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Fertility and Family Labor Supply*

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Abstract

We study the role of fertility adjustments for the labor market responsiveness of men and women. First, we use longitudinal Danish register data and tax reforms from 2009 to provide new empirical evidence on asymmetric fertility adjustments to tax changes of men and women. Second, we quantify the importance of these fertility adjustments for understanding the labor supply responsiveness of couples through a life-cycle model of family labor supply and fertility. Allowing fertility adjustments increases the labor supply responsiveness of women by 28%. These adjustments affect human capital accumulation and has permanent implications for the gender wage gap within couples.

Keywords: Fertility, Labor supply, Human capital accumulation, Gender inequality, Tax reform, Life-Cycle.
JEL Codes: J22, J13, D15, H24

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1 Introduction

Understanding labor supply is a key part of evaluating policy reforms. Labor supply choices are not made in isolation of other decisions, however, and in particular the arrival and rearing of children is deeply interlinked with labor market behavior.[1] This interlink appears especially strong for women, and children are today seen as one of the main drivers of labor market gender inequality (Kleven, Landais and Søgaard, 2019; Cortes and Pan, forthcoming). Endogenous fertility adjustments to wage and tax changes may thus affect the labor supply responsiveness of men and women through what we refer to as a "fertility multiplier" effect. While this fertility feedback channel is likely key for our understanding and evaluation of labor market reforms and their long run implications, we know very little about the importance of this fertility multiplier. We fill this gap by quantifying how important fertility adjustments are for the labor market responsiveness of men and women and long run gender inequality.

Our first contribution is to provide new empirical evidence that fertility responds to tax changes, not specifically targeted to parents with children. We use longitudinal register data on Danish couples and labor income tax reforms, especially in 2009/2010, to pin down the extent to which fertility decisions are affected by changes in the marginal net-of-tax wage rates of men and women, analogous to the elasticity of taxable income literature (see e.g. Gruber and Saez, 2002). We find strong asymmetric effects: Increased marginal net-of-tax wage rates of women tend to reduce fertility of couples while increased marginal net-of-tax wage rates of men tend to increase fertility. Our results imply that the substitution effect between children and labor supply is dominating for women while the income effect is dominating for men.

Our second contribution is to quantify the importance of such fertility adjustments for the labor market responsiveness of men and women. To quantify this, we estimate a dynamic life-cycle model of fertility choices and family labor supply. We compare behavior in our estimated model with a counterfactual scenario in which couples cannot adjust fertility plans in response to changes in economic incentives. Our results show that fertility adjustments are key to understand the labor supply responsiveness, especially of women. We show that the asymmetric effects we find have long run consequences. An increase in men's wage that increases fertility and reduces women's labor supply, leads to a reduced long-run wage for women and so a further worsening of the gender gap within the household. By contrast, the increase in the women's wage that reduces fertility and increases labor supply, leads to an increase in the long-run wage and further dampens the

[1] There is a vast literature on the impact of children on labor market outcomes, see for example Rosenzweig and Wolpin (1980); Waldfogel (1998); Simonsen and Skipper (2006, 2012); Bertrand, Goldin and Katz (2010); and Kleven, Landais and Søgaard (2019).

gender gap within the household. We refer to these amplification and dampening effects as the fertility multiplier.

In each period of the model, couples choose how much to consume or save, whether to try to conceive (more) children, and whether each household member should work and if so whether to work part or full time. Men and women accumulate human capital through on-the-job learning by doing and wages are thus endogenous to labor supply and connected to fertility decisions. Parents derive direct utility from children but the number and age of children also affect the dis-utility from market work, potentially in different ways for men and women. Biological fecundity declines with age and there is a tension between human capital accumulation motives and fertility in the model. This implies that both men and women trade off the number and timing of children with human capital investments.

We estimate the model using detailed Danish data on couples from 2010–2018 by Simulated Minimum Distance. We include moments related to labor market work and fertility over the life cycle along with changes in labor market work after childbirths. The latter moments are especially informative about the dis-utility from market work when children are present. Although not directly targeted, the model reproduces our empirical finding that increases in wages of women lead to reduced fertility while increases in wages of men lead to increases in fertility. Labor market elasticities simulated from the model are also within the ranges typically reported in the literature (see e.g. Keane, 2011, forthcoming; Attanasio, Levell, Low and Sánchez-Marcos, 2018). We estimate average life-cycle Marshallian hours elasticity of 0.60 for women and 0.09 for men. As most existing literature, we find responses along the extensive margin to be important with participation elasticities of 0.54 for women and 0.06 for men.

We quantify the importance of fertility adjustments through counterfactual simulations within our framework. Concretely, we compare the labor hours elasticities in the baseline estimated model with that from an alternative version of the model, in which couples *cannot* adjust their fertility. In the alternative model, fertility is exogenous and stochastic, as often assumed in existing studies estimating labor market elasticities, such as Attanasio, Levell, Low and Sánchez-Marcos (2018). We assume that couples in the alternative model have fertility expectations which are internally consistent with the baseline model. The life-cycle Marshallian hours elasticity of women in our baseline model is around 28% higher than that of women in the alternative exogenous fertility model. We find only small differences in the labor supply responses of men comparing across the two specifications but increased fertility is an important driver of the cross-elasticity of women with respect to the wages of men. There are substantial long run implications of this fertility channel on women's offer wage at age 55: The cross-elasticity on the offer wage is 25% (8pp) larger when fertility can respond. Combined, our suggests that fertility adjustments are key for labor supply responses of women in the short and long run and have substantial consequences for the gender wage gap.

Finally, we show that fertility adjustments are important for evaluating policies targeted families with children, such as childcare subsidies. Although these subsidies are transitory, we again find an asymmetric effect on men and women. Increased child subsidies tend to increase fertility, lower labor market work of women and increase labor market work of men. Increasing the newborn child subsidy by 3,000 Danish Kroner (approximately $550) reduces women's hours worked by around 2.2%, increases men's hours worked by 0.13%, and increases completed fertility by around 3.7%.[2] The labor market responses to the extra income are to a great extend exacerbated by increased fertility. Women's hours worked is only reduced by around 0.2% in the model if fertility cannot adjust. Ignoring behavioral responses to child care and subsidy reforms likely lead to substantial underestimates of the the long run gender wage gap and the fiscal cost of the increased child subsidies.

We contribute to two main strands of literature. We contribute to a growing literature analyzing women's labor supply through the lens of dynamic economic models.[3] Most existing studies of women's labor supply treat fertility as exogenous, with some notable exceptions including Moffitt (1984); Hotz and Miller (1988); Francesconi (2002); Sheran (2007); Keane and Wolpin (2010); Adda, Dustmann and Stevens (2017); Yamaguchi (2019); and Eckstein, Keane and Lifshitz (2019).[4] The studies closest related to ours are Adda, Dustmann and Stevens (2017) and Eckstein, Keane and Lifshitz (2019). Adda, Dustmann and Stevens (2017) model endogenous educational attainment, fertility, female labor supply and occupational choice of women along with endogenous wealth accumulation. Male labor supply is exogenous in their model, however. In our model, we treat men and women symmetrically to allow for non-separability between labor supply and fertility for both men and women. Eckstein, Keane and Lifshitz (2019) model both male and female labor supply as endogenous choices along with educational attainment, fertility and marriage and divorce. They do not allow households to save, however. More importantly, existing research has not quantified the importance of fertility adjustments

[2] This impact on fertility is comparable with existing studies (see e.g. Milligan, 2005; Haan and Wrohlich, 2011; Guner, Kaygusuz and Ventura, 2020; Wang, 2022).

[3] There is a large literature related to our paper which analyzes dynamic labor supply of men with endogenous human capital accumulation. See e.g. Eckstein and Wolpin (1989); Keane and Wolpin (1997); Imai and Keane (2004) and the reviews by Keane (2011, forthcoming).

[4] A non-exhaustive list of key contributions to the literature on women's labor supply: Heckman and Macurdy (1980); Rosenzweig and Wolpin (1980); Moffitt (1984); Hotz and Miller (1988); Eckstein and Wolpin (1989); Klaauw (1996); Francesconi (2002); Sheran (2007); Keane and Wolpin (2010); Guner, Kaygusuz and Ventura (2012); Bronson (2015); Bick (2016); Greenwood, Guner, Kocharkov and Santos (2016); Blundell, Dias, Meghir and Shaw (2016); Adda, Dustmann and Stevens (2017); Attanasio, Levell, Low and Sánchez-Marcos (2018); Yamaguchi (2019); Eckstein, Keane and Lifshitz (2019); Guner, Kaygusuz and Ventura (2020); Bronson and Mazzocco (2021) and Borella, De Nardi and Yang (forthcoming).

for the labor market responsiveness and the gender wage gap, which is our main focus.

We contribute to a growing literature on how fertility adjusts to financial incentives. Previous research has shown that the likelihood of giving birth responds to financial incentives directly targeted on fertility, such as child subsidies and tax reliefs (see e.g. Rosenzweig, 1999; Milligan, 2005; Brewer, Ratcliffe and Smith, 2012; Cohen, Dehejia and Romanov, 2013; Laroque and Salanié, 2014), and child care costs (Blau and Robins, 1989; Del Boca, 2002; Mörk, Sjögren and Svaleryd, 2013). Our focus is different: We provide the first empirical evidence that fertility responds to changes in taxes not directly linked to the presence of children.[5]

The paper proceeds as follows: In Section 2, we describe the Danish data used throughout and in Section 3 we present our empirical findings that increased female wages reduce fertility while increased male wages increases fertility. We present a life-cycle model of families that capture the trade-off between labor market supply and fertility in Section 4. In Sections 5 and 6, respectively, we discuss the calibration and estimation of the model. Our key findings are in Section 7 where we use our model to simulate counterfactual scenarios and dissect the importance of fertility adjustments for labor supply responses in the short and long-run. In Section 8 we explore the sensitivity and robustness of our results before we conclude in Section 9.

2 Data and Institutional Background

We use longitudinal Danish administrative register data on the universe of Danish individuals from 2004 through 2018. Our use of this high quality data is twofold. First, in Section 3, we utilize Danish tax reforms from 2009 to 2018 to document how fertility responds to changes in the marginal net-of-tax wage rate. Second, we use detailed labor market information from 2010–2018 to estimate our life-cycle model in Section 6. We refer to the two samples as the "tax sample" and "estimation sample", respectively.

We link individuals to potential partners through Statistics Denmark's definition of a family, including both married and cohabiting couples.[6] We link children through the mother and father ID of newborns and adoptees. The main variables used throughout,

[5] Heckman and Walker (1990) find that fertility is decreasing in wages of women but slightly decreasing in wages of men. More recently and exploring natural experiments, Black, Kolesnikova, Sanders and Taylor (2013) and Kearney and Wilson (2018) find that fertility is increasing in men's wages and Schaller (2016) and Autor, Dorn and Hanson (2019) find that improved local labor markets of men increase fertility while improved local labor markets of women decrease fertility. Keller and Utar (forthcoming) find that worsened Danish local labor market conditions from Chinese import competition lead to increased fertility and reduced labor market work of women but not of men.

[6] Families are both married and cohabiting couples. Cohabiting couples are defined as either two adults living at the same address who are registered as parents to a child, or two adults of opposite sex with an absolute age-difference less than 15 years, registered as living at the same address.

besides fertility and the number of children, are income and labor supply indicators, as we will describe below. Detailed variable definitions are given in Section A in the Supplemental Material.

In the estimation sample, used to estimate the model in Section 6, we utilize detailed monthly pay-slip information through the Danish eIndkomst register (BFL). This data is only available in the last part of our sample period and the estimation sample thus includes the years 2010–2018. This data is well suited for our purpose because we can define the degree of labor market participation based on hours worked as reported on the pay-slip and we can use the monthly frequency to accurately account for changes around childbirths. Unless otherwise noted, we aggregate monthly pre-tax labor income and working hours to the annual level using the calendar year. When we calculate estimation moments around child arrival, however, we center the year around the childbirth such that income and hours worked in the birth year is the sum of 12 months from the month of birth.

Depending on the employment contract, hours worked are either contracted hours or actual hours. Many employees are hired on fixed-pay contracts and do not get overtime pay.[7] We therefore primarily use hours worked to construct indicators of labor market participation and what we will refer to as part time and full time work. Concretely, an individual is said to participate in the labor market if working at least 481 hours annually (e.g. 37 hours in 13 weeks) with an annual income of at least 50,000DKK ($8900).[8] We denote part time work as working between 481 and 1,664 hours annually (e.g. 32 hours in 52 weeks) and full time work as working more than 1,664 hours annually. Although this measure is arguably one of "part year" work, we refer to this as "part time" work throughout.

In the tax sample (going back to 2004), the eIndkomst register is not available. We therefore construct labor income and personal income measures using the annual income tax data. The main components of personal income is labor earnings and transfers along with profits from own businesses. These measures are only used in the tax sample in the section below but are also constructed in the estimation sample for comparison.

We use additional characteristics such as educational attainment and labor market experience. Educational attainment is measured as the highest degree earned in the sample period and we define high skilled as those with at least a bachelor's degree (similar to a college degree in the US) and less skilled as those with less education than that.

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[7] Around 36 percent of the survey respondents in the Danish labor force survey (1st quarter, 2017), who worked overtime in the week of the survey, received overtime pay. Since there is likely a positive selection into overtime work of people with overtime pay, 36 percent is likely an upper bound for the share of workers with overtime pay.

[8] The normal full-time working week is 37 hours in Denmark and the average exchange rate in 2010 was 5.6DKK/USD or 8.7DKK/GBP.

Labor market experience is the definition used by Statistics Denmark using all previous special pension payments (ATP). These are mandatory payments for wage workers and is a function of hours worked. We focus on individuals aged 25 through 60 with an opposite-sex partner and discard observations where the individual is registered as mainly student, self-employed, retired or on disability pension. Finally, we drop observations with missing information on key variables.

In Table 1, we report descriptive statistics for the estimation and tax samples. In the estimation sample, for which descriptive statistics are reported in the columns 1–2 of Table 1, we further restrict attention to 2010–2018 and focus on couples with at most five years age difference. The restriction on age-differences between partners in the estimation sample is to increase homogeneity in age when matching moments from the model. In the tax sample, shown in columns 3–4, we use all years from 2004-2018, but focus on coupled women aged 25–40 where both household members have personal income in the range DKK50,000–600,000 ($8,900-$107,000).

In the estimation sample, the average personal income is around 293,770DKK ($52,400) for women and 401,740DKK ($71,700) for men. Around 88% of women work while around 92% of men do. The average age is around 43 for both men and women and the average number of children is around 1.8 for women and 1.7 for men. 74% of women and 72% of men are married and 47% of women and 40% of men are high skilled. The tax sample focuses on a more narrow group, namely 25-40 year old women and is thus slightly different from the estimation sample. Because the average age is mechanically lower, the share working is lower and so is the income, number of children and share married. The share of high skilled is slightly higher in this group.

2.1 Institutional Background

The Danish tax system is progressive in marginal tax rates, and has traditionally been composed of three income tax brackets, and is to a large extend individual. Figure 1 illustrates the main changes in the Danish statutory labor income tax schedule throughout the sample period. Panel a) shows the evolution of the marginal tax rate in the three main income tax brackets and panel b) shows the associated income thresholds (deductions) in real 2010 values. From 2004 until 2008, the tax system was quite stable in terms of both marginal tax rates and tax bracket thresholds, with one of the worlds highest marginal income tax rates in the top bracket of around 63%.

A 2009/2010 tax reform significantly reduced the marginal tax rate of a large share of the working population by increasing the middle tax bracket to the level of the top tax bracket in 2009, before removing the middle tax bracket completely in 2010. The marginal tax rate in the top bracket was also reduced in 2010 to around 58%. Another change in 2010, not depicted in the Figure, was how unused middle tax deductions could be

Table 1: Descriptive Statistics.

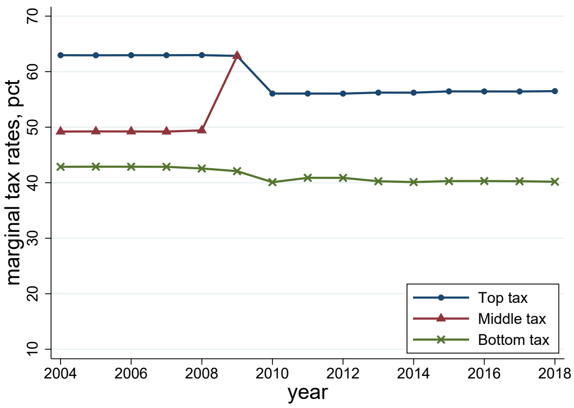
	Estimation sample: all aged 25–60 w. partners 2010–2018		Tax sample: women aged 25–40 w. partners 2004–2018	
	mean	std	mean	std
Personal income, women 203.95	293.77	200.95	262.16	81.22
Personal income, men 579.03	401.74	406.02		
Working, women 0.37	0.88	0.32	0.83	0.38
Working, men 0.30	0.92	0.26		
Age, women 9.70	43.09	9.73	33.24	4.39
Age, men 9.63	43.17	9.71		
Children, women 1.06	1.83	1.04	1.52	1.07
Children, men 1.11	1.73	1.07		
Married, women 0.43	0.74	0.44	0.61	0.49
Married, men 0.45	0.72	0.45		
High skilled, women 0.49	0.47	0.50	0.49	0.50
High skilled, men 0.48	0.40	0.49		
	BFL			
Labor income, women	316.78	161.93		
Labor income, men	433.86	358.24		
Working, women	0.88	0.32		
Working, men	0.92	0.26		
Part time, women	0.30	0.46		
Part time, men	0.17	0.38		
Full time, women	0.58	0.49		
Full time, men	0.75	0.43		
Observations	10,851,444		2,531,181	

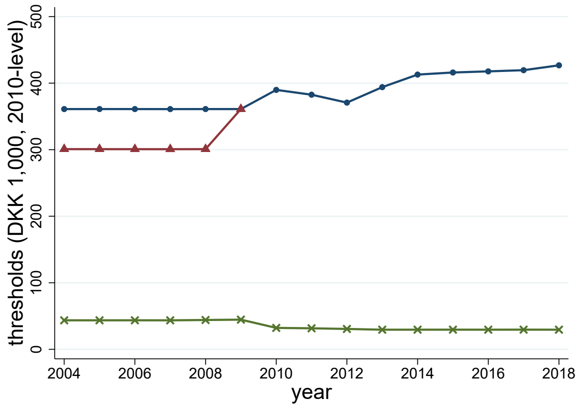
Notes: This table reports descriptive statistics and number of observations (individual-time) for the "estimation sample" used to estimate the model in Section 6 and the "tax sample" used to estimate the causal effect of tax changes on fertility in Section 3. Both are sub-samples of a baseline sample. The baseline sample is based on all individuals aged 25 through 60 with an opposite-sex partner who are not registered as mainly being a student, self-employed, retired or on disability pension. We also exclude individuals with missing information on key variables. The estimation sample is based on the baseline sample but only includes the years 2010–2018 and discard couple with more than five years age difference. The tax sample is also based on the baseline sample but restricts attention to women aged 25–40. All financial variables are in 1,000 of Danish kroner (DKK) and in 2010 real values using the Danish CPI.

Figure 1: Danish Tax Variation, 2004–2018.

(a) Marginal tax rates.

(b) Thresholds.





Notes: This Figure illustrates the main tax variation in the statutory tax thresholds and marginal tax rates from 2004 through 2018 (average across municipalities). Source: Jakobsen and Søgaard (2019).

transferred between spouses prior to the 2010 reform. Unused deductions in the bottom tax bracket can be transferred between spouses throughout. From 2012, the top tax bracket threshold increased steadily. The dip in 2010 stems from a fixed nominal threshold in 2009 and 2010. The marginal tax rate is lowered in the bottom tax bracket from 2010 as is the real threshold in that year. See also e.g. Kreiner, Leth-Petersen and Skov (2016) for a description of the 2010 reform.

3 Empirical Fertility Responses to Tax Changes

Our contribution to this literature is to study fertility adjustments to tax changes on households. We follow the large elasticity of taxable income literature (see e.g. Gruber and Saez, 2002; Kleven and Schultz, 2014; and Jakobsen and Søgaard, 2019 and reference therein). The large degree of individual taxation in the Danish tax system enables us to study responses on the household level from changes in member's marginal tax rates, something that would be virtually impossible in the US, for example, where taxation is joint on the household level.

Let $\tau_{i,t} = \tau_t(z_{i,t}, Z_{i,t})$ denote the marginal tax-rate given the tax schedule at time t, personal income $z_{i,t}$ and other characteristics $Z_{i,t}$ (such as marital status) and let $\tau_{partner(i,t)}$ similarly denote the marginal tax rate of the male partner. Furthermore, denote $y_{i,t}$ and $y_{partner(i,t)}$ as the virtual income of the woman and man, respectively, calculated as in e.g. Gruber and Saez (2002). We then estimate equations of the form

$$\begin{aligned}\Delta_4 N_{i,t} =& \eta_w \Delta_4 \log(1-\tau_{i,t}) + \gamma_w \Delta_4 \log(y_{i,t}) + \eta_m \Delta_4 \log(1-\tau_{partner(i,t)}) + \gamma_m \Delta_4 \log(y_{partner(i,t)}) \\ &+ \beta X_{i,t} + g(z_{i,t}, z_{partner(i,t)}) + \varepsilon_{i,t} \qquad (1)\end{aligned}$$

where $N_{i,t}$ is the number of children of woman i at time t and $\Delta_4 x_{i,t} = x_{i,t+4} - x_{i,t}$ are four-year forward differences. The parameters η_w and η_m are proportional to the compensated elasticity of fertility w.r.t women's and men's marginal net-of-tax wages, respectively, and γ_w and γ_m are proportional to the income elasticities.

