Can Future Uncertainty Keep Children Out of School?

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Abstract

There is little doubt in the literature, that poverty and liquidity constraints can drive children out of school and into child labour in developing countries. But are there other important explanations for low primary school enrolment rates? The child labour and schooling literature often ignores that uncertainty about future returns results in a need for risk diversification, that children function as old-age security providers when there are no available pension systems, that the human capital investment decision of one child is likely to be influenced by that of his/her siblings, and that rural parents face a choice of investing in either specific or general human capital of their children. In this paper, I investigate the effects of future income uncertainty on the joint human capital investment decision of children in a household. I develop and calibrate a simple illustrative human capital portfolio model and show that existing levels of uncertainty can indeed result in less than full school enrolment within a household, even in a world of perfect credit markets. The paper thus offers an alternative explanation for why it might be optimal for rural parents not to send all of their children to school.

Keywords: Schooling, child labour, specific human capital, traditional education, intergenerational transfers, old-age security, uncertainty, income source diversification, liquidity constraints. JEL codes: J13, J24, O15

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1 Introduction

Primary school enrolment rates are low in many developing countries, and generally lower than what policy makers aim for. This is problematic since schooling and human capital is central for economic development. In the economic literature on child labour and schooling, the main explanation for this lack of schooling is the inability of parents to borrow against the future enhanced earnings of children in order to finance their schooling today, e.g. Baland and Robinson (2000), Ranjan (2001), Edmonds (2007). Most rural households live in a high risk environment with incomplete credit and insurance markets, and virtually no social security system. Faced with poverty or periodic income short falls, households have to resort to informal insurance mechanisms to smooth consumption. It is often argued that one important mechanism is adjusting the labour supply of children as a means of ex-post risk coping. Liquidity constrained households thus borrow on the human capital market rather than on the incomplete financial capital market. The focus on the constraints and costs side of the human capital investment decision and on the use of child labour as a means of ex-post risk coping is the essence of the explanations given in the child labour and schooling literature on why enrolment rates are low and child labour widespread. Although these are valid explanations for why some children are kept out of school in rural areas of developing countries, they might not constitute the full explanation. It seems reasonable that households in risk prone environments will, apart from their ex-post risk coping strategies, also consider the possibilities of ex-ante risk diversification.

In this paper, I therefore ask the following question: Can future income uncertainty result in households keeping some of their children out of school as an optimal ex-ante risk diversification strategy? I hypothesise that when there is uncertainty about future income of children and when parents rely on this income for their old-age support, diversifying the future income sources of children becomes an important means of ex-ante risk management. In rural areas, the basis for such a diversification is laid already in the human capital investment decision. Formal schooling will direct children towards future urban employment, whereas traditional on-farm learning-by-doing will direct children towards the agricultural sector. With such a sectoral divide in returns to education, the need for risk diversification, due to future income uncertainty, can result in less than full enrolment into primary schools among siblings being an optimal human capital investment strategy for the household. I find that this is the case even if there are perfect credit markets and schooling is the most profitable human capital investment choice for the individual child.

My main argument, that uncertainty and thus the need for risk diversification influence the joint schooling decision of children in a household, primarily grew out of insights from literatures other than the child labour and schooling literature. These literatures will all be reviewed in turn below, but the key points follow here. When focusing on a broader perspective of the rural household rather than on the direct and indirect costs of schooling of the individual
child, it becomes clear that the following factors may also influence the joint human capital investment decision of children in a household. First, future income is generally uncertain and thus returns to education are uncertain. Second, in risk prone environments with very limited public pension schemes, children may not only play an important role in current ex-post consumption smoothing, but also function as future old-age security assets of their parents. Third, if there is uncertainty about the future income of children, ex-ante risk diversification is an important means of income smoothing. There is thus no apparent reason to assume that parents would consider the human capital investment decision of each child independently of his or her siblings. Rather, if children indeed are the old-age security providers, then parents should seek to optimize the portfolio of joint human capital investment decisions of their children, such that they balance future returns and risk exposure. Finally, work participation of children in household-based agricultural production systems may itself entail an important element of training and, as such, be part of a traditional education. In such a traditional rural environment, parents transfer specific human capital when working with their children, directing these towards future agricultural self-employment. Formal schooling, on the other hand, will direct them towards employment in the modern urban sector, where general human capital skills are needed.

