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Fast Solution of Dynamic Intra-Household Bargaining Models[*]

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Abstract

Dynamic household bargaining models are growing in popularity but are computationally demanding to solve and estimate. We propose a modification to the endogenous grid method (EGM) that allows this class of models to reap the benefits of the EGM. The method, which we refer to as interpolated EGM (iEGM), precomputes optimal intertemporal consumption as a function of the expected marginal value of wealth before solving the dynamic bargaining model. We illustrate the implementation of the iEGM in a simple example, where the iEGM is around 20 times faster than standard value function iterations. In a more complex quantitative model it is about 50 times faster, without compromising accuracy. We apply the iEGM to a rich household model to study how productivity shocks affect consumption inequality. Our results suggest that the degree of commitment of household members can affect the consequences of individual labor productivity shocks, such as e.g. skill-biased technological change.

JEL-codes: D13, D15, C61, C63, C78.

Keywords: Household Bargaining, limited commitment, life cycle, couples, numerical dynamic programming.

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1 Introduction

Most individuals go through life in and out of partnership relationships and many economically relevant policy analyses will miss important aspects if ignoring this element of life. For this reason, dynamic household bargaining models, which also allow for endogenous household formation and dissolution, have gained popularity. For example, dynamic household bargaining models have been used to study compulsory insurance arrangements (Attanasio and Ríos-Rull, 2000), the effect of divorce laws on labor supply, savings, and the marriage market equilibrium (Voena, 2015; Reynoso, 2024), social security policy and well-being (Low, Meghir, Pistaferri and Voena, 2025), joint taxation of couples (Bronson, Haanwinckel and Mazzocco, 2025), and effects of child support and alimony (Foerster, 2025).

A significant barrier to the use of dynamic bargaining models is computational. The state space often includes information on each household member along with other states such as match quality and the bargaining weight, and the number of endogenous choices are likewise often quite large. As a result, solving these models can be extremely time-consuming. Any numerical method that can reduce the solution time would thus be very valuable. One such method is the endogenous grid method (EGM), proposed by Carroll (2006), which has proved particularly efficient in alleviating the computational cost of solving single agent stochastic dynamic programming models (see e.g. Jørgensen, 2013; Fella, 2014 and White, 2015). Unfortunately, however, the speed gains from the EGM cannot generally be directly transferred to dynamic household bargaining models.

We propose a modification of the EGM that facilitates reaping the known benefits of EGM in this class of models. Our key idea is to note that the expected marginal value of wealth is a "sufficient statistic" for all dynamics related to the intertemporal consumption allocation: Only the level of the expected marginal value of wealth matters for consumption, not the dynamic mechanisms that generated it. This fact allows us to express intertemporal consumption as a function of the expected marginal value of wealth. We can, in turn, precompute an interpolator of the the optimal intertemporal consumption allocation *before* solving the model. This interpolator is then used repeatedly to approximate optimal intertemporal consumption allocations when subsequently solving the dynamic model by backwards induction, removing the need for potentially costly root finding operations. For this reason, we refer to the method as the interpolated EGM (iEGM).

To appreciate the idea of the iEGM, recall that the first order condition of a standard dynamic consumption problem states that the marginal utility of consumption should equal the discounted expected marginal value of wealth. The central trick in the EGM was to note that the *post-decision savings* is a sufficient statistic for intertemporal consumption. That is, conditional on post-decision savings, the marginal value of wealth is independent of consumption, and optimal consumption can be found by inverting the marginal utility for values of post-decision savings. In dynamic bargaining models, however, the marginal utility function cannot be inverted *analytically*, which almost all existing implementations of the EGM have relied on.[1] While the FOC can be inverted numerically, this approach is typically costly in the class of models we consider. The iEGM circumvents these numerical root finding operations and retain the attractive features of the EGM, even for this class of models.

Our contribution has three components. First, we illustrate the implementation of the iEGM through a simple example of dynamic consumption allocation in a two-person household. The iEGM is 15-20 times faster than standard value function iterations (VFI) without reducing numerical accuracy. While the analytical EGM is generally not applicable for solving this model, it can be applied for a particular set of parameter values. Using this restricted version of the model, we find that the iEGM delivers similar accuracy and speed as the analytical EGM. While this simple example highlights the computational advantages of the iEGM, the gains will typically be greater in richer models. For example, in our empirical application, the iEGM solves the model in a couple of minutes while VFI takes around 1.5 hours.

Secondly, we define and discuss the broad class of models for which the iEGM is applicable. While the iEGM applies to the same broad model class as the EGM[2], we focus on a relevant sub-class of dynamic household bargaining models in our general exposition of the implementation of the iEGM. Concretely, motivated by the empirical popularity, we restrict attention to models with one continuous intertemporal consumption choice, an arbitrary number of intertemporal discrete choices as well as an arbitrary number of continuous intra-temporal choices.[3] Within this model class, we highlight the construction of intra- and intertemporal parts of the dynamic program – a key distinction for the implementation of the EGM to this class of models.

Finally, we demonstrate the applicability of the iEGM in a rich quantitative household model. The model incorporates several mechanisms commonly found in state-of-the-art household models, such as e.g. Bronson, Haanwinckel and Mazzocco (2025). The model includes limited commitment bargaining, a discrete choice of labor supply, endogenous wages through human capital accumulation, home production and the allocation of pri-

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[1] A notable exception is Fella (2014) who inverts marginal utility numerically.

[2] See Druedahl and Jørgensen (2017) for general conditions.

[3] The model class includes bargaining with full-, limited- and no commitment (see e.g. Theloudis, Velilla, Chiappori, Giménez-Nadal and Molina, 2022).

vate consumption goods within the household. We show how the iEGM can be applied to solve this model around 50 times faster than a generous implementation of VFI. These gains make calibration of the model to the US economy more manageable.

With the calibrated model, we study how labor productivity shocks can propagate into consumption inequality. We find that the dynamic labor supply behavior following a productivity shock to human capital are markedly different in the baseline limited commitment model compared to a model in which household members can fully commit to future allocations. This is particularly so in the lower end of the skill distribution. This difference, in turn, leads to a larger change in consumption inequality in the limited commitment model. Our results thus suggest that the degree of commitment of household members can be an important part of understanding e.g. skill-biased technological change.

We add to two strands of literature. First, we complement the literature on extensions of the EGM. Since the method was initially proposed by Carroll (2006), it has been extended to models with e.g. multiple dimensions (Barillas and Fernández-Villaverde, 2007; Ludwig and Schön, 2014; White, 2015), multiple constraints (Hintermaier and Koeniger, 2010 and Druedahl and Jørgensen, 2017), and mixed discrete-continuous choices (Fella, 2014; Iskhakov, Jørgensen, Rust and Schjerning, 2017; and Druedahl and Jørgensen, 2017). The iEGM can be applied in combination with any of these previous extensions when standard EGM would require root-finding. The iEGM thus widens the class of models that can reap the benefit of the EGM, facilitating the estimation of richer models for a given computational infrastructure. All code used to generate the results are available online for researchers to modify and use in their own work.[4]

We also add to the literature on skill-biased technological change. While an influential literature has documented how productivity shocks, such as e.g. skill-biased technological changes, affects wages and labor market inequality (see e.g. the reviews by Acemoglu, 2002 and Card and DiNardo, 2002 and recent work by Macera and Tsujiyama, 2024), less is known about the transmission of such productivity shocks into consumption inequality.[5] Arvai and Mann (2025) shows that price and demand adjustments can dampen the productivity shock pass-through to consumption inequality. We add to this literature by showing how intra-household bargaining, something not included in the existing literature, can influence the transmission of productivity shocks into consumption inequality.

The paper proceeds as follows. In Section 2, we describe the idea, implementation

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[4] `https://github.com/ThomasHJorgensen/FastBargaining` contains code that generates all results herein.

[5] Another related literature is that on consumption insurance against transitory and permanent income shocks (see e.g. Kaplan and Violante, 2010; Blundell, Pistaferri and Saporta-Eksten, 2018).

and numerical performance of the iEGM through a simple illustrative model of intra-household consumption allocation. In section 3, we describe how to apply the iEGM to a more general model class. In Section 4, we apply the iEGM to study the dynamic consequences of individual productivity shocks and illustrate the performance of the iEGM in an empirical relevant model. Finally, we conclude with a discussion in Section 5.

2 The EGM without Analytical Inverse Marginal Utility

We propose an interpolated version of the endogenous grid method (EGM), originally proposed by Carroll (2006), which we refer to as the iEGM. The motivation for this approach is the fact that a large class of models do not feature closed form expressions for the inverse marginal utility function, which is a key element in the EGM. We discuss the general model class for which the iEGM applies in the next Section, and here use a simple illustrative model of a two-person household to demonstrate the approach.

In this example, couples live for T periods, and in each period choose individual consumption, $c_{j,t}$ for both members $j \in \{w, m\}$, and public consumption, c_t. We have one state variable: beginning of period t resources, M_t. For simplicity, we assume that couple can fully commit to future allocations, so the model is of a unitary type. The household utility function is a weighted sum of each household member's individual utility, with a constant bargaining weight μ on the utility of the woman.

In turn, the Bellman equation for couples is

$$V_t(M_t) = \max_{c_{w,t}, c_{m,t}, c_t} \mu u_w(c_{w,t}, c_t) + (1-\mu) u_m(c_{m,t}, c_t) + \beta \mathbb{E}_t[V_{t+1}(M_{t+1})] \tag{1}$$

$$\text{s. t.}$$

$$\begin{aligned} M_{t+1} &= RA_t + Y_{w,t+1} + Y_{m,t+1} \\ A_t &= M_t - c_t - c_{w,t} - c_{m,t} \geq 0 \end{aligned} \tag{2}$$

where R is the gross interest rate, $Y_{j,t}$ is exogenous income of member j, assumed to be log-normal mean one with variance σ_j^2. Individual preferences are of the CES type,

$$u_j(c_{j,t}, c_t) = \frac{1}{1-\rho_j} \left(\alpha_j c_{j,t}^{\phi_j} + (1-\alpha_j) c_t^{\phi_j} \right)^{1-\rho_j}. \tag{3}$$

The problem contains an intertemporal choice concerned with the allocation of resources into consumption and savings and an intra-temporal choice concerned with the allocation of consumption between goods. Key here is to note that, conditional on the total resources spent on consumption in a given period, there are no dynamic implications of the allocation of consumption between goods. This allows us to partition the problem into an intra- and intertemporal part. This reformulation is handy when solving the model with EGM-based methods, as we will see below.

Intra-temporal problem. Conditional on total consumption in period t, $C_t = c_t + c_{w,t} + c_{m,t}$, the intra-temporal allocation is

$$\tilde{c}_t(C_t), \tilde{c}_{w,t}(C_t), \tilde{c}_{m,t}(C_t) = \arg\max_{c_t, c_{w,t}, c_{m,t}} \mu u_w(c_{w,t}, c_t) + (1-\mu) u_m(c_{m,t}, c_t) \tag{4}$$

$$\text{s. t.}$$

$$C_t = c_t + c_{w,t} + c_{m,t}$$

and the household level utility can thus be defined as a function of total consumption,

$$U(C_t) = \mu u_w(\tilde{c}_{w,t}(C_t), \tilde{c}_t(C_t)) + (1-\mu) u_m(\tilde{c}_{m,t}(C_t), \tilde{c}_t(C_t))$$

Intertemporal problem. The intertemporal problem is then

$$C_t^\star(M_t) = \arg\max_{C_t} U(C_t) + \beta \mathbb{E}_t[V_t(M_{t+1})] \tag{5}$$

$$\text{s. t.}$$

$$M_{t+1} = R(M_t - C_t) + Y_{w,t+1} + Y_{m,t+1} \tag{6}$$

$$M_t - C_t \geq 0$$

with associated first order condition (FOC)

$$U'(C_t) = W_t \tag{7}$$

where

$$W_t = \beta R \mathbb{E}_t[V_t'(M_{t+1})] \tag{8}$$

is the expected marginal value of wealth. The Envelope Theorem then gives the standard Euler equation, where $W_t = \beta R \mathbb{E}_t[U_t'(C_{t+1}(M_{t+1}))]$.

2.1 The Endogenous Grid Method (EGM)

Inserting the transition equation for wealth (2) into the Euler equation, optimal consumption can be found by iterating as (time iteration):

$$C_t^{TI} = \{C_t : U'(C_t) - \beta R \mathbb{E}_t[U_t'(C_{t+1}(R(M_t - C_t) + Y_{w,t+1} + Y_{m,t+1}))] = 0\} \tag{9}$$

for all values of the state, M_t. The key insight of the EGM, as proposed by Carroll (2006), is to realize that rather than forming an exogenous grid over beginning-of-period resources, M_t, and iterating on C_t, we can formulate a grid over end-of-period wealth, A_t. Conditional on savings, $A_t = M_t - C_t$, the RHS of the FOC is independent of consumption, and we can thus use that

$$W_t(A_t) = \beta R \mathbb{E}_t[U_t'(C_{t+1}(RA_t + Y_{w,t+1} + Y_{m,t+1}))]$$

does not depend on C_t to form the EGM problem for a given value of $A_t = a$:

$$C_t^{EGM}(a) = \{C_t : U'(C_t) - W_t(a) = 0\}. \tag{10}$$

The EGM solution now depends on a as W_t is a function of a. We can then use the budget constraint to get an endogenous grid of beginning-of-period resources as $M_t = a + C_t^{EGM}$.

If the inverse marginal utility is known analytically, we can write the optimal consumption as

$$C_t^{EGM,analytical}(a) = U'^{-1}(W_t(a)). \tag{11}$$

All existing implementations of the EGM, with the exception of Fella (2014), rely on an analytically invertible utility function. For many models, however, our simple one included, there exists no analytical formulation of the inverse marginal household utility function, $U'^{-1}(W_t(a))$. In such cases, consumption can instead be obtained by using a numerical solver to solve eq. (10) for C_t given A_t. We refer to this approach as "numerical EGM".

Numerical EGM can be implemented to avoid recomputing $W_t(a)$ when searching over C_t, which is often computationally expensive since it involves taking an expectation over future states.[6] However, it requires repeated evaluations of the marginal utility, $U'(C_t)$. The computational gain from the numerical EGM therefore hinges on $U'(C_t)$ being relatively cheap to evaluate. This is often the case if it is known analytically. In the general case presented above, $U'(C_t)$ is not known analytically, and approximating it numerically requires solving the intra-temporal problem several times for each guess of C_t.

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[6] We discuss this in relation to Table 3 below.

For rich models, with a high dimensional intra-temporal problem and/or a large state space, this cost can be quite substantial, as we will show below.

We instead propose to replace the numerical inverse with a precomputed interpolator of the intertemporal consumption choice that typically can be constructed without a significant change in the computation speed and accuracy of the numerical solution. We call this the interpolated EGM, or iEGM.

2.2 The Interpolated Endogenous Grid Method (iEGM)

The idea of iEGM is to build on the EGM idea and realize that the EGM consumption solution can be thought of as a function of W_t rather than A_t. For any arbitrary expected marginal utility of wealth $W_t = w$, corresponding optimal consumption can be found by solving a problem like that in eq. (10). In other words, the optimal consumption depends only on the marginal value of wealth and not on the underlying dynamic processes generating a particular value of W_t. In that sense, W_t is a sufficient statistic for optimal consumption. What we propose is to precompute this function and interpolate the solution, when solving the intertemporal problem. In turn,

$$C_t^{iEGM} = C^{interp}(W_t) \tag{12}$$

where we construct the interpolator $C^{interp}(\bullet)$ *once* before solving the model.

Constructing the interpolator. There are several ways to construct the interpolator. One approach is to construct a grid over marginal values of wealth, $\overrightarrow{W}$ and use a numerical root finder to find the solution to eq. (10) and get a corresponding grid of optimal consumption points,

$$C(w) = \{C : U'(C) - w = 0\}.$$

This requires repeatedly solving the intra-temporal problem to get $U'(\bullet)$ but as this is done once before solving the dynamic model it typically would not be the computational bottleneck. This approach is especially useful if the interpolation approach employed requires direct control over the placement of the evaluation nodes in $\overrightarrow{W}$, such as e.g. Chebyshev interpolation.

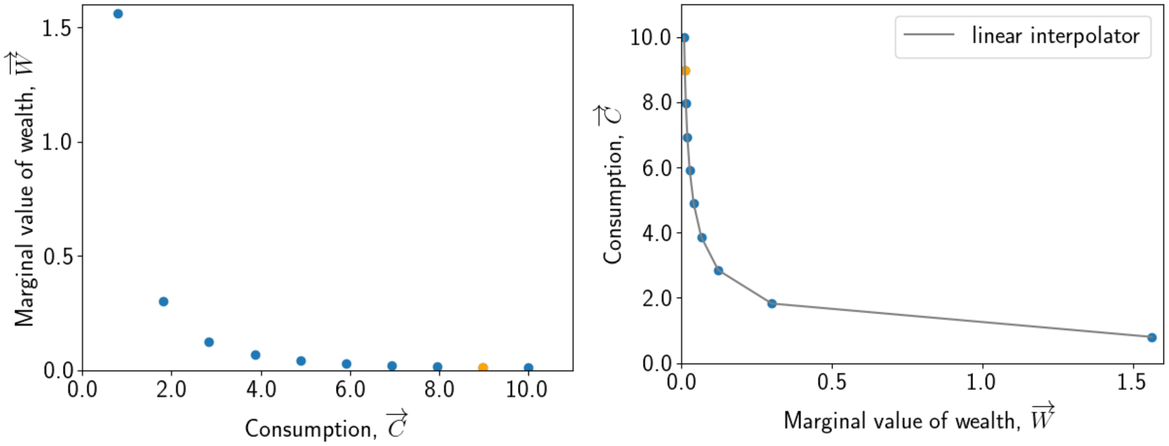
If such control is not required, a simpler approach is inverse interpolation, which we refer to as "flipping the axes". This approach avoids numerical inversion and is often straightforward to implement. While it is a well-known trick to construct inverse relationships, we describe it below for completeness.

We construct the interpolator through two simple steps. First, we construct a grid over consumption, $\overrightarrow{C}$, with $\#_C$ number of points. Evaluating the marginal utility for all points in this grid gives a grid of associated marginal utilities, $U'(\overrightarrow{C})$, which we know from equation (7) to be equal to $\overrightarrow{W}$ at the optimal intertemporal consumption level.[7] Panel a) of Figure 1 shows this relationship with the constructed consumption grid, $\overrightarrow{C}$, on the x-axis and the associated marginal utility on the y-axis. We use only $\#_C = 10$ points here for illustrative purposes. As we will see below, increasing the number of points and thus the accuracy of the interpolator is often not computationally costly.

Second, we use these associated grids to construct an interpolator for C as a function of W. We simply "flip the axes", such that we can use the calculated marginal utility grid points in $\overrightarrow{W}$ together with the associated consumption grid points in $\overrightarrow{C}$ to construct an interpolator, $C^{interp}(W)$, of optimal consumption for values of marginal values of wealth. We show this in panel b) of Figure 1. The line connecting the node points illustrates linear interpolation between these points.

In this example, this interpolator depends only on W, and not on other states and choices. In general, this interpolator should be constructed for all variables that are relevant for the marginal household utility of consumption. We discuss this in detail below in Section 3 and provide a concrete example in our application in Section 4.

Figure 1: Interpolator of the Inverse Marginal Utility, $C^{interp}(W)$.


(a) Grid over $\overrightarrow{C}$ gives $\overrightarrow{W} = U'(\overrightarrow{C})$.

(b) Flipped Axes and $C^{interp}(W)$.

Notes: The figure illustrates in panel a) how constructing a grid over consumption $\overrightarrow{C}$ can be used to evaluate the marginal utility to get the marginal utility of wealth, $\overrightarrow{W}$, as these are equal at an internal optimum. Panel b) shows how flipping the axes provides an interpolator for the object of interest, namely C^{interp} using the known points in $(\overrightarrow{W}, \overrightarrow{C})$. The orange-colored marker highlights the flip of axes. The solid lines represents a linear interpolator between the points.

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[7] If the marginal utility is not known analytically (which is the case we entertain), it can e.g. be approximated by finite differences of the utility function, $U(C_t)$.

Applying the interpolator. The interpolator is applied when finding optimal consumption behavior in the intertemporal problem (5) on a grid over post-decision assets $\overrightarrow{A_t}$, as in the standard EGM. Working backwards in time, the marginal value of wealth $W_t(\cdot)$ is known from the solution to the next period's problem.

For a given point in the post-decision assets grid, say $A_t = a$, the marginal value of wealth is $W_t(a) = \beta R \mathbb{E}_t[U_t'(C_{t+1}(Ra + Y_{w,t+1} + Y_{m,t+1}))]$, and optimal consumption is

$$C^{iEGM}(a) = C^{interp}(W_t(a)). \tag{13}$$

This replaces the numerical inversion step with a simple evaluation of the interpolator. The associated endogenous grid point over beginning-of-period resources, m, can then be found from the budget constraint as $m = a + C_t^{iEGM}(a)$ exactly as done when applying the EGM (therefore the name). The intra-temporal allocation is then found by inserting $C^{iEGM}(a)$ in the intra-temporal problem (4).

2.3 Computational Performance

We demonstrate the performance of different solution methods by solving the model above for $T = 20$ periods by standard VFI, EGM with analytical inverse (when available), the EGM with numerical inverse, and the iEGM.

In our simple example, letting $\alpha = \phi_w = \phi_m = 1$, and $\rho_w = \rho_m$, we can solve the intra-temporal problem in closed form, derive the household utility and implement the analytical EGM with the inverse marginal utility being

$$C_t^{EGM,analytical}(a) = U'^{-1}(W_t(a)) = \Psi^{\frac{1}{\rho}} W_t(a)^{-\frac{1}{\rho}}$$

where

$$\Psi = \left(\frac{1}{2}\left[\frac{\mu}{1-\mu}\right]^{\frac{1}{\rho}}\right)^{1-\rho}\mu + \left(1 - \frac{1}{2}\left[\frac{\mu}{1-\mu}\right]^{\frac{1}{\rho}}\right)^{1-\rho}(1-\mu).$$

This version serves as a standard analytical EGM benchmark in what we refer to as the "restricted" model. We also implement a "unrestricted" version where we cannot solve for an analytical inverse marginal utility. The parameter values of both models are given in Table 1. Bold numbers are the restrictions applied in the restricted model to facilitate the implementaion of the analytical EGM. For the "unrestricted model", the analytical EGM cannot be applied, only the numerical version and the iEGM.