The controls in $X_{i,t}$ include age and year dummies, number of children dummies, a high skilled dummy, and a quadratic polynomial in work experience. Besides year and children dummies, we include separate controls for men and women. To control for mean-reversion, we include flexible controls for both household member's income through $g(z_{i,t}, z_{partner(i,t)})$. Specifically, we follow the approach in Jakobsen and Søgaard (2019) and include DKK10,000-interval income dummies for both partners in the regressions.

Marginal tax rates are likely endogenous to the behavior of couples. We again follow the existing elasticity of taxable income literature and use mechanical tax rate changes as instruments. Let $\Delta_4 \tau_{i,t}^m \equiv \log(1 - \tau_{t+4}(z_{i,t}, Z_{i,t})) - \log(1 - \tau_t(z_{i,t}, Z_{i,t}))$ be the mechanical change in the net-of-tax rate change due to a change in the tax system from t to $t+4$ while keeping individual characteristics fixed at year-t values. We then instrument $\Delta_4 \log(1 - \tau_{i,t})$, $\Delta_4 \log(1 - \tau_{partner(i,t)})$, $\Delta \log(y_{i,t})$ and $\Delta \log(y_{partner(i,t)})$ using mechanical tax rate changes. We predict the mechanical tax rate changes using a Danish tax-simulator in the spirit of TAXSIM for the US, extended from Kleven and Schultz (2014) and Jakobsen and Søgaard (2019). The tax simulator includes the changes in the statutory tax rates discussed above but also differences across individuals due to e.g. family structure and capital income. We discuss the validity of the instrument in Supplemental Material A.1.

Our main 2SLS estimation results of the effect of marginal net-of-tax wages on fertility are reported in Table 2. Each column reports the estimated effect on the change in the number of children gradually adding more controls across columns. The first-stage results are reported in Tables A.1–A.4 in the Supplemental Material.

We estimate a significant negative compensated elasticity w.r.t. wages of women ($\hat{\eta}_w$ has a p-value of 0.021) and a significant positive income effect from wages of men ($\hat{\gamma}_m$ has a p-value of 0.0005). The compensated elasticity w.r.t. wages of men and the income effect from wages of women are insignificant (p-values of 0.589 and 0.096, respectively). In turn, this suggests that the fertility substitution effect dominates w.r.t. wages of women while the fertility income effect dominates w.r.t. wages of men.

Our results suggests that children are normal goods and are very much in line with predictions of Becker, 1973: If women are the primary caregiver for children, the income effect w.r.t. wages of men will likely dominate and increased male wages would lead to increased fertility, as we find empirically. Whether the income effect or the substitution effect would dominate w.r.t. wages of women is theoretically more unclear. Our results suggests that the substitution effect dominates.

The results are also in line with existing research showing that increased wages of women decrease fertility (see e.g. Haan and Wrohlich, 2011) and increased wages of men

Table 2: 2SLS Estimates: Number of Children.

	(1)	(2)	(3)
$\Delta_4 \log(1-\tau_{i,t})$, women	-0.035***	-0.023**	-0.023**
	(0.010)	(0.010)	(0.010)
$\Delta_4 \log(y_{i,t})$, women	0.003	0.004*	0.005*
	(0.003)	(0.003)	(0.003)
$\Delta_4 \log(1-\tau_{i,t})$, men	0.008	0.005	0.005
	(0.009)	(0.009)	(0.009)
$\Delta_4 \log(y_{i,t})$, men	0.020**	0.026***	0.028***
	(0.008)	(0.008)	(0.008)
Income dummies	Yes	Yes	Yes
Children dummies	Yes	Yes	Yes
Year dummies	Yes	Yes	Yes
Age dummies	Yes	Yes	Yes
Hum. cap. controls, women	No	Yes	Yes
Hum. cap. controls, men	No	No	Yes
Avg. dep. var. (y, level)	1.52	1.52	1.52
Obs.	2,531,181	2,531,181	2,531,181
First stage F-stat.	27,585.8	27,869.9	27,903.8

Notes: This table shows estimated parameters, η_w, γ_w η_m, and γ_m from equation (1) using 2SLS. As instruments for the change in the net of marginal tax rates and virtual income we use the mechanical net of marginal tax rate changes, fixing information as in the base-year. All regressions include DKK10,000 bin income dummies for both women and men, age dummies for both men and women, year dummies and number of children dummies. Column (2) also includes a quadratic polynomial in labor market experience of women and an indicator equal to one if she has at least a bachelor's degree. Column (3) adds similar human capital variables for men. The data used for estimation is the "tax sample" discussed in connection to Table 1. Robust standard errors in brackets are clustered at the individual level. $^*p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

increase fertility (Black, Kolesnikova, Sanders and Taylor, 2013 and Kearney and Wilson, 2018). Schaller (2016) and Autor, Dorn and Hanson (2019) find that fertility increases if the local labor market demand for men improves but decreases if the local labor market demand for women improves.

We have implemented several alternative specifications to investigate the robustness and heterogeneity of our results. Table A.5 in the Supplemental Material shows estimates from our preferred specification with different minimum income thresholds. Our baseline minimum income in base years are DKK50,000, which is reproduced in the second column. Reassuringly, the results are quite robust to changing this threshold.

We investigate whether there is heterogeneity in fertility responses across the income distribution in Table A.6 in the Supplemental Material. The first two columns show estimates from two groups of couples in which personal income of the woman is in the range $50,000$–$350,000$ in column (1) and in $350,000$–$600,000$ incolumn (2). We find that the effects are slightly stronger for the lower-income couples.

We report separate estimation results split by women's educational attainment in the third and fourth columns of Table A.6. The effects are slightly larger and more significant for less skilled women. For men of less skilled women we estimate a significantly positive compensated elasticity (and no income effect) while for men with high skilled partners, we find a significant positive income effect and insignificant compensated elasticity.

We report in Table A.7 in the Supplemental Material estimated labor income elasticities using a similar specification as above where the left hand side variable is either log-labor income of women or that of their male partners. We estimate a significant positive compensated labor income elasticity of around 0.21 for women and 0.20 for men, in the range of what other studies have found (see e.g. Gruber and Saez, 2002; Kleven and Schultz, 2014 and Jakobsen and Søgaard, 2019). We also estimate negative own income elasticities for both, but only significant for women.

4 Life-Cycle Model

We now turn to the life cycle model of fertility and family labor supply that we use to quantify the importance of fertility adjustments for labor market responsiveness of men and women. A period in the model is a year and in each period couples maximize the expected discounted sum of utility from the remaining part of their life by choosing how much to consume and save for the future, how much each household member should work, and whether to try to conceive more children.

We let $e_t = 1$ denote if a couple chose to exert effort to conceive a child, and $e_t = 0$ if not. Whether a child arrives in the following period is then probabilistic. We allow for three discrete levels of labor supply of women and men: Not working, part time work, and full time work, denoted as $l_{j,t} \in \{0, l_{PT}, 1\}$ for $j \in \{w, m\}$. Finally, we let C_t denote consumption and A_t the amount of savings.

Households make their decisions while taking a range of state variables into account. These are collected in $\mathcal{S}_t = (A_{t-1}, K_{w,t}, K_{m,t}, n_t, o_t, job_{w,t}, job_{m,t})$ where $A_{t-1} \geq 0$ is beginning-of-period wealth, $K_{w,t}, K_{m,t} \geq 0$, are human capital of women and men, $n_t \in \{0, 1, 2, 3\}$ is the number of children living in the household, and $o_t \in \{NC, 0, 1, 2, 3, 4, 5, 6+\}$ denotes the age of the youngest child, where NC indicates "no children" in case $n_t = 0$. Finally, $job_{j,t} \in \{0, 1\}$ indicates if member j is restricted in the job opportunities in the beginning of the period. If $job_{j,t} = 0$ the labor market supply is restricted to be $l_{j,t} = 0$ and member j is thus unemployed. We allow for partnership dissolution and denote $s_t \in \{0, 1\}$ as an indicator equal to one if single and zero otherwise. Below, we describe the decision problem of couples before discussing the environment for singles.

4.1 Preferences

Household utility is a weighted sum of individual utilities

$$U(C_t, n_t, o_t, e_t, l_{w,t}, l_{m,t}) = \lambda u_w(C_t, n_t, o_t, l_{w,t}) + (1-\lambda)u_m(C_t, n_t, o_t, e_t, l_{m,t})$$

where λ is the bargaining power of the woman in the household.[9] The utility of household member j in period t is

$$\begin{aligned} u_j(C_t, n_t, o_t, e_t, l_{j,t}) = & \frac{(C_t/\nu(n_t))^{1-\rho}}{1-\rho} \\ & + \sum_{k=1}^{3} \omega_k \mathbf{1}(n_t \geq i) \\ & + \eta_0 e_t \mathbf{1}(o_t = 0) + \eta_1 e_t \mathbf{1}(o_t = 1) \\ & + f_j(l_{j,t}, age_{j,t}) \\ & + q_j(l_{j,t}, n_t, o_t)\mathbf{1}(n_t > 0) \end{aligned}$$

consisting of five components. The first component is a standard constant relative risk aversion (CRRA) value of consumption with ρ being the CRRA coefficient and $\nu(n_t) = 1 + 0.5(1-s_t) + 0.3n_t$ being OECD equivalence scales. The second component relates to the direct value of having children and ω_1, ω_2 and ω_3 measures the value of having at least 1, 2 or 3 children, respectively. Children also affect the marginal utility of consumption through $\nu(n_t)$ and the dis-utility from market work through the fourth term, $q_j(l_{j,t}, n_t, o_t)$. The third term is related to the spacing of childbirths. Concretely, η_0 and η_1 captures potential dis-utility (if negative) of trying to conceive a child if the youngest child is zero or one years old, respectively.

The fourth term captures the dis-utility from working,

$$\begin{aligned} f_j(l_{j,t}, age_{j,t}) = & \mu_{PT,j}\mathbf{1}(l_{j,t} = l_{PT})\Big[1 + \mu_{PT,age,j}(age_{j,t} - 25)\Big] \\ & + \mu_{FT,j}\mathbf{1}(l_{j,t} = 1)\Big[1 + \mu_{FT,age,j}(age_{j,t} - 25)\Big] \end{aligned}$$

where $\mu_{PT,j}$ and $\mu_{FT,j}$ measures the utility from working part time and full time, respectively, relative to not working. We thus expect these to be negative. $\mu_{PT,age,j}$ and $\mu_{FT,age,j}$ allows for the dis-utility to change over the life cycle.

The fifth term is a flexible non-separable function of labor supply and children, aimed

[9] Since λ is fixed in our model, we do not allow household bargaining as in the limited commitment as done in e.g. Mazzocco (2007); Voena (2015); and Low, Meghir, Pistaferri and Voena (2018). We focus on the interaction between fertility and family labor supply. While allowing for limited commitment is an interesting avenue for further research, it is out of the scope of the current paper.

at capturing the main trade-offs between working and home production. Concretely, we let the utility from labor market work depend on age, the number of children and the age of the youngest child through

$$\begin{aligned}q_j(l_{j,t}, n_t, o_t) =& \mu_{PT,j}\mathbf{1}(l_{j,t} = l_{PT})\Big[\alpha_{PT,child,j} + \alpha_{PT,more,j}(n_t - 1) + \alpha_{PT,young,j}\mathbf{1}(o_t \in \{0,1,2,3\})\Big] \\ &+\mu_{FT,j}\mathbf{1}(l_{j,t} = 1)\Big[\alpha_{FT,child,j} + \alpha_{FT,more,j}(n_t - 1) + \alpha_{FT,young,j}\mathbf{1}(o_t \in \{0,1,2,3\})\Big]\end{aligned}$$

where $l_{j,t} = 0$ is again the reference alternative. Importantly, we allow for different parameters for men and women. All parameters are relative to the level parameters. For example, $\alpha_{PT,child,j} \cdot 100$, measures the percentage increase in the utility of part time work when a child is present. If $\mu_{PT,j} < 0$ and $\alpha_{PT,child,j} > 0$ (as we estimate below), then children tend to increase the dis-utility from part time work compared to not working.

4.2 Fertility and Children

In each period, couples choose whether to try to conceive an additional child. We denote $e_t = 1$ if a couple exerts effort to conceive a child and $e_t = 0$ else. Whether a subsequent childbirth in the following period occurs depends on the effort and the biological fecundity of the woman. The biological fecundity is falling in age and calibrated to match medical literature. This means that after a certain age, T_f, women are no longer fertile and cannot have more children. Furthermore, couples also have imperfect contraceptive control such that unintended childbirths can occur. The imperfect fertility control is similar to that in Adda, Dustmann and Stevens (2017) but we also allow for unintended pregnancies as in e.g. Ejrnæs and Jørgensen (2020).

Letting $b_t = 1$ denote the birth of a child in period t and $x_t = 1$ denote a child moving out, the number of children evolves as

$$n_{t+1} = n_t + b_{t+1}(e_t) - x_{t+1} \tag{2}$$

where $b_{t+1} = 0$ if $n_t = 3$ and otherwise

$$b_{t+1}(e_t) = \begin{cases} 1 & \text{with probability } \wp_t(e_t) \\ 0 & \text{with probability } 1 - \wp_t(e_t) \end{cases} \tag{3}$$

with the probability of a childbirth given as

$$\wp_t(e_t) = \begin{cases} \overline{\wp}_t & \text{if } e_t = 1 \\ \overline{\wp}_t\underline{\wp} & \text{if } e_t = 0 \end{cases}$$

in which $\overline{\wp}_t$ measures the biological fecundity and $\underline{\wp} > 0$ allows for unintended childbirths.

Children can also move out of the household and this process is governed by

$$x_{t+1} = \begin{cases} 1 & \text{with probability } q_t(n_t, o_t) \\ 0 & \text{with probability } 1 - q_t(n_t, o_t) \end{cases} \tag{4}$$

where we assume that children can move out once the fertile period ends, i.e. $q_t = 0$ when $\overline{\wp}_t > 0$ in the fertile period and $q_t \geq 0$ when $\overline{\wp}_t = 0$ in the infertile period. Concretely, we assume that x_{t+1} is a realization of a Binomial distribution with

$$q_t(n_t, o_t) = \begin{cases} P_{bin}(n_t) & \text{if } n_t > 0,\ t > T_f \text{ and } o_t \in \{6+\} \\ 0 & \text{else} \end{cases}$$

where

$$P_{bin}(n) = \frac{n!}{(n-1)!} p_x (1 - p_x)^{n-1}$$

is the binomial distribution yielding the likelihood of a child out of n_t children moving. p_x is the leave probability parameter. The restriction that the youngest child must be at least six years old ensures that not until all children in the household are above 6 years old does any children move out. This is a parsimonious way of delaying children moving out even in the infertile period.

Finally, the age of the youngest child, o_t, evolves deterministically as

$$o_{t+1} = \begin{cases} 0 & \text{if } b_{t+1} = 1 \\ o_t + 1 & \text{if } b_{t+1} = 0 \text{ and } o_{t+1} \in \{0, 1, 2, 3, 4, 5\} \\ o_t & \text{if } b_{t+1} = 0 \text{ and } o_t \in \{6+\} \\ NC & \text{if } b_{t+1} = 0 \text{ and } o_t \in \{NC\}. \end{cases} \tag{5}$$

4.3 Human Capital, Wages and Income

In the beginning of each period, each household member $j \in \{m, w\}$ either receives a job-offer (with probability p_{job}) or not. If $job_{j,t} = 1$ member j has the full choice-set available (not work, part time, or full time work) while if $job_{j,t} = 0$, member j are forced out of employment.

Labor income is given as

$$Y_{j,t} = w_{j,t} l_{j,t} \tag{6}$$

where the wage depends on the level of accumulated human capital through

$$\log w_{j,t} = \gamma_{j,0} + \gamma_{j,1} K_{j,t}. \tag{7}$$

Human capital evolves according to

$$K_{j,t+1} = [(1-\delta)K_{j,t} + l_{j,t}]\epsilon_{j,t+1} \tag{8}$$

where δ is the depreciation rate and $\epsilon_{j,t+1}$ is a log-normal mean one permanent shock to human capital, $\log \epsilon_j \sim \mathcal{N}(-0.5\sigma^2_{j,\epsilon}, \sigma^2_{j,\epsilon})$, similarly to the process in e.g. Keane and Wasi (2016). The declining biological fecundity and endogeneous wages create a trade-off between investing in human capital while young and postponing fertility too long.

Combining the two household member's labor market income, we denote $Y_t = Y_{m,t} + Y_{w,t}$ as the household income. We let $\mathcal{B}_{j,t}$ denote the unemployment benefits received by household member j and denote $\tilde{Y}_{j,t} = Y_{j,t} + \mathcal{B}_{j,t}$ as the total income received by member j. In turn, total household income is $\tilde{Y}_t = \tilde{Y}_{m,t} + \tilde{Y}_{w,t}$.

4.4 Budget Constraint and Institutions

In each period, choices must satisfy the inter-temporal budget constraint

$$C_t + A_t = RA_{t-1} + \tilde{Y}_t - \mathcal{T}(n_t, \tilde{Y}_{w,t}, \tilde{Y}_{m,t}, l_{w,t}, l_{m,t}, s_t) - \mathcal{C}(n_t, o_t, Y_t, s_t) \tag{9}$$

where $A_t \geq 0\ \forall t$ is end-of-period wealth, $\mathcal{C}(\bullet)$ is child care costs net of child subsidies, and $\mathcal{T}(\bullet)$ is total taxes paid.

The child care costs and subsidies along with the transfer and tax system are parsimonious versions of the Danish rules as they were in 2010. Below, we briefly describe the main features of the implemented institutions and defer the exact implementation and illustrations hereof to the Supplemental Material B.

Child care costs and subsidies. Child care costs are highly subsidized in Denmark such that at most around 25% of the cost of child care provision are held by the parents. In the model, child care costs, $\mathcal{C}_c(n_t, o_t, \tilde{Y}_t, s_t)$, depends on the partnership status of parents, the number of children, the age of the youngest child, and household income. Low to middle income households pay a reduced fee and child care is completely free if household pre-tax income is less than around 150,000 DKK (depending on the household composition).

Child-related *subsidies*, $\mathcal{C}_s(n_t, o_t, s_t)$, are subtracted from the child care costs. In the model, couples recieve child subsidies of 16,988DKK for one child if the youngest child is below age 6 and 10,580DKK for each of the remaining children. Singles receive additional benefits, as described in the Supplemental Material. Combining the child care costs and subsidies, the net child care cost is

$$\mathcal{C}(n_t, o_t, Y_t, s_t) = \mathbf{1}(n_t > 0)(\mathcal{C}_c(n_t, o_t, Y_t, s_t) - \mathcal{C}_s(n_t, o_t, s_t)) \tag{10}$$

which can be negative if the child subsidies exceed the costs.

Labor market transfers. We assume that all individuals not working receive $118,284$ DKK annually, equal to the basic income assistance level in Denmark in 2010 ("Kontanthjælp" in Danish). In turn, we let $\mathcal{B}_{j,t} = \mathbf{1}(l_{j,t} = 0) \cdot 118,284$.

Taxes. The Danish labor income tax system is individual with a relatively small link between couples where the unused labor participation tax deduction of around 43,000 DKK in 2010 can be transferred across spouses. We have implemented a version of the labor income tax system capturing the main statutory rates in 2010. We discuss the features and calculation in more detail in the Supplemental Material.

4.5 Partnership Dissolution

Couples transition into single-hood randomly, and the probability of partnership dissolution, $p_s(t, n_t)$, is a function of age and the number of children. We assume that single-hood is an absorbing state and abstract from re-partnering for simplicity.

The allocation of wealth, children, and custody after a divorce is as follows. We assume that a fraction κ_A of the household wealth goes to women and the reciprocal share $1 - \kappa_A$ goes to men after partnership dissolution. We further assume that children is with their mother a fraction κ_n of the time but that the mother bears all the child care costs and receives all the child subsidies. The father pays ζn_t in child support each year. The share of time spent with a parent affects the consumption equivalence scale through $\nu(\kappa_n n_t)$ for women and $\nu((1 - \kappa_n)n_t)$ for men.

4.6 Retirement

Retirement is exogenous at age T_r. We assume that individuals receive constant retirement benefits such that a single person receives the maximum amount of old-age pension ("Folkepension" in danish), which is DKK 122,712 annually in 2010. Couples receive a slight reduction (per person) and receive DKK 179,808 in total annually. While unlikely, children could still be living at home in retirement until they move out. For simplicity, we do not allow for divorce in retirement and ignore potential spousal death and bequests. The problem, in turn, becomes a simple consumption-savings model in this part of the life cycle with associated value function $\tilde{V}_{T_r}(K_{w,T_r}, K_{m,T_r}, n_t, o_t, A_{T_r})$. We let $T_r = 60$.

To adjust for the parsimonious description of life in retirement, we follow the approach in e.g. Keane and Wolpin (1997) and Gourinchas and Parker (2002) and discussed in Jørgensen and Tô (2020). Concretely, we introduce an adjustment factor, κ_V, multiplied to the retirement value function, such that the value in the first retirement period is

$V_{T_r}(K_{w,T_r}, K_{m,T_r}, n_t, o_t, A_{T_r}) = \kappa_V \tilde{V}_{T_r}(K_{w,T_r}, K_{m,T_r}, n_t, o_t, A_{T_r})$. We will estimate κ_V as a way of allowing for empirical deviations from this stylized formulation in retirement.