Building on these insights from the literature, I develop an illustrative portfolio model of the joint human capital investment decision of all children in a household. The model is a two-period unitary household model, where parents in the first period decide on the optimal human capital portfolio allocation of their \( N \) children, where the choice is between either general formal education (schooling) or specific traditional education (on-farm learning-by-doing). In the second period, parents depend on the income of their adult children for consumption. The formally educated children will earn income from the urban sector and the traditionally educated children will earn income in the agricultural sector. Second period income is uncertain. In the model I abstract from liquidity constraints and child labour in order to focus on the pure effects of future income uncertainty on schooling. My purpose is not to argue against the influence of poverty and credit constraints on schooling, but rather to complement these existing explanations by analysing the human capital investment decisions of siblings jointly and from an ex-ante risk management perspective. I wish to emphasise that the model is only applicable to rural households where children can be engaged in traditional agricultural production. Child labour is thus viewed solely as work participation in family-based farming. The analysis should not be applied to children working as wage workers or otherwise under hazardous or exploitative conditions.\(^1\)

The analytical results of the model show that future income uncertainty has a negative

\(^1\)See Edmonds (2007) for an overview of which types of economic activities working children engage in. Based on cross-country UNICEF data sources, he estimates that 8% of children are engaged in wage work outside the household.
effect on the proportion of children sent to school. However, this is a qualitative result and it does not indicate whether existing levels of uncertainty could potentially keep some children out of school purely due to future risk diversification, even if households are not liquidity constrained in any way. The model is therefore calibrated using numerical values based on household averages from a national household survey undertaken in Tanzania. As opposed to two recent papers, which have also introduced uncertainty about the returns to schooling\(^2\), I am able to show that a relatively small degree of uncertainty taken from a simple income spread measure is enough for the optimal portfolio choice of the average household to be less than full school enrolment, even in a world with perfect credit markets. Existing levels of uncertainty can indeed result in parents only sending some, but not all children to school. This negative effect on the optimal human capital portfolio allocation can be surprisingly large, even in the presence of perfect credit markets. For the average household, the pure effect of uncertainty is so strong that actual school enrolment rates could, in principle, be explained solely by the existence of uncertainty. Thus, the roots of child labour and lack of schooling need not lie solely with incomplete credit markets and poverty, but could also be caused by the fact that rural households are not only concerned with securing their current, but also their future old-age income. Future income uncertainty may constitute a very important element in the schooling decisions of households and the need for future income source diversification and ex-ante risk management can have direct implications for the optimal composition of a household’s human capital portfolio of children. This adds a new perspective to the child labour debate, which has previously been centered around the need for ex-post consumption smoothing in the liquidity constrained household. These findings have direct policy implications for educational policies, the aim of which tends to be full enrolment into primary school. Policies, which only act on the cost side of the human capital investment decision may be insufficient in terms of reaching full enrolment. It may well be necessary to supplement such policies with some that also act on the return side of the investment decision.

Before turning to the details of the model, the next section looks at how this paper links with existing papers on schooling and child labour, uncertainty about income, intergenerational transfers and sibling dependence. The model is presented in section 3. Three different types of preference structures are considered in slightly lengthly detail, mainly to ensure that prudence is not generating the results. However, there is no indication of this being the case and the use of standard CRRA preferences is probably the most appropriate choice. Calibration results are shown in section 4, and section 5 concludes.

\(^2\)See Pouliot (2005) and Estevan and Baland (2007). Although the latter focuses on mortality risk of young adults, this is in some sense also a source of uncertainty about returns to schooling seen from the parental point of view. However, as Estevan and Baland (2007) argue, young adult mortality risk may in regions of sub-Saharan Africa dominate the intrinsic uncertainty associated with returns to education.
2 Related Literature

As mentioned above, the idea that uncertainty and risk diversification can influence the joint schooling decision of all children in a household grew out of insights from literatures on uncertainty, income and consumption smoothing and risk diversification, on returns to specific versus general human capital, on sibling dependency, and on intergenerational transfers and children as old-age security assets. Drawing on these literature, a broader basis is formed for analysing the human capital investment decisions of a household as a whole, rather than for the individual child.

2.1 Income and consumption smoothing

It is well-known that most rural households in developing countries live in a high risk environment with incomplete credit and insurance markets, very limited public pension schemes and virtually no social security system. In such an environment, children may provide an important source of informal insurance, consumption smoothing and future old-age security. That is, they may play an important role both as providers of additional sources of income, when anticipated income of parents is low in old-age; and in the risk management strategies of the household aimed at shielding consumption from income variations. These strategies are generally two-fold; ex-ante risk management through income smoothing or ex-post risk coping through consumption smoothing, see e.g. Morduch (1995) and Dercon (2002), and for a more detailed analysis see Fafchamps (2003). I return to the role of children as old-age security providers in section 2.6 below.

Ex-ante, households smooth income by diversifying their income sources, labour supply and investments. The farm household diversifies income sources in part by diversifying the household labour supply between on-farm and non-farm economic activities, but also by diversifying the on-farm investments and production portfolio between a variety of crops, land holdings and animal stock. Examples of widespread use of on-farm/non-farm diversification of labour supply are found in Reardon (1997), C.B. Barrett and P.Webb (2001) and Dercon and Krishnan (1996). Morduch (1990), Rosenzweig and Binswanger (1993), and Dercon (1996) all show that both the composition of agricultural investments and the production portfolio are influenced by the degree of income variability faced by a farm household. This results in lower profitability when income variability is high, because production portfolios with less risk exposure and lower returns are chosen in high risk environment.

