicants in their admission rules; we call this effect *talent hoarding*, as it hoards talent away from other sectors and distorts the distribution of wages. The second effect reflects an incentive for the admissions agent to *increase* the number of admitted applicants as signals become noisy. This induces more informative and binding self-selection and showcases a mechanism by which increased noise in signals can actually improve efficiency, as it incentivises the admissions

[1] The Roy-model is the workhorse model for analysing sorting of individuals in e.g. education (see e.g. Willis & Rosen (1979), Saltiel (2023)), immigration (e.g. Borjas (1987), Dustmann et al. (2011)) and sectoral choice (e.g. Heckman & Honore (1990) and Almgren et al. (2023)).

[2] See e.g. Sievertsen (2022) for an overview of developments in grade inflation across countries revealing a systematic tendency to more lenient grading eroding grades as a signal of skills. A survey conducted by Intelligent.com (2021) found that 1 in 4 parents in the US cheated to get their kid into college. Recent papers have documented how generative AI have eroded written applications as a credible signal of skills (see e.g. Cui et al. (2025) and Galdin & Silbert (2025)).

agent to rationally give up some market power. We call this *the talent-separation effect*.

To illustrate the core dynamics of our model, consider the intuitive setting of college admissions. A population of individuals chooses between working in a white-collar and a blue-collar sector. In line with the standard Roy framework, individuals self-select (apply) into the sector that maximises their expected wage based on comparative advantage and private information about their skills. Now suppose that entry into the white-collar sector requires a college degree. We introduce an admissions agent representing a college, acting as gatekeeper to the white-collar sector, by deciding how many and which applicants to admit in order to maximise a function of aggregate white-collar skills, based on either perfectly observed or noisy signals (e.g. high school grades, test scores, etc.).

In our baseline model, we treat the number (mass) of admitted students as fixed to focus the analysis entirely on how admissions agents choose to weigh signals under perfect and imperfect information regarding applicants' skills. When skills are perfectly observed, there exists a unique Nash equilibrium in which the admissions agent gives weight only to signals of relevant white-collar skills. Less intuitively, when institutions make admission decisions based on noisy signals, they may rationally give weight to signals of skills that are unproductive in the target sector and even uncorrelated with skills that are productive in the target sector—an effect we call *talent hoarding*. The intuition behind this result is that while sector-irrelevant signals ex-ante do not hold any information about relevant skills, the expected sector-relevant skill changes conditional on application to the white-collar sector, hence revealing some private information about skills. The admissions agent rationally takes applicants' self-selection behaviour into account, leading to a positive weight on signals of sector-irrelevant skills when designing the admissions rule. The optimal weight for signals of sector-irrelevant skills increases with the relative wage rate in the two sectors. This behaviour systematically hoards talent from one sector to another, distorting comparative advantage, and reducing overall efficiency.

Aside from the behavioural effect, our model also reveals that the presence of noise mechanically increases *skill-mismatch* in the white-collar sector as it becomes harder for the admissions agent to know the true skills of the applicants in general. Although this mechanical skill-mismatch effect of noisy signals is not a novel discovery in itself but has been documented in other settings, e.g. Fredriksson et al. (2018) and Moscarini (2005), it strengthens the credibility of our model that it reproduces this very intuitive effect of noise in signals.

In an extended model, we allow the admissions agent to endogenously select the number of admitted students. We make this extension partially to understand how capacity constraints in, e.g. college admissions can arise endogenously, and to understand how this might be impacted by noisy signals of skills. We show that when skills are perfectly observed, capacity constraints can arise endogenously in admission procedures under our double-sided selection framework, even when all individuals are positively skilled and institutional objectives are monotonic in skills. As wage rates are decreasing in the number of admitted individuals, the admissions agent may optimally reduces the number of admitted applicants to increase the general wage level in the white-collar sector, thereby attracting many applicants to choose from. This offers a microfounded, potential explanation of how capacity constraints can be reconciled within a Roy-framework.

We find that in the presence of noise in signals, the admissions agent is incentivised to increase the number of admitted applicants, making the self-selection bind more, a behavioural response we call *talent separation*. The admissions agent optimally expands the capacity as the noise increases in order to induce more informative self-selection. The intuition behind this result is that, as it becomes more difficult for the admissions agent to identify skilled applicants, they have an incentive to lower the general wage level in the target sector by increasing admissions to attract higher-quality applicants on average. This decrease in the general sectoral wage levels, driven by decreasing marginal product of labour, ensures that applicants self-select relatively more on their individual skill-components, ensuring that fewer but higher-quality applicants are attracted to the white-collar sector.

Using Danish administrative data on admitted college students, we find evidence consistent with the key mechanisms of our model. Specifically, we show that when admitting students, there is a strong incentive to consider signals of seemingly study-specific irrelevant skills, and that this incentive is increasing in the wage rate associated with the irrelevant skill-dimension, highlighting an incentive to talent hoard. This finding holds even when other potential explanations are taken into account. We take this empirical exercise one step further and estimate a structural model of parts of the Danish admissions system using simulated method of moments[3], showcasing our theoretical findings, while also giving us a sense of the sizes of the behavioural effects. We show that when allowing admissions agents to act according to our model the behavioural effects of increasing noise in signals on the distribution of wages can be substantial and even larger than the effects of the mechanical skill-mismatch effect.

Our framework has direct implications for education policy and, more broadly, institutional design. Admissions rules focused on GPA can enhance outcomes for admitted students, but harm society by rejecting those with strengths in other areas, leading to talent hoarding. In addition, policies that limit seats in high-employment programmes to shift students to lucrative fields can backfire. While wages may rise in restricted fields, overall welfare may

[3] See e.g. Smith Jr (1993) and Jakobsen et al. (2022) for a description of this method.

decline as displaced students perform poorly in new sectors. With noisy screening, institutions might raise quality by making self-selection more binding, such as admitting more students or raising application costs. This talent separation mechanism can help counter the negative effects of imperfect information.

The remainder of the paper is structured as follows: Section 2 clarifies contributions to the related literature. Section 3 sets up the general model. Section 4 analyses the general model with exogenous capacity constraints. Section 5 extends the general model and imposes more structure to endogenise capacity constraints within the framework. Section 6 discusses the generality of our contributions. Section 7 empirically validates our main findings from Section 4. Section 8 validates and shows the magnitude of all our theoretical findings in a numerical example. Section 9 concludes.

2 Contribution to Literature

We take an offset in and contribute to the broad class of Roy (1951) by embedding two-sided selection, where institutions screen applicants based on noisy signals. These models have been extensively used to analyse the sorting of individuals and the allocation of skills, both theoretically and empirically. They have been used in various kinds of settings, most predominantly in educational sorting (e.g. Willis & Rosen (1979), Cicala et al. (2018), Mourifie et al. (2020), and Saltiel (2023)), immigration decisions (e.g. Borjas (1987) and Dustmann et al. (2011)), and sectoral choice (e.g. Heckman & Honore (1990), Taber & Vejlin (2020), and Almgren et al. (2023)). We contribute to this class of models by generalising and formalising the framework to account for the possibility that selection is not purely based on individuals' comparative advantages, but rather on a combination of self-selection *and* some kind of admission procedure. We show that allowing for this double-sided selection is important for the sorting of individuals and the allocation of skills and can ultimately alter the conclusions drawn from a standard Roy model. The model can be adjusted and applied to all situations in which individuals select amongst choices based on comparative advantages, and admissions agents admit applicants based on absolute advantages.

In addition, we contribute to the literature investigating the impact of noisy signals of skills and college admissions procedures in the presence of imperfectly observed skills. Fredriksson et al. (2018) and Moscarini (2005) show that noisy signals can increase skill mismatch. This is backed by Hansen et al. (2024) who find that employers initially use grades as a signal of skills but gradually learn about the skills of their employees. Our research identifies two novel behavioural effects that may arise from the presence of noisy signals in admission processes: specifically, *talent hoarding* and *talent separation.* In the domain of college admissions under noisy signals Bjerre-Nielsen & Chrisander (2022) find that in the presence of noisy signals of skills, applicants can benefit if they voluntarily share private information. Building on Spence (1973), MacLeod & Urquiola (2015) show that applicants can use college admission, and in particular the reputation of the school, as a signal of ability. In addition, they show that noisy signals of ability can lead applicants to overinvest in preparing for admissions tests. While these papers investigate applicant behaviour under noisy signals of skills, our main focus lies on how the presence of noise alters the behaviour of admissions agents assuming standard Roy-sorting of the applicants.

We also extend previous work by modelling applicant skills and signals in two dimensions, capturing both productive ability and signal quality, while much of the literature focusses on a single-dimensional signal or selection criterion. This richer environment allows us to identify novel behavioural responses to information frictions, including talent hoarding and talent separation, which shape selection decisions in ways that affect efficiency and equity. Other related contributions include models of decentralised admission systems, matching under constraints, and the interaction between student effort, standards, and sorting mechanisms (e.g. Costrell (1994), De Fraja (2002), Epple et al. (2006)).

Epple et al. (2006) and Fu (2014) build and estimate structural models of college admissions, modelling both applicant and college behaviour. Although their work focusses heavily on college competition and the use of tuition policies, we do not directly focus on these aspects. Instead, we focus on the use of signals when admitting applicants and on the dynamic interactions between self-selection based on comparative advantages and screening of applicants with the preferences of maximising absolute advantages. A thought for future research is to combine the multidimensional skill framework from the Roy-literature with a more rich modelling of the college admission side of the market incorporating, e.g. college competition, tuition policies, and differential returns to education.

While some of the existing literature on school admissions assumes that capacity constraints are exogenous, arising from physical, regulatory, or planning limitations, we further provide a behavioural micro-foundation for their emergence. Related work such as Jewitt & Ospina (2015), Chen & Kesten (2017), and Chade et al. (2014) examines admission procedures with fixed capacity. In contrast, our model endogenises the constraint itself, showing that institutions can optimally restrict access to improve selection outcomes when signals are noisy. This distinguishes our approach by showing that even purely skill-maximising institutions may find it optimal to limit capacity in the absence of any external restrictions.

Together, the contributions presented in this paper offer a unified framework for understanding how institutional screening under imperfect information reshapes talent allocation across a range of settings, from education to immigration and labour markets.

3 Model Setup

3.1 Application behaviour

We consider a standard Roy (1951)-type economy consisting of two sectors: a white-collar sector and a blue-collar sector. A continuum of individuals of unit mass indexed by $i \in [0,1]$ is each endowed with a two-dimensional skill vector:

$$\Theta(i) = (h_W(i),\ h_B(i)),$$

where $h_W(i)$ and $h_B(i)$ denote individual i's skills in the white-collar and blue-collar sectors, respectively.

Assumption 1: We assume that the skill levels of each individual i in each sector, $h_W(i), h_B(i)$, are the realisations of two independent random variables h_W, h_B that follow continuous distributions with finite means and support in $\mathbb{R}_+$, and that individuals observe their own skill levels perfectly.[4]

Assumption 1 states that, in the population, individuals' skills in the white-collar sector and blue-collar sector are independent. Although this assumption may appear restrictive, it is conceptually flexible. One can interpret it as each individual possessing a relevant skill for a sector and an independent residual component that is productive in another sector. This captures the idea that individuals may have comparative advantages across sectors without requiring strong assumptions about correlation structures.

Each individual chooses whether they prefer to work in the white-collar or blue-collar sector based on where they can maximise expected wages:

$$U(i) = \max\{\omega_W(h_W(i)),\ \omega_B(h_B(i))\},$$

where $\omega_j(h_j)$ denotes the wage function in sector $j \in \{W, B\}$.[5] We assume that wage functions are strictly increasing in skill, and for interpretability, we specify them as:

$$\omega_j(h_j(i)) = W_j h_j(i) \tag{3.1}$$

[4]Our results also apply to alternative assumptions in which individuals have imperfect knowledge of their skills, as long as they possess more accurate information than the institution. One could without loss of generality or implications for our findings set up the problem with a unit mass of agtens who each a productivity pair (W,B) from a given (joint) distribution *iid* for each agent.

[5]W stands for white-collar, while B stands for the blue-collar sector.

W_j is the wage rate in sector j. Log wages take the form:

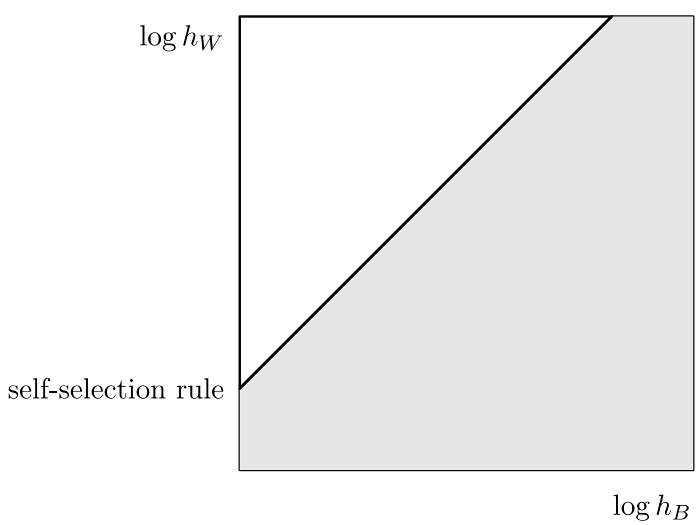
$$\log \omega_j(h_j) = w_j + \log h_j$$

with $w_j = \log W_j$. This form captures skill-proportional wages and preserves rank-ordering by skill within sectors. An individual prefers to work in the white-collar sector if and only if

$$\omega_W(h_W(i)) \geq \omega_B(h_B(i)) \iff \frac{h_W(i)}{h_B(i)} \geq \frac{W_B}{W_W} \tag{3.2}$$

This inequality defines a threshold in relative comparative advantage that governs sectoral sorting, as in the standard Roy model. The mass of individuals who end up in the white-collar sector is denoted by $|N_W| \in [0,1]$, while the mass in the blue-collar sector is $|N_B| \equiv 1 - |N_W|$. Throughout the paper, N_W and N_B refer to the corresponding sets of individuals.

Figure 1: Self-Selection in Competitive Equilibrium



NOTES: The square represents the skill space with $\log h_B$ on the horizontal axis and $\log h_W$ on the vertical axis, containing a continuum of individuals. Individuals with a comparative advantage in the blue-collar sector are located in the shaded grey area, while those with a comparative advantage in the white-collar sector are in the unshaded white area. In the competitive equilibrium, all individuals can pursue their comparative advantage.

Figure 1 illustrates this competitive environment (CE), in which allocation is governed purely by self-selection based on comparative advantage. The self-selection line marks those who are exactly indifferent, with a skill ratio of $\frac{h_W(i^*)}{h_B(i^*)}$.[6] Individuals above this line self-select into the white collar sector, and individuals below this line self-select into the blue-collar sector. Note that as sorting in the standard Roy-model is based *purely* on self-selection each

[6] In log skills, the self-selection line intersects the $\log h_W$ axis and has a slope of 1.

individual ends up in their preferred sector. We denote the mass of individuals who end up in the white-collar sector in the competitive equilibrium as $|N_W^{CE}|$.

3.2 Admissions Agent

The existing Roy literature assumes that everyone can follow their *comparative advantage* with free entry. In many contexts, however, applicants cannot freely choose amongst the different alternatives, and talent allocation is shaped not only by self-selection but also by some institutional decision maker who cares about *absolute advantages.* Prominent examples of this include immigration, where individuals cannot always freely choose where to migrate. Higher education also highlights the interplay between self-selection and institutional decision making. College admissions processes differ across countries, but a common feature is that applicants self-select based on their comparative advantages, and institutions select which applicants to admit based on certain signals regarding the applicants' skill levels.

To formalise and operationalise this double-selection dynamic between self-selection and admissions behaviour, suppose that access to working in the white-collar sector requires a college degree. Access to a representative college is governed by an admissions agent, representing this college, who admits a subset of applicants based on observed or inferred characteristics of applicants' skills in the relevant sector in order to maximise some objective function. The admissions agent selects a mass of individuals $N_W \in [0, N_W^{CE}]$ [7] The admission agent has preferences over admissible worker sets N_W, represented by the functional form: [8]

$$\Phi(N_W) = \int_{i \in N_W} f(h_W(i))\, di. \tag{3.3}$$

Hence, given two feasible sets N_W and N'_W, the agent weakly prefers N_W to N'_W if and only if $\Phi(N_W, h_W) \geq \Phi(N'_W, h_W)$.

Assumption 2: We assume that the function $f : \mathbb{R}_+ \to \mathbb{R}$ is strictly increasing in $h_W(i)$ and continuously differentiable. It represents a monotone transformation of skill and captures a range of institutional preferences.

Examples of the functional form of f can include that the institution may care about raw

[7]Note that he cannot admit more students than would enter the white-collar sector in the competitive equilibrium, as he can only admit students who want to enter the white-collar sector.

[8]The model can easily be extended to include a cost term $C(|N_W|)$, where $|N_W|$ denotes the mass of admitted applicants and $C(\cdot)$ is a strictly increasing and weakly convex function. Such a cost may reflect tuition subsidies, infrastructure and staffing costs, or reputational trade-offs associated with expanding access. As long as $C' > 0$ and $C'' \geq 0$, key results hold. Hence, the choice of excluding a cost term from the model does not drive our results. This includes the uniqueness and interiority of the optimal admission mass and its monotone response to signal noise.

productivity, in which case $f(h_W) = h_W$; weighted output, such as $f(h_W) = \log h_W$; or other nonlinear transformations such as $f(h_W) = h_W^\gamma$. f might also depend on other variables as long as it is strictly increasing in $h_W(i)$.

From equation 3.3 and Assumption 2, the payoff increases with the mass $|N_W|$ and the skills, $h_W(i)$, of individuals in N_W (since $h_W(i) > 0; \forall i$). This is standard in the college admission literature (see, e.g. Epple et al. (2006), Fu (2014) and Chade et al. (2014)).

Note, that for simplicity, we assume that the admissions agent and, thereby, the college does not add any value to its' students but only affects outcomes by acting as a gatekeeper to the white-collar sector. In addition, we follow MacLeod & Urquiola (2015) and assume a fixed tuition, which we normalise to zero. We make these simplifying assumptions to focus our analysis on admission behaviour and implications for sorting, and the model can easily be extended to account for these aspects.[9]

3.2.1 Information Structure

The admissions agent seeks to maximise utility defined in equation (3.3). However, the agent does not necessarily observe the underlying skills of the applicants directly at the time of the admission decision. Instead, there might be asymmetric information between the' true skills of the applicants and the signals they send, so that the admission agent observes the signals for each dimension of the skill, given by the following:

$$s_j(i) = \log h_j(i) + \varepsilon_j(i), \quad \varepsilon_j(i) \sim \mathcal{N}(0, \sigma_j^2), \quad \sigma_j > 0, \quad j \in \{W, B\}, \tag{3.4}$$

where $s_j(i)$ is the signal observed for skills $\log h_j(i)$, and $\varepsilon_j(i)$ is a mean-zero, independent, normally distributed noise term. This way of modelling noise in signals is common in the literature (see, e.g. MacLeod & Urquiola, 2015;Bjerre-Nielsen & Chrisander, 2022). Intuitively, signals can be thought of as high school grades reflecting an applicant's underlying ability in a course, standardized test scores, or any other signals of different types of abilities.[10] Note that while the admissions agent does not necessarily observe the true skills

[9] To see this, consider a small increase in wages from attending college such that white-collar wages is $\log \omega_W(h_W(i)) = w_W + \log h_j(i) + v$, where v is the expected effect on wages from attending college. Log wages thereby become: $\log \omega_W(h_W(i)) = \tilde{w}_W + \log h_j(i)$ where $\tilde{w}_W \equiv w_W + v$. This does not change the structure of the model. Note, however, that if the (perceived) effect on wages from attending college, e.g. varies with $\log h_W(i)$ or even $\log h_B(i)$, the results might change. In addition a known but non-zero tuition fee from attending college can be modelled by a common shift in the mean of h_W. In this case, the sorting equation of individuals in equation 3.2 can be viewed as net-of-tuition wage sorting.

[10] Note that while we choose to model the double-selection problem as college applicants deciding whether to apply to a white-collar college based on their comparative advantage, and an admissions agent choosing whom to admit to maximise a function of absolute skills, the same framework applies more broadly to settings such as migration and sectoral choice, as discussed in Section 6.

of the individuals, the aggregate distributions of h_j and ε_j are common knowledge.

3.2.2 Admissions Strategy

The strategy of the admissions agent consists of deciding how many to admit, $|N_W|$, and *who* to admit. In order to decide who of the applicants to admit, the admissions agent selects an admission rule $\mathbb{M}(s_W(i), s_B(i)) > \mu$ where $\mathbb{M} : \mathbb{R}^2 \to \mathbb{R}$ and selects both the functional form of the admissions rule and a threshold value μ. Hence, the admissions agent decides how to weight the signals of the applicants and a cutoff value for whom to admit. All applicants with values of $\mathbb{M}(s_W(i), s_B(i)) > \mu$ are admitted to the college and thereby in the white-collar sector. This rule reflects that the admissions agent considers different signals (grades from different high school courses, test results, extracurricular activities, etc.) in order to decide whom to admit, and note that the rule need not be linear.[11] Rejected applicants end up in the blue-collar sector.

3.3 Characterizing an Equilibrium

When solving the model, we consider pure-strategy Nash equilibria. An equilibrium consists of (i) an application strategy for all individuals in the economy and (ii) an admission strategy for the admissions agent, being the mass of admitted students $|N_W|$, and an admission rule $\mathbb{M}(s_W, s_B) \geq \mu$, which makes up the set of individuals in the white collar sector, N_W. In equilibrium, neither the applicants nor the admissions agent have an incentive to deviate, given the strategies of all other agents in the economy.

Given the structure of the model, the problem can be simplified. To reduce the strategy space, and without loss of generality, we first note that applicants have a weakly dominant strategy: given their skills, they apply to the sector that offers the highest expected wage. This result is formalised in Lemma 1.

[11] One could think of a college admissions agent using a linear selection rule based on the observed signals, similar to a weighted GPA, like:

$$\mathbb{M}(i) = s_W(i) + \lambda s_B(i) \geq \mu, \tag{3.5}$$

where $\lambda \in \mathbb{R}$ is the relative weight placed on the signal of the dimension of unproductive skills (blue-collar skill), and $\mu \in \mathbb{R}$ is the admission threshold relative to the signal of the relevant skill. The signal of white-collar skills $s_W(i)$ receives a normalised weight of one. An applicant is admitted if the weighted sum of signals exceeds the threshold

Lemma 1: Truthful Application behaviour

Each individual applies to the sector that maximizes their expected wage. If $w_W + \log h_W(i) \geq w_B + \log h_B(i)$, individual i (weakly, strictly if $>$) prefers to apply to the white-collar sector. If $w_W + \log h_W(i) < w_B + \log h_B(i)$, they do not apply for college but enter the blue-collar sector. Applicants who would prefer the white-collar sector but fall below the admission cutoff are indifferent between applying to college or not, since applications are costless and rejection yields blue-collar employment. By convention, we let such individuals apply to college, which has no effect on admissions, wages, or any equilibrium object and is hence without loss of generality.

Proof: See Appendix A.2.

Truthful application is a weakly dominant strategy: Even in the presence of an admissions agent, the individuals still follow the simple self-selection rule, as outlined in equation 3.2. That is, an individual applies to college if and only if her expected wage is weakly higher in the white-collar sector than in the blue-collar sector. If the reverse is true, the individual does not apply, but enters the unrestricted blue-collar sector. This allows us to treat application behaviour as exogenous in the analysis.

Turning to the strategy of the admissions agent, we can further simplify the set of feasible strategies. Lemma 2 indicates that the admissions agent's strategy space can be reduced without loss of generality, as any admission rule can be represented as a threshold rule, and choosing a threshold is equivalent to choosing an admitted mass. Ultimately, the admissions rule boils down to the admissions agent selecting how many applicants to admit and how to aggregate the blue-collar and white-collar signals of the applicants into a single value used to evaluate them.

Lemma 2: Equivalence of Threshold and Admission Mass

Let the admissions agent use a threshold rule

$$\mathbb{M}(s_W(i), s_B(i)) \;\geq\; \mu,$$

where the admission index is an unspecified continuous function of both signals,

$$s_j(i) = \log h_j(i) + \varepsilon_j(i), \quad \varepsilon_j(i) \sim N(0, \sigma_j^2), \quad j \in \{W, B\},$$

Assume $m : \mathbb{R}^2 \to \mathbb{R}$ is continuous and (weakly) increasing in each argument and not constant on any set of positive measure.

Conditional on truthful application behaviour (Lemma 1), there exists a unique, strictly decreasing, and continuous function $\mu = \mu(|N_W|)$ such that

$$|N_W| \;=\; \mathbb{P}\Big[\mathbb{M}(s_W(i), s_B(i)) > \mu\Big].$$

Equivalently, choosing a threshold μ is the same as choosing an admission mass $|N_W|$, and vice versa.

Proof: See appendix A.3.

Intuitively, as the aggregate distribution of skills and signals is known to the admissions agent, setting an admission cutoff μ is equivalent to choosing how many students to admit, $|N_W|$. Hence, the choice of the admissions agent consists of selecting a function $\mathbb{M}(s_W(i), s_B(i))$ and a mass $|N_W|$ such that $\mathbb{M}(s_W(i), s_B(i)) > \mu(|N_W|)$.

3.4 Timing of the Model

The sequence of events in the model unfolds as follows[12]:

1. Nature draws a continuum of individuals together with their underlying skill endowments, which each individual observes.

2. Individuals draw noisy signals of their white-collar and blue-collar skills.

3. The admissions agent chooses an admission rule $\mathbb{M}(s_W(i), s_B(i)) > \mu(|N_W|)$.

[12]Note that the relative timing of steps 2, 3 and 4 is not crucial to the logic of the model and can be rearranged without affecting the implications of equilibrium. We include a notational summary in Appendix A.1 to ease reading.

4. Individuals apply to college if the white-collar sector yields the highest expected payoff, given their observed skills.

5. Admission decisions are implemented according to the combination of self-selection and the admission rule: Applicants who satisfy the admissions rule are admitted to college and ends up the white-collar sector. The remaining individuals are allocated into the blue-collar sector.

