Do you have time to take a walk together?

Private and joint time within the household*

Martin Browning  Olivier Donni  Mette Gørtz

Subtitle (running head): PRIVATE AND JOINT TIME IN THE HOUSEHOLD

Abstract

We develop a theoretical model for the intra-household allocation of time and consumption that distinguishes between partners’ joint and private leisure. Estimating the model using time use data leads to five findings. First, the intra-household expenditure distribution correlates with relative wages, consistent with the collective model. Second, men put relatively more weight on private expenditure and composite leisure. Third, joint and private leisure are imperfect substitutes. Fourth, joint and private leisure are independent of the wage distribution, suggesting that togetherness does not substitute for economic factors. Fifth, higher female wages imply higher childcare hours for women, but lower for men.

Keywords: Intrahousehold; collective model; joint leisure

JEL classification: D1; D13; J1; J2; J22

* Corresponding author: Mette Gørtz, CEBI, University of Copenhagen, Øster Farimagsgade 5, 1353 Copenhagen K, Denmark. Email: mette.gortz@econ.ku.dk
Acknowledgements: Martin Browning and Mette Gørtz appreciate generous funding from the Carlsberg Foundation (2007-010462) and from the Danish National Research Foundation through its grant (DNRF-134) to the Center for Economic Behavior and Inequality (CEBI). Olivier Donni acknowledges the support of the consortium MME-DII (ANR11-LBX-0023-01) and the grant FAMINEQ (ANR-17-CE41-0007).
Economic models of marriage and household formation emphasize the role of marriage in organizing household specialization, sharing of public consumption goods, and risk sharing (Becker, 1994; Browning et al., 2013). This is in addition to marriage built on ideas of love and togetherness (Grossbard-Shechtmann, 2003; Hess, 2004). Empirical investigations in the economics of time use also acknowledge that an important part of the motivation for couples to cohabit or marry is the desire to spend time together (Hamer-mesh, 2002). Spending some time together not only generates direct utility to people, but may also serve as an important investment in marriage stability and a crucial component for child development. It is therefore interesting to establish in what way households’ choices of joint time varies with their economic circumstances.

Empirical models of time use have traditionally assumed a unitary household decision process in which decisions are implicitly taken by one decision maker who maximizes household welfare. In recent years, much of the modelling of intrahousehold decisions uses the collective model which simply assumes that household decisions, however they are arrived at, are efficient (Chiappori, 1988; Browning et al. 1994; Donni and Chiappori, 2011). Recent applications of the collective model treat the simultaneous allocation of leisure and consumption (Donni, 2007; Browning and Gørtz, 2012). Up to now, however, the distinction between time spent alone and time spent together has almost always been ignored. Individual time allocations are usually treated as decisions that affect individual utilities directly, while the time allocation of the partner only affects the spouse indirectly through the income raised or the public good produced by the partner’s work effort in the home or in the market, or through caring for the other partner’s felicity.¹

¹Exceptions are extremely rare. Fong and Zhang (2001) propose a collective model of labor supply in which they make a distinction between leisure spent alone and leisure
Thus the value of leisure or other uses of time spent away from market work – pure leisure, time spent caring for children or housework activities – does not depend on whether that time is spent alone, with the partner, with children or with friends and family.

The main contribution of our paper is that we develop a parametric structural model for the intrahousehold allocations of goods and time that explicitly incorporates the value of time together.\footnote{Blundell \textit{et al.} (2005) or Cherchye \textit{et al.} (2012) propose a collective model of time allocation with production activities, but they consider only one form of leisure.} We then show that the main structural components of the model are identifiable and present original empirical results. The model, which is an extension of Browning and Görts (2012), takes explicit account of heterogeneity within couples and between couples. To estimate the parameters of the model, we use a Danish time use survey from 2001. An important feature of our data is that we have information on ‘with whom’ time is spent. This allows us to take a closer look at time spent together in the couple. Compared to previous studies that have mainly relied on information on synchronous time (see discussion in section 2), our measure of joint time is a more direct measure of the time actually spent together in the couple. We thus distinguish between four uses of time: market work, housework, child care and leisure, the latter being broken down into private and joint leisure. Another important advantage of our data is that it includes information on the allocation of both time use and assignable goods in the household. This allows us to take account of trade-offs between time use and private and public consumption.

The paper is organized as follows. Section 2 gives a short summary of some previous empirical evidence of synchronization and jointness in time spent together by both partners but ignore the time used for domestic production. In addition they focus on a theoretical model without empirical evidence.
use. Section 3 develops and describes our theoretical model. Section 4 presents the scheme we employ for observable and unobservable heterogeneity, and we briefly discuss our indirect inference estimation method. Section 5 describes our data and presents the initial ‘reduced form’ estimates which are used as inputs in the form of auxiliary parameters for the structural estimation. Section 6 presents results and section 7 concludes.

1 Background

Individual utility derived from leisure time often benefits from others being present, either inside or outside the household. For married couples, a primary candidate for a complement to one’s own leisure is the time spent with the spouse (Hamermesh, 2000, 2002; Hallberg, 2003; Ruuskanen, 2004; Jenkins and Osberg, 2005). Apart from the classical economic benefits from marriage in the form of joint consumption of household public goods and intra-household division of labour (Lam, 1988; Becker, 1991; Weiss, 1997), enjoying the company of one another is certainly a prime reason for forming couples. Recent research on marriage formation and dissolution even suggests that the declining prevalence and stability of marriage in Western societies may be attributable to reduced production complementarities (i.e., reduced specialization). Instead, individual gains from marriage have become increasingly consumption-based (Lundberg, 2012; Stevenson and Wolfers, 2007). While production-based gains arise from economies of scale and returns to specialization in the household, consumption-based gains arise from risk pooling, joint consumption of household public goods (including children), and the direct utility of time spent together. Using American time use data (ATUS), Mansour and McKinnish (2014) find that couples with a smaller difference in working hours (i.e., a low degree of specialization) spend more time together,
which suggests they generate more gains from joint leisure. However, this result is not found for couples with young children.

Several empirical studies examine whether couples synchronize their time in market work, housework and leisure. Part of this synchronisation is due to an overall time synchronisation in society, that is, the organisation of the labour market, shop opening hours, television schedules etc. However, it seems that couples intentionally synchronize their activities to enjoy some time together. Hamermesh (2000, 2002) uses information on overlapping work hours from the US Current Population Surveys (CPS) from the 1970s to investigate how couples synchronize their market hours. As overlapping work hours say nothing about whether the overlap in spouses’ time at home is intentional, Hamermesh compares the actual distribution of work timing with a hypothetical time use as it would be in a ‘pseudo couple’ and shows that couples attempt to time their market work to provide themselves the opportunity to be together when they are not working. The method of constructing ‘pseudo couples’ is also used in Jenkins and Osberg (2005), using the British Household Panel Survey for survey years 1991–1999, and in Scheffel (2010) on German time use data. Hallberg (2003), using detailed data from time diaries of both spouses in couples taken from the Swedish survey on Household Market and Non-market Activities (HUS), examines the technique of using ‘pseudo couples’ further. Among other papers on joint time in couples are van Klaveren and van den Brink (2007), who use Dutch data, Bryan and Sevilla-Sanz (2014), who are using data from the British Household Panel Survey from 2003, and Cosaert et al. (2019) on Dutch data. Ruuskanen (2004) is among the few previous papers with access to direct survey information about with whom time was spent. Using Finnish time-use data, he finds that couples tend to spend around 20%-30% of their leisure together.
The previous empirical evidence does not fully agree on how joint time use co-varies with economic variables like income and individual wages. While Hamermesh (2000, 2002) found that time synchronization is rising with wage rates of husband and wife, Jenkins and Osberg (2005) find that work time synchronization is only rising in female wages, whereas the effect of male wages is insignificant. Hallberg (2003) finds that none of the economic variables have a significant impact on synchronous time, and Ruuskanen (2004) finds that the share of joint time of spouses is decreasing in wages. Scheffel (2010) finds that wages have opposing influences on joint time in leisure and other non-market activities for men and women. Finally, Bryan and Sevilla-Sanz (2014) find that synchronous working schedules are positively associated with men’s hourly wages and, for childless couples, with women’s hourly wages.

It seems, however, that there is a consensus on how the presence of children affects the timing of couples’ activities. Most previous empirical research finds a negative relationship between having young children and the amount of parents’ synchronous time (Hamermesh, 2000; Hallberg, 2003; Jenkins and Osberg, 2005; Ruuskanen, 2004; Scheffel, 2010; van Klaveren and Maasseen van den Brink, 2007; Bryan and Sevilla-Sanz, 2014).

2 Theoretical model

2.1 Individual utilities

We specify a parametric model of the allocation of time and goods in married and cohabiting couples which balances flexibility and empirical tractability. Each partner $s$ (with $s = a, b$) enjoys consumption of market goods; leisure; the output from household production and child utility. The total amount of
time \( T \) for partner \( s \) is divided between market work \( m_s \); private leisure \( l_s \); joint leisure \( L \); housework \( h_s \) and child care \( d_s \). The time budget constraint is:\(^3\)

\[
m_s + l_s + L + h_s + d_s = T. \tag{1}
\]

Total expenditure, \( X \), is spent on private assignable expenditures \( x_a \) and \( x_b \) and expenditures on a household public good, \( x_g \), that is an input into household production.\(^4\) The household budget constraint is given by:

\[
x_a + x_b + x_g = X. \tag{2}
\]

Households produce a home produced public good, \( G \), that is produced using inputs of housework from both partners and market purchases. The production function first combines the houseworks of the two spouses, \( h_a \) and \( h_b \), into a CES composite housework:

\[
h_g = \left\{ \alpha (h_a)^\varepsilon + (1 - \alpha) (h_b)^\varepsilon \right\}^{\frac{1}{\varepsilon}} \text{ with } \varepsilon \leq 1 \text{ and } \alpha \in (0, 1). \tag{3}
\]

The parameter \( \alpha \) governs the relative efficiency of partner \( a \) in housework.

The parameter \( \varepsilon \) determines the substitutability of the two houseworks with strong substitutability if \( \varepsilon \) is close to unity and strong complementarity if \( \varepsilon \) is very negative. The production function of the public good, \( G \), is taken to be a Cobb-Douglas form:

\[
G = h_g^\kappa * x_g^{(1-\kappa)} \text{ and } \kappa \in (0,1). \tag{4}
\]

The parameter \( \kappa \) determines the relative efficiencies of composite housework and market expenditures on the public good.

\(^3\)The total time we use is net of personal care which is fixed, in the empirical application, at seven hours per day for everyone.

\(^4\)Expenditures on children are included in total public expenditures in the household, \( x_g \).
In the utility function, we assume that each partner aggregates private and joint leisure into a composite leisure, denoted by $c_s$, with a CES form:

$$c_s = \left\{ \pi_s (l_s)^{\rho_s} + (1 - \pi_s) (L)^{\rho_s} \right\}^{\frac{1}{\rho_s}} \text{ with } \rho_s \leq 1 \text{ and } \pi_s \in (0, 1).$$  \hspace{1cm} (5)

The parameter $\pi_s$ governs how much partner $s$ likes private leisure as opposed to joint leisure and $\rho_s$ determines the substitutability of private and joint leisure.

To account for child care we distinguish between the number of young children ($ny$) and the number of older children ($nt$). We define an aggregator specific to partner $s$ as:

$$g_s (ny, nt) = (ny + \chi_s * nt)^\nu.$$  \hspace{1cm} (6)

where $\chi_s > 0$ allows that the mother and father may have different weights for spending time with different aged children. The parameter $\nu$ allows for economies of scale; we restrict $\nu$ to be the same across partners and to be between zero and unity. This aggregator is zero if there are no children present and increasing in the number of either age group of children.