Ex-post, households shield consumption from idiosyncratic income shocks by obtaining credit, depleting of assets and buffer stocks, readjusting the labour supply of household members, and seeking assistance from the extended family or other informal risk sharing arrangements, see Kotlikoff and Spivak (1981), Townsend (1994) and Udry (1994). However, as
Townsend (1994) showed, households are generally uninsured against covariate income shocks at village level, typically due to adverse weather events. Under such circumstances, spatial diversification of the extended family becomes an important informal insurance arrangement through intergenerational transfers and remittances, see Rosenzweig (1988), Rosenzweig and Stark (1989), and Appelbaum and Katz (1991).

Income and consumption smoothing mechanisms thus have important implications for the allocation of labour and the investment portfolios of a household. The child labour literature reviewed below has a strong emphasis on the role of children in achieving ex-post consumption smoothing through increased child labour rather than schooling. However, the child labour literature is virtually silent, when it comes to analysing the role of children in the ex-ante income smoothing strategies of a household through future income diversification and informal insurance possibilities, as suggested in the fertility literature, see below.

2.2 Child labour and schooling

There is, by now, an impressive number of articles in the child labour and schooling literature, so many that various literature surveys have already been undertaken, see for example Basu (1999), Andvig (2000), Brown, Stern, and Deardorff (2003), Bhalotra and Tzannatos (2003) and Edmonds (2007). I will therefore not even attempt at making an exhaustive review of the literature, but rather focus on the subjects that this paper links with directly.

In general, the literature on child labour and schooling has focussed on one major reason for children being sent to work: binding credit constraints which tend to go hand-in-hand with poverty. Households are not able to cover the current costs of schooling. Most of the literature is based on the intertemporal human capital investment model by Ben-Porath (1967). He simply suggests that each individual must invest in an additional year of education as long as the increase in the discounted future earnings is larger than the current direct costs (e.g. school fees) and indirect costs (foregone earnings) of schooling. It is assumed that the individual can borrow against his/her future earnings to finance each additional year of schooling at perfect capital markets.

However, in developing countries, financial capital markets are far from perfect and the banking sector is almost non-existent. Credit sources are therefore often informal social networks or local moneylenders with high interest rates, see Udry (1994) and Deaton (1997, ch.6.3). Such credit sources seldom provide a plausible means of financing long term human capital investments, although they can be used for smoothing consumption in the short run when faced with income shocks.

Basu and Van (1998), Baland and Robinson (2000) and Ranjan (1999, 2001) all analyse, theoretically, how liquidity constraints can increase child work and reduce schooling because parents are unable to reduce current consumption by the direct and indirect costs of schooling.
due to poverty and they are unable to borrow against the future earnings of their children. The fact that parents cannot borrow against the future income of their children, arise for two reasons. One is the incomplete credit market, which limits intertemporal transfers. The other is the problem of agency, or what Baland and Robinson (2000) model as insufficient levels of altruism between parents and children, which limits intergenerational transfers, see also Parsons (1984), and Becker and Murphy (1988). The agency problem arises from the fact that parents cannot strictly enforce repayment of the educational expenses when children become adults and experience returns to the human capital investments made by parents when young. However, although the theoretical papers, and in particular Baland and Robinson (2000), focus on these two main reasons for child labour and lack of schooling, the corresponding empirical literature has virtually only focussed on the effect of binding credit constraints and poverty. Few papers have analysed the link between child labour and intergenerational transfers, I will return to this below.

Although there is general agreement, theoretically, about the negative effects of poverty and credit constraints on schooling, causal effects and not mere correlations are hard to identify empirically. Some studies have found the expected negative correlations between credit constraints, poverty and schooling, but this is at best suggestive evidence consistent with theory, see Jacoby (1994), Jensen and Nielsen (1997) and Bhalotra (2007) for examples on household data, and Krueger (1996) and Dehejia and Gatti (2002) for cross-country evidence. Yet, other studies have found mixed evidence, no significant correlations or even significantly positive correlations between income or wealth and child labour, see Coulombe and Canagarajah (1998), and Ray (2000). Bhalotra and Heady (2003) emphasise that there can be a 'wealth paradox' in relation to child labour, which arise when there are imperfections in the land and labour markets. If the demand for labour cannot be met, farm households may have to use own labour resources, including those of their children.

A second group of studies have analysed the relationship between poverty and child labour over the full income range. They all find that it can be highly non-monotonic, locally. Edmonds (2005) and Bhalotra (2007) base their theoretical set-up on the notion from Basu and Van (1998) that only households which cannot afford otherwise in terms of subsistence, send their children to work. Edmonds (2005) finds that there is 'dramatic non-linearity' in the relationship between child labour and household expenditure in the neighbourhood of the poverty line. The expected negative relationship generally only appears for households above the poverty line. Bhalotra (2007) finds that sons in Pakistan do indeed engage in wage-work because of subsistence poverty. Rogers and Swinnerton (2004) take a theoretical approach and use the model in Baland and Robinson (2000) to show that rising incomes can lead to more child labour. This happens when income rises enough to reduce old-age transfers from adult children to parents, but not enough for the credit constraints not to bind and thus for parents to send
their young children to school purely out of altruism. The result is that the relationship between income and child labour may be ‘neither monotonically decreasing nor continuous’. All three papers show that there is an overall negative relationship between income and child labour, but local estimates can very well produce a positive or insignificant relationship due to local non-monotonics.

