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Jeppe Druedahl

Thomas H. Jørgensen

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CEBI
Department of Economics
University of Copenhagen
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Can Consumers Distinguish Persistent from Transitory Income Shocks?*

Jeppe Druedahl[†]
Thomas H. Jørgensen[‡]

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Abstract

We study whether households can distinguish persistent from transitory income shocks, and the implications for consumption-saving behavior. We construct a novel consumption-saving model where the household must infer the persistent component of its income process from actual income realizations together with an additional noisy private signal. We first show that the degree of imperfect information has important consequences for the interpretation of transmission parameters to persistent and transitory income shocks. A large transitory transmission parameter can e.g. be estimated despite of a low marginal propensity to consume because the short run covariance between income growth and consumption growth increases when households cannot distinguish persistent from transitory income shocks. We further show that the households' degree of knowledge can be identified from panel data on income and consumption. Finally, we estimate a high degree of knowledge in the Panel Study of Income Dynamics.

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[†]CEBI, Department of Economics, University of Copenhagen, Øster Farimagsgade 5, Building 26, DK-1353 Copenhagen K, Denmark. E-mail: jeppe.druedahl@econ.ku.dk. Website: http://econ.ku.dk/druedahl.

[‡]CEBI, Department of Economics, University of Copenhagen, Øster Farimagsgade 5, Building 26, DK-1353 Copenhagen K, Denmark. E-mail: thomas.h.jorgensen@econ.ku.dk. Website: http://www.tjeconomics.com/.

1 Introduction

The degree to which households can distinguish persistent from transitory income shocks have important implications for our interpretation of observed consumption-saving behavior. In particular, imperfect information with respect to the composition of income shocks can induce strong short run co-movements between income growth and consumption growth even if the true marginal propensity to consume is low. The standard assumption that households know the persistent component of their income process, and thus can perfectly distinguish persistent from transitory shocks, is rarely discussed explicitly, let alone tested empirically.[1]

In this paper, we construct a novel consumption-saving model where households know the true income process but need to infer the persistent component of their income process, and thus whether observed income changes are persistent or transitory, from the realization of their actual income path together with an additional noisy private signal. This allows us to consider a continuum of cases from the households perfectly knowing the persistent component of their income process, and thus the composition of their income shocks, to the household having the same information set as an econometrician, who needs to solve a filtering problem to infer the hidden persistent component from observed actual income.

In the first part of the paper, Section 2, we consider a simple certainty equivalence framework with quadratic utility and a permanent-transitory income process to build intuition on how households behave when they cannot perfectly distinguish permanent from transitory income shocks. We show that the assumption about the households' degree of knowledge about its permanent income has important consequences for the interpretation of transmission parameters to permanent and transitory income shocks estimated as in the influential studies by Blundell, Pistaferri and Preston (2008) (henceforth BPP) and Kaplan and Violante (2010).[2]

In line with intuition, the estimated response to transitory shocks rises above the true marginal propensity to consume (MPC) when the noise in the households' private signal increases and they cannot perfectly discriminate permanent from transitory shocks. Somewhat counter-intuitively, the estimated response to *permanent* shocks also rises above the true marginal propensity to consume out of

[1] In a recent survey, Blundell (2014) thus asks for research on "the consumer's ability to distinguish between permanent and transitory shocks" (p. 312).

[2] See also Blundell, Low and Preston (2013) and the surveys in Jappelli and Pistaferri (2010), Meghir and Pistaferri (2011) and Blundell (2014).

permanent shocks (MPCP) when the noise increase. This is due to an increasing bias in the estimator of the permanent transmission parameter proposed by BPP. Consistent with intuition, the true permanent transmission parameter is decreasing in the degree of noise in the private signal.

We further derive the joint covariance structure of consumption and income growth and show that the households' degree of knowledge, i.e. the precision of their private signal, can be point identified in closed form with panel data on income and consumption, even if the consumption data is subject to measurement error. With unknown measurement error in income, the variance of the transitory shock is, as previously noted in the literature, not point identified, which implies that the degree of knowledge is not point identified. However, we show that the estimated degree of knowledge will be increasing in the variance of measurement error in income. From an upper bound on the measurement error in income relative to the observed variance of income growth, we can thus derive an upper bound on the degree of knowledge; the assumption of no measurement error delivers a lower bound.

In the second part of the paper, Section 3, we consider a more general consumption-saving model where households have CRRA utility and face a life-cycle income process with potentially persistent rather than fully permanent shocks, and a MA(1) term. In the limit, where the household's information about their permanent income is perfect, the model nests the canonical buffer-stock model of Deaton (1991, 1992) and Carroll (1992, 1997, 2012). Through simulations we show that the theoretical results from the certainty equivalence framework also holds qualitatively for the more general model. In a Monte Carlo study we in particular show that the households' degree of knowledge can be estimated from panel data on income and consumption using a simulated method of moments (SMM) estimator. We jointly estimate the degree of knowledge together with income process and preference parameters. We show that the closed-form estimator derived from the certainty equivalence model also delivers reasonable estimates when used on data simulated from the more general model.

In the third, and final, part of the paper, Section 4, we apply the same estimator to data on income and consumption from the Panel Study of Income Dynamics(PSID). We conclude that we cannot reject that the PSID households know the persistent component of their own income process perfectly, and that they can fully distinguish persistent from transitory shocks. This result is robust to a wide range of specifications of the income process and allowing for measurement error in both income and consumption. We also find no evidence of differences in the degree of knowledge across different educational groups. We only find evidence of imperfect information when we impose a too high discount factor on the data. With very patient households the model cannot match the co-movements between income growth and consumption growth observed in the data without restricting them to have imperfect information.

Related literature. The notion that households might not have perfect information about the components of their own income process goes back to at least Muth (1960). Pischke (1995) analyzed the case where the households have access to less information than the econometrician about the aggregate state of the economy but observe their own income shocks perfectly.[3] Wang (2004, 2009) show in a continuous time model with CARA preferences that imperfect knowledge about permanent income implies larger precautionary saving. Blundell and Preston (1998), Cunha, Heckman and Navarro (2004), and Blundell (2014), also all briefly discuss the possibility that households cannot perfectly discriminate persistent from transitory shocks.

Our study is also related to the seminal papers by Guvenen (2007) and Guvenen and Smith (2014) studying consumption-saving behavior when households are gradually learning their individual-specific income growth rate. They assume that households only have imperfect information about their own latent income growth rate in form of an initial private signal, and otherwise need to infer it from the realization of their income path. This implies that the standard case, with homogeneous growth rates and perfect information about the persistent component of the income process, is not nested in their specification.[4] Our paper does not consider heterogeneous growth rates, but focus on the learning of the persistent component of income given a period-by-period private signal. Hereby the standard case with perfect information is nested as a special case in our framework.

Finally, our paper is related to Pistaferri (2001) and Kaufmann and Pistaferri (2009) who use subjective belief data to identify respectively transitory and permanent shocks. Their identification strategy relies on the assumption that the

[3] Goodfriend (1992) investigates aggregation bias when households face an income process consisting of an individual component and an aggregate component observed with a one period lag.

[4] In turn, the models in Guvenen (2007) and Guvenen and Smith (2014) cannot answer the question of the current paper.

households know their permanent income component.

2 Framework and Identification

In this section, we use a certainty equivalence consumption-saving model to build intuition on how households behave when they cannot perfectly distinguish between permanent and transitory income shocks. We consider a continuum of cases from the households perfectly knowing their permanent income, and thus the composition of their income shocks, to the households having the same information set as an econometrician only observing realized income. We show that point identification of the households degree of knowledge can be achieved with panel data on income and consumption allowing for an unknown degree of measurement error in consumption. When also allowing for measurement error in income, we can derive lower and upper bounds on the households' degree of knowledge.

2.1 Household problem

We assume that households have quadratic utility, like Hall (1978), face a permanent-transitory income process, and only observe actual income, y_t, and a noisy private signal of permanent income, z_t. Specifically, the households solve

$$\begin{aligned}
U &= \max_{c_t, c_{t+1}, \ldots} \tilde{\mathbb{E}}_t \sum_{k=0}^{\infty} \beta^k [\alpha c_{t+k} - \gamma \frac{c_{t+k}^2}{2}], \quad \alpha > 0, \gamma > 0 && (2.1) \\
&\text{s.t.} \\
a_t &= R(a_{t-1} + y_t - c_t), \; R > 0 && (2.2) \\
\lim_{t \to \infty} R^{-t} a_t &\geq 0 && (2.3) \\
y_t &= p_t + \xi_t, \qquad \xi_t \sim \mathcal{N}(0, \sigma_\xi^2), \quad \sigma_\xi > 0 && (2.4) \\
p_t &= p_{t-1} + \psi_t, \quad \psi_t \sim \mathcal{N}(0, \sigma_\psi^2), \; \sigma_\psi > 0 && (2.5) \\
z_t &= p_t + \epsilon_t, \qquad \epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon^2), \quad \sigma_\epsilon \geq 0, && (2.6)
\end{aligned}$$

where eq. (2.2) is the budget constraint and eq. (2.3) is the No-Ponzi game condition, eq. (2.4)-(2.5) are the permanent-transitory income process, z_t in eq. (2.4) is the private signal, and the expectations operator $\tilde{\mathbb{E}}_t$ is conditional on the history of actual income and the private signal, i.e.

$$\tilde{\mathbb{E}}_t[\bullet] \equiv \mathbb{E}[\bullet \,|\, y_t, y_{t-1}, \ldots, z_t, z_{t-1}, \ldots]. \tag{2.7}$$

If $\sigma_\epsilon = 0$ we are in the standard case with perfect information, $z_t = p_t$, and the households can perfectly distinguish permanent from transitory income shocks. For $\sigma_\epsilon > 0$ the households need to solve a filtering problem to form beliefs about their permanent income and the composition of income shocks. Because the income process is linear-Gaussian it is optimal for the household to infer its level of permanent income by the Kalman filter.[5] We denote a household's mean-belief of p_t by $\hat{p}_t$, and the variance of its mean-belief by $\hat{q}_t$.

A useful property of the Kalman filter is that the distribution of prediction errors, $\hat{p}_t - p_t$, is mean-zero and is Gaussian with the same variance as the mean-belief, i.e.

$$\kappa_t \equiv \hat{p}_t - p_t \quad \sim \quad \mathcal{N}(0, \hat{q}_t), \tag{2.8}$$

and uncorrelated with future shocks,

$$\forall k > 0: \ \mathrm{cov}(\kappa_t, \psi_{t+k}) = \mathrm{cov}(\kappa_t, \xi_{t+k}) = \mathrm{cov}(\kappa_t, \epsilon_{t+k}) = 0. \tag{2.9}$$

In the following section, we present central properties of the Kalman filter updating process before turning to the implied consumption-saving behavior.

2.2 Updating Beliefs

Given period t information the best predictions of $t+1$ variables are

$$\begin{aligned} \hat{p}_{t+1|t} &= \hat{p}_t && (2.10)\\ \hat{y}_{t+1|t} &= \hat{p}_{t+1|t} && (2.11)\\ \hat{z}_{t+1|t} &= \hat{p}_{t+1|t} && (2.12)\\ \hat{q}_{t+1|t} &= \hat{q}_t + \sigma_\psi^2. && (2.13) \end{aligned}$$

The mean-belief is optimally updated as

$$\hat{p}_{t+1} = \hat{p}_{t+1|t} + K_{t+1}\Delta_{t+1}, \tag{2.14}$$

[5] For a general treatment of the Kalman filter see e.g. Hamilton (1994).

where Δ_{t+1} is the vector of prediction errors given by

$$\Delta_{t+1} \equiv \begin{bmatrix} y_{t+1} \\ z_{t+1} \end{bmatrix} - \begin{bmatrix} \hat{y}_{t+1|t} \\ \hat{z}_{t+1|t} \end{bmatrix}, \tag{2.15}$$

and K_{t+1} is the optimal Kalman gain vector given by

$$\begin{aligned} K_{t+1} &\equiv \hat{q}_{t+1|t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}^T S_{t+1}^{-1}, \\ &= \frac{\hat{q}_t + \sigma_\psi^2}{(\sigma_\xi^2 + \sigma_\epsilon^2)(\hat{q}_t + \sigma_\psi^2) + \sigma_\xi^2 \sigma_\epsilon^2} \begin{bmatrix} \sigma_\epsilon^2 & \sigma_\xi^2 \end{bmatrix}, \end{aligned} \tag{2.16}$$

with

$$S_{t+1} \equiv \begin{bmatrix} 1 \\ 1 \end{bmatrix} \hat{q}_{t+1|t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}^T + \begin{bmatrix} \sigma_\xi^2 & 0 \\ 0 & \sigma_\epsilon^2 \end{bmatrix}. \tag{2.17}$$

The variance of the mean-belief is updated optimally as

$$\hat{q}_{t+1} = \left(1 - K_{t+1} \begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) \hat{q}_{t+1|t}. \tag{2.18}$$

Steady state. We can solve for the steady state value of $\hat{q}_t$ in eq. (2.18), as

$$q^\star = \left(\sqrt{\frac{\sigma_\xi^2 \sigma_\epsilon^2}{(\sigma_\xi^2 + \sigma_\epsilon^2)\sigma_\psi^2} + \frac{1}{4}} - \frac{1}{2} \right) \sigma_\psi^2. \tag{2.19}$$

The steady state Kalman gain vector consequently is

$$K^\star \equiv \begin{bmatrix} K_1^\star & K_2^\star \end{bmatrix} = \begin{bmatrix} q^\star \sigma_\xi^{-2} & q^\star \sigma_\epsilon^{-2} \end{bmatrix}. \tag{2.20}$$

Lemma 1 presents some important properties of the steady state variance of the mean-belief and Kalman gain vector. Note in particular that when all the noise in the private signal disappears, $\sigma_\epsilon \to 0$, we have perfect information and get $\hat{p}_t = p_t$ and $\hat{q}_t = 0$. On the other hand, $\hat{q}_t$ reaches an upper bound when the private signal becomes infinitely noisy, $\sigma_\epsilon \to \infty$. Note also that the sum of the elements of the Kalman gain vector, $\mathcal{K}^\star \equiv K_1^\star + K_2^\star$, is one in the perfect information case, and gradually declines towards a positive constant as $\sigma_\epsilon \to \infty$.

Lemma 1. *$q^\star$ and $K^\star$ have the following properties:*

1. *The steady state variance of the mean-belief is bounded by the minimum of the transitory shock variance and the variance of the noise in the*

private signal, and the steady state Kalman gains are always positive and sum to weakly less than one,

$$q^\star \leq \min\{\sigma_\xi^2, \sigma_\epsilon^2\} \tag{2.21}$$

$$K_1^\star, K_2^\star \geq 0 \tag{2.22}$$

$$\mathcal{K}^\star \equiv K_1^\star + K_2^\star \leq 1 \tag{2.23}$$

2. *In the limit as all noise in the private signal disappears (perfect information), $\sigma_\epsilon \to 0$, the variance of the mean-belief collapses and all of the Kalman gain is placed on the noiseless private signal,*

$$\lim_{\sigma_\epsilon \to 0} q^\star = 0 \tag{2.24}$$

$$\lim_{\sigma_\epsilon \to 0} K^\star = \begin{bmatrix} 0 & 1 \end{bmatrix} \tag{2.25}$$

$$\lim_{\sigma_\epsilon \to 0} \mathcal{K}^\star = 1 \tag{2.26}$$

3. *In the opposite limit, where the private signal becomes infinitely noisy, $\sigma_\epsilon \to \infty$, the variance of the mean-belief reaches an upper bound, and no weight is placed on the private signal*

$$\lim_{\sigma_\epsilon \to \infty} q^\star = \left(\sqrt{\frac{\sigma_\xi^2}{\sigma_\psi^2} + \frac{1}{4}} - \frac{1}{2}\right)\sigma_\psi^2 \equiv \overline{q}^\star > 0 \tag{2.27}$$

$$\lim_{\sigma_\epsilon \to \infty} K^\star = \begin{bmatrix} \overline{q}^\star \sigma_\xi^{-2} & 0 \end{bmatrix} \tag{2.28}$$

$$\lim_{\sigma_\epsilon \to \infty} \mathcal{K}^\star = \overline{q}^\star \sigma_\xi^{-2} \in (0, 1) \tag{2.29}$$

4. *The variance of the mean-belief and the elements of the Kalman gain changes monotonically with the noise in the private signal as*

$$\frac{\partial q^\star}{\partial \sigma_\epsilon^2} > 0 \tag{2.30}$$

$$\frac{\partial K_1^\star}{\partial \sigma_\epsilon^2} > 0 \tag{2.31}$$

$$\frac{\partial K_2^\star}{\partial \sigma_\epsilon^2} < 0 \tag{2.32}$$

$$\frac{\partial \mathcal{K}^\star}{\partial \sigma_\epsilon^2} < 0 \tag{2.33}$$

Proof. Follow from eq. (2.19) and (2.20). See Supplemental Material A. □

Lemma 2 presents the law of motion for the prediction error. An important implication is that the prediction errors under imperfect information have a positive autocovariance and are correlated with both past and current shocks. In particular, we see that permanent shocks lead to under-prediction of permanent income ($\frac{\partial \kappa_t}{\partial \psi_t} < 0$) and transitory shocks lead to over-prediction of permanent income ($\frac{\partial \kappa_t}{\partial \xi_t} > 0$). We also have mean-reversion in the prediction error, $\mathbb{E}[\kappa_t|\kappa_{t-1}] < \kappa_{t-1}$ because $(1-\mathcal{K}^\star) < 1$ when $\sigma_\epsilon > 0$.