6. True skills are revealed and payoffs are realised.

4 Baseline Model with Exogenous Capacity

In the first step of our analysis, we follow a large literature that takes capacity constraints in college, and thereby the white-collar sector, $|N_W|$, as exogenously given, which we label as $|\tilde{N}_W|$. Formally, the admissions agent's problem is:

$$\underset{\mathbb{M}(s_W,s_B)>\mu(|\tilde{N}_W|)}{\arg\max} \quad \Phi(N_W) = \int_{i\in\tilde{N}_W} f(h_W(i))\,\mathrm{d}i, \tag{4.1}$$

subject to

$$\log\omega_W(h_W(i)) \geq \log\omega_B(h_B(i)) \quad \text{(Self-selection rule)}, \tag{4.2}$$

$$\mathbb{M}(s_W(i), s_B(i)) > \mu(|\tilde{N}_W|) \quad \text{(Admissions rule)}. \tag{4.3}$$

In words, the objective of the admissions agent is to fill the $|\tilde{N}_W|$ seats with the applicants with the highest expected h_W. The applicants are all individuals for whom the self-selection rule is satisfied. The admissions agent decides on an admissions rule $\mathbb{M}(\cdot)$, choosing how to weigh the signals of the applicants' skills and ranks the applicants according to this rule. The limited number of seats is allocated to the $|\tilde{N}_W|$ applicants with the highest ranking according to the admissions rule, $\mathbb{M}(\cdot)$. To analyse the effect of noisy signals of skills on admission behaviour and the distribution of wages, we consider both the perfect information equilibrium (PI) and an imperfect information equilibrium (II) and compare the two.

4.1 Perfect Information Equilibrium

We first consider a benchmark case in which both applicants and the admissions agent perfectly observe skills, so that there is no asymmetric information. Formally, when $\sigma_j = 0, j \in (W, B)$, all noise terms vanish ($\varepsilon_j(i) = 0,\ \forall\ i, j$), and the signals reduce to the true skill levels:

$$s_j(i) = \log h_j(i),\ j \in (W, B). \tag{4.4}$$

In this setting, we can characterise the following about equilibrium admission behaviour:

Proposition 1: No weight on irrelevant signals under Perfect Information

Under perfect information of skills ($\varepsilon_j(i) = 0$), $\forall\ i, j$, the admissions agent admits exactly the $|\tilde{N}_W|$ individuals with the highest white-collar signals and hence skills $h_W(i)$. Equivalently, the perfect information (PI) equilibrium admission rule is linear: $\mathbb{M}^{PI}(s_W(i), s_B(i)) = s_W(i) > \mu(|\tilde{N}_W|)$ and ignores $s_B(i)$ completely.

Proof: See Appendix A.4.

With perfect information, the admissions agent applies a linear rule based solely on the relevant signal s_W. All applicants with s_W (and thus h_W) above a threshold $\mu(|\tilde{N}_W|)$ are admitted, where the threshold is set to fill available seats. Since productive signals fully reveal true skills, the admissions agent has no incentive to consider the unrelated and unproductive dimension, s_B.

4.2 Imperfect Information Equilibrium

We now consider the case of asymmetric information, where the admissions agent observes noisy signals rather than the true skills of the applicants, such that:

$$s_j(i) = \log h_j(i) + \varepsilon_j(i), \quad \varepsilon_j \sim N(0, \sigma_j^2), \quad \sigma_j > 0,\ j \in (W, B). \tag{4.5}$$

In this setting, we can characterise the equilibrium admission behaviour as follows:

Proposition 2.A: Positive Weight on Irrelevant Signals under Imperfect Information (Talent Hoarding)

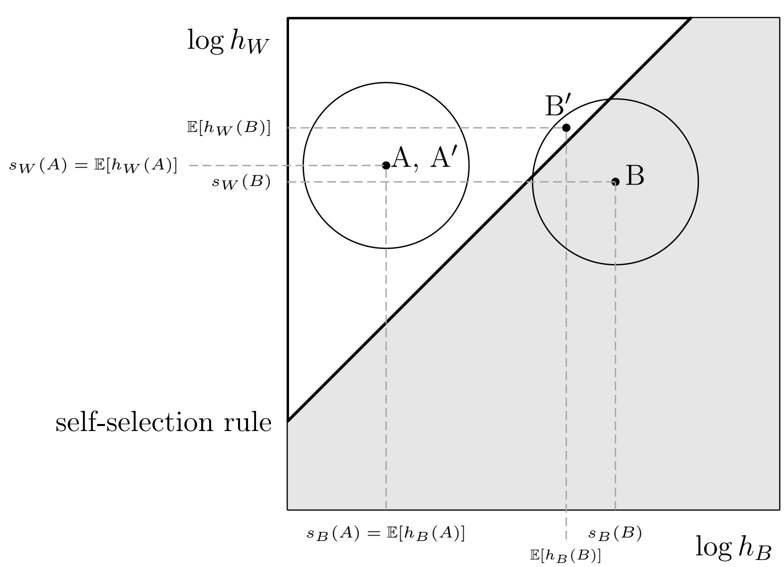
Under imperfect information of skills (II), such that the admissions agent does not observe skills perfectly, he sets an admission rule $\mathbb{M}^{II}(s_W(i), s_B(i)) > \mu(|\tilde{N}_W|)$ where he puts positive weight not only on the sector-relevant signal, s_W, but also on the signal of the independent and sector-irrelevant skill, s_B.

Proof: See appendix A.5.

Proposition 2.A formalises the effect we call *the talent-hoarding effect:* when institutions make admission decisions based on noisy signals, they may rationally give weight to signals of skills that are unproductive in the target sector, *and* even if they are uncorrelated with skills which are productive in the target sector. The reason for this counterintuitive result is that while blue-collar skills are unproductive in the white-collar sector, their signals can help the admissions agent infer applicants' white-collar skills when combined with observed self-selection behaviour. The result is proven under the assumption that noise is present in the signal of the productive skill (s_W). However, the same logic extends to the case where both signals are noisy. The general result even holds for more than two dimensions, so long as the admissions agent relies on noisy signals and applicants self-select based on private information. Utilising information where it is not put to its most productive use reflects a systematic incentive to *hoard talent* from other sectors into the white-collar sector, thereby distorting comparative advantage and reducing overall efficiency.

Intuitively, consider two applicants for college, A and B, illustrated in Figure 2. Applicant A has a higher signal in the white-collar dimension than applicant B. However, applicant B has a substantially higher signal in the blue-collar dimension. Uncertainty (noise) is represented as circles around the observed signals. While applicants know their true skills within this circle, the admissions agent only observes sector-specific signals and whether a student applied to the college and thereby the white-collar sector.

Figure 2: Talent Hoarding under Noisy Signals



NOTES: The square represents the skill space with $\log h_B$ on the horizontal axis and $\log h_W$ on the vertical axis. Point A marks applicant A's signals and true skills (A = A′). Point B marks applicant B's signals, and point B′ marks applicant B's expected skills conditional on applying to the white-collar sector. For illustrative purpose, the expected signal for A is unaffected by noise.

Under a horizontal admission rule (that is, selecting solely based on the observed white-collar signal), applicant A would be admitted over applicant B. However, since wages in each sector are strictly increasing in true sector-specific skills, and the admissions agent knows that applicant B's true skill lies above the self-selection line (as B chose to apply to the white-collar sector despite a weaker relative signal), B has a higher expected white-collar skill than A. The admissions agent prefers to admit B over A, even though B's strong signal originates from the blue-collar dimension. By admitting individuals like B, the admissions agent effectively hoards talent at the cost of a greater counterfactual loss for individuals like A. As a result, the admissions agent may marginally increase output in the restricted white-collar sector by admitting applicant B, but this comes at the cost of significantly reduced productivity in the unrestricted blue-collar sector, since A's expected productivity there is much lower than the marginal expected productivity B would have contributed. From the perspective of maximising overall efficiency, this misallocation leads to aggregate losses for the economy, which we formalise in Proposition 3.B.

4.3 Comparative Statics on Talent Hoarding

Turning to comparative statics, we can characterise the following about how the admissions rule is affected by the primitives of the model:

> **Proposition 2.B:** Comparative Statics on Weight to Irrelevant Signals
>
> In Equilibrium when skills are not perfectly observed, the optimal weight to the signal of the irrelevant skill, s_B increases with the relative log wage rate, $w_B - w_W$.

Proof: See Appendix A.6.

If the relative wage level increases in the sector where the signal is unproductive, that is, an increase in $\frac{W_B}{W_W}$, applicants with strong skills in that sector are more likely to opt out of applying altogether. Therefore, when an applicant with strong indicators in the unproductive dimension still chooses to apply for college, it suggests that they believe their white-collar potential is even higher. As the outside option (the blue-collar sector) becomes more attractive, the signal of applying to college becomes more informative and signals an even higher white-collar skill. Consequently, signals of blue-collar skills become more useful for inferring who possesses high white-collar skills. Such shifts are especially pertinent given projections in both the academic literature and the media that advances in artificial intelligence potentially can reallocate the demand of labour between white- and blue-collar sectors, reshaping outside options and the informational content of observed choices.

When the admissions agent receives a noisier signal about applicants' relevant skills, that is, an increase in σ, it becomes harder to distinguish high-ability from low-ability individuals based on that signal alone. As a result, the agent might rely more heavily on self-selection behaviour, which can reveal underlying abilities even through signals that are not related to performance in the target sector. In particular, observing a strong signal in the sector-irrelevant dimension becomes relatively more informative, not because it has direct value, but because it helps infer the applicant's relevant ability through their decision to apply. Hence, the incentive to consider blue-collar signals, s_B can intensify with increasing noise. As uncertainty increases, the inference that B may be "hiding" higher true white-collar skills becomes stronger. Further elaboration on this can also be found in A.7. Examples of increased noise in signals include grade inflation and the use of artificial intelligence in writing applications, making it harder to distinguish high-ability applicants from low-ability ones.

The results presented in this section show that, in the presence of noise, the admissions agent strategically utilises applicants' self-selection behaviour when designing an admissions rule. That is, by utilising indirect information about applicants' comparative advantages through self-selection behaviour, they can better infer the absolute advantages of the applicants.

4.4 Effect of Noise on Equilibrium Distribution of Wages

Next, we analyse how the introduction of noise in signals affects the distribution of wages in terms of realised mean log wages in each of the two sectors separately and in the entire economy. We define mean log wages in sector $j \in (W, B)$ for the individuals who end up in this sector as:

$$\bar{\omega}_j = \mathbb{E}[\log \omega_j(i) \mid i \in N_j] \tag{4.6}$$

The mean log wages in the entire economy is an average of realised mean log wages in each of the two sectors, weighted by the share of the population in each sector. We compare the distribution of wages in a situation with imperfect information (i.e., noise in signals) as described in section 4.2 to the perfect information benchmark described in section 4.1.

Introducing noise into the signal of skills has two effects on the equilibrium distribution of wages:

i. Mechanical Effect

ii. Behavioural Effect of Talent Hoarding

The mechanical effect captures how noise *per se* alters the distribution of wages. Holding the admission rule fixed at the perfect-information benchmark, $\mathbb{M}(s_W) = s_W > \mu(|\tilde{N}_W|)$, we isolate the pure impact of noise on wage outcomes. The talent-hoarding effect captures the behavioural adjustment of the admission rule for a fixed cohort size and its impact on wage distributions. Since noise is exogenous, we must separate this response from the mechanical effect of noise itself.

We first consider the mechanical effect of noisy signals in skills on the distribution of wages:

Proposition 3.A: Mechanical Effects of Noisy Signals on the Distribution of Wages

Holding the admission rule is held fixed at $\mathbb{M}^{PI}, (s_W) = s_W > \mu(|\tilde{N}_W|)$, then the presence of noise in signals, σ, implies that:

1. Mean log wages in the white-collar sector decreases.

2. Mean log wages in the blue-collar sector increases.

3. The effect on mean log wages in the entire economy is ambiguous.

Proof: See Appendix A.8.

Looking isolated at the effect of an increase in noise, the skill component becomes a less significant part of the signal, which increases the probability that the admissions agent admits individuals with low white-collar skills to the white-collar sector. This increased *skill mismatch* reduces mean log wages in the white-collar sector. The fact that noise in signals increases skill mismatch is not a novel result but a mechanical feature of our model and has been documented in other settings, e.g. Fredriksson et al. (2018) and Moscarini (2005). Nevertheless, it is reassuring that our model is able to replicate this well-known phenomenon. Although mean log wages fall in the white-collar sector, the mechanical effect raises mean log wages in the blue-collar sector. With perfect information, individuals who are highly skilled in both dimensions sort into white-collar jobs. When noise is introduced, some are misclassified into blue-collar positions, raising that sector's average wage. The mechanical effect of increased signal noise on mean log wages in the entire economy is, to our knowledge, ambiguous.

Now consider the effect of talent hoarding on the distribution of wages:

Proposition 3.B: Effects of Talent Hoarding on Distribution of Wages

Let $|N_W| = |\tilde{N}_W|$ be fixed. Suppose that noise σ is present in the signals. If the admissions agent, in accordance with Proposition 2.A, increases the relative weight on the unproductive signal s_B in the admissions rule $\mathbb{M}(s_W, s_B) > \mu(|\tilde{N}_W|)$, then compared to placing weight only on the relevant signal, it follows that:

1. Mean log wages in the white-collar sector increases.

2. Mean log wages in the blue-collar sector decreases.

3. Mean log wages in the entire economy decreases.

Proof: See Appendix A.9.

Given that the admissions agent optimally assigns weight to the blue-collar signal, as shown in Proposition 2.A, it is unsurprising that this raises mean log wages in the white-collar sector. However, it does lower mean log wages in both the blue-collar sector and the overall economy. [13]

The intuition behind this result is as follows: The admissions agent uses the blue-collar signal only to gain information about white-collar ability. By admitting students with relatively high blue-collar signals into the white-collar sector, they waste blue-collar skill compared to an admissions rule that only considers white-collar skills. This admissions behaviour changes the composition of admitted students, drawing high-ability individuals away from the blue-collar sector and into the white-collar sector. The admissions agent thus *hoards talent* away from the blue-collar sector.

While it is rational for the admissions agent to consider signals of irrelevant skills when admitting applicants, this behaviour generates an aggregate efficiency loss: Because the admission rule depends on both signals, weaker blue-collar skills, all else being equal, reduce the probability of admission to the white-collar sector (for a given white-collar signal) and paradoxically increase the likelihood of ending up in the blue-collar sector. Rejected applicants who are displaced into the blue-collar sector face a greater comparative disadvantage on average than those admitted to the white-collar sector. The cost of this admissions behaviour falls upon rejected applicants with the largest comparative disadvantages in the sector to which they are displaced.

The talent-hoarding effect is visible in admissions in the real-world. Selective programmes

[13] In the proof, we introduce noise in the relevant dimension to capture the informational frictions. Without noise in the blue-collar dimension, the conditional expectations would collapse to deterministic values in that sector. Our findings are further validated from our numerical example in Section 8.

and GPA-based admissions are based not only on signals directly relevant to succeeding in that particular programme, but also on signals which might be weakly or even unrelated to actual productivity in that particular programme. When true ability is only imperfectly observed, these practices can distort how talent is allocated across fields and institutions. For example, when extracurricular activities, standardised tests, general coursework, or stylistic features of applications weigh heavily in admission decisions, talented students may be diverted from the areas where their skills would be valuable, raising average quality within certain restricted programmes but impairing the overall allocation of talent. A numerical example illustrating the magnitude of both effects can be found in Section 8.

5 Extended Model with Endogenous Capacity

In the baseline model in Section 4, we assumed a fixed and binding admissions capacity in the white-collar sector. In practice, however, such constraints may be endogenous rather than externally imposed. Such constraints may arise for various reasons. In the context of college admission, factors such as limited infrastructure, insufficient teaching resources, and administrative rigidity could all be causes of capacity constraints. A large body of work on the economics of education treats these constraints as exogenous (see, e.g. Jewitt & Ospina (2015), Chen & Kesten (2017), and Chade et al. (2014)). If we adopt this perspective, the results in Section 4 can be used alone. In what follows, however, we take a more ambitious approach, as we endogenise capacity constraints by providing a microfoundation within the model, grounded in the Roy framework and aligned with the broader literature on educational selection.

There are two reasons why we provide such a microfoundation. The first and most obvious reason is that we observe such capacity constraints in many situations where sorting models such as the Roy model are typically used to analyse the allocation of individuals. Most predominantly in educational sorting and migration. Each year across the world, many applicants are rejected from the college they applied for. Similarly, in migration, many potential immigrants are rejected visas. In order to extend the Roy model and make a framework that is able to analyse the interplay between self-selection and admission behaviour, we therefore need to take the origins of such capacity constraints seriously. The second reason is that for many colleges, immigration policymakers, etc. choosing how many individuals to admit *is* a choice variable and potentially important for talent allocation. It is therefore in itself interesting to analyse how it is determined and how this choice is affected when noise in signals is present.

In this section, we therefore extend the baseline model from section 4 and endogenise capacity constraints in the admissions procedure, and analyse how they respond to changing information structures. In order to accomplish this, we need to impose slightly more structure on the model compared to the very general setup outlined in the baseline model.

5.1 Production Setup and Assumptions

Relative to the baseline model in Section 3, where the admissions agent's objective was left as a general monotone transformation $f(h_W)$, we now impose additional structure on the admissions agent's preferences. Specifically, we set $f(h_W(i)) = h_W(i)$ such that:

$$\Phi(N_W) = \int_{i \in N_W} h_W(i)\, di = |N_W| \cdot \bar{h}_W = |N_W| \cdot \mathbb{E}[h_W(i)|i \in N_W], \tag{5.1}$$

where $\bar{h}_W$ is the mean skill level of the white-collar skills in the white-collar sector. This specification nests the baseline formulation, but makes the function of preferences simple and analytically tractable, ensuring that the results are not driven by exotic choices of $f(\cdot)$ – it is also standard in the college admissions literature. The admissions agent cares about the aggregate skill level of the admitted individuals, which implies a preference for both larger cohorts and higher individual skills. This restriction is primarily a simplifying assumption, consistent with existing work (see, e.g. Epple et al. (2006), Fu (2014), Chade et al. (2014)), and ensures that our findings do not hinge on unusual objectives . Note that, as all individuals have positive skills, admitting one more applicant, all other things being equal, increases the admissions agent's utility. Thus, there is no *mechanical* incentive to restrict admissions built into the choice of utility functions.

Next, we impose some additional structure on the wage function $\omega_j(h_j(i)) = W_j \cdot h_j(i)$. Specifically, we impose the following two assumptions:

Assumption 3: The wage rate W_j is decreasing in the number of individuals in the same sector, $|N_j|$, i.e., $W_j \equiv W_j(|N_{j-}|)$. Intuitively, as the number of workers increases, the capital-labour ratio falls, and individual productivity decreases, leading to lower wages.

Assumption 4: The wage rate W_j is independent of the distribution of realised skills, h_j, in sector j.

Although assumption 3 is very standard and can be rationalised by decreasing capital per worker (see wage rates derived from standard Cobb-Douglas functions), assumption 4 is potentially more restrictive, as it excludes the possibility of peer effects: While the wages of individual i in sector j under assumptions 3 and 4 is equal to $\omega_j(h_j(i)) = W_j(|N_{j-}|) \cdot h_j(i)$ and depend on own skills, it cannot depend on the skill distribution of other individuals in the same sector. Although a higher skill level of peers might give rise to either negative congestion effects (see, e.g. Adão (2015), Cicala et al. (2018), Almgren et al. (2023), where a higher mean skill level of peers increases labour in efficiency units) or positive productivity spill-over effects (see Lucas Jr (1988), Mas & Moretti (2009), Cornelissen et al. (2017)), we wish not to take a firm position on which effect dominates. We note that assumption 4 is a sufficient condition for our results to hold. In practice, our results will likely be qualitatively robust to moderate peer effects in either direction, as long as they do not dominate the effect of N_j on the wage rate, W_j. In this analysis, however, we exclude the possibility of peer effects entirely, to focus on other aspects of admission dynamics. We note that this specification of wage rates under assumption 3 and 4 is consistent with the basic model in Acemoglu (1996), where it is shown that only the capital available to each worker, and not the skill distribution of other workers, affects the returns to own skills.[14] For further elaboration and discussion of this assumption and potential specific functional form formulation of the wage rate, see appendix A.13.

We can now characterise the extended model under assumptions 1-4 as follows:

$$\underset{\mathbb{M}(s_W,s_B)>\mu(|N_W|)}{\arg\max} \quad \Phi(N_W) = \int_{i\in N_W} h_W(i)\,\mathrm{d}i = |N_W|\cdot \bar{h}_W = |N_W|\cdot \mathbb{E}\left[h_W(i) \mid i\in N_W\right], \quad (5.2)$$

subject to

$$\log \omega_W(h_W(i)) \geq \log \omega_B(h_B(i)) \quad \text{(Self-selection rule)}, \quad (5.3)$$

$$\mathbb{M}(s_W(i), s_B(i)) > \mu(|N_W|) \quad \text{(Admissions rule)}. \quad (5.4)$$

Note that in this extended model, the admissions agent chooses both an admissions rule, $\mathbb{M}$ *and* how many students to admit $|N_W|$.[15] In practice, they select a cutoff value, $\mu(|N_W|)$ to admit the desired number of applicants (see Lemma 2). We can now characterise the equilibrium behaviour in this extended model.

[14]Using the structure of Acemoglu (1996), the wage rate in sector j can be formulated as:

$$W_j \equiv (1-\alpha)A_j k_j^{\alpha}$$

Where $k_j \equiv \left(\frac{K_j}{N_j}\right)$, and K_j and A_j are, respectively, capital and total factor productivity in sector j and α is the capital's share of total income. With this specification of wage rates, the the wages of individual i in sector j under assumption 3 and 4 is equal to $\omega_j(h_j(i)) = W_j(|N_{j-}|)\cdot h_j(i) = (1-\alpha)A_j k_j^{\alpha}\cdot h_j(i)$. Note that we do not require this specific formulation of the wage function in order for the following results to hold. They do so as long as assumptions 3 and 4 are satisfied. In our main specification, we do not allow for potential capital adjustments to focus on the key parts of the model. In appendix A.14 we specify a version of the model with free movement of capital.

[15]It is easy to see that if N_W is fixed, the problem collapses to the baseline model described in section 4, where $f(h_W(i)) = h_W(i)$.

5.2 Perfect Information Equilibrium

Like in the baseline model, we start with the case where skills are perfectly observed by both applicants and the admissions agent. In this benchmark, there is no asymmetric information, implying $\varepsilon_j(i) = 0,\ \forall\, i, j$, and the signals collapse to:

$$s_j(i) = \log h_j(i),\ j \in (W, B).$$

For the perfect information equilibrium, we can characterise the following:

Proposition 4: Perfect information – Ever-present restriction mechanism

Under perfect information of skills (PI) ($\varepsilon_j(i) = 0,\ \forall\, i, j$) the admissions agent sets an admission rule:

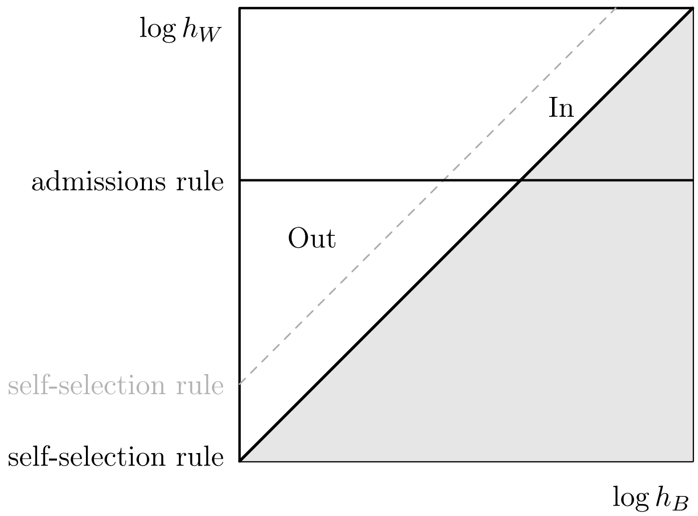
$$\mathbb{M}^{PI}(s_W(i), s_B(i)) = \mathbb{M}^{PI}(s_W(i)) = s_W(i) > \mu(|N_W^{PI}|),$$

i.e. a linear rule in s_W only.
In this environment, a scale–composition mechanism is always present: increasing $|N_W|$ expands the number of admits but lowers their average h_W. This trade-off is inherent to the admission problem, even if it does not necessarily translate into a smaller intake in equilibrium. This incentive may lead to a situation where $|N_W^{PI}| < |N_W^{CE}|$.

Proof: See Appendix A.10.

As outlined in Lemma 1, Lemma 2, and Proposition 1, all applicants apply truthfully, and the admissions agent sets $\mathbb{M}^{PI}(s_W(i), s_B(i) = s_W(i) > \mu(|N_W^{PI}|)$. However, what is new in proposition 4 relative to proposition 1, is that the admissions agent faces an incentive to restrict $|N_W^{PI}| < |N_W^{CE}|$, hence allowing for the possibility to endogenously creating capacity constraints in the number of college seats. The complete perfect information equilibrium is graphically depicted in figure 3 below:

Figure 3: Perfect Information Equilibrium



NOTES: The square depicts the skill space with $\log h_B$ on the horizontal axis and $\log h_W$ on the vertical axis. The solid diagonal represents the self-selection rule, while the horizontal line shows the admissions rule under perfect information. Applicants above the admissions rule are accepted ("In"), and those below are rejected ("Out").

Figure 3 illustrates the perfect information equilibrium, where the admissions agent restricts the number of admitted students below the competitive equilibrium level. The self-selection line (the black diagonal line) is pushed down relative to the competitive equilibrium in figure 1. The admissions rule found in proposition 4 is illustrated by the horizontal line.[16] While all individuals above the self-selection line applies to college, only individuals above the self-selection line *and* the admissions line are admitted to college and end up the white-collar sector. This outcome can arise even when the agent does not care about wages, there are no direct costs of admitting additional applicants, and all applicants have positive white-collar skills. The key mechanism is that the admissions agent cares about maximising the *aggregate* white-collar skills of admitted students.