The felicity function for partner $s$ is given by the addi-Box-Cox form of functions of $x_s$, $c_s$, $G$ and $d_s$:

$$u_s = \theta_s x_s^{1-\gamma_s} - \frac{1}{1 - \gamma_s} + \tau_s c_s^{1-\eta_s} - \frac{1}{1 - \eta_s} + \ln G + \delta_s \ln (1 + d_s) g_s (ny, nt).$$  \hspace{1cm} (7)

with $\theta_s > 0, \gamma_s > 0, \tau_s > 0, \eta_s > 0, \delta_s > 0$. The parameter $\theta_s$ determines how much partner $s$ cares about private consumption relative to the public good. The parameter $\gamma_s$ governs whether, relative to the home produced (public) good, the private assignable good for partner $s$ is a luxury ($\gamma_s < 1$) or a necessity ($\gamma_s > 1$). The limiting case as $\gamma_s \to 1$ gives that private consumption has a unit income elasticity.

The parameters $\eta_s$ and $\tau_s$ govern preferences for composite leisure relative to the public good with a higher value of $\tau_s$ giving a higher preference for
leisure (or, conversely, a higher distaste for total work). The parameter $\eta_s$ gives the curvature of partner $s$’s felicity function in composite leisure. If $\eta_s$ is high then leisure is unresponsive to wages and the level of composite leisure is largely determined by $\tau_s$; that is, the variation in leisure is mostly due to heterogeneity rather than state dependence. If there is no home production or children then $\eta_s$ is the inverse of the Frisch labour supply elasticity. With home production and children, however, it is entirely possible that market work, housework and child care are responsive to wage changes even though leisure is largely unaffected.

The direct inclusion of child care time spent by partner $s$ in the felicity function of $s$ reflects the notion that time spent with children in our dual-earner families is producing utility for that individual. This is also supported by Gørtz (2011) and Aguiar and Hurst (2007) who note that when individuals are asked in surveys to assess the satisfaction they receive from various activities, they consistently report that child care is more enjoyable than ordinary housework activities. The parameterisation for child care is chosen so that no weight is given to child care if there are no children present in which case the optimal child care $d_s$ is zero. Our accounting for time spent on child care represents something of a compromise. Ideally we would specify a child utility function along the lines of the home production function and include that outcome in the two utility functions. We do not have enough information to identify these components, hence the inclusion of a direct composite child time term. The term we include represents a combination of how much $s$ likes to spend time with the child; how effective $s$ is in producing child outcomes and how much $s$ values the child outcome.
2.2 The household utility function

We assume that the outcome of the decision process is Pareto efficient so that household utility can then be represented as a weighted function of the individual felicity functions:

\[ u = u_a + \mu u_b, \mu \geq 0. \]  

(8)

The \( \mu \) parameter is the Pareto weight which picks up how much influence \( b \) has on household decision making. In our full model we allow that the Pareto weight depends on observables such as wages and education so that we have a non-unitary model. Whilst it is tempting to attribute some significance to a value of unity for \( \mu \) (‘equal weights’), this temptation is to be resisted. If the preferences of men and women differ then we cannot make interpersonal comparisons and conclude that, for example, a value of \( \mu > 1 \) means that he is better off than she is. This will be clear if, instead of normalising the marginal utility of the home produced good to unity for both partners, we normalised on, say, the marginal utility of private consumption. That is equivalent to dividing her utility by \( \theta_a \) and his by \( \theta_b \); to keep the same household preferences (and the same observable choices) we have to take a transformation of the Pareto weight by \( \tilde{\mu} = \mu \theta_b / \theta_a \). This is not equal to \( \mu \) unless the two have the same weight for the private good. Similarly for normalising on composite leisure.

We set total expenditure equal to net earnings; this is a strong assumption that is necessitated by the absence of any expenditure information in our data, except for private expenditures, \( x_a \) and \( x_b \). Gross wages are denoted by \( w_a \) and \( w_b \). Given individual filing for taxes, we assume a tax system that maps gross earnings for each person into net earnings. If the tax function for
partner $s$ is denoted by $f(w_s m_s)$, the household budget constraint is:

$$X = f(w_a m_a) + f(w_b m_b) = \Phi(m_a, m_b; w_a, w_b).$$  \hfill (9)

Substituting the household budget constraint (9) and the individual time constraints (1) in the household production function gives:

$$\ln G = \kappa \ln \left( \left\{ \frac{\alpha(T - m_a - l_a - L - d_a)^{\varepsilon}}{(1 - \alpha)(T - m_b - l_b - L - d_b)^{\varepsilon}} \right\}^{1/\varepsilon} \right) + (1 - \kappa) \ln (\Phi(m_a, m_b) - x_a - x_b).$$  \hfill (10)

Substituting this expression in the felicity functions and then in the household utility function gives:

$$u(x_a, x_b, m_a, m_b, l_a, l_b, L, d_a, d_b) = \theta_a x_a^{1 - \gamma_a} - 1 + \mu \theta_b x_b^{1 - \gamma_b} - 1 + \frac{\tau_a}{1 - \eta_a} \left( \left\{ \pi_a (l_a)^{\rho_a} + (1 - \pi_a) (L)^{\rho_a} \right\}^{\frac{1 - \eta_a}{\rho_a}} - 1 \right) + \frac{\mu \tau_b}{1 - \eta_b} \left( \left\{ \pi_b (l_b)^{\rho_b} + (1 - \pi_b) (L)^{\rho_b} \right\}^{\frac{1 - \eta_b}{\rho_b}} - 1 \right) + \delta_a \ln (1 + d_a) g_a (ny, nt) + \mu \delta_b \ln (1 + d_b) g_b (ny, nt) + (1 + \mu) \ln G(x_a, x_b, m_a, m_b, l_a, l_b, L, d_a, d_b).$$  \hfill (11)

This is maximised by a choice of $(x_a, x_b, m_a, m_b, l_a, l_b, L, d_a, d_b)$. The parameters of the model for an individual household are:

$$\mu, \alpha, \kappa, \varepsilon \text{ and } (\theta_s, \gamma_s, \tau_s, \eta_s, \pi_s, \rho_s, \delta_s) \text{ for } s = a, b.$$  \hfill (12)

Our specification balances flexibility and empirical tractability. In particular, the felicity function nests the traditional collective model (which ignores
the distinction between private and joint leisure) as a particular case when \( \rho_s = 1 \) (perfect substitutability of private and joint leisure) and \( \pi_s = 1/2 \). Another important item to note is that the additivity assumption does not imply that preferences over labour supply and consumption are separable. Additivity with home production implies that market work and market expenditures are complements which is in accord with the empirical evidence; see Browning et al. (1999) for a discussion. Finally, as explained in the next subsection, the separability of private and joint leisure assumed in equation (5) is sufficient to identify the structural parameters of the model.

Since we shall be referring to three distinct sorts of parameters, we refer to the parameters presented in (12) as model parameters. For convenience we display the model parameters and their interpretation in Table 1. The final two columns show whether the parameters for our preferred set of estimates below display observable and/or latent variation. For example, we found that we did not need to allow for observable heterogeneity in \( \pi_a \) and \( \pi_b \) once we took latent heterogeneity into account. Conversely, once we allowed for observable heterogeneity in \( \rho_a \) and \( \rho_b \) there was no need for latent heterogeneity. All parameters for individual model parameters (that is, parameters with an \( s \) subscript) vary with gender.

### 2.3 Identification

As it can be easily checked, all the parameters of the model can be identified from the estimation of behavioural equations. Whether such identification is specific to our functional form or is valid for large families of utility functions is another issue that must be clarified, though. The question is whether the observation of household behaviour, that is, the choice variables expressed as a smooth function of state variables, uniquely defines preferences and the Pareto weight. The answer is complicated because only one relative price by individual (the wage) is variable,
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range</th>
<th>Interpretation</th>
<th>Heterogeneity</th>
<th>Observable</th>
<th>Latent</th>
</tr>
</thead>
<tbody>
<tr>
<td>θ_s</td>
<td>(0, ∞)</td>
<td>Private consumption weight</td>
<td>Yes</td>
<td>Yes</td>
<td></td>
</tr>
<tr>
<td>τ_s</td>
<td>(0, ∞)</td>
<td>Composite leisure weight</td>
<td>Yes</td>
<td>Yes</td>
<td></td>
</tr>
<tr>
<td>π_s</td>
<td>(0, 1)</td>
<td>Private leisure weight in composite</td>
<td>No</td>
<td>Yes</td>
<td></td>
</tr>
<tr>
<td>δ_s</td>
<td>[0, ∞)</td>
<td>Child care weight</td>
<td>Yes</td>
<td>Yes</td>
<td></td>
</tr>
<tr>
<td>γ_s</td>
<td>(0, ∞)</td>
<td>Consumption curvature</td>
<td>No</td>
<td>No</td>
<td></td>
</tr>
<tr>
<td>η_s</td>
<td>(0, ∞)</td>
<td>Composite leisure curvature</td>
<td>No</td>
<td>No</td>
<td></td>
</tr>
<tr>
<td>ρ_s</td>
<td>(−∞, 1)</td>
<td>Leisure complementarity</td>
<td>Yes</td>
<td>No</td>
<td></td>
</tr>
<tr>
<td>μ</td>
<td>(0, ∞)</td>
<td>Pareto weight for partner b</td>
<td>Yes</td>
<td>No</td>
<td></td>
</tr>
<tr>
<td>α</td>
<td>(0, 1)</td>
<td>Housework efficiency of a</td>
<td>Yes</td>
<td>No</td>
<td></td>
</tr>
<tr>
<td>ε</td>
<td>(−∞, 1)</td>
<td>Housework complementarity</td>
<td>No</td>
<td>No</td>
<td></td>
</tr>
<tr>
<td>κ</td>
<td>(0, 1)</td>
<td>Efficiency of housework</td>
<td>No</td>
<td>No</td>
<td></td>
</tr>
<tr>
<td>χ_s</td>
<td>(0, ∞)</td>
<td>Weight on age in child care</td>
<td>No</td>
<td>No</td>
<td></td>
</tr>
<tr>
<td>ν</td>
<td>(0, 1)</td>
<td>Child care scaling</td>
<td>No</td>
<td>No</td>
<td></td>
</tr>
</tbody>
</table>

Note: All parameters with an s subscript vary with gender.

which prevents us from recovering individual utility functions in a completely general framework. To obtain our result, we thus exploit the fact that private and public leisures are aggregated into a composite leisure. More precisely, we first ignore child care time (e.g., we consider a childless couple) and prove that, if felicity functions have a general form such as $u_s = u_s(x_s, G(h_a, h_b, X), c_s(l_s, L))$, and if regularity conditions are satisfied, the main structural components of the model can be identified. The formal proof of this result is a little technical and is relegated to Appendix A. The main idea goes as follows. First, it is shown that the home produced good $G$ and the composite leisure $c_s$ can be identified.
as functions of \((w_a, w_b, y)\) up to monotonic transformations from the observation of \((h_a, h_b, X)\) and \((l_s, L)\), respectively. The choice of the transformations is arbitrary and irrelevant in terms of welfare analysis, but it affects the marginal rate of substitution between \(x_s\), on the one hand, and \(G\) and \(c_s\), on the other hand. Secondly, it is shown that, once these transformations have been picked up, individual felicity functions can be identified up to a monotonic transformation from the observation of \((x_s, G, c_s)\). This identification result is based, in particular, on the condition that private leisure is an essential good, that is, whatever its price, its consumption is necessarily positive. This property is satisfied by the CES specification. For the more general case, we then consider felicity functions of the form:

\[
u_s = u_s(x_s, G(h_a, h_b, X), c_s(l_s, L)) + v_s(d_s).
\]

Identification of the function \(v_s(d_s)\) is then a straightforward consequence of the results that precede. In addition, identification can, in principle, be extended to an arbitrary number of different time uses as long as they enter additively in the individual utility functions.