Table 1: Model Parametrizations.

Parameter	Restricted model	Unrestricted model
β	0.98	0.98
ρ_w	2.5	2.5
ρ_m	**2.5**	1.5
ϕ_w	**1.0**	0.8
ϕ_m	**1.0**	0.5
α	**1.0**	0.5
μ	0.3	0.3
σ_w	0.1	0.1
σ_m	0.1	0.1
R	1.03	1.03

Notes: The Table reports the parameter values of the "restricted model" in the first column and the parameters of the "unrestricted model" in the last column. Bold numbers are the restrictions applied in the restricted model to facilitate a closed form solution of the intra-temporal problem. See the text for further details. Across all models and solution methods, we use the same number of discrete points to approximate the continuous state and number of Gaussian Quadrature nodes. Concretely, we use 40 points in the beginning of period resource grid in VFI and 40 points in the post-decision asset grid for EGM-type methods. We use 5 quadrature points to approximate each integral over individual income shocks (so 25 in total).

2.3.1 Solving the illustrative model

We solve the model using backwards induction, assuming no bequest motive in the terminal period, T. That means that all available resources is consumed in the final period and we have $C_T(M_T) = M_T$ and $V_T(M_T) = U(M_T)$. We solve the model in earlier periods using several approaches.

VFI. The standard VFI approach is to loop through all points in the state space and solve for optimal consumption. For a given level of resources, say $M_t = m$, optimal total consumption is found as

$$C_t^{VFI}[m] = \arg\max_{C_t} U(C_t) + \beta \mathbb{E}_t[V_{t+1}^{VFI}(M_{t+1}(m, C_t))] \tag{14}$$

where next-period resources depend on beginning-of-period resources, m, and total consumption, C_t. The expectation over future resources thus involves a joint expectation over $Y_{w,t}$ and $Y_{m,t}$, which we approximate with Gauss-Hermite quadrature, such that $\mathbb{E}_t[V_{t+1}^{VFI}(M_{t+1}(m, C_t)] \approx \sum_{j=1}^{\#_Y} \sum_{k=1}^{\#_Y} V_{t+1}^{VFI}(m_{t+1}^{j,k}(m, C_t))\omega_j \omega_k$. Here, next-period resources depends on realized income shocks $Y_w[j], Y_m[k]$, cf. eq. (2), $\#_Y$ denotes the number of quadrature nodes in each dimension, and ω_j and ω_k denote the quadrature weights. We let $\#_Y = 5$ throughout. Finally, the value function is constructed as

$$V_t^{VFI}(M_t) = U(C_t^{VFI}(M_t)) + \beta \mathbb{E}_t[V_{t+1}^{VFI}(M_{t+1}(M_t, C_t)).$$

EGM, numerical. Both the numerical EGM and the iEGM use the marginal household utility of total consumption, $U'(C)$, when solving the intertemporal problem. When this object is not known analytically (as in our unrestricted model), we calculate this object by finite differences as

$$U'(C) = \frac{U(C+\eta) - U(C-\eta)}{2\eta} \tag{15}$$

where each evaluation of $U(C)$ requires solving the intra-temporal problem in (4) numerically.

Rather than looping through all values of the beginning-of-period resources, M_t, as in VFI, we construct a post-decision grid of savings, $\overrightarrow{A}_t$. From the FOC, for a given level of end-of-period wealth, say $A_t = a$, optimal consumption solves the root finding problem

$$C_t^{EGMnum}[a] = \{C_t : U'(C_t) - W_t(a) = 0\} \tag{16}$$

where the marginal value of wealth is

$$W_t(a) = R\beta \mathbb{E}_t \left[U'\left(C_{t+1}^{EGMnum}\left(M_{t+1}(a)\right)\right)\right] \tag{17}$$

and $C_{t+1}^{EGMnum}(M_{t+1})$ is the optimal consumption next period, evaluated at next-period resources. As with the VFI implementation, we approximate the expectation over future income shocks by Gauss-Hermite quadrature. In turn, for each point in $\overrightarrow{A}$ and for each guess of C_t, we approximate expectations in eq. (17) as $\sum_{j=1}^{\#_Y}\sum_{k=1}^{\#_Y} U'(C_{t+1}^{EGMnum}(m_{t+1}^{j,k}(a)))\omega_j\omega_k$, where realized next-period resources are $m_{t+1}^{j,k}(a) = (1+r)a + Y_w[j] + Y_m[k]$.

Similarly as the standard EGM, beginning-of-period (endogenous) resources are then $m_t = a + C_t^{EGMnum}[a]$, creating the the object $C_t^{EGMnum}(m_t)$ used in the next backwards iteration.

iEGM. To implement the iEGM, we first construct a grid over total household consumption, $\overrightarrow{C}$, with $\#_C$ grid points. For each point in $\overrightarrow{C}$, we evaluate marginal household utility given by eq. (15). This gives us a grid over the marginal value of wealth $\overrightarrow{W}$, since $U'(C) = W$ in interior solutions by the FOC. Finally, by "flipping the axes" as per figure 1, we obtain an interpolator for optimal total consumption: $C^{interp}(W)$ over the grid $\overrightarrow{W}$. Importantly, this step is performed *only once*, before starting the backwards induction algorithm.

As in numerical EGM, we loop though a grid of post-decision savings, $\overrightarrow{A}_t$. For a given value of savings, say, $A_t = a$, we first compute the marginal value of wealth by interpolating next period's consumption and using the FOC:

$$W_t(a) = R\beta\mathbb{E}_t\left[U'^{interp}\left(C_{t+1}^{iEGM}\left(M_{t+1}(a)\right)\right)\right] \tag{18}$$

where $C_{t+1}^{iEGM}(M_{t+1}(a))$ is the optimal consumption next period, evaluated at next-period resources. This allows us to interpolate optimal intertemporal consumption in the present period as

$$C_t^{iEGM}[a] = C^{interp}(W_t(a)). \tag{19}$$

Similar to the standard EGM, beginning-of-period (endogenous) resources is then $m_t = a + C_t^{iEGM}[a]$, creating $C_t^{iEGM}(m_t)$.

There are two key differences between this approach and the numerical EGM above. First, we do not have to iterate on (16). Instead we interpolate the precomputed solution in (19). This removes the need for repeatedly constructing the marginal household utility in equation (15). Second, when calculating the expected marginal value of wealth in eq. (18) we again use interpolation to approximate the marginal household utility, $U'^{interp}(C)$, rather than repeatedly calculating it, as in the numerical EGM in eq. (16). The numerical EGM can also utilize precomputed $W_t(a)$ as we will discuss below, but still has to repeatedly solve for the marginal utility when searching over consumption.

We show below how the iEGM can greatly speed up the solution of this simple illustrative model without sacrificing accuracy.

2.3.2 Accuracy measures

Since the model does not lend itself to an analytical solution, we evaluate the accuracy of the solution methods by benchmarking against a solution using standard VFI on fine , dense grids. We refer to this as the "true" solution. All accuracy measures are relative to this solution.

We calculate several accuracy measures to show how our proposed iEGM compares to existing methods. First, we calculate the mean absolute difference (MAD) in period $t = 1$ optimal consumption across 100 values of resources in the interval $[0.1, 5]$ between each method j and the true solution, $MAD_j = \frac{1}{100}\sum_{m=1}^{100}|(C_1^{true}[m] - C_1^{j}[m])|$. Second, we calculate how much initial wealth compensation (in percent of expected household income of 2) all households should have to be indifferent between the consumption schedule of each method and the true solution. We find this value as the level of wealth that equates the simulated discounted sum of expected utilities from method j with the true solution,

$$A_j = \{A : \frac{1}{N}\sum_{i=1}^{N}[V_1^{true}(M_{1,i}) - V_1^{j}(M_{1,i} + A \cdot 2/100)] = 0\}.$$

This measure, which we refer to as "Comp(A)" in the tables below, is in nature similar to a consumption equivalence (CE) measure but does not require us to resolve the model when searching over A.[8] A value of $A = 1$ indicates that households would be willing to give up 1% of a full household level income (in expected terms) to replace the consumption function of method j with that of the true solution. As this number approaches zero, the agents are indifferent between the two solutions.

2.3.3 Computational Time and Accuracy

We show computation time and accuracy of all approaches solving the restricted model in the left panel of Table 2. Importantly, this parametrization facilitates the implementation of the analytical EGM. We show results for increasing degree of accuracy in the construction of the interpolator for iEGM, letting $\#_C$ vary from 25 to 200. All statistics are averages across 50 solution and simulation runs. We solve and simulate the model for $T = 20$ periods and simulate 10,000 households for the calculation of the "Comp(A)" accuracy measure.

First, the VFI and the analytical EGM have almost the same accuracy but the analytical EGM solves the model in 7% of the time it takes VFI. Likewise, the numerical EGM, EGMnum, delivers identical accuracy to the analytical EGM, but is slightly slower, since it iterates on the Euler equation. Finally, the iEGM solves the restricted model with around the same speed as the EGM, independent of the degree of accuracy, $\#_C$. However, as expected, the quality of the solution is poor when using very few interpolation points. For example, consumers should be compensated with around 60% of household expected income in the first period of life to follow the consumption profile produced by the iEGM with only 25 interpolation points (and linear interpolation between them). However, as the number of points increases to 200, time and accuracy are almost identical to the analytical EGM. This shows that the iEGM is able to reproduce the significant speed up of the EGM documented in e.g. White (2015); Fella (2014); Jørgensen (2013).

In the right panel of Table 2, we report similar statistics for the unrestricted model. For

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[8] A similar measure is used in e.g. Borella, De Nardi and Yang (2023).

this parametrizaiton, the analytical EGM cannot be applied. The iEGM again performs very well both in terms of accuracy and time and delivers comparable accuracy to standard VFI using only around 2% of the computation time. In the unrestricted model, computation time increases moderately with the number of points in the interpolation grid $\#_C$. It is worth noting that for this relatively simple model, the precomputation step takes up a larger part of total solution time than it would in a more complex model with more states and choices. In a more complex model, the precomputation is typically a smaller part of the total solution, so computation time should depend less on $\#_C$.

While the analytical EGM cannot be applied to solve this model, the numerical EGM is again able to produce comparable accuracy to VFI (if anything, slightly better). However, the solution time is an order of magnitude slower than that of VFI. This stems from the fact that numerically solving the Euler equation now requires a very high number of evaluations of the marginal household utility, $U'(C)$, which is computationally costly in this parametrization. This implementation should be viewed as the simplest naive implementation of numerical EGM. We explain and unpack this further below.

Table 2: Computational Performance.

	Restricted model				Unrestricted model			
	time (s)	rel.	MAD(C)	Comp(A)	time (s)	rel.	MAD(C)	Comp(A)
VFI	1.129	1.000	0.015	0.066	17.590	1.000	0.004	0.000
EGM	0.066	0.058	0.013	0.073	N.A.	N.A.	N.A.	N.A.
EGM (num)	0.109	0.097	0.013	0.073	158.816	9.041	0.001	0.000
iEGM								
$\#_C$=25	0.066	0.059	0.218	15.692	0.163	0.009	0.013	0.001
$\#_C$=50	0.066	0.059	0.064	1.803	0.264	0.015	0.003	0.000
$\#_C$=75	0.067	0.059	0.039	0.514	0.373	0.021	0.002	0.000
$\#_C$=100	0.066	0.058	0.028	0.270	0.492	0.028	0.002	0.000
$\#_C$=200	0.066	0.058	0.016	0.103	0.919	0.052	0.001	0.000

Notes: The Table reports timings and numerical accuracy of different solution methods. Timings are reported in seconds and relative to the VFI benchmark. Accuracy is measured in terms of 1) MAD(C): Mean absolute difference of the consumption profile to the "true" solution (VFI on very dense grids), and 2) Comp(A): initial wealth compensation to ensure indifference to the "true" solution. All statistics are based on 50 Monte Carlo runs, where we solve (and store timing) and simulate 10,000 households for 20 periods.

2.3.4 Time Decomposition through Precomputation

The computational benefits of the iEGM stem from its use of pre-comptations. Once we entertain the possibility of precomputating parts of the problem, we believe iEGM to be the natural choice of solution method. That said, other methods may also benefit from precomputations. Particularly i) the precomputation of the intra-temporal allocation of resources used in the iEGM is applicable across methods, and ii) the expected (marginal) continuation values can likewise be precomputed. In this section, we illustrate how alternative methods perform when incorporating these precomputations. This will provide valuable insights into the bottlenecks in each of the alternative methods and clearly show which parts of the problem, the iEGM greatly speeds up. We only focus on the unrestricted model here and report all results in Table 3. These numbers should be compared with the right panel of Table 2.

Expectations. First, we implement versions in which we precompute the expected continuation value $\mathbb{E}_t[V_{t+1}(M_{t+1})]$ at backwards iteration step. For VFI, we pre-compute

$$EV_t(a) = \sum_{j=1}^{\#_Y} \sum_{k=1}^{\#_Y} V_{t+1}^{VFI}(m_{t+1}^{j,k}(a))\omega_j\omega_k$$

on a grid of post-decision assets and replace the expected value with an interpolator in equation (14),

$$C_t^{VFI,pre\,E}[m] = \arg\max_{C_t} U(C_t) + \beta EV_t^{interp}(m - C_t). \tag{20}$$

Likewise, for EGM-related methods, we store the expected marginal value $W_t(A_t) = \beta R \mathbb{E}_t[U_t'(C_{t+1}(RA_t + Y_{w,t+1} + Y_{m,t+1}))]$ rather than re-calculating it for all guesses of C_t and then interpolate this object through $W_t^{interp}(a)$. For the numerical EGM, this means that

$$C_t^{EGMnum,pre\,E}[a] = \{C_t : U'(C_t) - W_t^{interp}(a)\}. \tag{21}$$

These approaches removes the need for recalculating expectations when applying numerical optimization or root finding to determine C_t.

The results from this exercise are shown in Table 3 with post-fix "pre E". Precomputing the expectations does not lead to a significant speed-ud for the VFI implementation. For the numerical EGM implementation, precomputing expectations leads to a modest speed-up. This is because the main computational burden is not computing the expectations. Instead, most of the time is likely spent evaluating $U(C_t)$ in VFI and $U'(C_t)$, which requires *two* evaluations of $U(C_t)$, cf. eq. (15) in the numerical EGM.

Intra-temporal problem. For the VFI implementation, we can also precompute the solution to the intra-temporal problem and interpolate from that rather than resolving every time $U(C)$ is called. This solution method, referred to as "VFI, pre E + intra", is shown in Table 3 to be as accurate as the standard VFI but in 5% of the time. This shows that for the standard VFI, the bulk of the computation time is spent solving the intra-temporal problem numerically for every call of $U(C)$.

This results also suggests that solving the household problem in two stages – an intertemporal problem over total household consumption C_t and an intra-temporal allocation problem across goods $c_{w,t}$, $c_{m,t}$ and c_t – is not necessarily efficient when solving using standard VFI in this simple example. The nested structure requires repeatedly solving the intra-temporal problem, and the numerical solver does not make efficient use of information contained in the marginal values of individual consumption goods.

For comparison, we also implement a simultaneous version of VFI where the levels of each consumption good are found directly without conditioning first on total household consumption C_t. Expectations are precomputed in this implementation. This version (referred to as "VFI (simul.), pre E" in Table 3 is five times faster than the standard VFI approach. However, among the VFI-type methods considered here, the two-step approach with precomputed intra-temporal solutions remains by far the fastest.

We also report results for the numerical EGM with precomputed expectations and intra-temporal allocations, referred to as "EGMnum, pre E + intra" in Table 3. Since this is now almost identical to the iEGM, the time and accuracy are very similar. The main difference is that in the iEGM the solution is constructed once before solving and then interpolated, and in the EGMnum, the solution and expectations are interpolated for all evaluations of the root-finding algorithm when searching over C_t. In this simple example, when precomputing everything, the numerical root finding is not that computationally expensive. Note, however, that when the state space grows, the cost of the numerical root finding for each point in the state space can become costly. We will see an example of this in our application in Section 4 below. Furthermore, if we are willing to do all these pre-computations, we might as well implement the iEGM.

Table 3: Computational Performance. Precomputed Versions.

	Unrestricted model			
	time (s)	rel.	MAD(C)	Comp(A)
VFI, pre E	13.550	0.771	0.004	0.052
EGM (num), pre E	63.454	3.611	0.001	0.000
VFI (simul.), pre E	2.899	0.165	0.004	0.051
VFI, pre E + intra	0.567	0.032	0.004	0.052
EGM (num), pre E + intra	0.199	0.011	0.001	0.000

Notes: The Table reports timings and numerical accuracy of different solution methods and variation where model elements are precomputed. For methods with "pre E", the expected continuation value – or marginal continuation value for EGM-type methods – is precomputed. For methods with "intra", the intra-temporal problem is precomputed. Timings are reported in seconds and relative to the VFI benchmark in Table 2. Accuracy is measured in terms of 1) MAD(C): Mean absolute difference of the consumption profile to the “true” solution (VFI on very dense grids), and 2) Comp(A): initial wealth compensation to ensure indifference to the “true” solution. All statistics are based on 50 Monte Carlo runs, where we solve (and store timing) and simulate 10,000 households for 20 periods.

3 General Model Class and the iEGM

The model class and required conditions for the applicability of the EGM can be found in Druedahl and Jørgensen (2017). That class includes multidimensional models, allowing for several continuous and discrete choices along with potentially binding constraints. While the iEGM applies to this general class of models, the notation and exposition quickly becomes quite involved due to the unlimited continuous choices and constraints.[9] For completeness, we include here a general description of an empirically relevant sub-class of models and detail the required assumptions for the EGM and thus the iEGM to be applicable.

We will focus here on two-agent dynamic bargaining models in which only one of the *intertemporal* choices, referred to throughout as total consumption, C_t, is continuous. We allow for an unlimited number of discrete choices and put no restrictions on the *intratemporal* choices. Importantly, this sub-class includes non-convex elements (such as discrete choices), as allowed by existing extensions of the EGM, proposed and studied in Hintermaier and Koeniger (2010); Fella (2014); Iskhakov, Jørgensen, Rust and Schjerning (2017); Druedahl and Jørgensen (2017). We will build heavily on these existing studies

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[9] The convergence property of the EGM, and thus the iEGM, has been studied in e.g. Iskhakov, Jørgensen, Rust and Schjerning (2017) and Druedahl and Jørgensen (2017). See also Rendahl (2015) for convergence of Euler equation based methods.

and apply their results to this setting.[10]

Our general model formulation encompasses the so-called limited commitment model, originating from work by, among others, Coate and Ravallion (1993); Kocherlakota (1996); Ligon, Thomas and Worrall (2002); Ligon (2002) and Mazzocco (2007). Our exposition above follows that of e.g. Voena (2015) and Bronson (2015), based on the recursive formulation provided by Marcet and Marimon (2019). We include in the Supplemental Material A an application of the results of Marcet and Marimon (2019) to the two-member household models that we focus on here.

This sub-class is widely used in the literature (see e.g. Voena, 2015; Bronson, Haanwinckel and Mazzocco, 2025; Low, Meghir, Pistaferri and Voena, 2025) and allows us to focus our notation and exposition on the implementation of the EGM (and the iEGM) to this model framework, while keeping the notation manageable. It also allows us to clearly define the construction of the intra- and intertemporal elements, which is a central part of the application of the iEGM to this class of models. In our empirical application in Section 4 below, we provide a concrete example within this class and connect it to the general notation provided here.

3.1 Notation and Setup

An individual lives for T periods and can, in a given period, be either single or in a couple. At the beginning of period t, a couple is characterized by resources M_t, a vector of other states $\mathcal{S}_t$, and the beginning-of-period bargaining weight μ_{t-1}, which acts as a co-state. We will refer to μ_{t-1} as the *pre-bargaining* weight throughout to stress that the bargaining weight may evolve endogenously within period t according to a researcher-chosen bargaining protocol $\mu_t^\star(M_t, \mathcal{S}_t, \mu_{t-1})$. As discussed in e.g. Theloudis, Velilla, Chiappori, Giménez-Nadal and Molina (2022), the bargaining protocol depends on the degree to which couples are assumed to be able to commit to future allocations. The bargaining protocol provides the *post-bargaining* weight

$$\mu_t = \mu_t^\star(M_t, \mathcal{S}_t, \mu_{t-1}), \tag{22}$$

which is the weight used to determine optimal choices in period t and is subsequently carried on to period $t+1$. When discussing the numerical solution below, we will return to this point as it proves beneficial to articulate the household problem conditional on the *post-bargaining* weight, rather than the *pre-bargaining* weight.

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[10] For further background on the EGM we refer the reader to the review by Fella (2025).

We focus on the problem of a couple here since that is often the most involved part of the model and the part that is less likely solvable using the analytical EGM. We denote the value associated with partnership dissolution (i.e. transitioning from marriage to singlehood) as

$$V_{j,t}^{m\to s}(M_t, \mathcal{S}_t, \mu_{t-1}). \tag{23}$$

In section 4 we provide a detailed specification of this value in the context of the application. The value associated with starting period t as a couple is then

$$\begin{aligned} V_{j,t}^{m}(M_t, \mathcal{S}_t, \mu_{t-1}) &= (1 - D_t^\star) V_{j,t}^{m\to m}(M_t, \mathcal{S}_t, \mu_t) + D_t^\star V_{j,t}^{m\to s}(M_t, \mathcal{S}_t, \mu_{t-1}) \\ \text{s.t. } \mu_t &= \mu_t^\star(M_t, \mathcal{S}_t, \mu_{t-1}) \end{aligned} \tag{24}$$

where $V_{j,t}^{m\to m}(M_t, \mathcal{S}_t, \mu_t)$ is the value of remaining married *post-bargaining* and $D_t^\star$ denotes the endogenous choice of divorcing, which follows from the bargaining protocol.