By restricting capacity, the agent effectively increases the expected wages in the white-collar sector. This shifts the self-selection line down, attracting more individuals to the white-collar sector, and thereby applying to college. By shifting the self-selection line down, the admissions agent can incentivise some high-white-collar skilled individuals to apply for college who are also high-blue-collar skilled individuals and would otherwise not have applied. As there is no noise in the signals, the admissions agent can perfectly screen the applicants based on their true white-collar skills. Hence, by reducing the number of college seats, the admissions agent can increase the aggregate amount of white-collar skills in the white-collar

[16] It is horizontal as it loads only on s_W, and under perfect information, $s_W = \log h_W$. The admissions line intersects the $\log h_W$-axis at $\mu(|N_W^{PI}|)$.

sector by rejecting many applicants at the bottom of the distribution to attract a few high-skilled individuals who would otherwise not have applied. While the mechanism will always be present, the primitives of the model will be present. determine if the optimum lies below or above the competitive equilibrium.

Although reminiscent of monopoly logic, this mechanism is fundamentally different. In classic monopoly models, restricting access raises wages, which directly enters the admissions agent's preferences. Here, in contrast, the admissions agent does not value wages directly, but uses wage increases in the white-collar sector as a tool to attract more applicants from whom the admissions agent can choose. If the agent also valued student output or faced per-student costs, the incentive to restrict admissions would be even stronger. Similarly, institutional competition or poor planning could further reinforce these endogenous constraints.

5.3 Imperfect Information Equilibrium

We now analyse how equilibrium behaviour changes under asymmetric information when the admissions agent no longer observes the' true skills of the applicants but instead receives noisy signals. Formally, signals are given by:

$$s_j(i) = \log h_j(i) + \varepsilon_j(i), \quad \varepsilon_j \sim N(0, \sigma^2), \quad \sigma > 0, \quad j \in (W, B).$$

Note that this is similar to the equation 4.5 in section 4.2, except that, for simplicity, we assume that the magnitude of noise, σ, is the same in both sectors. When the admissions agent does not perfectly observe the skills of applicants, we can characterise the following about equilibrium admission behaviour:

Proposition 5: Imperfect Information Equilibrium (Talent Hoarding and Talent Separation)

Under imperfect information (II), the admissions agent sets an admission rule $\mathbb{M}^{II}(s_W, s_B) > \mu(|N_W^{II}|)$, placing positive weight on both the signal of the sector-relevant skill, s_W and the signal of the independent, sector-irrelevant skill, s_B (Proposition 2.A). Given a restriction in $|N_W^{PI}|$, the admissions agent selects a set N_W^{II}, whose mass lies in the interval $|N_W^{II}| \in [|N_W^{PI}|; |N_W^{CE}|]$ and is weakly increasing in noise of signals, σ.

Proof: See Appendix A.11.

As shown previously, the presence of noise in signals gives the admissions agent an incentive to rely on signals of irrelevant and skills. That is, the talent-hoarding effect is still present. Proposition 5 further demonstrates that the presence of noise creates an additional counterintuitive behavioural effect: the agent has an incentive to *increase* the number of admitted individuals relative to the perfect information scenario. We refer to this as *talent separation*.

The logic is as follows. The admissions agent faces a trade-off between admitting few versus many applicants. With few admissions, high wage rates W_W in the white-collar sector attract many applicants, but of mixed quality. With many admissions, the sector becomes less attractive, deterring weaker applicants, leaving a pool of fewer but stronger candidates.

Under perfect information, screening is easy. Under PI, the agent prefers to maximise the number of applicants, as the admissions agent can perfectly select individuals with the highest white-collar skills. With noisy signals, screening becomes more difficult and as it becomes harder to separate highly skilled applicants from low-skilled applicants, the *average quality* of the applicants matters more. By expanding admissions, the agent reduces the wage premium in the white-collar sector, discouraging low-skilled applicants from applying, and strengthens the self-selection mechanism. In effect, the admissions agent uses capacity as a strategic tool and is more reliant on the self-selection of applicants by making it more binding and informative, thus improving the average skills of applicants.

It is worth noting that in our model, a higher reliance on more informative self-selection is achieved by increasing $|N_W|$. However, the key mechanism, more generally, is that the admission agent reduces the attractiveness of the common component in the payoff function of the applicants. In our model, this translates into a decrease in W_W, which can be achieved by increasing $|N_W|$. This means that applicants will self-select more heavily on the individual component h_W, hence increasing the average skill level of applicants, which is what the admissions agent ultimately cares about. In general, institutions could achieve the same goal of reducing the attractiveness of the common component through other channels than increasing capacity, such as increasing tuition fees, relocating programmes to unattractive locations, or imposing other indirect costs common to all applicants which discourage low-skill applicants in the relevant dimension from applying. These measures arguably shift the self-selection threshold in a manner equivalent to our modelled increase in $|N_W|$.

This logic is consistent with the seminal signaling model of Spence (1973), in which agents undertake costly actions not to directly improve productivity but to credibly reveal their type. In our context, by reducing the attractiveness of applying to college, the admissions agent induces only those with sufficiently high expected productivity in the white-collar sector to apply for college, thereby improving the quality of the applicant pool when screening is noisy.

5.4 Effect of Noise on Equilibrium Distribution of Wages

As found, the effect of talent-separation increases the total skills in the target white-collar sector. Turning to the partial effect of talent-separation on the distribution of wages, we now investigate the mean log wages in both the white-collar sector, blue collar sector, and in the entire economy.

Proposition 6: Effect of Talent-Separation on the Distribution of Wages

With noisy signals (σ) and a fixed admissions-rule $\mathbb{M}(s_W, s_B) = s_W > \mu(|N_W|)$, increasing $|N_W^{II}|$ relative to $|N_W^{PI}|$ implies that:

1. The effect on mean log wages in the white-collar sector is ambiguous.
2. Mean log wages in the blue-collar sector increases.
3. When total wages in sector j is concavely increasing in $|N_j|$ $\forall j \in (W, B)$: Mean log wages in the entire economy increases.

Proof: See Appendix A.12.

As the extended model is nested in the baseline model presented in section 4, the talent-hoarding effect and the mechanical effect of noise in signals presented in Section 4.4 are also present in our extended model. The partial effect from talent-separation on mean log wages in the white-collar sector is ambiguous: On the one hand, the increase in the mass of admitted students, lowers the log wage rate, w_W, thus making the sector less attractive. On the other hand, this makes the self-selection constraint more binding, improving the expected $\log h_W$ of the applicants. Which effect dominates depends on the distribution of h_W and on the functional form of the wage rate. Mean log wages in the blue collar sector increases: As more individuals are admitted to college, fewer are pushed into the blue-collar sector despite having a comparative disadvantage there. This shift raises mean log wages in the blue-collar sector.

The aggregate effect on mean log wages in the economy is positive: relaxing the binding admission constraint allows a larger share of individuals to self-select based on their comparative advantage, thereby increasing mean log wages. In Section 8, we present a numerical example of these findings, validating and quantifying the magnitude of the effect of talent-separation on wages.

6 Generality

Although our framework can be easily understood through the lens of college admissions, its relevance extends far beyond this setting. At its core, the model captures the interaction between self-selection and admission procedures under perfect and imperfect information. As such, it applies to any context where individuals choose whether to apply, but a gatekeeping institution ultimately decides who gains access. This includes settings where the classical Roy model has traditionally been applied, such as sectoral choice, geographic migration, and occupational sorting, but where individuals cannot freely select into their preferred sector.

Beyond the margin of college versus non-college attendance, our findings also apply to horizontal educational decisions: In many countries, some colleges and specific college programmes have unrestricted access, and admission is governed almost exclusively by self-selection. In other colleges or specific programmes, applicants face capacity constraints and selective admissions. As we show in Sections 7 and 8, our model can also be applied to this scenario.

Another natural use of our model is on the topic of immigration policies. Consider a visa programme in which individuals apply to migrate to a high-income country. The immigration authority evaluates applicants based on observed characteristics such as education, language proficiency, and prior earnings—noisy proxies for true productivity. Like in our college admissions setting, the institution ends up hoarding talent away from countries (sectors) where it would have been more productive. The result can be an inefficient allocation of skills in the labour market of across countries and a misalignment between migrant ability and job placement.

Similarly, our framework applies to hiring in segmented labour markets. Employers often face a flood of applications and must screen candidates using imperfect information, such as CVs, test results, interviews, and referrals. Individuals self-select (apply) into firms or job types, but the firms decide who gets hired. This double-sided selection process, under noisy signals, can lead to talent hoarding: firms favour candidates with strong but potentially irrelevant signals. For example, a consulting firm may overemphasise GPAs, hoarding individuals who might have thrived in more technical or operational roles elsewhere.

In all these cases, the same mechanisms apply. When institutions make decisions based on noisy signals, they may rationally give weight to unproductive dimensions to improve screening. This gives rise to the talent-hoarding effect, where the use of such signals distorts the overall allocation of talent. Conversely, institutions may lower admission thresholds or adapt selection rules to elicit better self-sorting, leading to talent separation. The strength of our framework lies in its ability to unify these scenarios in a single conceptual model.

In sum, the generality of our framework lies in its structural insight: when sorting of individuals is a function of both self-selection and strategic behaviour by an external institution, new and previously unknown behavioural mechanisms can arise, which ultimately affect the allocation of talent. Understanding these dynamics is crucial for designing policies that allocate opportunity fairly and efficiently—whether in education, immigration, or labour markets.

7 The Empirical Validation of the Roy Model With Double-sided Selection

Ideally, we would like to test whether the behavioural effects are present in real-life admissions. Specifically, we would want to test whether (exogenously) introduced noise in signals would cause admissions agents to rely more on signals of not-directly relevant skills (talent hoarding) and increase the number of admitted students - or at least reduce the desirability of the study programme (talent separation). In Denmark, however, this is not feasible, as admission rules into programmes are often governed centrally and/or politically determined. Hence, admissions agents at individual college programmes have very limited discretion over how to select students. Instead, we settle for validating the underlying incentives driving the talent-hoarding mechanism. We investigate whether the mechanisms predicted by our model are present in the data such that admissions agents have an incentive to act according to our model *if they could*. Specifically, we investigate two key parts of the talent hoarding mechanisms outlined in section 4.2: i) College admissions agents have an incentive to put weight on signals of irrelevant skills when admitting students (proposition 2.A), and ii) this incentive is increasing in the wage rate associated with the irrelevant skill (proposition 2.B).

We consider applicants to BSc programmes in English and Mathematics across all Danish universities offering these fields. This setting provides an almost ideal environment, mimicking a controlled admission experiment, for our purpose to validate the talent-hoarding mechanism, as: First, all applicants to these two programmes were admitted in our sample period. This is crucial since we wish to approximate a scenario in which an admissions agent faces a pool of self-selected applicants and decides whom to admit. That is, currently, the setting of choosing between English and Mathematics in Denmark closely resembles the situation in a competitive equilibrium Roy-model depicted in figure 1. If admission were already restricted, we would only observe the admitted students, which would bias the analysis. Second, the mapping between skills and programmes is transparent, as english skills are directly relevant for success in the English programme, while math skills are directly relevant for success in the Mathematics programme. Third, for many applicants, we can obtain their high school exam grades in both English and Mathematics. These exams are taken prior to university admission and serve as noisy signals of applicants' underlying skills in each domain. Fourth, english and math are likely less correlated than, for example, math and physics or english and danish. In Appendix B.2 we explicitly address the potential concern of residual correlation between the two skills. Finally, there is a clear and substantial wage differential across the two fields. Graduates in Mathematics earn on average roughly 25 percent more than graduates in English. [17] As there are meaningful differences in wage rates between the two choices, this allows us to investigate proposition 2.B.

7.1 Empirical Strategy and Data

Using administrative data from Statistics Denmark, we collect a sample of individuals who applied and were admitted to Mathematics and English in the period 2002-2017. We link this to their written high school exam grades in english and math, respectively. The sample consists of 3581 students (1547 in Mathematics and 2034 in English) for whom we observe high school exam grades in both english and math. We standardise each grade to have mean zero and standard deviation of one. Our main outcome is the completion of the studies.

Then we estimate the following equations:

$$Y_{ij} = \beta_0^j + \beta_1^j Grade_{ij} + \beta_2^{-j} Grade_{i,-j} + \varepsilon_{ij}, \quad j \in (Math, English), \tag{7.1}$$

where Y_{ij} is a dummy for whether individual i completes study j where j is either B.Sc. in Mathematics or BA in English. β_1 measures how predictive the corresponding high school grade is in terms of predicting completion of the degree (i.e. High school english grade on completion of English Degree and vice versa). β_2 measures how predictive the *other* high school grade is for predicting the completion of the degree (that is, the high school math grade for the completion of the English degree and vice versa).

The aim of this exercise is to mimic an admissions scenario where an admissions agent has to weigh noisy signals (high school grades) of relevant and irrelevant skills to admit the best students. Hence, we wish to investigate whether an admissions agent would have an incentive to put weight on the signal of the irrelevant skill, when skills are not observed perfectly, as it is arguably the case with high school grades.

The ratio $\frac{\hat{\beta}_2}{\hat{\beta}_1}$ can be interpreted as the relative predictive power of the signal of the irrelevant skill relative to the signal of the relevant skill, and hence the strength of the

[17] See CEPOS (2025).

incentive to hoard talent. That is, a higher ratio implies a higher optimal relative weight to the signal of the irrelevant skill.[18]

Our theoretical results from the model would predict the following:

1. $\beta_1 > 0$: The relevant signal is a predictor of success.

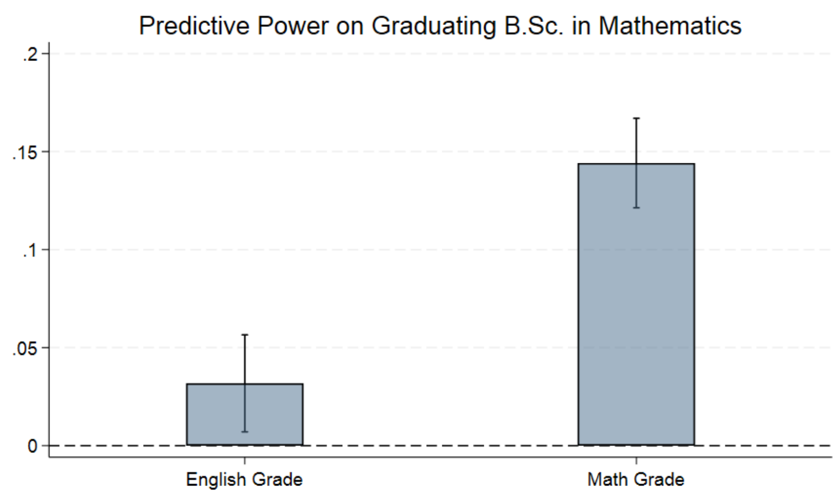
2. $\beta_2 > 0$: As there is noise in the exam grades, the "other" signal should also hold predictive power (proposition 2.A).

3. $\frac{\beta_2^{math}}{\beta_1^{english}} > \frac{\beta_2^{english}}{\beta_1^{math}}$: As the mean wage is higher for Mathematics graduates, the high school math grade should hold more predictive power for completion of English studies than the other way around (proposition 2.B).

7.1.1 Empirical Results

The coefficients of both regressions on completion of B.Sc. in Mathematics and B.Sc. in English, respectively, are reported in figures 5 and 4 below:

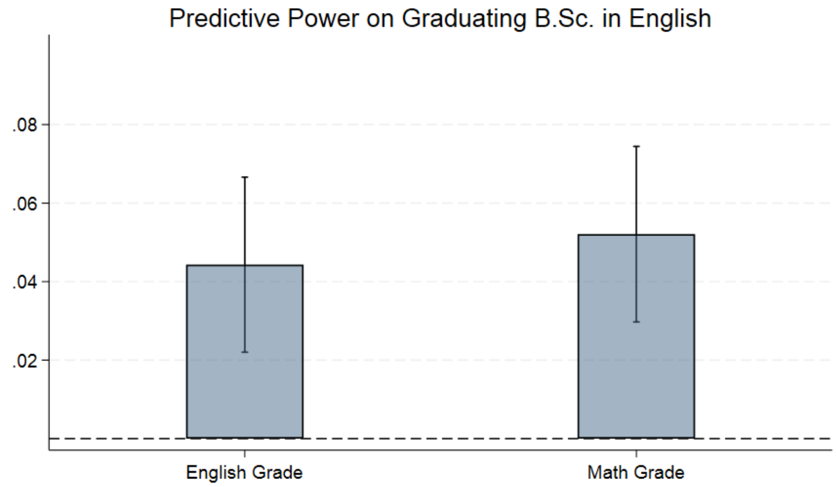
Figure 4: University Study: Math



NOTES: The figure shows coefficients from a regression of completion of the B.Sc. in Mathematics on standardized high school grades in english and math. The regression is based on 1547 observations. R^2 is 0.101. Source: Statistics Denmark and own calculations.

[18]If we consider a linear admission rule as the one in equation 3.5 equivalent to admitting students based on a weighted GPA, the ratio $\frac{\hat{\beta}_2}{\hat{\beta}_1}$ is an empirical estimate of relative weight to the irrelevant signal λ.

Figure 5: University Study: English



NOTES: The figure shows coefficients from a regression of completion of the B.Sc. in English on standardised high school grades in english and math. The regression is based on 2034 observations. R^2 is 0.026. Source: Statistics Denmark and own calculations.

Figure 5 shows the predictive power of the high school exam grades for english and math, respectively, for those studying English, and figure 4 shows the predictive power of the high school exam grades for english and math, respectively, for those studying Mathematics.

First, we note that all coefficients are positive and statistically significant. In addition, we notice that the relevant signal generally seems to be as large, or larger than the seemingly irrelevant signal. Most notably, we notice that the relative size of the prediction power is much larger for math on performance in the English study programme than the other way around. Consider the B.Sc. in Mathematics $\frac{\hat{\beta}_2^{english}}{\hat{\beta}_1^{math}} = 0.23$, meaning that the seemingly irrelevant signal holds around one-quarter the predictive power of the relevant signal. When considering B.Sc. in English $\frac{\hat{\beta}_2^{math}}{\hat{\beta}_1^{english}} = 1.18$.

Overall, we find that the signal of the irrelevant skills has positive predictive power in explaining graduation, even conditional on the relevant signal. In addition, we find that the signal of the irrelevant skills has higher predictive power if the wage associated with that particular skill is higher. These results are consistent with the theoretical findings in Proposition 2.A and proposition 2.B and are an intuitive example of the talent-hoarding effect: The combination of a potentially irrelevant skill and self-selection reveals something about the individuals' true relevant skills and can therefore be used as a predictor.

Although the results presented above are consistent with the model, two alternative interpretations could in principle generate similar empirical patterns. First, math and english skills may be intrinsically relevant for success in the opposite program. In particular, math skills could be hypothesised to be important for success on the English programme (see, e.g. Finseraas et al. (2024)). Second, the underlying joint distribution of math and english skills may be correlated, such that each grade contains information about the other due to noise in high school grading. To mitigate the first concern, we control for the exam grades of high school courses, which the English programme administrators *themselves* think are important for success on the English programme. Even in this regression, the coefficient to the math grade is positive, significant, and virtually unchanged. To mitigate the second concern, we utilize the full distribution of relevant high school grades to construct a residualised grade, purged of correlations between the underlying distribution of skills. While attenuated, we find that the coefficients to these residualised grade measures of the "irrelevant" grade is still positive, and the residualised math grade significantly predicts completion of the English programme in line with the predictions from proposition 2.A and proposition 2.B. These results suggest that our main predictions hold even when addressing potential competing explanations. In Appendix B.2 we describe in depth how we address these two concerns and report the specific results.

8 Numerical Example and Simulation

Having shown that the incentives proposed by our model are present in real-life admission data, we can take this empirical exercise one step further. In this section, we estimate a structural Roy model of the admissions system to which we introduce an admissions agent acting accordingly to our model, by optimally selecting an admissions rule and how many students to admit. Next we introduce noise in signals and conduct counterfactual simulations, to see how the behaviour of the admissions agent and the resulting distribution of wages changes. The purpose of this exercise is to provide an intuitive and empirically grounded example that highlights how strategic admission behaviour, specifically talent hoarding and talent separation, can distort wage distributions.

We again consider a pool of college applicants in Denmark who choose to study English or Mathematics. This setting allows us to apply the model not only to the decision to attend college but also to the choice of field-of-study. Again, we focus on these two programmes because, unlike many others in Denmark, they do not have restrictive admissions: all individuals who meet the general requirements are admitted. This situation therefore resembles a standard unrestricted competitive Roy model without an admission agent similar to the one depicted in figure 1.

We begin by estimating a simple competitive equilibrium Roy model without an admissions agent using simulated method of moments (SMM), based on individuals choosing between English and Mathematics, to anchor the parameters in our model to a real-life scenario. Using this structural Roy model, we introduce an admissions agent on the English programme and allow him to act according to our model in a situation where no noise is present. That is, he selects an admissions rule to maximise the amount of english skills admitted to the English programme. Next, we conduct counterfactual simulations in which we increase the noise in the signals and allow the admissions agent to adjust the number of admitted students and the relative weight on the math signal sequentially. We then analyse how admissions behaviour and the distribution of wages change as noise increases, isolating the talent-hoarding, and the talent-separation effect.

8.1 Selecting the Parameters in the Baseline Model

We set up a simple version of a standard competitive Roy-model with no admissions agent where individuals can either apply for English or Mathematics. The structure of the model is similar to our extended model outlined in section 5, except that when we estimate the model, we omit the admissions agent as both Mathematics and English have free admission. We model the wage rate as $W_j = (1-\alpha)A_j \left(\frac{K_j}{N_j}\right)^{\alpha}$ and normalise capital and total factor productivity to be equal to 1 in both sectors. We assume that $\alpha = 0.33$, as is standard in the literature. Further, we assume that skills in both math and english follow a joint log-normal distribution, and normalise the underlying normal distribution of english skills to be standard normal with mean equal to zero and variance equal to one.

We calibrate the correlation coefficient ρ of mathematics and english skills to be equal to 0.3 based on reduced form regressions.[19] While our theoretical framework assumes that skills are independently drawn across sectors, in practice the skills are often correlated amongst sectors. This is not a problem for the core intuition of our model. Our mechanisms remain valid as long as there exists a residual component of skills which is productive in one sector but unproductive in the other sector. While we can model this component theoretically, it is harder to disentangle empirically. However, we also perform a different calibration where ρ=0.0 and ρ=0.6, which we report in Appendix C. The qualitative conclusions of our model are similar in all cases.

We estimate the distribution of math-skills (mean and variance) using Simulated Method of Moments (SMM) (See e.g. Smith Jr (1993) and Jakobsen et al. (2022)). I.e. we estimate the parameters by minimizing the difference between simulated moments from our model and their empirical counterparts. According to Heckman & Honore (1990) the distribution of skills in a standard log-normal Roy-model can be identified from information on choices

[19] See appendix C for an elaboration.

and a single cross section of wage distributions.

We use the sample of admitted students in the period 2002-2017 on both Mathematics and English programmes in Denmark and their log of disposable income in 2022, based on Statistics Denmark's administrative registries, to calculate the empirical moments. The empirical moments we try to match are 1) the share of individuals choosing to study English, relative to the full population of both Mathematics and English students (0.67) and 2) the difference in post-graduation mean log disposable income between the English students and Mathematics students (0.106). We loop over potential parameter values, solving and simulating the model for different candidate values of the mean and variance of math skills until the simulated moments are equal to the empirical moments. We keep the difference in log variance in disposable income (0.11) as a validation moment.

The (SMM) estimation yields an estimated mean and variance of the underlying normal distribution of math skill to be -0.84 and 1.57, respectively. The implied difference in log variance in wages in the model is 0.139, which is close to the empirical moment of 0.11. For more information on the structural estimation procedure, see Appendix C.

8.2 Introducing The Admissions Agent and Counterfactual Simulations

Having parameterised the standard competitive Roy-model, we introduce an admissions agent on the English Programme and allow him to act according to our model by maximising the amount of expected english skills admitted into the English programme by selecting an admissions rule, similar to the model explained in equation 5.2-5.4.

We assume that the admissions agent uses a linear admissions rule,

$$\mathbb{M}(s_E, s_M) = s_E + \lambda s_M \geq \mu(|N_E|), \tag{8.1}$$

where s_E and s_M denote the english and math signals, respectively.

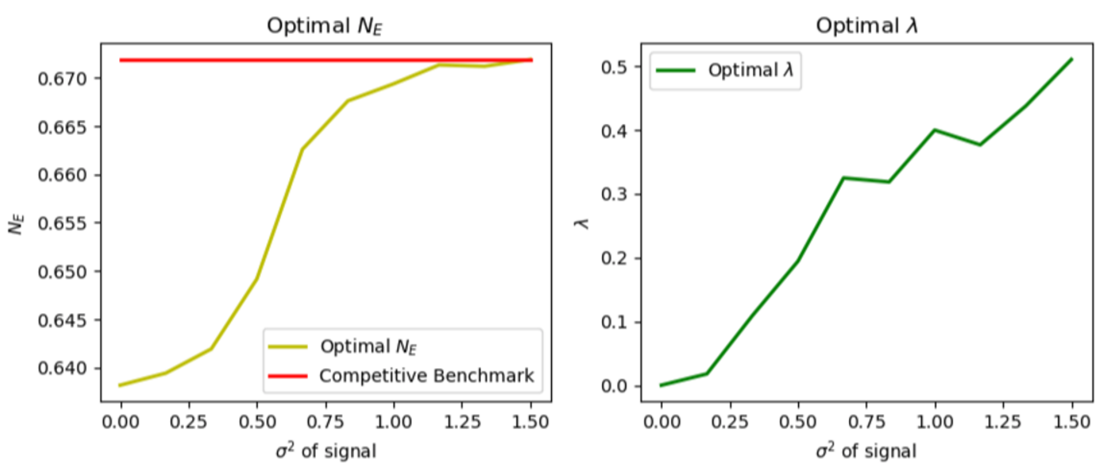
In our simulations, the admissions agent selects an admission capacity $|N_E|$ (share of total individuals that the admissions agent admits) and a relative weighting of the signals λ to maximize total expected english skills in the English programme. We impose a linear admissions rule, where the admissions agent weighs signals linearly, as this is typically used by admissions agents in practice, such as a weighted GPA calculation. In addition, it adds analytical simplicity to our numerical example. Finally, a linear admissions rule is particularly suited to show our key mechanisms in an intuitive way: In a linear admissions rule λ denotes how much weight to put on the sector-irrelevant signal of math skills, s_M relative to the signal of the relevant English skills, s_E. Hence, the λ selected by the admissions agent captures the size of the incentive to consider unproductive signals in admissions relative to the relevant signal and, therefore, the strength of the talent-hoarding effect.