4.7 Recursive Formulation

The state variables for a single individual $j \in \{w, m\}$ is $\mathcal{S}_{j,t} = (A_{t-1}, K_{j,t}, n_t, o_t, job_{j,t})$ and the recursive problem prior to retirement can be formulated as, with $s_t = 1$,

$$\begin{aligned}
V_{w,t}(\mathcal{S}_{w,t}) = \max_{C_t, l_{w,t}} & \; u_w(C_t, \kappa_n n_t, o_t, 0, l_{w,t}) + \beta \mathbb{E}_t[V_{w,t+1}(\mathcal{S}_{w,t+1})] \\
\text{s.t.} & \\
& A_t = RA_{t-1} - C_t + \tilde{Y}_{w,t} - \mathcal{T}(n_t, \tilde{Y}_{w,t}, 0, l_{w,t}, 0, 1) - \mathcal{C}(n_t, o_t, Y_{w,t}, 1) + \zeta n_t \\
& \text{and eqs. (4)–(8)}
\end{aligned}$$

for women and similarly for single men

$$\begin{aligned}
V_{m,t}(\mathcal{S}_{m,t}) = \max_{C_t, l_{m,t}} & \; u_m(C_t, (1-\kappa_n) n_t, o_t, 0, l_{m,t}) + \beta \mathbb{E}_t[V_{m,t+1}(\mathcal{S}_{m,t+1})] \\
\text{s.t.} & \\
& A_t = RA_{t-1} - C_t + \tilde{Y}_{m,t} - \mathcal{T}(n_t, 0, \tilde{Y}_{m,t}, 0, l_{m,t}, 1) - \zeta n_t \\
& \text{and eqs. (4)–(8)}
\end{aligned}$$

where mothers bear all child care costs and receive all subsidies, and fathers pay child support.

The recursive problem for a couple is

$$\begin{aligned}
V_t(\mathcal{S}_t) = & \max_{C_t, l_{w,t}, l_{m,t}, e_t} U(C_t, n_t, o_t, e_t, l_{w,t}, l_{m,t}) + \beta \mathbb{E}_t[p_s(t, n_t) V^s_{t+1}(\mathcal{S}_{t+1}) + (1 - p_s(t, n_t)) V_{t+1}(\mathcal{S}_{t+1})] \\
& \text{s.t. eqs. (2)–(10)}
\end{aligned}$$

where $V^s_{t+1}(\mathcal{S}_{t+1}) = \lambda V_{w,t+1}(\mathcal{S}_{w,t+1}) + (1-\lambda) V_{m,t+1}(\mathcal{S}_{m,t+1})$ is the weighted value of becoming single in the following period. The expectation is with respect to the arrival and moving of children and human capital shocks of both women and men. The model is solved numerically using the extension of the endogenous grid method (proposed by Carroll, 2006) in Iskhakov, Jørgensen, Rust and Schjerning (2017) and Druedahl and Jørgensen (2017), as described in detail in the Supplemental Material D.

5 Calibrated Parameters

We employ a standard two-step approach to estimating the parameters of the model. Some parameters, listed in Table D.1 in the Supplemental Material, are calibrated outside the model in a first step. We denote these calibrated parameters as $\boldsymbol{\phi}$. We estimate the remaining parameters, collected in $\boldsymbol{\theta}$, below in a second step. We investigate the sensitivity of our results to the calibrated parameters in Section 8, following the approach suggested in Jørgensen (forthcoming).

We fix the constant relative risk aversion, ρ, is set to 1.5 based on e.g. Attanasio and Weber (1995) and the relative loading on the utility of women, λ, to 0.5, as done in e.g. Eckstein, Keane and Lifshitz (2019). We let the discount factor, β, be 0.97 based on estimates from Danish data in Jørgensen (2017) and fix the gross real interest rate R at 1.03, as in that study.

The human capital process is calibrated based on values in Keane and Wasi (2016). In particular, we calibrate the human capital depreciation rate to 10 percent, $\delta = 0.1$ and let $\sigma_{j,\epsilon} = 0.1$ for $j \in \{m, w\}$. We fix the unemployment probability, $1 - p_{job}$, to 3 percent based on the low unemployment rate in Denmark in the sample period and the fact that we focus on individuals with a relatively close attachment to the labor market. Part time work, l_{PT}, is calibrated to be 75% of full time work, motivated by the Danish labor market in which the normal working week is 37 hours and part time work often is 28–32 hours a week.

The biological fecundity, $\overline{\wp}_t$, is calibrated based on medical literature (Leridon, 2004) and falling in age (see Figure C.1 in the Supplemental Material). We calibrate $\underline{\wp}$ in the unplanned pregnancy probability, $\overline{\wp}_t\underline{\wp}$, to 5% based on the estimated values in Ejrnæs and Jørgensen (2020) in the range from 3.8% to 6.1%. We fix the probability parameter in the binomial distribution related to child moving out to $p_x = 0.08$ based on the Danish data.

We estimate the probability of partnership dissolution as a function of age and number of children from the Danish data. The resulting probabilities are presented in Figure C.1b in the Supplemental Material. We assume that children spend 80 percent of the time with their mother in case of a dissolution, $\kappa_n = 0.8$, and that wealth is shared fifty-fifty, $\kappa_A = 0.5$, which is the default in Denmark.

6 Estimated Parameters

We estimate the remaining parameters governing the wage process, $\boldsymbol{\gamma}$, the value of children, $\boldsymbol{\omega}$ and $\boldsymbol{\eta}$, the utility of labor market work, $\boldsymbol{\mu}$, the interlink between labor market work and children, $\boldsymbol{\alpha}$, and the retirement value adjustment parameter, κ_V by Simulated Method of Moments. (Smith, 1993; Gouriéroux, Monfort and Renault, 1993). Collecting these parameters in $\boldsymbol{\theta}$, we estimate these as

$$\hat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta}} g(\boldsymbol{\theta})' W g(\boldsymbol{\theta})$$

where $g(\boldsymbol{\theta}) = m^{data} - m^{sim}(\boldsymbol{\theta})$ is a $J \times 1$ vector of differences between the J empirical moments calculated from the "estimation sample" discussed in Section 2 and the same moments calculated from simulated data for a given $\boldsymbol{\theta}$. W is a $J \times J$ symmetric positive definite weighting matrix. We use a diagonal matrix with the inverse of the variance of the empirical moments on the diagonal.

To construct the moments in $m^{sim}(\boldsymbol{\theta})$ we solve the model for a given $\boldsymbol{\theta}$ and simulate synthetic data for N_{sim} households and use these simulated observations to calculate the same moments as in the Danish data. See the Supplemental Material for details on how we numerically solve and simulate the model for a given value of $\boldsymbol{\theta}$. We use a modified version of the so-called "TikTak" global search algorithm in Arnoud, Guvenen and Kleineberg (2019). Below, we discuss which moments we include to estimate θ and how these moments are informative about (identify) $\boldsymbol{\theta}$.

6.1 Moments Matched and Identification

Here we list all included moments and discuss which moments are likely key for identification of particular parameters in $\boldsymbol{\theta}$. We also supplement our intuitive discussion with a more formal analysis of the informativeness of (groups of) included moments to further substantiate our claim of identification of $\boldsymbol{\theta}$.

The moment informativeness measures reported in Figure 2 is motivated by Honoré, Jørgensen and de Paula (2020) and measures the percentage change in the asymptotic variance of elements of $\hat{\boldsymbol{\theta}}$ from removing groups of moments in $g(\boldsymbol{\theta})$, enumerated below. Concretely, the informativeness measure measures the percentage change in the asymptotic variance of $\hat{\boldsymbol{\theta}}$, Σ, from excluding a set of estimation moments. The measure thus is an extension of what Honoré, Jørgensen and de Paula (2020) refers to as M_4. Concretely, we calculate the informativeness as

$$I_k = \text{diag}(\tilde{\Sigma}_k - \Sigma)/\text{diag}(\Sigma) \cdot 100, \tag{11}$$

where

$$\begin{aligned}
\tilde{\Sigma}_k &= (G'\tilde{W}_k G)^{-1} G'\tilde{W}_k S \tilde{W}_k G (G'\tilde{W}_k G)^{-1} \\
\tilde{W}_k &= W \odot (\iota_k \iota_k')
\end{aligned}$$

with $\odot$ denoting element-wise multiplication and ι_k is a $J \times 1$ vector with ones in all elements except the kth *group* of moments which are zeros.

The six groups of moments included in the estimation are plotted below in Figures 3–4 and listed here:

1. **Labor.** Share working and the share working full time conditional on working, split by age and gender. This gives in total $2 \times 36 \times 2 = 144$ moments that should

be especially informative about the baseline value of leisure over the life cycle, $\mu_{PT,j}, \mu_{PT,age,j}$ and $\mu_{FT,j}, \mu_{FT,age,j}$ for $j \in \{w, m\}$. This is also evident in Figure 2 in which the informativeness measure suggests that the asymptotic variance of $\mu_{PT,w}$ and $\mu_{PT,age,w}$ would increase significantly if we left out this set of moments.

2. **Income.** Average labor income when working, split by age and gender. This gives $36 \times 2 = 72$ moments that are meant to be informative about the wage process parameters $\gamma_{0,j}, \gamma_{1,j}$ for $j \in \{w, m\}$. Figure 2 confirms that these parameters are sensitive to this set of moments.

3. **Children.** Share with at least 1 through 3 children, split by age. This gives in total $3 \times 36 = 108$ moments that should be informative about the value of having (more) children, ω_1, ω_2, and ω_3. Figure 2 confirms that these moments are very informative about the value of children.

4. **Spacing.** Distribution of years between first and second childbirths. This gives in total 15 moments that should be informative about the dis-utility of fertility effort when an infant is present, η_0 and η_1. Figure 2 confirms this with enormous increases in the variances of these parameters if these moments were excluded. These moments are naturally also informative about the overall value of children in $\boldsymbol{\omega}$.

5. **Work/fertility interaction.** Share working and share working full time after first and second childbirth, split by gender. We use up to and including 7 years after birth and measure all moments in percent relative to the year prior to birth. This gives in total $2 \times 2 \times 2 \times 7 = 56$ moments that should be informative about the trade-off between work and leisure when children are present, $f_j(\bullet)$, i.e. $\alpha_{l,child,j}$, $\alpha_{l,young,j}$, and $\alpha_{j,more,j}$ for $l \in \{FT, PT\}$, $j \in \{w, m\}$. Figure 2 suggests that these moments are informative about many of the parameters but especially the before mentioned parameters. For example, the measure suggests that the variances of many of these parameters would increase with more than 100% if we left out this group of moments.

6. **Wealth.** Average wealth split by age. This gives 36 moments that are meant to be informative about the retirement adjustment factor, χ_V. Figure 2 indicates that other moments, e.g. labor market participation, might be more informative about this parameter.

All age profile moments are based on age-dummies estimated from a regression including age and cohort dummies. All work/fertility interaction moments are time-since birth coefficients from a regression also including cohort dummies and empirical measures are based on 12 months of observations centered around the first or second childbirth.

Figure 2: Informativeness of Estimation Moments.

	labor	income	children	spacing	interaction	wealth
ω_1	-10.56	18.24	29.48	1109.15	15.11	0.40
ω_2	27.00	110.57	58.82	475.62	-11.79	2.21
ω_3	7.83	-23.95	96.01	-6.82	61.59	-37.16
η_0	13.06	-5.93	39.57	15774.30	17.84	0.42
η_1	3.92	9.24	22.65	353.17	102.92	4.82
$\mu_{FT,w}$	-7.74	-2.85	107.28	40.02	18.06	0.59
$\mu_{FT,age,w}$	0.33	12.98	16.23	165.79	55.95	-32.34
$\mu_{PT,w}$	9.20	2.70	95.98	37.24	18.21	5.25
$\mu_{PT,age,w}$	71.68	6.55	2.45	154.39	85.75	-0.22
$\mu_{FT,m}$	-23.04	144.48	32.41	108.97	66.70	-35.45
$\mu_{FT,age,m}$	-29.33	-51.84	51.15	88.23	82.23	-39.90
$\mu_{PT,m}$	-34.52	119.62	13.71	120.73	106.64	-42.33
$\mu_{PT,age,m}$	38.07	0.15	39.73	111.14	87.84	-17.89
$\alpha_{FT,child,w}$	64.27	39.36	30.51	56.87	50.23	-18.31
$\alpha_{FT,more,w}$	42.69	30.05	27.11	25.29	138.32	-0.47
$\alpha_{FT,young,w}$	-9.90	-2.63	90.04	6.66	1200.37	-9.41
$\alpha_{PT,child,w}$	51.28	3.42	33.36	22.06	202.65	-0.61
$\alpha_{PT,more,w}$	39.94	8.16	25.11	-24.61	31.68	-0.10
$\alpha_{PT,young,w}$	-12.58	2.91	21.50	79.86	530.55	-5.89
$\alpha_{FT,child,m}$	54.31	27.22	-2.08	260.49	84.84	-42.63
$\alpha_{FT,more,m}$	12.99	35.59	-1.67	-17.71	164.29	0.05
$\alpha_{FT,young,m}$	14.76	9.85	-8.56	22.74	228.53	-4.25
$\alpha_{PT,child,m}$	-19.26	-14.65	-11.54	0.17	606.30	-4.95
$\alpha_{PT,more,m}$	12.23	52.32	-21.11	-14.28	254.42	0.56
$\alpha_{PT,young,m}$	-54.56	4.36	-0.11	-20.16	72.64	-1.90
$\gamma_{0,w}$	38.84	21.11	-27.53	46.71	10.41	-20.03
$\gamma_{1,w}$	51.53	25.46	-11.85	11.63	-26.44	7.40
$\gamma_{0,m}$	-32.14	122.85	-10.45	31.11	27.86	-4.90
$\gamma_{1,m}$	-25.55	7.09	-26.02	20.68	-9.21	-0.75
κ_V	27.27	4.95	36.31	42.38	65.47	-53.17

Notes: The figure illustrates the percent change in the asymptotic variance of $\hat{\theta}$ from removing groups of estimation moments based on the extension of the informativeness measure M_4 proposed by Honoré, Jørgensen and de Paula (2020) in eq. (11). To calculate this measure, we use the estimated model parameters in Table D.2.
Moments in group 1 (labor) are the share working and the share working full time conditional on working split by age and gender. Moments in group 2 (income) are the average labor income when working split by age and gender. Moments in group 3 (children) are the share with at least 1, 2 or 3 children split by age. Moments in group 4 (spacing) are the distribution of years between first and second childbirths. Moments in group 5 (interaction) are the share working and share working full time after first and second childbirth split by gender. Moments in group 6 (wealth) are the average wealth split by age.

6.2 Model Fit

Figures 3–4 show the model fit by plotting the empirical moments along with moments calculated from simulated data based on the estimated model. The model fit is quite good especially considering the relatively few number of parameters estimated and the many different aspects of behavior matched.

Labor. The overall level and age profile of the share working and the share working full time are similar in the data and simulated from the model in panels a)–d) in Figure 3. Labor market participation of particularly women in the model around age 35–40 is a bit lower in the model compared to the data.

Income. The simulated age profiles of labor market earnings before taxes are very similar to the observed age profiles in the data, as seen in panels e) and f) of Figure 3.

Children. The estimated model fits well the empirical fertility patterns in panels a)–c) in Figure 4. The model predicts slightly too many childless couples but fits the share with at least two and three children well.

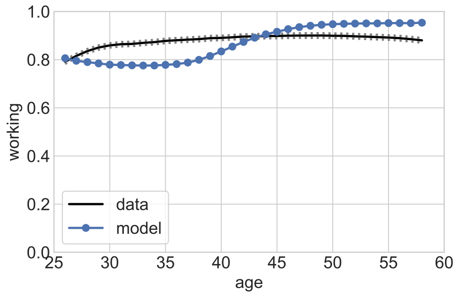
Spacing. The spacing between the first and second childbirth is quite similar in the model as in the data. The arrival of the second child is slightly earlier in the model, however.

Work/fertility interaction. Focusing on the labor market responses around childbirth in Figure 5, the model fits the empirical patterns quite well. The empirical moments are aggregated to 12-month measures centered around the month of childbirth. The model re-produces the labor market participation of women and men after the first and second childbirth very well in panels a), b), e) and f). While a large share of women reduce labor market participation after the first childbirth, the effect of men are small and even slightly positive. We do not match the dip in labor market participation of women around three years after the first birth. This difference is primarily due to the arrival of the second child and parental leave. The reduction in the share of women working full time three years after the second childbirth is larger in the model compared to the data.

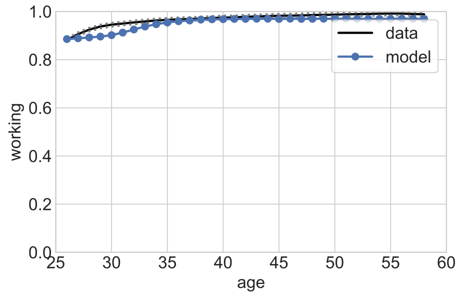
Wealth. Finally, the level and evolution of the average age profiles of labor income and wealth in panel g) of Figure 3 are also matched well although the model overshoots the amount of wealth late in life a bit.

Figure 3: Model Fit. Age Profiles.

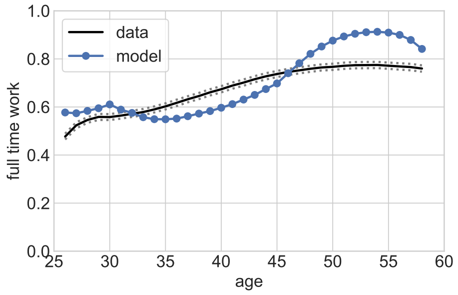
(a) Share Working, Women.



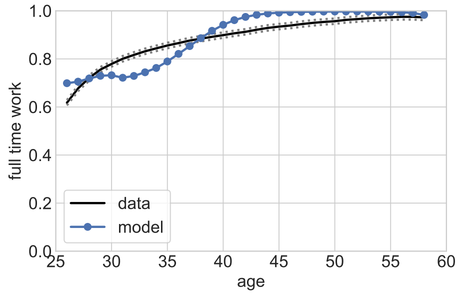
(b) Share Working, Men.



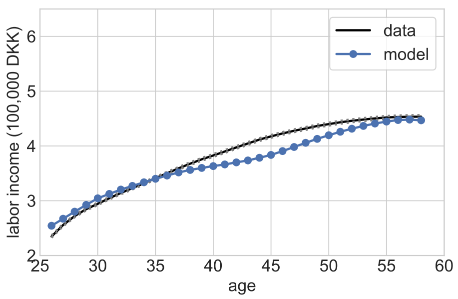
(c) Full time when working, Women.



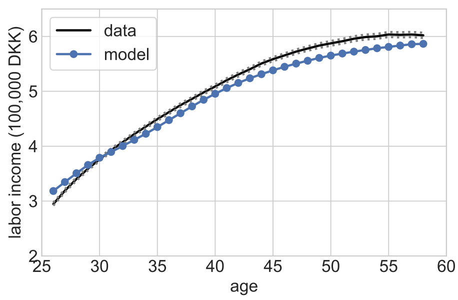
(d) Full time when working, Men.



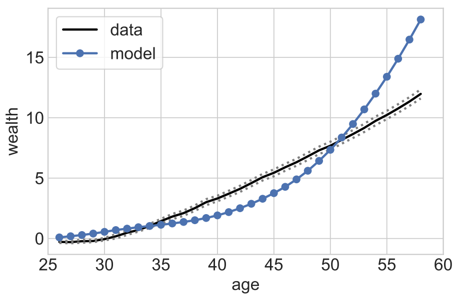
(e) Labor Income when Working, Women.



(f) Labor Income when Working, Men.



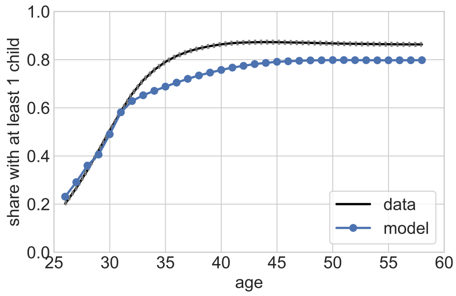
(g) Wealth (household).



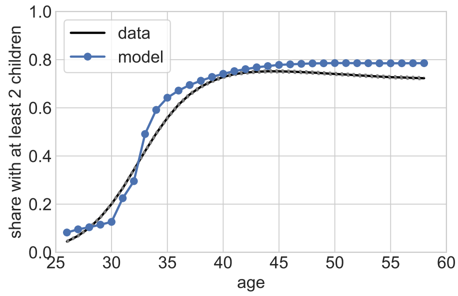
Notes: The figure illustrates the model fit. All empirical moments are regression coefficients from a regression including also cohort dummies. Financial outcomes are measured in 100,000s DKK.

Figure 4: Model Fit: Children.

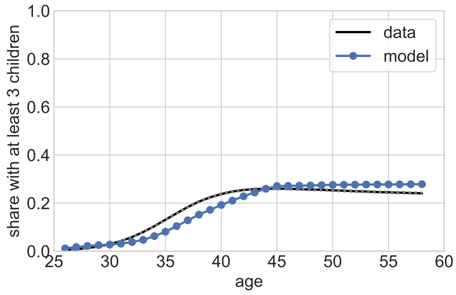
(a) Share with at least one child.