We restrict attention to models with a single asset class A_t and a single continuous choice of total consumption C_t. Accordingly, we explicitly single out the beginning-of-period resources, M_t, as a state variable throughout and let $\mathcal{S}_t$ denote any other state variables of the problem. In addition to the consumption/savings decision, we allow for an arbitrary number of discrete choices $\mathcal{D}_t$ with potential dynamic implications, and an arbitrary number of continuous intra-temporal choices $\mathcal{C}_t$. We focus on this case to streamline notation, but the iEGM extends to more general settings with multiple continuous intertemporal choices.

We define the value of member j from remaining married post-bargaining as

$$V_{j,t}^{m\to m}(M_t, \mathcal{S}_t, \mu_t) = \tilde{u}_j(C_t^\star, \mathcal{D}_t^\star, \mathcal{C}_t^\star, \mathcal{S}_t) + \beta \mathbb{E}_t[V_{j,t+1}^{m}(M_{t+1}, \mathcal{S}_{t+1}, \mu_t)] \tag{25}$$

where the optimal choices are found by maximizing the *post-bargaining* weighted individual values,

$$\begin{aligned} &C_t^\star(M_t, \mathcal{S}_t, \mu_t), \mathcal{D}_t^\star(M_t, \mathcal{S}_t, \mu_t), \mathcal{C}_t^\star(M_t, \mathcal{S}_t, \mu_t) = \\ &\quad \max_{C_t, \mathcal{D}_t, \mathcal{C}_t} \mu_t \tilde{u}_1(C_t, \mathcal{D}_t, \mathcal{C}_t, \mathcal{S}_t) + (1 - \mu_t)\tilde{u}_2(C_t, \mathcal{D}_t, \mathcal{C}_t, \mathcal{S}_t) \\ &\qquad + \beta \mathbb{E}_t[\mu_t V_{1,t+1}^{m}(M_{t+1}, \mathcal{S}_{t+1}, \mu_t) + (1 - \mu_t) V_{2,t+1}^{m}(M_{t+1}, \mathcal{S}_{t+1}, \mu_t)] \end{aligned} \tag{26}$$

subject to a host of constraints that we will clearly define below.

To apply the ideas of the EGM to solve this class of models, we will now reformulate the household problem in (26) into *i)* an intra-temporal problem for a subset of choices that has no intertemporal effects, denoted $\mathcal{C}_t$, throughout, and *ii)* an intertemporal problem that then deals with all choices with dynamic implications. The key reason for this is that the intertemporal problem (where total consumption and savings are determined) then will take a form quite similar to the type of models where the EGM has been found to perform very well (see e.g. Jørgensen, 2013; Fella, 2014; White, 2015). The challenge, however, is that the inverse marginal household utility will generally not be analytically invertible in this class of models. Fortunately, the iEGM can then be applied to reap the time and accuracy benefits of the EGM.

3.1.1 The intra-temporal problem and the household utility function.

In order to define the intra-temporal part of the problem in (26), we introduce two new vector variables. First, we let $\mathcal{Q}_t \subseteq (\mathcal{S}_t, \mathcal{D}_t)$ denote a vector of relevant conditioning variables in excess of total consumption, C_t, that are sufficient to make the intra-temporal problem independent of the dynamic problem. These variables are needed to ensure that the intra-temporal problem does not depend on future states, such that we can solve the intra-temporal problem separately from the intertemporal problem. In our simple example above, total consumption, $C_t = c_t + c_{w,t} + c_{m,t}$, was the only variable that we needed to condition on in order for the consumption allocation $c_t, c_{w,t}, c_{m,t}$ to have no intertemporal impact. In turn, $\mathcal{Q}_t$ is empty in that example. In more general settings, additional variables may be required. For instance, in our application in Section 4, we include discrete labor supply ($\mathcal{D}_t$ in the application) of both members in $\mathcal{Q}_t$, since labor supply affects future states through human capital accumulation and must therefore be conditioned on to make the intra-temporal problem static.

Second, we define $\mathcal{U}_t$ as a vector of variables that are important for the instantaneous utility over and above the intra-temporal choices, $\mathcal{C}_t$. In our example above, the only thing that matters for utility is the intra-temporal decisions, $\mathcal{C}_t = (c_t, c_{w,t}, c_{m,t})$, and $\mathcal{U}_t$ is thus empty in that example. In the application below, $\mathcal{U}_t$ includes a home produced good and time spent on leisure of both partners.

The function $f_{\mathcal{U}}(C_t, \mathcal{C}_t, \mathcal{Q}_t)$ defines utility-relevant objects, $\mathcal{U}_t$ as a function of total consumption, intra-temporal choices and conditioning variables. This allows us to rewrite individual utility in eq. (26) as

$$\tilde{u}_j(C_t, \mathcal{D}_t, \mathcal{C}_t, \mathcal{S}_t) = u_j(\mathcal{C}_t, \mathcal{U}_t), \quad \text{with} \quad \mathcal{U}_t = f_{\mathcal{U}}(C_t, \mathcal{C}_t, \mathcal{Q}_t)$$

This reformulation is without loss of generality, as we allow $f_{\mathcal{U}}(C_t, \mathcal{C}_t, \mathcal{Q}_t)$ to be the identity function and for $\mathcal{Q}_t = (\mathcal{S}_t, \mathcal{D}_t)$. By expressing all utility relevant objects as functions of $\mathcal{C}_t$, C_t, and $\mathcal{Q}_t$, we separate intra-temporal choices from the remaining part of the problem.

Conditional on $(C_t, \mathcal{Q}_t, \mu_t)$, the only optimization left is over $\mathcal{C}_t$. In our application below, $f_{\mathcal{U}}(C_t, \mathcal{C}_t, \mathcal{Q}_t)$ includes, among other components, a public good production function taking as inputs market purchased goods and time allocated to home production, similar to e.g. Bronson, Haanwinckel and Mazzocco (2025).

Given this formulation, the intra-temporal problem of a couple conditional on remaining together and conditional on a post-bargaining weight of μ_t can then be formulated as

$$\begin{aligned} U(C_t, \mu_t, \mathcal{Q}_t) = \max_{\mathcal{C}_t} \mu_t u_1(\mathcal{C}_t, \mathcal{U}_t) + (1-\mu_t) u_2(\mathcal{C}_t, \mathcal{U}_t) \\ \text{s. t.} \\ \mathcal{U}_t = f_{\mathcal{U}}(C_t, \mathcal{C}_t, \mathcal{Q}_t) \end{aligned} \tag{27}$$

defining also what we refer to as the *household utility function*, $U(C_t, \mu_t, \mathcal{Q}_t)$. As discussed above, this will in general not be known in closed form nor will its derivative. The key point of the iEGM is to realize that we can precompute these key objects on (potentially dense) evaluation nodes over $(C_t, \mu_t, \mathcal{Q}_t)$ and interpolate these when solving the intertemporal problem. This is what we turn to now.

3.1.2 The intertemporal problem of a couple.

With the household utility function in (27) we can formulate the *post-bargaining* intertemporal problem of a couple under the assumption that they remain together as,

$$\begin{aligned} & C_t^{\star}(M_t, \mathcal{S}_t, \mu_t), \mathcal{D}_t^{\star}(M_t, \mathcal{S}_t, \mu_t) = \\ & \arg\max_{C_t, \mathcal{D}_t} U(C_t, \mu_t, \mathcal{Q}_t) \\ & + \beta \mathbb{E}_t[\mu_t V_{1,t+1}^m(M_{t+1}, \mathcal{S}_{t+1}, \mu_t) + (1-\mu_t) V_{2,t+1}^m(M_{t+1}, \mathcal{S}_{t+1}, \mu_t)] \\ & \text{s. t.} \end{aligned} \tag{28}$$

$$\mathcal{Q}_t = f_{\mathcal{Q}_t}(A_t, \mathcal{D}_t, \mathcal{S}_t) \tag{29}$$

$$M_{t+1} \sim f_{M_t}(A_t, \mathcal{D}_t, \mathcal{S}_t) \tag{30}$$

$$\mathcal{S}_{t+1} \sim f_{\mathcal{S}_t}(A_t, \mathcal{D}_t, \mathcal{S}_t) \tag{31}$$

$$A_t = f_{A_t}(C_t, \mathcal{D}_t, M_t, \mathcal{S}_t) \tag{32}$$

where $\mathcal{D}_t \in \{d_1, d_2, \ldots, d_D\}$ are discrete choices over D alternatives with potential dynamic implications, and $f_{\mathcal{Q}_t}(A_t, \mathcal{D}_t, \mathcal{S}_t)$ is a function returning the vector of conditioning variables in $\mathcal{Q}_t$ as a function of all state and intertemporal choices. The state transitions are governed by $f_{M_t}(A_t, \mathcal{D}_t, \mathcal{S}_t)$ and $f_{\mathcal{S}_t}(A_t, \mathcal{D}_t, \mathcal{S}_t)$.

The applicability of the EGM and thus the iEGM rests on the assumption that end-of-period savings, A_t in equation (32), is a sufficient statistic for M_t and C_t (Assumption 1). In our example above, we have $f_{A_t}(C_t, \mathcal{D}_t, M_t, \mathcal{S}_t) = M_t - C_t$. In addition, we require the marginal utility function $U'(C_t, \mu_t, \mathcal{Q}_t)$ to be invertible in C_t (Assumption 2), and the budget function $f_{A_t}(C_t, \mathcal{D}_t, M_t, \mathcal{S}_t)$ to be invertible in M_t (Assumption 3). Finally, the first order condition (FOC) of the objective function in eq. (28) should be at least necessary (Assumption 4). These assumptions are standard in the existing literature on the EGM (see e.g. Iskhakov, Jørgensen, Rust and Schjerning, 2017 and Druedahl and Jørgensen, 2017). We discuss the assumptions and their importance below, when outlining how to solve the model using the iEGM.

Assumption 1 (Sufficient Statistic)**.** *The end-of-period level of assets, A_t, is a sufficient statistic for beginning-of-period resources, M_t, and total consumption, C_t.*

This means that the set of conditioning variables, and all state transitions in eqs. (29)–(31) can depend on A_t but neither M_t nor C_t. Further, denoting the relationship between the post-decision state and beginning-of-period states and choices as $A_t = f_{A_t}(C_t, \mathcal{D}_t, M_t, \mathcal{S}_t)$ we must have that $f_{A_t}(C_t, \mathcal{D}_t, M_t, \mathcal{S}_t)$ is differentiable and that $\frac{\partial^2 f_{A_t}(C_t, \mathcal{D}_t, M_t, \mathcal{S}_t)}{\partial^2 C_t} = \frac{\partial^2 f_{A_t}(C_t, \mathcal{D}_t, M_t, \mathcal{S}_t)}{\partial M_t \partial C_t} = 0$.

Assumption 2 (Invertible Marginal Utility)**.** *The marginal household utility, $U'(C, \mu, \mathcal{Q})$, is a bijective function in C, conditional on μ and $\mathcal{Q}$, such that the inverse, $U'^{-1}(W, \mu, \mathcal{Q})$, exists.*

Assumption 3 (Monotonic Savings)**.** *End-of-period savings, $A_t = f_{A_t}(C_t, \mathcal{D}_t, M_t, \mathcal{S}_t)$, must be monotonic in beginning-of-period resources, M_t, such that it can be inverted to get $M_t = f_{A_t}^{-1}(C_t, \mathcal{D}_t, A_t, \mathcal{S}_t)$.*

Assumption 4 (The FOC is at Least Necessary)**.** *The objective function in eq. (5) is differentiable w.r.t C_t and the first order condition is at least necessary for an interior optimal total consumption, C_t.*

3.2 Solving the Intertemporal Problem with iEGM

The solution of the household bargaining model contains two main steps.[11] First, conditional on the bargaining outcome μ_t, we solve for optimal consumption C_t and discrete choices $\mathcal{D}_t$. In this stage, we can disregard any bargaining in period t and instead condition directly on the *post-bargaining* weight μ_t. This conditional problem can be solved

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[11] Shephard (2019) similarly splits the formulation and solution into a post-bargaining solution where μ_t is fixed and a subsequent update of the bargaining state.

using any suitable solution method, including standard VFI. In the following section, we show how to implement the iEGM to solve this problem efficiently.

Second, we apply a bargaining protocol to determine the link between the beginning-of-period bargaining weight, μ_{t-1}, and the post-bargaining weight, μ_t, which yields the solution to the model. For instance, with limited commitment, this involves evaluating, for each state in $(M_t, \mathcal{S}_t)$, whether the household will either *i)* remain at their *pre-bargaining* weight, *ii)* adjust the bargaining weight to a new value, or *iii)* divorce. The outcome of this stage determines the relevant bargaining weight and marital status used in the household's decision problem. We describe in detail how to implement the limited commitment bargaining protocol below after first outlining how the *post-bargaining* solution can be uncovered using the iEGM.

3.2.1 Solving the Intertemporal Problem Conditional on Bargaining Outcome, μ_t.

In this section, we solve the household problem conditional on *post-bargaining* weight μ_t. Hence, we focus on the stage where all states relevant for the choice of consumption C_t and discrete choices $\mathcal{D}_t$ are pre-determined, and bargaining has already happened such that μ_t is fixed. Focusing on post-bargaining allocations is instructive as we can separate how optimal allocations are found from the implementation of the bargaining protocol. A similar strategy is employed in e.g. Shephard (2019).

Denote the discounted expected continuation value as

$$w_t(A_t, \mathcal{D}_t, \mathcal{S}_t, \mu_t) = \beta \mathbb{E}_t[\mu_t V^m_{1,t+1}(M_{t+1}, \mathcal{S}_{t+1}, \mu_t) + (1-\mu_t) V^m_{2,t+1}(M_{t+1}, \mathcal{S}_{t+1}, \mu_t)]$$

where we use that A_t is a sufficient statistic for M_t and C_t (Assumption 1) and we explicitly condition on a post-bargaining bargaining weight, μ_t, and discrete choice $\mathcal{D}_t$. The first order condition (FOC) for optimal consumption is (due to Assumption 4)

$$\begin{aligned} U'(C_t, \mu_t, \mathcal{Q}_t) &= -\frac{dw_t(A_t, \mathcal{D}_t, \mathcal{S}_t, \mu_t)}{dA_t} \frac{\partial f_{A_t}(C_t, \mathcal{D}_t, M_t, \mathcal{S}_t)}{\partial C_t} \\ &= W_t(A_t, \mathcal{S}_t, \mathcal{D}_t, \mu_t) \end{aligned} \tag{33}$$

where $U'(C_t, \mu_t, \mathcal{Q}_t) = \frac{\partial}{\partial C_t} U(C_t, \mu_t, \mathcal{Q}_t)$ is the marginal household utility.

The key point of the iEGM is to realize that W_t is a sufficient statistic in the FOC and that we can back out the inverse marginal utility i.e. optimal consumption C_t, as a function of W_t from the optimality condition. Concretely, it does not matter for optimal consumption whether a value of the expected marginal value of wealth originates due to any of the states, choices or preferences of the agents. All that matters for optimal consumption is the value of W_t. In turn, we can write optimal consumption as a function of the marginal value of wealth

$$C(W, \mu, \mathcal{Q}) = \{C : U'(C, \mu, \mathcal{Q}) - W = 0\}. \tag{34}$$

where we remove time subscripts to highlight that the intertemporal consumption allocation is *independent* of everything not included in $\mathcal{Q}$ (and μ) once we know W. If e.g. age enters the marginal household utility, then that information should be included in $\mathcal{Q}$. The existence of a unique solution to (34) is a requirement for the EGM and is satisfied by assumption 2 (invertability of the household utility function).

Implementing the iEGM first involves constructing an interpolator, $C^{interp}(W, \mu, \mathcal{Q})$, over W and μ (and any continuous elements in $\mathcal{Q}$) by precomputing the solution to (34) on grids over these variables. This can be done by constructing a grid of marginal values of wealth, $\overrightarrow{W}$ and solve (34) numerically for all these points to get a grid over optimal intertemporal consumption, $\overrightarrow{C}$. Since this is done *once* before solving the model, the use of numerical root finding might not be too computationally costly. Alternatively, the construction can be done without any root finding by "flipping the axis", as discussed in the example in Section 2. This would entail constructing a grid over consumption, $\overrightarrow{C}$, and inserting that into the marginal household utility, $U'(\bullet)$, to get corresponding values of the expected marginal wealth, $\overrightarrow{W}$, because the two should be identical at interior solutions to the FOC. This also delivers a combination of grids that can be used to construct the interpolator.[12] Independent of which method is used, this is done for each combination of values in grids over the other conditioning variables, $\overrightarrow{\mu}$ and $\overrightarrow{\mathcal{Q}}$.

The second element of the iEGM is then to use this interpolator when solving the model by backwards induction, to approximate optimal intertemporal consumption for a given set of states, discrete choices, and end-of-period savings, A_t, as

$$\begin{gathered} C_t^{\star}(A_t, \mathcal{D}_t, \mathcal{S}_t, \mu_t) = C^{interp}(W_t(A_t, \mathcal{D}_t, \mathcal{S}_t, \mu_t), \mu_t, \mathcal{Q}_t) \\ \text{s. t.} \\ \mathcal{Q}_t = f_{\mathcal{Q}_t}(A_t, \mathcal{D}_t, \mathcal{S}_t) \end{gathered}$$

where the precomputed interpolator is now evaluated at concrete expected marginal values of wealth arising in the concrete dynamic context of the model.

The optimal consumption can then be thought of as a function of the state variable M_t by inverting the relationship $f_{A_t}(C_t, \mathcal{D}_t, M_t, \mathcal{S}_t)$ such that $M_t = f_{A_t}^{-1}(C_t^{\star}(A_t, \mathcal{D}_t, \mathcal{S}_t, \mu), \mathcal{D}_t, A_t, \mathcal{S}_t)$.

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[12] Note, that if flipping the axis is used to construct the interpolator, the resulting grid points in the W grid will generally be irregular. This means that if e.g. linear interpolation is used, W should be the last dimension of the interpolator.

The invertability of this equation follows from Assumption 3. Most commonly, and also in our simple example above, we had $A_t = f_{A_t}(C_t, \mathcal{D}_t, M_t, \mathcal{S}_t) = M_t - C_t$ and thus $M_t = f_{A_t}^{-1}(C_t, \mathcal{D}_t, A_t, \mathcal{S}_t) = A_t + C_t$. We now have the object of interest, $C_t^\star(M_t, \mathcal{D}_t, \mathcal{S}_t, \mu)$.

Finally, we can find the discrete choices as

$$\mathcal{D}_t^\star(M_t, \mathcal{S}_t, \mu_t) = \arg\max_{\mathcal{D}_t} v_t(M_t, \mathcal{S}_t, \mu_t | \mathcal{D}_t)$$

where

$$\begin{aligned} v_t(M_t, \mathcal{S}_t, \mu_t|\mathcal{D}_t) &= U(C_t^\star, \mu_t, \mathcal{Q}_t^\star) + w_t(A_t^\star, \mathcal{D}_t, \mathcal{S}_t, \mu_t) \\ &\text{s. t.} \\ A_t^\star &= f_{A_t}(C_t^\star, \mathcal{D}_t, M_t, \mathcal{S}_t) \\ \mathcal{Q}_t^\star &= f_{\mathcal{Q}_t}(A_t^\star, \mathcal{D}_t, \mathcal{S}_t) \end{aligned}$$

and we get the final solution $C_t^\star(M_t, \mathcal{S}_t, \mu_t)$ and we can find the intertemporal choices by inserting optimal consumption in the intertemporal problem (27) to get $\mathcal{C}_t^\star(M_t, \mathcal{S}_t, \mu_t)$. As mentioned, this exposition is conditional on the post-bargaining weight, μ_t. After discussing the differentiability of the FOC, we will use this to implement the bargaining protocol of models of different degrees of commitment, linking the beginning-of-period bargaining weight, μ_{t-1} to μ_t.

Differentiability and Non-Sufficiency of the FOC. The model class outlined here includes several elements that might threaten differentiability of the expected marginal value of wealth in the FOC in eq. (33). First, the expected marginal value of wealth on the right hand side of eq. (33) includes the derivative of each member's value of entering a period as married with a given bargaining weight, μ_t,[13]

$$\frac{dw_t}{dA_t} = \beta \mathbb{E}_t \left[\mu_t \frac{d}{dA_t} V_{1,t+1}^m(M_{t+1}, \mathcal{S}_{t+1}, \mu_t) + (1-\mu_t) \frac{d}{dA_t} V_{2,t+1}^m(M_{t+1}, \mathcal{S}_{t+1}, \mu_t) \right]. \tag{35}$$

From eq. (24) we see that there are multiple channels through which a marginal change in wealth affects individual values of being married. Concretely, for continuously married

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[13] Since we condition on μ_t the derivative of μ_t w.r.t. savings does not enter this expression. As we describe below, the derivative of the next-period bargaining weight enters, though.

individuals $\frac{dV^{m}_{j,t+1}}{dA_t}$ will generally include, among other things,

$$\frac{dV^{m\to m}_{j,t+1}}{dA_t} = \frac{\partial V^{m\to m}_{j,t+1}}{\partial M_{t+1}}\frac{\partial M_{t+1}}{\partial A_t} + \frac{\partial V^{m\to m}_{j,t+1}}{\partial \mathcal{S}_{t+1}}\frac{\partial \mathcal{S}_{t+1}}{\partial A_t} + \frac{\partial V^{m\to m}_{j,t+1}}{\partial \mu_{t+1}}\left(\frac{\partial \mu_{t+1}}{\partial M_{t+1}}\frac{\partial M_{t+1}}{\partial A_t} + \frac{\partial \mu_{t+1}}{\partial \mathcal{S}_{t+1}}\frac{\partial \mathcal{S}_{t+1}}{\partial A_t}\right) \tag{36}$$

where the third term captures the effect on the endogenous bargaining weight through the bargaining protocol in eq. (22). As we will see below, the bargaining protocol might have kink-points.