In the first simulation, skills are perfectly observed; then we increase the noise of the math and english signals so that $\sigma_E^{Signal} = \sigma_M^{Signal}$ ranges from 0.0 to 1.5 and allow the admissions agent to adjust $|N_E|$ and λ. We simulate the model for 100,000 individuals for each level of noise in signals and report the optimal $|N_E|$ and λ chosen by the admissions agent and the corresponding output in the form of mean log wages, variance in log wages, and mean log wages in the Mathematics sector and English sector. We also report the corresponding output for a standard competitive Roy model as a reference. The choices and output are reported in figure 6 and figure 7.

8.2.1 Equilibrium Admission behaviour

Figure 6 shows how the admissions agent on the English programme chooses $|N_E|$ and λ under perfect information and as noise in the signals increases. These panels show both the talent-separation effect (increased $|N_E|$) and the talent-hoarding effect (increased λ) in play. The red line in figure 6 denotes the competitive equilibrium benchmark (no admissions agent).

Figure 6: Optimal $|N_E|$ and λ selected by English Admissions Agent



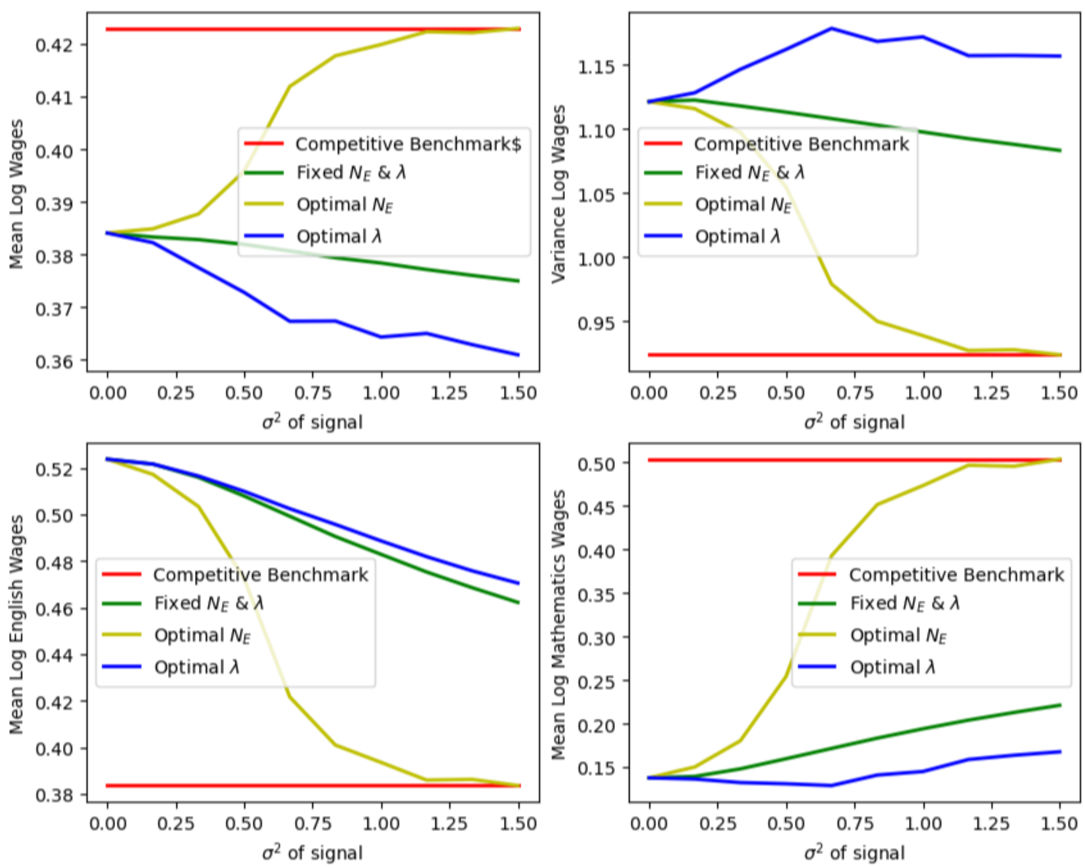
NOTES: The panel to the left shows the share of admitted individuals out of 100,000 into the English programme. The red line denotes the competitive benchmark and the yellow line denotes the optimal $|N_E^*|$ selected by the admissions agent. The panel to the right shows the λ chosen by the admissions agent. Source: Own model simulations. Model parameters are calibrated and estimated using data from Statistics Denmark.

When skills are perfectly observed (no noise in the signals of skills), the English Admissions agent sets $|N_E^*|$ to 0.635, which is below the 0.673, as was the share of individuals studying English in the competitive benchmark. From the right panel in Figure 6 we observe that when there is no noise in the signal, the admissions agent chooses λ of 0.0. As noise increases, both optimal $|N_E|$ and λ increase, consistent with both *the talent-hoarding effect* (increased λ) and *the talent-separation effect* (increased $|N_E|$). We note that $|N_E|$ converges to the competitive equilibrium benchmark as noise in the signals increases.

8.2.2 Equilibrium Distribution of Wages

Next, we investigate how the admissions agent and increasing noise in the signals impact sectoral wages.

Figure 7: Log Mean Wages and Log Variance



Notes: The top left panel shows development in mean log wages as noise in signals increase. The top right panel shows the development in variance in log wages. The two bottom panels show mean log wages in the two sectors (English left, Mathematics right) as noise increases. The red line represents the competitive benchmark. The green line represents a simulation where $|N_E|$ and λ is kept fixed at their no noise levels (mechanical effect). The blue line represents a simulation where the admissions agent adjusts λ as noise increases. The yellow line represents a simulation where the admissions agent adjusts $|N_E|$ as noise increases. Hence the difference between the green line and the blue line denotes the talent-hoarding effect on output, and the difference between the green line and the yellow line denotes the talent-separation effect on output. Source: Own model simulations. Model parameters are calibrated and estimated using data from Statistics Denmark.

Figure 7 shows the development in mean log wages (top left panel), the variance in log wages (top right panel), the mean log English wages (bottom left panel), and mean log Mathematics wages (bottom right panel) as noise in the signal increases. The red line represents the competitive equilibrium benchmark. The green line represents a simulation where both $|N_E|$ and λ are fixed at the optimal values of perfect information (the mechanical effect). The blue line represents a simulation where the admissions agent adjusts λ as noise in signals increases (the talent-hoarding effect). The yellow line represents a simulation where the admissions agent adjusts $|N_E|$ (the talent-separation effect) as the noise increases.

We first consider the distribution of wages under perfect information, when the noise of the signal is 0, and compare it to the competitive equilibrium. From the top left panel, we can see that introducing an admissions agent reduces mean log wages by around 0.04 log points compared to the competitive equilibrium benchmark. This is the result of an increase in the mean log wages in English by 0.14 log points, but a substantial reduction in mean log Mathematics wages. Notably, we can also observe from the top right panel that the variance in log wages has increased by around 0.15 log points compared to the competitive equilibrium benchmark.

Next, we consider how increasing the noise in the signals affects the distribution of wages. We start by considering the mechanical effect of increasing noise, that is, we keep $|N_E|$ and λ fixed at the perfect information values and increase the noise in the signal. The green line in figure 7 shows *the mechanical effect* of increasing noise on the distribution of wages. From the top left panel, we see that increasing noise decreases mean log wages by approximately 0.01 log points. This covers a decrease in mean log English wages of 0.05 log points and an increase in Mathematics wages of around 0.07 log points. From the top right panel, we notice that the mechanical effect marginally reduces the variance in log earnings.

We now consider *the talent-hoarding effect*. We investigate how changing the weight to the math signal, λ, as noise increases affects the distribution of earnings. The difference between the blue line and the green line quantifies this effect. From the top left panel, we see that allowing the admissions agent to adjust λ reduces mean log wages by around 0.02 log points when the noise in the signal is 1.50. This covers a slightly higher mean log English wage but a substantially lower mean log wage in Mathematics of around 0.05 log points. Interestingly, we also see that the talent-hoarding effect leads to a higher variance in log wages. This is caused by the fact that applicants with low mathematics skills are disproportionately pushed into the mathematics sector.

Finally, we consider *the talent-separation effect*. That is, we investigate how changing the number of admitted students as the noise increases affects output. The difference between the yellow line and the green line quantifies this effect. From the top left panel, we can see that allowing the admissions agent to increase admissions increases the mean log wage by approximately 0.04 log points when the noise in the signals is 1.50, converging to the competitive equilibrium. This covers a dramatic decrease in mean log English wages, but an even higher increase in mean log Mathematics wages. We also see that the talent-separation effect reduces the variance in log wages as more individuals are allowed to pursue their comparative advantage.

This numerical exercise shows that for reasonable and partially empirically founded parameter values, the two potential behavioural consequences of increased noise in signals (talent hoarding and talent separation) can have a substantial effect on mean log wages and the variance in log wages. We show that when noise increases, the impact of the two behavioural effects; talent hoarding and talent separation, matters *even more* for output than the mechanical *skill-mismatch* effect, which is already known from previous literature.

8.3 Policy Implications

This numerical exercise presented above is not a specific policy experiment, but rather an intuitive validation to showcase the impact of strategic admission behaviours and noisy signals. Yet, parallels can be drawn to actual educational policies. In Denmark, for instance, it was decided in 2014, that the capacity on educational programmes with relatively poor employment outcomes, predominately humanities studies, should be restricted to push applicants into educational programmes which had better employment outcomes for STEM-degrees.[20] This qualitatively corresponds to introducing an admissions agent in the English programme in our numerical exercise, which restricts access to the English programme and pushes applicants over in the Mathematics programme. Our simulations show that while such policies may appear to be effective as they increase the mean log wages for individuals enroled in the English programme, it risks introducing an aggregate wage loss as the individuals who are rejected for their preferred programme are pushed into other educational programmes where they have even worse potential outcomes, which reduces their earnings.

This negative selection effect on aggregate mean log wages risks being further aggravated by the procedure by which individuals are selected for restrictive study programmes. In Denmark, like in many other countries, college seats in restricted programmes are allocated to the individuals with the highest high school GPA's, such that individuals are selected into restricted humanities programmes by both their high school grades in humanities courses *and* high school STEM-courses. This phenomenon corresponds to the talent-hoarding effect in our model, which shows that while this increases the mean log wages in the restricted

[20] See Uddannelses- og Forskningsministeriet (2018).

programmes, it means that the individuals who are rejected for their preferred programmes are individuals who on average have even lower skills in other domains. Hence, by using this admission rule, we risk rejecting individuals from humanities programmes who are least likely to do well in other study programmes. This highlights another key insight from our model: Even though it from the perspective of the individual study programmes makes sense to select students based on signals of both relevant and less relevant skills, it risks introducing an aggregate wage loss, as the rejected individuals on average have a higher comparative disadvantage in the sectors they are pushed into.

One might argue that, contrary to our stylised two-sector model, individuals in reality face many study programmes. Hence, applicants rejected from the English programme are not forced into mathematics, but could substitute into more similar fields such as Literature or Journalism, thereby mitigating their loss from not being able to self-select. However, this reasoning overlooks the ripple effects in centralised admission systems: when seats in one programme are reduced, displaced applicants crowd into their second-best options, potentially displacing others from those programmes, and so forth. Gandil (2025) shows that in Denmark, such ripple effects can nearly double the number of applicants affected by capacity reductions. As many of these alternative programmes already have binding capacity constraints, the aggregate misallocation from restricting self-selection may therefore be even larger in a multi-programme environment compared to our stylised two-sector framework presented in the numerical exercise above.

9 Concluding Remarks

This paper develops a unified framework for analysing how institutional selection under imperfect information reshapes the allocation of talent. By extending the Roy model to include an admissions agent who selects applicants based on noisy signals, we uncover mechanisms with first-order implications for efficiency.

Under perfect information, with no asymmetric information, the admission agent observes true ability and admits students based only on relevant signals and skills. When noise is present (under imperfect information), institutions may rationally give weight to unproductive signals, a behaviour that we call *talent hoarding*. This behaviour raises output in the restricted sector but reduces aggregate efficiency and misallocates high-ability individuals.

Secondly, we show that capacity constraints can arise endogenously even when institutional objectives are monotonic in skill, as admissions agents strategically restrict access to improve selection quality. We demonstrate that noisier signals induce institutions to increase reliance on self-selection, a mechanism we call *talent separation*, which can partly offset informational frictions by improving the applicant pool.

In addition, we document the well-known *skill-mismatch* effect of noisy signals during admission processes, which subsequently distorts wage structures. This observation is consistent with the findings of existing scholarly literature. The effects on the wage distribution attributable to increased noise levels are systematically presented in Table 1.

These insights carry important policy implications. Attempts to redirect students across fields by tightening admissions may backfire: while wages rise in the restricted sector, aggregate output falls as talent is displaced from its most productive use. Similarly, admissions rules that rely heavily on proxyes such as GPA or extracurricular activities may benefit the institution locally but distort the overall allocation of skills. Policies that increase the cost of applying, such as tuition increases or geographic relocation, may improve selection under noisy signals but must be weighed against broader equity concerns.

Table 1: Effects of Noise in Signals, σ, on Mean Log Wages

Mechanism	$\bar{\omega}_W$	$\bar{\omega}_B$	$\bar{\omega}$
Talent Hoarding	↑	↓	↓
Talent Separation	?	↑	↑
Mechanical effect	↓	↑	?

NOTES: Arrows indicate the direction of change in mean log wages relative to the perfect information equilibrium. $\bar{\omega}_W$ and $\bar{\omega}_B$ denote mean log wages in the white-collar and blue-collar sectors, respectively, while $\bar{\omega}$ denotes aggregate mean log wages. "?" indicates theoretical ambiguity. Each row isolates a distinct mechanism by holding other variables fixed as described in the text.

In sum of our theoretical contributions, our framework nests the standard Roy model but extends it to account for the strategic behaviour of gatekeeping institutions. The resulting distortions; *talent hoarding*, *endogenous capacity restriction*, and *talent separation*, have measurable effects on wage distributions and sectoral sorting.

Using Danish administrative data, we show, in an empirical example, that the main predictions of the model are present in real-life admissions data. In particular, we show that signals of irrelevant skills hold predictive power for success in unrelated fields, especially when the wage premium in the irrelevant sector is high. We further demonstrate the dynamics of our model by estimating a structural model on the Danish admissions system. Through counterfactual simulations, we show that the behavioural effects of noisy signals on the distribution of wages can be substantial. In particular, we find that talent hoarding and talent separation have a larger impact on aggregate wages than the well-known mechanical mismatch effect.

More broadly, our framework applies to a wide range of settings beyond education, including immigration, hiring, and occupational sorting. In all these domains, institutions make decisions based on imperfect information, and individuals self-select based on private knowledge. The interaction between these forces gives rise to new behavioural dynamics that are not captured by traditional models.

In summary, this paper contributes a generalisable and empirically grounded framework for analysing talent allocation under imperfect information. It highlights the importance of institutional behaviour in shaping economic outcomes and offers new tools for evaluating policy interventions. As societies grapple with increasing inequality, skill mismatches, and constrained opportunities, understanding the dual role of self-selection and institutional screening becomes ever more critical. Our model provides a foundation for such analysis and opens the door to future research on designing institutions that allocate talent more efficiently and fairly.

References

Abdi, H. & Williams, L. J. (2010). Principal component analysis. *Wiley interdisciplinary reviews: computational statistics*, 2(4), 433–459.

Acemoglu, D. (1996). A microfoundation for social increasing returns in human capital accumulation. *The Quarterly Journal of Economics*, 111(3), 779–804.

Adão, R. (2015). Worker heterogeneity, wage inequality, and international trade: Theory and evidence from brazil. *Unpublished paper, MIT*, 98.

Almgren, M., Kramer, J., & Sigurdsson, J. (2023). *It runs in the family: Occupational choice and the allocation of talent*. Technical report, Mimeo.

Bjerre-Nielsen, A. & Chrisander, E. (2022). Voluntary information disclosure in centralized matching: Efficiency gains and strategic properties. *arXiv preprint arXiv:2206.15096*.

Borjas, G. J. (1987). Self-selection and the earnings of immigrants. *The American Economic Review*, (pp. 531–553).

Carrell, S. E., Fullerton, R. L., & West, J. E. (2009). Does your cohort matter? measuring peer effects in college achievement. *Journal of Labor Economics*, 27(3), 439–464.

Cattan, S., Salvanes, K. G., & Tominey, E. (2022). First generation elite: the role of school networks.

CEPOS (2025). Se listen: Hvad tjener man med forskellige lange videregående uddannelser? `https://cepos.dk/artikler/0311-se-listen-hvad-tjener-man-med-forskellige-lange-videregaende-uddannelser/`. Accessed 28 November 2025.

Chade, H., Lewis, G., & Smith, L. (2014). Student portfolios and the college admissions problem. *Review of Economic Studies*, 81(3), 971–1002.

Chen, Y. & Kesten, O. (2017). Chinese college admissions and school choice reforms: A theoretical analysis. *Journal of Political Economy*, 125(1), 99–139.

Cicala, S., Fryer, R. G., & Spenkuch, J. L. (2018). Self-selection and comparative advantage in social interactions. *Journal of the European Economic Association*, 16(4), 983–1020.

Cornelissen, T., Dustmann, C., & Schönberg, U. (2017). Peer effects in the workplace. *American Economic Review*, 107(2), 425–456.

Costrell, R. M. (1994). A simple model of educational standards. *The American Economic Review*, 84(4), 956–971.

Cui, J., Dias, G., & Ye, J. (2025). Signaling in the age of ai: Evidence from cover letters. *arXiv preprint arXiv:2509.25054*.

De Fraja, G. (2002). The design of optimal education policies. *The Review of Economic Studies*, 69(2), 437–466.

Dustmann, C., Fadlon, I., & Weiss, Y. (2011). Return migration, human capital accumulation and the brain drain. *Journal of Development Economics*, 95(1), 58–67.

Epple, D., Romano, R., & Sieg, H. (2006). Admission, tuition, and financial aid policies in the market for higher education. *Econometrica*, 74(4), 885–928.

Epple, D. & Romano, R. E. (1998). Competition between private and public schools, vouchers, and peer-group effects. *American Economic Review*, (pp. 33–62).

EVA (2016). Karaktergivning i gymnasiet. `https://eva.dk/Media/638379657332597157/Karaktergivning%20i%20gymnasiet.pdf`. Accessed 28 November 2025.

Finseraas, H., Nyhus, O. H., Salvanes, K. V., & Sandsør, A. M. J. (2024). *Sustained Effects of Small-Group Instruction in Mathematics*. Technical report, CESifo Working Paper.

Fredriksson, P., Hensvik, L., & Skans, O. N. (2018). Mismatch of talent: Evidence on match quality, entry wages, and job mobility. *American Economic Review*, 108(11), 3303–3338.

Fu, C. (2014). Equilibrium tuition, applications, admissions, and enrollment in the college market. *Journal of Political Economy*, 122(2), 225–281.

Galdin, A. & Silbert, J. (2025). Making talk cheap: Generative ai and labor market signaling. *arXiv preprint arXiv:2511.08785*.

Gandil, M. H. (2025). Trickle down education - ripple effects in college admissions. `https://www.dropbox.com/scl/fi/oar9p63bgpblw7jotazm5/RippleEffects.pdf?rlkey=m1f5bc6d0b7jo00ljvbjavoi6&e=1&dl=0`. Accessed 07 August 2025.

Hansen, A. T., Hvidman, U., & Sievertsen, H. H. (2024). Grades and employer learning. *Journal of Labor Economics*, 42(3), 000–000.

Heckman, J. J. & Honore, B. E. (1990). The empirical content of the roy model. *Econometrica: Journal of the Econometric Society*, (pp. 1121–1149).

Humlum, M. K. & Thorsager, M. (2021). The importance of peer quality for completion of higher education. *Economics of Education Review*, 83, 102120.

Intelligent.com (2021). 1 in 4 parents cheated to get their child into college. `https://www.intelligent.com/1-in-4-parents-cheated-to-get-their-child-into-college/`. Accessed 20 November 2025.

Jakobsen, K. M., Jørgensen, T. H., & Low, H. (2022). Fertility and family labor supply.

Jewitt, I. & Ospina, G. (2015). Selection by selection. *Working paper*.

Kleven, H., Landais, C., Muñoz, M., & Stantcheva, S. (2020). Taxation and migration: Evidence and policy implications. *Journal of Economic Perspectives*, 34(2), 119–142.

Københavns Universitet (2025). Kvote 2 på engelsk. `https://www.ku.dk/studier/bachelor/kvote-2/kvote-2-interview/kriterier/kvote-2-i-engelsk`. Accessed 28 November 2025.

Lucas Jr, R. E. (1988). On the mechanics of economic development. *Journal of monetary economics*, 22(1), 3–42.

MacLeod, W. B. & Urquiola, M. (2015). Reputation and school competition. *American Economic Review*, 105(11), 3471–3488.

Mas, A. & Moretti, E. (2009). Peers at work. *American Economic Review*, 99(1), 112–145.

Moscarini, G. (2005). Job matching and the wage distribution. *Econometrica*, 73(2), 481–516.

Mourifie, I., Henry, M., & Meango, R. (2020). Sharp bounds and testability of a roy model of stem major choices. *Journal of Political Economy*, 128(8), 3220–3283.

Rothschild, M. & White, L. J. (1995). The analytics of the pricing of higher education and other services in which the customers are inputs. *Journal of political Economy*, 103(3), 573–586.

Roy, A. D. (1951). Some thoughts on the distribution of earnings. *Oxford economic papers*, 3(2), 135–146.

Saltiel, F. (2023). Multi-dimensional skills and gender differences in stem majors. *The Economic Journal*, 133(651), 1217–1247.

Sievertsen, H. H. (2022). Assessments in education. *arXiv preprint arXiv:2208.05826*.

Smith Jr, A. A. (1993). Estimating nonlinear time-series models using simulated vector autoregressions. *Journal of Applied Econometrics*, 8(S1), S63–S84.

Spence, M. (1973). Job market signaling. *The Quarterly Journal of Economics*, 87(3), 355–374.

Taber, C. & Vejlin, R. (2020). Estimation of a roy/search/compensating differential model of the labor market. *Econometrica*, 88(3), 1031–1069.

Uddannelses- og Forskningsministeriet (2018). Evalueringen af den ledighedsbaserede dimensionering - sammenfatning af evalueringsrapporten. https://ufm.dk/uddannelse/institutioner-og-drift/styring-af-uddannelsesudbud/dimensionering/ledighedsbaseret-dimensionering/evaluering/sammenfatning.pdf. Accessed 28 November 2025.

Willis, R. J. & Rosen, S. (1979). Education and self-selection. *Journal of political Economy*, 87(5, Part 2), S7–S36.

Appendix

A Proofs of Propositions and Lemmas

A.1 Notational Summary

Notation Summary

Symbol	**Description**
N_W	Set of individuals admitted to the white-collar sector.
$\lvert N_W \rvert$	Mass of individuals admitted to the white-collar sector.
N_W^{CE}	Set of individuals in the white-collar sector under the competitive equilibrium.
$\lvert N_W^{CE} \rvert$	Admission mass under the competitive equilibrium.
N_W^{PI}	Set of individuals admitted to the white-collar sector under perfect information.
$\lvert N_W^{PI} \rvert$	Admission mass under perfect information.
N_W^{II}	Set of individuals admitted to the white-collar sector under imperfect information.
$\lvert N_W^{II} \rvert$	Admission mass under imperfect information.
$\tilde{N}_W$	Set of individuals admitted under an exogenous capacity constraint.
$\lvert \tilde{N}_W \rvert$	Exogenous admission capacity (mass).
$M^{PI}(s_W, s_B)$	Admission rule under perfect information.
$M^{II}(s_W, s_B)$	Admission rule under imperfect information.
$\mu(\lvert N_W \rvert)$	Admission threshold as a function of the admitted mass.
$s_j(i)$	Signal observed for skill $h_j(i)$, with $j \in \{W, B\}$.
$h_W(i)$, $h_B(i)$	Individual i's skills in the white- and blue-collar sectors.
$\omega_j(h_j)$	Wage function in sector j.
$w_j = \log W_j$	Log wage rate in sector j.
$W_j(\lvert N_{j-} \rvert)$	Wage rate in sector j, decreasing in the mass of individuals in sector j.

A.2 Proof of Lemma 1

For perfect information of own skills, the optimal application behaviour is to apply to the sector that offers the higher wage given one's skills.

Each individual i observes $(h_W(i), h_B(i))$. Sector-j wages satisfy:

$$\log \omega_j(h_j(i)) = w_j + \log h_j(i), \quad j \in \{W, B\}.$$

When $w_W + \log h_W(i) \geq w_B + \log h_B(i)$, the individual (weakly, strictly if $>$) prefers to apply to the white-collar sector. Therefore, whenever the white-collar sector pays (weakly) more, applying (weakly) dominates not applying; if it pays strictly more, applying is strictly better. When the inequality does not hold, not applying is preferred, and these individuals always end up in the blue-collar sector.

When $w_W + \log h_W(i) \geq w_B + \log h_B(i)$ but the individual's signals fall below the admission cutoff $\mathbb{M}(s_W(i), s_B(i)) < \mu$, they are indifferent, since applying are costless and rejection simply yields blue-collar employment. By convention, we let such individuals apply to the white-collar sector, which has no effect on admissions, wages, or any equilibrium object. This constitutes truthful application.

Robustness: If admission instead were stochastic but assigned any positive acceptance probability to types with $w_W + \log h_W(i) \geq w_B + \log h_B(i)$, the expected gain from applying would be strictly positive for such types.