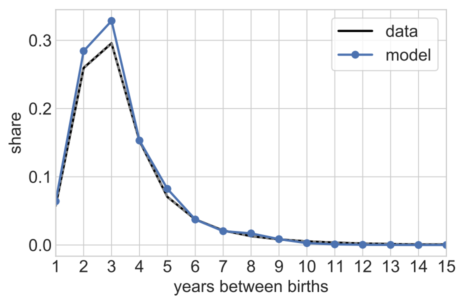
(b) Share with at least two children.



(c) Share with at least three children.



(d) Years between first and second birth.

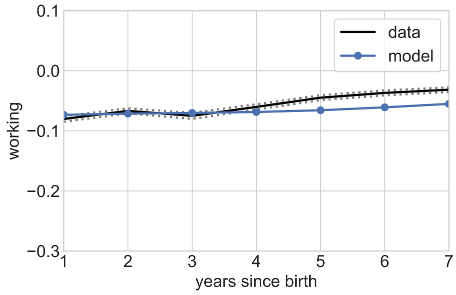


Notes: The figure illustrates the model fit. All empirical moments are regression coefficients from a regression including also cohort dummies. Wages are measured in 1,000s DKK and wealth is measured in 100,000s DKK.

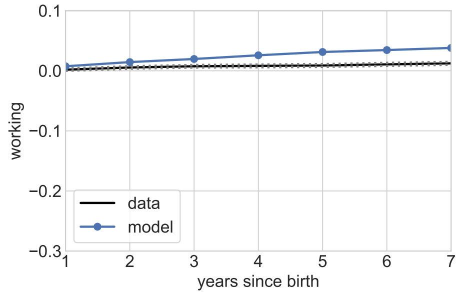
Figure 5: Model Fit. First and Second Childbirth Events.

First childbirth

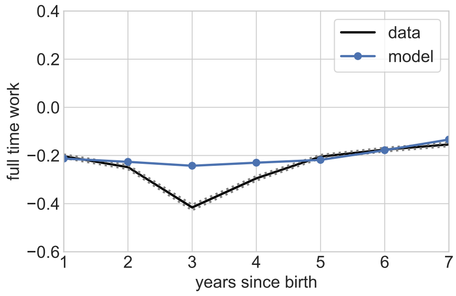
(a) Share Working, Women.



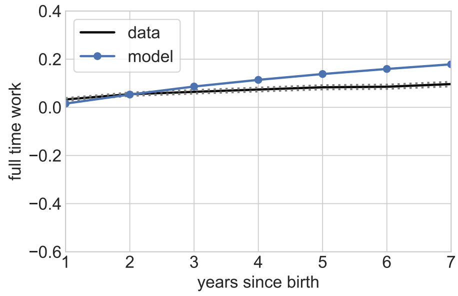
(b) Share Working, Men.



(c) Full time, Women.

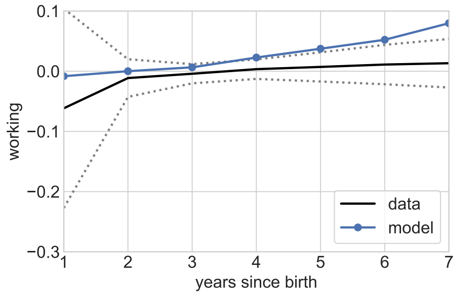


(d) Full time, Men.

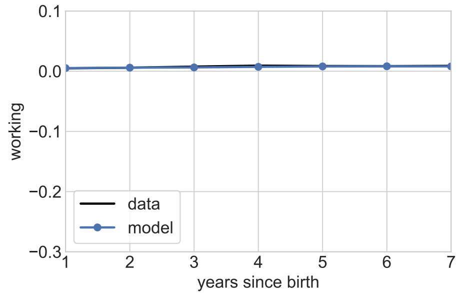


Second childbirth

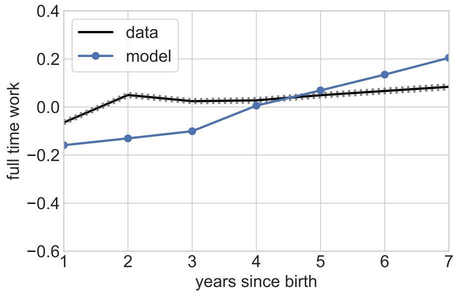
(e) Share Working, Women.



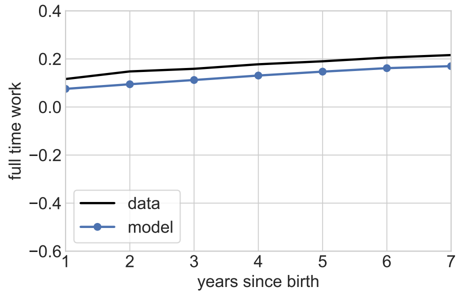
(f) Share Working, Men.



(g) Full time, Women.



(h) Full time, Men.



Notes: The Figure shows empirical and simulated labor market outcomes after first and second childbirth. All empirical outcomes are 12-month aggregated values centered around the the relevant childbirth (first or second). All empirical moments are regression coefficients from a regression including cohort dummies. We only use couples with the first birth in the age interval 26 through 59.

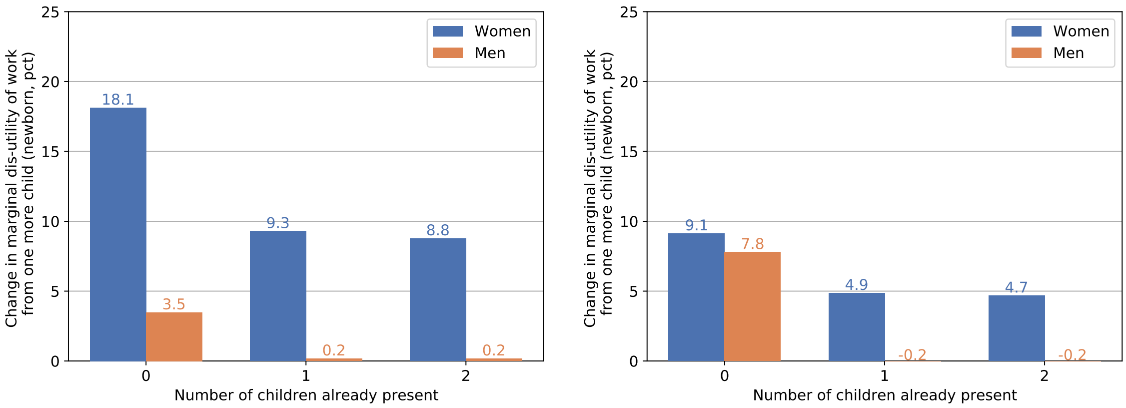
6.3 Estimation Results

Table D.2 in the Supplemental Material reports the estimated parameters in $\boldsymbol{\theta}$. We estimate a significant dis-utility from labor market hours since $\mu_{FT,j}$ and $\mu_{PT,j}$ for $j \in \{w, m\}$ are all estimated to be negative. The dis-utility of working is falling in age since $\mu_{FT,age,j}$ and $\mu_{PT,age,j}$ for $j \in \{w, m\}$ are all estimated negative.

We estimate a significant increase in the dis-utility of work when children are present. The dis-utility of full and part time work of women increases by around 11.4% and 14.2%, respectively when a child is present, relative to not working. For men, the dis-utility of full and part time work increases with 5.4% and 3.5% when children are present, respectively. The dis-utility from work is slightly higher for women, but not for men, when young children are present. The first child has a much larger effect on the the dis-utility from work than do subsequent children. Concretely, the dis-utility of full and part time work of women increases with around 5.6% and 6.7%, respectively, when subsequent children are born. There is not really an effect for men from having several children.

Figure 6: Change in Marginal Dis-Utility from Work from Additional Children.

(a) From not working to part time. (b) From part time to full time.



Notes: The figure illustrates the change in the marginal dis-utility of work from having additional children as calculated in eq. (12). Panel (a) reports $\Delta_j(PT, n)$ Panel (b) reports $\Delta_j(FT, n)$ for $j \in \{w, m\}$ and $n = 0, 1, 2$.

To get a sense of the utility trade-offs between working and having more children, we plot in Figure 6 the percentage *change in the marginal dis-utility from work form having additional children*,

$$\Delta_j(l, n) = \frac{\Delta_l U_j(n+1, 0) - \Delta_l U_j(n, 6+)}{\Delta_l U_j(n, 6+)} \cdot 100 \tag{12}$$

for $l \in \{PT, FT\}$, where the marginal dis-utility of work is

$$\Delta_{PT}U_j(n,o) = -q_j(PT,n,o) + q_j(NT,n,o)$$
$$\Delta_{FT}U_j(n,o) = -q_j(FT,n,o) + q_j(PT,n,o).$$

The change in the marginal dis-utility of work when not working (panel a) from the first child is 18.1% for women, corresponding to the sum estimated parameters $\alpha_{PT,child,w}+\alpha_{PT,young,w}$. Subsequent children change the marginal dis-utility of work with around 9%. For men, the change in the marginal utility from work is around 3.5% from the first child and practically zero for higher order parities. In panel (b), we similarly show the change in the marginal dis-utility of work from having more children, when already working part time. We see that the dis-utility from work is concave in the amount worked. For example, the change in the marginal dis-utility from work of women from having their first child when already working part time is 9.8%, about half that when not working. For men, we find hardly any extensive margin effects of children on the marginal utility from work (panel a) but we do find an intensive margin effect of having the first child (in panel b).

We estimate significant labor market returns to human capital investments of around 0.105 for men and 0.085 for women. The constant is a bit higher for women compared to men. We estimate significant costs associated with fertility effort if a small child is already present in the household, $\eta_0, \eta_1 < 0$. Finally, we estimate the retirement value function adjustment parameter, κ_V, to be around 0.52, which is consistent with a reduction in the marginal utility of consumption in retirement.

7 Simulated Fertility and Labor Market Responses

In this section, we analyze the interaction between fertility and family labor supply in the estimated model. We focus on how labor market and fertility responses to changes in men's and women's wages have long lasting impacts on human capital accumulation and the gender wage gap. Further, we simulate counterfactual reforms of unconditional cash transfers at age 25 and at childbirths. Together, these counterfactual simulations illustrate the fertility and labor market behavior within the model and validates the implied behavior with existing research.

We start by simulating responses to unanticipated permanent wage changes. In Table 3, we report life-cycle Marshallian elasticities of participation, hours worked, offered wage at age 55, childbirth and completed fertility simulated from the estimated model. Elasticities are calculated based on an unanticipated permanent 5% increase from different ages for either women (in Panel A) or men (in Panel B). All elasticities are "long run" in the sense that they measure the change in the remaining working life. The average (avg.) is calculated as the average elasticity across all ages.

Table 3: Unanticipated Permanent Wage Changes. Elasticities.

	Participation		Hours		Wage at 55		Child	Comp.
Age	Women	Men	Women	Men	Women	Men	birth	fertility
				A. Elasticities w.r.t. wages of women				
26	1.34	-0.10	1.54	-0.40	1.64	-0.15	-1.17	-0.72
30	0.95	-0.08	1.04	-0.25	1.48	-0.10	-0.58	-0.28
35	0.82	-0.02	0.84	-0.12	1.39	-0.05	-0.40	-0.07
40	0.57	-0.00	0.59	-0.04	1.27	-0.02	-0.17	-0.01
45	0.31	-0.00	0.35	-0.02	1.13	-0.01	–	–
50	0.21	-0.00	0.24	-0.02	1.05	-0.00	–	–
avg.	0.54	-0.02	0.60	-0.10	1.24	-0.04	-0.37	-0.11
				B. Elasticities w.r.t. wages of men				
26	-0.81	0.31	-1.26	0.44	-0.56	1.14	3.12	1.89
30	-0.42	0.16	-0.80	0.23	-0.40	1.08	3.28	1.56
35	-0.37	0.03	-0.64	0.06	-0.32	1.03	2.32	0.42
40	-0.28	0.00	-0.52	0.02	-0.25	1.01	3.94	0.23
45	-0.11	0.00	-0.29	0.01	-0.12	1.00	–	–
50	-0.03	0.00	-0.16	0.00	-0.04	1.00	–	–
avg.	-0.25	0.06	-0.48	0.09	-0.21	1.03	3.46	0.46

Notes: The Table illustrates elasticities of labor market participation, hours, full-time wage offer at age 55, likelihood of childbirth, and completed fertility w.r.t. an unanticipated permanent wage change of either women or men. Participation and hours elasticities are "long run" in the sense that they measure the change in the remaining working life. All elasticities are calculated based on an unanticipated permanent 5% increase at different points in the life cycle. The average (avg.) is calculated across all ages. Panel A shows elasticities w.r.t. women's wages and Panel B shows elasticities w.r.t. men's wages.

Women are more responsive to permanent wage changes than men and are also affected more by men's wages than men are by women's wages. The average hours elasticity (over age) is 0.60 for women and 0.09 for men. These elasticities are in the range reported in previous studies, see e.g. Keane (forthcoming). Attanasio, Levell, Low and Sánchez-Marcos (2018) find an aggregate life-cycle Marshallian elasticity of 0.91 for women. As existing studies, we also find the extensive margin to be very important in generating these elasticities. Human capital accumulation is an important driver of long term effects of wage changes, especially for women. Increasing the wage rate of women by 10% from age 30 leads to a 15% increase in the offered wage at age 55 because human capital accumulation leads to an additional 5pp increase in the offer wage. Increasing wages of men by 10% from age 30 only leads to a 0.8pp increase in the offered wage of men at age 55 from human capital accumulation (elasticity is 1.08) while the wage offer of women at age 55 drops by 4% because of the cross elasticity. These results on the own and cross effects suggest that human capital accumulation is an important driver of long run inequality within couples.

Fertility adjusts asymmetrically to wage changes of men and women. Increased wages of women leads to lower completed fertility while increased wages of men leads to higher completed fertility. Although not targeted directly, these results are consistent with our reduced form findings in Section 3. A permanent increase in women's wages changes the shadow price of child-rearing throughout the entire remaining working life due to a sizable interaction effect between children and the dis-utility from market work, estimated in $\boldsymbol{\alpha}$. In turn, the substitution effect dominates w.r.t. wages of women. For men, we estimate a much weaker interaction effect and the income effect dominates.

Our fertility responses are of similar magnitudes as existing studies. Francesconi (2002) simulates completed fertility elasticities w.r.t. wages of women mostly in the range -1.25 to -0.65. Haan and Wrohlich (2011) simulate a fertility (birth probability) elasticity of -0.48. Wang (2022) simulates a completed fertility elasticity of -1.4. None of these studies investigate the fertility response to wages of men. Using variation in coal reserve values Black, Kolesnikova, Sanders and Taylor (2013) and Kearney and Wilson (2018) find a positive fertility elasticity w.r.t. wages of men of around 0.75 and 1.24, respectively.

7.1 The Role of Fertility Adjustments

Couples trade of investments in human capital with having children. Increasing incentives for human capital investments thus lead to reduced fertility that further facilitates human capital investments that in turn endogenously increase wages further. This fertility multiplier exacerbates the long term consequences of wage changes. Here, we quantify the importance of such endogenous fertility adjustments. We do so by comparing labor supply responses in our baseline model with an alternative scenario in which fertility cannot respond to wage changes.

In the alternative model, fertility is exogenous and stochastic, as is assumed in much existing literature.[10] Childbirth expectations in this alternative model are consistent with the realized arrival rate in the baseline model, conditional on age and the number of children. That is, couples in the alternative model expect children to arrive exogenously following a process estimated from the endogenously chosen fertility in the baseline model. Consequently, the simulated age profiles of fertility are identical across the baseline and the alternative model.

In Figure 7 we report fertility and hours elasticities from an unanticipated permanent wage increase for both the baseline model (solid lines) and the alternative exogenous fertility model (dashed lines). The horizontal axis denotes the age at which the unanticipated permanent wage increase occurred and the vertical axis shows the life-cycle elasticity of

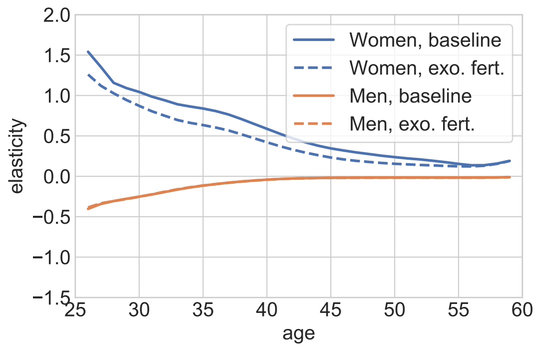
[10] For some recent examples see e.g. Blundell, Dias, Meghir and Shaw (2016); Low, Meghir, Pistaferri and Voena (2018); Guner, Kaygusuz and Ventura (2020); Bronson and Mazzocco (2021); and Borella, De Nardi and Yang (forthcoming).

hours worked and the elasticity of completed fertility. In Table 4, we report elasticities of participation, hours worked, and offered wage at age 55 for the baseline model (End.) and the exogenous fertility model (Exo.). We show responses of both women and men and show elasticities w.r.t women's wages in Panel A and elasticities w.r.t. men's wages in Panel B.

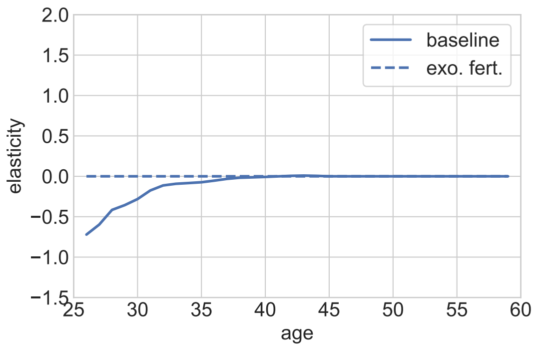
Figure 7: Quantifying the Role of Fertility Responses.

Increased wage of women

(a) Hours.

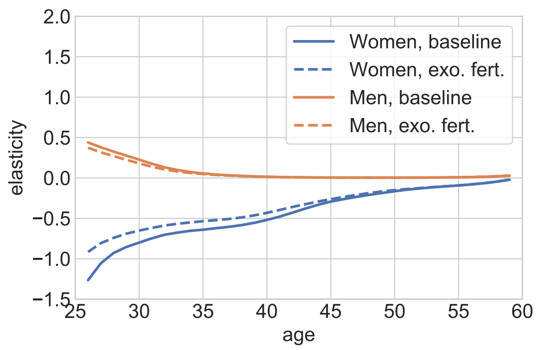


(b) Number of children.

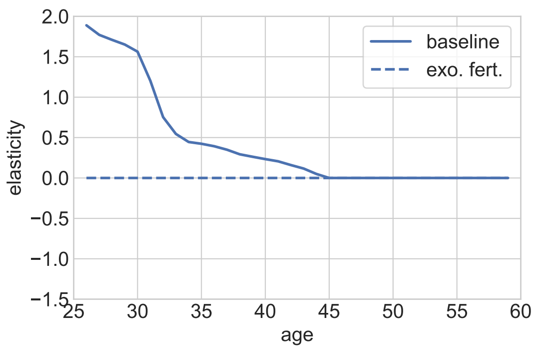


Increased wage of men

(c) Hours.



(d) Number of children.



Notes: The figure shows the elasticities of several outcomes from an unanticipated permanent wage increase for both the baseline model (solid lines) and the alternative exogenous fertility model (dashed lines). On the x-axis is the age at which the unanticipated permanent wage increase occurred, s_1, and on the y-axis we show the elasticity of completed fertility (number of children at age 45) and the elasticity of hours worked, calculated as the average number of hours in the remainder of the working life from the age at which the permanent shock occurred.

Fertility choices are a substantial driver of women's labor supply responses. Comparing the baseline model with the alternative exogenous fertility model, we can quantify how allowing fertility to adjust affects the labor market sensitiveness of men and women. The average (across age) hours elasticity of women is 13 percentage points, or 28%, higher (0.60 vs. 0.47) in the baseline model compared to the alternative model in which fertility cannot adjust. For men the difference is only 1.2 percentage points (16% of a low baseline).

When increasing the wage rate of men we find a similar pattern: The cross-elasticity of women is much larger when fertility can adjust. This thus points to fertility playing an important role in understanding labor market responses of women and help explain why women's labor supply elasticities are often found to be higher than those for men.

Table 4: Unanticipated Permanent Wage Changes. Exogenous Fertility.