Second, the discounted expected continuation value in eq. (28) can have kinks and non-concave regions, arising from future discrete relationship choices and the discrete choices in $\mathcal{D}_t$. Iskhakov, Jørgensen, Rust and Schjerning (2017) provides a detailed discussion on the consequences of non-convex elements, such as discrete choices.

While these features pose a threat to differentiability, Clausen and Strub (2020) provide results that show that problems of this form are often still differentiable.[14] The results of Clausen and Strub (2020) has already been used extensively in the discrete-continuous generalizations of the EGM (see e.g. Fella, 2014; Iskhakov, Jørgensen, Rust and Schjerning, 2017; Druedahl and Jørgensen, 2017), and are likewise relevant for the iEGM.

The before mentioned kinks and non-concave regions of the expected continuation value renders the FOC only necessary but not sufficient. This means that several values of consumption for a given level of resources, M_t, might solve the FOC while only one level is the optimal.[15] Fella (2014); Iskhakov, Jørgensen, Rust and Schjerning (2017); Druedahl and Jørgensen (2017) have developed so-called upper-envelope algorithms to remove all non-optimal solutions to the FOC when implementing the EGM. These algorithms are directly applicable to the iEGM too. In our application below, we use the approach proposed in Druedahl and Jørgensen (2017). See their discussion around their Figure 1 for a description of the algorithm in one dimension (as is the case in our setting here).

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[14] In principle, each model can be checked for differentiability using their "Differentiable Sandwich Lemma" and "Reverse Calculus". However, the type of problem outlined herein will typically satisfy the conditions for differentiability. See also the supplemental material of Druedahl and Jørgensen (2017).

[15] The EGM (and iEGM) maps out all these candidate solutions because there is a unique level of consumption for each level of post-decision savings, A_t. Importantly, this is a feature of the model class, and approaches such as VFI suffers from the same problem of local maxima. To deal with this in VFI, multistart or other "global" searches might have to be applied. The iEGM finds all the candidate values without any root-finding but we then afterwards have to locale the global max, which the "upper envelope" algorithms do quite quickly.

3.2.2 Implementing the Bargaining Protocol.

With the solution approach above, we can now implement the bargaining protocol. This depends on whether household members are assumed to have full, limited or no commitment regarding future allocations. While this part does not depend on whether the iEGM or any other solution approach is used, for completeness, we describe all three below. We put special emphasis on the limited commitment case as that is the most involved case.

To this end, we will denote the individual surplus from remaining a couple with post-bargaining weight μ as $S_{j,t}(M_t, \mathcal{S}_t, \mu)$. If singlehood is the outside option (threat point) and the value associated with this transition is denoted $V_{j,t}^{m\to s}(M_t, \mathcal{S}_t, \mu)$, the surplus after bargaining would be

$$S_{j,t}(M_t, \mathcal{S}_t, \mu) = u_j(\mathcal{C}_t^\star, \mathcal{U}_t^\star) + \beta \mathbb{E}_t[V_{j,t+1}^m(M_{t+1}, \mathcal{S}_{t+1}, \mu)] - V_{j,t}^{m\to s}(M_t, \mathcal{S}_t, \mu) \tag{37}$$

where $\mathcal{U}_t^\star = f_{\mathcal{U}_t}(C_t^\star, \mathcal{C}_t^\star, \mathcal{Q}_t^\star)$ and, importantly, optimal choices depend on the value of μ.

Full Commitment. Under full commitment, household members commit to a plan for μ_t upon forming the marriage and never renegotiate. Here, μ_t can remain fixed at some value such that $\mu_t = \mu_{t-1}$ for all t, or it can evolve according to some known process, see e.g. Bruze, Svarer and Weiss (2012). The key implication, following Theloudis, Velilla, Chiappori, Giménez-Nadal and Molina (2022), is that bargaining power can only depend on information available at the time of relationship formation. If the couple forms at time τ then $\mu_t = \mu_t^{FC}(M_\tau, \mathcal{S}_\tau)$ for all $t \geq \tau$. Conditional on $(M_\tau, \mathcal{S}_\tau)$, subsequent states $(M_t, \mathcal{S}_t)$ have no effect on μ_t.

In this situation, the relevant post-bargaining solution can simply be used. In our simple example above, we just let this be a fixed parameter and imagine only one level of bargaining power, μ.

No Commitment. If household members have no commitment, they renegotiate the allocation in every period. Hence, post-bargaining allocations depend on the current state, but are independent of the previous allocations, $\mu_t = \mu_t^\star(M_t, \mathcal{S}_t)$, see Theloudis, Velilla, Chiappori, Giménez-Nadal and Molina (2022). This function can be determined by e.g. assuming cooperative Nash bargaining, such that

$$\mu_t^\star(M_t, \mathcal{S}_t) = \arg\min_{\mu_t} S_{1,t}(M_t, \mathcal{S}_t, \mu_t)^\omega S_{2,t}(M_t, \mathcal{S}_t, \mu_t)^{1-\omega} \tag{38}$$

where ω is the weight put on member 1. With the post-bargaining weight determined, the relevant solution from above can readily be used.

Limited Commitment. Under limited commitment, household members commit to the current allocation only up to the point where participation constraints bind. Hence, the bargaining weight depends both on the current allocation AND the current state, and we need to uncover the relationship $\mu_t = \mu_t^\star(M_t, \mathcal{S}_t, \mu_{t-1})$ by checking the forward-looking participation constraints. To this end, we use that the bargaining updating rule has a well-known structure (see e.g. Mazzocco, 2007; Voena, 2015; Bronson, 2015; Bronson, Haanwinckel and Mazzocco, 2025).

Let $\tilde{\mu}_{j,t} = \tilde{\mu}_{j,t}(M_t, \mathcal{S}_t)$ be the level of bargaining power that makes member j indifferent between the outside option and being married, such that $S_{j,t}(M_t, \mathcal{S}_t, \tilde{\mu}_{j,t}) = 0$. Intuitively, this means that for all pre-bargaining weights that satisfy $\tilde{\mu}_{1,t} \leq \mu_{t-1} \leq \tilde{\mu}_{2,t}$, we know that $\mu_t = \mu_{t-1}$ as neither of the two forward looking participation constraints are binding. This means that the post-bargaining solution found by, e.g., the iEGM above, can be directly used for these values of μ_{t-1}. For pre-bargaining weights not satisfying this, bargaining takes place, as we describe below.

Formally, the bargaining power updating rule can then be formulated as (see e.g. Shephard, 2019)

$$\mu_t^\star(M_t, \mathcal{S}_t, \mu_{t-1}) = \begin{cases} \mu_{t-1} & \text{if } S_{1,t}(M_t, \mathcal{S}_t, \mu_{t-1}) \geq 0 \text{ and } S_{2,t}(M_t, \mathcal{S}_t, \mu_{t-1}) \geq 0 \\ \tilde{\mu}_{1,t} & \text{if } S_{1,t}(M_t, \mathcal{S}_t, \mu_{t-1}) < 0 \text{ and } S_{2,t}(M_t, \mathcal{S}_t, \tilde{\mu}_{1,t}) \geq 0 \\ \tilde{\mu}_{2,t} & \text{if } S_{1,t}(M_t, \mathcal{S}_t, \tilde{\mu}_{2,t}) \geq 0 \text{ and } S_{2,t}(M_t, \mathcal{S}_t, \mu_{t-1}) < 0 \\ \emptyset & \text{else} \end{cases} \tag{39}$$

where the first case represents no updating, the second case represents the case in which member 1 gains more bargaining power, the third case represents the case in which member 2 gains bargaining power, and the fourth case represents divorce where no bargaining solution is possible.

The problem has now been reduced to a few steps. For completeness, we include a novel algorithm in the Supplemental Material B that can be used to implement the following procedure. In general, there are three relevant situations depending on the marital surplus:[16]

1. If both spouses have a positive marital surplus with the beginning-of-period bargaining weight, μ_{t-1}, i.e. $S_{j,t}(M_t, \mathcal{S}_t, \mu_{t-1}) \geq 0$ for $j \in \{1,2\}$, they remain mar-

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[16] See Chiappori and Mazzocco (2017, Figure 1) for a graphical illustration of the bargaining process.

ried and $D_t^\star = 0$. Furthermore, the bargaining weight is not updated and $\mu_t = \mu_{t-1}$. Thus, the value of starting as married and remaining married is identical, $V_{j,t}^m(M_t, \mathcal{S}_t, \mu_{t-1}) = V_{j,t}^{m\to m}(M_t, \mathcal{S}_t, \mu_{t-1})$.

2. If none of the household members have positive marital surpluses, $S_{j,t}(M_t, \mathcal{S}_t, \mu_{t-1}) < 0$ for $j \in \{1, 2\}$, the couple divorces and $D_t^\star = 1$. The value of entering the period as married is then $V_{j,t}^m(M_t, \mathcal{S}_t, \mu_{t-1}) = V_{j,t}^{m\to s}(M_t, \mathcal{S}_t, \mu_{t-1})$.

3. If one of the spouses, say $j = 1$, has a negative marital surplus while the other spouse has a positive surplus, the couple renegotiates the bargaining power. Let $\tilde{\mu}_{1,t}$ be the level of bargaining power that puts $S_{1,t}(\mathcal{S}_t, \tilde{\mu}_{1,t}) = 0$. If the other member has a positive surplus with this updated bargaining power, $S_{2,t}(\mathcal{S}_t, \tilde{\mu}_{1,t}) \geq 0$, the couple remains married, $D_t^\star = 0$. The bargaining weight is then updated to $\mu_t = \mu_t^\star = \tilde{\mu}_{1,t}$, and the value of entering and remaining married is $V_{j,t}^m(M_t, \mathcal{S}_t, \mu_{t-1}) = V_{j,t}^{m\to m}(M_t, \mathcal{S}_t, \tilde{\mu}_{1,t})$.[17] If, on the other hand, member 2 prefers the outside option with this alternative allocation, $S_{2,t}(\mathcal{S}_t, \tilde{\mu}_{1,t}) < 0$, there is no feasible bargaining power allocation that can sustain marriage and the couple divorces with similar consequences as above. The process is symmetric if it is partner 2 that is unsatisfied with the initial bargaining weight.

4 Application: Labor Productivity Shocks and Consumption Inequality

In this section, we apply the iEGM to analyze how individual labor productivity shocks affect consumption inequality. To this end, we formulate a dual-earner limited commitment model of labor supply, heterogeneous skill-types, endogenous wages through human capital accumulation, home production, and dynamic consumption and savings allocations. We calibrate the model to the US economy and conduct counterfactual simulations and impulse-response analyses of shocks to individual productivity and wages.

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[17] Ligon, Thomas and Worrall (2002) show that this updating scheme is optimal. The intuition comes from the min-max saddle-point problem in the Supplementary Material. The shadow price, $\gamma_{1,t}$, should be chosen as the *lowest* value such that the forward-looking participation constraint is satisfied. This corresponds to the lowest value of μ (since that is the weight on member 1) that puts $S_{1,t}(\mathcal{S}_t, \mu) = 0$.

4.1 Model

In the model, individuals choose how to allocate their time towards paid market work, $l_{j,t}$, home production, $h_{j,t}$, and leisure, $f_{j,t}$, subject to the available time constraint

$$\overline{T} = l_{j,t} + h_{j,t} + f_{j,t}, \tag{40}$$

where we normalize $\overline{T}$ to 1. Labor market work can take three values; no market work, part time market work, and full time market work, while home production and leisure can take values in $[0, \overline{T}]$ subject to the time constraint.[18]

Together with hours allocated to home production, $h_{w,t}, h_{m,t}$, households choose how much to spend on market purchased goods, c_t. These inputs together produce a public good, Q_t. Couples also choose how to allocate resources into private consumption, $c_{w,t}, c_{m,t}$, and savings, A_t, subject to an intra-temporal budget constraint, described below.

Each member accumulates human capital, $K_{j,t}$, from labor market work, which in turn affects their market wages, $w_{j,t}$, which also depends on their individual-specific time-invariant skill-type, denoted e_j. Couples experience random match-quality shocks, ψ_t. In turn, the state variables of couples are $\mathcal{S}_t = (K_{w,t}, K_{m,t}, e_w, e_m, \psi_t)$, in excess of beginning-of-period wealth, A_{t-1} and the beginning-of-period bargaining power, μ_{t-1}. We do not allow for remarriage for simplicity. Below, we describe the model components in some detail and relegate the details on the numerical solution to the Supplemental Material C.

4.1.1 Preferences and Home Production Technology

Individual preferences are given by

$$u_j(c_{j,t}, f_{j,t}, Q_t, s_t) = \frac{c_{j,t}^{1-\rho}}{1-\rho} + \phi_j \frac{f_{j,t}^{1-\eta_j}}{1-\eta_j} + \alpha_Q \log(Q_t) + (1 - s_t)\psi_t \tag{41}$$

where $s_t = 1$ indicates singlehood, ρ is the constant relative risk aversion coefficient, ϕ_j is the weight on the utility from leisure relative to private consumption, η_j is the curvature on the amount of leisure, and α_Q is the weight on the partially home produced public good relative to private consumption.

The home-produced good, Q_t, is given by a nested CES production function similar to

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[18] Full-time work is set to $\frac{40}{7 \cdot 16} \approx 0.36$ and part-time work to half of that, corresponding to a 40-hour work week out of 16 waking hours, 7 days per week. The time endowment $\overline{T}$ is thus interpreted as total waking hours.

that used by Bronson, Haanwinckel and Mazzocco (2025),

$$Q_t = (\alpha_c c_t^{\omega} + (1-\alpha_c) H_t^{\omega})^{\frac{1}{\omega}} \tag{42}$$
$$H_t = (\alpha_h h_{w,t}^{\zeta} + (2-\alpha_h) h_{m,t}^{\zeta})^{\frac{1}{\zeta}}$$

where α_c is the weight on market purchased goods relative to home produced goods, ω is the substitutability between these two gods, α_h is the productivity of the women's time input into home production relative to the man's, and ζ is the degree of substitutability between the time inputs of the two household members.

Singles ($s_t = 0$) do not experience a match quality shock, ψ_t and their home production function is

$$Q_{j,t} = (\alpha_c c_t^{\omega} + (1-\alpha_c)(\alpha_j h_{j,t})^{\omega})^{\frac{1}{\omega}}$$

where $\alpha_j = \alpha_h$ if $j = w$ and $\alpha_j = 2 - \alpha_h$ if $j = m$.

4.1.2 Wages and Human Capital Accumulation

Wages follow a Mincer-type process

$$\log w_{j,t} = e_j + \gamma_j K_{j,t} \tag{43}$$

where γ_j is the return to experience and e_j is the individual-specific skill-type, assumed to be drawn from a Normal distribution, i.e. $e_j \sim \mathcal{N}(\lambda_j, \sigma_e^2)$. Experience, which drives individual productivity, follows a stochastic Markov process

$$K_{j,t+1} = [(1-\delta)K_{j,t} + l_{j,t}]\varepsilon_{j,t+1} \tag{44}$$

where δ is the depreciation rate and $\log \varepsilon_{j,t} \sim iid\mathcal{N}(-\frac{1}{2}\sigma_{\varepsilon,j}^2, \sigma_{\varepsilon,j}^2)$ is a mean-one shock to human capital.

4.1.3 Marriage Dynamics and Divorce

In each period couples remain together, they receive a match quality shock, φ_t. Match quality thus evolves according to

$$\psi_{t+1} = \psi_t + \varphi_{t+1} \tag{45}$$

where $\varphi_t \sim iid\mathcal{N}(0, \sigma_{\psi}^2)$ is an i.i.d. couple-specific match quality shock.

Couples bargain with limited commitment and update their bargaining weight, μ_t, according to the law of motion laid out by the limited commitment framework,

$$\mu_t = \mu_t^\star(A_{t-1}, K_{w,t}, K_{m,t}, e_w, e_m, \psi_t, \mu_{t-1}). \tag{46}$$

If a couple divorces they split wealth equally among them.

4.1.4 Budget Constraint

Individuals and couples are subject to a no-borrowing constraint, $A_{w,t}, A_{m,t}, A_t \geq 0$, and the intertemporal budget for couples,

$$A_t = (1+r)A_{t-1} + w_{w,t}l_{w,t} + w_{m,t}l_{m,t} - \tau^m - c_{w,t} - c_{m,t} - c_t, \tag{47}$$

must be satisfied in all periods. Likewise for singles,

$$A_{j,t} = (1+r)A_{j,t-1} + w_{j,t}l_{j,t} - \tau^s - c_{j,t} - c_t. \tag{48}$$

Here, τ^m and τ^s denotes total income taxes for couples and singles, respectively. The tax functions are described in appendix D.

4.1.5 The Value of Remaining a Couple

Here, we describe how we reformulate the key part of the problem into an intra- and intertemporal component. This serves as intuition for how we numerically solve the model using the approaches outlined above. We provide a full description of the remaining components of the model in the Supplemental Material E.

Couples who remain married and enter the period with states $\mathcal{S}_t = (K_{w,t}, K_{m,t}, e_w, e_m, \psi_t)$ along with savings, A_{t-1}, solve the problem

$$\begin{aligned}\max_{\substack{c_{w,t}, c_{m,t}, c_t, A_t \\ l_{w,t}, l_{m,t} \\ h_{w,t}, h_{m,t}, f_{w,t}, f_{m,t}}} \quad & \mu_t \Big(u_w(c_{w,t}, f_{w,t}, Q_t) + \mathbb{E}_t[V^m_{w,t+1}(A_t, K_{w,t+1}, K_{m,t+1}, e_w, e_m, \psi_{t+1}, \mu_t)] \Big) \\ & + (1-\mu_t) \Big(u_m(c_{m,t}, f_{m,t}, Q_t) + \mathbb{E}_t[V^m_{m,t+1}(A_t, K_{w,t+1}, K_{m,t+1}, e_w, e_m, \psi_{t+1}, \mu_t)] \Big) \\ & \text{s. t.} \\ & \text{eq. } (40) - (47)\end{aligned}$$

conditional on the post-bargaining weight of μ_t. This is a high dimensional problem for each combination of state variables. To ease computation and to use the proposed iEGM approach, we reformulate this problem into an intra-temporal part and an intertemporal part below.

The intra-temporal part of the problem can be formulated by noting that conditional on μ, discrete labor supply of each partner, l_w, l_m, and total consumption expenditures, $C = c_t + c_{w,t} + c_{m,t}$, the allocation of time between leisure and home production $h_{w,t}, h_{m,t}, f_{w,t}, f_{m,t}$ and expenditures towards private consumption, $c_{w,t}, c_{m,t}$ and marked purchased goods for home production, c_t has *no* consequences for the discounted expected continuation value in the problem just stated above.

In turn, we can find these objects as functions of the conditioning variables as

$$
\begin{aligned}
&\tilde{c}_w(C, l_w, l_m, \mu), \tilde{c}_m(C, l_w, l_m, \mu), \tilde{h}_w(C, l_w, l_m, \mu), \tilde{h}_m(C, l_w, l_m, \mu) \\
&= \arg \max_{\substack{c_w, c_m \\ h_w, h_m}} \; \mu u_w(c_w, f_w, Q) + (1-\mu) u_m(c_m, f_m, Q) \\
&\quad \text{s. t.} \\
&\text{eq. } (40) - (42) \\
&\quad c = C - c_w - c_m \\
&\quad \overline{T} = l_j + h_j + f_j, \; j \in \{w, m\} \\
&\quad c_j > 0, \; j \in \{w, m\}
\end{aligned}
$$

where we have dropped time-subscripts to clearly indicate that these objects only depend on time through the input variables. We can then define the *household utility function* as a function of the total consumption, labor supply and bargaining power,

$$
\begin{aligned}
U(C_t, l_{w,t}, l_{m,t}, \mu_t) &= \mu_t u_w\left(c_{w,t}, f_{w,t}, Q_t\right) + (1-\mu_t) u_m\left(c_{m,t}, f_{m,t}, Q_t\right) \\
&\quad \text{s. t.} \\
&\text{eq. } (42) \\
&\quad c_t = C_t - \tilde{c}_w(C_t, l_{w,t}, l_{m,t}, \mu_t) - \tilde{c}_m(C_t, l_{w,t}, l_{m,t}, \mu_t) \\
&\quad h_{j,t} = \tilde{h}_j(C_t, l_{w,t}, l_{m,t}, \mu_t), \; j \in \{w, m\} \\
&\quad f_{j,t} = \overline{T} - l_{j,t} - j_{j,t}, \; j \in \{w, m\}.
\end{aligned}
\tag{49}
$$

The intertemporal part of the problem is concerned with optimal labor supply and the allocation of resources into total consumption and savings. Concretely, the intertemporal problem of a couple remaining married wiht post-bargaining weight μ_t is thus

$$
\begin{aligned}
& C_t^\star(A_{t-1}, \mathcal{S}_t, \mu_t), l_{w,t}^\star(A_{t-1}, \mathcal{S}_t, \mu_t), l_{m,t}^\star(A_{t-1}, \mathcal{S}_t, \mu_t) && (50)\\
& = \arg \max_{C_t, l_{w,t}, l_{m,t}} U(C_t, l_{w,t}, l_{m,t}, \mu_t) && (51)\\
& + \beta \mathbb{E}_t[\mu_t V_{w,t+1}^m(A_t, K_{w,t+1}, K_{m,t+1}, e_w, e_m, \psi_{t+1}, \mu_t) \\
& + (1-\mu_t) V_{m,t+1}^m(A_t, K_{w,t+1}, K_{m,t+1}, e_w, e_m, \psi_{t+1}, \mu_t)] \\
& \text{s. t.} \\
& \text{eq. } (40) - (47) \\
& l_{w,t}, l_{m,t} \in \{0, 0.18, 0.36\}
\end{aligned}
$$

where $V_{j,t+1}^m(\bullet)$ denotes the value of member j associated with entering period $t+1$ as married. Solving this continuous-discrete problem can be done by applying the iEGM discussed above and elaborated on in the Supplemental Material, using the insights and upper envelope algorithms described in Iskhakov, Jørgensen, Rust and Schjerning (2017) and Druedahl and Jørgensen (2017). Finally, this intertemporal solution can then be used to calculate the optimal intra-temporal time and consumption allocations by inserting these in the intra-temporal problem above.