3 Heterogeneity

Our next step is to allow for heterogeneity in the parameters in (12). We distinguish between observed heterogeneity and unobserved heterogeneity. We also make a distinction between within (household) heterogeneity and between (household) heterogeneity. Let \(i\) index a household so that \(a_i\) refers to the wife in household \(i\) (and \(b_i\) to the husband). For example, taking the parameter \(\theta\), within heterogeneity allows that:

\[
\theta_{a_i} \neq \theta_{b_i}, \tag{13}
\]

and between (women) heterogeneity gives that, for households \(i\) and \(j\):

\[
\theta_{a_i} \neq \theta_{a_j}. \tag{14}
\]

This structure allows us to consider that, for example, all women have the same preferences for private goods relative to the public good, \(\theta_{a_i} = \theta_a\) for all \(i\), (between
homogeneity) and all men have the same $\theta_b$ but these differ, $\theta_a \neq \theta_b$ (within heterogeneity).

We take account of observable heterogeneity in the education levels of the two partners and the numbers and ages of children in the household. Denote the vector of demographics (including a constant) for household $i$ by $z_i$. We model the dependence using index models with parameterisations to impose the range constraints from Table 1:

$$
\theta_{si} = \exp(z_i'\beta_{\theta_s}), \\
\tau_{si} = \exp(z_i'\beta_{\tau_s}), \\
\delta_{si} = \exp(z_i'\beta_{\delta_s}), \\
\rho_{si} = 1 - \exp(z_i'\beta_{\rho_s}), \\
\alpha_i = \ell(z_i'\beta_\alpha), \\
\mu_i = \exp(z_i'\beta_\mu + \beta_{\mu1} \ln(w_a) + \beta_{\mu2} \ln(w_b)).
$$

(15)

for $s = a, b$ and $\ell(y) = e^y/(1 + e^y)$. The interpretation of the coefficients in the education parameters in $z_i$ is that these reflect time invariant differences in preferences. The interpretation of the children coefficients is more subtle since they represent dependence on a time varying state. Since the child care aggregator in equation (6) takes account of the presence of children, we do not allow for variation in the child care coefficients, $\delta_s$, with children, only with education. Finally, we allow that the Pareto weight depends on the (log) wages and education, but not children.

To capture unobserved heterogeneity we posit a nonlinear factor structure. Let $N_1,..,N_6$ be six standard Normals, each independent of the other and all other variables.\footnote{A preliminary specification search indicated that six factors is sufficient.} Denote the draw from $N_k$ for household $i$ by $N_{ki}$. We supplement the scheme in (15) for those parameters that display latent heterogeneity in Table 1,
using a truncated triangular scheme:

\[
\begin{align*}
\theta_a &= \exp \left( z_i' \beta_{\theta_a} + \exp (\sigma_{\theta a 1}) N_{1i} \right), \\
\theta_b &= \exp \left( z_i' \beta_{\theta b} + \sigma_{\theta b 1} N_{1i} + \exp (\sigma_{\theta b 2}) N_{2i} \right), \\
\tau_a &= \exp \left( z_i' \beta_{\tau a} + \sigma_{\tau a 1} N_{1i} + \sigma_{\tau a 2} N_{2i} + \exp (\sigma_{\tau a 3}) N_{3i} \right), \\
\tau_b &= \exp \left( z_i' \beta_{\tau b} + \sigma_{\tau b 1} N_{1i} + \sigma_{\tau b 2} N_{2i} + \sigma_{\tau b 3} N_{3i} + \exp (\sigma_{\tau b 4}) N_{4i} \right), \\
\pi_a &= 0.2 + 0.6 \ell (\beta_{\pi a 0} + \sigma_{\pi a 1} N_{1i} + \ldots + \exp (\sigma_{\pi a 5}) N_{5i}), \\
\pi_b &= 0.2 + 0.6 \ell (\beta_{\pi b 0} + \sigma_{\pi b 1} N_{1i} + \ldots + \sigma_{\pi b 5} N_{5i} + \exp (\sigma_{\pi b 6}) N_{6i}), \\
\delta_a &= \exp \left( z_i' \beta_{\delta a} + \sigma_{\delta a 1} N_{1i} + \ldots + \sigma_{\delta a 5} N_{5i} + \sigma_{\delta a 6} N_{6i} \right), \\
\delta_b &= \exp \left( z_i' \beta_{\delta b} + \sigma_{\delta b 1} N_{1i} + \ldots + \sigma_{\delta b 5} N_{5i} + \sigma_{\delta b 6} N_{6i} \right). 
\end{align*}
\] (16)

The parameters to be estimated are the vectors \((\beta_{\theta_a}, \ldots, \beta_{\mu})\), the factor loadings \((\sigma_{\theta a 1}, \ldots, \sigma_{\delta b 6})\) and the Pareto weight wage parameters \((\beta_{\pi a 0}, \beta_{\pi a 1})\). Since these govern the distribution of the model parameters we refer to them as *distribution parameters*. This factor scheme allows for a rich dependence between unobserved preference parameters. For instance, if we find \(\sigma_{\theta b 1} > 0\) then there is a positive correlation between the latent components of preferences of husbands and wives for private consumption. This scheme also provides simple tests for within and between homogeneity.

We estimate the distribution parameters given in this section using indirect inference. This requires a specification of an auxiliary model that is estimated on the data and simulated data. The parameters of the auxiliary model are referred to as *auxiliary parameters* (ap’s). Indirect inference proceeds by minimising the weighted distance between the data ap’s and the simulated ap’s. For weights we take the inverse of the covariance matrix of the data ap’s.
4 Data

Our data are from the Danish Time Use Survey (DTUS) from 2001. In the DTUS, each respondent was asked to fill in a time diary stating his/her activities in two 24 hour periods, one a week-day and the other a weekend day. Respondents were asked to report their activities by 10 minute intervals choosing from a detailed list of activities. For married/cohabiting respondents, the partner/spouse was given a similar time-diary. For each time interval, all respondents (and partners) were asked with whom the activity was carried out; whether it was an activity carried out with the partner/spouse, one of the children, others from the household or others from outside the household. Furthermore, the primary respondent of each household was given a questionnaire (by interview). The questionnaire has two important features which we use in our empirical analysis. Firstly, the main respondent was asked to state usual expenditures on three groups of assignable goods for him/herself and the partner/spouse. Very few surveys provide detailed information on both time use and consumption of assignable goods for both partners in the household. Secondly, the respondent was asked to provide information on usual hours worked in the labour market, at home and commuting for both him/herself and the spouse. Finally, the questionnaire covers a wide range of background variables for family background, marital status, children, income, housing, education, and employment status at the time of the interview.

\[\text{The DTUS survey is representative of the Danish population and complies with methodologies developed at the EU level for conducting time use surveys; see Bonke (2005) for a detailed description.}\]

\[\text{An assignable good is a private good for which we can observe how much each partner consumes.}\]

\[\text{The module was designed by Jens Bouke and Martin Browning in collaboration with Statistics Denmark who ran the survey. Browning et al. (2003) present a discussion of the pros and cons of using information on ‘usual’ expenditures from general purpose surveys. The broad conclusion from their analysis is that although survey recall expenditure measures are noisy as compared to diary measures, they do contain a useful signal.}\]
The survey data has been linked to administrative register information from Statistics Denmark. This gives us access to a vast amount of background information on each respondent and spouse for 2001 and for some years preceding the survey year. An attractive feature of the register data is that it contains a wage measure for employed individuals that is constructed independently of the time use collected in the survey and is consequently free from division bias; details are given in Appendix B.

Our sample consists of married or cohabiting couples for whom we have complete information on both partners’ time use for both a weekday and a weekend-day, questionnaire information on assignable expenditure and register information. We treat married and cohabiting couples as one group as cohabitation is widespread in Denmark.\textsuperscript{9} In addition, we focus on households where both spouses are 23–64 years old and worked at the time of the survey, so we exclude households where the survey or the register data told us that one of the spouses was a student, on early retirement, disability pension, long-term sick leave, maternity leave, unemployed or participating in an activation programme for the unemployed. We focus on dual earner households where the husband works at least 30 hours per week. Labour force participation and employment rates are high in Denmark for both men and women, in 2001, employment rates among people in working age was 81% for men and 72% for women. Finally, we also exclude households with missing information on expenditure for one of the spouses or where assignable expenditures were zero (around 5% of the households). This leads to a sample of 551 households.\textsuperscript{10}

\textsuperscript{9}In 2001, one in four Danish couples were cohabiting rather than married, and the share of cohabiting parents was even higher.

\textsuperscript{10}Our sample of 551 observations is selected from time use data on 1000 couples with two spouses of different gender. First, we drop elderly over 65 → 866 observations. Second, we drop if one of spouses is on early retirement or disabled → 792 observations. Third, we drop if one partner is student or apprentice → 716 observations. Fourth, we drop if one of spouses is unemployed or in activation → 665 observations. Fifth, we drop if on long-term sickness → 658 observations. Sixth, we drop if on maternity leave or other leave
4.1 Time use

The DTUS provides us with information on usual time use from the questionnaire and time use on two specific days (a weekday and a weekend day) in the time diary. We combine these two sources of information to obtain information on total time use for the two spouses. We have chosen to use usual time use rather than the diary information to avoid the infrequency problems in the latter. In general, surveys asking about usual time use have a smaller variance but larger bias than diary studies, see Juster and Stafford (1991) and Bonke (2005). Frazis and Stewart (2012) argue that it is problematic to base inference of individual long-run time allocation on observations of short-run time use from time diaries.

We construct variables for market work of the two spouses combining information from the questionnaire, the diaries and the register data. The questionnaire, which gives information on usual market hours, is our primary source of information, providing information on market work for more than 90% of our sample, while information from the time diaries in combination with information on the registers provides information on market work for the remaining 10%. Our measure of market work includes commuting. The questionnaire is also our primary source of information for housework. Housework time is specified to include normal housework such as cleaning, laundry, shopping, cooking etc. and also gardening, repairs, other do-it-yourself work and transport of children, but not child care. The construction of variables for market work and housework is explained further in Appendix C.

Child care is constructed by combining information from the questionnaire and the diaries. We calculate residual home time (leisure and child care) for each man and woman as the residual of total number of hours (excluding 7 daily hours of sleep and personal care, i.e. a total of \(7 \times 17 = 119\) hours per week) minus total

\[\rightarrow 624\] observations. Seventh, we drop if no sensible consumption or time use information \[\rightarrow 572\] observations. Eight, we drop if not full-time (according to our definition) \[\rightarrow 564\] observations. Ninth, we drop if zero hourly wage \[\rightarrow 551\] observations.
market hours minus total housework:

\[
\text{Residual home time} = (7 \times 17) - \text{market work} - \text{housework}.
\]

We divide this residual home time into time spent on child care and in (pure) leisure combining information from the diary about time spent on specific child care related activities with information from the questionnaire about usual (residual) home time. The diary contains information on caring for children, reading to children and accompanying children to activities as well as (pure) leisure activities. We use the split between child care and leisure in the diary to calculate our measures of child care and pure leisure based on questionnaire based total residual home time.

For pure leisure, our model distinguishes between private leisure of the two spouses, which is the leisure time they spend without their spouse (alone or with others, e.g. children, friends etc.), and joint leisure, which is time spent with the spouse (and possibly others). This information is taken from the diary about with whom the time was spent. For each individual, we calculate the proportion of their (diary) leisure time that was spent with their spouse. This proportion is combined with our measure of residual leisure calculated above to create a measure of joint leisure of the two spouses. Since both partners in the household independently reported the time they spent with their spouse, these two sources of information may actually give different reports of joint leisure. If the two partners reported differently on their joint leisure, we use the lowest number of hours of joint leisure reported by the two partners in the household. In general, men and women report fairly equal numbers of hours with joint leisure, and for most couples, the reporting difference is not large; the median difference in reported joint leisure is less than 1%. The average relative reporting difference, however, is around 10% (if we ignore top and bottom 5%).\footnote{Spouses’ reports on joint leisure can be different for a number of reasons: First, time diaries for husband and wife may have been collected for different weekdays and weekend} We define the difference between total leisure and joint leisure as private leisure.
Table 2 shows the time usage of couples. The familiar picture appears that while men work more in the market than women, they do less housework. Child care hours are an average of all households in sample including couples without children (which by definition have zero child care time). Child care activities include specific care-oriented activities as well as accompanying children to activities, but not more leisure-oriented activities such as e.g. eating, watching tv with children. Men have on average two hours more private leisure than women. Couples divide their total leisure more or less in two, spending a little more than half of the time as private leisure (i.e. without their spouse, but alone, with children, with friends or others) while the other half of their leisure is spent with their spouse.