	Women's response						Men's response					
	Participation		Hours		Wage at 55		Participation		Hours		Wage at 55	
Age	End.	Exo.	End.	Exo.	End.	Exo.	End.	Exo.	End.	Exo.	End.	Exo.
	A. Elasticities w.r.t. wages of women											
26	1.34	1.06	1.54	1.26	1.64	1.50	-0.10	-0.11	-0.40	-0.39	-0.15	-0.15
30	0.95	0.77	1.04	0.87	1.48	1.39	-0.08	-0.08	-0.25	-0.25	-0.10	-0.10
35	0.82	0.59	0.84	0.63	1.39	1.30	-0.02	-0.02	-0.12	-0.11	-0.05	-0.05
40	0.57	0.39	0.59	0.42	1.27	1.20	-0.00	-0.00	-0.04	-0.04	-0.02	-0.02
45	0.31	0.19	0.35	0.23	1.13	1.10	-0.00	-0.00	-0.02	-0.02	-0.01	-0.01
50	0.21	0.11	0.24	0.15	1.05	1.04	-0.00	-0.00	-0.02	-0.02	-0.00	-0.00
avg.	0.54	0.39	0.60	0.47	1.24	1.19	-0.02	-0.03	-0.10	-0.11	-0.04	-0.04
	B. Elasticities w.r.t. wages of men											
26	-0.81	-0.50	-1.26	-0.91	-0.56	-0.41	0.31	0.27	0.44	0.37	1.14	1.11
30	-0.42	-0.32	-0.80	-0.65	-0.40	-0.32	0.16	0.13	0.23	0.18	1.08	1.06
35	-0.37	-0.25	-0.64	-0.54	-0.32	-0.27	0.03	0.02	0.06	0.05	1.03	1.02
40	-0.28	-0.19	-0.52	-0.43	-0.25	-0.21	0.00	0.00	0.02	0.01	1.01	1.01
45	-0.11	-0.08	-0.29	-0.26	-0.12	-0.11	0.00	0.00	0.01	0.01	1.00	1.00
50	-0.03	-0.02	-0.16	-0.15	-0.04	-0.04	0.00	0.00	0.00	0.00	1.00	1.00
avg.	-0.25	-0.17	-0.48	-0.39	-0.21	-0.17	0.06	0.05	0.09	0.07	1.03	1.02

Notes: The Table illustrates elasticities of labor market participation, hours, wage offer at age 55, likelihood of childbirth, and completed fertility w.r.t. an unanticipated permanent wage change of either women or men. Elasticities are calculated based on an unanticipated permanent 5% increase at different points in life. Participation and hours elasticities are "long run" in the sense that they measure the change in the remaining working life. The average (avg.) is calculated across all ages from 25 through 60. Columns denoted with "End." shows elasticities simulated from the baseline model with endogenous fertility. Columns denoted with "Exo." shows elasticities simulated from an alternative model in which fertility is exogenous and random, with ex-ante fertility expectations consistent with realized fertility simulated from the baseline model.

The long run consequences of fertility adjustments can be inferred from women's offer wages at age 55. Without the fertility multiplier effect, in the exogenous fertility model, increasing the wage rate of women with 10% from age 30 leads to a 14% increase in the offered wage at age 55. This is one percentage point or around 7% lower than in the baseline model. Fertility adjustments are especially important drivers of the cross-effects from men's wage to women's human capital accumulation. The long term effect on women's offer wage at 55 is 25% higher in the endogenous fertility model: With exogenous fertility, increasing wages of men by 10% from age 30 reduces women's wage offer at age

55 with 3.2% compared to 4.0% in the baseline model. Together, this suggests that the interlink between fertility and human capital investments can exacerbate gender labor market inequality.

Fertility adjustments happen on both the extensive and intensive margin. In Figure D.2 in the Supplemental Material we show the elasticity of the age at first birth, the share remaining childless and the share with one child at completed fertility. We see that increased wages of women early in the life cycle tend to increase the age of first child markedly. In addition to this, we also see that the share remaining childless and couples with only one child also increases substantially. This suggests that the results are driven by a combination of two forces. A group of women will focus on careers and remain childless and another group will still have children, but later and fewer of them. The picture is the opposite for increased wages of men: The age at first child decreases, the share of childless couples reduces, and the number of children increases.

Labor supply responses to wage changes can differ between the baseline model and the alternative exogenous fertility model for two reasons. First, there is a direct fertility effect from changes in the relative price of having children. Second, couples self-select into parenthood and, as a result, into labor supply, affecting the labor market responsiveness to wage realizations. To separate the direct effect from the selection effect, we simulate an additional version of the alternative stochastic fertility model in which realized fertility for a particular household remains fixed at their chosen fertility in the baseline model. In this simulation, realized fertility is thus identical on the household level. This removes the selection effect and the only remaining difference in the labor supply response from the baseline model is due to the change in relative price of having children and thus direct fertility adjustments.

In Table D.3 in the Supplemental Material we show the labor market elasticities when simulated from this alternative model with fixed fertility. The average (across age) hours elasticity of women is 4 percentage points, or 7%, higher (0.60 vs. 0.56) in the baseline model compared to this second alternative model. This suggests that both fertility adjustments and also self-selection into parenthood are important drivers of labor market responsiveness.

7.2 Child Subsidy Reform

Here, we investigate responses to policies targeted directly fertility. Concretely, we introduce an unconditional cash transfer of 3000DKK or 9000DKK at the time of birth. We report the percentage change in labor market supply and fertility relative to the baseline model in Panel A of Table 5. We find that labor market participation and hours worked of women is reduced with 2.23% if child subsidies are increased with 3000DKK. The childbirth probability increases with 4.97% and the completed fertility with 3.66%.

Interestingly, the labor market response of men goes in the opposite direction and are smaller in magnitudes compared to those of women. Human capital accumulation leads to a 0.5% reduction in offer wage of women at age 55.

Table 5: Increased Child Subsidy. Percentage Changes.

	Participation		Hours		Wage at 55		Child	Comp.
	Women	Men	Women	Men	Women	Men	birth	fertility
A. Baseline model								
3000	-2.23	0.03	-2.23	0.13	-0.53	0.02	4.97	3.66
9000	-3.08	0.11	-3.21	0.34	-0.87	0.05	12.29	9.11
B. Alternative exogenous fertility model								
3000	-0.14	-0.02	-0.15	-0.03	-0.03	-0.01	0.00	0.00
9000	-0.26	-0.04	-0.24	-0.12	-0.07	-0.02	0.00	0.00

Notes: The Table illustrates the percentage change labor market participation, hours worked, full-time offer wage at age 55 for women and men, the percentage change in the childbirth probability and completed fertility from an unconditional cash transfer at childbirth.

The fertility responses are in line with existing studies. Milligan (2005) estimates a 16.9% increase in fertility from an increased child subsidy of 1,000 Canadian Dollars (~5,500DKK) using policy reforms in Quebec. Haan and Wrohlich (2011) evaluate a similar counterfactual reform in Germany, increasing child subsidies by €360 (~2700DKK) until age three and find that fertility increases by 4.6% in their model. Wang (2022) also finds reduced labor supply of women and increased fertility from childcare subsidies in line with our results. Bick (2016) does not find significant fertility effects from increased childcare availability and Adda, Dustmann and Stevens (2017) hardly find any fertility response from increasing child subsidies in Germany.

Fertility adjustments are extremely important drivers of the labor supply effects. In Panel B of Table 5 we report percentage changes in labor market outcomes from child subsidies in the alternative exogenous fertility model. Controlling for selection in Table D.4 in the Supplemental Material hardly changes the responses. In the alternative exogenous fertility model, women's labor market response from the child subsidy is an order of magnitude lower than in the baseline model. This is because the child subsidy has no substitution effect in this model and thus only an income effect. Because fertility would increase in response to such a reform, ignoring this adjustment leads to under-prediction of the labor market response and thus the impact on governmental budgets from such reforms. Our results thus suggest that women's labor supply responses from increased child subsidies in Guner, Kaygusuz and Ventura (2020) is a lower bound.

7.3 Wealth Effects

To investigate how labor supply and fertility responds to general wealth changes not targeted fertility, we simulate responses to an unconditional (also on childbirth) cash transfer of DKK 50,000 (roughly \$10,000) at age 25. Panel A in Table 6 shows the percentage change in labor market participation, hours worked, wage offer at age 55, child birth probability and completed fertility from this reform, simulated from the estimated model.

Table 6: Responses to Wealth Changes.

	Participation		Hours		Wage at 55		Child	Comp.
	Women	Men	Women	Men	Women	Men	birth	fertility
				A. Baseline model				
50000	-3.25	-0.16	-3.32	-0.49	-0.70	-0.07	3.09	2.03
				B. Alternative exogenous fertility model				
50000	-0.96	-0.27	-0.84	-0.55	-0.18	-0.09	0.00	0.00

Notes: The Table illustrates the percentage change labor market participation, hours worked, full-time offer wage at age 55 for women and men, the percentage change in the childbirth probability and completed fertility from an unconditional cash transfer of DKK50000 at age 25.

Fertility responds in an interesting way. In particular, couples change the timing of their children, leading to an increase in the probability of birth of around 3%. However, a significant part of this is timing and the completed fertility increases with less, around 2%. These responses are in line with a growing literature documenting how fertility responds to wealth-changes. For the US, Lovenheim and Mumford (2013) estimate that a \$100,000 wealth increase increases the birth probability by 16–18%. Dettling and Kearney (2014), also for the US, find a 5% increase in the likelihood of birth from a \$10,000 wealth increase. For Australia Atalay, Li and Whelan (2017) estimate that \$100,000 additional wealth increases the likelihood of having a child by 7.5%. For Denmark, Daysal, Lovenheim, Siersbæk and Wasser (forthcoming) estimate that a DKK100,000 increase in housing wealth increases the likelihood of birth with 2.35%.

Women reduce the hours worked with 3.3% while men reduce their hours worked with 0.5%. For women, most of the response is on the extensive margin while the intensive margin response is driving most of the response of men. The reduction in labor market participation of women translates into a sizable long term effect and a 0.7% reduction in their offer wage at age 55.

The labor market reduction of women is markedly lower when couples cannot increase fertility. Panel B in Table 6 shows the labor market responses simulated from the alternative model in which fertility is exogenous. Interestingly, the labor market response of men is larger when fertility cannot adjust due to the added worker effect of their partners.

8 Sensitivity Analysis

We now investigate how sensitive our results are to the calibrated parameter values from Section 5 following the approach suggested in Jørgensen (forthcoming). Collecting all calibrated parameters in $\boldsymbol{\phi}$, we approximate marginal changes in the estimated parameters w.r.t. the calibrated parameters, $\frac{\partial \hat{\boldsymbol{\theta}}}{\partial \boldsymbol{\phi}'}$, as

$$\hat{S} = -(G'WG)^{-1}G'D \tag{13}$$

in which $G = \frac{\partial g(\hat{\boldsymbol{\theta}}|\boldsymbol{\phi})}{\partial \hat{\boldsymbol{\theta}}'}$ and $D = \frac{\partial g(\hat{\boldsymbol{\theta}}|\boldsymbol{\phi})}{\partial \boldsymbol{\phi}'}$ are Jacobians of the estimation moments with respect to the estimated and calibrated parameters, respectively. $\hat{S} \approx \frac{\partial \hat{\boldsymbol{\theta}}}{\partial \boldsymbol{\phi}}$ is a $\dim(\boldsymbol{\theta}) \times \dim(\boldsymbol{\gamma})$ matrix containing the sensitivity of the estimated parameters to the calibrated parameters.

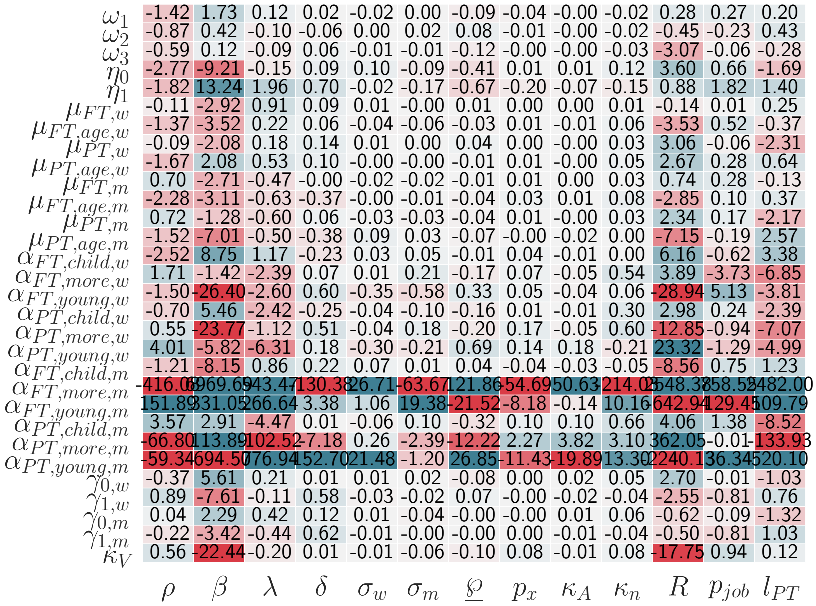
We calculate the elasticities of the kth estimated parameter in $\boldsymbol{\theta}$ to the jth calibrated parameter in $\boldsymbol{\phi}$ as

$$\hat{S}_{(k,j)} \cdot |\boldsymbol{\phi}_{(j)}/\hat{\boldsymbol{\theta}}_{(k)}|$$

and report the elasticities in Figure 8. Four estimated parameters stand out as being very sensitive: $\alpha_{FT,more,m}$, $\alpha_{FT,young,m}$, $\alpha_{PT,more,m}$ and $\alpha_{PT,young,m}$, all related to the dis-utility of work of men when more children arrive and when a young child is present. However, these elasticities are large simply because the parameters are estimated to be very close to zero. We will thus not discuss these further.

The calibrated parameters affecting the estimates the most are the CRRA coefficient, ρ, the discount factor, β, the intra-household weight on female utility, λ, the human capital accumulation in part time work, l_{PT}, and the gross interest rate, R. Unsurprisingly, the latter has almost identical effects as the discount factor. Likewise, elasticities w.r.t. l_{PT} are very similar to elasticities w.r.t. λ. Since it is mostly women in the model who work part time increasing either their bargaining power, through λ, or their wage rate, through human capital accumulation of l_{PT}, has almost similar effects. The probability of unemployment, $1 - p_{job}$, and the human capital depreciation rate, δ, also influence several parameters but to a lesser degree. The estimates are quite robust to the human capital shock variances, σ_w^2 and σ_m^2, the child leaving probability, p_x, the sharing rules in partnership dissolution, κ_A and κ_n, and the likelihood of unintended pregnancies, $\underline{\wp}$.

Figure 8: Sensitivity of $\hat{\boldsymbol{\theta}}$ to Calibrated Parameters in $\boldsymbol{\phi}$.



Notes: The figure shows the elasticity of $\hat{\boldsymbol{\theta}}$ from calibrated parameters in eq. 13 based on the approximation proposed by Jørgensen (forthcoming). On the horizontal axis are the calibrated parameters in $\boldsymbol{\phi}$ and on the vertical axis is the estimated parameters in $\hat{\boldsymbol{\theta}}$.

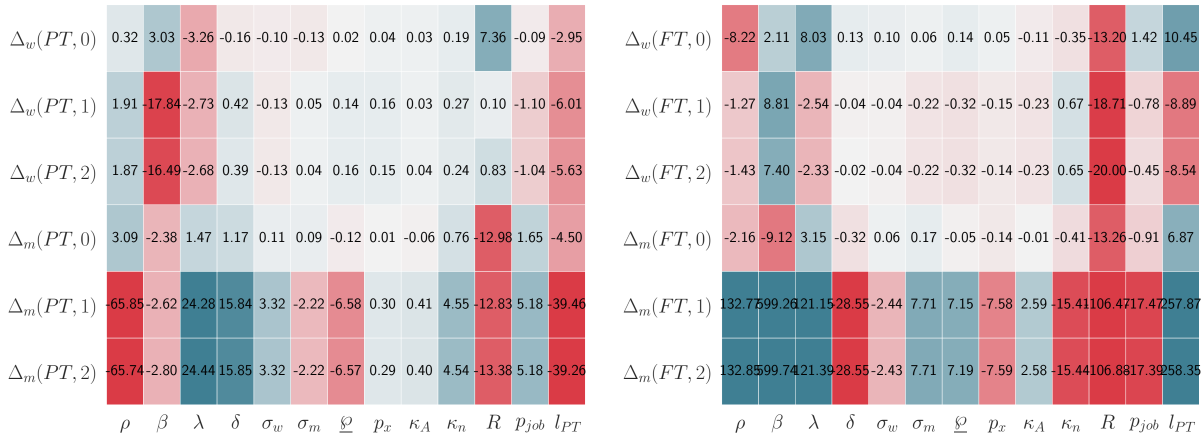
Increasing the discount factor, β, would lead to more wealth accumulation. To match the empirical wealth profile, the retirement adjustment factor, κ_V, would decrease substantially, as seen in the second column of Figure 8. Further, to maintain the observed spacing between births, the dis-utility of a low spacing would have to increase (η_0 should be more negative).

The interpretation of the sensitivity of parameters related to the dis-utility of labor work, $\boldsymbol{\alpha}$, is complicated by the fact that the parameters are measured relative to baseline parameters $\mu_{PT,w}$ and $\mu_{FT,w}$ for women and $\mu_{PT,m}$ and $\mu_{FT,m}$ for men. Using the chain-rule, we report in Figure 9 the sensitivity of $\Delta_j(l,n)$ i.e. the change in the marginal dis-utility of labor work from additional children. Because $\alpha_{FT,more,m}$, $\alpha_{FT,young,m}$, $\alpha_{PT,more,m}$ and $\alpha_{PT,young,m}$ are all estimated close to zero, so is $\Delta_m(l,n)$ for $l \in \{PT, FT\}$ and $n \in \{1,2\}$. This again explains the large elasticities in the two bottom rows of Figure 9. We will thus ignore these in our discussion below. Furthermore, because the signs are mostly flipped between $\Delta_j(PT,n)$ and $\Delta_j(FT,n)$, we focus on $\Delta_j(PT,n)$ in panel (a) in Figure 9.

The same parameters are important for the non-seperability between labor market work and children. The CRRA coefficient is positively related to $\Delta_j(PT,n)$ for all $n \in \{0,1,2\}$ and $j \in \{w,m\}$. The discount factor is positively related to the marginal dis-utility of work around first childbirth, $\Delta_w(PT,0)$, but negatively related to to the marginal dis-utility of work when having more children. Increasing the job finding probability, p_{job},

Figure 9: Sensitivity of $\Delta_j(l,n)$ with respect to calibrated parameters in $\boldsymbol{\phi}$.

(a) Part Time. (b) Full Time.



Notes: The Figure reports the sensitivity to the change in the marginal dis-utility of work from additional children in Figure 6. The elasticities are based on $\frac{d\Delta_j(l,n)}{d\boldsymbol{\phi}'} = \frac{\partial\Delta_j(l,n)}{\partial\boldsymbol{\theta}'}\frac{\partial\boldsymbol{\theta}}{\partial\boldsymbol{\phi}'} \approx \frac{\partial\Delta_j(l,n)}{\partial\boldsymbol{\theta}'}\hat{S}$, using the approximation proposed in Jørgensen (forthcoming).

will reduce the link between children and labor market work of women but increase it for men. Interestingly, the bargaining power of women, λ, is negatively related to $\Delta_w(PT,n)$ and positively related to $\Delta_m(PT,n)$ which suggests that increasing the bargaining power of women would reduce the link between children and labor market work of women and increase it for men.

9 Conclusion

The labor supply responsiveness of men and women are key to understanding welfare reforms and optimal tax policy. In this paper, we show that fertility adjustments are an important driver of especially women's labor supply elasticities. This has implications for how we design optimal policy and shows that we should think carefully about fertility adjustments as potentially playing an important role in counterfactual policy evaluations.

We provide new evidence that fertility responds to general tax changes not specifically targeted families with children. Using detailed Danish register data and a series of tax reforms from 2009, we show that increases in women's marginal net-of-tax wages tend to decrease fertility while increases in men's marginal net-of-tax wages tend to increase fertility. Our results suggest that this asymmetric response stems from the fertility substitution effect dominating w.r.t. wages of women while the fertility income effect dominates w.r.t. wages of men.

We then estimate a dynamic model of fertility and family labor supply that replicates our empirical finding, without explicitly being targeted in estimation. Our main contribution is to quantify the importance of fertility adjustments through counterfactual simulations within our framework. Concretely, we compare the labor hours elasticities in the baseline estimated model with that from an alternative version of the model, in which couples *cannot* adjust their fertility. The life-cycle Marshallian hours elasticity of women in our baseline model is around 28% higher than that of women in the alternative exogenous fertility model. This suggests that fertility adjustments are a key component of labor supply responses of women and an important driver of long term gender wage inequality.

Our results guide several avenues for future research. The estimated model is rich in the sense that couples in each period chose how much to save, whether each member should work and how much to work and whether to try to conceive a child. Our analysis is not without caveats, however. In particular, we abstract from intra-household bargaining and time-allocation decisions over leisure and home production (child care). This is partly motivated by data limitations and computational considerations but also by our explicit goal of formulating the model as standard as possible, by combining a dual-earner standard labor market model with a realistic model of fertility choices. Finally, we abstract from the important quantity-quality trade-of in child rearing (Becker, 1960). Interesting alternative frameworks include collective models (Chiappori, 1992; Bourguignon and Chiappori, 1994), non-coorperative models (Konrad and Lommerud, 2003), and limited commitment bargaining models (Mazzocco, 2007; Doepke and Kindermann, 2019). Including time-allocation of non-market work in the spirit of e.g. Blundell, Pistaferri and Saporta-Eksten (2018) would also be an interesting future extension of the model.