A third group of studies have focussed on estimating the effect of exogenous transitory variations in income on child labour and schooling. By choosing such an estimation strategy, these studies come closer to identification of a causal relation between child labour and income and, thus, of the possible effect of credit constraints and consumption smoothing. Jacoby and Skoufias (1997), Jensen (2000) and Beegle, Dehejia, and Gatti (2006) all estimate the effect of current transitory income shocks, either due to adverse weather or accidental unanticipated crop loss (e.g. due to insects or fire), on human capital investment or child labour. They find clear indications of self-insurance strategies resulting in a reduction of human capital investments and/or increasing levels of child work. These adverse effects of income shortfalls are contributed to the lack of ex-post consumption smoothing possibilities on the local incomplete credit market. Edmonds (2006) propose an alternative way of estimating the effects of credit constraints on child labour and schooling. He uses the timing of a fully anticipated age-dependent increase in income, pensions. If credit markets are complete, the announcement of a permanent increase in income should have an immediate effect on schooling. If credit markets are incomplete and households face borrowing constraints, the effect on schooling will only occur after the increase in income has actually taken place. He finds indications of credit constraints, especially in rural areas.

The literature on how poverty and/or credit constraints affect child labour and schooling decisions concentrates on the need for ex-post consumption smoothing to overcome income fluctuations and current uncertainty. However, in this paper, I argue that the ex-ante need for risk diversification might also be an important factor in the allocation of children’s time between schooling and work. If schooling is considered an investment, any future uncertainty about its return should have an impact on the decision to invest.

2.3 Uncertainty about future returns

A recent issue of Labour Economics (vol 14, issue 6) was devoted to research on education and risk. Although the papers focus on education in the context of a developed country, several points stand out. It is noted that even though investments in human capital are often thought of in the same way as investments in financial or physical capital, the concept of risk in returns or future uncertainty is often missing in the discussion of schooling decisions, e.g. Hogan and Walker (2007). And, importantly, Cunha and Heckman (2007) point to the fact that ex-ante, not ex-post, returns are what agents act on, when making their schooling decision.
In the literature on child labour and schooling in developing countries, very few papers have looked at the effect of future uncertainty. Fitzsimons (2007) estimates the effect of future uncertainty in parental income, predicted by past rainfall variability, on education choices of children. Appelbaum and Katz (1991) analyse a similar problem theoretically. Both papers find negative effects of future uncertainty in parental income on schooling when credit markets are incomplete. Pouliot (2005) uses the Baland and Robinson (2000) model to show that when there is incomplete insurance and uncertainty about returns to education, then the level of child labour will be inefficiently high, even when there are perfect credit markets and no poverty (positive bequests from parents to children in old-age). However, Pouliot (2005) does not consider the effects of uncertainty on schooling and child labour, when parents rely on the income of their children for old-age support, nor does he consider how much uncertainty is necessary for child labour to dominate schooling. Estevan and Baland (2007) argue that only high mortality rates among adult children can generate enough uncertainty for parents to alter their human capital investment decision.

Although this paper is closely related to the models of Pouliot (2005) and Estevan and Baland (2007), it differs in two fundamental ways. First, because the negative effect of uncertainty of schooling is established not only analytically, but also numerically by calibrating the model using household survey data showing that existing levels of income variation is indeed enough to predict strong negative effects of uncertainty on schooling. Second, because the effect of future uncertainty on schooling is analysed for the full set of children at household level.

2.4 Siblings

Allowing for sibling dependency and portfolio effects, which can yield very different predictions compared to one-child models and, not least, provide an alternative explanation for sibling differences. There is a variety of papers analysing sibling differences in educational attainment and child labour. These papers are roughly grouped by two different approaches. One group focus on explaining positive birth order effects on schooling. Different explanations, which are not simply attributed to parental preferences, have been given. If the household faces credit constraints, older children might have to work to help finance the education of the younger siblings, see Willis and Parish (1993), Emerson and Souza (2002) and Manacorda (2006). The birth order effects could also be due to the fertility decision being ruled by the genetic endowment of the last born child. If the youngest child is high-ability, Ejrnæs and Pörtner (2004) argue, then parents are more likely not to have additional children compared to a situation where the youngest child is low ability. This results in a higher probability of schooling among the youngest children. Edmonds (2006b) argue that older siblings (lower birth order) have a comparative advantage over the younger ones in household production and therefore are less likely to be sent to school.
The other group of papers focus on explaining sibling differences in general. Horowitz and Wang (2004) also point to the fact that there might be heterogeneity in the ability of children, which can lead to one child having a comparative advantage over other children in the accumulation of human capital. Dahan and Gaviria (2003) show that differences can also arise, even for completely identical siblings, as long as households are credit constrained and there are increasing returns to human capital investment (e.g. due to sheepskin effects of school diplomas). Their model has a clear empirical implication, very poor households will not be educating any children, middle income households will be educating some and rich households will be educating all children. Their findings from Latin America are broadly consistent with this prediction of the model. Morduch looks at, what he terms, 'sibling rivalry', see Garg and Morduch (1998) and Morduch (2000). He argues that the competition for resources within the household is gender specific and finds that moving from an all-brothers to an all-sisters household can be beneficial in terms of schooling (in Tanzania) or health (in Ghana). Bommier and Lambert (2004) follow up on this and propose a test for whether such dependency among siblings is due to competition for resources or a result of more complicated interactions between siblings, say as being substitutes or complements for each other in the household production function or in the parental utility function. Their empirical findings are in favour of a model with interaction, although their test does not allow them to identify where these interactions originate from.