Lemma 2. *The law of motion for the steady state prediction error is*

$$\kappa_t \quad = \quad (1-\mathcal{K}^\star)\kappa_{t-1} + (\mathcal{K}^\star - 1)\psi_t + K_1^\star \xi_t + K_2^\star \epsilon_t \tag{2.34}$$

and consequently the autocovariance of the prediction errors is

$$cov(\kappa_t, \kappa_{t-1}) = (1-\mathcal{K}^\star)q^\star \geq 0 \tag{2.35}$$

Proof. Follow from eq. (2.14) and eq. (2.8)–(2.9). See Supplemental Material A. □

Note further that despite the prediction errors have a positive autocovariance the income growth forecast errors, defined as $e_t \equiv (y_{t+1|t} - y_{t|t}) - (y_{t+1} - y_t)$, still have zero autocovariance as shown in Lemma 3.

Lemma 3. *The income growth forecast error is mean zero and has excessive variance, but have zero autocovariance.*

1. $e_t \equiv (y_{t+1|t} - y_t) - (y_{t+1} - y_t) = \kappa_t - \psi_{t+1} - \xi_{t+1}$
2. $\mathbb{E}[e_t] = 0$
3. $var(e_t) = q^\star + \sigma_\psi^2 + \sigma_\xi^2 \geq \sigma_\psi^2 + \sigma_\xi^2$
4. $cov(e_t, e_{t+k}) = 0$*, for all* $k > 0$

Proof. See Supplemental Material A. □

This implies that the household's degree of knowledge cannot easily be inferred from survey information on their income growth forecasts. In principle the variance of the income growth forecast errors could be used, but it would require strong assumptions on the measurement error in the reported forecasts. Here, we instead pursue identification through the use of consumption data.

2.3 Consumption-Saving

We can now derive an analytical formula for the change in consumption under imperfect information. The result is provided in Theorem 1.

Theorem 1. *Consider a household solving the problem in eq. (2.1) where the variance of the mean-belief has converged to $q^\star$. If $\beta R = 1$ then*

$$\Delta c_t \;=\; \phi_\psi(\psi_t - \kappa_{t-1}) + \phi_\xi \xi_t + \phi_\epsilon \epsilon_t \tag{2.36}$$

where

$$\phi_\psi \;\equiv\; R^{-1}(R - 1 + q^\star(\sigma_\xi^{-2} + \sigma_\epsilon^{-2})) \tag{2.37}$$
$$\phi_\xi \;\equiv\; R^{-1}(R - 1 + q^\star \sigma_\xi^{-2}) \tag{2.38}$$
$$\phi_\epsilon \;\equiv\; R^{-1} q^\star \sigma_\epsilon^{-2} \tag{2.39}$$

Proof. See Supplemental Material A. □

Using the consumption growth result in Theorem 1, we can derive the autocovariance structure of consumption growth and the covariance of consumption and income growth. These covariances are useful for determining identifying moments for the variance of the households' private signal, σ_ϵ^2, i.e. their degree of knowledge. Corollary 1 shows that the standard result of consumption growth being a random walk is preserved under imperfect information about permanent income.[6] However, the variance of consumption growth, given $R > 1$, is increasing in the noise of the private signal, σ_ϵ, through its positive effect on the variance of the mean-belief, $q^\star$.

Corollary 1. *The variance and autocovariances of consumption growth are*

$$cov(\Delta c_t, \Delta c_{t+k}) \;=\; \begin{cases} \sigma_\psi^2 + \frac{(R-1)^2}{R^2}\sigma_\xi^2 + \frac{R^2-1}{R^2} q^\star & \text{if } k = 0 \\ 0 & \text{else} \end{cases}$$

Proof. Follows from Theorem 1 and Lemma 2. See Supplemental Material A. □

Corollary 2 shows the covariances of consumption growth with past, current and future income growth. The standard results that consumption growth is uncor-

[6] The random walk result is only broken if we assume that the household does not observe its own income perfectly.

related with past income growth (i.e. no excess sensitivity[7]) and future income growth beyond the first lead (i.e. no indication of advance information[8]) is preserved.

The covariance of consumption growth with current income, however, increases with the noise of the private signal, σ_ϵ, though its positive effect on the variance of the mean-belief, $q^\star$. The covariance of consumption growth with next-period income, on the other hand, becomes even more negative through the effect of σ_ϵ on the variance of the mean-belief $q^\star$.

Corollary 2. *The covariances of consumption growth and income growth are*

$$cov(\Delta c_t, \Delta y_{t+k}) = \begin{cases} -R^{-1}(\sigma_\xi^2(R-1) + q^\star) & \text{if } k = 1 \\ \sigma_\psi^2 + \frac{R-1}{R}\sigma_\xi^2 + q^\star & \text{if } k = 0 \\ 0 & \text{else} \end{cases}$$

Proof. Follows from Theorem 1 and eq. (2.4). □

In sum, we have thus shown in Corollary 1 and 2 that three central moments for identifying σ_ϵ are var(Δc_t), cov$(\Delta c_t, \Delta y_t)$ and cov$(\Delta c_t, \Delta y_{t+1})$. We return to identification of σ_ϵ after discussing the implications of σ_ϵ on the transmission parameters in eq. (2.37)–(2.39).

2.4 Transmission parameters

The parameters ϕ_ψ, ϕ_ξ and ϕ_ϵ in the consumption growth eq. (2.36) are informative with respective to how consumption responds to different shocks. In particular, ϕ_ψ and ϕ_ξ have similar interpretations as the transmission parameters estimated in BPP. Corollary 3 shows that in the limit with perfect information, $\sigma_\epsilon \to 0$, the household optimally responds one-to-one to permanent shocks, $\phi_\psi = 1$, and only marginally to transitory shocks, $\phi_\xi = \frac{R-1}{R}$. This is the standard result from BPP. In the limit where the information sets of the household and the econometrician coincide, $\sigma_\epsilon \to \infty$, we instead have that $\phi_\psi = \phi_\xi$ such that the

[7] Excess sensitivity can e.g. be due to liquidity constraints (Flavin, 1981) or precautionary saving (Commault, 2017) which we have both ruled out here. In the general CRRA model studied below we allow for both these elements in the model.

[8] Primiceri and van Rens (2009) and Kaufmann and Pistaferri (2009) argue in favor of advance information about one year ahead shocks. BPP do not find any evidence for advance information.

transmission parameters to the permanent and transitory shocks are the same. Note, however, that when $\sigma_\epsilon > 0$ the interpretation of ϕ_ψ and ϕ_ξ as transmission parameters is not correct in the usual sense. Specifically, we always have, irrespective of σ_ϵ, that the marginal propensity to consume (MPC) is $\frac{R-1}{R}$ and the marginal propensity to consume out of permanent shocks (MPCP) is 1, while ϕ_ψ and ϕ_ξ vary systematically with σ_ϵ. The imperfect information thus opens up a wedge between the MPC and MPCP and the respective transmission parameters.

Corollary 3. *The transmission parameters ϕ_ψ, ϕ_ξ and ϕ_ϵ vary with σ_ϵ as follows:*

1. *For ϕ_ψ we have*

$$\begin{aligned}
\lim_{\sigma_\epsilon \to 0} \phi_\psi &= 1 \\
\lim_{\sigma_\epsilon \to \infty} \phi_\psi &= R^{-1}(R - 1 + \overline{q}^\star \sigma_\xi^{-2}) \\
\frac{\partial \phi_\psi}{\partial \sigma_\epsilon} &< 0
\end{aligned}$$

2. *For ϕ_ξ we have*

$$\begin{aligned}
\lim_{\sigma_\epsilon \to 0} \phi_\xi &= \frac{R-1}{R} \\
\lim_{\sigma_\epsilon \to \infty} \phi_\xi &= R^{-1}(R - 1 + \overline{q}^\star \sigma_\xi^{-2}) \\
\frac{\partial \phi_\xi}{\partial \sigma_\epsilon} &> 0
\end{aligned}$$

3. *For ϕ_ϵ we have*

$$\begin{aligned}
\lim_{\sigma_\epsilon \to 0} \phi_\epsilon &= \frac{1}{R} \\
\lim_{\sigma_\epsilon \to \infty} \phi_\epsilon &= 0 \\
\frac{\partial \phi_\epsilon}{\partial \sigma_\epsilon} &< 0
\end{aligned}$$

Proof. Follows from Theorem 1 and Lemma 1. See Supplemental Material A. □

BPP showed that the transmission parameters were identified under the assumption that the household has perfect information about its permanent income, $\sigma_\epsilon = 0$. Using the same moment condition, the parameter ϕ_ξ can still be recovered under the assumption of imperfect information. This follows from eq. (2.40) in Corollary 4. The coefficient ϕ_ψ can, however, not be recovered using the moment condition suggested by BPP when information is imperfect. The reason is that the lagged shocks have a non-zero covariance with the lagged prediction error, κ_{t-1}, present in the equation for the change in consumption, eq. (2.36). Specifically, the BPP estimate of ϕ_ψ is upwards biased as seen in eq. (2.41) Corollary 4.

Corollary 4. *Using the moments proposed by BPP to estimate the transmission parameters we get*

$$\hat{\phi}_\xi^{BPP} \equiv \frac{cov(\Delta c_t, -\Delta y_{t+1})}{cov(\Delta y_t, -\Delta y_{t+1})} = \phi_\xi, \tag{2.40}$$

and

$$\begin{aligned} \hat{\phi}_\psi^{BPP} &\equiv \frac{cov(\Delta c_t, \Delta y_{t-1} + \Delta y_t + \Delta y_{t+1})}{cov(\Delta y_t, \Delta y_{t-1} + \Delta y_t + \Delta y_{t+1})} \\ &= \phi_\psi[1 + (1 - \mathcal{K}^\star)(1 + K_1^\star \frac{\sigma_\xi^2}{\sigma_\psi^2})] > \phi_\psi. \end{aligned} \tag{2.41}$$

Proof. Follows from Theorem 1 and eq. (2.4). See Supplemental Material A. □

This further implies the surprising result in Corollary 5 that while the actual ϕ_ψ is decreasing in the noise of the private signal, $\frac{\partial \phi_\psi}{\partial \sigma_\epsilon} < 0$ (Corollary 3), the estimated $\hat{\phi}_\psi^{BPP}$ is increasing in it, $\frac{\partial \hat{\phi}_\psi^{BPP}}{\partial \sigma_\epsilon} > 0$.

Corollary 5. *We have*

$$\begin{aligned} \frac{\partial \hat{\phi}_\xi^{BPP}}{\partial \sigma_\epsilon^2} &= \frac{\partial \phi_\xi}{\partial \sigma_\epsilon^2} = \frac{1}{R} Q^\star \\ \frac{\partial \hat{\phi}_\psi^{BPP}}{\partial \sigma_\epsilon^2} &= \frac{R-1}{R} \frac{\sigma_\xi^2}{\sigma_\psi^2} Q^\star, \end{aligned}$$

where

$$Q^\star \equiv \frac{\sigma_\xi^2 (2q^\star + \sigma_\psi^2)}{(\sigma_\epsilon^2 + \sigma_\xi^2)(4\sigma_\epsilon^2 + (\sigma_\epsilon^2 + \sigma_\xi^2)\sigma_\psi^2)}.$$

Proof. See Supplemental Material A. □

2.5 Identification of σ_ϵ^2

We now turn to identification of the households' degree of knowledge with panel data on income and consumption. Lemma 4 shows how to estimate the variance of the private signal, σ_ϵ^2, given estimates of the variance of the mean-belief $\hat{q}^\star$, and the income shocks variances, $\hat{\sigma}_\psi^2$ and $\hat{\sigma}_\xi^2$.

Lemma 4. *Given estimates $\hat{q}^\star$, $\hat{\sigma}^2_\psi > 0$ and $\hat{\sigma}^2_\xi > 0$ the variance of the private signal is*

$$\hat{\sigma}^2_\epsilon = q^{\star-1}(\hat{q}^\star, \hat{\sigma}^2_\psi, \hat{\sigma}^2_\xi), \tag{2.42}$$

where

$$q^{\star-1}(q^\star, \sigma^2_\psi, \sigma^2_\xi) \equiv \begin{cases} 0 & \text{if } q^\star \leq 0 \\ \frac{q^\star \sigma^2_\xi (q^\star + \sigma^2_\psi)}{\sigma^2_\xi \sigma^2_\psi - q^\star (q^\star + \sigma^2_\psi)} & \text{if } q^\star \in (0, \left(\sqrt{\sigma^2_\xi/\sigma^2_\psi + \frac{1}{4}} - \frac{1}{2}\right) \sigma^2_\psi). \end{cases} \tag{2.43}$$

If $q^\star \geq \left(\sqrt{\sigma^2_\xi/\sigma^2_\psi + \frac{1}{4}} - \frac{1}{2}\right) \sigma^2_\psi$ there does not exist any σ^2_ϵ for the given σ^2_ξ and σ^2_ψ that is consistent with $q^\star$.

Proof. $q^{\star-1}(\bullet)$ is the solution in σ^2_ϵ to eq. (2.19). For $\sigma_\epsilon > 0$, we always have $\sigma^2_\xi \sigma^2_\psi - q^\star(q^\star + \sigma^2_\psi) > 0$. □

Theorem 2 next shows that the degree of noise in the private signal, σ^2_ϵ, is point identified with panel data on income and consumption even when consumption is subject to measurement error.

Theorem 2. *Consider a panel data set where income, $\tilde{y}_t$, is observed without measurement error, and consumption, $\tilde{c}_t$, is observed with additive iid measurement error with variance σ^2_c.*

The variance of the mean-belief, $q^\star$, is point identified as

$$\hat{q}^\star = -R cov(\Delta\tilde{c}_t, \Delta\tilde{y}_{t+1}) - (R-1)\hat{\sigma}^2_\xi. \tag{2.44}$$

where

$$\hat{\sigma}^2_\xi = \max\{cov(\Delta\tilde{y}_t, -\Delta\tilde{y}_{t+1}), 0\} \tag{2.45}$$

$$\hat{\sigma}^2_\psi = \max\{cov(\Delta\tilde{y}_t, \Delta\tilde{y}_{t-1} + \Delta\tilde{y}_t + \Delta\tilde{y}_{t+1}), 0\} \tag{2.46}$$

$$\hat{\sigma}^2_c = \max\{cov(\Delta\tilde{c}_t, -\Delta\tilde{c}_{t+1}), 0\}. \tag{2.47}$$

The degree of noise in the private signal, σ^2_ϵ, is point identified as

$$\hat{\sigma}^2_\epsilon = q^{\star-1}(\hat{q}^\star, \hat{\sigma}^2_\psi, \hat{\sigma}^2_\xi). \tag{2.48}$$

Proof. See Supplemental Material A. □

With unknown measurement in income, the transitory shock variance is not point identified. However, Corollary 6 shows that the estimate of σ^2_ϵ is monotonically decreasing in the variance of measurement error of income. Assuming no measurement error in income thus provides a lower bound on σ_ϵ^2. In practice, the amount of measurement in income can be bounded by the observed variance of income growth, and we can thus also provide an upper bound.

Corollary 6. *Consider the same case as Theorem 2, but assume that there additionally also is additive iid measurement error in income, $\tilde{y}_{it}$, with variance σ_y^2. For any $\tilde{\sigma}_y^2 \in [0, cov(\Delta\tilde{y}_t, -\Delta\tilde{y}_{t+1}))$ we can estimate the degree of knowledge by*

$$\hat{\sigma}_\epsilon^2(\tilde{\sigma}_y^2) = q^{\star-1}(\hat{q}^\star(\hat{\sigma}_\xi^2(\tilde{\sigma}_y^2)), \hat{\sigma}_\psi^2, \hat{\sigma}_\xi^2(\tilde{\sigma}_y^2)), \tag{2.49}$$

where

$$\hat{\sigma}_\xi^2(z) = cov(\Delta\tilde{y}_t, -\Delta\tilde{y}_{t+1}) - z \tag{2.50}$$

$$\hat{q}^\star(x) = -Rcov(\Delta\tilde{c}_t, \Delta\tilde{y}_{t+1}) - (R-1)x. \tag{2.51}$$

We have that the estimated degree of knowledge will be increasing in the measurement error of income

$$\frac{\partial\hat{\sigma}_\epsilon^2(\tilde{\sigma}_y^2)}{\partial\tilde{\sigma}_y^2} = \frac{\partial q^{\star-1}(\hat{q}^\star(\tilde{\sigma}_y^2), \hat{\sigma}_\psi^2, \hat{\sigma}_\xi^2(\tilde{\sigma}_y^2))}{\partial\tilde{\sigma}_y^2} > 0. \tag{2.52}$$

Proof. See Supplemental Material A. □

3 Buffer-stock Model and Monte Carlo

In this section, we present a more general model where households have CRRA preferences and face a life-cycle income process with potentially persistent rather than fully permanent shocks, and a MA(1) term. In the limit where the households information about their permanent income is perfect, the model nests the canonical buffer-stock model of Deaton (1991, 1992) and Carroll (1992, 1997, 2012). After describing the model details, we show how to estimate the degree of knowledge with the Simulated Method of Moments (SMM) using panel data on consumption and income, and present an encouraging Monte Carlo study.