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Supplemental Material

A Data Appendix

We construct our variables as follows:

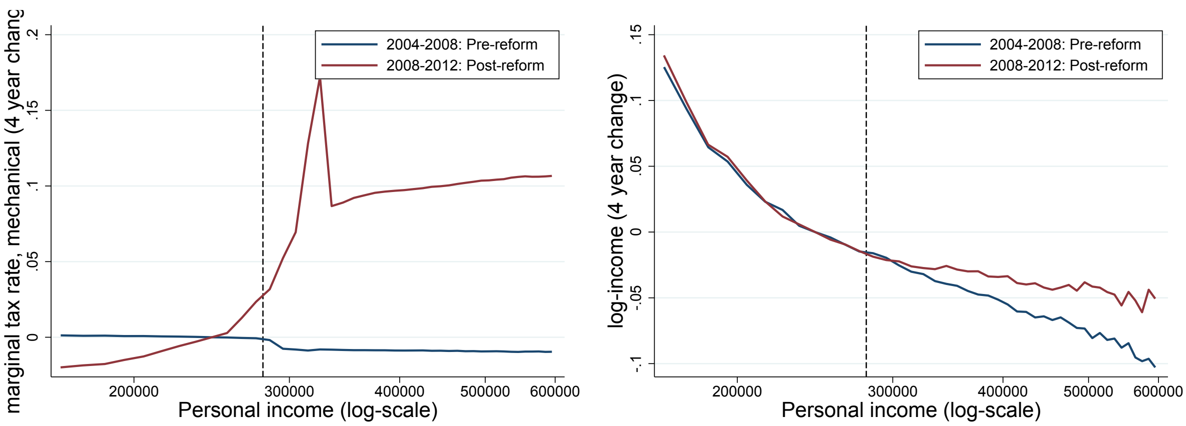
- Couples are constructed using *EFALLE* (from the register BEF).
- Birth year (cohort) and gender is based on FOED_DAG and KOEN (from BEF). The age of a couple is the age of the woman.
- Household wealth includes deposits in bank accounts, bonds, stocks and properties net of loans and mortgages and are measured at the end of the calendar year for tax purposes FORM and FORMREST_NY05 (after 1996) from IND). Property values are based on the public valuation and is in many cases underestimated. We thus follow the approach in, e.g. Leth-Petersen (2010) and increase registered property values with 10%. Wealth is deflated using the Danish CPI and in 1000DKK.
- We use two measures of income. The measure of personal income (PERINDKP from IND) includes labor income through wage work and profit from own firms and labor market transfers during a given year. Labor income is constructed as 12-month sum of monthly labor income in the BFL registry. In age profiles, the 12 months are the calandar year, running from January through December. For event-study moments, we center the 12-months around childbirth. For example, if a firstborn child arrives in March 2010, all income from January and February will be included in annual income one year prior to childbirth (-1). This has the effect that in event-study-moments, all births happen in the beginning of the year, which is what we assume in the model. All income measures are deflated using the Danish CPI and in 1000DKK.
- Self-employed, retired, students and individuals on disability pension is identified through SOCSTIL and SOCSTIL_KODE measuring the main activity in end of November.
- An individual is classified as high-skilled if the individual has at least 180 months of education (using HFPRIA from UDDA).
- The age of children is linked through the parents identifiers in the register BEF. Almost all children can be linked to a mother.
- Labor market experience is based on the accumulated payments to ATP (ERHVER and ERHVER79 from the IDAP register). Part time employment here is equivalent to 2/3 of that of full time.

A.1 Additional info and Results Related to Section 2

Similar in spirit to Gruber and Saez (2002) and Jakobsen and Søgaard (2019), we restrict attention to couples in which both members have personal income in the range 50,000–600,000 in the baseline years of the forward differences.[11] This leaves a significant part of the income distribution in the "validation region" without significant mechanical marginal tax changes even after the 2009/2010 reform. In Figure A.1, we illustrate this point by plotting the mechanical tax change in panel a) for base years 2004 and 2008 and the change in personal income in panel b), both across the income distribution in the base year. We clearly see that in the validation region (to the left of the dashed vertical line in panel a)), the mechanical tax change is zero both before and after the reform. Likewise, in panel b) we see that the change in personal income was very similar in this region while quite different higher up in the income distribution where the mechanical net-of-tax change was much larger after the reform. Combined, this indicates that the tax variation used here constitutes a valid instrument, once we flexibly control for base year income (Jakobsen and Søgaard, 2019).

Figure A.1: Verification: 4-year differences across the income distribution.

(a) Mechanical tax change. (b) Log income.



Notes: This figure illustrates the tax variation and the plausibility of the variation in generating exogeneous variation.

[11]In turn, while $z_{i,t}$ and thus $\tau_t(z_{i,t}, Z_{i,t})$ are in a restricted range, $z_{i,t+4}$ and $\tau_{t+4}(z_{i,t+4}, Z_{i,t+4})$, are unrestricted and can be outside the specified range.

Table A.1: First-stage estimates, $\Delta_4 \log(1-\tau_{i,t})$, Women.

	(1)	(2)	(3)
$\Delta_4 \tau^m_{i,t}$, women	0.428***	0.426***	0.426***
	(0.002)	(0.002)	(0.002)
$\Delta_4 \log(y^m_{i,t})$, women	0.010***	0.010***	0.010***
	(0.000)	(0.000)	(0.000)
$\Delta_4 \tau^m_{i,t}$, men	0.019***	0.019***	0.019***
	(0.001)	(0.001)	(0.001)
$\Delta_4 \log(y^m_{i,t})$, men	0.028***	0.027***	0.027***
	(0.001)	(0.001)	(0.001)
Income dummies	Yes	Yes	Yes
Children dummies	Yes	Yes	Yes
Year dummies	Yes	Yes	Yes
Age dummies	Yes	Yes	Yes
Hum. cap. controls	No	Yes	Yes
Male partner controls	No	No	Yes
Avg. dep. var. (y, level)			
Obs.	2531181	2531181	2531181
First stage F-stat.			

Notes: This Table reports first-stage estimation results of $\Delta_4 \log(1-\tau_{i,t})$ using mechanical tax rate changes, $\Delta_4 \log(\tau^m_{i,t})$, $\Delta_4 \log(\tau^m_{partner(i,t)})$, $\Delta_4 \log(y^m_{i,t})$, and $\Delta_4 \log(y^m_{partner(i,t)})$ as instruments. Robust standard errors in brackets are clustered at the individual level. $^*p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$. See notes to Table 2.

Table A.2: First-stage estimates, $\Delta_4 \log(1-\tau_{i,t})$, Men.

	(1)	(2)	(3)
$\Delta_4 \tau^m_{i,t}$, women	0.015***	0.013***	0.014***
	(0.001)	(0.001)	(0.001)
$\Delta_4 \log(y^m_{i,t})$, women	0.008***	0.009***	0.008***
	(0.000)	(0.000)	(0.000)
$\Delta_4 \tau^m_{i,t}$, men	0.407***	0.407***	0.406***
	(0.001)	(0.001)	(0.001)
$\Delta_4 \log(y^m_{i,t})$, men	0.006***	0.005***	0.006***
	(0.001)	(0.001)	(0.001)
Income dummies	Yes	Yes	Yes
Children dummies	Yes	Yes	Yes
Year dummies	Yes	Yes	Yes
Age dummies	Yes	Yes	Yes
Hum. cap. controls	No	Yes	Yes
Male partner controls	No	No	Yes
Obs.	2531181	2531181	2531181

Notes: This Table reports first-stage estimation results of $\Delta_4 \log(1-\tau_{partner(i,t)})$ using mechanical tax rate changes, $\Delta_4 \log(\tau^m_{i,t})$, $\Delta_4 \log(\tau^m_{partner(i,t)})$, $\Delta_4 \log(y^m_{i,t})$, and $\Delta_4 \log(y^m_{partner(i,t)})$ as instruments. Robust standard errors in brackets are clustered at the individual level. $^*p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$. See notes to Table 2.

Table A.3: First-stage estimates, $\Delta_4 \log(y_{i,t})$, Women.

	(1)	(2)	(3)
$\Delta_4 \tau^m_{i,t}$, women	0.428***	0.426***	0.426***
	(0.002)	(0.002)	(0.002)
$\Delta_4 \log(y^m_{i,t})$, women	0.010***	0.010***	0.010***
	(0.000)	(0.000)	(0.000)
$\Delta_4 \tau^m_{i,t}$, men	0.019***	0.019***	0.019***
	(0.001)	(0.001)	(0.001)
$\Delta_4 \log(y^m_{i,t})$, men	0.028***	0.027***	0.027***
	(0.001)	(0.001)	(0.001)
Income dummies	Yes	Yes	Yes
Children dummies	Yes	Yes	Yes
Year dummies	Yes	Yes	Yes
Age dummies	Yes	Yes	Yes
Hum. cap. controls	No	Yes	Yes
Male partner controls	No	No	Yes
Avg. dep. var. (y, level)			
Obs.	2531181	2531181	2531181
First stage F-stat.			

Notes: This Table reports first-stage estimation results of $\Delta_4 \log(y_{i,t})$ using mechanical tax rate changes, $\Delta_4 \log(\tau^m_{i,t})$, $\Delta_4 \log(\tau^m_{partner(i,t)})$, $\Delta_4 \log(y^m_{i,t})$, and $\Delta_4 \log(y^m_{partner(i,t)})$ as instruments. Robust standard errors in brackets are clustered at the individual level. $^*p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$. See notes to Table 2.

Table A.4: First-stage estimates, $\Delta_4 \log(y_{i,t})$, Men.

	(1)	(2)	(3)
$\Delta_4 \tau^m_{i,t}$, women	0.015***	0.013***	0.014***
	(0.001)	(0.001)	(0.001)
$\Delta_4 \log(y^m_{i,t})$, women	0.008***	0.009***	0.008***
	(0.000)	(0.000)	(0.000)
$\Delta_4 \tau^m_{i,t}$, men	0.407***	0.407***	0.406***
	(0.001)	(0.001)	(0.001)
$\Delta_4 \log(y^m_{i,t})$, men	0.006***	0.005***	0.006***
	(0.001)	(0.001)	(0.001)
Income dummies	Yes	Yes	Yes
Children dummies	Yes	Yes	Yes
Year dummies	Yes	Yes	Yes
Age dummies	Yes	Yes	Yes
Hum. cap. controls	No	Yes	Yes
Male partner controls	No	No	Yes
Obs.	2531181	2531181	2531181

Notes: This Table reports first-stage estimation results of $\Delta_4 \log(y_{partner(i,t)})$ using mechanical tax rate changes, $\Delta_4 \log(\tau^m_{i,t})$, $\Delta_4 \log(\tau^m_{partner(i,t)})$, $\Delta_4 \log(y^m_{i,t})$, and $\Delta_4 \log(y^m_{partner(i,t)})$ as instruments. Robust standard errors in brackets are clustered at the individual level. $^*p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$. See notes to Table 2.

Table A.5: 2SLS Estimates: Number of Children. Varying Minimum Income.

	≥ 0 (1)	≥ 50 (2)	≥ 100 (3)
$\Delta_4 \log(1-\tau_{i,t})$, women	-0.022**	-0.023**	-0.019*
	(0.010)	(0.010)	(0.010)
$\Delta_4 \log(y_{i,t})$, women	0.005*	0.005*	0.004
	(0.003)	(0.003)	(0.003)
$\Delta_4 \log(1-\tau_{i,t})$, men	0.006	0.005	0.008
	(0.009)	(0.009)	(0.009)
$\Delta_4 \log(y_{i,t})$, men	0.027***	0.028***	0.028***
	(0.008)	(0.008)	(0.010)
Income dummies	Yes	Yes	Yes
Children dummies	Yes	Yes	Yes
Year dummies	Yes	Yes	Yes
Age dummies	Yes	Yes	Yes
Hum. cap. controls	Yes	Yes	Yes
Male partner controls	Yes	Yes	Yes
Avg. dep. var. (y, level)	1.52	1.522	1.533
Obs.	2541455	2531181	2475451
First stage F-stat.	27662.7	27903.8	27885.3

Notes: This Table reports 2SLS estimates when varying the minimum level of base year income allowed in the estimation sample. Robust standard errors in brackets are clustered at the individual level. *$p < 0.10$, **$p < 0.05$, ***$p < 0.01$. The first column reproduces the preferred specification from Table 2. See notes from that table.

Table A.6: 2SLS Estimates: Number of Children. Heterogeneity across Income and Educational Attainment.

	income $\in$ $[50, 350]$ (1)	income $\in$ $(350, 600]$ (2)	less skilled (3)	high skilled (4)
$\Delta_4 \log(1-\tau_{i,t})$, women	-0.030***	-0.048	-0.048***	-0.019
	(0.010)	(0.038)	(0.015)	(0.013)
$\Delta_4 \log(y_{i,t})$, women	0.005*	0.009	0.002	0.003
	(0.003)	(0.016)	(0.003)	(0.004)
$\Delta_4 \log(1-\tau_{i,t})$, men	0.007	0.004	0.038***	-0.026*
	(0.010)	(0.027)	(0.012)	(0.014)
$\Delta_4 \log(y_{i,t})$, men	0.048***	0.040***	0.000	0.025**
	(0.016)	(0.010)	(0.013)	(0.011)
Income dummies	Yes	Yes	Yes	Yes
Children dummies	Yes	Yes	Yes	Yes
Year dummies	Yes	Yes	Yes	Yes
Age dummies	Yes	Yes	Yes	Yes
Hum. cap. controls	Yes	Yes	Yes	Yes
Male partner controls	Yes	Yes	Yes	Yes
Avg. dep. var. (y, level)	1.526	1.496	1.664	1.372
Obs.	2205258	325923	1299908	1231273
First stage F-stat.	19869.3	1996.9	11197.1	15910.2

Notes: This Table reports 2SLS estimates when varying the female income or educational level. Robust standard errors in brackets are clustered at the individual level. $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$. See notes to Table 2.

Table A.7: 2SLS Estimates: Log-Labor Income.

	Women (1)	Men (2)
$\Delta_4 \log(1-\tau_{i,t})$, women	0.213***	0.111***
	(0.015)	(0.013)
$\Delta_4 \log(y_{i,t})$, women	-0.016***	0.003
	(0.005)	(0.003)
$\Delta_4 \log(1-\tau_{i,t})$, men	-0.004	0.200***
	(0.015)	(0.014)
$\Delta_4 \log(y_{i,t})$, men	0.006	-0.019
	(0.011)	(0.016)
Income dummies	Yes	Yes
Children dummies	Yes	Yes
Year dummies	Yes	Yes
Age dummies	Yes	Yes
Hum. cap. controls	Yes	Yes
Male partner controls	Yes	Yes
Avg. dep. var. (y, level)	5.454	5.728
Obs.	2316021	2396584
First stage F-stat.	28173.6	27295.1

Notes: This Table reports 2SLS estimates with four-year log-labor income changes as dependent variable. Robust standard errors in brackets are clustered at the individual level. $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$. See notes to Table 2.

B Danish Institutions in the Model

All institutions described here are based on simplified versions of the Danish institutions as they were in 2010.

B.1 Child care costs and transfers, $\mathcal{C}(\bullet)$.

In each period, households with children pay child-care costs of

$$\mathcal{C}(n_t, o_t, Y_t, s_t) = \mathbf{1}(n_t > 0)(\mathcal{C}_c(n_t, o_t, Y_t, s_t) - \mathcal{C}_b(n_t, o_t, s_t)) \tag{B.1}$$

where $\mathcal{C}_b(n_t, o_t, s_t)$ denotes child-related benefits (discussed below), $\mathcal{C}_c(n_t, o_t, Y_t, s_t) = \pi_t(n_t, Y_t, s_t) \cdot \tilde{\mathcal{C}}_c(n_t, o_t)$ denotes child care costs where the term π_t accounts for an income rebate such that a reduced percentage is paid if the before tax household income is in a certain range,

$$\pi_t(n_t, Y_t) = \begin{cases} 0 & \text{if } Y_t < Y_{min}(n_t, s_t) \\ \min\{1, \zeta_0 + \zeta_1 \cdot (Y_t - Y_{min}(n_t, s_t))\} & \text{if } Y_t \geq Y_{min}(n_t, s_t) \end{cases}$$

where $\zeta_0 = 0.05$ and $\zeta_1 = 0.26 \times 10^{-5}$. The income floor, $Y_{min}(n_t)$ was in 2010 around DKK150,000 and increased with DKK60,000 for singles and with DKK7,000 for every child in excess of the first, $Y_{min}(n_t, s_t) = 150 + 7 \cdot (n_t - 1) + 60 s_t$. Household income Y_t includes all labor market income and transfers before taxes. The cost can be negative if the household is a net-receiver of child care benefits.

The child care costs in Denmark are highly subsidized such that at most 25% (per child) of the underlying cost of child care provision can be held by the parents.[12] The cost per child in practice vary slightly with the type of care each child receives and across municipalities in Denmark. For example, in the largest municipality of Copenhagen (capitol) in 2018 the cost of sending one child to home nursery (in Danish “dagpleje”) was around DKK3,254 per month including lunch while regular nursery (in Danish “vuggestue”) was DKK3,732 per month if lunch was included and DKK3,107 per month without lunch. Whether the services includes lunch or not is often decided through a democratic process at the institutional level. For kindergarten, the monthly costs was DKK2,402 per month including lunch and DKK1,754 without lunch. Children usually enters school around the age of 6 and public schools are universal and free. After school, around 1PM, the children can enter after-school care (“SFO” or “Fritidshjem” in Danish) at a cost of around DKK1,000 per month until around age 10 where the cost reduces to around DKK500 per

[12]Ministry for Children and Social Affairs (in Danish): https://socialministeriet.dk/arbejdsomraader/dagtilbud/tilskud-og-egenbetaling/

month ("Fritidsklub" in Danish) until age 14 where the cost goes to zero ("Ungdomsklub" in Danish).

We model a simplified version of the age-dependence of child care costs. We do this for several reasons. The primary reason is for computational simplicity because we otherwise would have to keep track of the age of all children. In particular, we calculate the child care costs as

$$\tilde{\mathcal{C}}_c(n_t, o_t) = a_0 + a_1 \cdot \mathbf{1}(o_t \notin \{NC\}) + \frac{a_0}{2}(n_t - 1)$$

where we account for the fact that there is a rebate if households have several children such that the full amount is paid for the most expensive child and the remaining costs are reduced by 50%. We calculate a_0 and a_1 as the average across all municipalities in Denmark in 2010 by age-groups. We then let a_0 be the average costs for child care of children aged 0–6 and let a_1 be the average for children aged 7–18. In turn, we let $a_0 = 6,109$ and $a_1 = 27,236 - a_0 = 21,127$. The series RES88 at Statistics Denmark, http://www.statistikbanken.dk/10557, contains an overview of the different costs in different municipalities across different child-care services.

Child-related *subsidies*, $\mathcal{C}_s(n_t, o_t, s_t)$, are subtracted from the child care costs in eq. (10). We include approximate rules to capture the main part of the biggest subsidy in Denmark, called "Børne- og Ungeydelse" or "børnecheck" in Danish. Child care transfers are typically paid into the mothers account automatically (by a fourth of the annual amount each quarter) and depend on the age of each child. In particularly, the household receives DKK16,988 per 0–2 year old child, DKK13,448 per 3–6 year old child and DKK10,580 per 7–17 year old child (2010 rules and prices). Since we only keep track of the presence of at least one child in the age of 0–6, we will use 16,988 as the child benefit level for one child and 10,580 for the remaining children, $n_t - \mathbf{1}(o_t \in \{1, 2, 3, 4, 5\})$. In turn, the tax-exempt child care transfer is

$$\mathcal{C}_{1,s}(n_t, o_t) = c_0 \cdot \mathbf{1}(o_t \in \{0, 1, 2, 3, 4, 5\}) + c_1(n_t - \mathbf{1}(o_t \in \{0, 1, 2, 3, 4, 5\}))$$

where $c_0 = 16,988$ and $c_1 = 10,580$.[13]

We also include another child-subsidy applicable to singles in our model. This "Børnetilskud" as it is called in Danish is independent of the household income and tax-exempt. The amount is

$$\mathcal{C}_{2,s}(n_t, s_t) = [c_2\mathbf{1}(n_t > 0) + c_3 n_t]\mathbf{1}(s_t = 1)$$

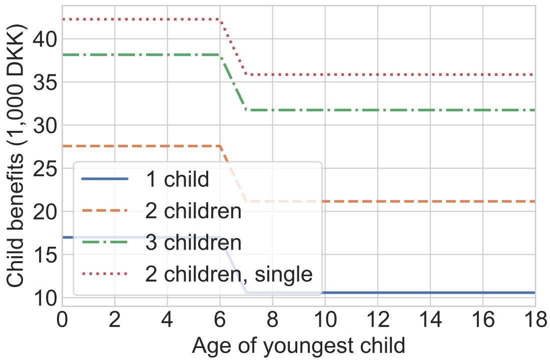
[13] The levels are adjusted with the consumer price index and can be found for different years here (in Danish): https://www.skm.dk/skattetal/statistik/tidsserieoversigter/boerne-og-ungeydelse-en-historisk-oversigt.

where $c_2 = 4,956$ and $c_3 = 4,868$

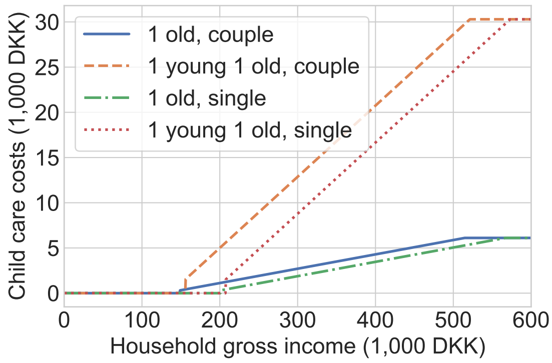
In total, the combined child-related transfers are

$$\mathcal{C}_s(n_t, o_t, s_t) = \mathcal{C}_{1,s}(n_t, o_t) + \mathcal{C}_{2,s}(n_t, s_t)$$

(a) Child Benefits.



(b) Child Care Costs.



(c) Costs net of Benefits, $\mathcal{C}(n_t, o_t, Y_t, s_t)$.