If a small application cost $c > 0$ were introduced, only individuals who both expect to be admitted and have a comparative advantage in the white-collar sector would apply. Previously indifferent applicants would no longer apply, leaving admissions, wages, and all equilibrium allocations unchanged as they would, regardless, end up in the blue-collar sector.

■

A.3 Proof of Lemma 2

Any admission rule can, without loss of generality, be written as a threshold rule and that choosing a threshold is equivalent to choosing the admitted mass, given truthful application behaviour.

Let $\mathbb{M}(i) = m(s_W(i), s_B(i))$ denote the admission index for individual i, where $\mathbb{M}(\cdot)$ is continuous and weakly increasing in both arguments. This index is used to rank applicants by aggregating their observed signals into a single value, which the admissions agent uses to determine admission. Since the signals $(s_W(i), s_B(i))$ are continuously distributed with full support, the index $\mathbb{M}(i)$ is also continuously distributed with cumulative distribution function $F_{\mathbb{M}}(\mu)$.

For any cutoff μ, the share of admitted applicants is:

$$|N_W|(\mu) = \Pr(\mathbb{M}(i) \geq \mu) = 1 - F_{\mathbb{M}}(\mu).$$

Because F_M is continuous and strictly increasing, $|N_W(\mu)|$ is continuous and strictly decreasing from 1 to 0 as μ increases. Hence, there exists a unique inverse function $\mu = \mu(|N_W|)$, which implies that setting a cutoff μ is equivalent to choosing the admission mass $|N_W|$.

Intuition: The cutoff μ determines a quantile of the applicant pool. Choosing where to cut the distribution uniquely determines how many applicants are admitted. ■

A.4 Proof of Proposition 1

Under perfect information, the admissions agent admits the top $|\tilde{N}_W|$ individuals ranked by white-collar skill. The optimal admission rule is a threshold rule based solely on $s_W(i)$ and places zero weight on $s_B(i)$.

Under perfect information, the signals perfectly reveal true skills:

$$s_W(i) = \log h_W(i), \quad s_B(i) = \log h_B(i).$$

The admissions agent chooses an admission rule $M(s_W(i), s_B(i))$ to maximise total white-collar skill among the $|\tilde{N}_W|$ admitted individuals:

$$\max_{\mathbb{M}(s_W(i), s_B(i))} \int_{i \in \tilde{N}_W} h_W(i)\, di.$$

Because the objective depends only on $h_W(i)$, any variation in $s_B(i)$ is irrelevant: if $h_W(i) > h_W(i')$, replacing i' with i in the admitted set strictly increases total white-collar skill, regardless of s_B values.

Hence, the optimal ranking is determined entirely by $h_W(i)$, which under perfect information is one-to-one with $s_W(i)$. This implies that the optimal rule is a monotone linear function of s_W; there exists a cutoff $\mu(|\tilde{N}_W|)$ such that

$$\mathbb{M}^{PI}(s_W(i), s_B(i)) = s_W(i) \geq \mu(|\tilde{N}_W|).$$

By Lemma 2 (equivalence of threshold and admission mass), the threshold $\mu(|\tilde{N}_W|)$ is uniquely determined by $|\tilde{N}_W|$. Any admission rule that placed positive weight on s_B would admit some applicants with lower h_W than some rejected applicants, strictly reducing the objective. Hence, the linear rule in s_W is uniquely optimal. ■

A.5 Proof of Proposition 2.A

When signals are noisy, it is optimal for the admissions agent to assign a positive weight to the signal of the *sector-irrelevant* skill, even if the two skills are uncorrelated and the blue-collar signal is unproductive in the white-collar sector.

Setup: Consider two applicants to the white-collar college:

$$\begin{aligned}\text{Applicant A:}\quad & s_W(A) = p + \gamma, \quad s_B(A) = \psi,\\ \text{Applicant B:}\quad & s_W(B) = p, \quad s_B(B) = \psi + \tau.\end{aligned}$$

Assume $p, \gamma, \psi, \tau > 0$. Assume further that $s_B = \log h_B$ is perfectly informative (i.e., no noise), and that the only noise is in the white-collar signal:

$$\log h_W(i) = s_W(i) + \varepsilon_W(i), \qquad \varepsilon_W(i) \sim \mathcal{N}(0, \sigma^2).$$

In words, the notation used in the proofs for the entire section is as follows: γ denotes how much higher Applicant A's white-collar signal s_W is relative to applicant B; τ denotes how much higher Applicant B's blue-collar signal s_B is relative to applicant A; ψ is the baseline blue-collar signal; and p is the baseline white-collar signal. We define $c \equiv w_B - w_W$, which is the relative difference in log wage rates between sectors.

Expected $f(h_W(i))$ conditional on application: On the margin, the admissions agent prefers to admit the applicant with the highest expected $f(h_W(i))$, where f is a monotonically increasing function. For simplicity and transparency, we use $f(h_W(i)) = \log h_W(i)$, and as f is a monotonically increasing function, the inequality holds regardless of the choice of f. We now investigate which applicant is preferred by the admissions agent.

$$\text{Applicant A is preferred if } \mathbb{E}[\log h_W(A)] > \mathbb{E}[\log h_W(B)].$$

The conditional expectation for A, given signals and the fact that she applied (i.e. $\log h_W(A) > \psi + c$), is:

$$\mathbb{E}[\log h_W(A) \mid s_W(A), s_B(A), \text{apply}] = \mathbb{E}[\log h_W(A) \mid s_W(A), s_B(A), w_W + \log h_W(A) \geq w_B + \log h_B(A)]$$

$$= p + \gamma + \sigma \cdot \frac{\phi\left(\frac{\psi + c - p - \gamma}{\sigma}\right)}{1 - \Phi\left(\frac{\psi + c - p - \gamma}{\sigma}\right)},$$

where ϕ and Φ denote the standard normal pdf and cdf, respectively.

Similarly, for B:

$$\mathbb{E}[\log h_W(B) \mid s_W(B), s_B(B), \text{apply}] = p + \sigma \cdot \frac{\phi\left(\frac{\psi+\tau+c-p}{\sigma}\right)}{1-\Phi\left(\frac{\psi+\tau+c-p}{\sigma}\right)},$$

The latter term in both equations above denotes the expected white-collar skill premium from applying, such that the self-selection constraint is satisfied. It follows directly from the rule of expected values from truncated normal distributions.

When ignoring s_B is suboptimal: If the admissions rule only considers s_W, applicant A would be admitted when $s_W(A) > s_W(B)$. We now show that it can be optimal instead to admit B.

Applicant B is preferred whenever:

$$\mathbb{E}[\log h_W(B) \mid s_W(B), s_B(B), \text{apply}] > \mathbb{E}[\log h_W(A) \mid s_W(A), s_B(A), \text{apply}]$$

which holds if and only if:

$$\sigma\left[\frac{\phi\left(\frac{c+\psi+\tau-p}{\sigma}\right)}{1-\Phi\left(\frac{c+\psi+\tau-p}{\sigma}\right)} - \frac{\phi\left(\frac{c+\psi-p-\gamma}{\sigma}\right)}{1-\Phi\left(\frac{c+\psi-p-\gamma}{\sigma}\right)}\right] - \gamma > 0.$$

Existence of such applicants: Define $\lambda(z) \equiv \phi(z)/(1-\Phi(z))$, which is strictly increasing in z. Then we get:

$$\sigma\left[\lambda\left(\frac{\psi+\tau+c-p}{\sigma}\right) - \lambda\left(\frac{\psi+c-p-\gamma}{\sigma}\right)\right] > \gamma.$$

Since $c + \psi + \tau - p > c + \psi - p - \gamma$, the bracketed terms are positive. Moreover, as $\lambda(z) \to \infty$ when $z \to \infty$, for any finite γ there exists τ sufficiently large such that the inequality above holds. Because the applicant pool is a continuum, there always exist pairs (γ, τ) such that the admissions agent strictly prefers B. Hence, the optimal admission rule must load positively on s_B.

Robustness to noisy s_B and multiple signals: When both signals are noisy, $s_W = \log h_W + \varepsilon_W$ and $s_B = \log h_B + \varepsilon_B$ (cf. Equation (3.4)), the same logic applies. It suffices that s_B is incrementally informative about h_W conditional on s_W and application, i.e.,

$$\frac{\partial \mathbb{E}[h_W \mid s_W, s_B, \text{apply}]}{\partial s_B} > 0.$$

Then there exists a rule that assigns positive weight to s_B strictly increases the admissions agent's objective, since f is strictly increasing. More generally, in any multidimensional signal environment, whenever the productive signal s_W is sufficiently noisy, other signals s_j, $j \neq W$, contain incremental information through applicant' self-selection, when the other signals are productive in other sectors, implying that it remains optimal to assign positive weight to such signals.

So in conclusion, for any $\sigma > 0$ and non-minus infinity wage differential $w_B - w_W$, there exist applicants A and B as defined above for which:

$$\mathbb{E}[\log h_W(B) \mid \text{apply}] > \mathbb{E}[\log h_W(A) \mid \text{apply}].$$

Hence, an applicant with a weaker observed white-collar signal s_W but a stronger irrelevant signal s_B may be strictly preferred. It follows that the optimal admission rule assigns positive weight to s_B even when s_B is unproductive in the white-collar sector and uncorrelated with h_W.

Generalization: If there exists a pair of applicants where one has a lower white-collar signal but a higher blue-collar signal, and yet a higher expected white-collar skill, the same trade-off arises throughout the applicant pool. Because both the applicant distribution and the mapping from signals to underlying skills vary smoothly, an argument that holds at one margin extends to a neighbourhood of similar cases.

This completes the proof that under imperfect information, the admissions agent may optimally assign positive weight to an irrelevant signal due to its informational value through self-selection.

■

A.6 Proposition 2.B

The incentive to assign weight to the irrelevant signal s_B increases in the relative log wage ratio $w_B - w_W$.

Set-up: We define the incentive to accept applicant B over applicant A:

$$\delta \equiv \sigma \left[\lambda\left(\frac{c + \psi + \tau - p}{\sigma}\right) - \lambda\left(\frac{c + \psi - p - \gamma}{\sigma}\right)\right] - \gamma,$$

where $\lambda(z) \equiv \frac{\phi(z)}{1-\Phi(z)}$ denotes the inverse Mills ratio. δ expresses the relative difference in expected $\log h_W$ between applicant B, who has lower observed s_W, and applicant A, who has a lower s_B. This expression is directly tied to the incentive to consider s_B in admissions: for a given combination of (observed) advantage by applicant A in the white-collar sector, γ, and by applicant B in the blue-collar sector, τ, the expression δ denotes the difference in expected $\log h_W$ between applicant B and applicant A. In other words, it describes how large the incentive is to admit applicant B over applicant A. When δ increases, it implies an increased incentive to consider the applicant who has a higher s_B despite having a lower s_W. We can now investigate how the primitives affect δ. If a term increases δ, it increases the incentive to consider s_B relative to s_W.

When the Relative Wages Change: Let's consider the effect on δ as c increases.

We have that $\log\left(\frac{W_B}{W_W}\right) = w_B - w_W \equiv c$. Investigating how $w_B - w_W$ affects δ is equivalent to investigating how c affects δ. We have that:

$$\frac{\partial \delta}{\partial c} = \lambda'\left(\frac{c + \psi + \tau - p}{\sigma}\right) - \lambda'\left(\frac{c + \psi - p - \gamma}{\sigma}\right),$$

where $\lambda'(z) = \lambda(z)(\lambda(z) - z)$. As $\lambda'(z)$ is positive and strictly increases in z and $c + \psi + \tau - p > c + \psi - p - \gamma$, it follows that $\frac{\partial \delta}{\partial c} > 0$. Intuitively, this is the case, as a given increase in c eliminates a relatively larger mass of potential $\log h_W$ at the bottom of the distribution for applicant B than for applicant A, thus increasing the expected value of $\log h_W$ relatively more for applicant B than for applicant A.

We can see that δ, and thus the incentive to consider s_B relative to s_W, is increasing in $w_B - w_W$, which concludes the proof. ■

A.7 Weight to Unproductive Signal when Noise Increases

Consider δ from A.6. We have $\frac{\partial \delta}{\partial \sigma} > 0$, meaning that the incentive to assign weight to the irrelevant signal s_B increases as the noise in the white-collar signal s_W increases. This holds under the sufficient condition that $p + \gamma - (c + \psi) \geq \frac{\tau+\gamma}{2}$, which ensures that Applicant A's advantage in s_W, adjusted for the wage differential and baseline s_B, is not too large relative to Applicant B's advantage in s_B. This is true for some applicants A and B when there is enough heterogeneity in the underlying distribution of skills and signals.

Recall that τ is applicant B's additional blue collar signal, and γ is applicant A's additional white collar signal. Hence, $\frac{\tau+\gamma}{2}$ is a balance point, which A's distance over the cut-off $p + \gamma - (c + \psi)$ must be greater than. Intuitively, the sufficient condition can be understood as follows: if A is far away from the cut-off, her self-selection reveals less information in it-self, which indirectly increases the incentive to admit B.

When this is true, then for a given admission mass $|N_W|$, an increase in σ reduces the informativeness of s_W, making it harder for the admissions agent to infer true white-collar skills. As a result, the optimal admission rule shifts toward placing more weight on s_B, which becomes incrementally informative through the self-selection behaviour of applicants. Consequently, some applicants with lower s_W but higher s_B are more likely to be admitted.

A.8 Proof of Proposition 3.A

When noise in signals, σ, is introduced and the admission rule is kept fixed at $\mathbb{M}^{PI}(s_W(i)) = s_W(i) > \mu(|\tilde{N}_W|)$, then mean log wages in the white-collar sector decrease, mean log wages in the blue-collar sector increase, and the effect on mean log wages in the entire economy is ambiguous.

Mean log wages in the white-collar sector: Consider a situation without noise in the signals ($\sigma = 0$) and a given mass of admitted applicants, $|\tilde{N}_W|$. We define $c \equiv w_B - w_W$. The mean log wages for individuals admitted into the white-collar sector are:

$$
\begin{aligned}
&\mathbb{E}[w_W + \log h_W(i) \mid i \in N_W] \\
&= \mathbb{E}[w_W + \log h_W(i) \mid \log h_W(i) > \log h_B(i) + c,\ s_W(i) > \mu(|\tilde{N}_W|)] \\
&= w_W + \mathbb{E}[\log h_W(i) \mid \log h_W(i) > \log h_B(i) + c,\ \log h_W(i) > \mu(|\tilde{N}_W|)].
\end{aligned}
$$

Now consider what happens when the noise in the signal σ is introduced such that $s_W(i) = \log h_W(i) + \varepsilon_W(i)$. For the admission rule fixed on the perfect information equilibrium admission rule, $\mathbb{M}(s_W(i), s_B(i)) = s_W(i) > \mu(N_W^{PI})$, the mean log wages in the white-collar sector become:

$$
w_W + \mathbb{E}[\log h_W(i) \mid \underbrace{\log h_W(i) > \log h_B(i) + c}_{\textit{Self-selection}},\ \underbrace{\log h_W(i) + \varepsilon_W(i) > \mu(|\tilde{N}_W|)}_{\textit{Admission Rule}}].
$$

As $\sigma \uparrow$ the informative part of the signal in the admission rule $\log h_W(i)$ becomes less and less informative relative to the noise term $\varepsilon_W(i)$. As $\sigma \to \infty$, the admission rule collapses to $\varepsilon_W(i) > \mu(\tilde{N}_W)$, which is completely uninformative of $\log h_W(i)$. The expected log wages in the white-collar sector therefore become:

$$
\begin{aligned}
&w_W + \mathbb{E}[\log h_W(i) \mid \log h_W(i) > \log h_B(i) + c] \\
&< w_W + \mathbb{E}[\log h_W(i) \mid \log h_W(i) > \log h_B(i) + c,\ \log h_W(i) > \mu(|\tilde{N}_W)|], \quad \forall |\tilde{N}_W| < |N_W^{CE}|.
\end{aligned}
$$

This is true as the extra condition on the right hand side puts a minimum value on $h_W(i)\ \forall\ i \in |\tilde{N}_W|$, which is not present on the left hand side of the inequality. Intuitively, under perfect information very low h_W applicants are rejected. This is not guaranteed as $\sigma \to \infty$ as some very low h_W applicants are admitted who self-select into the white-collar sector because they also have very low values of h_B.

Mean log wages in the blue-collar sector: All individuals who are below the self-selection line and hence do not apply for college are never admitted anyways. Hence, increasing noise in signals does not affect their wages. The only impact increasing noise in signals has on mean log wages in the blue-collar sector is through the difference in the blue-collar skills of those who are rejected without noise in the signals relative to a situation with noise in the signals. We therefore restrict the focus to those individuals:

Mean log blue-collar wages for those who are rejected for college when there is no noise in signals:

$$\mathbb{E}[w_B + \log h_B(i) \mid \log h_W(i) > \log h_B(i) + c,\ \log h_W(i) < \mu(|\tilde{N}_W)|].$$

Now consider mean log blue-collar wages for those who are rejected for college when $\sigma \to \infty$:

$$\mathbb{E}[w_B + \log h_B(i) \mid \log h_W(i) > \log h_B(i) + c].$$

The mean log wages of the ones rejected become the mean log wages of the ones applying, when the admissions rule becomes uninformative of $\log h_W$ as $\sigma \to \infty$. We compare the difference in expected blue-collar wages from a situation with complete noise and without noise, and notice that:

$$\mathbb{E}[\log h_B(i) \mid \log h_W(i) > \log h_B(i) + c] > \mathbb{E}[\log h_B(i) \mid \log h_W(i) > \log h_B(i) + c,\ \log h_W(i) < \mu(|\tilde{N}_W)|].$$

As $\log h_W$ and $\log h_B$ are independent, this inequality holds. The reason is that the latter term includes an upper bound on the distribution of $\log h_W$. As $\log h_B(i) + c < \log h_W(i)$ and the two distributions are independent, the condition $\log h_W(i) < \mu(|\tilde{N}_W|)$ shifts the conditional expectation of $\log h_B(i)$ down in the second term relative to the first term. As both distributions are continuous, the inequality holds strictly.

Mean log wages in the entire economy: While increasing noise in signals raises mean log wages in the blue-collar sector and lowers them in the white-collar sector, the aggregate effect on mean log wages in the economy is, to our knowledge, theoretically ambiguous. It depends on the relative magnitudes of the reported effects across sectors, which depends on the underlying distributions of skills, h_W, h_B. ■

A.9 Proof of Proposition 3.B

For a fixed $|\tilde{N}_W|$ and increasing noise in the signals, σ, increasing the relative weight in the admissions rule to the signal of the irrelevant skill, s_B increases mean log wages in the white-collar sector, decreases mean log wages in the blue-collar sector, and decreases mean log wages in the entire economy.

Mean log wages in the white-collar sector: To evaluate the impact of the admission rule in the white-collar sector, we begin with defining the mean log wages in the white-collar sector:

$$\mathbb{E}[w_W + \log h_W(i) | i \in \tilde{N}_W] = w_W + \mathbb{E}[\log h_W(i) | i \in \tilde{N}_W].$$

In proposition 2.A, we show that considering s_B in the admissions rule increased $\mathbb{E}[\log h_W(i)]$ of the admitted students. w_W is unaffected by the admissions rule. Hence, mean log wages in the white-collar sector increase.

Mean log wages in the blue-collar sector:

To evaluate the impact of the admission rule in the blue-collar sector, consider the two applicants to the white-collar college as outlined in the proof of proposition 2.A, with the following signals:

$$\begin{aligned} \text{Applicant A:} \quad & s_W(A) = p + \gamma, \quad s_B(A) = \psi, \\ \text{Applicant B:} \quad & s_W(B) = p, \quad s_B(B) = \psi + \tau, \end{aligned}$$

where $p, \gamma, \psi, \tau > 0$. Assume that $s_W = \log h_W$ is perfectly informative (i.e., without noise) and that the only noise is in the blue-collar signal:

$$\log h_B(i) = s_B(i) + \varepsilon_B(i), \qquad \varepsilon_B(i) \sim \mathcal{N}(0, \sigma^2).$$

We can think of this as keeping the noise fixed in the irrelevant dimension, allowing us to evaluate how each sector is affected by the admission rule. In the absence of, to our knowledge, a formal solution with noise in both dimensions, we fix noise in the white-collar sector in this proof to analyse the impact on blue-collar log wages. Let $c \equiv w_B - w_W$ denote the relative sector wage differential.

We want to show that accepting B in white-collar instead of A, which means that applicant A ends in the blue-collar sector instead of applicant B, which is what a rule which loads positively on s_B as explained in proposition 2.A implies, makes the mean log wages decrease in the blue-collar sector. We hence compare the expected blue-collar wages for respectively A and B, conditional on both of them applying to the white-collar sector. Throughout the proof, we use that $\frac{\phi(z)}{\Phi(z)} = \frac{\phi(-z)}{1-\Phi(-z)} = \lambda(-z)$.

First, consider the conditional expectation of $\log h_B(A)$:

$$
\begin{aligned}
\mathbb{E}[\log h_B(A)|s_W(A), s_B(A), \text{Applied}] &= \mathbb{E}[\log h_B(A)|s_W(A), s_B(A), \log h_W(A) > \log h_B(A) + c] \\
&= \psi - \sigma \left(\frac{\phi\left(\frac{p+\gamma-c-\psi}{\sigma}\right)}{1-\Phi\left(\frac{p+\gamma-c-\psi}{\sigma}\right)} \right).
\end{aligned}
$$

Now consider the conditional expectation of $\log h_B(B)$:

$$
\begin{aligned}
\mathbb{E}[\log h_B(B)|s_W(B), s_B(B), \text{Applied}] &= \mathbb{E}[\log h_B(B)|s_W(B), s_B(B), \log h_W(B) > \log h_B(B) + c] \\
&= \psi + \tau - \sigma \left(\frac{\phi\left(\frac{p-c-\psi-\tau}{\sigma}\right)}{1-\Phi\left(\frac{p-c-\psi-\tau}{\sigma}\right)} \right).
\end{aligned}
$$

Defining the inverse Mills ratio as:

$$
\lambda(z) = \frac{\phi(z)}{1-\Phi(z)},
$$

we can rewrite the expectations as:

$$
\begin{aligned}
\mathbb{E}[\log h_B(A) \mid \text{Applied}] &= \psi - \sigma \cdot \lambda \left(\frac{p+\gamma-c-\psi}{\sigma} \right), \\
\mathbb{E}[\log h_B(B) \mid \text{Applied}] &= \psi + \tau - \sigma \cdot \lambda \left(\frac{p-c-\psi-\tau}{\sigma} \right).
\end{aligned}
$$

The latter term in the two equations denotes the penalty of expected blue-collar skills from having applied to the white-collar sector. We compare these two expressions. The difference is:

$$
\mathbb{E}[\log h_B(B) \mid \text{Applied}] - \mathbb{E}[\log h_B(A) \mid \text{Applied}] = \tau + \sigma \cdot \left[\lambda \left(\frac{p+\gamma-c-\psi}{\sigma} \right) - \lambda \left(\frac{p-c-\psi-\tau}{\sigma} \right) \right].
$$

Since $\lambda(z)$ is strictly increasing in z, and given that $\gamma, \tau > 0$, we have:

$$
\frac{p+\gamma-c-\psi}{\sigma} > \frac{p-c-\psi-\tau}{\sigma},
$$

which implies:

$$
\lambda \left(\frac{p+\gamma-c-\psi}{\sigma} \right) > \lambda \left(\frac{p-c-\psi-\tau}{\sigma} \right).
$$

Therefore, the entire expression is strictly positive and we conclude:

$$\mathbb{E}[\log h_B(B) \mid \text{Applied}] > \mathbb{E}[\log h_B(A) \mid \text{Applied}].$$

This implies that if Applicant B is admitted to the white-collar sector instead of Applicant A, the expected blue-collar skill of the rejected individual (A) is lower than that of B. Repeating this across each pair of applicants, affected by the changed admission rule as noise increases, proven in proposition 2.A, it follows that mean log wages in the blue-collar sector decreases.

Mean log wages in the entire economy:

In order for mean log wages in the entire economy to fall, it must be such that the mean log wage loss in the blue-collar sector is higher than the mean log wage gain in the white-collar sector. To show that the wages in the entire economy decrease, we consider the same applicants A and B as in the proof of proposition 2.A. We show that accepting Applicant B into the white-collar sector instead of Applicant A gives lower aggregate wages than admitting A into white-collar and letting B enter the blue-collar sector. In this scenario we assume that there is both noise in the white-collar signal and the blue-collar signal which is equal to σ.

We define aggregate log wages as the sum of expected log wages across both sectors when A is admitted into the white-collar sector and B enters the blue-collar sector and vice versa as:

$$\begin{aligned}
\log \omega^{A\to W, B\to B} &= \mathbb{E}[\log h_W(A) \mid \text{Applied}] + \mathbb{E}[\log h_B(B) \mid \text{Applied}], \\
\log \omega^{B\to W, A\to B} &= \mathbb{E}[\log h_W(B) \mid \text{Applied}] + \mathbb{E}[\log h_B(A) \mid \text{Applied}].
\end{aligned}$$

We want to show that:

$$\log \omega^{B\to W, A\to B} < \log \omega^{A\to W, B\to B}.$$

I.e. that following the admissions rule which loads positively on s_B and admits B into the white-collar sector over A reduces mean log wages in total relative to the case of the admissions rule which only considers s_W and admits A into the white-collar sector over B.