3.1 General model

Specifically, we consider the following specification for log-income

$$p_t = \Gamma_t + \alpha p_{t-1} + \mu_\psi + \psi_t, \quad \alpha \in [-1, 1] \tag{3.1}$$

$$y_t = p_t + \xi_t + \omega \xi_{t-1}. \tag{3.2}$$

Because the income process is still linear-Gaussian it is still optimal for the household to use the Kalman filter. The transition and measurement equations can be written as

$$\begin{bmatrix} p_t \\ \eta_t \\ \xi_t \end{bmatrix} = \underbrace{\begin{bmatrix} \alpha & 0 & 0 \\ 0 & 0 & \omega \\ 0 & 0 & 0 \end{bmatrix}}_{\equiv F} \begin{bmatrix} p_{t-1} \\ \eta_{t-1} \\ \xi_{t-1} \end{bmatrix} + \underbrace{\begin{bmatrix} \Gamma_t + \mu_\psi \\ \mu_\xi \\ \mu_\xi \end{bmatrix}}_{\equiv \mu} + \underbrace{\begin{bmatrix} \sigma_\psi & 0 & 0 \\ 0 & \sigma_\xi & 0 \\ 0 & \sigma_\xi & 0 \end{bmatrix}}_{\equiv \mathcal{W}} \begin{bmatrix} \pi_{1t} \\ \pi_{2t} \\ 0 \end{bmatrix} \tag{3.3}$$

$$\begin{bmatrix} y_t \\ z_t \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}}_{\equiv H} \begin{bmatrix} p_t \\ \eta_t \\ \xi_t \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & \sigma_\epsilon \end{bmatrix}}_{\equiv \mathcal{R}} \begin{bmatrix} 0 \\ \pi_{3t} \end{bmatrix} \tag{3.4}$$

$$\pi_t^j \sim \text{iid.}\mathcal{N}(0, 1).$$

The prediction step becomes

$$\begin{bmatrix} \hat{p}_{t+1|t} \\ \hat{\eta}_{t+1|t} \\ \hat{\xi}_{t+1|t} \end{bmatrix} = F \begin{bmatrix} \hat{p}_t \\ \hat{\eta}_t \\ \hat{\xi}_t \end{bmatrix} + \mu \tag{3.5}$$

$$\hat{Q}_{t+1|t} = F \hat{Q}_t F^T + \mathcal{W}\mathcal{W}^T. \tag{3.6}$$

The optimal Kalman gain vector becomes

$$K_{t+1} = \hat{Q}_{t+1|t} H^T S_{t+1}^{-1}, \tag{3.7}$$

where

$$S_{t+1} = H \hat{Q}_{t+t|t} H^T + \mathcal{R}\mathcal{R}^T. \tag{3.8}$$

The update step becomes

$$\begin{bmatrix} \hat{p}_{t+1} \\ \hat{\eta}_{t+1} \\ \hat{\xi}_{t+1} \end{bmatrix} = \begin{bmatrix} \hat{p}_{t+1|t} \\ \hat{\eta}_{t+1|t} \\ \hat{\xi}_{t+1|t} \end{bmatrix} + K_{t+1}\Delta_{t+1} \tag{3.9}$$

$$\hat{Q}_{t+1} = (I - K_{t+1}H)\,\hat{Q}_{t+1|t}. \tag{3.10}$$

where

$$\Delta_{t+1} = \begin{bmatrix} y_{t+1} \\ z_{t+1} \end{bmatrix} - \begin{bmatrix} \hat{p}_{t+1|t} + \hat{\eta}_{t+1|t} \\ \hat{p}_{t+1|t} \end{bmatrix}. \tag{3.11}$$

The vector of belief errors is

$$\kappa_t \equiv \begin{bmatrix} \kappa_t^p \\ \kappa_t^\eta \\ \kappa_t^\xi \end{bmatrix} \equiv \begin{bmatrix} \hat{p}_t \\ \hat{\eta}_t \\ \hat{\xi}_t \end{bmatrix} - \begin{bmatrix} p_t \\ \eta_t \\ \xi_t \end{bmatrix} \sim \mathcal{N}(0, \hat{Q}_t)$$

We denote the diagonal matrix of the sorted (low to high) eigenvalues of $\hat{Q}_t$ by D and the associated matrix with the eigenvectors as the columns by V. We make the following conjecture which we test numerically in practice

Conjecture 1. *The smallest eigenvalue of $\hat{Q}_t$ is zero for all t if it is zero for $\hat{Q}_0$.*

The household retires in period T_R, and hereafter they receive retirement benefits equal to a fixed ratio of the permanent income,

$$t > T_R : \; y_t = p_{T_R} + \log\lambda \tag{3.12}$$

The full recursive formulation of the household's problem then becomes

$$
\begin{aligned}
V_t(M_t, \hat{p}_t, \hat{\xi}_t) &= \max_{C_t} \frac{C_t^{1-\rho}}{1-\rho} + \beta \tilde{\mathbb{E}}_t \left[V_{t+1}(M_{t+1}, \hat{p}_{t+1}, \hat{\xi}_{t+1}) \right] \\
\text{s. t.} & \\
A_t &= M_t - C_t \\
\begin{bmatrix} p_{t+1} \\ \eta_{t+1} \\ \xi_{t+1} \end{bmatrix} &\sim F \left(\begin{bmatrix} \hat{p}_t \\ \hat{\eta}_t \\ \hat{\xi}_t \end{bmatrix} + V D^{\frac{1}{2}} \begin{bmatrix} \iota_{1t+1} \\ \iota_{2t+1} \\ 0 \end{bmatrix} \right) + \mu + \mathcal{W} \begin{bmatrix} \iota_{3t+1} \\ \iota_{4t+1} \\ 0 \end{bmatrix} \\
\begin{bmatrix} y_{t+1} \\ z_{t+1} \end{bmatrix} &\sim \begin{bmatrix} p_{t+1} + \eta_{t+1} \\ p_{t+1} \end{bmatrix} + R \begin{bmatrix} 0 \\ \iota_{5t+1} \end{bmatrix} \\
\begin{bmatrix} \hat{p}_{t+1} \\ \bullet \\ \hat{\xi}_{t+1} \end{bmatrix} &= F \begin{bmatrix} \hat{p}_t \\ \hat{\eta}_t \\ \hat{\xi}_t \end{bmatrix} + \mu \begin{bmatrix} \hat{p}_{t+1|t} \\ \hat{\eta}_{t+1|t} \\ \hat{\xi}_{t+1|t} \end{bmatrix} + K_{t+1} \Delta_{t+1} \\
M_{t+1} &= R \cdot A_t + \exp(y_{t+1}) \\
A_t &\geq 0 \\
\iota_{jt} &\sim \text{i.i.d.} \mathcal{N}(0,1), \ j \in \{1,2,3,4,5\}.
\end{aligned}
\tag{3.13}
$$

where R is the return factor and the households are not allowed to borrow. We always assume $\hat{Q}_0 = \mathbf{0}$.

The standard Euler-equation applies,

$$
C_t^{-\rho} = \beta R \tilde{\mathbb{E}}_t \left[C_{t+1}^{-\rho} \right], \tag{3.14}
$$

and the model can be solved using the Endogenous Grid Method (EGM) proposed by Carroll (2006) extended to allow for multiple states. The model is, however, still computationally demanding as it contains three continuous states and the expectation is a five dimensional integral.[9] Assuming $\omega = 0$ reduces the dimensionality of the state space to two and make the expectation three dimensional. Further assuming $\alpha = 1$ reduce the state space to just one dimension and the expectation to just two dimensions.

[9] Numerically, we approximate the integral with Gauss-Hermite quadrature.

3.2 Estimation

We imagine having panel data on income and consumption for $i = 1, \ldots, N$ individuals in $t = 1, \ldots, T$ periods with potentially missing observations of either income or consumption or both. We define $w_{it} \equiv (y_{it}, c_{it})$ such that $\mathbf{w}$ denotes the stacked data. In addition to the standard deviation of the private signal, σ_ϵ, we also wish to estimate preferences parameters and the income process parameters. We denote the vector of parameters to be estimated by $\theta \in \Theta \subset \mathbb{R}^{\dim\theta}$. In our empirical investigation below we have $\theta = (\sigma_\epsilon, \beta, \sigma_c, \sigma_\xi, \sigma_\psi, g_0, g_1, \alpha, \omega)$, where we allow income growth to be age dependent through $\Gamma_t = g_0 + g_1(age_t - 25)/100$.

We use the Simulated Method of Moments (SMM) pioneered by McFadden (1989) to estimate θ. Let $\Lambda(\mathbf{w})$ be a $K \times 1$ vector of moments calculated based on observed data. For each value of θ, we solve the model and simulate income and consumption trajectories for the N households forward from age 25 through 65. We use data from age 30 to mimic the PSID data used in the empirical investigation below.[10] We can then calculate the same moments using the simulated data. Denote as $= \frac{1}{J}\sum_{j=1}^{J}\Lambda_j(\theta)$ the average of the same K moments calculated from J simulated data sets from the model for a given value of θ.

We estimate θ as

$$\hat{\theta} = \arg\min_{\theta\in\Theta}(\Lambda(\mathbf{w}) - \overline{\Lambda}(\theta))' W (\Lambda(\mathbf{w}) - \overline{\Lambda}(\theta)),$$

where W is a $K \times K$ weighting matrix. As a baseline we choose W as the inverse of the covariance matrix of $\Lambda(\mathbf{w})$.

We explicitly utilize our theoretical results from the certainty equivalence case in the previous section for identification of σ_ϵ. Particularly, we use the following 82 ($K = 82$) moments to uncover the parameters in θ:

- **Moments 1-31:** Age profile of log income from age 30 through 60 (demeaned), $\text{mean}(y)|_{t=a} - \text{mean}(y)$, $a = 30, \ldots, 60$. These moments are included primarily for identification of the income growth parameters, g_0 and g_1.

- **Moments 32-62:** Age profile of log consumption from age 30 through 60 (demeaned), $\text{mean}(c)|_{t=a} - \text{mean}(c)$, $a = 30, \ldots, 60$. These moments are included primarily for identification of the discount factor, β.

[10] Households are initialized with $\hat{q}_0 = 0$, $A_t = 0$ and $p_0 = 1$.

- **Moments 63-68:** The autocovariances of log income growth, $\text{cov}(\Delta y_t, \Delta y_{t+k})$ for $k = 0, \ldots, 5$. These moments are primarily included for identification of the transitory and permanent income shock variances and the AR(1) and MA(1) parameters of the income process, σ_ξ^2, σ_ψ^2, α and ω, respectively.

- **Moments 69-74:** The autocovariances of log consumption growth, $\text{cov}(\Delta c_t, \Delta c_{t+k})$ for $k = 0, \ldots, 5$. These moments are primarily included for identification of the measurement error in consumption, σ_c^2.

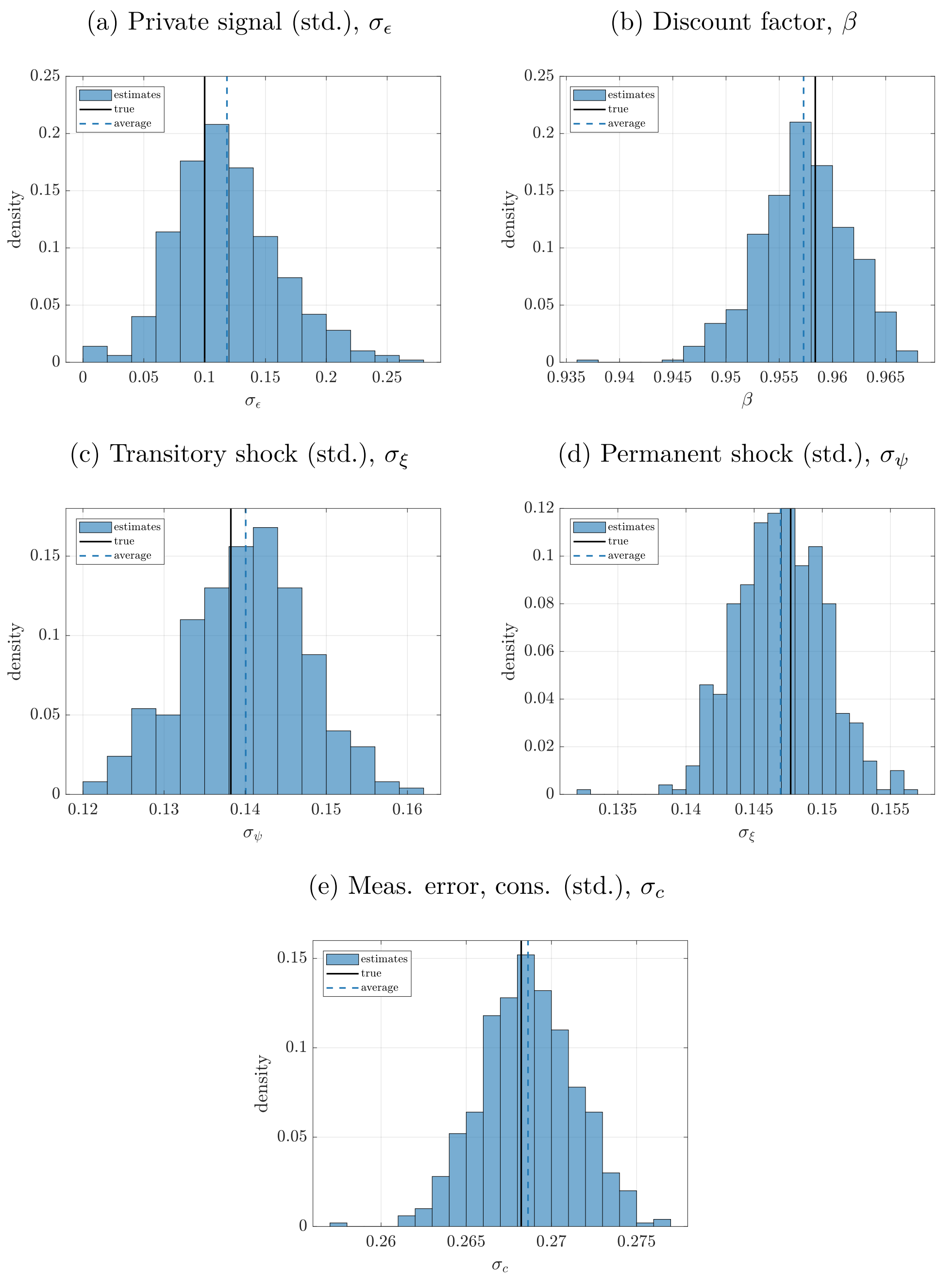
- **Moments 75-82:** The covariances of log consumption growth with log income growth, $\text{cov}(\Delta c_t, \Delta y_{t+k})$ for $k = -2, \ldots, 5$. These moments are primarily included for identification of the noise in the private signal, σ_ϵ^2.

3.3 Monte Carlo

We now present results from a Monte Carlo study mimicking our empirical analysis on PSID in Section 4. Specifically, we simulate $N = 1,765$ households from age 25 through 65 from the general CRRA model. We impose the same patterns of missing observations as in the PSID resulting in a total of 7,604 household-time observations.

Due to the computational complexity, we focus our attention on a restricted model with a unit root and no MA(1) term, i.e. $\alpha = 1$ and $\omega = 0$. Furthermore, we do not estimate the income growth parameters, g_0 and g_1, or the measurement error in income, σ_y^2, but keep them fixed at their true values. In sum, we estimate $\theta = (\sigma_\epsilon, \beta, \sigma_c, \sigma_\xi, \sigma_\psi)$ by SMM using the moments specified above. The parameter values are chosen according to our calibration and empirical estimates from the PSID, see column one of Table 4.2. The sole exception is the standard deviation of the noise in the private signal, which we pick to be $\sigma_\epsilon = 0.10$.

Figure 3.1: Monte Carlo Results.



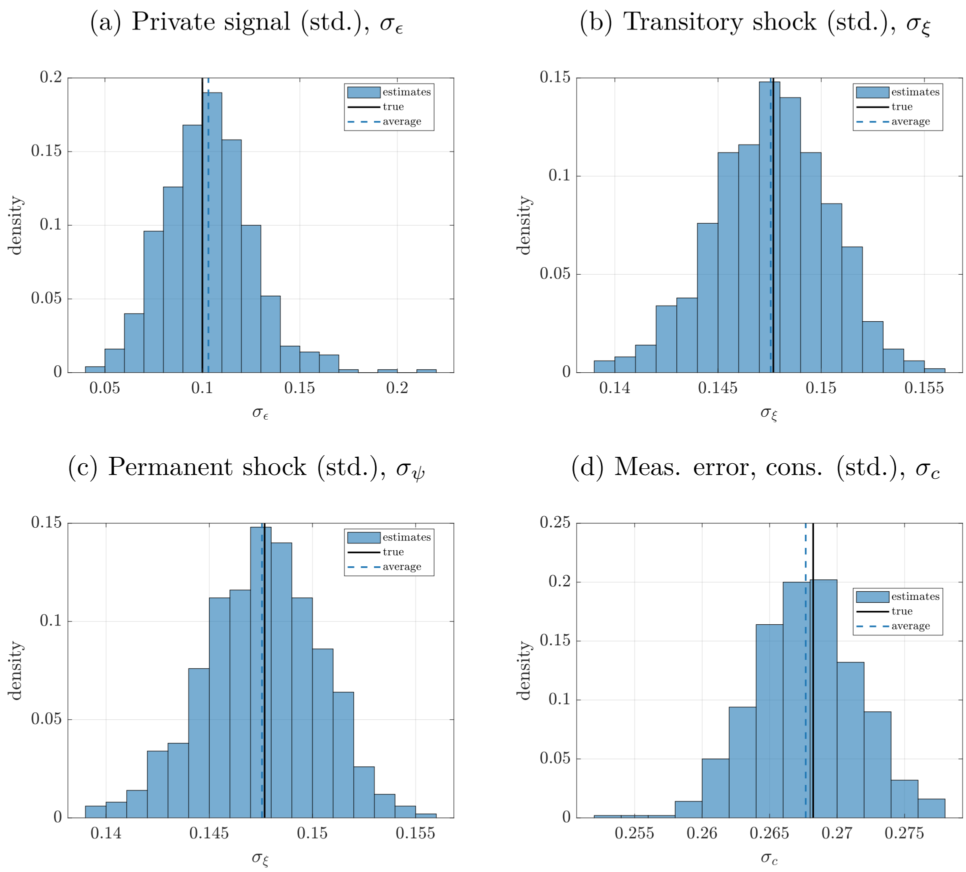
(a) Private signal (std.), σ_ϵ (b) Discount factor, β (c) Transitory shock (std.), σ_ξ (d) Permanent shock (std.), σ_ψ (e) Meas. error, cons. (std.), σ_c

Notes: Figure 3.1 reports histograms of Monte Carlo estimates of the five parameters $\theta = (\sigma_\epsilon, \beta, \sigma_\xi, \sigma_\psi, \sigma_c)$. We use 200 replications, fix $\alpha = 1$, $\omega = 0$, and assume g_0, g_1 and σ_y^2 to be known. We pick a standard deviation for the noise in the private signal of $\sigma_\epsilon = 0.10$, and choose the remaining parameter values from our calibration and estimates on PSID data, see column one of Table 4.2.