4.1.6 Relation to General Model Class

In our formulation of the empirical model we have $M_t = A_{t-1}$ and the transition for this state is simply $M_{t+1} = f_{M_t}(A_t, \mathcal{D}_t, \mathcal{S}_t | \varepsilon_{t+1}) = A_t$ in the notation of the general model class in Section 3. The remaining states are $\mathcal{S}_t = (K_{w,t}, K_{m,t}, e_w, e_m, \psi_t)$ and the transition is independent of A_t:

$$
\mathcal{S}_{t+1} = f_{\mathcal{S}_t}(A_t, \mathcal{D}_t, \mathcal{S}_t | \varepsilon_{t+1}, \varphi_{t+1}) = \begin{pmatrix} [(1-\delta)K_{w,t} + l_{w,t}]\, \varepsilon_{w,t+1} \\ [(1-\delta)K_{m,t} + l_{m,t}]\, \varepsilon_{m,t+1} \\ e_w \\ e_m \\ \psi_t + \varphi_{t+1} \end{pmatrix}.
$$

Besides the intertemporal choice of total consumption, the model includes discrete choices over labor supply of each spouse,

$$\mathcal{D}_t = \begin{pmatrix} l_{w,t} \\ l_{m,t} \end{pmatrix}$$

and intra-temporal choices related to the consumption allocation and time allocated to housework for each spouse,

$$\mathcal{C}_t = \begin{pmatrix} c_{w,t} \\ c_{m,t} \\ c_t \\ h_{w,t} \\ h_{m,t} \end{pmatrix}.$$

The post-decision savings are governed by

$$A_t = f_{A_t}(C_t, \mathcal{D}_t, M_t, \mathcal{S}_t) = (1+r)M_t + w_{w,t}l_{w,t} + w_{m,t}l_{m,t} - \tau^m - C_t$$

where we recall that $M_t = A_{t-1}$. Since labor supply has dynamic effects through human capital accumulation, we must condition on labor supply of each household member to formulate the intra-temporal problem. In turn we have

$$\mathcal{Q}_t = f_{\mathcal{Q}_t}(A_t, \mathcal{D}_t, \mathcal{S}_t) = \mathcal{D}_t.$$

Finally, the individual utility functions take as input

$$\mathcal{U}_t = \begin{pmatrix} f_{w,t} \\ f_{m,t} \\ Q_t \end{pmatrix}$$

which, given the intra-temporal allocation and conditioning variables, follow from time constraints and the household production function:

$$f_{\mathcal{U}_t}(C_t, \mathcal{C}_t, \mathcal{Q}_t) = \begin{pmatrix} \bar{T} - l_{w,t} - h_{w,t} \\ \bar{T} - l_{m,t} - h_{m,t} \\ (\alpha_c c_t^{\omega} + (1-\alpha_c)H_t^{\omega})^{\frac{1}{\omega}} \end{pmatrix}$$

with $H_t = \left(\alpha_h h_{w,t}^{\zeta} + (1-\alpha_h)h_{m,t}^{\zeta}\right)^{\frac{1}{\zeta}}$.

4.2 Solving the Intertemporal Problem using the iEGM

We describe the numerical solution algorithm in detail in the Supplemental Material C and here outline how the iEGM interpolator is constructed. Conditional on labor supply choices, $l_{w,t}, l_{m,t}$, and post-bargaining weight, μ_t, optimal total consumption, C_t, satisfies the FOC of the problem (51),

$$U'(C_t, l_{w,t}, l_{m,t}, \mu_t) = W_t(A_t, K_{w,t}, K_{m,t}, e_w, e_m, \psi_t, \mu_t) \tag{52}$$

where the expected marginal value of wealth is[19]

$$W_t(A_t, K_{w,t}, K_{m,t}, e_w, e_m, \psi_t, \mu_t) = R\beta \mathbb{E}_t \left[\mu_t \frac{dV_{w,t+1}}{dA_t} + (1 - \mu_t) \frac{dV_{m,t+1}}{dA_t} \right] \tag{53}$$

and $U'(C_t, l_{w,t}, l_{m,t}, \mu_t)$ is the marginal household utility. This object is not known analytically but can be found by finite difference of the household utility function, solving the intra-temporal problem (49). For this reason, the analytical EGM cannot be applied. But the iEGM can.

To implement the iEGM, we first note that W_t is a sufficient statistic for optimal consumption: If we know what the marginal value of wealth is, there is one level of consumption that lets marginal utility equal that number (for each value of the conditioning variables $\mathcal{Q}_t = (l_{w,t}, l_{m,t})$ and bargaining weight, μ_t). That means that we can construct a grid over optimal consumption, $\overrightarrow{C}$, and insert it into the marginal utility to construct the marginal value of wealth, $\overrightarrow{W}$. This is completely analogous to the approach we took in the illustrative example in Section 2, but we now do that for all nine combinations of labor supply of both members and for a grid of bargaining weights, $\overrightarrow{\mu}$, too. By flipping the axis, we have constructed an interpolator for optimal consumption as a function of the marginal value of wealth, the bargaining weight, and labor supply of both members,

$$C^{interp}(W, \mu | l_w, l_m). \tag{54}$$

Figure 2 shows this interpolator for one of the nine combinations of labor supply, namely the case when both members work full time. There are eight other planes like this (not reported). Panel a) shows the expected marginal value of wealth, W, on the z-axis from inserting values of consumption and bargaining power. Panel b) flips the consumption and expected marginal value of wealth axis to get an interpolator for consumption as a function of the expected marginal value of wealth (and μ and (l_w, l_m)).

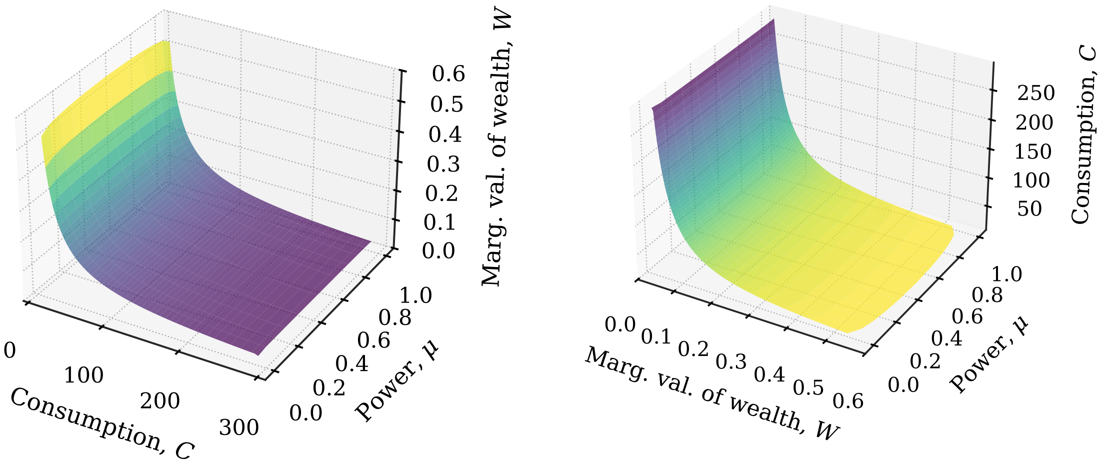
---

[19] We use forward differences to approximate this derivative, see also the discussion in Section 3 above.

Figure 2: Interpolator, $C^{interp}(W, \mu | l_w, l_m)$ for $l_w = 1, l_m = 1$.

(a) Grid over $\overrightarrow{C}$ and $\overrightarrow{\mu}$ gives $\overrightarrow{W}$.

(b) Flipped Axes and $C^{interp}(W, \mu | 1, 1)$.



Notes: The figure illustrates in panel a) how evaluating the marginal household utility for varying levels of consumption and bargaining power generates accompanying marginal values of wealth, W_t. This is because these two objects should be equal at an interior optimum. In panel b) we flip the axis such that the x-axis now has the marginal value of wealth and the z-axis has the optimal total consumption. This is the interpolator, that will be used when solving the model. The figure is constructed using parameter values calibrated to the US economy, presented below, for the case in which both household members work, i.e. $l_w = 1, l_m = 1$.

When solving the intertemporal problem for a given point in the state-space, we will calculate concrete values of the expected marginal value of wealth, W_t, and use bilinear interpolation in these planes to back out optimal total consumption without having to apply any root-finding. As this interpolator is constructed once before solving the model, we can construct this on a relatively fine grid without increasing the computational time significantly.[20]

4.2.1 Timings and accuracy

Table 4 reports timing and accuracy measures for the iEGM, and generous implementations of the VFI, and the numerical EGM. Concretely, since the state space is quite large for this model, we only report results from the implementations of VFI and numerical EGM in which we precompute all expectations and intra-temporal solutions. The reported re-

---

[20] We, in fact, only interpolate in the direction of the expected marginal value of wealth as we precompute the intertemporal consumption function on the same grid as we use to solve the dynamic model on.

sults are thus for what we refereed to as "VFI, pre E + intra" and "EGMnum, pre E + intra" in Table 3 in Section 2.

For each solution method, we solve the model over grids over all state variables: We use 30 grid points in grids over assets for couples and singles, $\overrightarrow{A}$ and $\overrightarrow{A}_j$, 11 grid points in the grid over match quality $\overrightarrow{\psi}$, 11 grid points in the grid over bargaining power $\overrightarrow{\mu}$, and 10 grid points in grids over human capital $\overrightarrow{K}$ for each household member. Finally, we allow for four skill types (of each partner). In total, we have 5,808,000 points in the the state space for each of the 40 time periods. We precompute the intra-temporal solution over a grid of total consumption $\overrightarrow{C}$ with 100 points and use this in all the implemented methods to reduce the computation time. For the numerical EGM and the iEGM, we additionally construct grids over post-decision assets $\overrightarrow{A}_{pd}$ with 30 grid points. See Supplementary Material C for details on the construction of grids.

Timings are in minutes and reported in absolute terms and relative to the VFI benchmark. Accuracy is measured along five dimensions. 1) Error(l): Share of discrete labor choices deviating from the “true” solution (VFI on dense grids)[21]; 2) Error(divorce): Error rate in the discrete divorce choice; 3) MAD(μ): Mean absolute deviation of the post-bargaining weight from the true solution, conditional on bargaining having occurred in that period; 4) MAD(C): Mean absolute deviation of the consumption function from the true solution; and 5) Comp(A): Initial wealth compensation required for workers to be indifferent between the implied behavior of a given solution method and that of the true solution, computed by comparing utility across simulations.[22]

Except for Comp(A), all accuracy measures are computed by evaluating policy functions for couples conditional on remaining married in the first period. For each labor choice, policy functions are evaluated on a tensor grid of 13 points in each of the five dimensions (initial bargaining weight, match quality, female human capital, male human capital, and assets), yielding over one million evaluation points in total. To mitigate boundary effects, we exclude the bottom and top 20% of each grid. Policy functions are evaluated using linear interpolation. All statistics are based on one Monte Carlo run, where we solve (and store timing) and simulate 100,000 households for 40 periods using the parameters calibrated to the US economy below.[23]

---

[21] To compute the accuracy of each solution method, we approximate the true model solution by solving the model with VFI using denser grids. Here, we use 50 grid points in $\overrightarrow{A}$ and $\overrightarrow{A_j}$, 15 grid points in $\overrightarrow{\mu}$, 15 grid points in $\overrightarrow{\psi}$, and 12 points in $\overrightarrow{K}$. For the pre-computation of the intra-temporal solution, we use 1024 points in $\overrightarrow{C}$.

[22] In our simulations, couples bargain and remain together in 0.58% of the time periods.

[23] All results are generated on a Dell PowerEdge R640 Server with 36 dual-core Intel Xeon Gold 6154 3.0GhZ processors and 768GB RAM. All methods are implemented in C++ and parallelized using 40 threads in OpenMP.

While VFI takes around 1.5 hours to solve, the numerical EGM is almost an order of magnitude faster, and the iEGM is almost an order of magnitude faster than the numerical EGM. In turn, the iEGM is around 50 times faster than the VFI while also producing a more accurate solution on all metrics. This speed gain is what makes the calibration below feasible as each objective function evaluation takes a couple of minutes using the iEGM.

Table 4: Computational Performance: Calibrated Limited Commitment Model.

	time (m)	rel.	Error(l)	Error(divorce)	MAD(μ)	MAD(C)	Comp(A)
VFI†	91.86	1.000	0.127	0.000	0.019	0.882	159.1
EGM (num)†	11.12	0.121	0.127	0.000	0.019	0.855	158.9
iEGM							
$\#_C$=25	2.81	0.031	0.106	0.000	0.019	2.321	168.2
$\#_C$=50	2.09	0.023	0.080	0.000	0.019	0.380	57.5
$\#_C$=75	2.09	0.023	0.050	0.000	0.019	0.382	39.8
$\#_C$=100	2.02	0.022	0.002	0.000	0.019	0.304	35.1
$\#_C$=200	1.95	0.021	0.000	0.000	0.019	0.054	32.9

Notes: The table reports timing and accuracy measures for the iEGM, standard VFI, and the numerical EGM. Timings are in minutes and also reported relative to the VFI benchmark. Accuracy is measured along five dimensions. 1) Error(l): Share of discrete labor choices deviating from the true solution (VFI on dense grids); 2) Error(divorce): Error rate in the discrete divorce choice; 3) MAD(μ): Mean absolute deviation of the post-bargaining weight from the true solution, conditional on bargaining having occurred in that period; 4) MAD(C): Mean absolute deviation of the consumption profile from the true solution; and 5) Comp(A): Initial wealth compensation required for indifference to the true solution, computed by comparing utility across simulations.

† Both VFI and numerical EGM is implemented with precomputed expectations and intra-temporal allocations. This is what we refer to as "pre E + intra" in our discussion in Section 2.

4.3 Calibration

We calibrate the model to match the US economy. Some parameters of the model are calibrated based on existing studies in a first step. We report these parameter values in Table 5 along with sources.

The remaining parameters, collected in $\theta = (\lambda_w, \lambda_m, \gamma_w, \gamma_m, \sigma_e, \eta_w, \eta_m, \phi_w, \phi_m, \alpha_Q, \alpha_c, \alpha_h, \sigma_\psi)$, are calibrated to match key moments of the US economy. Concretely, let $\mathcal{M}^{data}$ denote this vector of empirical moments and let $\mathcal{M}^{model}(\theta)$ denote the vector of moments calculated based on simulated model behavior as a function of the parameters of the model. We then calibrate θ as the vector that minimizes the distance

$$\hat{\theta} = \arg\min_{\theta}(\mathcal{M}^{data} - \mathcal{M}^{model}(\theta))' W(\mathcal{M}^{data} - \mathcal{M}^{model}(\theta)). \tag{55}$$

We include as many moments as parameters in θ and let the weighting matrix, W, be the identity.[24] The included moments are picked to be informative about particular sets of parameters. In Table 6 we report the included moments and indicate which parameters a set of moments should be informative about. We also include in this table their source, empirical value and the simulated model counterpart. We will refrain from listing them all here but highlight one. Since we are interested in the pass-through of productivity to income inequality, we include a measure of consumption inequality, the 10:90 percentile ratio from Meyer and Sullivan (2023). This should be informative about the degree of heterogeneity in the productivity distribution, σ_e. Generally, we see a quite good fit of the calibrated model to the included moments. The calibrated parameters are reported in Table 7.

Table 5: Externally Calibrated Parameters.

parameter	value	source
	Preferences	
β	0.98	Bronson, Haanwinckel and Mazzocco (2025); Attanasio, Low and Sánchez-Marcos (2008)
ρ	1.5	Bronson, Haanwinckel and Mazzocco (2025); Attanasio, Low and Sánchez-Marcos (2008)
	Human capital	
δ	0.1	Jakobsen, Jørgensen and Low (2025); Keane and Wasi (2016)
$\sigma_{\varepsilon,j}$ for $j \in \{w, m\}$	0.1	Jakobsen, Jørgensen and Low (2025)
	Household	
ζ	0.4	Bronson, Haanwinckel and Mazzocco (2025)
ω	0.38	Bronson, Haanwinckel and Mazzocco (2025)
	Interest rate	
r	0.03	Voena (2015); Jakobsen, Jørgensen and Low (2025)

Notes: The table reports parameters calibrated based on existing studies in a first step along with the sources.

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[24] Due to scale differences, we put a weight of 10 on consumption inequality.

Table 6: Moments Matched and Model Fit.

moment	model	data	source
Wage process, $\lambda_w, \lambda_m, \gamma_w, \gamma_m, \sigma_e$			
Wages, women aged 25-34	39.15	40.1	Eckstein, Keane and Lifshitz (2019)
Wages, women aged 35-44	51.17	49.3	Eckstein, Keane and Lifshitz (2019)
Wages, men aged 25-34	48.36	50.4	Eckstein, Keane and Lifshitz (2019)
Wages, men aged 35-44	68.49	67.8	Eckstein, Keane and Lifshitz (2019)
10:90 percentile ratio, consumption	3.68	3.65	Meyer and Sullivan (2023)
Dis-utility from work, $\eta_w, \eta_m, \phi_w, \phi_m$			
Employment, women aged 35-44	0.647	0.64	Eckstein, Keane and Lifshitz (2019)
Employment, men aged 35-44	0.891	0.88	Eckstein, Keane and Lifshitz (2019)
Working hours, women	1,465	1,674	Authors calculations using ATUS
Working hours, men	1,835	2,062	Authors calculations using ATUS
Home production function, $\alpha_Q, \alpha_C, \alpha_h$			
Home production hours, women	1,540	1,535	Authors calculations using ATUS
Home production hours, men	1,074	1,035	Authors calculations using ATUS
Total consumption	42,775	42,716	Blundell, Pistaferri and Saporta-Eksten (2018)
Match quality, σ_ψ			
Share married, aged 35-44	0.688	0.69	Eckstein, Keane and Lifshitz (2019)

Notes: The gross wage rates from Eckstein, Keane and Lifshitz (2019) are in $1,000 (2012 level) and based on married individuals from the 1965 cohort working full time full year. The employment rates are based on the same sample (see their Supplemental Table J).

Time allocation numbers are based time spent in primary activities in the American Time Use Survey (ATUS) for 25–41 year olds in 2024. Home production includes "household activities", "purchasing goods and services", "caring for and helping household members", and "caring for and helping nonhousehold members". Work includes educational activites. Numbers are daily averages multiplied by 365 found in `https://www.bls.gov/charts/american-time-use/activity-by-agew.htm` and `https://www.bls.gov/charts/american-time-use/activity-by-agem.htm`.

Total consumption from Blundell, Pistaferri and Saporta-Eksten (2018) is measured in 2010 prices and excludes durables.

Table 7: Internally Calibrated Parameters

Parameter	Value
Wage process	
λ_w	1.823
λ_m	1.969
γ_w	0.124
γ_m	0.263
σ_e	0.533
Dis-utility from work	
η_w	2.284
η_m	2.750
ϕ_w	3.218
ϕ_m	3.290
Home production function	
α_Q	12.031
α_c	0.596
α_h	0.951
Match quality	
σ_ψ	7.185

Notes: The table presents calibrated parameter values for internally calibrated parameters. We calibrate parameters by minimizing the distance between model and data moments. The exact moments and their simulated counterparts are listed in Table 6.

4.3.1 Simulation details

We simulate a population of 100,000 women and men (200,000 individuals in total) for 40 time periods, from age 25 to 65. In the initial period, 80% are assigned to couples and 20% are singles. We index simulated women by i, men by j, and ij denotes a household consisting of woman i and man j.

Individual skill types, $e_{w,i}$ and $e_{m,j}$, are assigned by independent random draws from the estimated Normal distribution with gender-specific means, λ_w and λ_m, given in Table 6. Concretely, we allow for four discrete types, with the share in the population given be the corresponding Gaussian quadrature weights. Initial assets and human capital are set equal to zero for all individuals.

For individuals who start in a couple, the initial match quality is normalized to $\psi_{ij,0} = 0$. Conditional on all other initial states, initial bargaining power, $\mu_{ij,0}$, for those in couples is determined by Nash Bargaining. Specifically, we determine initial bargaining power given all other states by choosing $\mu_{ij,0}$ to maximize the product of surpluses:

$$\mu_{ij,0} = \arg\max_{\mu} S_{w,i,0}(\mu) S_{m,j,0}(\mu)$$

where $S_{w,i,0}(\mu)$ and $S_{m,j,0}$ denote the marital surplus for woman i and man j at time $t = 0$:

$$\begin{aligned} S_{w,i,0}(\mu) &= V_{w,t}^{m\to m}(A_{i,-1}, K_{w,i,0}, K_{m,j,0}, e_{w,i}, e_{m,j}, \psi_{ij,0}, \mu) - V_{w,0}^{m\to s}(A_{i,-1}, K_{w,i,0}, e_{w,i}) \\ S_{m,j,0}(\mu) &= V_{m,t}^{m\to m}(A_{j,-1}, K_{w,i,0}, K_{m,j,0}, e_{w,i}, e_{m,j}, \psi_{ij,0}, \mu) - V_{m,0}^{m\to s}(A_{j,-1}, K_{m,j,0}, e_{m,j}). \end{aligned}$$

Households are then simulated forward using the model's policy functions for total consumption as well as intertemporal and discrete choices. For couples, we additionally evaluate the participation constraint and potentially update their bargaining power, $\mu_{ij,t}$, or terminate the relationship if no agreement is possible. Singlehood is absorbing, as in the solution. Period-by-period shocks to match quality and human capital are drawn independently across individuals and time from their respective continuous distributions, and all state variables are updated accordingly.

4.4 Counterfactual Simulations

We now conduct counterfactual simulations to illustrate how productivity shocks affect workers in the estimated model, and ultimately affect long run consumption inequality. In doing so, we compare the baseline limited commitment model outcomes with the same outcomes from an alternative model with full commitment. In this model, couples know with certainty that the bargaining weight will remain fixed throughout their entire life at the value negotiated through the initial Nash bargaining in the first period of their relationship. This alternative model is intended to illustrate how different types of bargaining protocols can matter for the propagation of productivity shocks.[25]

First, we document how a large negative shock to the productivity (human capital) at age 30 affects different outcomes throughout working life. We plot the average change from the baseline calibrated economy from a reduction in human capital of 25% for individuals of low and high productivity types, defined as below or above the average, λ_j, in Figure 3 and Figure 4, respectively.