From earlier results, we know:

$$\begin{aligned}
\mathbb{E}[\log h_W(A) \mid \text{Applied}] &= p + \gamma + \sigma \cdot \lambda\left(\frac{\psi + c - p - \gamma}{\sigma}\right), \\
\mathbb{E}[\log h_W(B) \mid \text{Applied}] &= p + \sigma \cdot \lambda\left(\frac{\psi + \tau + c - p}{\sigma}\right), \\
\mathbb{E}[\log h_B(A) \mid \text{Applied}] &= \psi - \sigma \cdot \lambda\left(\frac{p + \gamma - c - \psi}{\sigma}\right), \\
\mathbb{E}[\log h_B(B) \mid \text{Applied}] &= \psi + \tau - \sigma \cdot \lambda\left(\frac{p - c - \psi - \tau}{\sigma}\right).
\end{aligned}$$

such we get:

$$\begin{aligned}
\log \omega^{B\to W, A\to B} &= \mathbb{E}[\log h_W(B) \mid \text{Applied}] + \mathbb{E}[\log h_B(A) \mid \text{Applied}] \\
&= \left(p + \sigma \cdot \lambda\left(\frac{\psi + \tau + c - p}{\sigma}\right)\right) + \left(\psi - \sigma \cdot \lambda\left(\frac{p + \gamma - c - \psi}{\sigma}\right)\right)
\end{aligned}$$

$$\begin{aligned}
\log \omega^{A\to W, B\to B} &= \mathbb{E}[\log h_W(A) \mid \text{Applied}] + \mathbb{E}[\log h_B(B) \mid \text{Applied}] \\
&= \left(p + \gamma + \sigma \cdot \lambda\left(\frac{\psi + c - p - \gamma}{\sigma}\right)\right) + \left(\psi + \tau - \sigma \cdot \lambda\left(\frac{p - c - \psi - \tau}{\sigma}\right)\right)
\end{aligned}$$

The inequality holds when

$$\log \omega^{A\to W, B\to B} - \log \omega^{B\to W, A\to B} > 0.$$

The expression can be simplified to:

$$\tau + \gamma > \sigma\left[\lambda\left(\frac{c + \psi + \tau - p}{\sigma}\right) - \lambda\left(\frac{c + \psi - p - \gamma}{\sigma}\right) - \left(\lambda\left(\frac{p + \gamma - c - \psi}{\sigma}\right) - \lambda\left(\frac{p - c - \psi - \tau}{\sigma}\right)\right)\right].$$

The expression states that in order for there to be an aggregate log wage loss the sum of *observed* comparative advantages is greater than the differences in the revealed skill premium from admitting applicant B in the white-collar sector instead of A and the loss in the blue-collar sector from admitting applicant B in the white-collar sector and instead having applicant A in the blue-collar sector. We notice that:

$$\tau + \gamma > \sigma \left[\underbrace{\lambda\left(\frac{c+\psi+\tau-p}{\sigma}\right)}_{z_1} - \underbrace{\lambda\left(\frac{c+\psi-p-\gamma}{\sigma}\right)}_{z_2} - \left(\underbrace{\lambda\left(\frac{p+\gamma-c-\psi}{\sigma}\right)}_{-z_2} - \underbrace{\lambda\left(\frac{p-c-\psi-\tau}{\sigma}\right)}_{-z_1} \right) \right] \leftrightarrow$$

$$\frac{\tau+\gamma}{\sigma} > \left[\underbrace{(\lambda(z_1) + \lambda(-z_1))}_{G(z_1)} - \underbrace{(\lambda(z_2) + \lambda(-z_2))}_{G(z_2)} \right] \leftrightarrow$$

$$\frac{\tau+\gamma}{\sigma} > G(z_1) - G(z_2),$$

I.e. we need to find the maximum distance: $G(z_1) - G(z_2)$.

Consider $G'(z) = \lambda'(z) - \lambda'(-z)$, where $\lambda'(z) = \lambda(z)(\lambda(z) - z)$. We know that $\lambda'(z) \in (0, 1)$, $\forall z$. This means that

$$-1 < \lambda'(z) - \lambda'(-z) < 1 \leftrightarrow$$
$$|G'(z)| < 1, \ \forall z.$$

We use the mean value theorem which states that for some z_B which lies between z_1 and z_2 we have:

$$G'(z_B) = \frac{G(z_1) - G(z_2)}{z_1 - z_2} \leftrightarrow$$
$$G(z_1) - G(z_2) = G'(z_B)(z_1 - z_2) \leftrightarrow$$
$$|G(z_1) - G(z_2)| = |G'(z_B)| \cdot |(z_1 - z_2)|.$$

As $|G'(z_B)| < 1$ this implies:

$$|G(z_1) - G(z_2)| < |z_1 - z_2|.$$

Finally, we calculate: $|z_1 - z_2|$:

$$|z_1 - z_2| = \frac{|c + \psi + \tau - p - (c + \psi - p - \gamma)|}{\sigma} = \frac{|\tau + \gamma|}{\sigma} = \frac{\tau + \gamma}{\sigma}.$$

This means we get:

$$\begin{aligned} |G(z_1) - G(z_2)| <& \frac{\tau + \gamma}{\sigma} \leftrightarrow \\ -\left(\frac{\tau + \gamma}{\sigma}\right) < G(z_1) - G(z_2) <& \frac{\tau + \gamma}{\sigma} \end{aligned}$$

The second inequality was exactly what we set out to show. This means that whenever the admissions agent admits an applicant over another applicant which has higher observed white-collar signal, there is an aggregate log wage loss. As this is exactly what the admission behaviour in proposition 2.A implies, putting weight on the signal of the irrelevant skill, s_B decreases mean log wages in the entire economy. ■

A.10 Proof of Proposition 4

There exists a mechanism that gives the admissions agent an incentive to restrict $|N_W|$, even when all the skills admitted are positive. Increasing $|N_W|$ admits more applicants (scale effect) but also lowers the threshold and changes who applies (composition effect). This can reduce the average skill of those admitted enough to outweigh the extra admits.

Proof of Mechanism: Let $\mu(|N_W|)$ be the admissions cutoff corresponding to a mass $|N_W|$, such that:

$$|N_W| = 1 - F(\mu(|N_W|)).$$

The admissions agent's objective is:

$$\pi(|N_W|) = N_W \cdot \mathbb{E}[\log h_W \mid \log h_W \geq \mu(|N_W|)].$$

Differentiating gives:

$$\frac{d\pi}{dN_W} = \underbrace{\mu(|N_W|)}_{\text{scale effect}} + \underbrace{|N_W| \cdot \frac{d\mu(|N_W|)}{d|N_W|}}_{\text{composition effect}}.$$

While the scale effect is always positive, the derivative of the admitted average is negative (composition effect). When $|N_W|$ increases, the cutoff $\mu(|N_W|)$ falls and the applicant pool worsens due to self-selection, so $\frac{d\mu(|N_W|)}{d|N_W|} < 0$.

When the slope changes sign exactly once, so that $\pi(|N_W|)$ is strictly increasing and then decreasing, there exists a unique interior maximum $|N_W^{PI}| \in (0, 1)$. In the competitive equilibrium, all applicants who pass the self-selection condition are admitted, yielding an admission mass $|N_W^{CE}|$. The maximum $|N_W^{PI}|$ can lie above or below $|N_W^{CE}|$ depending on the primitives.

■

A.11 Proof of Proposition 5

Given an equilibrium with an endogenous capacity restriction, we want to show that as noise in signals, σ, is present, the admissions agent will choose an admission mass; $|N_W^{II}| \in [|N_W^{PI}|, |N_W^{CE}|]$, which increases weakly in noise in signals, $\sigma \in [0;\infty)$. This is an incentive to increase the mass (number) of admitted students, to make the self-selection bind more, when noise is present compared to a world with perfect information.

Setup:

Consider the general objective function of the admissions agent:

$$\pi(|N_W|,\ \mathbb{E}[h_W(i) \mid i \in N_W])$$

Where π is weakly increasing in $|N_W|$ and strictly increasing in $\mathbb{E}[(h_W(i))]$. As π increases weakly in $|N_W|$, increasing admissions will always mechanically increase the objective through $|N_W|$. Therefore, it is sufficient to focus on whether the increase in signal noise, σ, gives the admissions agent an incentive to increase $|N_W|$ through a higher $\mathbb{E}[(h_W(i))]$ among the admitted students.

Consider the expected white-collar skill of the admitted students:

$$\mathbb{E}\big[h_W(i) \mid i \in N_W^{II}\big] = \mathbb{E}\left[h_W(i) \;\middle|\; h_W(i) \geq h_B(i)\left(\frac{W_B\big(|1 - N_W^{II}|_{-}\big)}{W_W(|N_W^{II}|_{-})}\right),\ \mathbb{M}(s_W, s_B) \geq \mu\big(|N_W^{II}|\big)\right].$$

where $W_j,\ j \in (W,B)$ is decreasing in the number of individuals in sector j, and $s_j = \log h_j + \varepsilon_j(i), \quad \varepsilon_j \sim N(0,\sigma^2)$. We consider a situation where the restriction mechanism described in proposition 4 binds such that under perfect information $\sigma = 0$, $|N_W^{PI}| < |N_W^{CE}|$.

When $\sigma \to \infty$:

We show that $\lim_{\sigma\to\infty}$ leads to $|N_W^{II}| = |N_W^{CE}|$. As σ grows large, the noise terms $\varepsilon_W(i)$ and $\varepsilon_B(i)$ dominate the signals, and the admission constraint $\mathbb{M}(s_W, s_B) \geq \mu(|N_W|)$ no longer convey information about $h_W(i)$. The expected skill of admitted students therefore reduces to:

$$\mathbb{E}\left[h_W(i) \;\middle|\; h_W(i) \geq h_B(i)\left(\frac{W_B\big(|1 - N_W^{II}|_{-}\big)}{W_W(|N_W^{II}|_{-})}\right)\right].$$

Hence, since the noise term dominates the signals s_W and s_B, the admissions agent can no longer affect $\mathbb{E}[(h_W(i))|i \in N_W^{II}]$ by changing the structure of the admissions rule $\mathbb{M}(s_W, S_B)$ and the only way the admissions agent can change $\mathbb{E}[(h_W(i))|i \in N_W^{II}]$ is through changing

$|N_W^{II}|$.

This expression increases in $|N_W^{II}|$, since raising $|N_W^{II}|$ decreases the denominator and increases the numerator on the right side of the inequality. Intuitively, lowering the wage rate in the white-collar sector relative to the blue collar sector by admitting more students tightens the self-selection condition, as it is only satisfied for higher levels of h_W. The admissions agent cannot raise $|N_W^{II}|$ over $|N_W^{CE}|$ as setting $|N_W^{II}| > |N_W^{CE}|$ would reduce the number of applicants below $|N_W^{CE}|$. That is, the admissions agent cannot attract more applicants than would apply under the competitive equilibrium with no admissions restriction. If they tried to admit more students than would enter without restriction to admissions, they would simply not be able to fill all seats. Hence, as $\sigma \to \infty$ the admissions agent will choose $|N_W^{II}| = |N_W^{CE}|$ in the limit.

Next, we consider the admissions behaviour **when** $\sigma < \infty$: To clarify the trade-off between admitting many versus a few applicants, it is useful to decompose the expected white-collar skills of admitted students as follows:

$$\begin{aligned}
\mathbb{E}\big[h_W(i) \mid i \in N_W^{II}\big] &= \mathbb{E}\left[h_W(i) \,\middle|\, h_W(i) \geq h_B(i)\left(\frac{W_B\big(|1-N_W^{II}|_-\big)}{W_W(|N_W^{II}|_-)}\right),\ \mathbb{M}(s_W, s_B) \geq \mu\big(|N_W^{PI}|\big)\right] \\
&= \mathbb{E}[h_W \mid \text{Application, Admission}] \\
&= \left(\underbrace{\mathbb{E}[h_W \mid \text{Application, Admission}] - \mathbb{E}[h_W \mid \text{Application}]}_{\textit{Admissions premium}} + \underbrace{\mathbb{E}[h_W \mid \text{Application}]}_{\textit{Applicant quality}}\right).
\end{aligned}$$

That is, the expected white-collar skills of admitted students can be split into i) expected applicant quality and ii) expected admissions premium.

The expected quality of the applicant denotes the expected white-collar skills of all individuals who apply to the white-collar college. The admissions premium captures how much the admissions agent improves the expected skills of admitted students relative to the general pool of applicants. That is, by choosing an admission rule, the agent selects students with higher expected skills than if they were picked at random from the applicant pool.

Applicant quality:

$$\text{Applicant quality} = \mathbb{E}\left[h_W(i) \,\middle|\, h_W(i) \geq h_B(i)\left(\frac{W_B\big(|1-N_W^{II}|_-\big)}{W_W(|N_W^{II}|_-)}\right)\right].$$

This expression is increasing monotonically in $|N_W|$, as h_W and h_B are independent.

Increasing $|N_W|$ raises the threshold in the self-selection condition, which implies that only individuals with higher h_W choose to apply. The applicant quality term is independent of the noise in signals, σ.

Next, we consider the **Admissions Premium:**

$$\text{Admission premium} = \mathbb{E}\left[h_W(i) \,\middle|\, h_W(i) \geq h_B(i)\left(\frac{W_B\big(|1-N_W^{II}|_-\big)}{W_W(|N_W^{II}|_-)}\right),\ \mathbb{M}(s_W, s_B) \geq \mu\big(|N_W^{PI}|\big)\right] - \mathbb{E}\left[h_W(i) \,\middle|\, h_W(i) \geq h_B(i)\left(\frac{W_B\big(|1-N_W^{II}|_-\big)}{W_W(|N_W^{II}|_-)}\right)\right].$$

The admissions premium is by definition decreasing in $|N_W|$. It is the only term in the admissions agent's objective function that incentivises the admissions agent to reduce admissions. That is, the fewer applicants the admissions agent chooses to admit, the better the admitted individuals are relative to the average applicants. (This is both the case as a lower $|N_W|$ loosens the self-selection constraint and tightens the admission constraint through a higher μ.).

Additionally, we see that the admissions premium depends on the noise of the signals, σ. To see this, consider the admissions constraint $\mathbb{M}(s_W, s_B) \geq \mu(|N_W^{PI}|)$, which is what creates the admissions premium. As $s_j = \log h_j(i) + \varepsilon_j(i), \quad \varepsilon_j \sim N(0, \sigma^2),\ j \in (W, B)$ increasing noise in signals, σ, increases the relative importance of the noise term in the signal, it decreases the possibility for the admissions agent to select the applicants with the highest skills. The signals s_j consist of both a skill component $\log h_j$ and a noise component ε_j. The higher the noise in the signal, the more the admissions agent is selecting students based on noise rather than skills. For a given $|N_W|$, higher noise in the signals, σ impairs the admissions agent's ability to select the best students relative to the average applicants. Hence, increasing noise decreases the admissions premium.

When $\sigma = 0$:

Now consider a situation without noise in the signal. As previously shown, the admissions agent selects $|N_W| = |N_W^{PI}|$. If the admissions agent lowers N_W marginally, this will marginally increase the admissions premium and marginally decrease the applicant quality. As we are initially in optimum, these two effects will exactly cancel out. (If the objective function is also increasing in $|N_W|$ in itself, the negative effects from decreasing $|N_W|$ and from decreasing the applicant quality will cancel out the positive effect of the admissions premium.). The opposite is the case if the admissions agent chooses to increase $|N_W|$ marginally.

Now consider the same situation where we are initially in $|N_W| = |N_W^{PI}|$ but let $\sigma \uparrow$ be such that $\sigma = \tau$, where $\tau > 0$ is a constant. Let us consider now what happens if the admissions agent chooses to lower $|N_W|$ marginally: The applicant quality will decrease similarly as this is independent of the noise, but the admissions premium will increase by weakly less compared to the situation without noise. Hence, when noise increases, the admissions agent faces no incentive to reduce $|N_W|$ below $|N_W^{PI}|$. On the other hand, by increasing $|N_W|$ marginally, the applicant quality will increase similarly compared to the no-noise situation but the admissions premium will decrease by weakly less compared to the no noise situation. Hence, the admissions agent is weakly better off by increasing $|N_W|$. This means that the admissions agent has no incentive to restrict admission more than $|N_W^{PI}|$ as noise increases. This means that for all levels of σ: $|N_W^{II}| \in [|N_W^{PI}|; |N_W^{CE}|]$.

When $\sigma \in (0; \infty)$**:**

Now consider a situation where the noise in the signals is $\sigma = \upsilon$, where $\upsilon > 0$ is a positive constant. We know from above that for this level of noise the admissions agent will select some optimal $|N_W^{II}| \in [|N_W^{PI}|; |N_W^{CE}|]$ where changing $|N_W|$ marginally means that the effect of the admissions premium and the effect of applicant quality exactly cancels out. Again, consider what happens as $\sigma \uparrow$ such that $\sigma = \upsilon + \tau$. We can repeat the argument above: As the noise increases, the incentive to increase $|N_W|$ is unchanged, but the incentive to reduce $|N_W|$ is weakly lower. That is, for a given $|N_W^{II}| \in [|N_W^{PI}|; |N_W^{CE}|]$, a given increase in noise of signals will incentivise the admissions agent to weakly increase admissions.

This completes the proof that $|N_W^{II}|$ is weakly increasing in the noise of signals, σ and that $|N_W^{II}| \in [|N_W^{PI}|; |N_W^{CE}|]$. ■

A.12 Proof of Proposition 6

In this proposition, we prove that when noise in signals are present, selection of the admissions agent of $|N_W^{II}|$ relative to $|N_W^{PI}|$ implies that: i) The effect on mean log wages in the white-collar sector is ambiguous, ii) Mean log wages in the blue-collar sector increases, iii) When the total log wages in each sector is concavely increasing in $|N_j|$: the mean log wages in the entire economy increases.

We continue under the condition that the restriction mechanism binds when there is no noise in the signals, $\sigma = 0$ such that $|N_W^{PI}| < |N_W^{CE}|$. We know from proposition 5 that when noise is present, the admissions agent selects $|N_W^{II}| \in [|N_W^{PI}|; |N_W^{CE}|]$.

Mean log wages in the white-collar sector:

First, we consider total log wages in sector $j \in (W, B)$:

$$\bar{\omega}_j \cdot |N_j| = |N_j| \left(w_j(|N_{j-}|) + \mathbb{E}[\log h_j(i) \mid i \in N_j] \right).$$

We now examine generally how this changes as the number of individuals in each sector changes:

$$\frac{\partial \left(\bar{\omega}_j \cdot |N_j| \right)}{\partial |N_j|} \equiv \underbrace{w_j'(|N_j|) \cdot |N_j|}_{\text{Scale effect on others}} + \underbrace{\left(\underbrace{w_j(|N_j|) + \mathbb{E}[\log h_j(i) \mid i \in N_j]}_{\text{One extra individual}} + \underbrace{|N_j| \cdot \frac{\partial \, \mathbb{E}[\log h_j(i) \mid i \in N_j]}{\partial |N_j|}}_{\text{Mean log skill change}} \right)}_{\text{Skill effect}}. \tag{A.1}$$

There are two effects to consider: the scale effect on others and the skill effect. The scale effect denotes the change in the wage rate as more individuals enter the sector. The skill effect consists of both the total log wages changing as more individuals enter the sector and a mean skill effect due to compositional changes.

Now we instead consider mean log wages:

$$\frac{\partial \bar{\omega}_j}{\partial |N_j|} \equiv \underbrace{w_j'(|N_j|)}_{\text{Scale effect on others}} + \underbrace{\frac{\partial \, \mathbb{E}[\log h_j(i) \mid i \in N_j]}{\partial |N_j|}}_{\text{Mean log skill change}}. \tag{A.2}$$

Now there is no effect from one extra individual entering, but only the scale effect and the mean log skill change.

Going from $|N_W^{PI}|$ to $|N_W^{II}|$ (weakly) decreases mean log wages in the white-collar sector through the scale effect as $|N_W^{II}| \geq |N_W^{PI}|$ and as $w_W'(|N_W|) < 0$ this draws in the direction of decreased mean log wages in the white-collar sector.

However, as illustrated in the proof of proposition 5, an increase in $|N_W|$ may lead to a higher expected (log) skill of the admitted students. Consider, for instance, the limiting case where $\sigma \to \infty$. In this setting, the expected log white-collar skills of the admitted students is simply the expected log white-collar skills of the applicants. This will increase as $|N_W|$ increases (as illustrated in the proof of proposition 5). This leads to a positive mean skill effect from increasing $|N_W|$ from $|N_W^{PI}|$ to $|N_W^{II}|$. The aggregate effect on log white-collar wages depends on the distribution of h_W and the specification of the wage rate function. It is ambiguous and can be either positive or negative.

Mean log Wages in the Blue Collar Sector: Next we consider mean log wages in the blue-collar sector. Again, the effect on mean log wages from changing the number of individuals in each sector can be decomposed as in equation A.2. As $|N_W^{II}| > |N_W^{PI}|$ meaning that $|N_B^{II}| < |N_B^{PI}|$. This means that the scale effect in the blue-collar sector is positive. Hence, it is sufficient to show that the mean skill effect going from $|N_W^{PI}|$ to $|N_W^{II}|$ is positive.

Intuitively, the log mean skill level of individuals in the blue-collar sector as long as $|N_W^{II}| < |N_W^{CE}|$ consists of a weighted average of individuals who have self-selected into the blue-collar sector and individuals who are rejected from the white-collar sector. There are individuals who always self-select into the blue-collar sector under both $|N_W^{PI}|$ and $|N_W^{II}|$. Their behaviour is not of interest, and we therefore focus on the two subsets where behaviour actually changes.

Going from $|N_W^{PI}|$ to $|N_W^{II}|$ changes the composition of individuals and thereby the mean log blue-collar skills in two ways:

(A): Fewer individuals are rejected for the white-collar sector. Those individuals therefore leave the blue-collar sector.

(B): More individuals self-select into the blue-collar sector under $|N_W^{II}|$ compared to $|N_W^{PI}|$ as the wage rate in the blue-collar sector has increased.

Note that the mass of individuals in (A) is larger than the mass of individuals in (B), as the total number of individuals in the blue-collar sector decreases.

To prove that the mean log blue-collar skill level in the blue-collar sector increases, when noise is present as we go from $|N_W^{PI}|$ to $|N_W^{II}|$ it is sufficient to prove the following two conditions:

1. The expected log blue-collar skills of individuals in (A) are lower than that of individuals in (B).

2. Removing the individuals in (A) from the blue-collar sector increases the expected log blue-collar skills in the blue-collar sector all other things equal.

The first condition ensures that those entering the blue-collar sector going from $|N_W^{PI}|$ to $|N_W^{II}|$ have higher expected log blue-collar skills than those leaving. The second condition ensures that those who leaves the blue-collar sector going from $|N_W^{PI}|$ to $|N_W^{II}|$ draws the mean log blue-collar skills in the blue-collar sector up.

We start out by showing that the first condition is always satisfied. First, we define $c(N_{W+}) \equiv w_B(N_{W+}) - w_W(N_{W-})$.

The expected log blue-collar skills of individuals in (B) is given by: $\mathbb{E}[\log h_B(i)| \log h_W(i) - c(N_{W+}^{II}) < \log h_B(i) < \log h_W(i) - c(N_{W+}^{PI}), \log h_W(i) + \varepsilon_W(i) > \mu(|N_W^{PI}|)]$.

The expected log blue-collar skills of individuals in (A) is given by: $\mathbb{E}[\log h_B(i)| \log h_W(i) - c(N_{W+}^{II}) > \log h_B(i), \mu(N_W^{II}) < \log h_W(i) + \varepsilon_W(i) < \mu(|N_W^{PI}|)]$.

We can see that

$$
\begin{aligned}
&\mathbb{E}[\log h_B(i)| \log h_W(i) - c(N_{W+}^{II}) > \log h_B(i), \mu(N_W^{II}) < \log h_W(i) + \varepsilon_W(i) < \mu(|N_W^{PI}|)] \\
<&\mathbb{E}[\log h_B(i)| \log h_W(i) - c(N_{W+}^{II}) < \log h_B(i) < \log h_W(i) - c(N_{W+}^{PI}), \log h_W(i) + \varepsilon_W(i) > \mu(|N_W^{PI}|)].
\end{aligned}
$$

By looking at the first condition we condition that $\log h_B$ on the left side on the inequality is smaller than $\log h_W(i) - c(N_{W+}^{II})$, which we on the right side of the inequality condition $\log h_B$ to be greater than. In the second condition on the left side we condition on $\log h_W(i) + \varepsilon_W(i)$ to be smaller than $\mu(|N_W^{PI}|)$ which we again condition $\log h_W(i) + \varepsilon_W(i)$ to be greater than on the right side of the inequality. As all the random variables are independent this inequality is satisfied.

Next we turn to condition 2. We prove this under limiting cases and argue why it must hold in other cases as well: First we prove condition 2 as noise in signals, $\sigma \to 0$. Next we prove it as noise in signals, $\sigma \to \infty$:

When $\sigma \to 0$: As noise tends towards 0, admissions, is approximately deterministically based on $\log h_W$. We can prove the condition by using the following example:

Divide the $\log h_W$ axis into intervals of length $\mu(|N_W^{PI}|) - \mu(|N_W^{II}|)$. Now we consider the expected $\log h_B$ for all individuals who end up in the blue-collar sector for each interval on the $\log h_W$ axis starting from below: For all intervals below $\mu(|N_W^{II}|)$, this is just equal to $\mathbb{E}[\log h_B]$. That is, as noise is infinitesimally small, almost everyone with $\log h_W$ below $\mu(|N_W^{II}|)$ will end up in the blue-collar sector. As $\log h_W$ and $\log h_B$ are independent, conditioning on a given interval of $\log h_W$ does not change the expected value of $\log h_B$.

Now consider the actual interval on the $\log h_W$ axis where $\log h_W(i) \in [\mu(|N_W^{II}|), \mu(|N_W^{PI}|]$: When the admissions agent selects $|N_W^{PI}|$ all individuals in this interval are rejected for college, and end up in the blue-collar sector. In this case, the expected log blue-collar skills of individuals who end up in the blue-collar sector is also equal to $\mathbb{E}[\log h_B]$.

However, when the admissions agent selects $|N_W^{II}|$ expected $\log h_B$ for all individuals who end up in the blue-collar sector in this interval is approximately equal to: $\mathbb{E}[\log h_B(i)| \log h_B(i) + c(N_{W+}) > \log h_W(i), \mu(N_W^{II}) < \log h_W(i) < \mu(|N_W^{PI}|)] > \mathbb{E}[\log h_B]$. The reason is that under $|N_W^{II}|$ individuals for whom this condition is not satisfied can self-select into the white-collar sector but not under $|N_W^{PI}|$. For the range on the $\log h_W$ axis above this interval, he expected $\log h_B$ for all individuals who end up in the blue-collar sector is approximately equal to: $\mathbb{E}[\log h_B(i)| \log h_B(i) + w_B(N_{B-}) > \log h_W(i) + w(N_{W-}), \log h_W(i) > \mu(|N_W^{PI}|)] > \mathbb{E}[\log h_B]$. That is, removing individuals who can self-select into the white-collar sector under N_W^{II} but not under N_W^{PI} all else equal increases the expected log blue-collar skills in the blue-collar sector relative to $|N_W^{PI}|$ all other things equal. That is, condition 1 is satisfied when noise in signals $\sigma \to 0$.