<table>
<thead>
<tr>
<th></th>
<th>Females</th>
<th>Males</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market work ((m))</td>
<td>38.2 (7.3)</td>
<td>43.6 (7.9)</td>
</tr>
<tr>
<td>Housework ((h))</td>
<td>18.0 (8.4)</td>
<td>12.9 (7.3)</td>
</tr>
<tr>
<td>Child care ((d))</td>
<td>5.2 (9.3)</td>
<td>2.7 (5.2)</td>
</tr>
<tr>
<td>Private leisure ((l))</td>
<td>30.3 (15.0)</td>
<td>32.5 (15.2)</td>
</tr>
<tr>
<td>Joint leisure ((L))</td>
<td>27.3 (14.2)</td>
<td>27.3 (14.2)</td>
</tr>
</tbody>
</table>

**Note:** Standard deviations in brackets. \(m + h + d + l + L = 119\)

days. This was the case for only a handful of observations. Secondly, differences may be due to measurement error, as the time diary (and in particular the ‘with whom questions’) may have been filled in incorrectly by one (or both) of the spouses. Thirdly, spouses may have different perceptions regarding when the two have actually been ‘together’. For example, it is not obvious whether two persons are together if they are in the same room, but doing different things or if the question was understood in such a way that being together means spending some time together sharing the same activity.
4.2 Personal expenditures

The primary objective of the DTUS was to collect information on time use but the survey also collected some information on personal expenditures. The following questions were asked of the main respondent in the household: ‘When you think of your own personal expenditures, how much do you estimate it is normally on the following items during one month’:

- ‘Clothing and shoes’.
- ‘Leisure activities, hobbies etc.’ (e.g. sports, sports equipment and club memberships).
- ‘Other personal consumption’ (e.g. cigarettes, perfumes, games, magazines, sweets, bars and cinema).

The respondent was then asked the same questions for the spouse or cohabitant. We have verified the validity of these responses using the Danish Household Expenditure Survey (DHES). The DHES is a conventional diary based survey of expenditures with the unconventional feature that married respondents keeping an expenditure diary record who each item was bought for (‘the household’, ‘husband’, ‘wife’, ‘children’ and ‘other’). Comparing the information in our survey (the DTUS) and the DHES, we find that total expenditures on goods bought for the husband and wife (total assignable expenditures) in the DTUS are equal to about two thirds of the corresponding expenditures in the DHES and that the wife’s mean share of assignable expenditure is close to that computed from the DHES.

Table 3 presents the mean reported expenditures for husbands and wives. As can be seen, wives spend more (in mean) on clothing and less on recreation and other goods. The mean total figures are very similar. Despite the coincidence of means, the two total measures are quite dispersed with some households having high relative shares for one or other of the spouses.
Table 3: *Expenditures on individual goods for wife and husband*

<table>
<thead>
<tr>
<th></th>
<th>Wife</th>
<th>Husband</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clothing</td>
<td>540 (429)</td>
<td>370 (295)</td>
</tr>
<tr>
<td>Recreation</td>
<td>203 (339)</td>
<td>307 (486)</td>
</tr>
<tr>
<td>Other</td>
<td>455 (572)</td>
<td>470 (614)</td>
</tr>
<tr>
<td>Total</td>
<td>1198 (902)</td>
<td>1146 (936)</td>
</tr>
</tbody>
</table>

*Note:* All values are DKK/month. Standard deviations in parentheses.

## 5 Auxiliary parameters

In this section, we present an extended description of our sample. Using indirect inference to estimate the distribution parameters in our model requires that we specify a set of auxiliary parameters (ap’s) which provide a rich description of the data. It is important to emphasise that these auxiliary parameter estimates do not necessarily have any ‘direct’ interpretation. The logic of indirect inference is that if the model is well specified then the ap’s are consistent estimates of the same population values even though the latter are of no intrinsic interest.

In all, we specify 108 auxiliary parameters (ap’s) which reflect partial correlations in our reduced form model. Specifically, the log of each choice variable, $m_a, m_b, x_a, x_b, l_a, l_b, L, d_a, d_b$ is regressed on a set of explanatory variables\(^{12}\): her and his wage, her and his education (measured in years above minimum of 10 years), a dummy for the presence of young children (children aged 0-6), and a dummy for the presence of older children (aged 7-17).\(^{13}\) These OLS estimates give

\(^{12}\)The child care dependent variables are actually $\ln (1 + d_s)$ to allow for zeros for those without children. There are no zeros for the other time use variables.

\(^{13}\)This set of covariates is a subset of those we tried. For example, their ages and the duration of the marriage were never significant and hence are not included or reported.
63 regression parameters and 45 residual variances and correlations; see tables 4 and 5 respectively. This gives a total of 108 auxiliary parameters for fitting.

In table 4 we observe that female market hours are positively related to her hourly wage rate, and negatively related to her husband’s wage rate. Moreover, female market hours are rising in her education level and decreasing in his education level. The number of young children (aged 0-6) has an insignificant impact on female market hours, once we condition on wages and educations.\footnote{One explanation for this surprising result may be that these estimates reflect a cross-sectional finding across different age groups. If we include his and her age in the auxiliary regression, we do find that the presence of children is associated with less market work for women. However, the age variables are excluded from our analysis since they are never jointly significant.} Turning to male market hours, we find that these are not responsive to most of the factors that we include. The husband’s market hours are positively related to his education level and the presence of older children, but not statistically significant at a 5% level.

For private expenditures, \( x_a \) and \( x_b \), we find that both wages have a positive impact on both levels with the own wage having a higher value than the cross-wage effect, but only statistically significant for own wage. Both young children and older children have a negative impact (of similar size) on both male and female expenditure on assignable goods. Moreover, we observe that female private leisure is negatively correlated with her wage rate and number of young children and positively (although not significantly) related to the male wage rate. In general, the estimation of male private leisure and joint leisure seems to be only weakly related to our explanatory variables, except for the parameter estimates of young and older children on log joint leisure, which is negative and significant. This finding is consistent with Hamermesh (2000) and Hallberg (2003).
Table 4: OLS parameter estimates

<table>
<thead>
<tr>
<th>Log of $→$</th>
<th>$m_a$</th>
<th>$m_b$</th>
<th>$x_a$</th>
<th>$x_b$</th>
<th>$l_a$</th>
<th>$l_b$</th>
<th>$L$</th>
<th>$d_a$</th>
<th>$d_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ln female</td>
<td>0.16</td>
<td>0.00</td>
<td>0.24</td>
<td>0.13</td>
<td>−0.20</td>
<td>−0.01</td>
<td>0.01</td>
<td>0.02</td>
<td>0.09</td>
</tr>
<tr>
<td>wage</td>
<td>(5.6)</td>
<td>(0.2)</td>
<td>(2.5)</td>
<td>(1.3)</td>
<td>(2.3)</td>
<td>(0.0)</td>
<td>(0.1)</td>
<td>(0.9)</td>
<td>(0.6)</td>
</tr>
<tr>
<td>ln male</td>
<td>−0.05</td>
<td>−0.01</td>
<td>0.13</td>
<td>0.34</td>
<td>0.11</td>
<td>0.05</td>
<td>−0.04</td>
<td>−0.01</td>
<td>0.02</td>
</tr>
<tr>
<td>wage</td>
<td>(2.2)</td>
<td>(0.3)</td>
<td>(1.7)</td>
<td>(3.9)</td>
<td>(1.3)</td>
<td>(0.5)</td>
<td>(0.3)</td>
<td>(−0.2)</td>
<td>(1.1)</td>
</tr>
<tr>
<td># young</td>
<td>0.00</td>
<td>−0.00</td>
<td>−0.23</td>
<td>−0.21</td>
<td>−0.30</td>
<td>−0.02</td>
<td>−0.28</td>
<td>0.31</td>
<td>0.19</td>
</tr>
<tr>
<td>children</td>
<td>(0.2)</td>
<td>(0.4)</td>
<td>(3.7)</td>
<td>(3.0)</td>
<td>(4.2)</td>
<td>(0.6)</td>
<td>(3.5)</td>
<td>(12.4)</td>
<td>(9.9)</td>
</tr>
<tr>
<td># older</td>
<td>−0.02</td>
<td>0.021</td>
<td>−0.13</td>
<td>−0.12</td>
<td>−0.05</td>
<td>0.00</td>
<td>−0.19</td>
<td>0.09</td>
<td>0.05</td>
</tr>
<tr>
<td>children</td>
<td>(1.2)</td>
<td>(1.6)</td>
<td>(2.8)</td>
<td>(2.4)</td>
<td>(0.6)</td>
<td>(0.3)</td>
<td>(3.1)</td>
<td>(6.2)</td>
<td>(4.7)</td>
</tr>
<tr>
<td>Female</td>
<td>0.12</td>
<td>0.021</td>
<td>−0.23</td>
<td>0.20</td>
<td>0.06</td>
<td>0.10</td>
<td>−0.23</td>
<td>0.02</td>
<td>0.03</td>
</tr>
<tr>
<td>education</td>
<td>(3.1)</td>
<td>(0.7)</td>
<td>(0.3)</td>
<td>(1.3)</td>
<td>(1.0)</td>
<td>(0.8)</td>
<td>(1.3)</td>
<td>(0.7)</td>
<td>(1.0)</td>
</tr>
<tr>
<td>Male</td>
<td>−1.00</td>
<td>0.05</td>
<td>0.20</td>
<td>−0.02</td>
<td>0.15</td>
<td>−0.05</td>
<td>0.12</td>
<td>0.05</td>
<td>0.01</td>
</tr>
<tr>
<td>education</td>
<td>(2.6)</td>
<td>(1.4)</td>
<td>(1.4)</td>
<td>(0.3)</td>
<td>(1.1)</td>
<td>(0.3)</td>
<td>(0.7)</td>
<td>(1.6)</td>
<td>(0.3)</td>
</tr>
</tbody>
</table>

Note: Absolute values of t-values in parentheses. Constant not shown. $m$: market work, $x$: private assignable expenditure, $l$: private leisure, $L$: joint leisure, $d$: child care. Subscripts $a$ and $b$ for female and male choices, respectively.