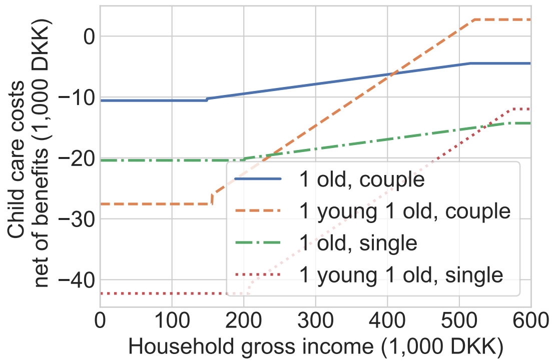


Figure B.1: Child Related Costs and Benefits.

Notes: Figure B.1 shows in panel (a) the child-related benefits in the model as a function of the age of the youngest child and the total number of children. Panel (b) shows the child-related costs in the model as a function of the household gross income (labor market income and transfers before taxes of all household members) and the number and age of children for both couples and singles. Panel (c) shows the child-related costs net of transfers as a function of the household gross income and the number and age of children for both couples and singles. A negative cost refers to situations in which the household is net receivers of child-benefits.

B.2 Taxes, $\mathcal{T}(\bullet)$.

The Danish tax system is individual with a relatively small link between couples. Concretely, the only link between couples is that capital income is measured at the household level in the government tax and unused labor participation tax deduction (around 43,000 DKK in 2010) can be transferred across spouses. The tax schedule is progressive with one of the highest marginal top tax rate in the world. We have implemented a parsimonious version of the system aimed at capturing the main features of the system in 2010.

The after-tax income of an individual j with income $Y_{j,t}$ and potential spousal income of $Y_{-j,t}$ can be calculated based on the following equations:

$$
\begin{aligned}
\tau_{\max} &= \tau_l + \tau_u + \tau_c + \tau_h - \bar{\tau}, \\
\text{personal income} &= (1 - \mathbf{1}(l_{j,t} > 0)\tau_{LMC}) \cdot Y_{j,t}, \\
\text{taxable income} &= \text{personal income} - \min\{WD \cdot Y_{j,t}, \overline{WD}\}, \\
\underset{\sim}{\mathrm{y}}_l &= \underset{\sim}{\mathrm{y}} + \max\{0, \underset{\sim}{\mathrm{y}} - Y_{-j,t}\}, \\
\mathrm{T}_c &= \max\{0, \tau_c \cdot (\text{taxable income} - \underset{\sim}{\mathrm{y}})\}, \\
\mathrm{T}_h &= \max\{0, \tau_h \cdot (\text{taxable income} - \underset{\sim}{\mathrm{y}})\}, \\
\mathrm{T}_l &= \max\{0, \tau_l \cdot (\text{personal income} - \underset{\sim}{\mathrm{y}})\}, \\
\mathrm{T}_u &= \max\{0, \min\{\tau_u, \tau_{\max}\} \cdot (\text{personal income} - \underset{\sim}{\mathrm{y}}_u)\}, \\
\text{tax} &= \mathbf{1}(l_{j,t} > 0)\tau_{LMC} \cdot Y_{j,t} + \mathrm{T}_c + \mathrm{T}_h + \mathrm{T}_l + \mathrm{T}_u,
\end{aligned}
$$

where the values from 2010 along with descriptions are given in Table B.1. Historical rates and relevant thresholds can be found at the web page of the Danish tax authorities.[14] Note that capital income in the municipal tax is individual while in the bottom–top tax is on the household level.

Table B.1: Tax System Parameters in 2010.

Symbol	Value in 2010, DKK	Description
$\bar{\tau}$	.515	Maximum tax rate, »Skatteloft«
τ_{LMC}	.08	Labor Market Contribution, »Arbejdsmarkedsbidrag«
WD	.0425	Working Deduction, »Beskæftigelsesfradrag«
$\overline{WD}$	13, 600	Maximum working deduction possible
τ_c	.2564	Average municipal tax rate (including .074 in church tax)
$\underset{\sim}{\mathrm{y}}$	42, 900	Amount deductible from bottom and muni. tax
$\underset{\sim}{\mathrm{y}}_u$	389, 900	Amount deductible from top tax bracket
τ_h	.08	Health contribution tax (in Danish »Sundhedsbidrag«)
τ_l	0.0367	Tax rate in lowest tax bracket
τ_u	0.15	Tax rate in upper tax bracket

The marginal tax rate as a function of gross income in two cases depending on whether unused spousal deductions are transferred is illustrated in panel (a) of Figure **??**. Note that although the middle tax was abolished from 2010 there is still a small change in the marginal tax rate around the location of the middle tax in earlier years. This stems from the cap on the labor marked deduction, $\overline{WD}$, binding at that level of income.

[14] https://www.skm.dk/skattetal/statistik/tidsserieoversigter/centrale-skattesatser-i-skattelovgivning

B.3 Child Support, $\mathcal{D}(\bullet)$

In case of divorce, we assume that the man pays child support to his former wife (which is by far the predominant situation in Denmark). In Denmark, child support is payed until the child is 18. In our model, we will assume that all children living at home are under 18 and all children moved from home are above. The child support payments are tax-deductible (excess of the supplemental amount).

The child support consists of three parts; a base level ($\underline{s}$), a supplement (s) and an elevated amount ($\overline{s}$) where the latter is income-dependent while the two former are not. Since the elevated amount is relatively minor and requires relatively high incomes, we ignore that element here. We do this because it would otherwise require us to keep track of the former husband's income when determining the child support received by the woman.

The total annual amount of child support the man has to transfer is thus

$$\zeta n_t$$

where $\zeta = 14,040$ Danish kroner in 2010. Out of this, the supplement is 1,608 DKK.[15]

C Calibration of Exogenous processes

We estimate the biological fecundity, i.e. the probability of a child arriving given a couple tries to become pregnant using the medical evidence in Leridon (2004). He finds that 75 percent of women starting to try to conceive at age 30 will have given birth within 1 year, 66 percent at age 35 and 44 percent at age 40. We add to these numbers the assumptions that the "success-rate" is 90 percent at age 20 and 0 percent at age 4 and estimate the complete age-profile from these five data-points. Figure C.1 panel a) shows the used biological fecundity, $\overline{\wp}_t$.

In panel b) we show estimated partnership dissolution probabilities based on Danish data.

D Numerical Implementation Details

D.1 Post Retirement Solution

After retirement, the state variables are the retirement benefits of each spouse and the only choices are how much to consume or save. In turn, we have a standard consumption

[15] http://www.statsforvaltningen.dk/site.aspx?p=8331

(a) Biological Fecundity, $\overline{\wp}_t$.

(b) Partnership Dissolution Probabilities.

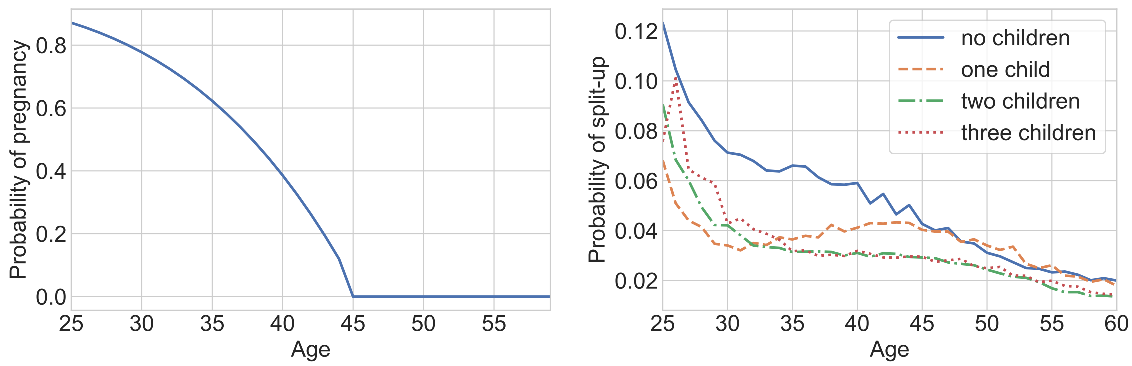


Figure C.1: Biological Fecundity and Dissolution Probabilities.

Notes: Figure C.1 shows in panel (a) the biological fecundity, $\overline{\wp}_t$, based on Leridon (2004). Panel (b) shows the probability of partnership dissolution as a function of the age of the woman and the existing number of children, based on Danish register data.

savings problem without any uncertainty. Note, then that the state variables here are $\mathcal{S}_t = (M_t, K_w, K_m)$. The problem can be written in recursive form as (for $t > T_r$)

$$\begin{aligned}
\tilde{V}_t(\mathcal{S}_t) &= \max_{C_t \in (0, M_t)} U(C_t, n_t, o_t, l_{w,t}, l_{m,t}) + \beta \tilde{V}_{t+1}(\mathcal{S}_{t+1}) \\
&\text{s.t.} \\
M_{t+1} &= RA_t + 2 \cdot B_{ret}
\end{aligned}$$

where B_{ret} are retirement benefits. The Euler equation must hold for $A_t > 0$:

$$\nu(n_t)^{-(1-\rho)} C_t^{-\rho} = \beta R \nu(n_{t+1})^{-(1-\rho)} C_{t+1}^{-\rho}$$

where we use that

$$\begin{aligned}
\frac{\partial U(C_t, n_t, o_t, l_{w,t}, l_{m,t})}{\partial C_t} &= \lambda \frac{\partial u_w(C_t, n_t, o_t, l_{w,t})}{\partial C_t} + (1-\lambda) \frac{\partial u_m(C_t, n_t, o_t, l_{m,t})}{\partial C_t} \\
&= \frac{\partial}{\partial C_t} \frac{(C_t/\nu(n_t))^{1-\rho}}{1-\rho} \\
&= \nu(n_t)^{-(1-\rho)} C_t^{-\rho}
\end{aligned}$$

We can invert the Euler equation to get

$$C_t = (\beta R)^{-\frac{1}{\rho}} [\nu(n_{t+1})/\nu(n_t)]^{\frac{1-\rho}{\rho}} C_{t+1}.$$

We use the Endogenous Grid Method (EGM) proposed by Carroll (2006) to solve for optimal consumption, $C_t^{\star}(\mathcal{S}_t)$ by using a post-decision grid over savings, $\overrightarrow{A}$ with the lowest point being (close to) zero. We then use this to construct the value function

$\tilde{V}_t(\mathcal{S}_t)$. When we interpolate next period consumption and value function we use that the constrained region, $A = 0$, correspond to the solution found for the lowest point in our $\overrightarrow{A}$ grid. Denoting the value function in this point as $\tilde{V}_t^0$, we can construct the value function in the constrained region for some $M_t < M_t^*$ where M_t^* is the point at which the credit constraint just binds (i.e. the endogenous level of resources found for the lowest point in the grid). In particular, we can construct the value at $M_t < M_t^*$ as $\tilde{V}_t(\mathcal{S}_t) = U(M_t, n_t, o_t, l_{w,t}, l_{m,t}) + \tilde{V}_t^0 - U(M_t^*, n_t, o_t, l_{w,t}, l_{m,t})$. For consumption, we simply put $C^\star = M_t$ in the constrained region.

We employ a similar strategy when solving the model for singles. For computational efficiency, we solve the model on a grid $\overrightarrow{A}_w = \kappa_A \overrightarrow{A}$ and $\overrightarrow{A}_m = (1 - \kappa_A)\overrightarrow{A}$ such that interpolation is not needed in the wealth direction for that model when constructing the post-decision continuation value, as we describe below.

To adjust for the parsimonious description of reality in retirement, we follow the approach in e.g. Gourinchas and Parker (2002) and Jørgensen (2017) and add an adjustment factor, $\varkappa_V$, multiplied to the retirement value function such that the value in the first non-working period is $V_{T_r}(\mathcal{S}_{T_r}) = \varkappa_V \tilde{V}_{T_r}(\mathcal{S}_{T_r})$.

D.2 Pre-Computation of Continuation Values: Singles

We solve the model for singles using value function iteration but rather than computing expectations for all guesses of discrete and continuous choice variables we pre-compute the discounted expected continuation value. The problem can be re-formulated as

$$V_{j,t}(\mathcal{S}_{j,t}) = \max_{l_{j,t} \in \ell(job_{j,t})} v_{j,t}(\mathcal{S}_{j,t} | l_{j,t})$$
$$v_{j,t}(\mathcal{S}_{j,t} | l_{j,t}) = \max_{C_t} u_j(C_t, n_t, o_t, l_{j,t}) + \beta \mathbb{E}_t[V_{j,t+1}(\mathcal{S}_{j,t+1})]$$

where

$$\ell(job_{j,t}) = \begin{cases} \{0\} & \text{if } job_{j,t} = 0 \\ \{0, 0.75, 1\} & \text{if } job_{j,t} = 1. \end{cases}$$

Defining $\overline{K}_{j,t} = [(1 - \delta)K_{j,t} + l_{j,t}]$ as the post-decision (before shock) human capital, we compute the expectation as

$$
\begin{aligned}
w_{j,t}(job_{j,t}, n_t, o_t, A_t, \overline{K}_{j,t}) &= \mathbb{E}_t[V_{j,t+1}(job_{j,t+1}, n_{t+1}, o_{t+1}, K_{j,t+1}, A_t)] \\
&= \beta \int_0^{\infty} \Big[q_t(n_t, o_t) V_{j,t+1}(job_{j,t+1}, n_t - 1, o_{t+1}, \overline{K}_{j,t} \cdot \varepsilon, A_t) \\
&+ (1 - q_t(n_t, o_t)) EV_{j,t+1}(job_{j,t+1}, n_t, o_{t+1}, \overline{K}_{j,t} \cdot \varepsilon, A_t) \Big] g(\varepsilon) d\varepsilon \\
&\approx \beta \sum_{k=1}^{K} \omega_{j,k} \Big[q_t(n_t, o_t) EV_{j,t+1}(job_{j,t+1}, n_t - 1, o_{t+1}, \overline{K}_{j,t} \cdot \varepsilon, A_t) \\
&+ (1 - q_t(n_t, o_t)) EV_{j,t+1}(job_{j,t+1}, n_t, o_{t+1}, \overline{K}_{j,t} \cdot \varepsilon, A_t) \Big]
\end{aligned}
$$

over a grid of end-of-period wealth and human capital. We use $K = 5$ Gauss-Hermite quadrature nodes to approximate the numerical integral. Note that we assume that singles do not have more children but the number of children can decline due to existing children moving out. When finding optimal $l_{j,t}, C_t$ we interpolate $w_{j,t}(n_t, o_t K_{c,t}, A_t, \overline{K}_{j,t})$ rather than re-calculating the expectations. We assume that singles enjoy an "effective number of children" $\kappa_N n_t$ for single women and $(1-\kappa_n) n_t$ for single men. Thus, in the flow utility function of singles, these enter rather than the number of children.

D.3 Pre-Computation of Continuation Values: Couples

Like for singles, we pre-compute the discounted expected continuation value on post-decision grids for couples. Recalling that children only move after the fertile period, the expected continuation value is in the infertile periods, $t > T_f$

$$
\begin{aligned}
& w_t(job_{w,t}, job_{m,t}, n_t, o_t, A_t, \overline{K}_{w,t}, \overline{K}_{m,t}) \\
&= \sum_{j_w=0}^{1} \sum_{j_m=0}^{1} p_{job,w} p_{job,m} \Bigg(p^s_{t+1} \int \int \Big[q_t(n_t, o_t) V_{t+1}(j_w, j_m, A_t, \overline{K}_{w,t} \cdot \varepsilon_w, \overline{K}_{m,t} \cdot \varepsilon_m, n_t - 1, o_{t+1}) \\
&+ (1 - q_t(n_t, o_t)) V_{t+1}(j_w, j_m, A_t, \overline{K}_{w,t} \cdot \varepsilon_w, \overline{K}_{m,t} \cdot \varepsilon_m, n_t, o_{t+1}) \Big] g(\varepsilon_w) g(\varepsilon_m) d\varepsilon_w d\varepsilon_m \\
&+ (1 - p^s_{t+1}) \lambda \int \Big\{ q_t(n_t, o_t) V_{w,t+1}(j_w, \kappa_A A_t, \overline{K}_{w,t} \cdot \varepsilon_w, n_t - 1, o_{t+1}) \\
&+ (1 - q_t(n_t, o_t)) V_{w,t+1}(j_w, \kappa_A A_t, \overline{K}_{w,t} \cdot \varepsilon_w, n_t, o_{t+1}) \Big\} g(\varepsilon_w) d\varepsilon_w \\
&+ (1 - p^s_{t+1})(1 - \lambda) \int \Big\{ q_t(n_t, o_t) V_{m,t+1}(j_m, (1 - \kappa_A) A_t, \overline{K}_{m,t} \cdot \varepsilon_m, n_t - 1, o_{t+1}) \\
&+ (1 - q_t(n_t, o_t)) V_{m,t+1}(j_m, (1 - \kappa_A) A_t, \overline{K}_{m,t} \cdot \varepsilon_m, n_t, o_{t+1}) \Big\} g(\varepsilon_m) d\varepsilon_m \Bigg)
\end{aligned}
$$

with $\overline{K}_{j,t} = [(1-\delta)K_{j,t} + l_{j,t}]$ being post-decision human capital. In the fertile periods, $t \leq T_f$, the expectation can be formulated as

$$
\begin{aligned}
&w_t(job_{w,t}, job_{m,t}, n_t, o_t, A_t, \overline{K}_{w,t}, \overline{K}_{m,t}|e_t) \\
&= \sum_{j_w=0}^{1}\sum_{j_m=0}^{1} p_{job,w}p_{job,m}\Bigg(p^s_{t+1}\int\int\bigg[\wp_t(e_t)\cdot V_{t+1}(j_w, j_m, A_t, \overline{K}_{w,t}\cdot\varepsilon_w, \overline{K}_{m,t}\cdot\varepsilon_m, n_t+1, o_{t+1}) \\
&+ (1-\wp_t(e_t))\cdot V_{t+1}(j_w, j_m, A_t, \overline{K}_{w,t}\cdot\varepsilon_w, \overline{K}_{m,t}\cdot\varepsilon_m, n_t, o_{t+1})\bigg]g(\varepsilon_w)g(\varepsilon_m)d\varepsilon_w d\varepsilon_m \\
&+ (1-p^s_{t+1})\wp_t(e_t)\lambda\int V_{w,t+1}(j_w, \kappa_A A_t, \overline{K}_{w,t}\cdot\varepsilon_w, n_t+1, o_{t+1})g(\varepsilon_w)d\varepsilon_w \\
&+ (1-p^s_{t+1})(1-\wp_t(e_t))\lambda\int V_{w,t+1}(j_w, \kappa_A A_t, \overline{K}_{w,t}\cdot\varepsilon_w, n_t, o_{t+1})g(\varepsilon_w)d\varepsilon_w \\
&+ (1-p^s_{t+1})\wp_t(e_t)(1-\lambda)\int V_{m,t+1}(j_m, \kappa_A A_t, \overline{K}_{m,t}\cdot\varepsilon_m, n_t+1, o_{t+1})g(\varepsilon_m)d\varepsilon_m \\
&+ (1-p^s_{t+1})(1-\wp_t(e_t))(1-\lambda)\int V_{m,t+1}(j_m, \kappa_A A_t, \overline{K}_{m,t}\cdot\varepsilon_m, n_t, o_{t+1})g(\varepsilon_m)d\varepsilon_m\Bigg)
\end{aligned}
$$

These expectations can then be pre-computed for each combination of discrete choice over fertility, e_t, and discrete states, $job_{w,t}, job_{m,t}, n_t, o_t$, on grids of $A_t, \overline{K}_{w,t}, \overline{K}_{m,t}$ and then re-used in the optimization over consumption and discrete choices. We again use $K = 5$ Gauss-Hermite quadrature nodes for each of the household member's shock to human capital. Recall, the single's solution is on gender-specific grids, $\overrightarrow{A}_w = \kappa_A\overrightarrow{A}$ and $\overrightarrow{A}_m = (1-\kappa_A)\overrightarrow{A}$, such that interpolation is not needed in that direction when looking up in the next-period value in case of divorce. We use linear interpolation to interpolate between known points in the grids.

D.4 EGM

We use the Endogenous Grid Method (EGM) propsoed by Carroll (2006) extended to allow for discrete choices as in Iskhakov, Jørgensen, Rust and Schjerning (2017) and Druedahl and Jørgensen (2017). We here describe how the approach is implemented for singles. The approach is similar for couples but the notation becomes cluttered and is left out. Let

$$
w'_t(job_{w,t}, job_{m,t}, n_t, o_t, A_t, \overline{K}_{w,t}, \overline{K}_{m,t}|e_t) \equiv \frac{\partial w_t(job_{w,t}, job_{m,t}, n_t, o_t, A_t, \overline{K}_{w,t}, \overline{K}_{m,t}|e_t)}{\partial A_t}
$$

be the marginal value of saving. We have that optimal consumption (away from the credit constraint) must satisfy the first order condition

$$
u'_j(C_t, n_t, o_t, l_{j,t}) = w'_t(job_{w,t}, job_{m,t}, n_t, o_t, A_t, \overline{K}_{w,t}, \overline{K}_{m,t}|e_t)
$$

such that given a grid of end-of-period wealth, $\overrightarrow{A}_t$, consumption can be found in closed form as

$$C_t = u_j'^{-1}[w_t'(n_t, o_t, \overrightarrow{A}_t, \overline{K}_{w,t}, \overline{K}_{m,t}|e_t)]$$

and endogeneous resources can be found as

$$M_t = C_t + \overrightarrow{A}_t.$$

We use the DC-EGM proposed in Iskhakov, Jørgensen, Rust and Schjerning (2017) with the upper envelope algorithm proposed in Druedahl and Jørgensen (2017) to back out which solutions to the FOC that maximizes the value function.