In the majority of these papers, sibling differences stem from poverty or binding credit constraints and the need for ex-post consumption smoothing. Only Bommier and Lambert (2004) discuss the possibility that sibling differences could arise due to explicit dependencies, rather than dependency arising because of a common credit constraint.

By analysing the joint human capital investment decision for all children in a household, I allow for dependency among siblings. The dependency in the model of this paper stems purely from the need for future risk diversification. Uncertainty about future returns affects the optimal human capital portfolio choice of the household in their balancing of risk exposure against the level of returns. If there is no uncertainty about future returns, the model collapses to a model of $N$ identical and independent children for whom the educational choice will all be the same and thus directly resembles standard child labour models in the literature.

2.5 Specific vs. general human capital

In some of the early economics literature on child labour and schooling, one can still come across more positive aspects of child labour. For instance, in their classic survey, Rodgers and Standing emphasise that '(...) it is important not to confuse schooling with education. Many other activities contribute to education, and some forms of economic activity are among them.’ and '(...) work itself may be an important component of "education" especially in household-based
Bonnet (1993) notes that work participation is part of a traditional educational process in Africa and that this traditional education may offer the best survival prospects for the future, i.e. also better than formal education. Here Bonnet, implicitly, touches upon two different aspects of why children are working. One is the social anthropological aspect of work participation being an important component of the traditional education and the ‘socialisation’ of a child; the other is the economic aspect focusing on the returns to traditional education compared to formal education.

In the social anthropological literature, there is a clear distinction between traditional education based on indigenous knowledge, and formal education based on Western principles. In traditional education, children learn by participating in the work of, in the early years, their mothers and, later for the boys, in the work of their fathers, Bradley (1993). Child labour is regarded as the accumulation of specific human capital through learning-by-doing; it is a way of ‘socialising’ the child, i.e. of adapting it to its environment and teaching it the life skills necessary for survival, Andvig (2000). African parents term it ‘responsibility training’, Agiobu-Kemmer (1992). However, it should be emphasised that this type of traditional education is concentrated in rural areas and less applicable to children in urban areas. Bekombo (1981) notes, ‘the productive activity of a child living in a rural and traditional environment is a means of social integration and cannot be likened to paid work.’ But in a modern urban environment, ‘when children’s work is no longer integrated into an educational system it becomes a “deviant” and "delinquent" activity (...),’ Bekombo (1981, p.114).

Bock (1998, 2002) takes the analysis of the educational element in child work participation one step deeper. He notes that parents are faced with a choice, when allocating their children’s time to different tasks. Some tasks are more complex than others and therefore have a higher learning potential. Parents thus have to make the trade-off between letting their children do simple (often boring) tasks with low learning but an immediate return, or letting them do more difficult tasks with high learning, more supervisory needs and only future returns in the form of higher specific human capital. Child work may therefore not always bring immediate returns as it is generally assumed in the recent economic child labour literature, but might even be costly and time consuming for parents, the stronger the educational element. Bock emphasises that there is a trade-off between task complexity and immediate output within traditional education and that parents are well aware of the need for generating learning opportunities for their children to ensure future agricultural returns.

According to the social anthropological literature, the introduction of formal education based on Western principles has not been unproblematic in Africa. The traditional concept of knowledge was suddenly questioned. Western knowledge is seen as de-contextual and rational, rather than ethical, Daun (1992). It is argued that Western education has induced unfavourable
changes in the behaviour of students away from the African sense of collective concern towards Western individualism, it has weakened the gerontocracies, threatened the continuation of traditional values and way of life, and resulted in brain drain of the rural villages, see for instance Schildkrout (1981), Daun (1992) and Odora (1992). Equally problematic, though, is the perceived lack of returns of schooling, Rodgers and Standing (1981) and Bonnet (1993). Agiobu-Kemmer (1992) notes that where traditional education hardly ever left an individual jobless, formal Western education entails a risk of future urban unemployment. If this is, indeed, the perception or even the reality of formal education in rural Africa that it ‘broadens your mind, but it does not tell you how to survive’ as an African commentator puts it\(^3\), then local reservations toward schooling and a continued emphasis on traditional specific learning is fully understandable.