In table 5 we show the standard deviations (in diagonal) and correlation coefficients (below diagonal) of the residuals of the 9 choice equations. There are some significant correlations among the choice variables, many of which are driven by the time and money budgets. Both female and male market work ($m_a$ and $m_b$) are negatively correlated with own child care ($d_a$ and $d_b$, respectively), own private leisure ($l_a$ and $l_b$, respectively) and joint leisure ($L$). Private leisures are also highly positively correlated with each other and negatively correlated with joint leisure. Private assignable expenditures ($x_a$ and $x_b$) are highly positively correlated, which surely partially reflects the dependence of these on total monetary resources.
Table 5: Choice equation residuals

<table>
<thead>
<tr>
<th></th>
<th>ln $m_a$</th>
<th>ln $m_b$</th>
<th>ln $x_a$</th>
<th>ln $x_b$</th>
<th>ln $d_a$</th>
<th>ln $d_b$</th>
<th>ln $l_a$</th>
<th>ln $l_b$</th>
<th>ln $L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ln $m_a$</td>
<td>0.19</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ln $m_b$</td>
<td>-0.01</td>
<td>0.16</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ln $x_a$</td>
<td>0.03</td>
<td>0.07</td>
<td>0.63</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ln $x_b$</td>
<td>0.10</td>
<td>0.09</td>
<td>0.51</td>
<td>0.69</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ln $d_a$</td>
<td>-0.06</td>
<td>0.02</td>
<td>-0.03</td>
<td>-0.04</td>
<td>0.15</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ln $d_b$</td>
<td>0.05</td>
<td>-0.12</td>
<td>0.00</td>
<td>-0.00</td>
<td>0.20</td>
<td>0.11</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ln $l_a$</td>
<td>-0.21</td>
<td>0.03</td>
<td>0.07</td>
<td>0.01</td>
<td>-0.30</td>
<td>-0.06</td>
<td>0.65</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ln $l_b$</td>
<td>0.05</td>
<td>-0.28</td>
<td>-0.01</td>
<td>-0.03</td>
<td>0.12</td>
<td>-0.19</td>
<td>0.49</td>
<td>0.64</td>
<td></td>
</tr>
<tr>
<td>ln $L$</td>
<td>-0.10</td>
<td>-0.10</td>
<td>-0.02</td>
<td>-0.09</td>
<td>-0.17</td>
<td>0.01</td>
<td>-0.42</td>
<td>-0.56</td>
<td>0.70</td>
</tr>
</tbody>
</table>

Note: $m$: market work, $x$: private assignable expenditure, $d$: child care, $l$: private leisure, $L$: joint leisure. Subscripts $a$ and $b$ for female and male choices, respectively.
6 Results

6.1 The goodness of fit

We have 108 auxiliary parameters. In our preferred model we have 57 distribution parameters to characterise the distribution of the model parameters. This is the result of a specification search that started from a more general model and sequentially removed distribution parameters that were ‘insignificant’. Thus we have 51 degrees of over-identification. The fit of the model (the minimised criterion value) is 91.0 which has a $\chi^2 (51)$ distribution. Although this has a low formal probability level, the fit for the individual auxiliary parameters are generally acceptable. More transparently, Table 6 gives the values for the means and standard deviations for each of the dependent variables. As can be seen, the fit for the means is good but the model standard deviations are generally under-predicted.

6.2 The parameter estimates

The 57 distribution parameter estimates are difficult to interpret so we present only the implications for model parameters and choices. Table 7 shows the median values of the model parameters for a benchmark household with median wages, medium education for both partners and no children. The values shown are medians over 2,000 simulations of the model. When considering the model parameter estimates it is important to keep in mind that, for the ‘slope parameters’ $(\theta_s, \tau_s, \delta_s)$, these are preference weights relative to the log of the public good, see equation (7). We find that women value both private consumption and composite leisure less than men, relative to the public good. Men have a slightly higher weight for spending time with their children than do women, relative to the public good. Men and women have weights for private leisure against joint leisure that are both above one half (which represents equal weights). Women are more efficient at housework than men. We find that the curvature parameters $\gamma_s$ and
Table 6: *Goodness of fit statistics for dependent variables*

<table>
<thead>
<tr>
<th>Variable</th>
<th>Data</th>
<th>Predicted</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_a$</td>
<td>Her market work</td>
<td>165.09</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(31.38)</td>
</tr>
<tr>
<td>$m_b$</td>
<td>His market work</td>
<td>188.92</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(34.36)</td>
</tr>
<tr>
<td>$l_a$</td>
<td>Her private leisure</td>
<td>131.16</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(65.14)</td>
</tr>
<tr>
<td>$l_b$</td>
<td>His private leisure</td>
<td>140.90</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(65.47)</td>
</tr>
<tr>
<td>$L$</td>
<td>Joint leisure</td>
<td>118.22</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(61.63)</td>
</tr>
<tr>
<td>$d_a$</td>
<td>Her child care</td>
<td>22.67</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(40.13)</td>
</tr>
<tr>
<td>$d_b$</td>
<td>His child care</td>
<td>11.49</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(22.39)</td>
</tr>
<tr>
<td>$x_a$</td>
<td>Her private consumption</td>
<td>1.200</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.903)</td>
</tr>
<tr>
<td>$x_b$</td>
<td>His private consumption</td>
<td>1.149</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.936)</td>
</tr>
</tbody>
</table>

*Note: For each variable, the first value is the mean and the second value (in brackets) is the standard deviation.*
\( \eta_b \) are all high with values well above unity, suggesting that private consumption and composite leisure are necessary goods (relative to the public good). The very high value for \( \eta_b \) relative to \( \eta_a \) implies that men’s labour supply is significantly less responsive to wages than that of women, consistent with the auxiliary parameters estimates in Table 4 and previous literature on labour supply of men and women.

Finally and most importantly, the estimates for \( \rho_a \) and \( \rho_b \) imply that individual leisure and joint leisure are anything but perfect substitutes. To formally test for perfect substitution, we estimate with the restriction \( \rho_a = \rho_b = 1 \) which has a \( \chi^2(2) \) distribution value of 7,123 which suggests a massive rejection of the perfect substitution restriction. The reason for this strong result is that if, for example, both partners prefer private leisure (\( \pi_a \) and \( \pi_b \) both above 0.5) then they will choose to have only private leisure, an outcome which is never seen in the data in which both types of leisure are always positive for all households. To give some indication of the precision of our estimates, we also test for the less extreme restriction that \( \rho_a = \rho_b = 0.9 \) which constitutes more substitution than the estimated parameters but less than perfect substitution; the \( \chi^2(2) \) value for this restriction is 50.0 which suggests reasonable precision.

Table 8 shows the variation of model parameters with demographics. The top third of the table gives the variation of parameters with education combinations for childless couples, the middle third of the table shows implications for the same education combinations and one young child, while the bottom third of the table shows variations of number of young and older children for different combinations of his and her education level. The second row (both partners having medium-long education and no children) is the benchmark used in Table 7.

For couples without children (top third), the parameter estimates are fairly robust to combinations of his and her education; however, when both partners have more education, the composite leisure weight is decreasing for women, while constant for men (see the \( \tau_a \) and \( \tau_b \) columns). Interestingly, we also find that the child care weight is lower for women in couples where both have more education, but higher for men (see the \( \delta_a \) and \( \delta_b \) columns). The most striking variation
Table 7: Medians of Model Parameter estimates

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range</th>
<th>Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_a$</td>
<td>Private consumption $(0, \infty)$</td>
<td>1.09</td>
</tr>
<tr>
<td>$\theta_b$</td>
<td>weight $(0, \infty)$</td>
<td>3.43</td>
</tr>
<tr>
<td>$\tau_a$</td>
<td>Composite leisure $(0, \infty)$</td>
<td>2.32</td>
</tr>
<tr>
<td>$\tau_b$</td>
<td>weight $(0, \infty)$</td>
<td>6.52</td>
</tr>
<tr>
<td>$\pi_a$</td>
<td>Weight on private $(0, 1)$</td>
<td>0.58</td>
</tr>
<tr>
<td>$\pi_b$</td>
<td>leisure $(0, 1)$</td>
<td>0.52</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Productivity at home for $a$ $(0, 1)$</td>
<td>0.84</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Efficiency of housework $(0, 1)$</td>
<td>0.23</td>
</tr>
<tr>
<td>$\delta_a$</td>
<td>Child care $(0, \infty)$</td>
<td>0.51</td>
</tr>
<tr>
<td>$\delta_b$</td>
<td>weight $(0, \infty)$</td>
<td>0.54</td>
</tr>
<tr>
<td>$\gamma_a$</td>
<td>Curvature of private $(0, \infty)$</td>
<td>2.39</td>
</tr>
<tr>
<td>$\gamma_b$</td>
<td>consumption $(0, \infty)$</td>
<td>2.92</td>
</tr>
<tr>
<td>$\eta_a$</td>
<td>Curvature of composite $(0, \infty)$</td>
<td>6.68</td>
</tr>
<tr>
<td>$\eta_b$</td>
<td>leisure $(0, \infty)$</td>
<td>12.48</td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>Substitutability of private $(-\infty, 1)$</td>
<td>0.06</td>
</tr>
<tr>
<td>$\rho_b$</td>
<td>and joint leisure $(-\infty, 1)$</td>
<td>0.29</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Pareto weight for $b$ $(0, \infty)$</td>
<td>0.87</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Housework complementarity $(-\infty, 1)$</td>
<td>$-3.95$</td>
</tr>
</tbody>
</table>
across education combinations is for $\mu$: the Pareto weight for $b$ is lower in couples with higher education; higher male education seems to be associated with a lower Pareto weight. For couples with one young child (middle section of Table 8), both private consumption taste parameters ($\theta_s$) fall with the presence of children (the implications of this for the choices are given in the next subsection), and the weight on composite leisure versus consumption ($\tau_s$) is reduced for both men and women. Finally, holding education constant at medium education for both partners in the bottom third of Table 8, we observe that the weights for private consumption ($\theta_s$) and the composite leisure weights ($\tau_s$) reduce significantly with the presence of children for both women and men, with the lowest parameter estimates for women.

### 6.3 Dependence of choices on demographics

In this subsection we present the variation in hours and expenditures with variation in demographics. We take the same variants as in Table 8 and fix wages at their median values. Results are shown in Table 9. At the benchmark values (the second row), husbands have higher market hours than wives, almost the same level of private leisure, and do less housework. For both men and women, market hours are increasing with own education, holding the other partner’s education fixed. However, the results for private leisure and housework go in opposite directions, with high educated men having less private leisure and housework, when holding his wife’s education fixed. Private consumption does not vary much with education (but recall that we are holding gross wages fixed; the next subsection reports on variation with gross wages). Husbands work about the same number of hours in the market irrespective of whether there are young children present, while wives with lower or medium education work less. For both men and women, both private and joint leisure decrease with having younger children as child care hours increase, while housework is not affected much by having children. Finally, private consumption falls for both husband and wife if there are children present,
Table 8: Variation of parameter estimates with demographics