Figure 3.1 reports the marginal distributions of the estimated parameters. For all parameters, we find that our estimator on average uncovers the true value. This is reassuring because it indicates that we can uncover the parameters of the model with the suggested moments. The income shock variances and the measurement error in consumption are moreover very precisely estimated, while the estimates of the discount factor and the noise in the private signal is a bit more dispersed.

We now turn to the closed-form results derived for the CEQ model in Corollary 6. While the data is still simulated from the CRRA model, and is identical to that used to generate Figure 3.1, we see in Figure 3.2 that the degree of noise in the private signal, σ_ϵ, is on average estimated close to the true value. We also estimate permanent and transitory income shock variances and the degree of measurement error in consumption to be close to their true values. In total, this is encouraging for the use of this simple estimation strategy.

Figure 3.2: Monte Carlo Results. CEQ.



Notes: Figure 3.2 reports histograms of Monte Carlo estimates of the five parameters $\theta = (\beta, \sigma_\epsilon, \sigma_\xi, \sigma_\psi, \sigma_c)$ using the CEQ model, but with data simulated from the CRRA model studied in Figure 3.1. We use 200 replications. We pick a standard deviation for the noise in the private signal of $\sigma_\epsilon = 0.10$, and choose the remaining parameter values from our calibration and estimates on PSID data, see column one of Table 4.2.

4 Empirical Investigation

In this section, we present an empirical investigation regarding whether consumers can distinguish persistent from transitory income shocks. We use data from the Panel Study of Income Dynamics (PSID) and the model and estimation approach validated by the Monte Carlo study in the previous section.

4.1 Data

We use the same PSID data as BPP, including their exact variable definitions and sample selection criteria. We use their imputed total non-durable consumption as our consumption measure and the income measure is total family income net of financial income and taxes. Both measures are deflated using the CPI. We exclude the low-income sample (SEO) and focus on stable married households of opposite sex in which the head is male and aged 30-65. We keep households in which the husband was born between 1920 and 1959. We discard a few income observations due to top-coded tax or income. In turn we have 17,604 household-time observations from 1,765 households. We refer the reader to the Appendix of BPP for more details on the data and sample selection criteria.

We follow BPP and remove predictable variation due to demographics from the income and consumption series through regression. As BPP, we include dummies for educational level, race, number of children, number of family members, region of residence, employment status and external income source. All these dummies are allowed to be varying over time. Finally, we include time and age dummies in the regressions.[11] Throughout the remainder of this paper we refer to y_t and c_t as log income and log consumption, respectively, with all predictable variation removed.

4.2 Estimation Results: CEQ Model

As a first step, we apply the closed form estimator derived in Corollary 6 under the assumption of a certainty equivalence (CEQ) model. We assume that the real interest rate is 3 percent ($R = 1.03$) like in e.g. Gourinchas and Parker (2002), and

[11] We include age dummies rather than birth cohort dummies as done in BPP. This is, however, identical because year dummies are also included. The age dummies provide us with an estimate of the life-cycle profiles of income and consumption which we utilize as moments in the estimation of the model.

that measurement error in income accounts for 25 percent ($\tau = 0.25$) of the total variance of log income growth in the PSID data ($\sigma_y^2 = 0.5 \cdot \tau \cdot 0.0907 = 0.0113$). This strategy is identical to that employed in Meghir and Pistaferri (2004) and is in part based on the result in Bound, Brown, Duncan and Rodgers (1994) that around 22 percent of the overall income growth variance in the PSID is attributed to measurement error.[12]

Table 4.1: Estimates, CEQ Model.

		Whole sample	No college	College
Parameter		(1)	(2)	(3)
σ_ϵ	Private signal (std.)	0.031	0.043	0.011
		(0.026)	(0.039)	(0.025)
σ_c	Meas. error, cons. (std.)	0.264	0.296	0.230
		(0.008)	(0.013)	(0.007)
σ_ψ	Persistent shock (std.)	0.165	0.172	0.158
		(0.005)	(0.008)	(0.008)
σ_ξ	Transitory shock (std.)	0.135	0.143	0.126
		(0.005)	(0.007)	(0.007)

Notes: Bootstrapped standard errors based on 5000 bootstrap replications reported in brackets.

Table 4.1 reports the reduced-form estimates of the CEQ model. The first column reports estimates for the whole sample and columns (2) and (3) report estimation results for the no college and college groups. In all cases we get small positive estimates of the standard deviation of the private signal, σ_ϵ, which, however, are clearly insignificant at the 5 percent level.[13] The point estimate is higher for the group with no college education.

The other estimated parameters are in the ranges typically found in the literature. The measurement error in consumption is substantial with a variance around 0.07, which is in the same range as reported in BPP.[14] The transitory and permanent income shock variances are estimated to be around 0.018 and 0.027, also close to the estimates reported in BPP.

[12] The results are robust to these calibrations.

[13] Figure C.1 in the Supplemental Material report the distributions of bootstrapped estimates showing that we with $\tau = 0.25$ get $\sigma_\epsilon = 0$ in 30 percent of the samples.

[14] The measurement error variance in BPP is allowed to be time-varying and the authors write that the imputation error variance is estimated in the range 0.05 to 0.10.

In the Supplemental Material, Table C.1 and C.2, we report the effect of assuming either no measurement error in income ($\tau = 0.0$) or a higher measurement error ($\tau = 0.50$). As expected, we find that the estimated transitory income shock variance is falling in the assumed degree of measurement error in income, while the noise in the private signal is increasing in it. In the high measurement error case we estimate $\sigma_\epsilon = 0.040$ with a standard deviation of 0.046, i.e. still small and insignificant.

As the estimates above is derived under the (arguably unrealistic) assumptions of the CEQ model, we now turn to the more general model with CRRA preferences and a more general income process

4.3 Estimation Results: General CRRA Model

We fix a few parameters of the model while estimating the remaining. Particularly, we set the real interest rate to three percent ($R = 1.03$) and fix the degree of measurement error in income to account for 25 percent of the total variance of log income growth in the PSID data as above. The replacement rate in retirement is fixed at 50 percent ($\lambda = 0.5$) and retirement happens with certainty at age 65 and agents die with certainty at age 85. Finally, we fix the constant relative risk aversion coefficient at $\rho = 1.5$ (Attanasio and Weber, 1995). We perform robustness checks below in sub-section 4.5 where changing these calibrated parameters does not change our results.

We estimate the parameters, $\theta = (\sigma_\epsilon, \beta, \sigma_c, \sigma_\xi, \sigma_\psi, g_0, g_1, \alpha, \omega)$, by Simulated Method of Moments (SMM) pioneered by McFadden (1989) as discussed in Section 3. Table 4.2 reports the estimated parameters. The first three columns are based on the whole PSID sample. In column (1), we consider a model with a permanent-transitory income process with a unit root and no MA(1) term ($\alpha = 1$ and $\omega = 0$). In column (2), we allow for a non-unit root (α free, $\omega = 0$), and in column (3) we additionally allow for a MA(1) term (α free, ω free). Columns (4) and (5) are, respectively, for the sub-sample with no college degree and with a college degree using the preferred model from column (2).

Table 4.2: Estimates, General Model.

Parameter		Whole sample (1)	(2)	(3)	No college (4)	College (5)
σ_ϵ	Private signal (std.)	0.000	0.000	0.000	0.000	0.000
		–	–	–	–	–
β	Discount factor	0.958	0.962	0.963	0.958	0.967
		(0.001)	(0.002)	(0.002)	(0.003)	(0.002)
σ_c	Meas. error, cons. (std.)	0.268	0.272	0.272	0.305	0.237
		(0.004)	(0.004)	(0.004)	(0.007)	(0.004)
σ_ψ	Persistent shock (std.)	0.138	0.172	0.150	0.191	0.155
		(0.006)	(0.008)	(0.010)	(0.012)	(0.010)
σ_ξ	Transitory shock (std.)	0.148	0.129	0.156	0.126	0.127
		(0.003)	(0.006)	(0.008)	(0.010)	(0.007)
g_0	Income growth, constant	0.031	0.087	0.061	0.127	0.076
		(0.002)	(0.019)	(0.013)	(0.052)	(0.021)
g_1	Income growth, age	−0.096	−0.036	−0.053	−0.016	−0.046
		(0.010)	(0.015)	(0.015)	(0.022)	(0.021)
α	AR(1) component	–	0.835	0.883	0.777	0.861
		–	(0.029)	(0.029)	(0.048)	(0.036)
ω	MA(1) component	–	–	0.154	–	–
		–	–	(0.035)	–	–
Objective		115.772	70.046	60.891	52.296	63.614
p-value for $\sigma_\epsilon = 0$		0.497	0.500	0.499	0.500	0.500

Notes: Asymptotic standard errors reported in brackets for all parameters in the interior of their domain.

The low estimated noise in the private signal, $\sigma_\epsilon \approx 0$, suggests that PSID households have a high degree of knowledge about their own permanent income. In fact, we cannot reject the one-sided hypothesis that $\mathcal{H}_0 : \sigma_\epsilon = 0$ against $\mathcal{H}_A : \sigma_\epsilon > 0$ with any standard confidence levels.[15] This is true for all specifications and across all robustness checks, reported below.

[15] Because our hypothesis is on the boundary of the parameter space, we employ a modified quasi likelihood ratio (QLR) test. Particularly, define the objective function to be minimized as $Q = (\Lambda(\mathbf{w}) - \overline{\Lambda}(\theta))' V^{-1} (\Lambda(\mathbf{w}) - \overline{\Lambda}(\theta))$ where the inverse of $V = var(\Lambda(\mathbf{w}))$ is the optimal weighting matrix (see the Supplemental Material on the calculation of V). The test statistic $QLR = Q(\hat{\theta}_{\sigma_\epsilon = 0}) - Q(\hat{\theta})$ where $Q(\hat{\theta}_{\sigma_\epsilon=0})$ and $Q(\hat{\theta})$, are the estimated objective functions under the null and with all parameters estimated, respectively, follows a mixture of two χ^2 distributions such that the p-value related to the null hypothesis can be found as $1 - F(QLR)$ where $F(z) = \frac{1}{2} + \frac{1}{2}\chi_1^2(z)$ and $\chi_1^2(z)$ is the CDF of a χ^2 distribution with 1 degree of freedom evaluated at z (Andrews, 2001, Theorem 4).

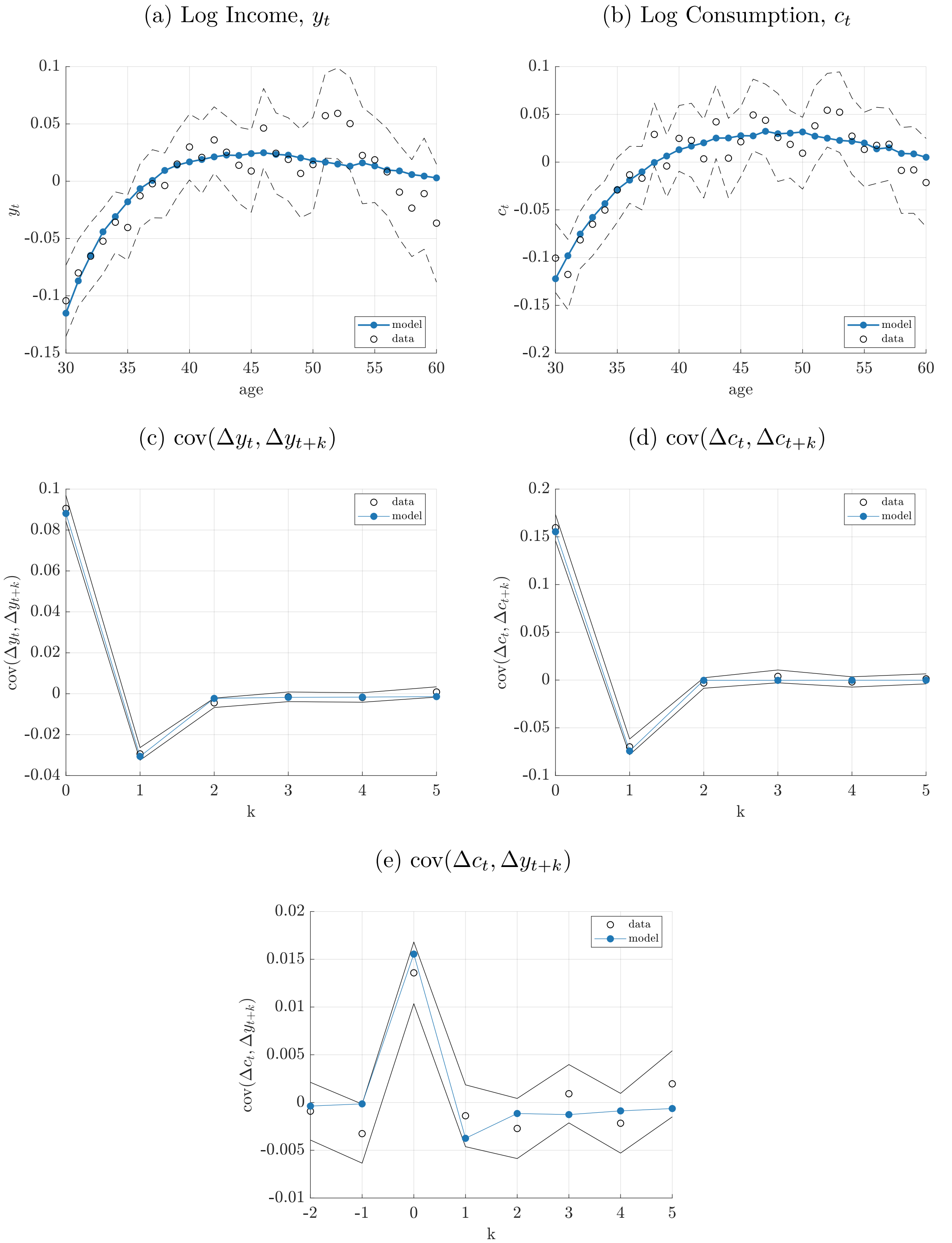
While the estimated variance of the private signal is zero (i.e. perfect information) for both educational groups, we estimate a slightly higher discount factor for the college group. The no college group have a higher degree of measurement error in consumption and a more volatile but less persistent shocks. The income growth rates also differ. Since we do not find important differences in the estimated degree of knowledge across educational groups, our preferred specification, and the focus in the remainder of the paper, is the results from the whole sample in column (2) of Table 4.2. The improvement in fit from adding the MA(1) term in column (3) is limited, and makes the model computationally much more demanding.

4.4 Model Fit and Sensitivity Analysis

Figure 4.1 reports the used moments calculated from the PSID with bootstrapped 95% confidence bands along with the same moments calculated using simulated data from the estimated model for the specification with α free and $\theta = 0$.[16] Given the amount of structure the estimated model places on the data, the model seems to fit the PSID data quite well. The moments calculated using synthetic data simulated from the estimated model is very close to the moments calculated using the PSID. Particularly, all moments except two are within the bootstrapped 95 percent confidence intervals. The only two moments outside the 95% confidence bands are two moments of the income age profile.

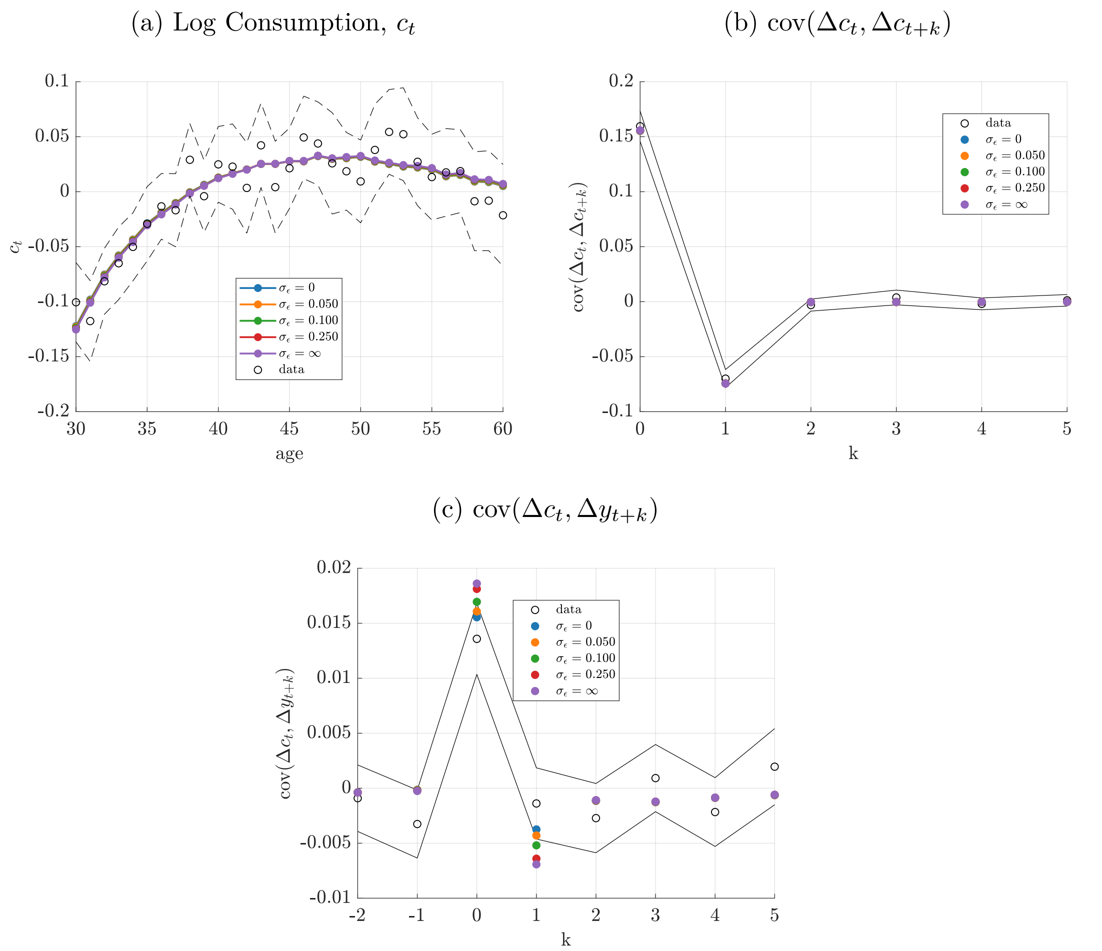
[16] The fit of the remaining specifications are shown in the Supplemental Material.