The economics literature on returns to schooling confirms that there are limited or even no returns to formal education in simple traditional agricultural production systems. A key contribution in this area is Rosenzweig (1995). He argues that there has to be ‘productive learning opportunities’ for schooling to result in positive returns. When the production technology is simple, schooling does not increase productivity. Children accumulate the necessary human capital through specific experience when working along side their parents, Rosenzweig and Wolpin (1985). This is typically the case in traditional agricultural household-based production systems, where best practises have been known for and passed on by generations, Rosenzweig (1996). Returns to formal education are only positive, when new complex technologies are introduced, creating an environment for productive learning opportunities. An example of this is the introduction of high-yielding variety seeds under the Green Revolution in India, where Foster and Rosenzweig (1996) find increasing returns to primary education during periods of technical progress. Fafchamps and Quisumbing (1999) and Jolliffe (2004) confirm the findings by Rosenzweig of low or no returns when agricultural technologies are simple. They use data from rural Pakistan and rural Ghana and show that on-farm returns to education are low, but off-farm returns can be high. This results in a shift of educated labour resources within the farm household away from farm activities and towards non-farm economic activities. Likewise, Fafchamps and Wahba (2006) find that on-farm child labour drops and schooling attendance increases with urban proximity, which they interpret as a reflection of local labour market possibilities. They note that ‘participation in subsistence work - primarily farming - may be seen as a beneficial activity by parents, probably because it teaches important skills to children’, (Fafchamps and Wahba (2006)).

From this dispersed literature on the training component in on-farm child work, there are two main points to emphasise; first that child labour may be an important element in a traditional educational system, which emphasises the accumulation of specific human capital

\(^3\)Agiobu-Kemmer (1992, p.7)
through experience; and second that returns to specific human capital might match or even be higher than returns to general human capital acquired through formal schooling in traditional rural environments. These two points seem largely ignored in the child labour literature, only Bommier and Lambert (2000) and de Vreyer, Lambert, and Magnac (1999) have followed the line of thought of distinguishing between specific and general human capital to explain delayed enrolment into primary schools and sibling differences in educational attainment. Surprisingly, child work is generally modelled purely as an additional current income source, e.g. Basu and Van (1998) and Baland and Robinson (2000) and the papers, which have followed in their wake. Bommier and Dubois (2004) even go one step further and introduce disutility of labour among children without adding the investment aspect. These approaches are highly relevant, when considering disturbing images of hazardous and exploitative child labour or even simple wage work. Less so, when considering children engaged in traditional agricultural work on the familily-run farms or household plots. Indeed, the vast majority of the many working children in Sub-Saharan Africa are engaged in these household-based production systems, see Bhalotra and Tzannatos (2003).

In this paper, there is a clear distinction between traditional and formal education that is between specific human capital aimed at the agricultural sector and general human capital aimed at the modern urban sector. Child labour is thus seen as an educational alternative to formal schooling with different future prospects. My purpose is not to argue against the importance of child work in overcoming poverty, credit constraints and income shocks, but simply to point to the fact that the role of children and their economic activities might be more complex than that in a traditional agricultural environment.

### 2.6 Intergenerational transfers and children as old-age security

A central assumption in the portfolio model in section 3 is that parents depend on the income of their children for old-age support. This assumption is based on the fertility literature, and supported by empirical literature on intergenerational transfers.

In the fertility literature, the argument for having children often extends beyond a pure consumption argument of parents deriving utility from having children, just as they derive utility from consuming goods. This is especially the case, when analysing fertility decisions of households faced with considerable risk, incomplete credit and insurance markets and highly inadequate or no public pension or social security schemes. In such an environment, it is often argued that children may function as security assets. Generally, the old-age security aspect of children is emphasised and Nugent (1985) is, by now, a classic reference on the subject. Children may also function as security assets in terms of insurance, because their future income sources represent additional risk diversification possibilities, in particular Appelbaum and Katz (1991)

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4They also do not consider the possibility that children might experience disutility of schooling.
emphasise the risk diversification aspect, but Cain (1981, 1983) and Pörtner (2001) also discuss the insurance role of children. In the fertility literature, children are thus naturally considered as part of the ex-ante risk management strategies of a household. If children indeed play the role of security assets, this is likely not only to affect fertility, but also the human capital investments in these children.

In the child labour and schooling literature, the old-age security motive for investing in the general human capital of children, has often been dismissed due to agency problems, see e.g. Udry (2004). That is, it is impossible for parents and children to engage in an intergenerational enforceable contract of parents financing the human capital investments of children in return for future old-age transfers, Parsons (1984) and Becker and Murphy (1988). Thus, unless there are high degrees of altruism between parents and children, old-age support is not seen as a motive for human capital investments, e.g. Baland and Robinson (2000).

Nugent (1985) is aware of the problems of agency, in what he terms, loyalty of children to their parents in old-age. He claims, however, that there is scope for loyalty training, which, he argues, is facilitated by cultural norms in traditional societies. Norms is often argued to be an effective means of overcoming agency problems, see for instance De Vos (1985) and Lucas and Stark (1985), but also Lassen and Lilleør (2008) for a more recent discussion.