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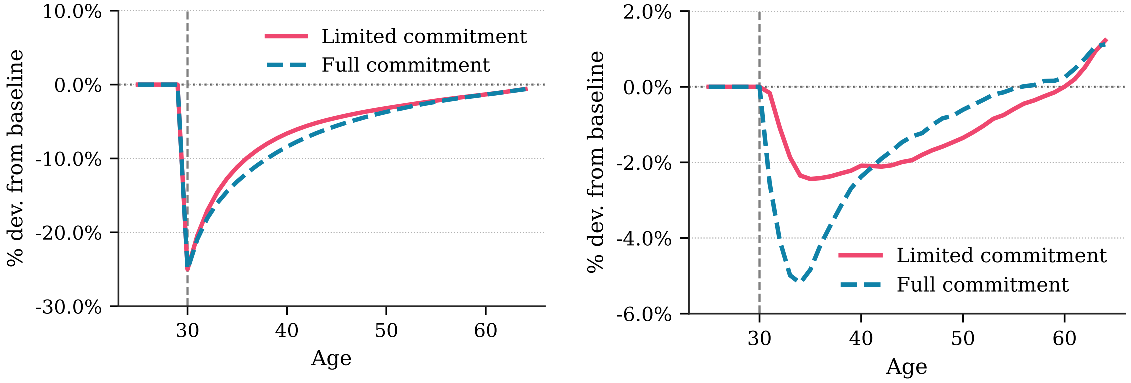
[25] All parameters of the model are kept identical to the ones estimated under the limited commitment assumption.

Figure 3: Productivity Shock: Less Skilled.


(a) Human capital

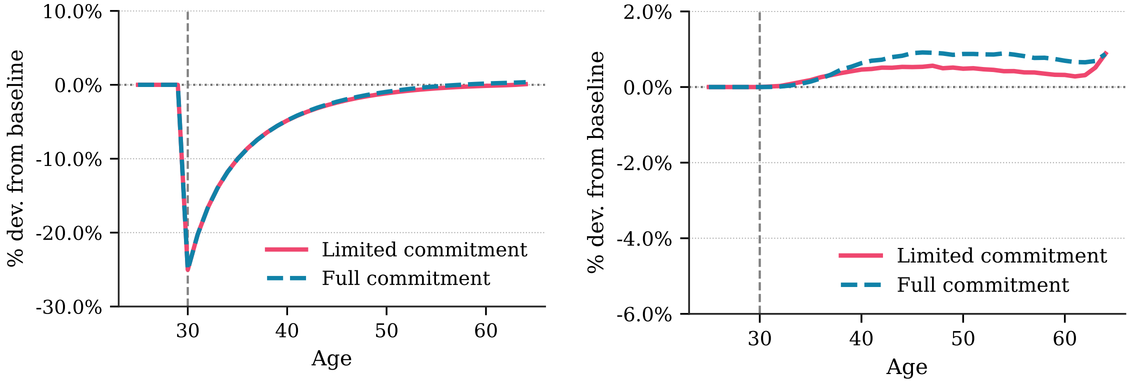
(b) Labor supply

(c) Wage income

(d) Consumption

Notes: The plot shows impulse-response functions of a simulated negative shock the less skilled individual's human capital. Specifically, human capital is reduced by 25% at age 30. Panel (a) shows human capital, panel (b) labor supply, panel (c) wage income, and panel (d) household equivalized consumption for the affected individuals. All outcomes are in percent relative to the baseline. Moreover, outcomes are shown for two different models: One, where couples bargain with limited commitment and divorce endogenously (fully drawn lines), and one where couples are fully committed to their bargained allocation and do not divorce (dashed lines).

Figure 4: Productivity Shock: High Skilled.



(a) Human capital

(b) Labor supply

(c) Wage income

(d) Consumption

Notes: The plot shows impulse-response functions of a simulated negative shock the high skilled individual's human capital. Specifically, human capital is reduced by 25% at age 30. Panel (a) shows human capital, panel (b) labor supply, panel (c) wage income, and panel (d) household equivalized consumption for the affected individuals.. All outcomes are in percent relative to the baseline. Moreover, outcomes are shown for two different models: One, where couples bargain with limited commitment and divorce endogenously (fully drawn lines), and one where couples are fully committed to their bargained allocation and do not divorce (dashed lines).

Focusing on less skilled workers, panel a) in Figure 3 shows how the human capital is reduced by 25%. Panel b) shows how labor supply is drastically reduced due to the reduced incentive to work (shown in panel c). The dashed line shows that the response is initially much larger for less skilled workers in the full commitment regime. The difference stems from the fact that the household member adversely hit by decreased productivity cannot rely as much on the support from their partner in the limited commitment case. In turn, they cannot "afford" to reduce labor supply as much as workers who have full commitment from their partner. In turn, the insurance value from the added worker effect is larger in the full commitment version of the model. Although the labor supply eventually recovers faster in the full commitment model, it takes around 20 years for the human capital to reach the same percentage drop as in the limited commitment model (panel a). Panel d) shows that equivalized total consumption responds quite similarly in the two models.

Interestingly, for high skilled we hardly see any difference between the limited commitment model and the full commitment model in Figure 4. While wages also go down for these workers, the income effect seems to dominate, leading to a slight increase in the labor supply in both versions of the model.

Finally, to investigate the aggregate consequences of skill-biased productivity shocks, we calculate in Table 8 the effect of productivity shocks on consumption inequality for all workers in the model economy. Our measure of consumption inequality is the 90:10 percentile ratio in consumption across individuals, as studied in e.g. Meyer and Sullivan (2023), and matched in our calibration of the model. We see that the degree of commitment matters for the aggregate consequences on consumption inequality. Again primarily for shocks to the productivity of less skilled workers. Combined, our results suggest that the degree of commitment between household members and the assumptions on the bargaining protocol can affect how individual productivity shocks propagate into aggregate consumption inequality. Especially in the low end of the skill distribution.

Table 8: Change in Wage and Consumption Inequality after Skill-Biased Shock.

	Less Skilled		High Skilled	
	Wage	Consumption	Wage	Consumption
Limited Commitment	+0.061	+0.062	-0.079	-0.072
Full Commitment	+0.075	+0.079	-0.072	-0.073

Notes: The table shows the simulated effect of a negative shock to human capital of the high skilled and less skilled, respectively, on consumption and wage inequality. For both measures of inequality, we use the 90:10 percentile ratio across individuals, and report absolute differences relative to the baseline case. Consumption is measured as household equivalized consumption. We report the effect for two different models: One, where couples bargain with limited commitment and divorce endogenously, and one where couples are fully committed to their bargained allocation and do not divorce.

5 Conclusion

We propose a modification to the endogenous grid method (EGM), originally proposed by Carroll (2006), which builds on precomputation and interpolation of total consumption as a function of the marginal value of wealth. We refer to this method as interpolated EGM (iEGM). The method is particularly useful in dynamic household bargaining models, where the lack of analytical invertibility of the marginal household utility function imply that standard EGM must rely on potentially numerical root-finding operations. In a simple example of intra-household consumption allocation, we show that the iEGM outperforms traditional value function iteration (VFI) by a factor of around 20 in terms of computation time, without sacrificing accuracy.

We demonstrate the applicability of the iEGM in a rich quantitative household model featuring limited commitment, endogenous divorce, labor supply, human capital accumulation, and home production. In this setting, iEGM outperforms VFI by a factor of around 50, again without loss of accuracy. We calibrate the model to the U.S. economy and use it to study how labor productivity shocks can propagate into consumption inequality. Comparing our model to a an alternative model with full commitment between household members, we find that the nature of household commitment plays an important role in shaping labor supply responses to productivity shocks, particularly in the lower part of the skill distribution.

In sum, the iEGM relaxes the computational constraints and widens the class of models that can be feasibly estimated for a given computational infrastructure. Building on this, future research analyze even more complex environments.

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Supplementary Material

A Recursive Formulation

To derive the recursive formulation of a couple, we follow the exposition in Marcet and Marimon (2019), but adopt our own notation. Let $V_{j,t}^{m\to s}(\mathcal{S}_t)$ denote the value of transitioning from marriage to singlehood (divorce) of household member j as a function of state variables in $\mathcal{S}_t$. Note that this set contains all relevant state variables in excess of marital status (which we will denote with superscripts) and the states measuring the bargaining power of each household member (as we will derive below). Given the set of state variables, couples choose the vector $\mathcal{C}_t$. We assume that the states transition following some known distribution,

$$\mathcal{S}_{t+1} \sim \Gamma(\mathcal{S}_t, \mathcal{C}_t)$$

with $\mathcal{S}_0$ given, and that choices potentially have to satisfy some additional constraints, such as a budget constraint. The important complication in the limited commitment framework is the presence of forward-looking participation constraints, and thus we will ignore all other constraints in the following exposition. In the example below, we will include all constraints in the formulation to be precise.

The Pareto problem of a newly formed couple in period zero can be formulated as

$$\max_{\{\mathcal{C}_t\}_0^T} \lambda_{1,0}\mathbb{E}_0\left[\sum_{t=0}^{T}\beta^t u_1(\mathcal{C}_t,\mathcal{S}_t)\right] + \lambda_{2,0}\mathbb{E}_0\left[\sum_{t=0}^{T}\beta^t u_2(\mathcal{C}_t,\mathcal{S}_t)\right]$$

s.t.

$$\mathbb{E}_t\left[\sum_{\tau=0}^{T-t}\beta^\tau u_j(\mathcal{C}_{t+\tau},\mathcal{S}_{t+\tau})\right] \geq V_{j,t}^{m\to s}(\mathcal{S}_t) \text{ for } t=0,\ldots,T \text{ and } j=1,2 \tag{A.1}$$

where $\mathbb{E}_t[\bullet] = \mathbb{E}[\bullet|\mathcal{S}_t,\mathcal{C}_t]$ denotes a conditional expectation, $\lambda_{1,0}$ and $\lambda_{2,0}$ are the initial weight put on the expected discounted utility of the first and second household member, respectively, and (A.1) are the forward looking marital participation constraints.[26] These constraints ensure that each household member finds it optimal to remain in a couple. If, in a given period, there is no allocation $\mathcal{C}_t$ that can satisfy these constraints, the couple will transition to singlehood. The key here is that couples cannot commit to future actions,

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[26] Note the slight difference in the sum in the participation constraint compared to e.q. (3) in Marcet and Marimon (2019). This difference leads to a slight difference in the timing: In the current formulation, the bargaining weight is updated in the current period as a consequence of a participation constraint being binding.

they cannot transfer utility to other household members, and there is unilateral divorce.

The Lagrangian

Again ignoring all other constraints than the forward looking ones in (A.1), the Lagrangian can be formulated as

$$\begin{aligned}\mathcal{L}(\boldsymbol{\mathcal{C}},\gamma,\boldsymbol{\lambda},\mathcal{S}_0) =&\mathbb{E}_0\Bigg[\sum_{t=0}^{T}\beta^t\sum_{j=1}^{2}\Bigg\{\lambda_{j,0}u_j(\mathcal{C}_t,\mathcal{S}_t)+\\ &+\gamma_{j,t}\Bigg(\mathbb{E}_t\Bigg[\sum_{\tau=0}^{T-t}\beta^\tau u_j(\mathcal{C}_{t+\tau},\mathcal{S}_{t+\tau})\Bigg]-V_{j,t}^{m\to s}(\mathcal{S}_t)\Bigg)\Bigg\}\Bigg]\end{aligned}$$

with $\boldsymbol{\mathcal{C}}=(\mathcal{C}_0,\mathcal{C}_1,\ldots,\mathcal{C}_T)$, $\boldsymbol{\lambda}=(\lambda_{1,0},\lambda_{2,0})$ and the associated shadow prices of the forward looking constraints in $\gamma=(\gamma_{1,0},\gamma_{1,1},\ldots,\gamma_{1,T},\gamma_{2,0},\gamma_{2,1},\ldots,\gamma_{2,T})$.[27]

The Lagrangian can be greatly simplified. First, due to the law of iterated expectations, the inner expectation can be dropped, i.e., we have that[28]

$$\mathbb{E}_0\left[\gamma_{j,t}\mathbb{E}_t\left[\sum_{\tau=0}^{T-t}\beta^\tau u_j(\mathcal{C}_{t+\tau},\mathcal{S}_{t+\tau})\right]\right]=\mathbb{E}_0\left[\gamma_{j,t}\sum_{\tau=0}^{T-t}\beta^\tau u_j(\mathcal{C}_{t+\tau},\mathcal{S}_{t+\tau})\right]$$

and we can write the Lagrangian as

$$\begin{aligned}\mathcal{L}(\boldsymbol{\mathcal{C}},\gamma,\boldsymbol{\lambda},\mathcal{S}_0) =&\mathbb{E}_0\Bigg[\sum_{t=0}^{T}\beta^t\sum_{j=1}^{2}\Bigg\{\lambda_{j,0}u_j(\mathcal{C}_t,\mathcal{S}_t)+\\ &+\gamma_{j,t}\Bigg(\sum_{\tau=0}^{T-t}\beta^\tau u_j(\mathcal{C}_{t+\tau},\mathcal{S}_{t+\tau})-V_{j,t}^{m\to s}(\mathcal{S}_t)\Bigg)\Bigg\}\Bigg].\end{aligned}$$

Second, we can collect terms to eliminate the double sum. To see how the same period utility for member j enters several times, think of some time-period $\bar{t}$ and write out the elements in the expectation as

$$\beta^{\bar{t}}\lambda_{j,0}u_j(\mathcal{C}_{\bar{t}},\mathcal{S}_{\bar{t}})+\beta^{\bar{t}}\gamma_{j,\bar{t}}\sum_{\tau=0}^{T-\bar{t}}\beta^\tau u_j(\mathcal{C}_{\bar{t}+\tau},\mathcal{S}_{\bar{t}+\tau})-\beta^{\bar{t}}\gamma_{j,\bar{t}}V_{j,\bar{t}}^{m\to s}(\mathcal{S}_{\bar{t}})$$

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[27] Since the elements in γ are within a conditional expectation, conditional on information in period zero, γ is *normalized* shadow prices, based on the expected path of state variables. Chiappori and Mazzocco (2017) explicitly account for this in their formulation.

[28] because $\gamma_{j,t}$ is a function of information at time t it can be included in the inner conditional expectation.

which means that $u_j(\mathcal{C}_{\bar{t}}, \mathcal{S}_{\bar{t}})$ will be multiplied by

$$\beta^{\bar{t}}\lambda_{j,0} + \beta^{\bar{t}}\gamma_{j,\bar{t}} + \beta^{\bar{t}-1}\beta\gamma_{j,\bar{t}-1} + \beta^{\bar{t}-2}\beta^2\gamma_{j,\bar{t}-2} + \cdots + \beta^0\beta^{\bar{t}}\gamma_{j,0} = \beta^{\bar{t}}\underbrace{\left[\lambda_{j,0} + \sum_{\tau=0}^{\bar{t}}\gamma_{j,\bar{t}-\tau}\right]}_{\text{call this } M_{j,\bar{t}}}.$$

We can thus finally write the Lagrangian as

$$\mathcal{L}(\boldsymbol{\mathcal{C}}, \boldsymbol{\gamma}, \boldsymbol{\lambda}, \mathcal{S}_0) = \mathbb{E}_0\left[\sum_{t=0}^{T}\beta^t\sum_{j=1}^{2}\left\{M_{j,t}u_j(\mathcal{C}_t, \mathcal{S}_t) - \gamma_{j,t}V_{j,t}^{m\to s}(\mathcal{S}_t)\right\}\right] \tag{A.2}$$

with a recursive formulation for the weights (often referred to as co-states) on current period utility as

$$\begin{aligned} M_{j,t} &= M_{j,t-1} + \gamma_{j,t} \\ M_{j,-1} &= \lambda_{j,0} \end{aligned} \tag{A.3}$$

for $j = 1, 2$. The formulation in eq. (A.2) is similar to that in Mazzocco (2007) and Attanasio and Ríos-Rull (2000).[29] Concretely, the formulation could be expressed in terms of surpluses

$$\mathcal{L}(\boldsymbol{\mathcal{C}}, \boldsymbol{\gamma}, \boldsymbol{\lambda}, \mathcal{S}_0) = \mathbb{E}_0\left[\sum_{t=0}^{T}\beta^t\sum_{j=1}^{2}\left\{M_{j,t-1}u_j(\mathcal{C}_t, \mathcal{S}_t) + \gamma_{j,t}(u_j(\mathcal{C}_t, \mathcal{S}_t) - V_{j,t}^{m\to s}(\mathcal{S}_t))\right\}\right]$$

as in Attanasio and Ríos-Rull (2000).

As noted in e.g. Attanasio and Ríos-Rull (2000), the first-order conditions w.r.t. individual consumption (assumed here to be the first input in the utility function) yields

$$\frac{u_1^1(\mathcal{C}_t, \mathcal{S}_t)}{u_2^1(\mathcal{C}_t, \mathcal{S}_t)} = \frac{M_{2,t}}{M_{1,t}}$$

showing that the weights must be equal (in the interior) if the marginal utilities of consumption are identical. If, say, member 1 has a higher marginal utility from consumption, that must be associated with a relatively lower bargaining power of member 1.

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[29] The option value is not explicitly discounted in Mazzocco (2007), however.

The Recursive (Saddle-Point) Formulation

A solution to this model involves minimizing the Lagrangian with respect to γ and maximizing it with respect to $\boldsymbol{\mathcal{C}}$. This min-max problem is referred to as a saddle-point problem and can be formulated recursively like a standard Bellman equation.

Concretely, the value of a newly formed couple is

$$W_0(\mathcal{S}_0, M_{1,-1}, M_{2,-1}) = \inf_{\{\gamma_{j,t}\geq 0\}_{j=1,t=0}^{2,T}} \sup_{\{\mathcal{C}_t\}_{t=0}^{T}} \mathcal{L}(\boldsymbol{\mathcal{C}}, \boldsymbol{\gamma}, \boldsymbol{\lambda}, \mathcal{S}_0)$$

where, due to the law of iterated expectations, the Lagrangian can be written as

$$\begin{aligned}\mathcal{L}_0 = &\sum_{j=1}^{2}\left\{M_{j,0}u_j(\mathcal{C}_0, \mathcal{S}_0) - \gamma_{j,0}V_{j,0}^{m\to s}(\mathcal{S}_0)\right\}\\ &+ \beta\mathbb{E}_0\underbrace{\left[\mathbb{E}_1\left[\sum_{t=0}^{T-1}\beta^t\sum_{j=1}^{2}\left\{M_{j,t+1}u_j(\mathcal{C}_{t+1}, \mathcal{S}_{t+1}) - \gamma_{j,t+1}V_{j,t+1}^{m\to s}(\mathcal{S}_{t+1})\right\}\right]\right]}_{=\mathcal{L}_1}\end{aligned}$$

with a slight abuse of notation. Denoting $W_0(\mathcal{S}_0, M_{1,-1}, M_{2,-1}) = \mathcal{L}_0(\boldsymbol{\mathcal{C}}^\star, \boldsymbol{\gamma}^\star, \boldsymbol{\lambda}, \mathcal{S}_0)$ as the value when optimal choices are inserted, the problem can be written as

$$\begin{aligned}W_0(\mathcal{S}_0, M_{1,-1}, M_{2,-1}) = &\inf_{\{\gamma_{j,0}\geq 0\}_{j=1}^{2}} \sup_{\mathcal{C}_0} \sum_{j=1}^{2}\left\{M_{j,0}u_j(\mathcal{C}_0, \mathcal{S}_0) - \gamma_{j,0}V_{j,0}^{m\to s}(\mathcal{S}_0)\right\}\\ &+ \beta\mathbb{E}_0[W_1(\mathcal{S}_1, M_{1,0}, M_{2,0})].\end{aligned}$$

In turn, for an arbitrary period t we have the recursive formulation

$$\begin{aligned}W_t(\mathcal{S}_t, M_{1,t-1}, M_{2,t-1}) = &\inf_{\{\gamma_{j,t}\geq 0\}_{j=1}^{2}} \sup_{\mathcal{C}_t} \sum_{j=1}^{2}\left\{(M_{j,t-1} + \gamma_{j,t})u_j(\mathcal{C}_t, \mathcal{S}_t) - \gamma_{j,t}V_{j,t}^{m\to s}(\mathcal{S}_t)\right\}\\ &+ \beta\mathbb{E}_t[W_{t+1}(\mathcal{S}_{t+1}, M_{1,t}, M_{2,t})] \qquad \text{(A.4)}\end{aligned}$$

where $M_{j,t}$ transitions as in eq. (A.3) and subject to state-transitions and other constraints.

A useful fact is that the individual weights can be scaled in any way (Marcet and Marimon, 2019). A convenient normalization, which we will use below, is

$$\mu_{t-1} = \frac{M_{1,t-1}}{M_{1,t-1} + M_{2,t-1}}$$

since we then only need to know one co-state, μ_{t-1}, rather than the two $M_{1,-1}, M_{2,-1}$.

Increasing μ_t then corresponds to increasing the shadow price on the participation constraint of member one, $\gamma_{1,t}$. In turn, all relevant state-variables of a couple are $\mathcal{S}_t$ and μ_{t-1}, optimal endogenous choices are $\mathcal{C}_t^\star(\mathcal{S}_t, \mu_{t-1})$ and $\mu_t^\star(\mathcal{S}_t, \mu_{t-1})$ and states transition according to

$$\begin{aligned} \mathcal{S}_{t+1} &\sim \Gamma(\mathcal{S}_t, \mathcal{C}_t) \\ \mu_t &= \mu_t^\star(\mathcal{S}_t, \mu_{t-1}). \end{aligned}$$

The updated bargaining weight, $\mu_t^\star(\mathcal{S}_t, \mu_{t-1})$, is a result of the intra-household bargaining process and is discussed in detail in the main text.