Next we consider the other extreme case, where the noise in signal, $\sigma \to \infty$: Again, we consider condition 2. Under this situation, the admission rule is uninformative about the skills of the individuals. The mean log blue-collar skill in the blue-collar sector is a weighted average of individuals who have self-selected into the blue collar sector and individuals rejected for the white-collar sector:

$$\begin{aligned}\mathbb{E}[\log h_B(i)|i \in N_B^{PI}] =& p \cdot \mathbb{E}[\log h_B(i)| \log h_B(i) + c(N_{W+}^{PI}) > \log h_W(i)] \\ &+ (1-p) \cdot \mathbb{E}[\log h_B(i)| \log h_B(i) + c(N_{W+}^{PI}) \leq \log h_W(i)],\end{aligned}$$

where the expectation in the former term (self-selectors) is higher than in the second term (rejectees). Now consider what happens as we move from $|N_W^{PI}|$ to $|N_W^{II}|$: The expected blue-collar skill of the individuals who now no longer are rejected from the white-collar sector and the blue-collar sector looses (A) is: $\mathbb{E}[\log h_B(i)|\log h_B(i)+c(N^{II}_{W+})\leq \log h_W]$ Where we have that:

$$\begin{aligned}&\mathbb{E}[\log h_B(i)|\log h_B(i)+c(N^{II}_{W+})\leq \log h_W]\\&\leq \mathbb{E}[\log h_B(i)|\log h_B(i)+c(N^{PI}_{W+})\leq \log h_W]\\&< \mathbb{E}[\log h_B(i)|\log h_B(i)+c(N^{PI}_{W+})> \log h_W].\end{aligned}$$

This is the case as $|N^{II}_W|\geq|N^{PI}_W|$. That is, the individuals in (A) have even lower expected blue-collar skills compared to those who were rejected under $|N^{PI}_W|$. This is the case as moving from $|N^{PI}_W|$ to $|N^{II}_W|$ moves the self-selection condition. Hence, Condition 2 is satisfied.

We have now shown that condition 2 is satisfied in the limiting cases where $\sigma\to 0$ and $\sigma\to\infty$. There are two things that can cause condition 2 to be violated. Going from $|N^{PI}_W|$ to $|N^{II}_W|$ we remove individuals from the blue-collar sector who are rejected for the white-collar sector under $|N^{PI}_W|$ but not under $|N^{II}_W|$. Removing these individuals can potentially increase the mean log blue-collar skills in the blue-collar sector (and hence violate condition 2) if either the expected blue-collar skills of rejectees in general is sufficiently high or if the ones who are removed going from $|N^{PI}_W|$ to $|N^{II}_W|$ are very distinct from the remaining pool of rejectees (i.e. have higher log-blue collar skills). The two limiting cases specifically address these situations: The expected log blue-collar skills of rejected applicants is maximised when $\sigma\to\infty$. In general, the expected log blue-collar skills of individuals who are rejected for white-collar college are: $\mathbb{E}[\log h_B(i)|\log h_B(i)+c(N_{W+})>\log h_W(i), \log h_W(i)+\varepsilon_W(i)<\mu(|N_W|)]$. As $\sigma\to\infty$ the latter condition becomes uninformative, and the expression reaches the global maximum in this limit at $\mathbb{E}[\log h_B(i)|\log h_B(i)+c(N_{W+})>\log h_W(i)]$. That is, even in the case where the expected log blue-collar skills of rejected applicants for the white-collar sector is maximised, condition 2 still holds. Turning to the other point, the ones who are removed going from $|N^{PI}_W|$ to $|N^{II}_W|$ are on average most distinct from the remaining pool of rejectees as $\sigma\to 0$. In this limiting case, the admissions agent can perfectly screen applicants based on actual skills. As noise increases, the admissions agent selects more and more on noise instead of skills, and the rejected applicants become on average more similar. Above, I have shown that condition 2 also holds in this limiting case. That is, by having shown that condition 2 holds in the two cases where we would most fear it to be violated, it must hold for other values of noise in signals, σ, as well.

Mean Log Wages in Entire Economy: In order to prove that mean log wages is increasing we make an additional sufficient assumption that total wages in each sector is concavely increasing in the number of individuals in each sector.[21]

First we show that mean wages are larger under competitive equilibrium than under perfect information equilibrium and that they are concave. Next, we argue that this is also the case when we consider total log wages instead.

Consider mean wages in the entire economy:

$$\bar{\omega} = |N_W|\, W_W(|N_{W-}|)\; \mathbb{E}[h_W(i) \mid i \in N_W] + |N_B|\, W_B(|N_{B-}|)\; \mathbb{E}[h_B(i) \mid i \in N_B]\,.$$

We know by definition that there is a competitive equilibrium in N_W^{CE}. Further, as total wages in each sector is concave, mean wages in the entire economy is also concave, as the sums of two concave functions are also concave. Note that, as $|N_W| + |N_B| = 1$, the sum of total wages in each sector is equal to the mean wages in the entire economy. This means that mean wages has one unique global maximum in $|N_W^{CE}|$. As the mean wages are concave and $|N_W^{PI}| \leq |N_W^{II}| \leq |N_W^{CE}|$, this means that increasing $|N_W^{II}|$ increases the mean wages throughout the entire economy.

To see that this also applies to mean log wages consider how the admissions agent specifically deviates from the competitive equilibrium: He does so by lowering wages in the blue-collar sector and especially amongst those who are rejected for the white-collar sector, whereas wages for the remaining white-collar workers are increased. However, this increase is not enough to offset the fall in wages for the other groups, so that the mean wages fall relative to the competitive equilibrium. And as the mean wage function is concave, it falls more the further away from the competitive equilibrium we are. Taking logs requires an even larger increase to offset a given loss (due to Jensen's inequality) as the log transformation is a strictly concave transformation. As the mean wages in levels fall as we move away from the competitive equilibrium, this must also be the case for mean log wages. That is, if the increase in white-collar wages is not enough to offset the loss in blue-collar wages and the wage loss for the rejectees, the increase in log white-collar wages is definitely not enough to offset the loss in log wages, as losses are punished more than gains are rewarded when wages are log transformed. Once again, as $|N_W^{II}|$ is closer to $|N_W^{CE}|$ than $|N_W^{PI}|$, the deviation from the competitive equilibrium is smaller than under the perfect information equilibrium. That is, mean log wages in the entire economy increases as $|N_W^{II}|$ moves closer to $|N_W^{CE}|$. ■

[21] This is e.g. satisfied in a standard Cobb-Douglass function or in a CES function with substitution elasticity of at least 1.

A.13 Elaboration of Specification of Wage Rate under Assumption 3 and 4

To further elaborate on the specification of the wage rate under Assumptions 3 and 4, we can model the production function and derive the wage rates explicitly within the Roy- and human capital framework.

In early Roy models, no specific production function was modelled (see, e.g. Roy (1951), Borjas (1987), and Heckman & Honore (1990)). However, in recent years, it has become conventional to model production as a standard function $Y_j(A_j, K_j, L_j)$ with Cobb-Douglas properties, where sector-specific output Y_j is a function of sector-specific technology A_j, sector-specific capital K_j, and sector-specific labour inputs L_j, measured in efficiency units such that $L_j = \int_{i \in N_j} h_j(i)\,\mathrm{d}i$, and where output displays diminishing returns to each input individually, e.g. Adão (2015), Cicala et al. (2018), Almgren et al. (2023). Individual wages are then given by $\omega_j(h_j(i)) = \frac{\partial Y_j}{\partial L_j} \cdot h_j(i)$. While we fully intend to follow this convention, this specific formulation of the production function implicitly assumes that the mean skill level of peers negatively affects wages through higher L_j, and thereby lower capital–labour ratios. This would be a violation of assumption 4.

This may be a reasonable property in some cases, as higher peer skill levels may cause congestion, leading to a downward pressure on wages when the mean skill level of one's peers increases. However, this contradicts the general literature on peer effects in many settings where the Roy model is typically applied, which has consistently shown that the mean skill level of peers positively affects own wages. Kleven et al. (2020) discuss the possibility that immigrants exhibit positive productivity spillover effects, and Mas & Moretti (2009) and Cornelissen et al. (2017) find that the productivity of workers in the same occupation positively affects own productivity.

Especially in our model, where sectoral choice and educational choice are effectively bundled into a single decision, the idea that higher peer skill levels reduce individual wages seems unrealistic. It would imply that individuals become *less* willing to enter a sector when more high-skilled peers also choose it. This stands in stark contrast to the broader economics of education and labour literature, which consistently finds that individuals have strong preferences for environments with highly skilled peers (see, e.g. Epple & Romano (1998), Epple et al. (2006), Rothschild & White (1995), MacLeod & Urquiola (2015)) and that the mean skill level of peers positively affects both academic and labour market outcomes (see, e.g. Lucas Jr (1988), Mas & Moretti (2009), Cornelissen et al. (2017)). Although the underlying mechanisms vary between settings, a robust empirical fact is that higher levels of peer skills, either in education or in the workplace, raise both individual performance and subsequent wages (see, e.g. Carrell et al. (2009), Humlum & Thorsager (2021), Cattan et al. (2022)).

Instead of taking a firm stand on whether the mean skill level of peers exhibits positive or negative externalities on own wages, we take a different approach. We acknowledge that the mean skill level of individuals choosing the same sector as individual i may exhibit both positive productive spillover effects and negative congestion effects on wages. Which effect dominates is ultimately an empirical question. Following Lucas Jr (1988), we model the production function in a way that is broad enough to capture both potential effects:

$$Y_j = A_j K_j^{\alpha} L_j^{1-\alpha} \bar{h}_j^{\gamma}, \tag{A.3}$$

where $L_j = \int_{i \in N_j} h_j(i)\, \mathrm{d}i$ and $\bar{h}_j$ are the average skill levels deployed in sector j. To see why this formulation is broad enough to capture both negative and positive spillover effects on own productivity, consider the following rewriting of equation (A.3):

Individual wages are then given by:

$$\omega_j(h_j(i)) = \frac{\partial Y_j}{\partial L_j} \cdot h_j(i) = (1-\alpha) A_j \left(\frac{K_j}{|N_j|} \right)^{\alpha} \bar{h}_j^{\gamma - \alpha} \cdot h_j(i). \tag{A.4}$$

When $\gamma < \alpha$, the negative peer effects stemming from lower available capital per efficiency unit of labour (congestion) dominate the positive productivity spillover effects from working (or attending college) with higher-skilled peers. In the extreme case of $\gamma = 0$, we are in a standard Roy model. When $\gamma > \alpha$, the positive spillover effects dominate, consistent with the traditional economics of education literature.[22]

Although we do not wish to take a firm position on which effect dominates, in the main analysis we specify that $\gamma = \alpha$, such that the mean skill level of peers in the same sector does not affect the own wages negatively or positively. By imposing this structure, we also show that our results are not driven by explicitly modelled peer effects. Under this structure, the wage in sector j collapses to:

$$\omega_j(h_j(i)) = (1-\alpha) A_j k_j^{\alpha} \cdot h_j(i), \tag{A.5}$$

where $k_j = \frac{K_j}{|N_j|}$ denotes capital per worker in sector j. Hence $W_j \equiv (1-\alpha) A_j k_j^{\alpha}$. This formulation of individual wages is equivalent to the basic model in Acemoglu (1996), where

[22] In Lucas Jr (1988), it is suggested that $\gamma = 0.412$, although this analysis is done for entire economies rather than for individual sectors.

it is shown that only the capital available to each worker, and not the skill distribution of other workers, affects the returns to own skills. This wage function satisfies assumptions 3 and 4. Note that we do not require this specific formulation of the wage function in order for the following results to hold. They do so as long as assumptions 3 and 4 are satisfied.

A.14 Robustness to Model Extensions and Alternative Specifications

We now turn to test whether the key result from Proposition 4, that admissions agents restrict capacity to raise white-collar wages, still holds when capital is mobile across sectors rather than fixed. Consider the model described in section 5, but Instead of assuming fixed capital in each sector, we now assume that capital is not fixed but can instead move freely in the world market and, hence, also across sectors:

$$\frac{\partial Y_j}{\partial K_j} = \bar{r}, \ \ j \in (W, B),$$

where $\bar{r}$ is the real rental rate of capital in the global capital market. Assume further that we are in a small open economy with a free flow of capital.

In order for proposition 4 to hold (restriction of $|N_W|$), it must be such that the admissions agent can increase the wage level in the white-collar sector by restricting the capacity of college seats. In other words, if the restriction of $|N_W|$ is followed by a decrease in K_W, then capital-labour ratios will be fixed, and incentives, and hence the application behaviour of the individuals, are left unchanged. To investigate this, consider the competitive equilibrium benchmark where:

$$\frac{\partial Y_j}{\partial K_j} = \alpha A_j K_j^{\alpha-1}(|N_j|\bar{h}_j)^{1-\alpha}\bar{h}_j^{\gamma} = \bar{r}, \quad j \in \{W, B\}.$$

From this, the following holds true for both the blue-collar and white-collar sectors in equilibrium when there is a free flow of capital:

$$\left(\frac{|N_W|\bar{h}_W}{|N_B|\bar{h}_B}\right)^{1-\alpha}\left(\frac{\bar{h}_W}{\bar{h}_B}\right)^{\gamma} = \left(\frac{K_W}{K_B}\right)^{1-\alpha}$$

The key question is whether restricting $|N_W|$ leads to a decrease in K_W that neutralises the intended wage effect. Let us consider what happens when the admissions agent begins to act marginally in accordance with Proposition 4:

$|N_B|\bar{h}_B$ increases: This can occur when the admissions agent seeks to maximise aggregate skills. He does so by reducing $|N_W|$, while ensuring that $\bar{h}_B$ increases proportionally more in his objective function.

$\left(\frac{\bar{h}_W}{\bar{h}_B}\right)^{\gamma}$ increases: The admissions agent reallocates individuals who had self-selected into the white-collar sector towards the blue-collar sector. This raises the average deployed skill level in the white-collar sector relative to the blue-collar sector.

It is now sufficient to show that $\frac{|N_W|\bar{h}_W}{|N_B|\bar{h}_B}$ increases when the admissions agent marginally moves away from the competitive equilibrium benchmark.

We know that when the admissions agent restricts admissions, he increases $|N_W|\bar{h}_W$ by trading off a positive contribution stemming from the individuals he induces to apply for the white-collar sector with the negative contribution stemming from the total skills of the individuals he rejects. By the envelope theorem, at the margin the skill gain in the white-collar sector equals the skill loss in the blue-collar sector, since those individuals are indifferent between sectors. This is the case as those individuals are on the margin between applying to the white-collar or blue-collar sector. However, the gain that the blue-collar sector receives from the individuals who are rejected from the white-collar sector is less than the loss incurred from their rejection from a white-collar job. The reason is that individuals who are rejected are not on the margin between applying for the blue-collar sector.

As the gain in total white-collar skills is equal to the loss in total blue-collar skills, and the loss in white-collar skills is greater than the gain in total blue-collar skills, and the gain in total white-collar skills is higher than the loss in white-collar skills (by optimisation), it follows that $\frac{|N_W|\bar{h}_W}{|N_B|\bar{h}_B}$ increases.

As both $\left(\frac{\bar{h}_W}{\bar{h}_B}\right)^{\gamma}$ and $\left(\frac{|N_W|\bar{h}_W}{|N_B|\bar{h}_B}\right)^{1-\alpha}$ increase when the admissions agent starts acting optimally, then $\left(\frac{K_W}{K_B}\right)^{1-\alpha}$ must also increase to ensure that the marginal product of capital is equal in both sectors. This implies a relative increase in $\left(\frac{K_W}{K_B}\right)$, which indicates that when the admissions agent restricts admission to the college, the capital movements reinforce rather than offset the incentives created by the admissions agent, further aggravating wage differentials between the two sectors in the two sectors even further, inducing even more individuals to apply to the white-collar sector relative to the fixed-capital benchmark.[23]

[23]Note that assumption 4 corresponds to a situation where $\gamma = \alpha$, but this result presented here in appendix A.14 also holds for some values of γ and α even when $\gamma \neq \alpha$.

B Empirical Content

B.1 Regression table

The regression coefficients used in the empirical validation of the talent-hoarding effect are reported in table 2 below. In the first four columns, the regressions are based on the admitted students in the B.Sc. in English, and in columns 5 and 6, the regressions are based on the admitted students in the B.Sc. in Mathematics. The number of observations across regressions within each study programme varies, as it was not possible to obtain the residualised grades and/or the Danish grade for all the admitted students. The reason is that in Danish high school, only some courses are drawn out for examination.

Table 2: Regression Coefficients

	(1) Completes English	(2) Completes English	(3) Completes English	(4) Completes English	(5) Completes Math	(6) Completes Math
English Grade (Std.)	0.0443***	0.0641***	0.0415***	0.0591***	0.0318*	
	(0.0114)	(0.0123)	(0.0123)	(0.0138)	(0.0126)	
Math Grade (Std.)	0.0521***		0.0533***		0.144***	0.170***
	(0.0114)		(0.0121)		(0.0116)	(0.0130)
Math Grade (Residualised)		0.0424**		0.0420**		
		(0.0139)		(0.0147)		
Danish Grade (Std.)			0.00143	0.0103		
			(0.0124)	(0.0141)		
English Grade (Residualised)						0.0184
						(0.0154)
Constant	0.569***	0.652***	0.570***	0.656***	0.415***	0.472***
	(0.0109)	(0.0137)	(0.0112)	(0.0143)	(0.0119)	(0.0136)
N	2034	1538	1930	1438	1547	1207
R^2	0.026	0.024	0.025	0.024	0.101	0.111

Robust standard errors in parentheses.
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

Notes: The table shows regression coefficients from regressions on completion of studies on high school grades. The english grade, math grade, and danish grade are simply the standardised written high school exam grade. The residualised math (english) grade are the residuals from a regression of the math (english) grade on the first principal component of all english (math) grades obtained in high school for all Danish high school students. Source: Statistics Denmark and own calculations.

B.2 Addressing Alternative Explanations

B.2.1 Concern 1

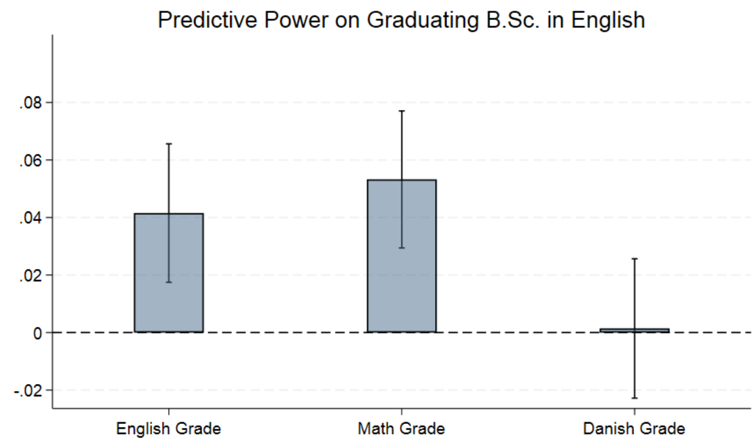
One might be worried that some of the high predictive power of the math grade on success in the English programme is caused by the fact that math skills are intrinsically important for succeeding in the English programme – for instance, through a more structured approach to problem solving or a stronger capacity for abstract reasoning. To mitigate this concern, we control for performance in other high school courses that the English programme *itself* reports as important for success in the degree.

Although admission to the English programme is in principle open for all applicants who meet the general entry requirements, Danish universities retain discretion over a small share of seats (approximately 10 percent), which are allocated through so-called *Quota 2* admissions. In Quota 2, applicants are assessed on criteria beyond their high school GPA, such as performance in specific courses, motivation letters, or entrance exams. In practice, this provides the programme with some limited flexibility, while the vast majority of applicants are automatically admitted under *Quota 1* (purely GPA-based admission).

In Quota 2, admissions officers may consider high school courses and corresponding exam grades that they believe are most predictive of success in the English programme. Since institutions are financially incentivised based on student completion rates, they have a strong incentive to admit the most capable students. The idea behind our empirical strategy is that, once we include these degree-relevant grades in the estimation equation, we effectively control for the skills that are intrinsically important for completing the B.Sc. in English. Any remaining predictive power of the math grade should therefore reflect the self-selection mechanism described in our model, rather than math skills being directly relevant. Based on information from the English programme at the University of Copenhagen, the relevant grades include those obtained in english and danish courses (see Københavns Universitet (2025)). We estimate equation 7.1 and include the grades obtained in danish. The results are reported in figure 8.

The results show that including the danish grade in the equation does not change the results. If the high predictive power of the math grade was caused by the math grade picking up some skills which are intrinsically important for success on the English programme, then we would expect that the inclusion of the danish grade would reduce the coefficient to the math grade. This, however, is not the case.

Figure 8: University Study: English (Controlling for Danish)



NOTES: The figure shows coefficients from a regression of completion of the B.Sc. in English on standardised high school grades in english and math and danish. The regression is based on 1930 observations. R^2 is 0.025. Source: Statistics Denmark and own calculations.

B.2.2 Concern 2

In addition, one might be concerned that the underlying distribution of math and english skills is correlated. As exam grades are noisy signals of true skills, the math grade will have predictive power on english performance (and vice versa) mechanically through the correlation of math skills and english skills. To illustrate this concern, think of the underlying underlying math- and english skills, which follow some joint distribution: $h_E, h_M \sim P$.

We can write the underlying math skills as a combination of a part which is correlated with the english skills and a part which is uncorrelated with the english skills:

$$h_M = \frac{\rho}{s} h_E + \tilde{h}_M \tag{B.1}$$

Where ρ is the correlation coefficient between the english- and math skills, and s is some scaling coefficient to reflect that English and math skills might follow different scales (we normalise them both to have mean zero, however, and standard deviation 1, so in this case, the scale is the same so $s = 1$.).

Similarly, we can write the math signal (math grade) for individual i as:

$$Grade_{M,i} = h_{M,i} + \varepsilon_{M,i} = \frac{\rho}{s} h_E + \tilde{h}_M + \varepsilon_{M,i}. \tag{B.2}$$

Using this, we can rewrite the estimation equation 7.1 as follows:

$$Y_{E,i} = \beta_0^E + \beta_1^E Grade_{E,i} + \beta_2^M \left(\underbrace{\frac{\rho}{s} h_E}_{Correlated\ part} + \underbrace{\tilde{h}_M + \varepsilon_{M,i}}_{Uncorrelated\ Part} \right) + u_{E,i} \tag{B.3}$$

The concern is that the entire reason $\hat{\beta}_2^M$ is positive is the correlated part. Our model, however, claims that the uncorrelated part in combination with the self-selection is important in itself.

If we knew $\frac{\rho}{s}$ and $h_{E,i}$ for each individual, we could separate them out to have:

$$Grade_{M,i}^{Residual} = Grade_{M,i} - \frac{\rho}{s} h_E = \tilde{h}_M + \varepsilon_{M,i}, \tag{B.4}$$

we could then estimate the following regression equation:

$$Y_{E,i} = \beta_0^E + \beta_1^E Grade_{E,i} + \beta_2^M Grade_{M,i}^{Residual} + u_{E,i}, \tag{B.5}$$

we could then test whether $\beta_2^M > 0$ even after removing the part of the math grade that is correlated with the English skill.

As we do not know $\frac{\rho}{s}$ or $h_{E,i}$ this is not directly feasible. Instead, the idea is to utilise the full data infrastructure of the entire population of high school students and all relevant grades to *estimate* $\frac{\rho}{s}$ and $h_{E,i}$. We carry out this analysis in four steps:

1. We use the entire population of Danish high school students and other relevant grades obtained in high school to estimate $h_{E,i}$

2. We estimate the correlation coefficient between maths skills and english skills , ρ for the entire population of high school students. As we normalise both the grades and the estimates of the underlying skill distribution, we set $s = 1$.

3. We estimate $\widehat{Grade}_{M,i}^{Residual} = Grade_{M,i} - \hat{\rho}\hat{h}_{E,i}$ for the relevant sample, where $\hat{h}_{E,i}$ and $\hat{\rho}$ are the estimates obtained in step 1 and 2, respectively.

4. We estimate the outcome regression with the residualised math grade: $Y_{E,i} = \beta_0^E + \beta_1^E Grade_{E,i} + \beta_2^M \widehat{Grade}_{M,i}^{Residual} + u_{E,i}$

The exact procedure for each of the steps is outlined below:

Step 1: In the first step, we obtain an estimate of the underlying english skill, $h_{E,i}$. In order to do this, we conduct a principal component analysis using other relevant english grades obtained in high school for *all* Danish high school students graduating in the period

1990-2017 and for whom we observe both written exam grade and written and oral year grades. Hence, we include not only the exam grade obtained in english but also the oral and written english year grades obtained by each student. Including these grades has several benefits in terms of obtaining a more accurate measure of the true underlying english skill. First, we include two more data points for each student, which reduces the amount of random variation in the measurement of english grades. Second, the year grades are given by the teacher, and although previous studies have shown that these might be biased, a large survey conducted by EVA amongst Danish high school teachers reveals that they strive to make the year grades match the students' skills in that given subject (EVA (2016)). We conduct a principal component analysis of these three grades. We use the first principal component as the estimate for underlying english skill. This component has an eigenvalue of 2.36 and is the only component with an eigenvalue of more than one. The first component explains 79 pct. of the total variation in the data. The written year grade, the oral year grade, and the exam grade load 0.60, 0.58, and 0.55 on the first component, respectively. The principal component analysis is based on 341,011 Danish high school students. Although this is not a perfect measure of underlying English skills , it is as good a measure as it is possible to obtain, given the available data. The results from the principal component analyses can be seen in table 3 and 4.[24]

Step 2: In the second step, we estimate the correlation coefficient between the math skills and english skills by running the following regression using the entire body of high school students:

$$Grade_{M,i} = \gamma_0 + \rho \hat{h}_{E,i} + \varepsilon_{M,i}. \tag{B.6}$$

The idea is to get an estimate for how much of the math exam grade, which is correlated with the underlying english skills for the entire population. This regression is based on 200,126 high school students. We run the regression and get $\hat{\rho} = 0.341$. The results are reported in table 7.