Figure 4.1: Model Fit (α free, $\theta = 0$).



Notes: Figure 4.1 illustrates the average age profiles of log income and log consumption together with the covariance moments. Both age profile series are normalized by the overall mean of each series. Hollow dots are calculated using the PSID, $\Lambda(\mathbf{w})$, solid lines are bootstrapped 95% confidence intervals, and solid dots are calculated using simulated data from the model, $\Lambda(\hat{\theta})$.

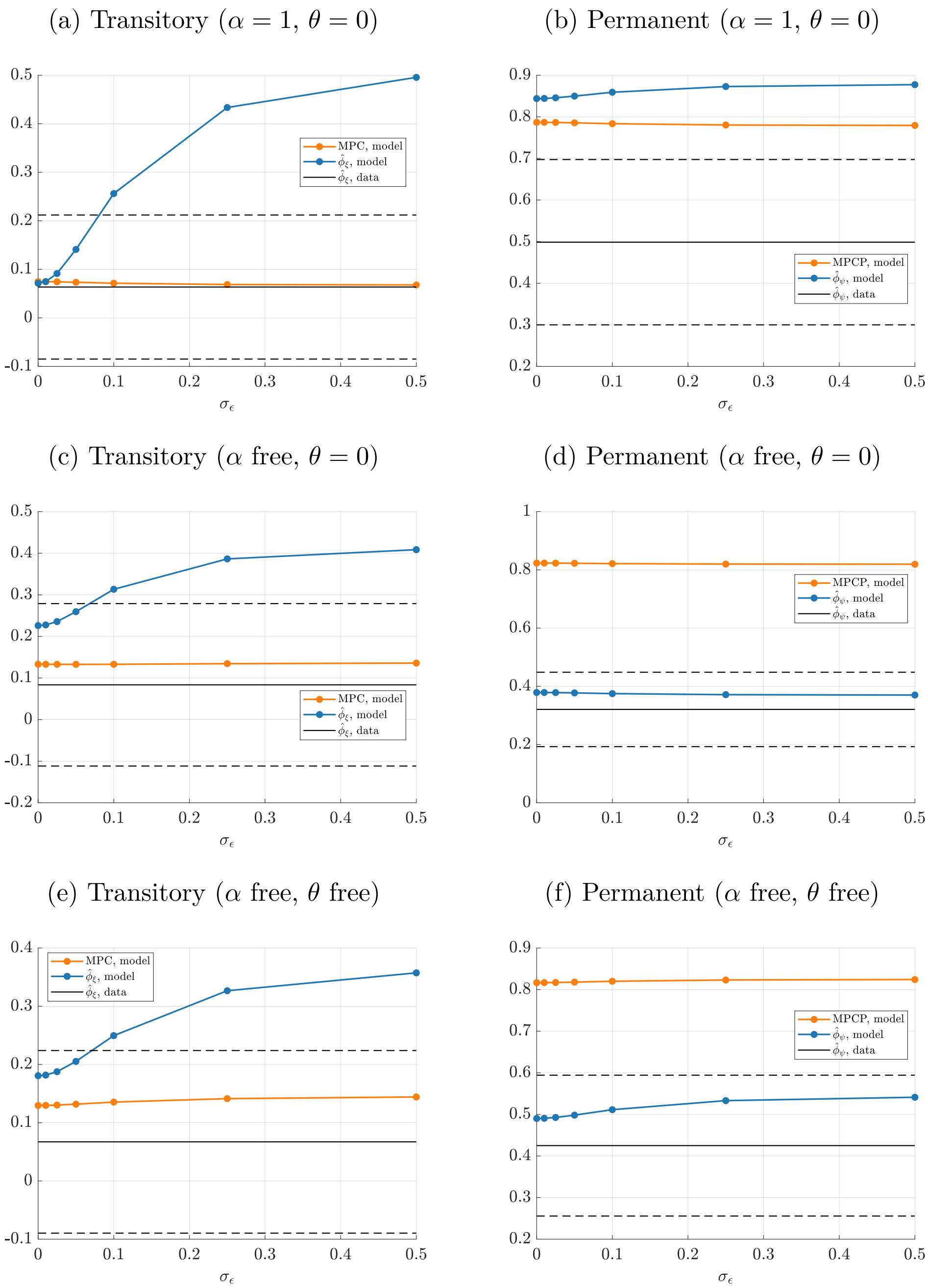
Figure 4.2: Model Sensitivity (α free, $\theta = 0$).



Notes: Figure C.4 illustrates the average age profiles of log income and log consumption together with the covariance moments. Both age profile series are normalized by the overall mean of each series. Hollow dots are calculated using the PSID, $\Lambda(\mathbf{w})$, solid lines are bootstrapped 95% confidence intervals and solid colored dots are calculated using simulated data from the model, $\Lambda(\theta)$.

To investigate which moments the degree of knowledge, σ_ϵ, is sensitive to, Figure 4.2 illustrates the effect of increasing σ_ϵ on the moments involving consumption. While decreasing the degree of knowledge, we keep all other parameters at their estimated values in column (2) in Table 4.2. We see that $\text{cov}(\Delta c_t, \Delta c_{t+k})$ changes insignificantly when varying the degree of knowledge. The same is true for the consumption age profile. The moments that seem to be changing the most are the covariances between consumption and income growth, $\text{cov}(\Delta c_t, \Delta y_{t+k})$. Especially for $k = 0$ and $k = 1$. This is in line with the theoretical results from the certainty equivalence model. We see that even with $\sigma_\epsilon = 0$ the covariance between current income growth and current consumption growth is already a bit too large in the estimated model, and decreasing the degree of knowledge, a higher σ_ϵ, only exacerbates this problem. Likewise, the covariance between current consumption growth and future income growth is too negative in the model, and decreasing the degree of knowledge, a higher σ_ϵ, also only worsens this problem.

Figure 4.3: Transmission parameters and propensities to consume.



Notes: Figure 4.3 report estimates of the transitory ($\hat{\phi}_\xi$) and permanent ($\hat{\phi}_\psi$) transmission parameters in the PSID data and in simulated data. We calculate the transmission parameters using $\hat{\phi}_\xi = \text{cov}(\Delta c_t, -\Delta y_t)/((1-\hat{\theta})\hat{\sigma}_\xi^2)$ and $\hat{\phi}_\psi = \text{cov}(\Delta c_t, \Delta y_{t-1} + \Delta y_t + \Delta y_{t+1})/\hat{\sigma}_\psi^2$. We also report the corresponding average marginal propensity to consume (MPC) and average marginal propensity to consume out of a persistent income shock, (MPCP).

The intuition for these results are that reducing the degree of knowledge first and foremost increase the transmission of income growth to consumption growth, but that the model already fit this transmission with perfect information, $\sigma_\epsilon = 0$. This is also illustrated in Figure 4.3. In the left column we show the transitory transmission parameters estimated as in BPP by $\hat{\phi}_\xi = \text{cov}(\Delta c_t, -\Delta y_t)/((1-\hat{\theta})\hat{\sigma}_\xi^2)$ in both the data (using the estimated income parameters) and simulations from the model with varying degrees of knowledge.[17] In the right column, we similarly show the permanent transmission parameters estimated by $\hat{\phi}_\psi = \text{cov}(\Delta c_t, \Delta y_{t-1} + \Delta y_t + \Delta y_{t+1})/\hat{\sigma}_\psi^2$. In the first row we consider the model with a permanent-transitory income process ($\alpha = 1$ and $\theta = 0$), where the BPP estimator is known to work well in the perfect information case. We see that the estimated transitory transmission parameter should be much larger than what we see in the data if there was even limited imperfect knowledge. If e.g. $\sigma_\epsilon = 0.10$ the transitory transmission parameter should be around 0.25 rather than the 0.08 observed in the data. This is also the case with the more general formulations of the income process in the remaining rows.

Figure 4.4 additionally shows the results of an experiment, where the discount factor, β, is fixed at various values prior to re-estimating the model (and α is not re-estimated). The left column shows the resulting objective function, while the right column shows the estimate of the degree of noise in the private signal, σ_ϵ. We see that when the discount factor is fixed at a high value, we estimate a substantial degree of imperfect information. The explanation is that when the households are forced to be patient the covariances between income growth and consumption growth falls, which implies that imperfect information is then required to fit the observed covariances.[18] Note, however, that fixing the discount factor just slightly above the estimated value leads to a substantial increase in the objective function.

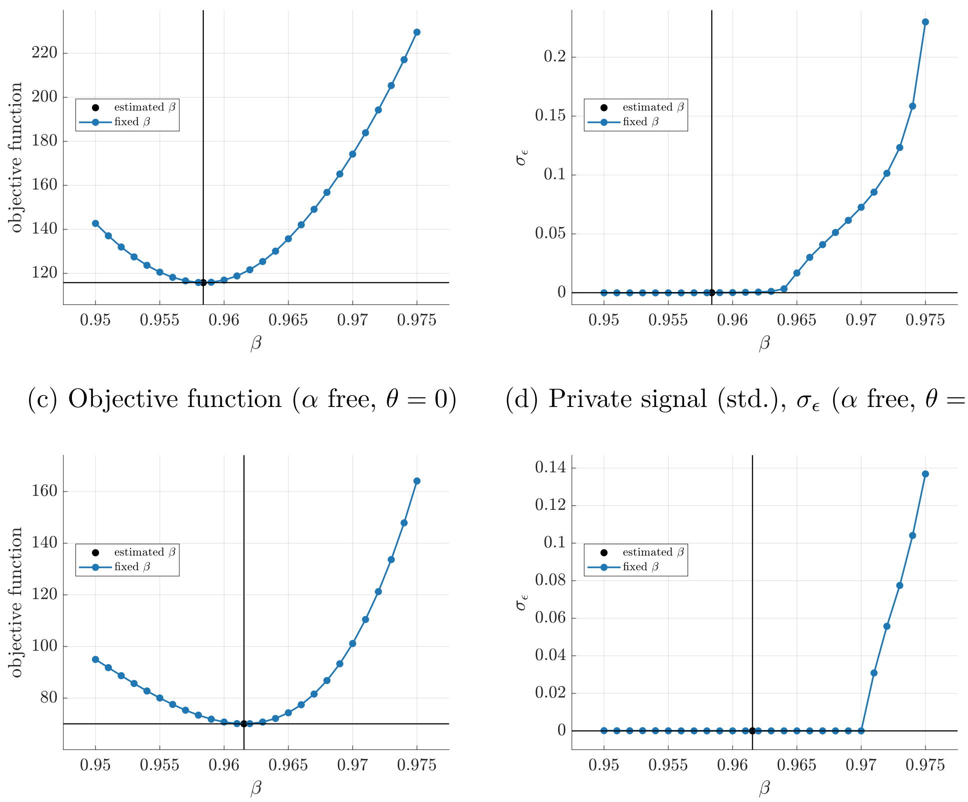
[17] All remaining parameters remain fixed at their estimated values while varying σ_ϵ.

[18] If we re-estimate α (not reported) we find that this parameter change drastically when changing β, while σ_ϵ remain around zero.

Figure 4.4: Identification. Fixing β.

(a) Objective function ($\alpha = 1$, $\theta = 0$) (b) Private signal (std.), σ_ϵ ($\alpha = 1$, $\theta = 0$)

(c) Objective function (α free, $\theta = 0$) (d) Private signal (std.), σ_ϵ (α free, $\theta = 0$)



Notes: The figure reports the objective function and the estimate of σ_ξ when fixing β to various values prior to the estimation and not re-estimating α.

4.5 Robustness Checks

Table 4.3 reports a series of robustness checks. Firstly, we see in column (1)-(6) that neither varying the choice of risk aversion, ρ, the measurement error in income, τ, or the replacement rate in retirement, λ, affect the main result of perfect information about permanent income. Secondly, in columns (7)–(8), we consider an extension of the model where the household each period is allowed to borrow up to a fraction ζ of the mean-belief of its persistent income; again we estimate no noise in the private signal. Finally, in columns (9)–(10) we see that our results are not affected by either choosing a diagonal weighting matrix with the inverse of the variances of the moments on the diagonal, or the identity matrix. In the Supplemental Material we show that our results survive the same robustness checks when fixing $\alpha = 1$ and $\omega = 0$.

Table 4.3: Estimates, Robustness (α free, $\theta = 0$).

	Parameter	$\rho = 2.0$ (1)	$\rho = 4.0$ (2)	$\tau = 0.0$ (3)	$\tau = 0.5$ (4)	$\lambda = 0.25$ (5)	$\lambda = 0.75$ (6)	$\zeta = 0.10$ (7)	$\zeta = 0.20$ (8)	Diag. W (9)	Equal W (10)
σ_ϵ	Private signal (std.)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.003
		–	–	–	–	–	–	–	–	–	–
β	Discount factor	0.957	0.928	0.963	0.960	0.964	0.958	0.948	0.869	0.962	0.941
		(0.003)	(0.008)	(0.002)	(0.002)	(0.002)	(0.003)	(0.002)	(0.009)	(0.002)	(0.010)
σ_c	Meas. error, cons. (std.)	0.272	0.272	0.272	0.272	0.273	0.271	0.272	0.271	0.272	0.262
		(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.008)
σ_ψ	Persistent shock (std.)	0.173	0.171	0.167	0.177	0.186	0.150	0.169	0.145	0.172	0.212
		(0.008)	(0.008)	(0.008)	(0.008)	(0.007)	(0.008)	(0.007)	(0.009)	(0.008)	(0.025)
σ_ξ	Transitory shock (std.)	0.129	0.130	0.168	0.070	0.123	0.138	0.132	0.138	0.129	0.108
		(0.006)	(0.006)	(0.004)	(0.011)	(0.006)	(0.005)	(0.005)	(0.005)	(0.006)	(0.022)
g_0	Income growth, constant	0.085	0.078	0.090	0.083	0.083	0.079	0.062	0.076	0.087	0.055
		(0.019)	(0.017)	(0.022)	(0.017)	(0.016)	(0.020)	(0.012)	(0.025)	(0.019)	(0.006)
g_1	Income growth, age	−0.036	−0.038	−0.034	−0.038	−0.042	−0.047	−0.047	−0.008	−0.036	−0.070
		(0.015)	(0.014)	(0.015)	(0.014)	(0.014)	(0.015)	(0.014)	(0.016)	(0.015)	(0.015)
α	AR(1) component	0.837	0.847	0.828	0.845	0.851	0.844	0.878	0.818	0.835	0.935
		(0.028)	(0.028)	(0.030)	(0.027)	(0.025)	(0.033)	(0.026)	(0.040)	(0.029)	(0.026)
ω	MA(1) component	–	–	–	–	–	–	–	–	–	–
		–	–	–	–	–	–	–	–	–	–
Objective		69.994	70.214	76.509	65.432	70.369	86.453	68.700	108.076	70.046	14.508
p-value for $\sigma_\epsilon = 0$		0.500	0.500	0.500	0.500	0.500	0.497	0.497	0.493	–	–

Notes: Asymptotic standard errors reported in brackets.

5 Conclusions

We have developed a novel consumption-saving model with a flexible specification of the households' ability to distinguish persistent from transitory income shocks. We showed that the assumption about the households' degree of knowledge has important implications for our interpretation of consumption-saving behavior in general and estimated transmission parameters in particular. If households, like an econometrician, need to solve a filtering problem to distinguish persistent from transitory income shocks, it eradicates the correspondence between transmission parameters estimated as in Blundell, Pistaferri and Preston (2008), and the true marginal propensity to consume. Based on a theoretical analysis of a certainty equivalence model we were able to show that the households' degree of knowledge is identifiable from panel data of income and consumption. In a Monte Carlo study we validated that our approach works for a general model with a complex income process using a sample similar to the Panel Study of Income Dynamics (PSID) both in terms of size and measurement error.

Finally, we estimated the general model using the PSID. We estimate a large degree of knowledge and cannot reject that households can distinguish persistent from transitory income shocks. This is reassuring in terms of validating the standard interpretation of estimated transmission parameters of persistent and transitory income shocks.

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A Proofs

A.1 Proof of Lemma 1

Taking the limit $\sigma_\epsilon \to \infty$ in eq. (2.19) using L'Hôpital's rule gives the result in eq. (2.27),

$$\lim_{\sigma_\epsilon\to\infty} q^\star = \left(\sqrt{\lim_{\sigma_\epsilon\to\infty}\frac{\sigma_\xi^2\sigma_\epsilon^2}{(\sigma_\xi^2+\sigma_\epsilon^2)\sigma_\psi^2}+\frac{1}{4}}-\frac{1}{2}\right)\sigma_\psi^2 \tag{A.1}$$
$$= \left(\sqrt{\frac{\sigma_\xi^2}{\sigma_\psi^2}+\frac{1}{4}}-\frac{1}{2}\right)\sigma_\psi^2 \leq \sigma_\xi^2.$$

Similarly, taking the limit $\sigma_\xi \to \infty$ gives

$$\lim_{\sigma_\xi\to\infty} q^\star = \left(\sqrt{\frac{\sigma_\epsilon^2}{\sigma_\psi^2}+\frac{1}{4}}-\frac{1}{2}\right)\sigma_\psi^2 \leq \sigma_\epsilon^2. \tag{A.2}$$

Further noting that the derivatives of $q^\star$ wrt. σ_ξ^2 and σ_ϵ^2 are always positive,

$$\frac{\partial q^\star}{\partial \sigma_\xi^2} = \frac{\partial q^\star}{\partial \sigma_\epsilon^2} = \frac{\sigma_\xi^2}{2(\sigma_\epsilon^2+\sigma_\xi^2)(q^\star/\sigma_\psi^2+\frac{1}{2})} > 0, \tag{A.3}$$

give the result $q^\star \leq \min(\sigma_\xi^2, \sigma_\epsilon^2)$ in eq. (2.21). The results in eq. (2.22) and (2.23) follow from eq. (2.20).