<table>
<thead>
<tr>
<th>Education</th>
<th>ych</th>
<th>och</th>
<th>$\theta_a$</th>
<th>$\theta_b$</th>
<th>$\tau_a$</th>
<th>$\tau_b$</th>
<th>$\delta_a$</th>
<th>$\delta_b$</th>
<th>$\rho_a$</th>
<th>$\rho_b$</th>
<th>$\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>low-low</td>
<td>0</td>
<td>0</td>
<td>1.09</td>
<td>3.43</td>
<td>2.13</td>
<td>6.52</td>
<td>0.56</td>
<td>0.48</td>
<td>0.06</td>
<td>0.29</td>
<td>1.04</td>
</tr>
<tr>
<td>med-med</td>
<td>0</td>
<td>0</td>
<td>1.09</td>
<td>3.43</td>
<td>2.32</td>
<td>6.52</td>
<td>0.51</td>
<td>0.54</td>
<td>0.06</td>
<td>0.29</td>
<td>0.87</td>
</tr>
<tr>
<td>high-high</td>
<td>0</td>
<td>0</td>
<td>1.09</td>
<td>3.43</td>
<td>2.51</td>
<td>6.52</td>
<td>0.46</td>
<td>0.61</td>
<td>0.06</td>
<td>0.29</td>
<td>0.70</td>
</tr>
<tr>
<td>low-high</td>
<td>0</td>
<td>0</td>
<td>1.09</td>
<td>3.43</td>
<td>2.51</td>
<td>6.52</td>
<td>0.56</td>
<td>0.61</td>
<td>0.06</td>
<td>0.29</td>
<td>0.70</td>
</tr>
<tr>
<td>high-low</td>
<td>0</td>
<td>0</td>
<td>1.09</td>
<td>3.43</td>
<td>2.13</td>
<td>6.52</td>
<td>0.46</td>
<td>0.48</td>
<td>0.06</td>
<td>0.29</td>
<td>1.04</td>
</tr>
<tr>
<td>low-low</td>
<td>1</td>
<td>0</td>
<td>0.62</td>
<td>1.70</td>
<td>0.41</td>
<td>1.58</td>
<td>0.56</td>
<td>0.48</td>
<td>0.06</td>
<td>0.29</td>
<td>1.04</td>
</tr>
<tr>
<td>med-med</td>
<td>1</td>
<td>0</td>
<td>0.62</td>
<td>1.70</td>
<td>0.44</td>
<td>1.58</td>
<td>0.51</td>
<td>0.54</td>
<td>0.06</td>
<td>0.29</td>
<td>0.87</td>
</tr>
<tr>
<td>high-high</td>
<td>1</td>
<td>0</td>
<td>0.62</td>
<td>1.70</td>
<td>0.48</td>
<td>1.58</td>
<td>0.46</td>
<td>0.61</td>
<td>0.06</td>
<td>0.29</td>
<td>0.70</td>
</tr>
<tr>
<td>low-high</td>
<td>1</td>
<td>0</td>
<td>0.62</td>
<td>1.70</td>
<td>0.48</td>
<td>1.58</td>
<td>0.56</td>
<td>0.61</td>
<td>0.06</td>
<td>0.29</td>
<td>0.70</td>
</tr>
<tr>
<td>high-low</td>
<td>1</td>
<td>0</td>
<td>0.62</td>
<td>1.70</td>
<td>0.41</td>
<td>1.58</td>
<td>0.46</td>
<td>0.48</td>
<td>0.06</td>
<td>0.29</td>
<td>1.04</td>
</tr>
<tr>
<td>med-med</td>
<td>1</td>
<td>0</td>
<td>0.62</td>
<td>1.70</td>
<td>0.44</td>
<td>1.58</td>
<td>0.51</td>
<td>0.54</td>
<td>0.06</td>
<td>0.29</td>
<td>0.87</td>
</tr>
<tr>
<td>med-med</td>
<td>2</td>
<td>0</td>
<td>0.49</td>
<td>1.27</td>
<td>0.22</td>
<td>0.88</td>
<td>0.51</td>
<td>0.54</td>
<td>0.06</td>
<td>0.29</td>
<td>0.87</td>
</tr>
<tr>
<td>med-med</td>
<td>1</td>
<td>1</td>
<td>0.48</td>
<td>1.10</td>
<td>0.33</td>
<td>1.12</td>
<td>0.51</td>
<td>0.54</td>
<td>0.06</td>
<td>0.29</td>
<td>0.87</td>
</tr>
<tr>
<td>med-med</td>
<td>0</td>
<td>2</td>
<td>0.76</td>
<td>1.86</td>
<td>1.50</td>
<td>4.00</td>
<td>0.51</td>
<td>0.54</td>
<td>0.06</td>
<td>0.29</td>
<td>0.87</td>
</tr>
<tr>
<td>med-med</td>
<td>0</td>
<td>1</td>
<td>0.85</td>
<td>2.23</td>
<td>1.70</td>
<td>4.61</td>
<td>0.51</td>
<td>0.54</td>
<td>0.06</td>
<td>0.29</td>
<td>0.87</td>
</tr>
</tbody>
</table>

Note: ‘Education’ is combinations of a’s and b’s education level. $\pi_a$ and $\pi_b$ are not reported since they depend only on latent heterogeneity. ych’: Number of young children; ‘och’: Older children in household.
consistent with most of the literature.

6.3.1 Responses to wage differences

We now examine the behavioural responses to variations in wages. Figure 1 shows the responses to variation in the wife's wage from the bottom decile to the top decile in our data (holding the husband’s wage at the median in the data). The top left panel indicates that her market hours are quite responsive to her wage changes: the wife’s market hours first fall with her wage until the median wage and then increases from around 150 hours to 160 hours per month. The top right panel shows neither joint leisure nor his private leisure change substantially. The bottom left panel indicates that housework is also fairly irresponsive to variations in her wage. Finally, we observe that her child care hours increase significantly with her wage (holding his wage constant), consistent with most of the previous literature, while his child care time is decreasing in her wage.
Table 9: Responses to demographics

<table>
<thead>
<tr>
<th>Education</th>
<th>ych</th>
<th>och</th>
<th>$x_a$</th>
<th>$x_b$</th>
<th>$m_a$</th>
<th>$m_b$</th>
<th>$l_a$</th>
<th>$l_b$</th>
<th>L</th>
<th>$d_a$</th>
<th>$d_b$</th>
<th>$h_a$</th>
<th>$h_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>low-low</td>
<td>0</td>
<td>0</td>
<td>1.03</td>
<td>1.05</td>
<td>167</td>
<td>187</td>
<td>142</td>
<td>152</td>
<td>124</td>
<td>0</td>
<td>0</td>
<td>82</td>
<td>56</td>
</tr>
<tr>
<td>med-med</td>
<td>0</td>
<td>0</td>
<td>1.06</td>
<td>1.01</td>
<td>161</td>
<td>190</td>
<td>147</td>
<td>149</td>
<td>125</td>
<td>0</td>
<td>0</td>
<td>81</td>
<td>56</td>
</tr>
<tr>
<td>high-high</td>
<td>0</td>
<td>0</td>
<td>1.10</td>
<td>0.97</td>
<td>156</td>
<td>192</td>
<td>151</td>
<td>146</td>
<td>126</td>
<td>0</td>
<td>0</td>
<td>80</td>
<td>55</td>
</tr>
<tr>
<td>low-high</td>
<td>0</td>
<td>0</td>
<td>1.11</td>
<td>0.97</td>
<td>151</td>
<td>199</td>
<td>151</td>
<td>146</td>
<td>125</td>
<td>0</td>
<td>0</td>
<td>86</td>
<td>48</td>
</tr>
<tr>
<td>high-low</td>
<td>0</td>
<td>0</td>
<td>1.02</td>
<td>1.04</td>
<td>175</td>
<td>180</td>
<td>143</td>
<td>152</td>
<td>124</td>
<td>0</td>
<td>0</td>
<td>74</td>
<td>64</td>
</tr>
<tr>
<td>low-low</td>
<td>1</td>
<td>0</td>
<td>0.81</td>
<td>0.82</td>
<td>151</td>
<td>188</td>
<td>108</td>
<td>141</td>
<td>105</td>
<td>63</td>
<td>19</td>
<td>81</td>
<td>56</td>
</tr>
<tr>
<td>med-med</td>
<td>1</td>
<td>0</td>
<td>0.84</td>
<td>0.80</td>
<td>151</td>
<td>188</td>
<td>111</td>
<td>139</td>
<td>105</td>
<td>61</td>
<td>22</td>
<td>80</td>
<td>56</td>
</tr>
<tr>
<td>high-high</td>
<td>1</td>
<td>0</td>
<td>0.88</td>
<td>0.77</td>
<td>151</td>
<td>190</td>
<td>115</td>
<td>136</td>
<td>106</td>
<td>58</td>
<td>23</td>
<td>80</td>
<td>56</td>
</tr>
<tr>
<td>low-high</td>
<td>1</td>
<td>0</td>
<td>0.88</td>
<td>0.77</td>
<td>151</td>
<td>198</td>
<td>112</td>
<td>138</td>
<td>104</td>
<td>68</td>
<td>24</td>
<td>79</td>
<td>46</td>
</tr>
<tr>
<td>high-low</td>
<td>1</td>
<td>0</td>
<td>0.82</td>
<td>0.83</td>
<td>176</td>
<td>177</td>
<td>109</td>
<td>140</td>
<td>106</td>
<td>39</td>
<td>21</td>
<td>77</td>
<td>65</td>
</tr>
<tr>
<td>med-med</td>
<td>1</td>
<td>0</td>
<td>0.84</td>
<td>0.80</td>
<td>151</td>
<td>188</td>
<td>111</td>
<td>139</td>
<td>105</td>
<td>61</td>
<td>22</td>
<td>80</td>
<td>56</td>
</tr>
<tr>
<td>med-med</td>
<td>2</td>
<td>0</td>
<td>0.76</td>
<td>0.72</td>
<td>151</td>
<td>182</td>
<td>98</td>
<td>135</td>
<td>98</td>
<td>84</td>
<td>43</td>
<td>79</td>
<td>55</td>
</tr>
<tr>
<td>med-med</td>
<td>1</td>
<td>1</td>
<td>0.76</td>
<td>0.69</td>
<td>151</td>
<td>189</td>
<td>111</td>
<td>142</td>
<td>93</td>
<td>69</td>
<td>29</td>
<td>82</td>
<td>57</td>
</tr>
<tr>
<td>med-med</td>
<td>0</td>
<td>2</td>
<td>0.91</td>
<td>0.82</td>
<td>151</td>
<td>190</td>
<td>145</td>
<td>156</td>
<td>104</td>
<td>20</td>
<td>0</td>
<td>80</td>
<td>56</td>
</tr>
<tr>
<td>med-med</td>
<td>0</td>
<td>1</td>
<td>0.96</td>
<td>0.87</td>
<td>153</td>
<td>193</td>
<td>147</td>
<td>154</td>
<td>110</td>
<td>5</td>
<td>0</td>
<td>83</td>
<td>57</td>
</tr>
</tbody>
</table>

Note: ‘Education’ is combinations of a’s and b’s education level. ‘ych’ is number of young children and ‘och’ is number of older children. Wages are held fixed at median values. $x$: private assignable expenditure, $m$: market work, $l$: private leisure, $L$: joint leisure, $d$: child care, $h$: housework. Subscripts $a$ and $b$ for female and male choices, respectively.
Figure 1: *Time use variation with wife’s wage.*

*Note:* Male wages held constant at median.
Figure 2: *Time use variation with husband’s wage.*

*Note:* Female wages held constant at median.
Figure 3: Private expenditures.

Note: Top panel - Male wages held constant at median. Bottom panel - Female wages held constant at median.

Figure 2 shows the responses to variation in the husband’s wage from the bottom decile to the top decile in our data (holding her wage at the median in the data). His market work is wholly independent of the higher wage, which was also suggested in the auxiliary regressions. Both private and joint leisure are largely independent of his wage. The wage effects on housework mirror those for women’s wages with his strongly decreasing and her’s strongly increasing. Finally, child care hours for both spouses are largely independent of his wage.

Figure 3 on the pattern for private consumption shows that both partners
experience an increase in own private expenditures when their wage increases, while private expenditures of the partner are fairly constant. Thus, both partners experience increasing shares of private expenditure in the household when their relative wage is increasing, consistent with the collective model, with the main effect coming from the variation in the Pareto weight.

7 Conclusion

This paper develops a parametric structural model of household allocation of time and consumption. The model extends the model developed in Browning and Gørtz (2012) in several important directions: First, our main contribution is to explicitly distinguish in this model between leisure spent jointly by husband and wife and individual (private) leisure. Secondly, we combine information from the diary and questionnaire to explicitly model the utility derived from caring for own children. We test our theoretical model on a Danish time use survey with information on couples’ allocation of time and expenditure in 2001. Our empirical investigations of the structural model lead to several interesting insights into both the properties of the structural parameters in the model and the implications for observed outcomes.

First, we verify that the Pareto weight (the weight of the husband’s utility in the household utility function), depends positively on the husband’s hourly wage rate and negatively on the wife’s wage rate. This finding, which is consistent with a collective model of household decision making, suggests that the bargaining power of the husband is increasing in his wage rate and is decreasing in his wife’s wage rate. As a consequence of the variations in the Pareto weight, individual spending on private goods is increasing in own wage but decreasing in the partner’s wage.

Secondly, we find that husbands value both private consumption (relative to public goods) and composite leisure (relative to public goods) more highly than wives.

The main focus of our empirical study, however, is on the partners’ time allocation. Our third result, which is one of our most important results in this
perspective, is that joint leisure and individual leisure are quite far from being perfect substitutes for partners. Thus summing them to construct a measure of total leisure, as it is implicitly made in most empirical investigations, may be invalid.