In essence, we solve the saddle-point problem in two steps. First, we check the corner solution, $\gamma_{1,t} = \gamma_{2,t} = 0$ and maximize over $\mathcal{C}_t$. We then check whether the forward-looking participation constraints are satisfied for both members. If they are, we are at the corner solution. If none of the participation constraints are satisfied, the couple divorces and if only one of the participation constraints are violated, of say member j, we find the lowest value of $\gamma_{j,t}$ (and associated optimal $\mathcal{C}_t$) that satisfies the participation constraint.

B Numerical Implementation of the Limited Commitment Bargaining Protocol

Here, we lay out an algorithm to implement the limited commitment bargaining protocol. We also propose a simple linear interpolation trick to improve the numerical accuracy for fixed number of points in the state space. First, we calculate the marital surplus for values of beginning-of-period bargaining, μ_{t-1}, on a grid, $\overrightarrow{\mu}$, with $\#_\mu$ points. We then go through the following algorithm for each point in the state space and each time period. To reduce clutter, the notation below ignores the dependence on state-variables, $M_t, \mathcal{S}_t$.

1. For a given state-combination, calculate the marital surplus for each household member for each value in $\overrightarrow{\mu}$ from eq. (37). Denote the resulting arrays as $\overrightarrow{S_{1,t}}$ and $\overrightarrow{S_{2,t}}$. These arrays are, respectively, ascending and descending grids in μ.

2. Check if bargaining is relevant for any μ_{t-1}.

    (a) If $\max(\overrightarrow{S_{j,t}}) < 0$ for any $j \in \{1, 2\}$ there is no room for bargaining as at least one member has negative surplus for all values of μ_{t-1}. In turn, they divorce and all values for entering a period as a couple are updated with values associated with being single. Terminate algorithm with $\mu_t = \emptyset$.

    (b) If $\min(\overrightarrow{S_{1,t}}) \geq 0$ and $\min(\overrightarrow{S_{2,t}}) \geq 0$ both members will remain together independent of the bargaining power. In turn, all values for entering a period as a couple are updated to the solutions to the "remaining couple" problem outlined in detail above. Terminate algorithm with $\mu_t = \mu_{t-1}$.

    (c) If $\min(\overrightarrow{S_{1,t}}) \geq 0$ and $\min(\overrightarrow{S_{2,t}}) \leq 0$ and $\max(\overrightarrow{S_{2,t}}) \geq 0$ member 1 is always satisfied and member 2 must have an indifference point, $\tilde{\mu}_{2,t}$. Member 1 does not formally have an indifference point, but for all practical purposes we can set $\tilde{\mu}_{1,t} = 0$.

    (d) If $\min(\overrightarrow{S_{2,t}}) \geq 0$ and $\min(\overrightarrow{S_{1,t}}) \leq 0$ and $\max(\overrightarrow{S_{1,t}}) \geq 0$ we likewise set $\tilde{\mu}_{2,t} = 1$.

3. Find the indifference points where each member is just indifferent between remaining and divorcing, if they exist. We propose to find the indifference points by linear interpolation, as illustrated in Figure B.1:

    (a) Find the lowest (highest) point in the grid $\overrightarrow{\mu}$ that makes member 1 (member 2) satisfied with remaining married,

$$\begin{aligned} i_1 &: \overrightarrow{S_{1,t}}[i_1] \geq 0, \overrightarrow{S_{1,t}}[i_1 - 1] < 0 \\ i_2 &: \overrightarrow{S_{2,t}}[i_2 + 1] < 0, \overrightarrow{S_{2,t}}[i_2] \geq 0 \end{aligned}$$

where square brackets indicate an element of an array.

(b) Use linear interpolation to approximate the point of indifference. Concretely, imagine that member 1 is unsatisfied with the current bargaining power. We then set member 1's surplus to zero using linear interpolation,

$$0 = \overrightarrow{S_{1,t}}[i_1 - 1] + \frac{\overrightarrow{S_{1,t}}[i_1] - \overrightarrow{S_{1,t}}[i_1 - 1]}{\overrightarrow{\mu}[i_1] - \overrightarrow{\mu}[i_1 - 1]}(\tilde{\mu}_{1,t} - \overrightarrow{\mu}[i_1 - 1])$$

by isolating

$$\tilde{\mu}_{1,t} = \overrightarrow{\mu}[i_1 - 1] - \overrightarrow{S_{1,t}}[i_1 - 1]\frac{\overrightarrow{\mu}[i_1] - \overrightarrow{\mu}[i_1 - 1]}{\overrightarrow{S_{1,t}}[i_1] - \overrightarrow{S_{1,t}}[i_1 - 1]}.$$

Similarly, for member 2, the indifference point is

$$\tilde{\mu}_{2,t} = \overrightarrow{\mu}[i_2] - \overrightarrow{S_{2,t}}[i_2]\frac{\overrightarrow{\mu}[i_2 + 1] - \overrightarrow{\mu}[i_2]}{\overrightarrow{S_{2,t}}[i_2 + 1] - \overrightarrow{S_{2,t}}[i_2]}.$$

The situation in which one member will always remain while the other member has a point of indifference is also a possibility. In such a situation, the indifference point of the member always willing to remain can be set to be outside the grid.

4. Check if bargaining is possible. If the indifference point of member 2 is lower than that of member 1, $\tilde{\mu}_{1,t} > \tilde{\mu}_{2,t}$, there is no room for bargaining and the couple divorces. All values associated with entering as a couple are thus set to that of singlehood.

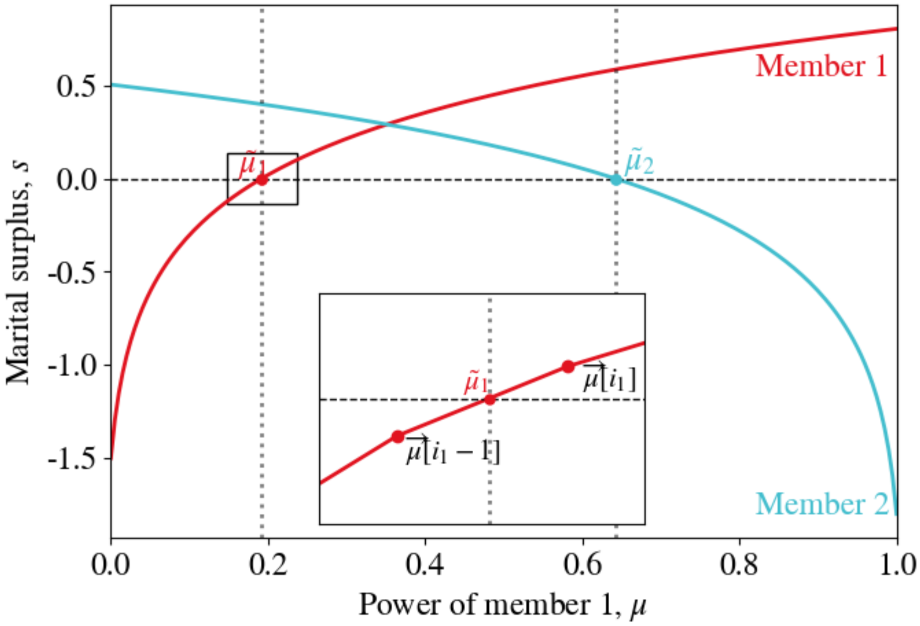
5. If bargaining is possible in step 4 above, determine the value of the updated bargaining power, $\mu_t^\star = \mu_t$ depending on the beginning-of-period bargaining state, μ_{t-1} in $\overrightarrow{\mu}$:

   (a) If $\tilde{\mu}_{1,t} \leq \mu_{t-1} \leq \tilde{\mu}_{2,t}$, they remain together with unchanged bargaining power, $\mu_t = \mu_{t-1}$, and all values for entering a period as a couple are updated to the solutions to the conditional problem.

(b) If $\mu_{t-1} \leq \tilde{\mu}_{1,t}$, set $\mu_t = \tilde{\mu}_1$ and interpolate all objects to values at $\tilde{\mu}_{1,t}$. Note that for all $\mu_{t-1} < \tilde{\mu}_{1,t}$ the bargaining power will be updated to $\tilde{\mu}_{1,t}$ and all solutions will be identical. Hence, we can reuse the solution for each $\mu_{t-1} < \tilde{\mu}_{1,t}$ in $\overrightarrow{\mu}$. This removes unnecessary interpolation.

(c) Similarly, if $\tilde{\mu}_{2,t} < \mu_{t-1}$, set $\mu_t = \tilde{\mu}_{2,t}$ and interpolate all objects to the values at $\tilde{\mu}_{2,t}$. Again, for all $\mu > \tilde{\mu}_{2,t}$, the solution will be identical and can be reused.

Figure B.1: Interpolation of the Indifference Point



Notes: The figure illustrates how the indifference point is found by linear interpolation of member 1's the marital surplus function. The interpolation is done between the grid point that makes member 1 satisfied with remaining married, i, and the point before that, $i-1$. The interpolated indifference point, $\tilde{\mu}_1$, is where the martial surplus is zero.

C Numerical solution

In this section, we describe how we solve the model of section 4 and highlight some specific tricks used. For further details, we refer to the GitHub repository with the accompanying code used to generate the results here.[30]

C.1 Construction of grids

We construct grids over all state variables. Couples' assets, $\overrightarrow{A}$, is constructed as a grid of 30 points between 0 and 300 with uneven spacing, where the majority of points are concentrated at lower values to account for higher curvature in this region and reduce interpolation errors. The monetary unit is 10,000 USD, so the top of the asset grid for couples corresponds to 3,000,000 USD. Asset grids for singles are constructed as $\overrightarrow{A_m} = \kappa_m \overrightarrow{A}$ and $\overrightarrow{A_w} = \kappa_w \overrightarrow{A}$, where we use $\kappa_w = \kappa_m = 0.5$.

We construct match quality, $\overrightarrow{\psi}$, as a grid of 11 grid points evenly distributed between -100 and 100. Bargaining power, $\overrightarrow{\mu}$, is a grid of 11 points between 0 and 1 with uneven spacing, where the

We approximate the distribution of time-invariant skill-types e_j through Gaussian quadrature, with 4 quadrature nodes.

Finally, human capital, $\overrightarrow{K}$ is a grid of 10 points between 0 and 10 with uneven spacing and a larger density for low values.

For the precomputation of the intra-temporal allocation of couple's consumption used for the iEGM and some variations in table 3, we construct a grid over total consumption, $\overrightarrow{C}$, with 100 unevenly spaced grid points from 0 to 300. For this precomputation, we also use the grid over bargaining power $\overrightarrow{\mu}$ constructed earlier, as well as discrete grids over labor supply choices, $\overrightarrow{l_w}$ and $\overrightarrow{l_m}$

For EGM implementations, we further construct a grid over post-decision assets, $\overrightarrow{A_{pd}}$ with 30 unevenly spaced grid points from 0 to 300. Finally, for the iEGM implementation, we construct another grid over total consumption, $\overrightarrow{C_{iEGM}}$. This grid runs from 0 to 300, and we vary the number of grid points, $\#_C$, between implementations to illustrate sensitivity with respect to this precomputation step.

Throughout this section, we let $\overrightarrow{X}[i]$ denote the value of the i'th index of grid $\overrightarrow{X}$.

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[30] https://github.com/ThomasHJorgensen/FastBargaining

C.1.1 Grids in the "true" model

To compute the accuracy of each solution method, we approximate the true model solution by solving the model with VFI using denser grids. Here, we use 50 grid points in $\overrightarrow{A}$ and $\overrightarrow{A_j}$, 15 grid points in $\overrightarrow{\mu}$, 15 grid points in $\overrightarrow{\psi}$, and 12 points in $\overrightarrow{K}$. For the precomputation of the intra-temporal solution, we use 1024 points in $\overrightarrow{C}$.

C.2 Precomputation of intra-temporal allocation

The problem of consumption and housework choice for couples can be divided into an intertemporal allocation of resources between periods and and intra-temporal allocation of resources. Conditional on total consumption expenditure, C, individual discrete labor supply choices, l_w and l_m, and bargaining power μ, in a given period, the latter amounts to the following time invariant problem:

$$
\begin{aligned}
&\tilde{c}_{w,t}(C, l_w, l_m, \mu), \tilde{c}_{m,t}(C, l_w, l_m, \mu), \tilde{h}_{w,t}(C, l_w, l_m, \mu), \tilde{h}_{m,t}(C, l_w, l_m, \mu)\\
&= \arg \max_{\substack{c_{w,t}, c_{m,t}\\ h_{w,t}, h_{m,t}}} \mu u_w(c_{w,t}, f_{w,t}, Q_t) + (1-\mu) u_m(c_{m,t}, f_{m,t}, Q_t)\\
&\text{s. t.}\\
\bar{T} &= l_{j,t} + h_{j,t} + f_{j,t}\\
Q_t &= Q(c_t, H_t) = (\alpha_c c_t^{\omega} + (1-\alpha_c) H_t^{\omega})^{\frac{1}{\omega}}\\
H_t &= H^m(h_{w,t}, h_{m,t}) = \left(\alpha_h h_{w,t}^{\zeta} + (1-\alpha_h) h_{m,t}^{\zeta}\right)^{\frac{1}{\zeta}}\\
c_t &= C - c_{w,t} - c_{m,t}\\
0 &\le h_{j,t} \le \overline{T} - l_j,\ j \in \{w, m\}\\
c_{j,t} &> 0,\ j \in \{w, m\}
\end{aligned}
$$

We solve this problem numerically using the BOBYQA algorithm on a tensor product grid for total consumption, $\overrightarrow{C}$, each spouse's labor supply, $\overrightarrow{l_w}$ and $\overrightarrow{l_m}$, and bargaining power, $\overrightarrow{\mu}$. For each grid point, we set starting values $h_j = \frac{1}{2}\left(\bar{T} - l_j\right)$ and $c_j = \frac{1}{3}C$ for $j = w, m$. Finally, we construct interpolators $c_w^{m,interp}(C, \mu | l_w, l_m)$, $c_m^{m,interp}(C, \mu | l_w, l_m)$, $h_w^{m,interp}(C, \mu | l_w, l_m)$ and $h_m^{m,interp}(C, \mu | l_w, l_m)$, where superscript m indicates the optimal choice in the married state. Note that we interpolate over continuous C and μ, conditional on discrete l_w and l_m

The singles' problem also involves an intertemporal allocation of resources between periods and and intra-temporal choice of consumption and housework. Conditional on total consumption and discrete labor supply choice, the single individual of gender j solves the problem:

$$\begin{aligned}
&\tilde{c}_{j,t}(C,l_j),\tilde{h}_j(C,l_j) = \arg\max_{c_{j,t},h_{j,t}} u_j(c_j, f_{w,t}, Q_t) \\
&\text{s.t.} \\
&\bar{T} = l_{j,t} + h_{j,t} + f_{j,t} \\
&Q_t = Q(c_t, H_t) = (\alpha_c c_t^{\omega} + (1-\alpha_c)H_t^{\omega})^{\frac{1}{\omega}} && \text{(C.1)} \\
&H_t = H_j^s(h_{j,t}) = \left(\alpha_j h_{j,t}^{\zeta}\right)^{\frac{1}{\zeta}}, \quad \alpha_w = \alpha_c, \alpha_m = 1-\alpha_c && \text{(C.2)} \\
&c_t = C - c_{j,t} \\
&0 \leq h_{j,t} \leq \bar{T} - l_j \\
&c_{j,t} > 0
\end{aligned}$$

Similar to the couple's problem, we solve the single's problem numerically over a tensor product grid over total consumption $\overrightarrow{C}$ and labor supply $\overrightarrow{l_j}$ for both genders $l = w, m$. For each grid point, we set starting values $h_j = \frac{1}{2}\left(\bar{T} - l_j\right)$ and $c_j = C/2$. Finally, we construct interpolators $c_j^{s,interp}(C|l_j)$ and $h_j^{s,interp}(C|l_j)$ for $j = w, m$, where superscript s indicates optimal choices in the single state.

C.3 Precomputation of the total consumption interpolator

Singles. We precompute total consumption for singles

$$U_j'(C, l_j) \approx \frac{U_j(C + 0.0001, l_j) - U_j(C, \mu)}{0.00001}$$

where ${U_j}'(C, l_j)$ contain the solution to the intra-temporal problem for singles:

$$\begin{aligned}
{U_j}'(C, l_j) = & \max_{c_j, c, h_j} u_j(c_j, f_j, Q) \\
& \text{s.t.} \\
& \bar{T} = l_j + h_j + f_j \\
& Q = Q(c, H) \\
& H = H_j^s(h_j) \\
& c = C - c_j \\
& 0 \leq h_j \leq \bar{T} - l_j \\
& c_j > 0
\end{aligned}$$

We solve this problem numerically using the BOBYQA algorithm on a tensor product grid of consumption and labor supply, $\overrightarrow{C} \times \overrightarrow{l_j}$ for each gender, $j = w, m$, resulting in a two-dimensional grid of ${U_j}'(C, l_j) = W_j^s(C, l_j)$. By "flipping the axes", we can then construct the interpolator of total consumption, $C_j^{s,interp}(W_j^s | l_j)$. We interpolate over the continuous W_j^s-dimension, conditional on discrete labor supply l_j.

Couples. Similarly, for couples we compute:

$$U'(C, l^i, l^j, \mu) \approx \frac{U(C + 0.0001, l_w, l_m, \mu) - U(C, l_w, l_m, \mu)}{0.00001}$$

where $U(C, l_w, l_m, \mu)$ solves the intra-temporal problem for couples:

$$\begin{aligned}
\max_{\substack{c_w, c_m, c, \\ h_w, h_m}} \quad & \mu u_w(c_w, f_w, Q) + (1 - \mu) u_m(c_m, f_m, Q) \\
& \text{s.t.} \\
& \bar{T} = l_j + h_j + f_j,\ j \in \{w, m\} \\
& Q = Q(c, H) \\
& H = H^m(h_w, h_m) \\
& c = C - c_w - c_m \\
& 0 \leq h_j \leq \bar{T},\ j \in \{w, m\} \\
& c_j > 0, j \in \{w, m\}
\end{aligned}$$

We solve this problem numerically using the BOBYQA algorithm on a tensor product grid of total consumption, each spouse's labor supply and bargaining power, $\overrightarrow{C} \times \overrightarrow{l_w} \times \overrightarrow{l_m} \times \overrightarrow{\mu}$, two-dimensional grid of $U'(C, l_w, l_m, \mu) = W^m(C, l_w, l_m, \mu)$. By "flipping the axes", we can then construct the interpolator of total consumption, $C^{interp}(W^m, \mu | l_w, l_m)$.

C.4 Solution

We solve the model by iterating backwards, starting in the terminal period T. To iterate from t to period $t-1$, we need to know the value of entering period t as single and couple. We describe the construction of these objects below.

C.4.1 Terminal period

The value of entering as single in the terminal period T is:

$$\begin{aligned} V^s_{j,T}(A_{j,T-1}, K_{j,T}) = &\max_{c_{j,T}, c_T, h_{j,T}, l_{j,T}} u_j(c_{j,T}, f_{j,T}, Q_t) \\ &\text{s.t.} \\ &\bar{T} = l_j + h_j + f_j \\ &Q = Q(c, H) \\ &H = H^s_j(h_j) \\ &c = C - c_j \\ &0 \leq h_j \leq \bar{T} - l_j \\ &c_j > 0 \end{aligned}$$

We first solve for optimal consumption and housework *conditional* on the discrete labor market choice. Conditional on labor supply $l_{j,T} = l_j$, the value of entering as single in the terminal period T is a function of total consumption C_T:

$$\begin{aligned} V_{j,T}^{S,l^i}(A_{j,T-1}, K_{j,T}) &= u_j(c_{j,T}, c_T, f_{j,T}, Q_T) \\ &\text{s.t.} \\ c_{j,T} &= c_j^{s,interp}(C_T|l_j) \\ c_T &= c^{s,interp}(C_T|l_j) \\ \bar{T} &= l^i + h_j^{s,interp}(C_T|l_j) + f_{j,T} \\ Q_T &= Q\left(c^{s,interp}(C_T|l_j), H_T)\right) \\ H_T &= H_j^s(h_j^{s,interp}(C_T|l_j)) \end{aligned}$$

where all choice values are interpolated from precomputed interpolators.

In the terminal period, optimal consumption is equal to total resources, $C_T = RA_{j,T-1} + w_{j,t}l_j$ We interpolate $c_{j,t}$ and c_t using the precomputed interpolators, and compute conditional value functions, $V_{j,T}^{S,l_j}(A_{j,T-1}, K_{j,t})$ over asset grids, $\overrightarrow{A_j}$ for $j = w, m$, and the human capital grid $\overrightarrow{K}$. We do this for every point l_j in the discrete labor supply grid, $\overrightarrow{l_j}$, $j = w, m$.

Next, for each point in the grid $\overrightarrow{A_j} \times \overrightarrow{K}$, we take the upper envelope over labor supply choices, $l_j \in \overrightarrow{l_j}$ to determine the optimal choice, and the corresponding value:

$$V_{j,T}^s(A_{j,T-1}, K_{j,T}) = \max_{l_j \in \overrightarrow{l_j}} V_{j,T}^{S,l_i}(A_{j,T-1}, K_{j,t}) \tag{C.3}$$

We define this object over $\overrightarrow{A_j}$, $j = w, m$, and $\overrightarrow{K}$.

Based on this, we can precompute the expected value of entering period T as single, before the realization of the shock to human capital. We do this over the grid of all states $\overrightarrow{A}_j \times \overrightarrow{K}$:

$$EV_{j,T-1}^s(A_{j,T-1}, K_{j,T-1}) = \mathbb{E}_{T-1}\left[V_{j,T}^s(A_{j,T-1}, K_{j,T})\right] = \sum_{q=1}^{\#_K} \omega^q \left[V_{j,T}(A_{j,T-1}, K^q)\right]$$

where we use $\#_K = 5$ Gauss-Hermite quadrature nodes to take expectation over future values of human capital and interpolate $V_{j,T}^s$ using linear interpolation.