Despite possible agency problems, there is ample empirical evidence that intergenerational transfers from children to parents do occur, e.g. Lee, Parish, and Willis (1994) and Lillard and Willis (1997, 2002). And some suggestive evidence that such transfers are in fact part of an informal old-age support system, Nugent and Gillaspy (1983) and De Vos (1985). More recent studies achieve better identification of this informal support system, because they show that the introduction of public security schemes, at least partially, crowd out private transfers, see Cox and Jimenez (1992) for evidence from Peru, and Jensen (2003) for even more robust evidence from South Africa. It therefore seems reasonable to assume that parents rely on some support from their children in old-age, although they might not be able to fully control it.

Recently, a few theoretical papers on child labour and schooling have acknowledged the importance of future intergenerational transfers for the human capital investment decisions today. Rogers and Swinnerton (2004) use the link between schooling and expected future transfers from children to parents to show that the relationship between child labour and parental income need not be monotonically decreasing, see above. Chakraborty and Das (2005) argue that there is positive relationship between life expectancy and human capital investment, because only parents that actually reach old-age will be able to benefit from their educational investments in their children. Raut and Tran (2005) suggest that if intergenerational transfers...
are simply an alternative means of financing schooling, then parental investment in education is socially optimal. Although, if intergenerational transfers are based on altruism and reciprocity, then some parents will underinvest in their children’s human capital and there is scope for policy intervention. Their findings, using Indonesian data, support the latter hypothesis. These three papers are the first attempts at establishing a link between the literatures on child labour and intergenerational transfers. By adding uncertainty about future income of children to the equation, this paper is an additional contribution to such a link.

3 Theoretical Framework

The model developed in this paper differs from most of the models in the existing child labour literature in four ways. First, the model introduces uncertainty about the future returns to education, i.e. about children’s future income. Second, parents rely on the future income of their children for old-age support. This gives parents a clear incentive to choose an optimal human capital portfolio of their children in terms of balancing returns and risk exposure, given their degree of risk aversion. Third, the model is not a one parent - one child model of human capital investment, but rather a one parent - \( N \) children model, where the human capital investment decision of children is modelled jointly, thus allowing for sibling dependence. Fourth, there is a clear distinction between general human capital acquired through schooling and specific human capital acquired through work experience. Child labour is thus modelled as an educational alternative, which directs children towards future agricultural income sources, whereas formal schooling directs children towards future urban income sources.

A theoretical framework is designed, which emphasises the effect of future uncertainty and the need for risk diversification on the allocation of children between schooling and labour in a household. To exhibit clearly what the effects of uncertainty and risk diversification are, I begin by abstracting from the conventional explanations for child labour and low school enrolment. That is, I assume that credit markets are perfect, such that households do not face any liquidity constraints, and that there are no agency problems between generations, such that parents can rely on full old-age support from their children. Later both liquidity constraints and child labour are introduced allowing me to compare model predictions under different scenarios.

The basic model set-up gives a general understanding of how uncertainty can affect the human capital investment allocation. By specifying a simple preference structure and the sources of uncertainty, it is possible to arrive at closed form solutions. It is straightforward to show analytically that uncertainty about future returns can have a negative effect on schooling both in a one-child model and for \( N \) children. However, the question of interest is whether the negative effect is large enough for the model to predict lower levels of schooling given realistic levels of uncertainty about children’s future income. In section 4, the model is therefore
calibrated using data driven numerical values for a variety of different preference structures and under different scenarios.

3.1 The basic model

The model is a two period unitary household model, where parents function as a unified sole decision maker. There is no discounting of the future and no interest rate on savings or credit. In the first period, parents earn agricultural income $Y_1$, which they allocate between first period household consumption $c_1$, savings $s$, and the education expenses for their $N$ children. $N$ is assumed to be exogenously given, since the emphasis here is not on the effect of uncertainty on fertility decisions, but on the effect of uncertainty on the joint human capital investment decision of children, given the fertility of the household.$^6$

There are two types of education in the model, general formal education achieved through primary schooling and specific traditional education achieved through on-farm learning-by-doing. Traditional education directs children towards future employment in the agricultural sector ($a$), whereas formal education directs children towards future employment in the non-agricultural urban sector ($b$) in the second period. Parents thus face a discrete choice for each of the $N$ children of whether he or she should be educated traditionally or formally. A child can only receive one type of education$^7$. In the second period, traditionally educated children earn agricultural income, $y_a^2$, whereas formally educated children earn urban income, $y_b^2$.