Fourth, we find that both joint and private leisure are largely constant over the distribution of both her wage and his wage rate. This empirical observation corroborates the study by Hallberg (2005) but contradicts those by Hamermesh (2000, 2002) and Jenkins and Osberg (2005) who have found that synchronous time in couples is rising with the wage rates of the two partners. Ruuskanen (2004), for his part, has found a negative relationship. By comparison with these studies, however, our data allows for a split of individual leisures into joint leisure and private leisure, and furthermore allows us to explicitly measure child care hours, and is thus a more direct measure of time actually spent together than synchronous time. Our conclusion is thus that joint leisure (when measured directly) is fairly constant across households – or at least not varying with the conventional distribution factors or socioeconomic variables. Thus, our results do not support the notion that a reduction in specialization through a diminished intrahousehold wage gap increase spouses’ time spent together. Instead, this may ultimately lead to speculation that married couples choose to spend time together to strengthen or sustain their marriage, suggesting perhaps that gains from marriage are indeed consumption based (as found in e.g. Lundberg, 2012; Stevenson and Wolfers, 2007). Moreover, as the wife’s wage increases, both partners do more housework, but the within-couple gender gap in housework is shrinking slightly for fairly high female wages. The latter result contrasts with the finding by Bertrand et al. (2015) that the gender gap in home production (how much more time the wife spends on non-market work than the husband) is in fact larger in couples where she earns more than he does.

Fifth, our model allows us to take explicit account of child care hours. We find that tastes for child care are positively correlated within households. Furthermore, consistent with previous studies (for the correlation with education; see,
for example, Guryan et al. 2008 and Kalil et al. 2012), we find that female child care hours are increasing in her wage, while her husband’s child care hours are decreasing in her wage. Generally, we observe that the structural parameters of the model vary with the presence of children in the household. Thus, the presence of children in the household is correlated with observable behaviour. Both private and joint leisure are lower if there are children in the house, for both husband and wife. Moreover, husbands work more in the market when there are children present. This overall picture for male and female time use is largely consistent with prior empirical investigations (a number of recent studies find that couples have less synchronous time if there are younger children present in the household, e.g. Hamermesh, 2000; Hallberg, 2003; Hallberg and Klevmarken, 2003; Bryan and Sevilla-Sanz, 2014; van Klaveren and Maassen van den Brink, 2007; Jenkins and Osberg, 2005). Also, private consumption falls for both husband and wife if there are children present, which is a familiar result in the literature.

University of Copenhagen, CEBI
University of Cergy-Pontoise
University of Copenhagen, CEBI, IZA
A Identification

To rigorously discuss identifiability issues we have to make general assumptions on utility and production functions. We first consider the benchmark case where utility is of the form \( u_s(x_s, l_s, L, G) \), i.e., the childless case, and then extend it to more general utility functions.

A.1. Individual utility functions \( u_s(x_s, l_s, L, G) \), with \( s = a, b \), are twice continuously differentiable, increasing, and strongly concave.

A.2. The production function \( G(h_a, h_b, X) \) is twice continuously differentiable, increasing, and concave.

Exogenous variables \((w_a, w_b, y)\) vary continuously in \( R = R^2_+ \times R \). The first order conditions of the optimization problem, after some arrangements, can then be written as:

\[
\frac{\partial G}{\partial h_a} = w_a, \quad (A.1)
\]
\[
\frac{\partial G}{\partial h_b} = w_b, \quad (A.2)
\]
\[
\frac{\partial G}{\partial x_g} = \mu, \quad (A.3)
\]
\[
w_a \frac{\partial u_a}{\partial L} + w_b \frac{\partial u_b}{\partial L} = w_a + w_b, \quad (A.4)
\]
\[
\frac{\partial u_a}{\partial a} = w_a, \quad (A.5)
\]
\[
\frac{\partial u_b}{\partial b} = w_b, \quad (A.6)
\]
\[
\frac{\partial u_a}{\partial x_a} + \frac{\partial u_b}{\partial x_b} = \left( \frac{\partial G}{\partial x_g} \right)^{-1}. \quad (A.7)
\]

Starting from these first order conditions, the first result is the following.

**Lemma 1.** Assume A.1-A.2. The production function \( G(h_a, h_b, x_g) \) is identified up to an increasing transformation \( F_G \). If the production function is linearly homogeneous, then the transformation \( F_G \) is linear.
**Proof.** The system of partial differential equations (A.1)-(A.2) defines a characteristics curve from which the production function can be retrieved up to an increasing transformation. That is, the general solution is \( G(h_a, h_b, x_g) = F_G(\bar{G}(h_a, h_b, x_g)) \), where \( \bar{G} \) is a particular solution of the aforementioned system and \( F_G \) is some arbitrary increasing transformation such that \( G \) is concave. If the function \( G \) is linearly homogeneous, then the function \( F_G \) is necessarily linear. Indeed, from the Euler Theorem, we have: \( F_G(\bar{G}) = \bar{G} \times F'_G(\bar{G}) \). The solution of this differential equation is a linear function with parameter \( K_G = F'_G(\bar{G}) \).

To obtain identification, we adopt the following separability assumption.

**A.3.** Individual utility functions \( u_s(x_s, l_s, L, G) \), with \( s = a, b \), have a separable structure and can be written as follows:

\[
u_s(x_s, l_s, L, G) = u_s(x_s, G, c_s(l_s, L)),
\]

where \( u_s \) and \( c_s \) are increasing and strongly quasi-concave functions.

This assumption is not sufficient to recover the composite leisure and we also need the following assumption.

**A.4.** The marginal rates of substitution between private and joint leisure, defined as

\[
\frac{\partial c_s(l_s, L)}{\partial L} / \frac{\partial c_s(l_s, L)}{\partial l_s} \quad \text{for} \quad s = a, b,
\]

converges to zero if private leisure tends to zero.

It means that private leisure is an essential good, the consumption of which is never zero.

Then the next lemma says that the marginal rate of substitution is identified.

**LEMMA 2.** Assume A.1-A.4 and some technical conditions listed in the proof. The composite leisure \( c_s(l_s, L) \) is identified up to an increasing transformation \( F_{c_s} \).
Proof. The proof follows in three steps.

First Step: The marginal rate of substitution between private and joint leisure minus 1 for individual \( s \) is defined as:

\[
R_s(l_s, L) = \frac{\partial c_s(l_s, L)}{\partial L} - 1,
\]

and, therefore, condition (A.4) becomes:

\[
R_a(l_a, L) + \frac{w_b}{w_a} R_b(l_b, L) = 0.
\]

We consider some open subset \( O \) of the domain of \((w_a, w_b, y)\) such that the determinant of the Jacobian matrix

\[
\begin{vmatrix}
\frac{\partial l_a}{\partial w_a}(w_a, w_b, y) & \frac{\partial l_a}{\partial w_b}(w_a, w_b, y) & \frac{\partial l_a}{\partial y}(w_a, w_b, y) \\
\frac{\partial l_b}{\partial w_a}(w_a, w_b, y) & \frac{\partial l_b}{\partial w_b}(w_a, w_b, y) & \frac{\partial l_b}{\partial y}(w_a, w_b, y) \\
\frac{\partial L}{\partial w_a}(w_a, w_b, y) & \frac{\partial L}{\partial w_b}(w_a, w_b, y) & \frac{\partial L}{\partial y}(w_a, w_b, y)
\end{vmatrix} \neq 0,
\]

meaning that leisure demand functions can be locally inverted (this condition is automatically satisfied in the unitary framework, i.e., if \( \mu \) is constant). Then, let us define \( r(l_a, l_b, L) = \frac{w_b}{w_a} \) as the relative wage expressed as a function of \((l_a, l_b, L)\). The first order condition can then be written as:

\[
R_a(l_a, L) + r(l_a, l_b, L) \times R_b(l_b, L) = 0. \tag{A.8}
\]

If we differentiate this expression with respect to \( l_b \), we obtain:

\[
\frac{\partial r}{\partial l_b} (l_a, l_b, L) \times R_b(l_b, L) + r(l_a, l_b, L) \times \frac{\partial R_b}{\partial l_b} (l_b, L) = 0.
\]

Because of the strong quasi-concavity of utility functions, \( R_b(l_b, L) \) is almost everywhere different from zero. The expression above can alternatively be written as:

\[
\frac{\partial \log |R_b(l_b, L)|}{\partial l_b} = - \frac{\partial \log r(l_a, l_b, X)}{\partial l_b}.
\]

42
If we integrate this expression with respect to $l_b$, we obtain:

$$\log |R_b(l_b, L)| = -\int_{\xi}^{l_b} \frac{\partial r(l_a, \ell, X)}{r(l_a, l_b, X)} \cdot d\ell + \nu(l_a, L),$$

where $\xi$ is an arbitrary limit of integration, and $\nu(l_a, L)$ is an arbitrary function; alternatively,

$$|R_b(l_b, L)| = \exp \left[-\int_{\xi}^{l_b} \frac{\partial r(l_a, \ell, X)}{r(l_a, l_b, X)} \cdot d\ell \right] \times \exp \nu(l_a, L).$$

That is, if $\bar{R}_b(l_b, L)$ is some particular solution of condition (A.4), then the general solution is of the form:

$$R_b(l_b, L) = K_b(l_a, L) \times \bar{R}_b(l_b, L),$$

for some (positive or negative) function $K_b(l_a, L) \equiv \pm \exp (\nu(l_a, L))$. Similarly, the general solution for $R_a(l_a, L)$ is:

$$R_a(l_a, L) = K_a(l_b, L) \times \bar{R}_a(l_a, L). \quad (A.9)$$

If we incorporate these expressions into condition (A.4), and simplify, we obtain:

$$\frac{K_a(l_b, L)}{K_b(l_a, L)} = -r(l_a, l_b, X) \times \frac{\bar{R}_b(l_b, L)}{\bar{R}_a(l_a, L)} = 1. \quad (A.10)$$

where the last equality results from the fact that the particular solutions must satisfy condition (A.4) as well. Hence $K_a(l_b, L) = K_b(l_a, L) = K_R(L)$, i.e., only one function of $L$ is not identified.

Second Step: The unidentified function $K_R(L)$ can then be retrieved from boundary conditions using A.4. Since the marginal rates of substitution tend to zero when private leisure tends to zero, i.e., $R(l_s, L) \to -1$ and $\bar{R}(l_s, L) \to -1$ when $l_s \to 0$, with $s = a, b$, then $K_R(L) = 1$ from (A.9). The marginal rates of substitution are exactly identified.

Third Step: From the marginal rates of substitution, the composite leisure is then identified up to a monotonic transformation $F_{c_s}$. The proof is similar to that of Lemma 1.■

We then prove the next result.
PROPOSITION 1. Assume A.1-A.4 and some additional technical conditions listed in the proof. Once the functions $F_G$ and $F_{c_s}$ are picked up, the utility functions $u_s(x_s, G, c_s)$, with $s = a, b$, are identified up to an increasing transformation $F_{u_s}$. For any choice of this transformation $F_{u_s}$, the bargaining weight $\mu(w_a, w_b, y)$ is identified as well.

Proof. The proof follows in two steps.

First Step: Once the functions $F_G$ and $F_{c_s}$ are picked up, we know from the preceding lemmas that $c_s$ and $G$ are known and can be expressed as a function of exogenous variables $(w_a, w_b, y)$. We then consider some open subset $O \subset \mathbb{R}$ such that $\partial G/\partial w_s' \neq 0$, with $s' = a, b$. The condition $G(w_a, w_b, y) = G$ is thus equivalent, by the implicit function theorem, to

$$w_{s'} = w_{s'}(w_s, G, y), \quad (A.11)$$

with $s = a, b$ and $s \neq s'$. For any $(w_a, w_b, y) \in O$, we can use the equation above and write (A.5) or (A.6) as a function of $(w_s, G, y)$ to obtain:

$$\frac{\partial u_s/\partial c_s}{\partial u_s/\partial x_s} = \phi_s(w_s, G, y), \quad (A.12)$$

where $\phi_s(w_s, G, y) = w_s \cdot (\partial c_s(l_s, L)/\partial l_s)^{-1}$ is a known function (thanks to Lemmas 1 and 2) that represents the implicit price of the composite leisure. The budget constraint can also be written as: $\kappa_s(w_s, G, y) = x_s + \phi_s(w_s, G, y)c_s$, where $\kappa_s(w_s, G, y)$ is a known function.