Step 3: In the third step, we only consider the sample of applicants to the B.Sc. in English. For each of the individuals in the sample we calculate the residualised math grade, according to the following formula: $\widehat{Grade}_{M,i}^{Residual} = Grade_{M,i} - \hat{\rho}\hat{h}_{E,i} = Grade_{M,i} - 0.341 \cdot \hat{h}_{E,i}$. The idea is to get a measure of math skills for each of the individuals in the sample, which is cleansed for the underlying correlation with English grades.

[24]For a detailed explanation of the principal component analysis method see Abdi & Williams (2010).

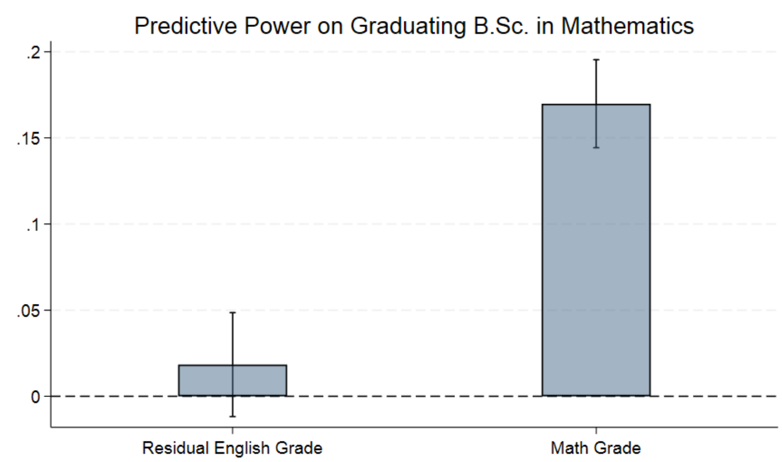
Step 4: In the fourth and final step, we estimate the following regression:

$$Y_{E,i} = \beta_0^E + \beta_1^E Grade_{E,i} + \beta_2^M \widehat{Grade}_{M,i}^{Residual} + u_{E,i}. \tag{B.7}$$

To the extent that $\hat{\rho}$ is a consistent measure of ρ and h_E, h_M has the same scale (which we have assumed they have), and $\hat{h}_{E,i}$ is both a consistent *and* sufficiently precise measure of the underlying english skill, for individual i, then $\hat{\beta}_2^M$ is a consistent estimate for the predictive power of the uncorrelated part of the math grade, and hence a test of our proposed mechanism. Although these are strong assumptions, this methodology is arguably an improvement relative to the regression in equation 7.1.

We conduct the same four steps for the applicants for B.Sc. in Mathematics. The results from the regression in B.7 and the corresponding regression for the B.Sc. in Mathematics are reported in figure 9 and 10.

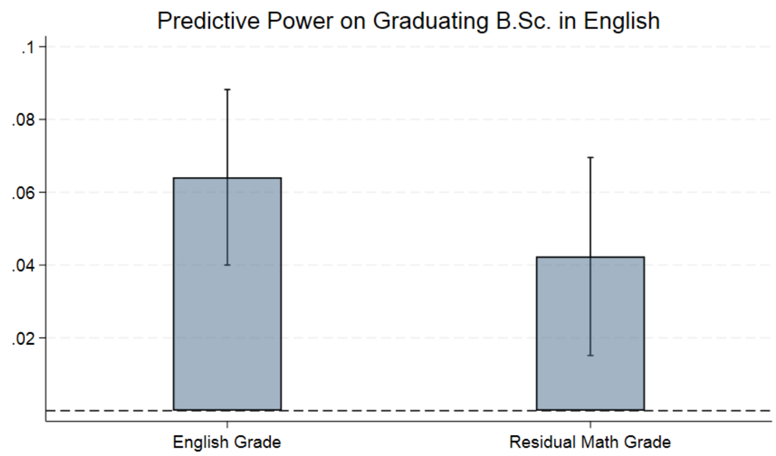
Figure 9: University Study: Mathematics (Residualised English grade)



NOTES: The figure shows coefficients from a regression of completion of the B.Sc. in Mathematics on standardised high school grades in math and the residualised grade in english. The regression is based on 1207 observations. R^2 is 0.111. Source: Statistics Denmark and own calculations.

The results show that residualising the grades of the irrelevant skills reduces the estimated coefficients. $\frac{\hat{\beta}_2^{english}}{\hat{\beta}_1^{math}} = \hat{\lambda} \approx 0.11$ and $\frac{\hat{\beta}_2^{math}}{\hat{\beta}_1^{english}} = \hat{\lambda} \approx 0.63$, meaning that the predictive power of the irrelevant signal relative to the relevant signal is reduced by around 40-50 pct. relative to the baseline estimation in equation 7.1. This indicates that a substantial part of the predictive power of the irrelevant signal was caused by a correlation between the underlying distribution of math and english skills . Both estimates, however, are positive, and $\hat{\beta}_2^{math}$ is still significant, even at the 1 pct. level. The results do not falsify our model predictions, but instead show that even when removing as much of the correlated part as possible with

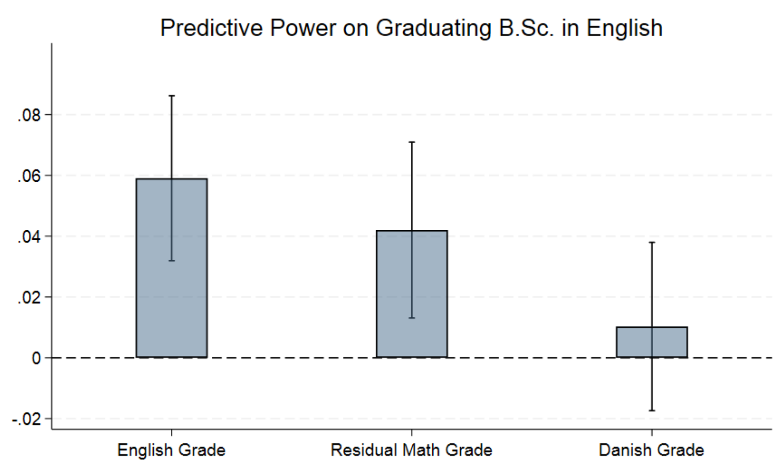
Figure 10: University Study: English (Residualised Math Grade)



NOTES: The figure shows coefficients from a regression of completion of the B.Sc. in English on standardised high school grades in english and the residualised grade in math. The regression is based on 1538 observations. R^2 is 0.024. Source: Statistics Denmark and own calculations.

the available data, the uncorrelated part of the irrelevant signal still holds predictive power, consistent with the talent-hoarding effect. As a final test, we estimate equation B.7 and also include the danish high school grade as an attempt to account for both concern 1 and concern 2 at the same time. The results are reported in figure 11 below and reveal that the findings are still robust to accounting for both concern 1 and 2.

Figure 11: University Study: English (Residualised Math grade)



NOTES: The figure shows coefficients from a regression of completion of the B.Sc. in English on standardised high school grades in english and the residualised grade in math and the danish grade. The regression is based on 1438 observations. R^2 is 0.024. Source: Statistics Denmark and own calculations.

Table 3: Results from Principal Component Analysis (English)

Principal Component Analysis (English)	Eigenvalue	Proportion of Variance Explained
Component 1	2.36	0.79
Component 2	0.42	0.14
Component 3	0.22	0.07

Loadings on Component 1	Loading
Written Year-Grade English	0.60
Oral Year-Grade English	0.58
Written Exam Grade English	0.55
N	341011

NOTES: The table shows the results of the principal component analysis used to calculate the underlying skill distribution of english skills. The results show the first three components, their eigenvalues, and the variance explained by each component. In addition it includes information on how each of the english grades loads on the first component used as the principal component. The analysis is based on the entire body of graduating high school students from 1990-2017 of whom we observe the three grades. Source: Statistics Denmark and own calculations.

Table 4: Results from Principal Component Analysis (Math)

Principal Component Analysis (Mathematics)	Eigenvalue	Proportion of Variance Explained
Component 1	2.58	0.86
Component 2	0.30	0.10
Component 3	0.12	0.04

Loadings on Component 1	Loading
Written Year-Grade Math	0.59
Oral Year-Grade Math	0.58
Written Exam Grade Math	0.56
N	277313

NOTES: The table shows the results of the principal component analysis used to calculate the underlying skill distribution of Math skills. The results show the first three components, their eigenvalues, and the variance explained by each component. In addition it includes information on how each of the math grades loads on the first component used as the principal component. The analysis is based on the entire body of graduating high school students from 1990-2017 of whom we observe the three grades. Source: Statistics Denmark and own calculations.

C Numerical Example

We estimate a standard unrestricted Roy Model using Simulated Method of Moments (SMM) (See, e.g. Smith Jr (1993) and Jakobsen et al. (2022)). In the outer loop, we search over the free parameters $\theta = (\mu_M, \sigma_M^2)$ to minimise the following criterion function $C = (m(\hat{\theta})^{Simulated} - m^{Data})'W(m(\hat{\theta})^{Simulated} - m^{Data})$. $m^{Simulated}$ is a two-dimensional vector of simulated moments where the first element is the share of individuals choosing English and the second element is the difference in mean log wages in Mathematics and English. m^{Data} is the vector of corresponding data moments. As the weighting matrix W, we use the unit vector. For initial values, we set $\theta = (-0.35, 1.5)$.

In the inner loop, we solve the standard Roy-model for the market clearing share of individuals who choose English using the calibrated parameters and for the trial values of θ. We simulate 20,000 individuals, find the share of individuals in the English sector that clears the market, and calculate the simulated moments. In both the inner loop and the outer loop, we use a Nelder-Mead routine, as this is generally robust and does not require the calculation of analytical or numerical gradients. The calibrated and estimated parameter values are reported in table 5 below.

In order to calculate the empirical moments, we use Danish Administrative Data on the mean and variance of log disposable income in 2022 for individuals admitted to the English and Mathematics programme in 2002-2017. The results are reported in table 6 below.

This gives the following data moments of share of English students and difference in log means on: $|N_E^{Data}| = 6915/(6915+3361) \approx 0.67$ and $\bar{\omega}^M - \bar{\omega}^E = 12.33356 - 12.2275 \approx 0.106$.

The correlation coefficient is based on estimates from a regression of normalised math (english) skills on the English (Mathematics) grade used for the residualization of grades in section B.2.2 on the entire population of Danish High School Graduates. The results from this regression are reported in table 7 below:

We also conduct the entire analysis for $\rho = 0.0,\ 0.6$. For this analysis, we use the calibrated parameters in table 5. The estimated mean and variance, however, become $(\hat{\mu}_M, \hat{\sigma}_M^2) = (-0.99, 1.89), (-0.68, 1.33)$ for $\rho = 0.0, 0.6$. The model output for these parameter values is reported in the figure below.

As is evident from these simulations, the results are not qualitatively different. However, we notice that the admissions agent restricts admission more when the skills are highly correlated. We also notice that as the noise increases, the admission agent will give a positive weight to the maths skill $\lambda > 0$ even when the skills are not correlated.

Table 5: Parameter values of the standard Roy model

Parameter	**Explanation**	**Value**	**Source / How set**
K_E	Capital in the English sector	1.0	Calibration / normalisation
K_M	Capital in the Math sector	1.0	Calibration / normalisation
A_E	TFP in the English sector	1.0	Calibration / normalisation
A_M	TFP in the Math sector	1.0	Calibration / normalisation
α	Capital share in each sector's Cobb–Douglas production	0.33	Calibration / normalisation
μ_E	Mean of (log-)skill distribution in English	0.0	Calibration / normalisation
σ_E^2	Variance of (log-)skill distribution in English	1.0	Calibration / normalisation
μ_M	Mean of (log-)skill distribution in Math	-0.838	Estimation (SMM)
σ_M^2	Variance of (log-)skill distribution in Math	1.573	Estimation (SMM)
ρ	Correlation between Math and English (log-)skills	0.30	External estimate / calibration

NOTES: The table shows and explains the parameters used to simulate the standard Roy Model in section 8. It also displays the value of each parameter as well as whether the parameter value was calibrated, estimated or has been derived from an estimation procedure outside this structural model. Source: Model parameters are calibrated and estimated using data from Statistics Denmark.

Table 6: Descriptive Statistics for Data Moments

	English Log(Disposable Income)	Math Log(Disposable Income)
Mean	12.2275	12.33356
Variance	0.7586	0.8583
N	6915	3361

NOTES: The table shows the data moments used for the estimation procedure using simulated method of moments (SMM). This includes the mean and variance of log disposable income in 2022 for individuals admitted to the English and Mathematics programmes on Danish Universities in the period 2002-2017. Source: Statistics Denmark and own calculations.

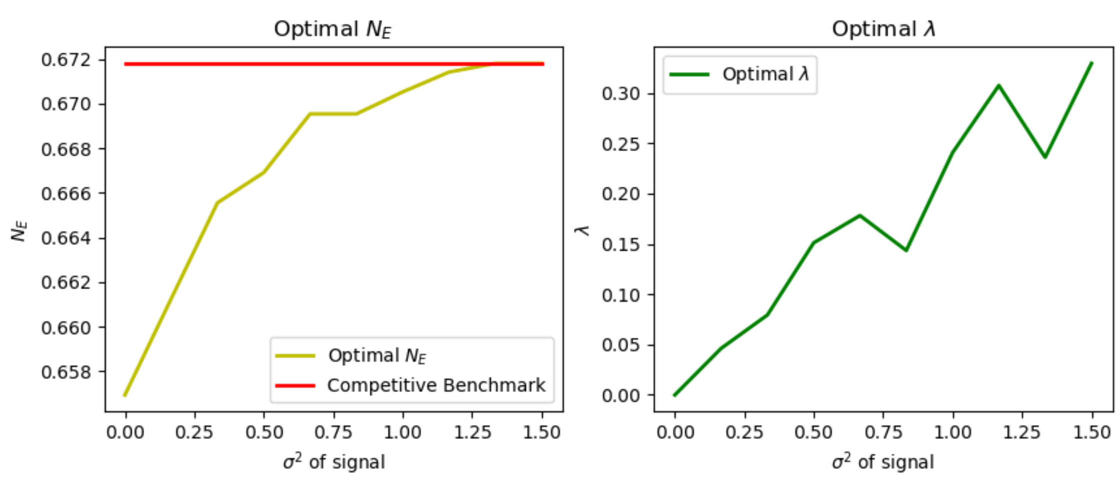
Table 7: Correlation between skills

	(1) Math Grade	(2) English Grade
$\hat{\rho}$	0.341***	
	(0.00112)	
$\hat{\rho}$		0.291***
		(0.00118)
Constant	0.00333	0.124***
	(0.00190)	(0.00195)
N	200126	200497
R^2	0.283	0.226

Standard errors in parentheses,
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

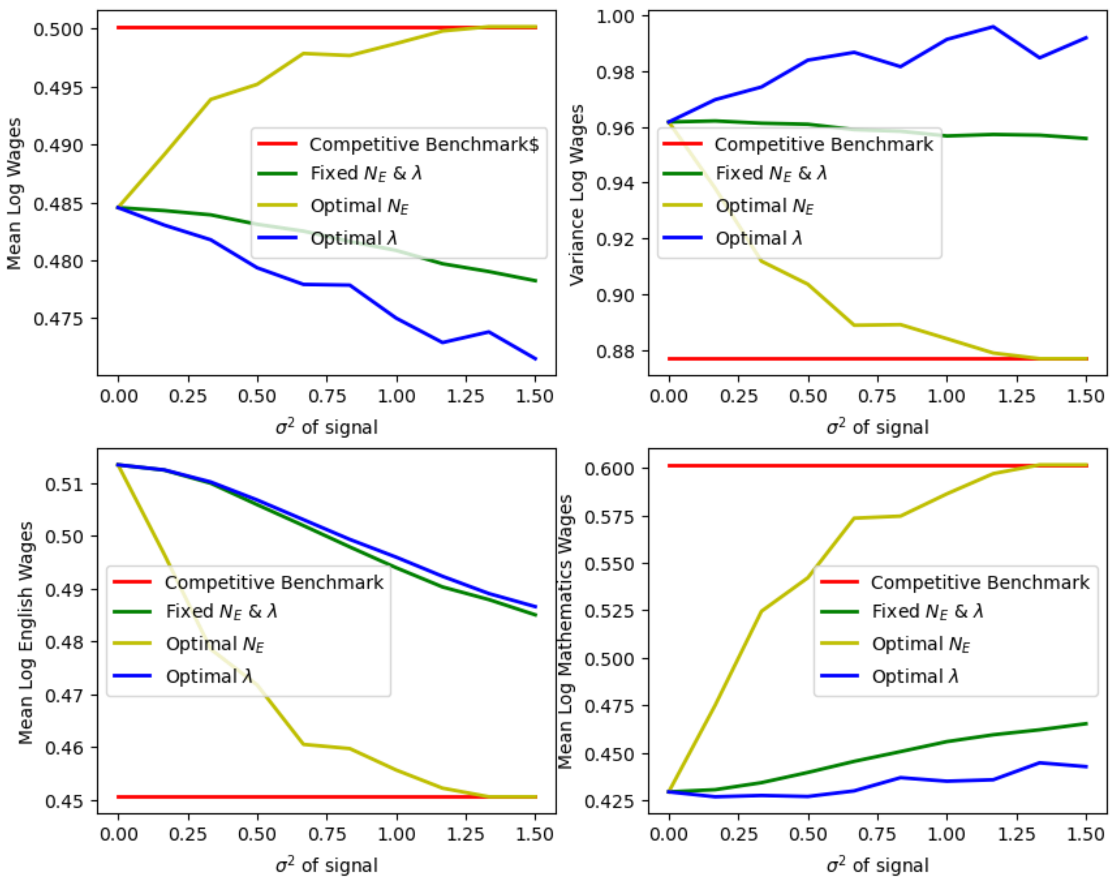
NOTES: The table shows estimates of the correlation coefficients between math skills and english skills. In the first column normalised math exam grades are regressed on a proxy of English skills which is comprised of the principal component of all English exam and year grades obtained in high school using all Danish high school students. In the second column normalised english exam grades are regressed on a proxy of math skills which is comprised of the principal component of all Math exam and year grades obtained in high school using all Danish high school students. Source: Statistics Denmark and own calculations.

Figure 12: Optimal N_E and λ selected by English Admissions Agent ($\rho = 0.0$)



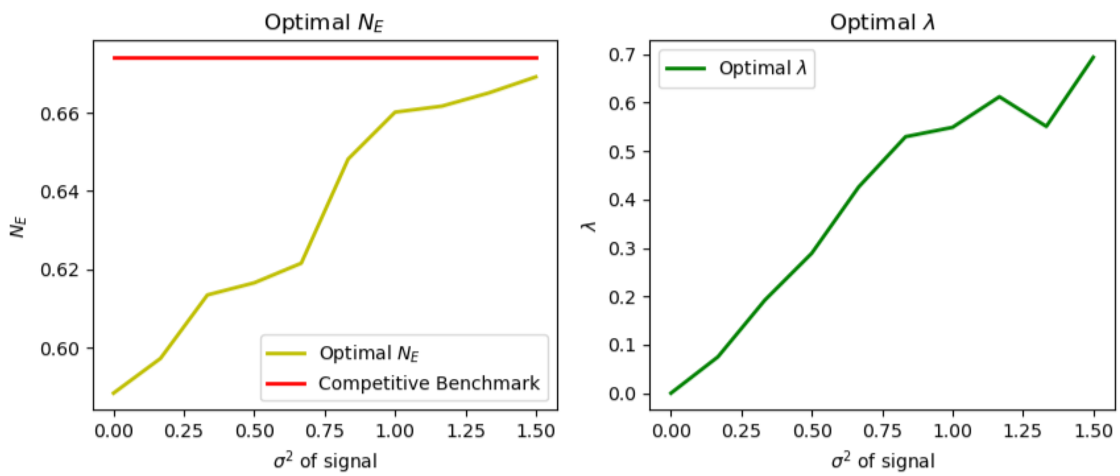
NOTES: The panel to the left shows the share of admitted individuals out of 100,000 into the English programme. The red line denotes the competitive benchmark and the yellow line denotes the optimal N_E^* selected by the admissions agent. The panel to the right shows the λ chosen by the admissions agent. The model has been simulated with a correlation coefficient ρ of 0 between the underlying normal distributions of math and English skills. Source: Own model simulations. Model parameters are calibrated and estimated using data from Statistics Denmark.

Figure 13: Log Mean Wages and Log Variance ($\rho = 0.0$)



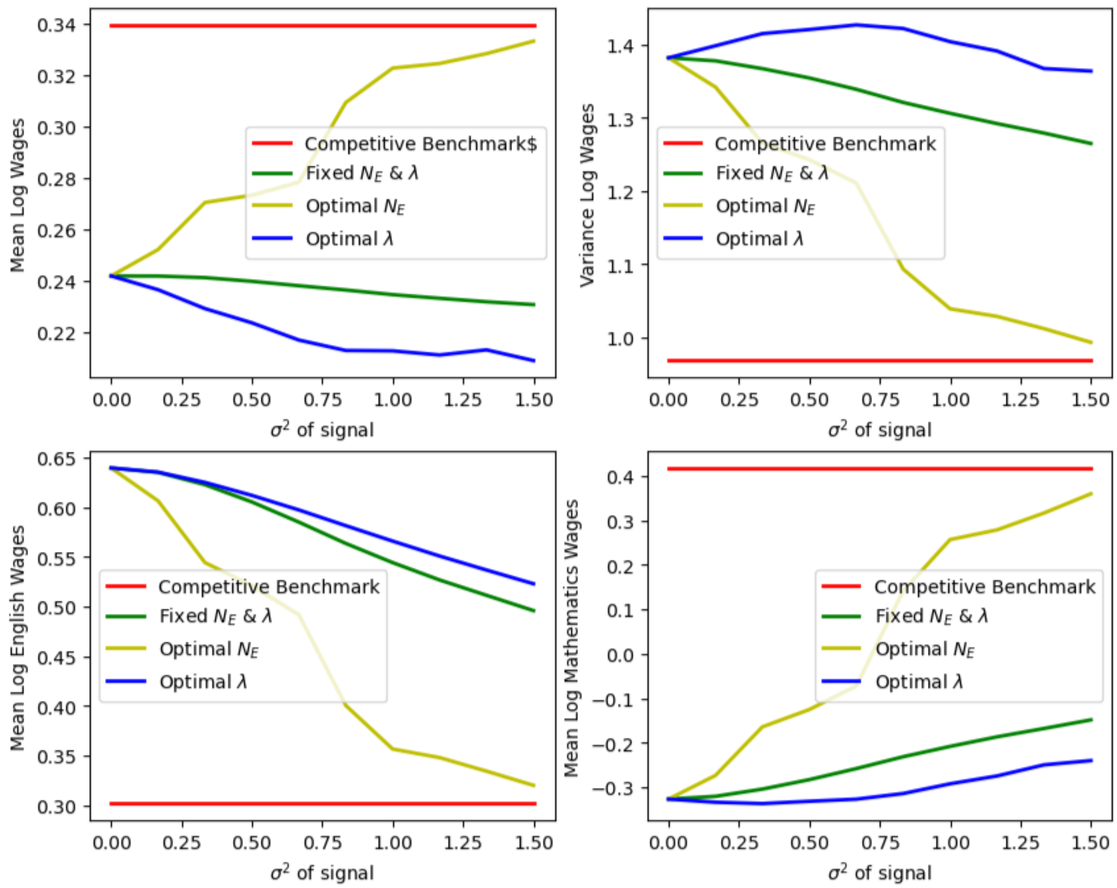
NOTES: The top left panel shows development in mean log wages as noise in signals increase. The top right panel shows the development in variance in log wages. The two bottom panels show mean log wages in the two sectors (English left, Mathematics right) as noise increases. The red line represents the competitive benchmark. The green line represents a simulation where N_E and λ is kept fixed at their no noise levels (mechanical effect). The blue line represents a simulation where the admissions agent adjusts λ as noise increases. The yellow line represents a simulation where the admissions agent adjusts N_E as noise increases. Hence the difference between the green line and the blue line denotes the talent-hoarding effect on output, and the difference between the green line and the yellow line denotes the talent-separation effect on output. The model has been simulated with a correlation coefficient ρ of 0 between the underlying normal distributions of math and english skills. Source: Own model simulations. Model parameters are calibrated and estimated using data from Statistics Denmark.

Figure 14: Optimal N_E and λ selected by English Admissions Agent ($\rho = 0.6$)



NOTES: The panel to the left shows the share of admitted individuals out of 100,000 into the English programme. The red line denotes the competitive benchmark and the yellow line denotes the optimal N_E^* selected by the admissions agent. The panel to the right shows the λ chosen by the admissions agent. The model has been simulated with a correlation coefficient ρ of 0.6 between the underlying normal distributions of math and English skills. Source: Own model simulations. Model parameters are calibrated and estimated using data from Statistics Denmark.

Figure 15: Log Mean Wages and Log Variance ($\rho = 0.6$)



NOTES: The top left panel shows development in mean log wages as noise in signals increase. The top right panel shows the development in variance in log wages. The two bottom panels show mean log wages in the two sectors (English left, mathematics left) as noise increases. The red line represents the competitive benchmark. The green line represents a simulation where N_E and λ is kept fixed at their no noise levels (mechanical effect). The blue line represents a simulation where the admissions agent adjusts λ as noise increases. The yellow line represents a simulation where the admissions agent adjusts N_E as noise increases. Hence the difference between the green line and the blue line denotes the talent-hoarding effect on output, and the difference between the green line and the yellow line denotes the talent-separation effect on output. The model has been simulated with a correlation coefficient ρ of 0.6 between the underlying normal distributions of math and English skills. Source: Own model simulations. Model parameters are calibrated and estimated using data from Statistics Denmark. ■