D.5 Simulating from the Model

To simulate synthetic data for N_{sim} households for T_{sim} periods from the model, we draw several stochastic draws. First, we draw initial conditions for all state variables, $\{\mathcal{S}_{i,0}\}_1^{N_{sim}}$ from an empirical distribution based on the Danish data. We assume that child-capital is zero for all households at beginning of adulthood. Secondly, we simulate $N_{sim} \times T_{sim}$ human capital shocks for each household member, uniform pregnancy draws, uniform child moving draws, and uniform divorce draws

$$\{\varepsilon_{w,i,t}, \varepsilon_{m,i,t}, \nu_{\wp,i,t}, \nu_{q,i,t}, \nu_{s,i,t}\}_{i,t}^{N_{sim},T_{sim}}.$$

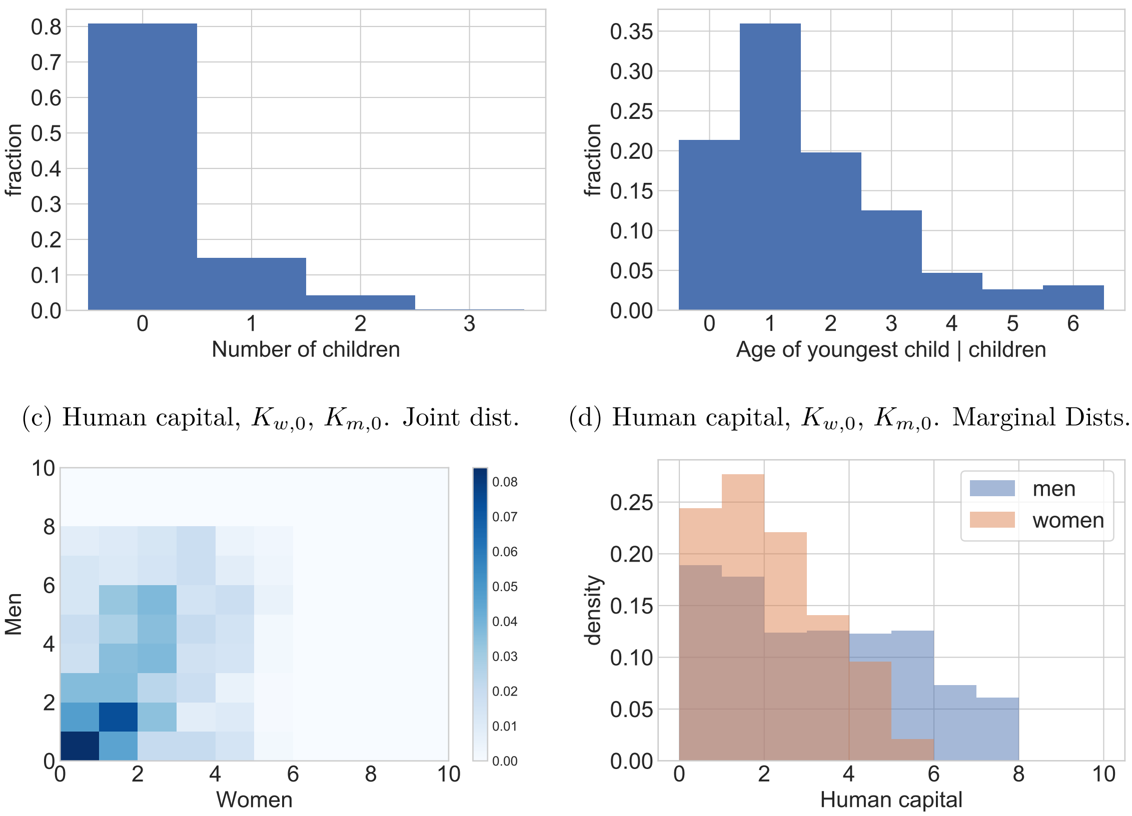
Finally, we also simulate $N_{sim} \times T_{sim}$ uniform draws to determine the optimal discrete choices

$$\{\nu_{l_w,i,t}, \nu_{l_m,i,t}, \nu_{f_w,i,t}, \nu_{f_m,i,t}, \nu_{e,i,t}\}_{i,t}^{N_{sim},T_{sim}}.$$

Optimal discrete choices are found by calculating choice-probabilities and comparing them with these uniform draws.

Initial Conditions.

When simulating when initialize all households at age 25 as couples with zero net wealth and the empirical joint distribution of number of children, age of youngest and human capital based on Danish couples in our sample at age 25. The latter is based on recorded labor market experience through payments to the labor market pension account (ATP). One year of full time employment gives a value of one and part time employment is equivalent to 2/3 of that of full time. Because this is a crude measure that does not take depreciation into account, we scale the register-based measure with 0.5. We truncate the empirical net-wealth distribution at the minimum amount allowed in the model. Figure D.1 shows the empirical distributions used as initial conditions when simulating data from the model.



(a) Number of children, n_0.

(b) Age of youngest child, o_0.

(c) Human capital, $K_{w,0}$, $K_{m,0}$. Joint dist.

(d) Human capital, $K_{w,0}$, $K_{m,0}$. Marginal Dists.

Figure D.1: Initial Distributions.

Notes: Figure D.1 shows the initial distribution of the number of children in panel (a), the age of the youngest child (given at least one child is present) in panel (b), and the simultaneous distribution of human capital level of both men and women in panel (c) and their marginal distributions in panel (d). All distributions are based on the Danish data for 25 year old women in couples.

Counterfactual simulations. We calculate the effect on an outcome y of a wage increase in the following way. The effect at age t is

$$\Delta y_t = y_t - \tilde{y}_t$$

where $y_t = n_t^{-1} \sum_i y_{i,t}$ is the average simulated optimal outcome under the baseline estimated model and $\tilde{y}_t^{(s_1:s_2)} = n_t^{-1} \sum_i \tilde{y}_{i,t}^{(s_1:s_2)}$ is the average simulated optimal outcome under the counterfactual setting in which wages are scaled by μ percent in periods s_1 through s_2. Formally, wages in the alternative model are given as

$$\tilde{w}_{i,t}^{(s_1:s_2)} = \begin{cases} (1+\mu) w_{i,t} & \text{if } s_1 \leq t \leq s_2 \\ w_{i,t} & \text{else.} \end{cases}$$

Unless otherwise explicitly stated, we use a five percent increase, $\mu = 0.05$. This specification is used both in the solution and simulation of the model.

For unanticipated chocks the model economy is identical before the shock and $y_t = \tilde{y}_t$ for $t < s_1$. The model economy is also identical for $t > s_2$ but the simulated paths might differ due to previous shocks. To generate transitory shocks we set $s_1 = s_2$ and to generate permanent shocks we set $s_2 = T_r$. Varying s_1 lets us investigate how responses differ as a function of the age at which the shock occurred. We refer to regime shifts as permanent changes from the first period of the model by letting $s_1 = 25$ and $s_2 = T_r$.

D.6 Estimation of θ

We estimate the remaining parameters primarily governing the wage process and utility parameters related to work and children, collected in θ by Simulated Method of Moments (Smith, 1993; Gouriéroux, Monfort and Renault, 1993). Concretely, we estimate θ as

$$\hat{\theta} = \arg\min_{\theta} g(\theta)' W g(\theta)$$

where $g(\theta) = m^{data} - m^{sim}(\theta)$ is a $J \times 1$ vector of differences between the J empirical moments in the data and the same moments calculated from simulated data for a given θ. W is a $J \times J$ symmetric positive definite weighting matrix and we use a diagonal matrix with the inverse of the variance of the empirical moments on the diagonal.

The minimization problem might have several local minima and be ill-behaved due to simulated discrete choices. Our estimation approach this use non-gradient based numerical solvers and involves several steps to increase the likelihood of finding the global minimum. Concretely, we use an extended version of the "TikTak" approach suggested in Arnoud, Guvenen and Kleineberg (2019). The parameters are estimated through the following steps, where $N_{threads}$ are the number of parallel threads used for estimation,

0. **Choose settings:** Define a set of initial guesses, θ_0 and set lower and upper bounds, $\underline{\theta}$ and $\overline{\theta}$.
   Set the number of initial (potential random) evaluation points per thread, N_{init}, the desired number of K-means groups per thread, N_{kmeans}, and the desired number of refinement estimation steps, N_{step}.
   Set also the maximum number of objective function evaluations, $N_{maxevals}$.

1. **Initialization:** Evaluate the objective function $Q(\theta) = g(\theta)' W g(\theta)$ for $N_{init} \times N_{threads}$ initial (pseudo) random points within the bounds. We use Sobel points to span the high-dimensional space of initial points weighted towards the initial guess with a weight w_0. This is easily done in parallel.

   (a) Sort the resulting parameters based in the value of the objective function in ascending order. This gives a set of parameter vectors $\{\theta^{(k)}\}_{k=1}^{N_{threads} \times N_{init}}$ where $Q(\theta^{(k+1)}) > Q(\theta^{(k)}) > Q(\theta^{(k-1)})$.

2. **K-means grouping:** (optional) Use K-means to group the best $N_{threads} \times N_{kmeans}$ initial parameters, $\{\theta^{(k)}\}_{k=1}^{N_{threads} \times N_{kmean}}$, into $N_{threads} \times N_{steps}$ groups based on the similarity of the parameters. Since the objective function is not evaluated in this step, this is almost cost-less.

   (a) Sort the resulting parameters based in the value of the objective function in ascending order and (optionally) include the specified initial guess θ_0 as the first element of the sorted list of parameters. If the initial guess is included, the parameters with the highest objective function is discarded.

   (b) This gives a set of parameter vectors $\{\theta^{(k)}\}_{k=1}^{N_{threads} \times N_{steps}}$ where $Q(\theta^{(k+1)}) > Q(\theta^{(k)}) > Q(\theta^{(k-1)})$ that we will initialize estimation from.

3. **Local estimations:** Successively apply local optimization from weighted vectors of starting values in parallel. For each step $s = 1, \ldots, N_{steps}$, do the following:

   (a) Construct initial values for each thread $h = 1, \ldots, N_{threads}$ as $z_{s,h} = \omega_s Z^\star + (1 - \omega_s)\theta^{((s-1) \cdot N_{threads} + h)}$ where
   $Z^\star = \arg\min_{j<s,k} Q(\hat{z}_{j,k})$ is the vector of parameters associated with the lowest objective function before the current step s and
   ω_s is a step-specific weight on the currently best parameter vector. We use a weight that increases linearly over the N_{steps} steps from 0.0 to 0.95.

   (b) Call local optimizer for each thread $h = 1, \ldots, N_{threads}$ using $z_{s,h}$ as starting values to get associated estimates $\hat{z}_{s,h}$.

4. **Final estimation:** (optional) Using the local estimated parameters with the lowest objective function across all $N_{threads} \times N_{steps}$, $Z^\star = \arg\min_{s,h} Q(\hat{z}_{s,h})$, as starting values, we call the local optimizer a final time and denote the resulting estimates as $\hat{\theta}$.

The step 3 is included as a modification to the approach in Arnoud, Guvenen and Kleineberg (2019) to try to circumvent that almost all of the $N_{threads} \times N_{steps}$ best initial values stem from the same local minima. We use the gradient-free Nelder-Mead as local solver and set $N_{threads} = 6$, $N_{ini} = 5,000$, $N_{kmeans} = 8$,$N_{steps} = 4$, $N_{maxevals} = 5,000$ and $w_0 = 0.1$. The (worst-case) computation time associated with this estimation setup is $N_{init} \times N_{threads} + N_{maxevals} \times N_{steps} \times N_{threads}$ evaluations of the objective function, $Q(\theta)$.

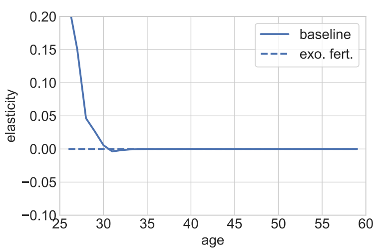
Table D.1: First Step Calibrations and Estimations.

Parameter		Value	Source
Utility function			
ρ	Constant relative risk aversion	1.5	Attanasio and Weber (1995)
β	Discount factor	0.97	Jørgensen (2017)
λ	Relative loading on women's utility	0.5	Eckstein, Keane and Lifshitz (2019)
Human capital accumulation			
δ	Human capital depreciation	0.1	Keane and Wasi (2016)
l_{PT}	Part time hours, relative to full time	0.75	Own calculations, see text.
$\sigma_{w,\epsilon}$	Human capital shocks, women	0.1	Keane and Wasi (2016)
$\sigma_{m,\epsilon}$	Human capital shocks, men	0.1	Keane and Wasi (2016)
Children process			
$\overline{\wp}_t$	Biological fecundity	Fig C.1	Leridon (2004)
$\underline{\wp}$	Unintended pregnancy probability	0.05	Ejrnæs and Jørgensen (2020)
p_x	Probability in child moving process	0.08	Own calculations, see text
Partnership dissolution			
$p_s(\bullet)$	Probability of dissolution	Fig C.1b	Own calculations, see text
κ_n	Share of time children are with mother	0.8	Own calculations, see text
κ_A	Share of wealth going to women	0.5	Own calculations, see text
Miscellaneous			
R	Gross after tax interest rate	1.03	Jørgensen (2017)
p_{job}	Probability of employment	0.97	Own calculations, see text

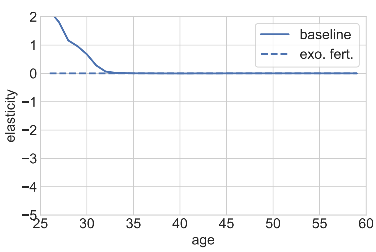
Figure D.2: Quantifying the Role of Fertility Responses: Mechanisms.

Increased wage of women

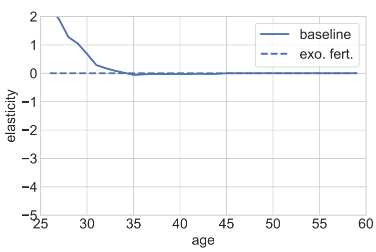
(a) Age at first birth.



(b) Share without children.

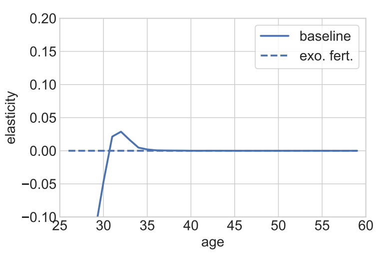


(c) Share with one child.

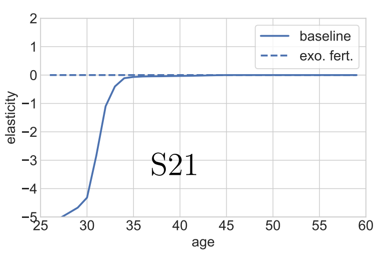


Increased wage of men

(d) Age at first birth.



(e) Share without children.



(f) Share with one child.

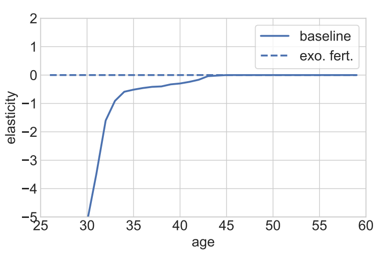


Table D.2: Parameter Estimates.

Parameter		estimate	se
Utility from children.			
ω_1	Value of having at least one child	11.698	(0.012)
ω_2	Value of having at least two children	13.002	(0.006)
ω_3	Value of having at least three children	9.591	(0.015)
η_0	Value of fertility effort when child aged 0 present	-0.064	(0.000)
η_1	Value of fertility effort when child aged 1 present	-0.015	(0.000)
Utility from market work, $f_w(\bullet)$ and $f_m(\bullet)$. Relative to not working.			
$\mu_{FT,w}$	Value of full time work, women	-0.511	(0.001)
$\mu_{FT,age,w}$	Value of full time work wrt. age, women (pct)	-2.060	(0.005)
$\mu_{PT,w}$	Value of part time work, women	-0.269	(0.000)
$\mu_{PT,age,w}$	Value of part time work wrt. age, women (pct)	-2.701	(0.006)
$\mu_{FT,m}$	Value of full time work, men	-0.670	(0.001)
$\mu_{FT,age,m}$	Value of full time work wrt. age, men (pct)	-1.966	(0.006)
$\mu_{PT,m}$	Value of part time work, men	-0.372	(0.001)
$\mu_{PT,age,m}$	Value of part time work wrt. age, men (pct)	-2.170	(0.008)
Utility from market work w. children, $q_w(\bullet)$ and $q_m(\bullet)$. Relative to not working.			
$\alpha_{FT,child,w}$	Value of full time work with children, women (pct)	11.394	(0.037)
$\alpha_{FT,more,w}$	Value of full time work with children, women (pct)	5.603	(0.031)
$\alpha_{FT,young,w}$	Value of full time work with young children, women (pct)	2.486	(0.029)
$\alpha_{PT,child,w}$	Value of part time work with more children, women (pct)	14.222	(0.064)
$\alpha_{PT,more,w}$	Value of part time work with more children, women (pct)	6.705	(0.060)
$\alpha_{PT,young,w}$	Value of part time work with young children, women (pct)	3.909	(0.073)
$\alpha_{FT,child,m}$	Value of full time work with children, men (pct)	5.363	(0.017)
$\alpha_{FT,more,m}$	Value of full time work with children, men (pct)	-0.005	(0.011)
$\alpha_{FT,young,m}$	Value of full time work with young children, men (pct)	0.033	(0.022)
$\alpha_{PT,child,m}$	Value of part time work with more children, men (pct)	3.451	(0.047)
$\alpha_{PT,more,m}$	Value of part time work with more children, men (pct)	0.157	(0.041)
$\alpha_{PT,young,m}$	Value of part time work with young children, men (pct)	0.026	(0.054)
Wage equations.			
$\gamma_{0,w}$	Wage: constant, women	0.773	(0.001)
$\gamma_{1,w}$	Wage: human capital, women	0.085	(0.000)
$\gamma_{0,m}$	Wage: constant, men	0.771	(0.001)
$\gamma_{1,m}$	Wage: human capital, men	0.103	(0.000)
Miscellaneous.			
κ_V	Retirement: value function adjustement	0.519	(0.004)

Notes: The table reports the Simulated Minimum Distance estimates of $\boldsymbol{\theta}$ using the "estimation sample". The moments matched are discussed in Section 6.1. Asymptotic standard errors are reported in brackets.

Table D.3: Unanticipated Permanent Wage Changes, Elasticities. Exogenous Fixed Fertility.

	Participation		Hours		Wage at 55		Child	Comp.
Age	Women	Men	Women	Men	Women	Men	birth	fertility
				A. Elasticities w.r.t. wages of women				
26	1.23	-0.10	1.41	-0.38	1.58	-0.14	0.00	0.00
30	0.92	-0.08	1.00	-0.25	1.45	-0.10	0.00	0.00
35	0.76	-0.02	0.79	-0.12	1.37	-0.05	0.00	0.00
40	0.53	-0.00	0.55	-0.04	1.25	-0.02	0.00	0.00
45	0.29	-0.00	0.32	-0.02	1.13	-0.01	–	–
50	0.19	-0.00	0.22	-0.02	1.05	-0.00	–	–
avg.	0.50	-0.03	0.56	-0.11	1.23	-0.04	0.00	0.00
				B. Elasticities w.r.t. wages of men				
26	-0.52	0.29	-0.91	0.39	-0.41	1.12	0.00	0.00
30	-0.38	0.15	-0.69	0.20	-0.33	1.07	0.00	0.00
35	-0.31	0.03	-0.57	0.05	-0.28	1.02	0.00	0.00
40	-0.23	0.00	-0.47	0.01	-0.22	1.01	0.00	0.00
45	-0.10	0.00	-0.28	0.01	-0.12	1.00	–	–
50	-0.03	0.00	-0.16	0.00	-0.04	1.00	–	–
avg.	-0.20	0.05	-0.41	0.08	-0.18	1.03	0.00	0.00

Notes: The Table illustrates elasticities of labor market participation, hours, full-time wage offer at age 55, likelihood of childbirth, and completed fertility w.r.t. an unanticipated permanent wage change of either women or men. Participation and hours elasticities are "long run" in the sense that they measure the change in the remaining working life. All elasticities are calculated based on an unanticipated permanent 5% increase at different points in the life cycle. The average (avg.) is calculated across all ages from 25 through 60. Panel A shows elasticities w.r.t. women's wages and Panel B shows elasticities w.r.t. men's wages.
All responses are simulated from an alternative model in which fertility is exogenous and random, with ex-ante fertility expectations consistent with realized fertility simulated from the baseline model and ex-post realized simulated fertility identical to that of the baseline model.

Table D.4: Increased Child Subsidy, Percentage Changes. Fixed Exogenous Fertility.

	Participation		Hours		Wage at 55		Child	Comp.
	Women	Men	Women	Men	Women	Men	birth	fertility
3000	-0.14	-0.01	-0.14	-0.02	-0.04	-0.01	0.00	0.00
9000	-0.25	-0.04	-0.23	-0.10	-0.07	-0.02	0.00	0.00

Notes: The Table illustrates the percentage change in labor market participation and hours worked for women and men, the percentage change in the pregnancy probability and completed fertility. All responses are simulated from an alternative model in which fertility is exogenous and random, with ex-ante fertility expectations consistent with realized fertility simulated from the baseline model and ex-post realized simulated fertility identical to that of the baseline model.