The remaining limits in eq. (2.24)-(2.29) follow directly from taking the respective limits of eq. (2.19) and (2.20).

The sign of the derivatives in eq. (2.31)-(2.33) are found using Mathematica. See Supplemental Material D under the heading "Lemma 1: sign of q and K derivatives".

A.2 Proof of Lemma 2

Using the definition of $\kappa_t = \hat{p}_t - p_t$ in eq. (2.8) with eq. (2.14) and the Kalman updating relations in eq. (2.10)-(2.12) gives

$$\begin{aligned}
\kappa_t &= \hat{p}_t - p_t \\
&= \hat{p}_{t|t-1} + K^\star \Delta_t - p_t \\
&= \hat{p}_{t|t-1} + K_1^\star (y_t - \hat{y}_{t|t-1}) + K_2^\star (z_t - \hat{z}_{t|t-1}) - p_t \\
&= \hat{p}_{t|t-1} + K_1^\star (y_t - \hat{p}_{t|t-1}) + K_2^\star (z_t - \hat{p}_{t|t-1}) - p_t \\
&= \hat{p}_{t|t-1}(1 - \mathcal{K}^\star) + K_1^\star y_t + K_2^\star z_t - p_t \\
&= \hat{p}_{t|t-1}(1 - \mathcal{K}^\star) + K_1^\star (p_t + \xi_t - \mu_\xi) + K_2^\star (p_t + \epsilon_t) - p_t \\
&= \hat{p}_{t|t-1}(1 - \mathcal{K}^\star) + p_t(\mathcal{K}^\star - 1) + K_1^\star \xi_t + K_2^\star \epsilon_t \\
&= \hat{p}_{t-1}(1 - \mathcal{K}^\star) + (p_{t-1} + \psi_t)(\mathcal{K}^\star - 1) + K_1^\star \xi_t + K_2^\star \epsilon_t - K_1^\star \mu_\xi \\
&= (\mathcal{K}^\star - 1)(p_{t-1} - \hat{p}_{t-1}) + (\mathcal{K}^\star - 1)\psi_t + K_1^\star \xi_t + K_2^\star \epsilon_t \\
&= (1 - \mathcal{K}^\star)\kappa_{t-1} + (\mathcal{K}^\star - 1)\psi_t + K_1^\star \xi_t + K_2^\star \epsilon_t.
\end{aligned}$$

The autocovariance then is

$$\begin{aligned}
\text{cov}(\kappa_t, \kappa_{t-1}) &= \text{cov}((1 - \mathcal{K}^\star)\kappa_{t-1} + (\mathcal{K}^\star - 1)\psi_t + K_1^\star \xi_t + K_2^\star \epsilon_t, \kappa_{t-1}) \\
&= (1 - \mathcal{K}^\star)\text{cov}(\kappa_{t-1}, \kappa_{t-1}) \\
&= (1 - \mathcal{K}^\star)q^\star.
\end{aligned}$$

A.3 Proof of Lemma 3

Inserting (2.11) and (2.4) and noting that $y_{t|t} = y_t$ (households perfectly observe their income) and using eq. (2.10) gives

$$\begin{aligned}
e_t &\equiv (y_{t+1|t} - y_{t|t}) - (y_{t+1} - y_t) \\
&= (\hat{p}_{t+1|t} - y_t) - (\psi_{t+1} + \xi_{t+1} - \xi_t) \\
&= (\hat{p}_t - y_t) - (\psi_{t+1} + \xi_{t+1} - \xi_t) \\
&= (\hat{p}_t - p_t) - (\psi_{t+1} + \xi_{t+1}) \\
&= \kappa_t - \psi_{t+1} - \xi_{t+1}.
\end{aligned}$$

We see that because all elements are mean zero that $\mathbb{E}_t[e_t] = 0$. Because all components are *iid*, the variance of the sum of components are the sum of the variances of the components, $\text{var}(e_t) = q^\star + \sigma_\psi^2 + \sigma_\xi^2$ and since $q^\star \geq 0$ we have that

$\text{var}(e_t) \geq \sigma_\psi^2 + \sigma_\xi^2$.

Finally, the autocovariance is

$$
\begin{aligned}
\text{cov}(e_t, e_{t+1}) &= \text{cov}(\kappa_t - \psi_{t+1} - \xi_{t+1}, \kappa_{t+1} - \psi_{t+2} - \xi_{t+2}) \\
&= \text{cov}(\kappa_t - \psi_{t+1} - \xi_{t+1}, \\
&\quad (1 - \mathcal{K}^\star)\kappa_t + (\mathcal{K}^\star - 1)\psi_{t+1} \\
&\quad + K_1^\star \xi_{t+1} + K_2^\star \epsilon_{t+1} - \psi_{t+2} - \xi_{t+2}) \\
&= \text{cov}(\kappa_t, (1 - \mathcal{K}^\star)\kappa_t) + \text{cov}(-\psi_{t+1}, (\mathcal{K}^\star - 1)\psi_{t+1}) + \text{cov}(-\xi_{t+1}, K_1^\star \xi_{t+1}) \\
&= (1 - \mathcal{K}^\star) q^\star - (\mathcal{K}^\star - 1)\sigma_\psi^2 - K_1^\star \sigma_\xi^2,
\end{aligned}
$$

where inserting $\mathcal{K}^\star$ and re-organizing gives

$$
\begin{aligned}
\text{cov}(e_t, e_{t+1}) &= q^\star + \sigma_\psi^2 - K_1^\star (q^\star + \sigma_\psi^2 + \sigma_\xi^2) - K_2^\star (q^\star + \sigma_\psi^2) \\
&= q^\star + \sigma_\psi^2 - \frac{\sigma_\epsilon^2 (q^\star + \sigma_\psi^2)(q^\star + \sigma_\psi^2 + \sigma_\xi^2)}{(\sigma_\xi^2 + \sigma_\epsilon^2)(q^\star + \sigma_\psi^2) + \sigma_\xi^2 \sigma_\epsilon^2} - \frac{\sigma_\xi^2 (q^\star + \sigma_\psi^2)(q^\star + \sigma_\psi^2)}{(\sigma_\xi^2 + \sigma_\epsilon^2)(q^\star + \sigma_\psi^2) + \sigma_\xi^2 \sigma_\epsilon^2} \\
&= q^\star + \sigma_\psi^2 - (q^\star + \sigma_\psi^2) \frac{\sigma_\epsilon^2 (q^\star + \sigma_\psi^2 + \sigma_\xi^2) + \sigma_\xi^2 (q^\star + \sigma_\psi^2)}{(\sigma_\xi^2 + \sigma_\epsilon^2)(q^\star + \sigma_\psi^2) + \sigma_\xi^2 \sigma_\epsilon^2} \\
&= q^\star + \sigma_\psi^2 - (q^\star + \sigma_\psi^2) \frac{(\sigma_\epsilon^2 + \sigma_\xi^2)(q^\star + \sigma_\psi^2) + \sigma_\xi^2 \sigma_\epsilon^2}{(\sigma_\epsilon^2 + \sigma_\xi^2)(q^\star + \sigma_\psi^2) + \sigma_\xi^2 \sigma_\epsilon^2} \\
&= 0.
\end{aligned}
$$

Since it holds for $t + 1$, it holds for $t + k$.

A.4 Proof of Theorem 1

The No-Ponzi game condition ensures that the intertemporal budget constraint (IBC) must hold with equality,

$$\sum_{k=0}^{\infty} R^{-(t+k)} c_{t+k} = a_0 + \sum_{k=0}^{\infty} R^{-(t+k)} y_{t+k}. \tag{A.4}$$

The Euler equation further implies that the expected future level of consumption is the same as today,

$$c_t = \mathbb{E}_t[c_{t+1}] \Rightarrow \mathbb{E}_t[c_{t+k}] = c_t, \; \forall k > 0. \tag{A.5}$$

Combining (A.4) and (A.5)

$$
\begin{aligned}
c_t \frac{1}{1-R^{-1}} &= a_0 + E_t\left[\sum_{k=0}^{\infty} R^{-(t+k)} y_{t+k}\right] \Leftrightarrow \\
c_t &= (1-\beta)\left(a_0 + E_t\left[\sum_{k=0}^{\infty} R^{-(t+k)} y_{t+k}\right]\right), \qquad \text{(A.6)}
\end{aligned}
$$

so that consumption differences evolve as

$$
\begin{aligned}
\Delta c_t &= (R-1)\sum_{k=0}^{\infty} R^{-k}(\mathbb{E}_t[y_{t+k}] - \mathbb{E}_{t-1}[y_{t+k}]) \\
&= \frac{R-1}{R}[y_t - \hat{p}_{t-1}] + \frac{R-1}{R}\sum_{k=1}^{\infty} R^{-k}(\hat{p}_t - \hat{p}_{t-1}) \\
&= \frac{R-1}{R}[y_t - \hat{p}_{t-1}] + R^{-1}(\hat{p}_t - \hat{p}_{t-1}). \qquad \text{(A.7)}
\end{aligned}
$$

From eq. (2.4) and (2.8) we derive

$$
y_t - \hat{p}_{t-1} = \kappa_{t-1} + \psi_t + \xi_t. \qquad \text{(A.8)}
$$

From eq. (2.8) and (2.34) we derive

$$
\hat{p}_t = \hat{p}_{t-1} + \mathcal{K}^\star(\psi_t - \kappa_{t-1}) + K_1^\star \xi_t + K_2^\star \epsilon_t. \qquad \text{(A.9)}
$$

Combing eq. (A.8) and (A.9) with eq. (A.7) yields the result in (2.36).

A.5 Proof of Corollary 1

See Mathematica output in Supplemental Material D under the heading "Corollary 1: $\text{cov}(\Delta c, \Delta c)$".

A.6 Proof of Corollary 2

First inserting for the leaded income growth, we get for $k = 1$,

$$
\begin{aligned}
\text{cov}(\Delta c_t, \Delta y_{t+1}) &= \text{cov}(\phi_\psi(\psi_t - \kappa_{t-1}) + \phi_\xi \xi_t + \phi_\epsilon \epsilon_t, \psi_{t+1} + \xi_{t+1} - \xi_t) \\
&= -\phi_\xi \sigma_\xi^2
\end{aligned}
$$

and for $k = 0$,

$$\begin{aligned} \text{cov}(\Delta c_t, \Delta y_t) &= \text{cov}(\phi_\psi(\psi_t - \kappa_{t-1}) + \phi_\xi \xi_t + \phi_\epsilon \epsilon_t, \psi_t + \xi_t - \xi_{t-1}) \\ &= \phi_\psi \sigma_\psi^2 + \phi_\xi \sigma_\xi^2 + \text{cov}(-\phi_\psi \kappa_{t-1}, -\xi_{t-1}) \\ &= \phi_\psi \sigma_\psi^2 + \phi_\xi \sigma_\xi^2 + \phi_\psi K_1^\star \sigma_\xi^2 \end{aligned}$$

which we use Mathematica to show that $\text{cov}(\Delta c_t, \Delta y_t) = \sigma_\psi^2 + \frac{R-1}{R}\sigma_\xi^2 + q^\star$. See Supplemental Material D under the heading "Corollary 2: $\text{cov}(\Delta c, \Delta y)$".

A.7 Proof of Corollary 3

The limits of ϕ_ψ, ϕ_ξ and ϕ_ϵ follow directly from the limits of $\mathcal{K}^\star$ in Lemma 1. Signing the derivatives of ϕ_ψ, ϕ_ξ and ϕ_ϵ with respect to σ_ϵ is done in Supplemental Material D showing Mathematica output under the heading "Corollary 3: sign of ϕ derivatives".

A.8 Proof of Corollary 4

Inserting the result from Corollary 2 and $\text{cov}(\Delta y_t, \Delta y_{t+1}) = -\sigma_\xi^2$, we get that

$$\begin{aligned} \hat{\phi}_\xi^{BPP} &\equiv \frac{\text{cov}(\Delta c_t, -\Delta y_{t+1})}{\text{cov}(\Delta y_t, -\Delta y_{t+1})} \\ &= \frac{\phi_\xi \sigma_\xi^2}{\sigma_\xi^2} \\ &= \phi_\xi, \end{aligned}$$

and for the permanent transmission parameter, we use that

$$\begin{aligned}
& \text{cov}(\Delta c_t, \Delta y_{t-1} + \Delta y_t + \Delta y_{t+1}) \\
= & \text{cov}(\phi_\psi(\psi_t - \kappa_{t-1}) + \phi_\xi \xi_t + \phi_\epsilon \epsilon_t, \\
& [\psi_{t-1} + \xi_{t-1} - \xi_{t-2}] + [\psi_t + \xi_t - \xi_{t-1}] + [\psi_{t+1} + \xi_{t+1} - \xi_t]) \\
= & \text{cov}(\phi_\psi \psi_t - \phi_\psi \kappa_{t-1} + \phi_\xi \xi_t + \phi_\epsilon \epsilon_t, \psi_{t-1} - \xi_{t-2} + \psi_t + \psi_{t+1} + \xi_{t+1}) \\
= & \text{cov}(\phi_\psi \psi_t - \phi_\psi[(1-\mathcal{K}^\star)\kappa_{t-2} + (\mathcal{K}^\star - 1)\psi_{t-1} + K_1^\star \xi_{t-1} + K_2^\star \epsilon_{t-1}] + \phi_\xi \xi_t + \phi_\epsilon \epsilon_t, \\
& \psi_{t-1} - \xi_{t-2} + \psi_t + \psi_{t+1} + \xi_{t+1}) \\
= & \phi_\psi \sigma_\psi^2 + \text{cov}(-\phi_\psi(1-\mathcal{K}^\star)\kappa_{t-2} + \phi_\psi(1-\mathcal{K}^\star)\psi_{t-1}, \psi_{t-1} - \xi_{t-2}) \\
= & \phi_\psi \sigma_\psi^2 + \phi_\psi(1-\mathcal{K}^\star)\sigma_\psi^2 + \phi_\psi(1-\mathcal{K}^\star)K_1^\star \sigma_\xi^2 \\
= & \phi_\psi \sigma_\psi^2[1 + (1-\mathcal{K}^\star)(1 + K_1^\star \frac{\sigma_\xi^2}{\sigma_\psi^2})],
\end{aligned}$$

and insert $\text{cov}(\Delta y_t, \Delta y_{t-1} + \Delta y_t + \Delta y_{t+1}) = \sigma_\psi^2$ to get

$$\begin{aligned}
\hat{\phi}_\psi^{BPP} & \equiv \frac{\text{cov}(\Delta c_t, \Delta y_{t-1} + \Delta y_t + \Delta y_{t+1})}{\text{cov}(\Delta y_t, \Delta y_{t-1} + \Delta y_t + \Delta y_{t+1})} \\
& = \frac{\phi_\psi \sigma_\psi^2[1 + (1-\mathcal{K}^\star) + (1-\mathcal{K}^\star)K_1^\star \frac{\sigma_\xi^2}{\sigma_\psi^2}]}{\sigma_\psi^2} \\
& = \phi_\psi[1 + (1-\mathcal{K}^\star)(1 + K_1^\star \frac{\sigma_\xi^2}{\sigma_\psi^2})] > \phi_\psi.
\end{aligned}$$

A.9 Proof of Corollary 5

See Mathematica output in Supplemental Material D under the heading "Corollary 5: Derivatives of Φ_ξ and Φ_ψ". We divide by

$$Q^\star \equiv \frac{\sigma_\xi^2(2q^\star + \sigma_\psi^2)}{(\sigma_\epsilon^2 + \sigma_\xi^2)(4\sigma_\epsilon^2 + (\sigma_\epsilon^2 + \sigma_\xi^2)\sigma_\psi^2)}$$

and then show that

$$\begin{aligned}
\frac{\partial \hat{\phi}_\xi^{BPP}/Q^\star}{\partial \sigma_\epsilon^2} & = \frac{1}{R} \\
\frac{\partial \hat{\phi}_\psi^{BPP}/Q^\star}{\partial \sigma_\epsilon^2} & = \frac{R-1}{R}\frac{\sigma_\xi^2}{\sigma_\psi^2}
\end{aligned}$$

implying the result.

A.10 Proof of Theorem 2

The assumptions on measurement error imply that

$$\begin{aligned}
\text{cov}(\Delta\tilde{y}_t, -\Delta\tilde{y}_{t+1}) &= \text{cov}(\Delta y_t, -\Delta y_{t+1}) = \sigma_\xi^2 \\
\text{cov}(\Delta\tilde{y}_t, \Delta\tilde{y}_{t-1} + \Delta\tilde{y}_t + \Delta\tilde{y}_{t+1}) &= \text{cov}(\Delta y_t, \Delta y_{t-1} + \Delta y_t + \Delta y_{t+1}) = \sigma_\psi^2 \\
\text{cov}(\Delta\tilde{c}_t, -\Delta\tilde{c}_{t+1}) &= \text{cov}(\Delta c_t, -\Delta c_{t+1}) + \sigma_c^2 = \sigma_c^2 \\
\text{cov}(\Delta\tilde{c}_t, \Delta\tilde{y}_{t+k}) &= \text{cov}(\Delta c_t, \Delta y_{t+k})
\end{aligned}$$

Eq. (2.4)–(2.5) and Corollary 1–2 imply the results in eq. (2.44). The results in eq. (2.48) hereafter follow from Lemma 4.