Similarly, we precompute the marginal expected value of assets when entering period

T as single:

$$W^s_{j,T-1}(A_{j,T-1},K_{j,T-1}) = \frac{\partial}{\partial A_{j,T-1}} EV^s_{j,T-1}(A_{j,T-1},K_{j,T-1})$$

We approximate this value through centered first differences on the asset grid $\overrightarrow{A_j}$:

$$W^s_{j,T-1}(A_{j,T-1},K_{j,T-1}) \approx \frac{EV^s_{j,T-1}(\overrightarrow{A}[i+1],K_{j,T-1}) - EV^s_{j,T-1}(\overrightarrow{A}[i-1],K_{j,T-1})}{\overrightarrow{A}[i+1]-\overrightarrow{A}[i-1]}$$

We precompute both expectations over the grid of all states $\overrightarrow{A}_j \times \overrightarrow{K}$, for both genders $j = w, m$.

The value of transitioning from marriage to singlehood is identical to the above, apart from a divorce cost,

$$V^{m\to s}_{j,T}(A_{j,T-1},K_{j,T}) = V^s_{j,T}(A_j, T-1) - \chi$$

We similarly define this object over $\overrightarrow{A_j}$, $j = w, m$, and $\overrightarrow{K}$.

The value remaining a couple conditional on post-bargaining weight μ_T is determined in multiple steps, by first maximizing the couple's value function and then inserting values into individual value functions. Since there is no continuation value, optimal choices are:

$$\begin{matrix} \tilde{c}_{w,T}, \tilde{c}_{m,T}, \tilde{c}_T \\ \tilde{l}_{w,T}, \tilde{l}_{m,T}, \tilde{h}_{w,T}, \tilde{h}_{j,T} \end{matrix} = \tag{C.4}$$

$$\arg \max_{\substack{c_{w,T}, c_{m,T}, c_T \\ l_{w,T}, l_{m,T}, h_{w,T}, h_{m,T}}} \mu_T u_w(c_{w,t}, f_{w,t}, Q_t) + (1-\mu_T) u_m(c_{m,t}, f_{m,t}, Q_t) \tag{C.5}$$

$$\text{s. t.}$$

$$\text{eq. } (40) - (47)$$

We first solve the problem conditional on discrete labor supply choice, $l_{w,t} = l_w$, $l_{m,t} = l_m$. Then, the conditional optimal choice becomes a function of total consumption C_T and post-bargaining weight, μ_T, and we can interpolate optimal choices based on the precomputed interpolators:

$$\begin{aligned}
\tilde{c}^{l_w,m}_{j,T} &= c^{m,interp}_{j}(C_T,\mu_T|l_w,l_m), \quad j=w,m \\
c^{l_w,l_m}_{T} &= c^{m,interp}(C_T,\mu_T|l_w,l_m) \\
h^{l_w,l_m}_{j,T} &= h^{m,interp}_{j}(C_T,\mu_T|l_w,l_m), \quad j=w,m
\end{aligned}$$

In the terminal period, total consumption is equal to total resources, $C_T = RA_{T-1} + w_{w,t}l_w + w_{m,t}l_m$. For each value of μ_T in $\overrightarrow{\mu}$, we can interpolate optimal choices based on precomputed interpolators and compute the value. We do this for each point in the grid over states $\overrightarrow{A} \times \overrightarrow{K} \times \overrightarrow{K} \times \overrightarrow{\psi} \times \overrightarrow{\mu}$, and for each point in the discrete labor supply grids $\overrightarrow{l_w} \times \overrightarrow{l_m}$. This defines individual conditional value functions:

$$V^{m\to m,l_w,l_m}_{j,T}(A_{T-1},K_{w,T},K_{m,T},\psi_T,\mu_T) = u_j(\tilde{c}^{l_w,l_m}_{j,T},f_{j,t},Q_t)+\psi_T, \quad j=w,m$$

where Q_t and $f_{j,t}$ follow from the public good production function and the time constraint.

Next, for each point in $\overrightarrow{A} \times \overrightarrow{K} \times \overrightarrow{K} \times \overrightarrow{\psi} \times \overrightarrow{\mu}$, we take the upper envelope of the household over labor supply choices, $l_w, l_m \in \overrightarrow{l_w} \times \overrightarrow{l_m}$ to determine the optimal choice:

$$\begin{aligned}
&\tilde{l}_{w,T},\tilde{l}_{m,T} = \\
&\arg\max_{l_w,l_m\in \overrightarrow{l}_w\times \overrightarrow{l}_m} \mu_T V^{m\to m,l_w,l_m}_{w,T}(A_{T-1},K_{w,T},K_{m,T},K_{w,T},\psi_T,\mu_T) \\
&\qquad +(1-\mu_T)V^{m\to m,l_w,l_m}_{m,T}(A_{T-1},K_{w,T},K_{m,T},K_{w,T},\psi_T,\mu_T)
\end{aligned}$$

and optimal choices $\tilde{c}_{j,t},\tilde{h}_{j,t}, j=w,m$ and $\tilde{c}_t$ follow directly. Finally, individual values are:

$$V^{m\to m}_{j,T}(A_{T-1},K_{w,T},K_{m,T},\psi_T,\mu_T) = u_j(\tilde{c}_{j,T},f_{j,T},Q_t)+\psi_T, \quad j=w,m$$

where Q_t and $f_{j,t}$ follow from the public good production function and the time constraint.

Finally, the marital surplus function is (for an arbitrary value of μ)

$$S_{j,T}(A_{T-1},K_{w,T},K_{m,T},\psi_T,\mu) = V^{m\to m}_{j,T}(A_{T-1},K_{w,T},K_{m,T},\psi_T,\mu) - V_{m\to s}(\kappa_j A_{T-1},K_{j,T})$$

The value of entering a period as a couple includes both the possibility of remaining married and divorcing, such that

$$\begin{aligned} &V^m_{j,T}(A_{T-1}, K_{w,T}, K_{m,T}, \psi_T, \mu_{T-1}) \\ &\qquad = D^\star_T V^{m\to s}_{j,T}(\kappa_j A_{T-1}, K_{j,T}) + (1 - D^\star_T) V^{m\to m}_{j,T}(A_{T-1}, K_{w,T}, K_{m,T}, \psi_T, \mu^\star_T) \end{aligned}$$

This value depends on the outcome of any potential bargaining, $\mu^\star_T$, and divorce, $D^\star_T$, which are updated according the the algorithm described in appendix B by interpolating the marital surplus function $S_{j,T}$ on the bargaining power grid $\overrightarrow{\mu}$. We do this over the grid of all states $\overrightarrow{A} \times \overrightarrow{K} \times \overrightarrow{K} \times \overrightarrow{\psi} \times \overrightarrow{\mu}$.

Knowing this value, we can precompute the expected value of entering period T as married, before the realization of the shocks to human capital and match quality. We do this over the grid of all states $\overrightarrow{A} \times \overrightarrow{K} \times \overrightarrow{K} \times \overrightarrow{\psi} \times \overrightarrow{\mu}$:

$$\begin{aligned} &EV^m_{j,T-1}(A_{T-1}, K_{w,T-1}, K_{m,T-1}, \psi_{T-1}, \mu_{T-1}) \\ &\qquad = \mathbb{E}_{t-1}\left[V^m_{j,T}(A_{T-1}, K_{w,T}, K_{m,T}, \psi_T, \mu_{T-1})\right] \\ &\qquad = \sum_{q=1}^{\#_\psi} \sum_{r=1}^{\#K} \sum_{s=1}^{\#K} \omega^i \omega^n \omega^m \left[V^m_{j,T}(A_{T-1}, K^q_w, K^r_m, \psi^s, \mu_{T-1})\right] \end{aligned}$$

where we use three dimensional Gauss-Hermite quadrature with $\#_\psi = 5$ nodes in the ψ-dimension and $\#_K = 5$ nodes in each K-dimension. We interpolate $V^m_{j,T}$ at the nodes using linear interpolation and construct the couple level expectation:

$$\begin{aligned} &EV^m_{T-1}(A_{T-1}, K_{w,T-1}, K_{m,T-1}, \psi_{T-1}, \mu_{T-1}) \\ &\qquad = \mu_{T-1} EV^m_{w,T-1}(A_{T-1}, K_{w,T-1}, K_{m,T-1}, \psi_{T-1}, \mu_{T-1}) \\ &\qquad + (1 - \mu_{T-1}) EV^m_{m,T-1}(A_{T-1}, K_{w,T-1}, K_{m,T-1}, \psi_{T-1}, \mu_{T-1}) \end{aligned}$$

This allows us to construct an interpolator for the expected value of entering period T in a couple, $EV^{m,interp}_{T-1}$ over the grid of all states $\overrightarrow{A} \times \overrightarrow{K} \times \overrightarrow{K} \times \overrightarrow{\psi} \times \overrightarrow{\mu}$.

Similarly, we can precompute the marginal expected value of assets when entering period T as married:

$$W_{j,T-1}(A_{T-1}, K_{w,T-1}, K_{m,T-1}, \psi_{T-1}, \mu_{T-1}) = \frac{\partial}{\partial A_{T-1}} EV^m_{T-1}(A_{T-1}, K_{w,T-1}, K_{m,T-1}, \psi_{T-1}, \mu_{T-1})$$

We approximate this object through centered first differences on the asset grid, $\overrightarrow{A}$. Let

$\overrightarrow{A}[i]$ and $\overrightarrow{A}[i+1]$ denote consecutive points in $\overrightarrow{A}$. Then:

$$W_{j,T-1}(\overrightarrow{A}[i], K_{w,T-1}, K_{m,T-1}, \psi_{T-1}, \mu_{T-1}) \\ \approx \frac{EV^m_{T-1}(\overrightarrow{A}[i+1], K_{w,T-1}, K_{m,T-1}, \psi_{T-1}, \mu_{T-1}) - EV^m_{T-1}(\overrightarrow{A}[i+1], K_{w,T-1}, K_{m,T-1}, \psi_{T-1}, \mu_{T-1})}{\overrightarrow{A}[i+1] - \overrightarrow{A}[i-1]}$$

where we extrapolate in the end points. We do this over the grid of all states, $\overrightarrow{A} \times \overrightarrow{K} \times \overrightarrow{K} \times \overrightarrow{\psi} \times \overrightarrow{\mu}$, allowing us to construct an interpolator $W^{m,interp}_{j,T-1}$.

C.5 Earlier periods

Solving earlier periods follows a similar approach as the terminal period, except that we use the iEGM to determine total consumption.

The value of starting as single is computed using iEGM. We first solve period t conditional on the discrete labor supply choice, l_j. For each point in the human capital grid $\overrightarrow{K}$ we loop over the grid of post decision assets, $\overrightarrow{A}_{pd}$ and interpolate the marginal expected value, $W^s_{j,t} \approx W^{s,interp}_{j,t}(A_t, K_{j,t})$. Then, we determine total consumption using the precomputed interpolator:

$$C^{l_j}_{j,t}(A_{j,t}, K_{j,t}) = C^{s,interp}_j(W^s_{j,t}|l_j) \tag{C.6}$$

With this, we construct an endogenous grid over resources:

$$\overrightarrow{M}_t = C_{j,t}(\overrightarrow{A}_{pd}) + \overrightarrow{A}_{pd}$$

from which we can now interpolate optimal consumption given beginning-of-period assets $A_{j,t-1} = \frac{1}{R}(M_{j,t-w} - w_{j,t}l_j)$. We enforce the credit constraint by setting consumption equal to total resources for all asset values below the first point in the endogenous grid.

Given this, we can determine optimal consumption and housework based on the preconstructed interpolators from section C.2 and compute the choice specific value:

$$V^{s,l^i}_{j,t}(A_{j,t-1}, K_{j,t}) = u_j(c^{s,interp}_j(C^{l_j}_{j,t}), f_{j,t}, Q_t) + \beta \mathbb{E}_t\left[V_{j,t+1}(A_{j,t}K_{j,t})\right]$$

where $Q_{j,t} = Q_j\left(c^{s,interp}(C^{l_j}_{j,t}|l_j), h^{s,interp}_j(C^{l_j}_{j,t}|l_j)\right)$, and $\bar{T} = l_j + h^{s,interp}_j(C^{l_j}_{j,t}|l_j) + f_{j,t}$, and we interpolate $\mathbb{E}_t\left[V_{j,t+1}(A_{j,t-1}, K_{j,t})\right]$ using $EV^{s,interp}_{j,t}$.

Due to the discrete choice of labor supply, the value function may have non-concave regions, in which case the first order condition is no longer sufficient for an optimum. We deal with this with a "secondary" upper envelope algorithm (see Iskhakov, Jørgensen, Rust and Schjerning (2017); Druedahl and Jørgensen (2017), while also interpolating all values back to the exogenous grid $\overrightarrow{A}$.

We do this for all labor supply choices $l_j \in \overrightarrow{l}_j$. After having constructed these objects over the grid $\overrightarrow{A}_j \times \overrightarrow{K}$ conditional on each labor supply choice, we take a "primary" upper envelope over the conditional value functions to determine the unconditional value:

$$V^s_{j,t}(A_{j,t-1}, K_{j,t}) = \max_{l_j \in \overrightarrow{l}_j} V^{s,l_j}_{j,t}(A_{j,t-1}, K_{j,t})$$

Optimal labor supply $l_{j,t}$ is the argument that maximizes this value, and optimal consumption $c_{j,t}, c_t$ and housework $h_{j,t}$ are the values corresponding to $l_{j,t}$. We compute and store these objects over $\overrightarrow{A_j} \times \overrightarrow{K}$.

The value of remaining a couple is similarly computed by first conditioning on all combinations of discrete labor supply choices, l_w, l_m. Given l_w, l_m, we loop through the grid $\overrightarrow{K} \times \overrightarrow{K} \times \overrightarrow{\psi} \times \overrightarrow{\mu}$ and a grid of post-decision assets $\overrightarrow{A}_{pd}$. Fo each point A_t in $\overrightarrow{A}_{pd}$, we interpolate the marginal expected value $W^{m\to m}_t \approx W^{m\to m,interp}_t(A_t, K_{w,t}, K_{m,t}, \psi_t, \mu_t)$ and determine total consumption using the precomputed interpolator:

$$C^{l_w,l_m}_t(A_t, K_{w,t}, K_{m,t}, \psi_t, \mu_t) = C^{interp}(W^{m\to m}_t, \mu_t | l_w, l_m)$$

From here, the approach is analogous to that of singles: interpolate intra-temporal allocations, perform secondary upper envelope to deal with non-concavities, interpolate to common grid, and determine optimal labor supply l_w, l_m using a "primary" upper envelope.

All other value functions are computed exactly as in the terminal period. The consecutive steps to compute the expected values EV^m_t and expected marginal values W^m_t follow the approach of the terminal period, and the steps can be iteratively repeated until the initial period.

D Taxes

Income taxes depend marital status and total household income, such that for singles the tax function is $\tau^s = \tau^s(y_{j,t})$, $y_{j,t} = w_{j,t}l_{j,t} + rA_{j,t-1}$, and for couples it is $\tau^m = \tau^m(y_t)$, $y_t = w_{w,t}l_{w,t} + w_{m,t}l_{m,t} + rA_{t-1}$. We base our tax functions on those of Bronson, Haanwinckel and Mazzocco (2025). Specifically, we let the income tax schedule have six brackets, each with its own marginal tax rate. Brackets and tax rates are based on the U.S. income tax regime in 2000. Contrary to Bronson, Haanwinckel and Mazzocco (2025), we model only income taxes and abstract from other social welfare-systems and transfers. Tax brackets and marginal tax rates are listed in table D.1.

Table D.1: Tax Brackets and Marginal Tax Rates

Tax Rate	*Singles*	*Couples*
15%	\$0 to \$26,250	\$0 to \$43,850
28%	\$26,250 to \$63,550	\$43,850 to \$105,950
31%	\$63,550 to \$132,600	\$105,950 to \$161,450
36%	\$132,600 to \$288,350	\$161,450 to \$288,350
39.6%	\$288,350 and up	\$288,350 and up

Notes: Brackets and tax rates based on the U.S. 2000 income tax regime, c.f. Bronson, Haanwinckel and Mazzocco (2025) Table A.1.

E Application: Singles and divorce

In this section, we describe the remaining value functions in the model of Section 4. Specifically, we define the value of being single, the value of transitioning from marriage to singlehood and the value of entering a period as married. Together with section 4, this fully describes the application model.

E.1 The Value of Being Single

For a single individual of gender j, the economic environment at time t is characterized by their time invariant type, e_j, assets $A_{j,t-1}$ and human capital $K_{j,t}$. The individual allocates resources between two consumption goods, $c_{j,t}$ and c_t, and savings, and time between market work $l_{j,t}$, housework $h_{j,t}$ and leisure $f_{j,t}$. The problem of the single individual is then

$$
\begin{aligned}
V^s_{j,t}(A_{j,t-1}, K_{j,t}, e_j) = & \max_{\substack{c_{j,t}, c_t, A_{j,t}, \\ l_{j,t}, h_{j,t}, f_{j,t}}} u_j(c_{j,t}, f_{j,t}, Q_{j,t}) + \mathbb{E}_t\left[V^s_{j,t+1}(A_{j,t}, K_{j,t+1}, e_j)\right] \\
s.t. & \\
\bar{T} &= l_{j,t} + h_{j,t} + f_{j,t} \\
Q_{j,t} &= \left(\alpha_c c_t^{\omega} + (1-\alpha_c)(\alpha_j h_{j,t})^{\omega}\right)^{\frac{1}{\omega}} \\
\log w_{j,t} &= e_j + \gamma_j K_{j,t} \\
K_{j,t+1} &= \left[(1-\delta)K_{j,t} + l_{j,t}\right]\varepsilon_{j,t+1}, \quad \log \varepsilon_{j,t} \sim iid\mathcal{N}\left(-\frac{1}{2}\sigma^2_{\varepsilon,j}, \sigma^2_{\varepsilon,j}\right) \\
A_{j,t} &= (1+r)A_{j,t-1} + w_{j,t}l_{j,t} - \tau^s - c_{j,t} - c_t
\end{aligned}
$$

where the uncertainty over the continuation value stems from shocks to human capital, $\varepsilon_{j,t+1}$. Note that singlehood is absorbing.

E.2 The Value of Transitioning from Marriage to Singlehood

The value of transitioning from marriage to singlehood is important for the limited commitment model, as it denotes the outside option value of each spouse and hence determines i) whether participation constraints bind and ii) how the bargaining power is updated, when they do. For an individual of gender j in a couple, this value depends on total couple assets A_{t-1}, as well as their own type e_j and human capital $K_{j,t}$. Given these states, it is similar to that of singlehood:

$$V_{j,t}^{m\to s}(A_{t-1}, K_{j,t}, e_j) = V_{j,t}^{s}(\kappa_j A_{t-1}, K_{j,t}, e_j)$$

where κ_j denotes the share of assets allocated to member j at divorce. We set κ_j equal to $\frac{1}{2}$ for $j \in \{w, m\}$ in the model.

E.3 The Value of Entering a Period as a Couple

The value of entering a period in a couple, *pre-bargaining*, has three central components: 1) the value of remaining a couple, 2) the value of transitioning to singlehood and 3) the bargaining protocol:

$$\begin{aligned}
V_{j,t}^{m}(A_{t-1}, &K_{w,t}, K_{m,t}, e_w, e_m, \psi_t, \mu_{t-1}) \\
&= (1 - D_t^\star) V_{j,t}^{m\to m}(A_{t-1}, K_{w,t}, K_{m,t}, e_w, e_m, \psi_t, \mu_t) + D_t^\star V_{j,t}^{m\to s}(A_{t-1}, K_{j,t}, e_j) \\
&s.t. \\
&\mu_t = \mu_t^\star(A_{t-1}, K_{w,t}, K_{m,t}, e_w, e_m, \psi_t, \mu_{t-1}) \\
&D_t^\star = D_t^\star(A_{t-1}, K_{w,t}, K_{m,t}, e_w, e_m, \psi_t)
\end{aligned}$$

where $\mu_t^\star$ and $D_t^\star$ denote the outcomes of limited commitment bargaining. For this purpose, let

$$S_{j,t}(A_{t-1}, \mathcal{S}_t, \mu) = V_{j,t}^{m\to m}(A_{t-1}, K_{w,t}, K_{m,t}, e_w, e_m, \psi_t, \mu) - V_{j,t}^{m\to s}(A_{t-1}, K_{j,t}, e_j)$$

denote the marital surplus of household member j as a function of bargaining power μ, assets, A_{t-1} and remaining states, $\{K_{w,t}, K_{m,t}, e_w, e_m, \psi_t\} = \mathcal{S}_t$. We define the "indifference point" $\tilde{\mu}_j = \tilde{\mu}_j(A_{t-1}, \mathcal{S}_t)$ as the value of μ that makes member j indifferent between staying married and transitioning to singlehood:

$$\tilde{\mu}_j = \{\mu : S_{j,t}(A_{t-1}, \mathcal{S}_t, \mu) = 0\}.$$

This gives the updating rule for bargaining power $\mu_t^\star$

$$\mu_t^\star = \begin{cases} \mu_{t-1} & \text{if} \quad S_{j,t}(A_{t-1}, \mathcal{S}_t, \mu_{t-1}) \geq 0 \quad \text{for } j \in \{w, m\} \\ \tilde{\mu}^w & \text{if} \quad S_{w,t}(A_{t-1}, \mathcal{S}_t, \mu_{t-1}) < 0 \quad \text{and} \quad S_{m,t}(A_{t-1}, \mathcal{S}_t, \mu_{t-1}) \geq 0 \\ \tilde{\mu}^m & \text{if} \quad S_{m,t}(A_{t-1}, \mathcal{S}_t, \mu_{t-1}) < 0 \quad \text{and} \quad S_{w,t}(A_{t-1}, \mathcal{S}_t, \mu_{t-1}) \geq 0 \\ \emptyset & \text{else} \end{cases}$$

along with the divorce choice

$$D_t^\star = \begin{cases} 1 & \text{if} \quad \mu_t^\star = \emptyset \\ 0 & \text{else} \end{cases}.$$