We then define $O'$ as some open subset of the domain of variation of $(w_s, G, y)$ such that $G = G(w_a, w_b, y)$ with $(w_a, w_b, y) \in O$ and

$$\frac{\partial \phi_s}{\partial w_s} \frac{\partial \kappa_s}{\partial y} \neq \frac{\partial \phi_s}{\partial y} \frac{\partial \kappa_s}{\partial w_s}, \quad (T.1)$$

i.e., $\kappa_s$ and $\phi_s$ may vary independently from each other when $w_s$ and $y$ vary. The condition is not strong since, in all likelihood, $\phi_s$ is very sensitive to $w_s$ while $\kappa_s$ is very sensitive to $y$. Then, from traditional results in integration theory, the partial
differential equation (A.12) defines a characteristics curve from which the function \( u_s(x_s, G, c_s) \) can be retrieved up to an increasing transformation for any choice of \( G \). In other words, if \( w_s(x_s, G, c_s) \) is a particular solution, then the general solution is of the form:

\[
\begin{align*}
    u_s(x_s, G, c_s) &= F_{u_s}(w_s(x_s, G, c_s), G),
\end{align*}
\]

where \( F_{u_s} \) is some arbitrary, increasing function.

**Second Step:** The next step then uses technical conditions that can be summarized as follows. For any \((w_a, w_b, y)\) ∈ \( \mathcal{O} \), we can use (A.11) and write utility and marginal utility and marginal productivity as a function of \( w_s, G \) and \( y \), i.e.,

\[
\begin{align*}
    u_s &= f_s(w_s, G, y) \quad \text{and} \quad u_s' = g_s(w_s, G, y),
\end{align*}
\]

and

\[
\begin{align*}
    \frac{\partial u_s}{\partial x_s} &= \mu_s^x(w_s, G, y), \quad (A.13) \\
    \frac{\partial u_s'}{\partial x_s'} &= \nu_s^x(w_s, G, y), \quad (A.14) \\
    \frac{\partial u_s}{\partial G} &= \mu_s^G(w_s, G, y), \quad (A.15) \\
    \frac{\partial u_s'}{\partial G} &= \nu_s^G(w_s, G, y), \quad (A.16)
\end{align*}
\]

and

\[
\begin{align*}
    \frac{\partial G}{\partial X} &= \pi^X(w_s, G, y), \quad (A.17)
\end{align*}
\]

with \( s = a, b \) and \( s \neq s' \). From condition (A.7), we can write:

\[
\frac{1}{\mu_a^2} \frac{\partial F_{u_a}}{\partial x_a} / \partial u_a + \frac{1}{\nu_b^2} \frac{\partial F_{u_b}}{\partial x_b} / \partial u_b = \frac{1}{\pi^X} - \frac{\mu_a^G}{\mu_a^x} - \frac{\nu_b^G}{\nu_b^x},
\]

where

\[
\frac{\partial F_{u_s}}{\partial G} / \partial F_{u_s} / \partial u_s
\]
is the marginal rate of substitution between \( u_s \) and \( G \) that has to be recovered. If\
\[
\frac{\partial f_s}{\partial w_s} \frac{\partial g_s}{\partial y} \neq \frac{\partial f_s}{\partial y} \frac{\partial g_s}{\partial w_s},
\]
\text{(T.2)}
we can write \( w_s = w_s(u_a, u_b, G) \) and \( y = y(u_a, u_b, G) \). If we incorporate these expressions in (A.13)-(A.16), we obtain:
\[
\alpha(u_a, u_b, G) T_a(u_a, G) + \beta(u_a, u_b, G) T_b(u_b, G) = \gamma(u_a, u_b, G),
\]
where
\[
\alpha(u_a, u_b, G) = \frac{1}{\mu_a^G(w_a(u_a, u_b, G), G, y(u_a, u_b, G))},
\]
\[
\beta(u_a, u_b, G) = \frac{1}{\nu_b^G(w_a(u_a, u_b, G), G, y(u_a, u_b, G))},
\]
\[
\gamma(u_a, u_b, G) = \frac{1}{\pi^X(w_s(u_a, u_b, G), G, y(u_a, u_b, G))} - \frac{\mu_a^G(w_a(u_a, u_b, G), G, y(u_a, u_b, G))}{\mu_b^G(w_a(u_a, u_b, G), G, y(u_a, u_b, G))} - \frac{\nu_b^G(w_a(u_a, u_b, G), G, y(u_a, u_b, G))}{\nu_a^G(w_a(u_a, u_b, G), G, y(u_a, u_b, G))}
\]
are known functions.

Blundell et al. (2005) show that such an equation has, in general, only one solution. The result comes from the fact that the unknowns are functions of only two variables, whereas the equation depends in general on four variables. To show this, we suppose that there are two different solutions \((H_a^0, H_b^0)\) and \((H_a^1, H_b^1)\). The differences \(M_a = H_a^0 - H_a^1\) and \(M_b = H_b^0 - H_b^1\) must satisfy the following homogeneous equation:
\[
M_a(u_a, G) + K(u_a, u_b, G) M_b(u_b, G) = 0,
\]
where \(K(u_a, u_b, G) = \beta(u_a, u_b, G)/\alpha(u_a, u_b, G)\). Note that \(K(u_a, u_b, G)\) is the ratio of marginal utilities; it coincides with the (inverse of the) bargaining weight that is consistent with utility functions \(u_a\) and \(u_b\). If we differentiate twice this expression with respect to \(u_b\) and \(u_a\), we obtain:
\[
\frac{\partial M_a(u_b, G)}{\partial u_b} K(u_a, u_b, G) + M_b(u_b, G) \frac{\partial K(u_a, u_b, G)}{\partial u_b} = 0,
\]
46
and
\[
\frac{\partial M_b(u_b, G)}{\partial u_b} \frac{\partial K(u_a, u_b, G)}{\partial u_a} + M_b(\bar{u}_b, G) \frac{\partial^2 K(u_a, u_b, G)}{\partial u_a \partial u_b} = 0.
\]

If
\[
K(u_a, u_b, G) \frac{\partial^2 K(u_a, u_b, G)}{\partial u_a \partial u_b} - \frac{\partial K(u_a, u_b, G)}{\partial u_b} \frac{\partial K(u_a, u_b, G)}{\partial u_a} \neq 0,
\]
then the only solution is:
\[
\frac{\partial M_b(u_b, G)}{\partial u_b} = 0, \quad M_b(u_b, G) = 0.
\]

In other words, if there are two solutions for \((H_a(u_a, G), H_b(u_b, G))\), then these solutions must necessarily coincide. Therefore, if the marginal rate of substitution is identified, then the function \(F_{u_s}(u_s, G)\) is identified up to an increasing transformation \(F_{u_s}\). Finally, from condition (A.3), we have:
\[
\frac{\partial u_b/\partial x_b}{\partial u_a/\partial x_a} = \mu(w_a, w_b, y).
\]

For a particular choice of \(F_{u_s}\), the left-hand side is identified; so is the right-hand side. ■

### A.1 Extension

The preceding identification result of identification is valid in the childless case under rather weak conditions. If utility functions are generalized to incorporate various time uses (e.g., child care time or transportation time) or disaggregated consumption, identification requires more structure. For instance, we can assume additive utility functions as:
\[
u_s(x_s, l_s, L, G) + v_s(d_s).
\]

The first order conditions of the household optimisation problem are (A.1)–(A.7) together with the following additional conditions:
\[
\frac{\partial v_a/\partial d_a}{\partial u_a/\partial x_a} = w_a,
\]
\[
\frac{\partial v_b/\partial d_b}{\partial u_b/\partial x_b} = w_b.
\]
Consequently, the preceding identification result continue to hold in this more general context. In particular, the functions $u_s$ can be identified up to a monotonic transformation from PROPOSITION 1. More precisely, using (A.18), the general solution has to be of the form:

$$u_s = F(\pi_s(x_s, l_s, L, G) + \tau_s(d_s)),$$

where $\pi_s$ and $\tau_s$ are particular solutions for $u_s$ and $v_s$, for some increasing function $F$. Thus,

$$\frac{\partial \tau_s}{\partial d_s} = w_s \frac{\partial \pi_s}{\partial x_s},$$  \hspace{1cm} (A.21)

i.e., the derivative of the function $v_s$ is identified for some particular solution $\pi_s$. Once the function $F$ is picked up, then the function $v_s$ is identified up to an additive constant.

\section*{B Wage rate imputation}

Our wage rate measure is derived from linked administrative Register data. In the register, the hourly wage rate is calculated for the population employed in a particular status week in November. The calculation of the hourly wage for this group is based on total annual salary divided by annual number of total working hours. The register does not observe actual number of hours worked; instead Statistics Denmark uses normal annual full-time hours (1702 annual hours = 37 hours a week for 46 weeks). Using this imputed register variable on hourly wage rate gives us information on hourly wages for the majority of our sample, while the rest of the sample has missing information on hourly wage rates. To avoid having to discard too many observations due to missing (register) wage rate information, we impute wage rates for the remaining part of the sample. First, for those with missing register information on hourly wages in 2001, but for whom we observe an hourly wage rate in 2000, we used the 2000-wage rate inflated by the consumer price index. Secondly, we calculate an hourly wage rate based
on information in the register on annual earnings (in primary job) divided by the number of usual market hours (excluding now commuting) from the survey. Thirdly, for the remaining group, we exploit our information on annual earnings (from register) and the number of weekly market hours given in the diary part of the DTUS questionnaire to arrive at an imputed hourly wage rate. Fourth, for a few (6) observations, the information on the hourly wage rate given directly in the register was very high (more than 400 DKK per hour). If our own measure of wage rates that we calculated based on annual earnings and questionnaire information is lower than 400 DKK, but higher than the minimum wage rate, we use this measure instead. In the end there are only very few (30) men and women for which we have an observed or imputed hourly salary of 0 in our selected sample of households. In order to be able to allocate a positive wage rate (shadow value of time) to all individuals in our sample, we regress log wages on background characteristics (age, education dummies, experience and partner’s wage) for the whole sample. Based on predicted wage rates from this set of regressions, we impute a positive wage rate for the remaining sample.\textsuperscript{15}

\section*{C Construction of market- and housework}

To avoid having to let the sample shrink even further, we fill in missing information on market work, using as a supplement the diary information on market work combined with information given in the questionnaire and registers on labour market status. A small group of individuals reported usual weekly market hours lower than 30, while stating in the diary that they worked more than 30 hours per week and stating in the questionnaire that they worked full time around the survey. In this situation, we use diary information on market hours instead of questionnaire information (6 observations were changed using this extra information). If there is no information in the survey on usual market hours, while respondents reported

\textsuperscript{15}Marginal tax rates from http://www.skm.dk/tal_statistik/tidsserieoversigter/
that they worked full time, and diary hours are above 20 hours per week, then we also use the diary information (this provided market hours for 49 men and 21 women). And if we have missing usual market hours, but the respondent reported working part-time, we also used diary information (this gave information on market hours for 6 observations). Furthermore, if usual market hours were missing, but respondents reported not working at the time of the survey, then market hours are set to 0 (6 households). Finally, for the remaining 13 observations with missing information on market hours for one of the spouses, we use average usual market hours for a comparable group. Consequently, we end up with a sample for which we have information on market hours for all households. We drop households where male market hours are below 30 hours per week (8 households were dropped).

Almost all households (more than 99%) provided information on usual housework. For the remaining few households, we use housework from the diary. One person reported 70 hours of housework per week; we censor housework hours at 56 hours per week.
References


51