A.11 Proof of Corollary 6

The composite function $q^{\star-1}(\hat{q}^\star(x), \hat{\sigma}_\psi^2, x)$ is strictly *decreasing* in x. This is shown in the Mathematica output in Supplemental Material D under the heading "Corollary 6".

B Estimation Details

In order to compute standard errors we use that

$$\hat{\theta} - \theta_0 \xrightarrow{d} N(0, (G'WG)^{-1}G'W[V \cdot (1 + \frac{1}{J})]WG(G'WG)^{-1}) \tag{B.1}$$

where G is the $\dim(\theta) \times \dim(\theta)$ Jacobian and V is the $K \times K$ covariance matrix of the moments. In practice, we compute standard errors as

$$\text{SE}(\hat{\theta}) = \sqrt{\text{diag}((\hat{G}'W\hat{G})^{-1}\hat{G}'W[\hat{V} \cdot (1 + \frac{1}{J})]W\hat{G}(\hat{G}'W\hat{G})^{-1})} \tag{B.2}$$

where

$$\hat{G} = \frac{\partial \Lambda(\mathbf{w}) - \overline{\Lambda}(\theta)}{\partial \theta}_{|\theta=\hat{\theta}} \tag{B.3}$$

and the elements in $\hat{V}$ are computed differently depending on whether it is an element related only to the life-cycle profile moments, the moments involving the covariances between income and consumption growth, or a combination.

Define the indicator function $1_q\{q \text{ is not missing}\}$. Let z_a^1 and z_b^2 be generic life-cycle profile variables (e.g. $\text{mean}(z_a^1) = \text{mean}(y)|_{t=a} - \text{mean}(y)$ and $\text{mean}(z_b^2) =$

mean$(c)|_{t=b}-$mean(c)). Let x^1, x^2, x^3 and x^4 be generic variables in the covariances (e.g. $\text{cov}(x^1, x^2) = \text{cov}(\Delta c_t, \Delta c_{t+2})$, and $\text{cov}(x^3, x^4) = \text{cov}(\Delta c_t, \Delta y_{t+1})$).
For elements in V only involving the life-cycle profiles, we then have

$$\hat{V}_{1:62,1:62} = \frac{\sum_{i=1}^{N} 1_{z_{ia}^1} 1_{z_{ib}^2} [z_{ia}^1 - \text{mean}(z_a^1)][z_{ib}^2 - \text{mean}(z_b^2)]}{[\sum_{i=1}^{N} 1_{z_{ia}^1}][\sum_{i=1}^{N} 1_{z_{ib}^2}]}$$

For elements in V only involving the covariances between income and consumption growth, we further have

$$\hat{V}_{63:81,63:81} = \frac{\sum_{i=1}^{N} \sum_{t=1}^{T} 1_{x_{it}^1} 1_{x_{it}^2} 1_{x_{it}^3} 1_{x_{it}^4} [x_{it}^1 x_{it}^2 - \text{cov}(x^1 x^2)][x_{it}^3 x_{it}^4 - \text{cov}(x^3, x^4)]}{(\sum_{i=1}^{N} \sum_{t=1}^{T} 1_{x_{it}^1} 1_{x_{it}^2})(\sum_{i=1}^{N} \sum_{t=1}^{T} 1_{x_{it}^3} 1_{x_{it}^4})}$$

Finally, for the cross elements we have

$$\hat{V}_{1:62,63:81} = \frac{\sum_{i=1}^{N} 1_{z_{ia}^1} 1_{x_{ia}^3} 1_{x_{ia}^4} [z_{ia}^1 - \text{mean}(z_a^1)][x_{ia}^3 x_{ia}^4 - \text{cov}(x^3, x^4)]}{(\sum_{i=1}^{N} 1_{z_{ia}^1})(\sum_{i=1}^{N} \sum_{t=1}^{T} 1_{x_{it}^3} 1_{x_{it}^4})}$$

$$\hat{V}_{64:81,1:62} = \frac{\sum_{i=1}^{N} 1_{x_{ib}^1} 1_{x_{ib}^2} 1_{z_{ib}^2} [x_{ib}^1 x_{ib}^2 - \text{cov}(x^1 x^2)][z_{ib}^2 - \text{mean}(z_b^2)]}{(\sum_{i=1}^{N} \sum_{t=1}^{T} 1_{x_{it}^1} 1_{x_{it}^2})(\sum_{i=1}^{N} 1_{z_{ib}^2})}$$

C Additional Tables and Figures

C.1 CEQ

Table C.1: Estimates, CEQ Model ($\tau = 0.0$).

		Whole sample	No college	College
Parameter		(1)	(2)	(3)
σ_ϵ	Private signal (std.)	0.024	0.037	0.000
		(0.023)	(0.034)	(0.022)
σ_c	Meas. error, cons. (std.)	0.264	0.296	0.230
		(0.008)	(0.013)	(0.007)
σ_ψ	Persistent shock (std.)	0.165	0.172	0.158
		(0.005)	(0.008)	(0.008)
σ_ξ	Transitory shock (std.)	0.172	0.181	0.162
		(0.005)	(0.006)	(0.007)

Notes: Bootstrapped standard errors based on 5000 bootstrap replications reported in brackets.

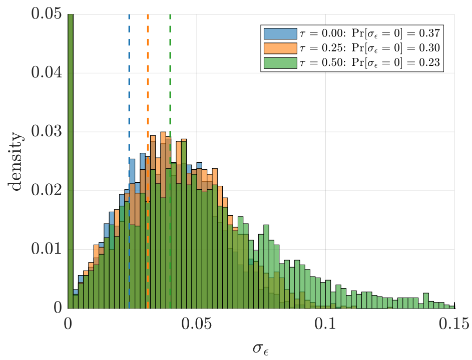
Table C.2: Estimates, CEQ Model ($\tau = 0.50$).

Parameter		Whole sample (1)	No college (2)	College (3)
σ_ϵ	Private signal (std.)	0.040	0.053	0.022
		(0.046)	(0.101)	(0.048)
σ_c	Meas. error, cons. (std.)	0.264	0.296	0.230
		(0.008)	(0.013)	(0.007)
σ_ψ	Persistent shock (std.)	0.165	0.172	0.158
		(0.005)	(0.008)	(0.008)
σ_ξ	Transitory shock (std.)	0.082	0.091	0.073
		(0.007)	(0.009)	(0.010)

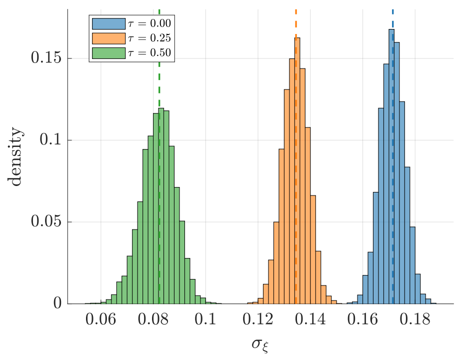
Notes: Bootstrapped standard errors based on 5000 bootstrap replications reported in brackets.

Figure C.1: Bootstrap, CEQ Model.

(a) Private signal (std.), σ_ϵ



(b) Transitory shock (std.), σ_ξ

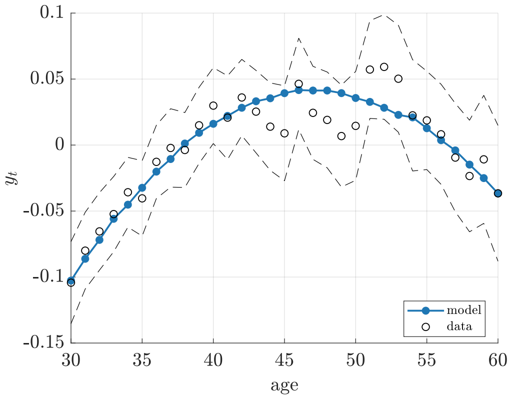


Notes: Figure C.1 reports histograms of estimates of σ_ϵ and σ_ξ for 5000 bootstrap replications and various assumption regarding the degree of measurement error in income, $\tau \in \{0.0, 0.25, 0.50\}$.

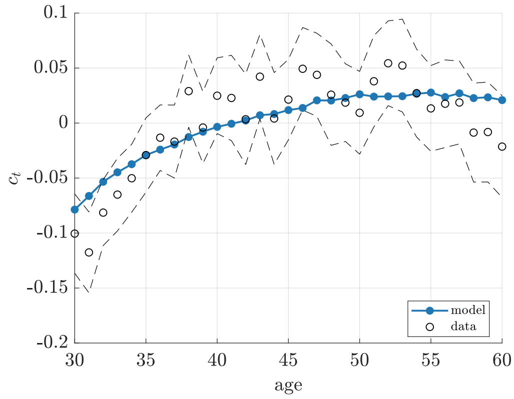
C.2 Fit

Figure C.2: Model Fit ($\alpha = 1$, $\theta = 0$).

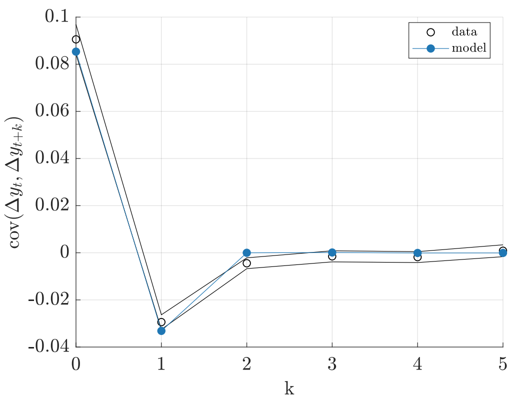
(a) Log Income, y_t



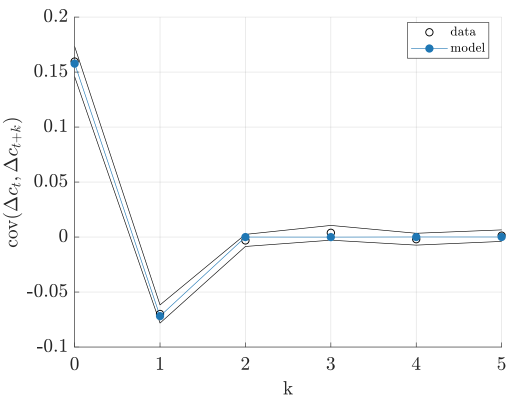
(b) Log Consumption, c_t



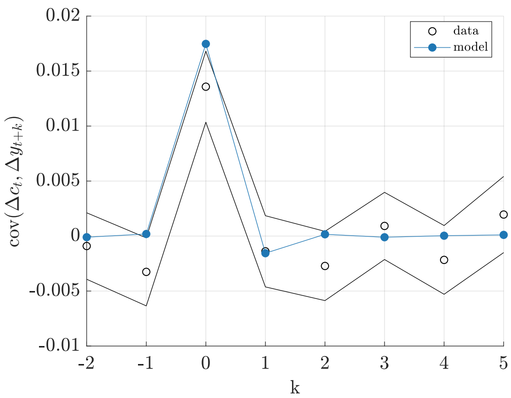
(c) $\text{cov}(\Delta y_t, \Delta y_{t+k})$



(d) $\text{cov}(\Delta c_t, \Delta c_{t+k})$

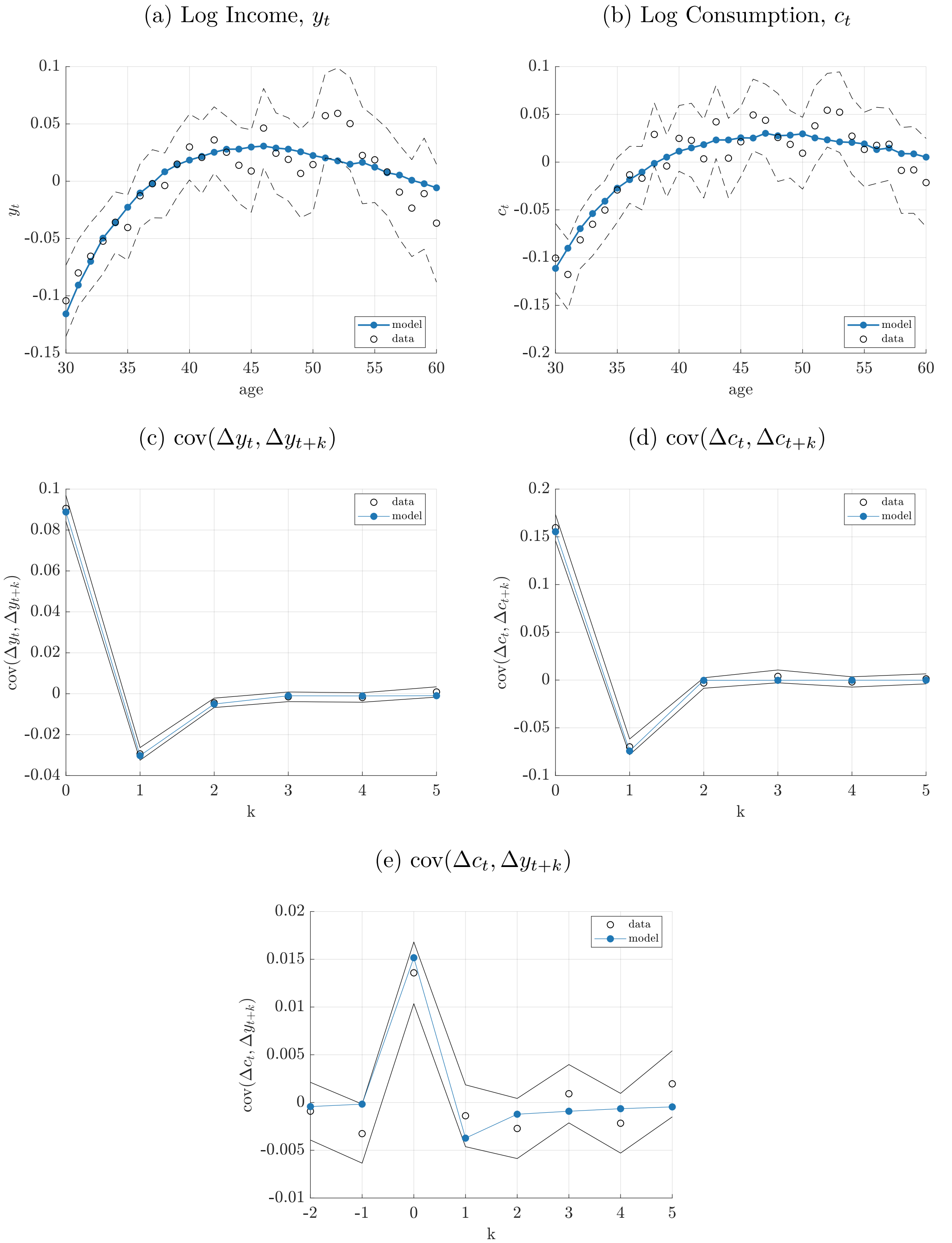


(e) $\text{cov}(\Delta c_t, \Delta y_{t+k})$



Notes: Figure C.2 illustrates the average age profiles of log income and log consumption together with the covariance moments. Both age profile series are normalized by the overall mean of each series. Hollow dots are calculated using the PSID, $\Lambda(\mathbf{w})$, solid lines are 95% confidence intervals, and solid dots are calculated using simulated data from the model, $\Lambda(\hat{\theta})$.

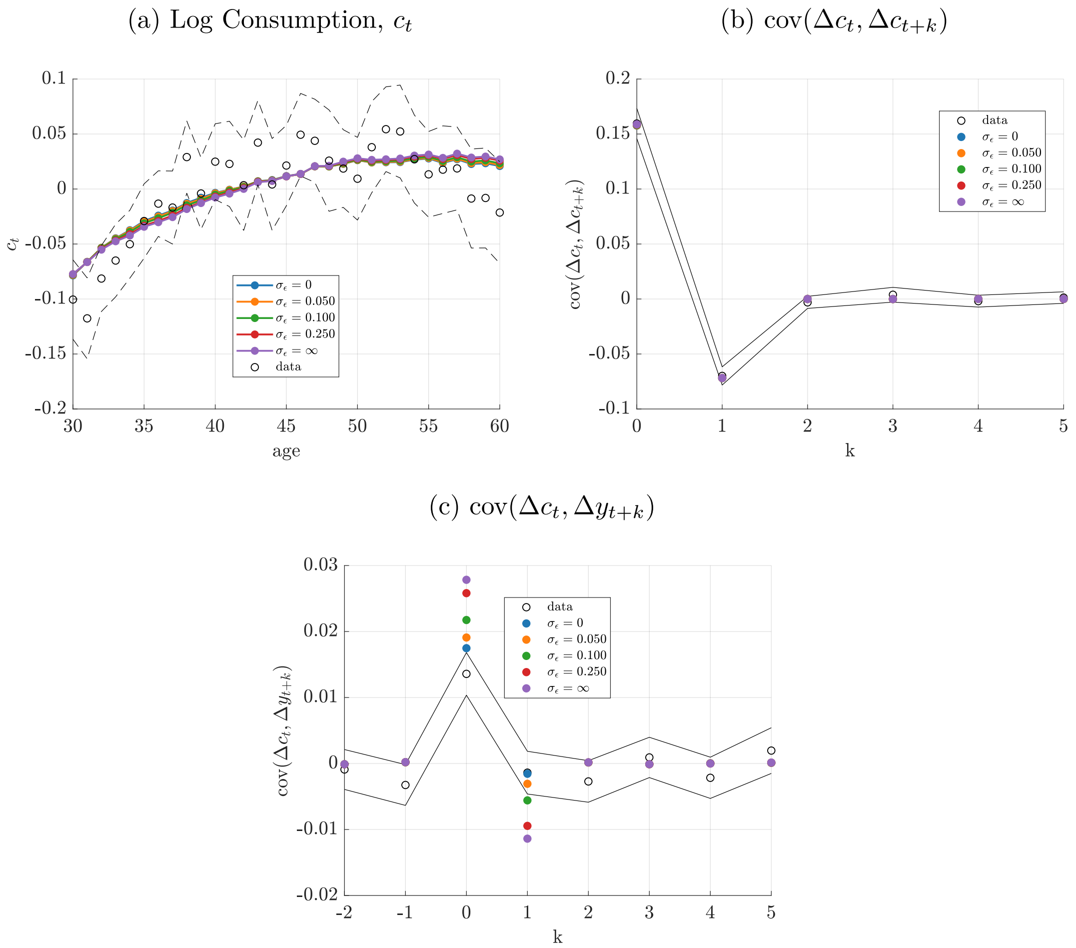
Figure C.3: Model Fit (α free, θ free).



Notes: Figure C.3 illustrates the average age profiles of log income and log consumption together with the covariance moments. Both age profile series are normalized by the overall mean of each series. Hollow dots are calculated using the PSID, $\Lambda(\mathbf{w})$, solid lines are 95% confidence intervals, and solid dots are calculated using simulated data from the model, $\Lambda(\hat{\theta})$.

C.3 Sensitivity

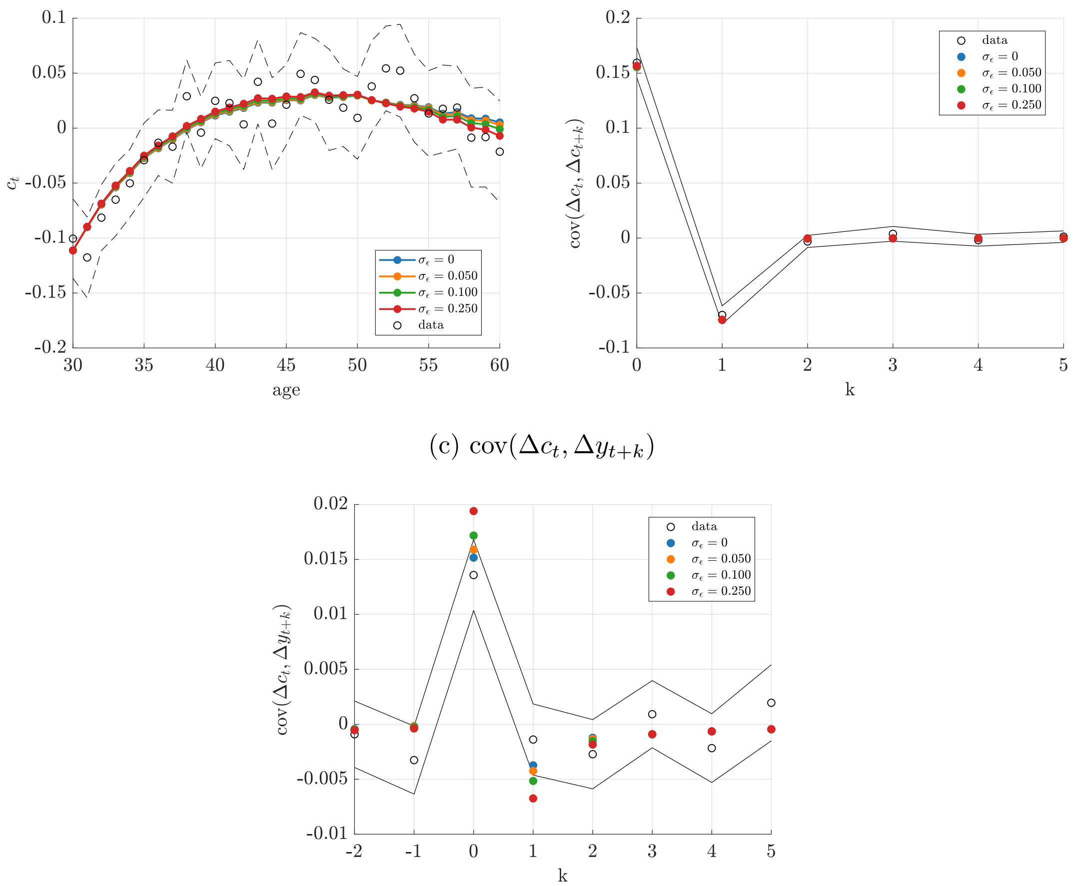
Figure C.4: Model Fit ($\alpha = 1$, $\theta = 0$).



Notes: Figure C.4 illustrates the average age profiles of log income and log consumption together with the covariance moments. Both age profile series are normalized by the overall mean of each series. Hollow dots are calculated using the PSID, $\Lambda(\mathbf{w})$, solid lines are 95% confidence intervals and solid colored dots are calculated using simulated data from the model, $\Lambda(\theta)$.

Figure C.5: Model Fit (α free, θ free).

(a) Log Consumption, c_t (b) $\text{cov}(\Delta c_t, \Delta c_{t+k})$ (c) $\text{cov}(\Delta c_t, \Delta y_{t+k})$



Notes: Figure C.4 illustrates the average age profiles of log income and log consumption together with the covariance moments. Both age profile series are normalized by the overall mean of each series. Hollow dots are calculated using the PSID, $\Lambda(\mathbf{w})$, solid lines are 95% confidence intervals and solid colored dots are calculated using simulated data from the model, $\Lambda(\theta)$.

C.4 Robustness

Table C.3: Estimates, Robustness ($\alpha = 1$, $\theta = 0$).

	Parameter	$\rho = 2.0$ (1)	$\rho = 4.0$ (2)	$\tau = 0.0$ (3)	$\tau = 0.5$ (4)	$\lambda = 0.25$ (5)	$\lambda = 0.75$ (6)	$\zeta = 0.10$ (7)	$\zeta = 0.20$ (8)	Diag. W (9)	Equal W (10)
σ_ϵ	Private signal (std.)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002
		–	–	–	–	–	–	–	–	–	–
β	Discount factor	0.947	0.877	0.959	0.903	0.959	0.956	0.959	0.959	0.958	0.928
		(0.002)	(0.008)	(0.001)	(0.007)	(0.001)	(0.002)	(0.001)	(0.001)	(0.001)	(0.009)
σ_c	Meas. error, cons. (std.)	0.268	0.269	0.269	0.265	0.269	0.268	0.268	0.268	0.268	0.249
		(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.008)
σ_ψ	Persistent shock (std.)	0.138	0.133	0.134	0.100	0.146	0.126	0.139	0.139	0.138	0.228
		(0.005)	(0.005)	(0.006)	(0.009)	(0.006)	(0.006)	(0.005)	(0.005)	(0.006)	(0.021)
σ_ξ	Transitory shock (std.)	0.148	0.149	0.183	0.105	0.145	0.151	0.147	0.147	0.148	0.132
		(0.003)	(0.003)	(0.003)	(0.005)	(0.004)	(0.003)	(0.003)	(0.003)	(0.003)	(0.017)
g_0	Income growth, constant	0.031	0.030	0.031	0.023	0.032	0.030	0.031	0.031	0.031	0.047
		(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.005)
g_1	Income growth, age	−0.096	−0.095	−0.096	−0.075	−0.093	−0.094	−0.096	−0.095	−0.096	−0.092
		(0.010)	(0.010)	(0.010)	(0.008)	(0.010)	(0.010)	(0.010)	(0.010)	(0.010)	(0.010)
α	AR(1) component	–	–	–	–	–	–	–	–	–	–
		–	–	–	–	–	–	–	–	–	–
ω	MA(1) component	–	–	–	–	–	–	–	–	–	–
		–	–	–	–	–	–	–	–	–	–
Objective		109.899	97.857	114.619	144.734	109.682	109.599	118.311	120.564	115.772	16.339
p-value for $\sigma_\epsilon = 0$		0.498	0.498	0.500	0.490	0.495	0.500	0.496	0.496	–	–

Notes: Asymptotic standard errors reported in brackets.

Table C.4: Estimates, Robustness (α free, θ free).

	Parameter	$\rho = 2.0$ (1)	$\rho = 4.0$ (2)	$\tau = 0.0$ (3)	$\tau = 0.5$ (4)	$\lambda = 0.25$ (5)	$\lambda = 0.75$ (6)	$\zeta = 0.10$ (7)	$\zeta = 0.20$ (8)	Diag. W (9)	Equal W (10)
σ_ϵ	Private signal (std.)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.005
		–	–	–	–	–	–	–	–	–	–
β	Discount factor	0.958	0.928	0.965	0.961	0.965	0.960	0.950	0.935	0.963	0.940
		(0.003)	(0.009)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.003)	(0.002)	(0.013)
σ_c	Meas. error, cons. (std.)	0.272	0.272	0.272	0.272	0.273	0.271	0.272	0.272	0.272	0.261
		(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.008)
σ_ψ	Persistent shock (std.)	0.150	0.149	0.141	0.158	0.171	0.115	0.149	0.131	0.150	0.217
		(0.010)	(0.009)	(0.010)	(0.009)	(0.010)	(0.009)	(0.009)	(0.009)	(0.010)	(0.030)
σ_ξ	Transitory shock (std.)	0.156	0.157	0.193	0.107	0.144	0.172	0.159	0.168	0.156	0.074
		(0.008)	(0.007)	(0.006)	(0.011)	(0.009)	(0.005)	(0.007)	(0.006)	(0.008)	(0.186)
g_0	Income growth, constant	0.061	0.056	0.059	0.062	0.069	0.041	0.047	0.030	0.061	0.057
		(0.013)	(0.011)	(0.013)	(0.012)	(0.014)	(0.007)	(0.008)	(0.004)	(0.013)	(0.007)
g_1	Income growth, age	−0.053	−0.056	−0.055	−0.052	−0.052	−0.074	−0.063	−0.080	−0.053	−0.070
		(0.015)	(0.014)	(0.015)	(0.014)	(0.014)	(0.013)	(0.013)	(0.012)	(0.015)	(0.015)
α	AR(1) component	0.885	0.896	0.886	0.884	0.878	0.936	0.919	0.982	0.883	0.935
		(0.029)	(0.028)	(0.030)	(0.027)	(0.027)	(0.026)	(0.024)	(0.020)	(0.029)	(0.025)
ω	MA(1) component	0.154	0.156	0.111	0.300	0.129	0.182	0.159	0.176	0.154	−0.500
		(0.035)	(0.035)	(0.025)	(0.066)	(0.044)	(0.027)	(0.033)	(0.029)	(0.035)	(3.809)
Objective		60.896	60.388	65.629	57.663	65.739	65.113	56.625	57.857	60.891	14.454
p-value for $\sigma_\epsilon = 0$		0.500	0.500	0.500	0.500	0.500	0.497	0.499	0.495	–	–

Notes: Asymptotic standard errors reported in brackets.

D Mathematica Calculations

Below we include output from Mathematica. These calculations involve a significant amount of tedious algebra and we have thus relegated these tasks to reliable software and report the results here. We refer to the relevant parts of the proofs in Appendix A.

```
Quit[]
```

The changes in notation are:

(1) all stars, ⋆, are omitted
(2) instead of e.g. σ_{ξ}^{2} we just have ξ
(3) the estimate of ϕ_{ψ} is denoted Φ_{ψ}

Lemma 1: sign of q and K derivatives

```
q[ϵ_] := (Sqrt[ξ ϵ/((ξ + ϵ) ψ) + 1/4] - 1/2) ψ;
FullSimplify[q'[ϵ] * (q[ϵ] / ψ + 1 / 2)]
fac[ϵ_] := (q[ϵ] + ψ)/((ξ + ϵ) (q[ϵ] + ψ) + ξ ϵ);
K1[ϵ_] := fac[ϵ] ϵ;
K2[ϵ_] := fac[ϵ] ξ;
K[ϵ_] := K1[ϵ] + K2[ϵ];
FullSimplify[K1[ϵ] / q[ϵ]]
FullSimplify[K2[ϵ] / q[ϵ]]
FullSimplify[K1'[ϵ] / q'[ϵ]]
Assuming[{ξ > 0, ψ > 0, ϵ > 0}, Refine[Reduce[q[ϵ] < Min[ϵ, ξ]]]]
Assuming[{ξ > 0, ψ > 0, ϵ > 0}, Refine[Reduce[Limit[q[ϵ], ϵ → ∞] < Min[ψ]]]]
Assuming[{ξ > 0, ψ > 0, ϵ > 0}, Refine[Reduce[q'[ϵ] > 0]]]
Assuming[{ξ > 0, ψ > 0, ϵ > 0}, Refine[Reduce[K1'[ϵ] > 0]]]
Assuming[{ξ > 0, ψ > 0, ϵ > 0}, Refine[Reduce[K2'[ϵ] < 0]]]
Assuming[{ξ > 0, ψ > 0, ϵ > 0}, Refine[Reduce[K'[ϵ] < 0]]]
```

$$\frac{\xi^2}{2\,(\epsilon+\xi)^2}$$

$$\frac{1}{\xi}$$

$$\frac{1}{\epsilon}$$

$$\frac{1}{\xi}$$

True

$$\psi > \frac{\xi}{2}$$

True

True

True

True

Corollary 3: sign of ϕ derivatives

```
ϕψ[ϵ_] := R^-1 (R - 1 + K1[ϵ] + K2[ϵ]);
ϕξ[ϵ_] := R^-1 (R - 1 + K1[ϵ]);
ϕϵ[ϵ_] := R^-1 K2[ϵ];
Assuming[{ξ > 0, ψ > 0, ϵ > 0, R > 0}, Refine[Reduce[ϕψ'[ϵ] < 0]]]
Assuming[{ξ > 0, ψ > 0, ϵ > 0, R > 0}, Refine[Reduce[ϕξ'[ϵ] > 0]]]
Assuming[{ξ > 0, ψ > 0, ϵ > 0, R > 0}, Refine[Reduce[ϕϵ'[ϵ] < 0]]]
```

```
True

True

True
```

Corollary 1: cov(Δc,Δc)

```
varc[ϵ_] := ϕψ[ϵ]^2 (q[ϵ] + ψ) + ϕξ[ϵ]^2 ξ + ϕϵ[ϵ]^2 * ϵ;
covc[ϵ_] := ϕψ[ϵ] (1 - K[ϵ]) q[ϵ] + ϕψ[ϵ] (1 - K[ϵ]) ψ - ϕξ[ϵ] K1[ϵ] ξ - ϕϵ[ϵ] K2[ϵ] ϵ;
FullSimplify[(varc[ϵ] - (ψ + ((R - 1)/R)^2 ξ)) / q[ϵ]]
FullSimplify[covc[ϵ]]
```

$$1 - \frac{1}{R^2}$$

```
0
```

Corollary 2: cov(Δc,Δy)

```
covcylead[ϵ_] := -ϕξ[ϵ] ξ;
covcy[ϵ_] := ϕψ[ϵ] ψ + (ϕξ[ϵ] + K1 [ϵ] ϕψ[ϵ]) * ξ;
covcylag[ϵ_, k_] := ϕψ[ϵ] (1 - K[ϵ])^k ψ - ϕψ[ϵ] K1 [ϵ] K[ϵ] (1 - K[ϵ])^(k-1) * ξ;
FullSimplify[(covcy[ϵ] - (ψ + (R-1)/R ξ)) / q [ϵ]]
FullSimplify[covcylag[ϵ, k]]
```

1

0

Corollary 4: Φ_ξ and Φ_ψ

```
Φξ[ϵ_] := covcylead[ϵ] / (-ξ);
Φψ[ϵ_] := (covcylag[ϵ, 0] + covcy[ϵ] + covcylead[ϵ]) / ψ;
FullSimplify[Φξ[ϵ] / ϕξ[ϵ]]
FullSimplify[Φψ[ϵ] / (ϕψ[ϵ] * (1 + (1 - K[ϵ]) + (1 - K[ϵ]) K1 [ϵ] ξ/ψ))]
```

1

1

Corollary 5: Derivatives of Φ_ξ and Φ_ψ

```
qbar[ϵ_] := ξ (2 q[ϵ] + ψ) / ((ϵ + ξ) (4 ϵ ξ + (ϵ + ξ) ψ));
FullSimplify[ϕξ'[ϵ] / qbar[ϵ]]
FullSimplify[Φψ'[ϵ] / qbar[ϵ]]
```

$$\text{qbar}[\epsilon\_] := \frac{\xi\,(2\,q[\epsilon] + \psi)}{(\epsilon + \xi)\,(4\,\epsilon\,\xi + (\epsilon + \xi)\,\psi)};$$

$$\text{FullSimplify}\left[\frac{\phi_\xi{}'[\epsilon]}{\text{qbar}[\epsilon]}\right]$$

$$\text{FullSimplify}\left[\frac{\Phi_\psi{}'[\epsilon]}{\text{qbar}[\epsilon]}\right]$$

$$\frac{1}{R}$$

$$\frac{(-1 + R)\,\xi}{R\,\psi}$$

Lemma 4: Estimate of ϵ given q

```
sol = Solve[q[ϵ] == y, ϵ]
Assuming[{ξ > 0, ψ > 0, ϵ > 0}, Refine[Reduce[q[ϵ]^2 + q[ϵ] ψ - ξ ψ < 0]]]
```

$$\left\{\left\{\epsilon \to \frac{-y^2\,\xi - y\,\xi\,\psi}{y^2 + y\,\psi - \xi\,\psi}\right\}\right\}$$

```
True
```

Corollary 6

```
f[q_, ξ_] := (-q^2 ξ - q ξ ψ) / (q^2 + q ψ - ξ ψ)
gcyp[ξ_] := f[-R * ζ - ξ (R - 1), ξ]
Assuming[{-R * ζ - ξ (R - 1) > 0, ξ > 0, ψ > 0, R > 1, ζ < 0,
  ((ξ - R (ζ + ξ))^2 - R (ζ + ξ) ψ)^2 ≠ 0}, Refine[FullSimplify[gcyp'[ξ] < 0]]]
Assuming[{ξ > 0, ψ > 0, R > 1, ∊ > 0},
 Refine[Reduce[(ξ - R (covcylead[∊] + ξ))^2 - R (covcylead[∊] + ξ) ψ < 0]]]
True

True
```