Parents do not generate any income in the second period, but rely fully on their savings and the joint agricultural and urban income transfers from their $N$ children for second period household consumption, $c_2$. Second period income is uncertain. Parents therefore maximise a joint von Neuman-Morgenstern expected utility function defined over and separable in household consumption, $c_t$, where $t = 1, 2$. The utility function is assumed to be concave, such that $U'(c) > 0$ and $U''(c) < 0$. The household solves the following maximisation problem

$$\max_{c_1, c_2} EW(c_1, c_2) = U(c_1) + EU(c_2)$$

subject to the budget constraints for period 1 and period 2, respectively

$$c_1 = Y_1 - (1 - \pi)N e^a - \pi Ne^b - s$$
$$c_2 = N^{-\alpha}((1 - \pi)Ny^a_2 + \pi Ny^b_2) + s$$

$^6$It is conceivable that the fertility decision and the human capital investment decision of the born and unborn children are both influenced by the parents’ preference for old-age security, which suggests modelling the two decisions jointly. However, to keep things simple, I focus on the effect of future income uncertainty on the human capital investment decision of children conditional on the household having completed their fertility.

$^7$This is a simplifying assumption. The choice here is not on how many hours a child spends in school or working, but rather whether he or she graduates with full primary school education or not.
where $\pi$ is the proportion of children, which parents have chosen to educate formally through schooling. That is, $\pi$ is the portfolio allocation of children between traditional and formal human capital investments. The number of children who receive schooling in the first period is thus given by $\pi N$ and the number who are educated within the traditional agricultural based system is $(1 - \pi)N$. The total amount of educational expenses is $(1 - \pi)Ne^a + \pi Ne^b$, where $e^a$ is the educational expenditure for each child in traditional education, e.g. supervisional costs of parents, and $e^b$ is the educational expenditure for each child in formal education, e.g. tuition fees and uniform costs. Educational expenditures are allowed to differ over the two sectors, and they are considered both non-negative.

Savings can be negative, and both the discount rate and the interest rate are normalised to unity and are thus explicitly left out of the model for simplicity. By assuming perfect credit markets, I can ignore any effect of liquidity constraints on the schooling decision and thus focus on the effect of future income uncertainty on the joint human capital portfolio decision of all $N$ children in the household. The question is: can this alone result in less than full school enrolment among siblings, i.e. a model prediction of $\pi < 1$ solely due to uncertainty.

Second period consumption will equal any capital transfers from period one in terms of savings or dissavings, $s$ plus a fraction, $1/N^\alpha$, of total income of all children, which is given by the income of children in the agricultural sector $(1 - \pi)Ny^a_2$, and the income of children in the urban sector $\pi Ny^b_2$. Children are thus assumed to transfer a certain fraction of their income to their parents. The fraction is the same for all children, irrespective of their sector of employment, but it depends on their number of siblings for $\alpha > 0$. In principle, $\alpha \in [0; 1]$, but in the following I will assume that $\alpha \in [0; 1]$ to ensure that there is a positive, but diminishing marginal effect of having more children on second period income. When $\alpha = 0$, children share all their income with their parents. When $\alpha = 1$ children share only a fraction $1/N$ of their income with their parents, resulting in parents receiving the equivalent of one full income from their children in total. If there is only one child in the household that child will be the sole breadwinner of the family in the second period and is forced to share his/her full income with the parents, irrespective of the size of $\alpha$.

Parents are faced with two choice variables; how much to save or dissave $s$, and which proportion of their children to educate formally through schooling $\pi$. The first order condition with respect to $s$ is

$$U'(c_1) = EU'(c_2)$$

---

8For analytical simplicity, $\pi$ is written as continuous in the theoretical model, but it will be treated as discrete in the calibrations and in the empirical model.

9While the literature on child labour and schooling generally set $e^a$ as negative and thus as a source of income, I here follow Bock (2002) in stating that the overall learning potential in the tasks completed by children in agriculture is higher than the immediate return. If children were only undertaking tasks with no learning, but high immediate output, such as fetching water or firewoods, there would be no transfer of farm-specific human capital from parents to children and therefore no future agricultural return from such activities.
That is, savings will be chosen such that marginal utility in period one equals the expected marginal utility of period two. The first order condition with respect to $\pi$ is given by equation (4), where $\pi^*$ is the optimal solution for the maximisation problem above

\[
N(e^b - e^a)U'(c_1) = E[N^{1-\alpha}(y_2^b - y_2^a)U'(c_2)], \quad \text{for } 0 < \pi^* < 1
\]
\[
N(e^b - e^a)U'(c_1) > E[N^{1-\alpha}(y_2^b - y_2^a)U'(c_2)], \quad \text{for } \pi^* = 0
\]
\[
N(e^b - e^a)U'(c_1) < E[N^{1-\alpha}(y_2^b - y_2^a)U'(c_2)], \quad \text{for } \pi^* = 1
\]

where

\[
E[N^{1-\alpha}(y_2^b - y_2^a)U'(c_2)] = E(N^{1-\alpha}(y_2^b - y_2^a))EU'(c_2) - \text{cov}(N^{1-\alpha}y_2^b, U'(c_2))
\]

Uncertainty about second period income results in two covariance terms, both negative, between the second period income variables, $y_2^a$ and $y_2^b$, and marginal utility, $U'(c_2)$. These terms will, when they are strong enough, pull the optimal portfolio allocation, $\pi^*$ away from each of the two corner solutions. Uncertainty in the agricultural sector will have a positive effect on $\pi^*$ because it will increase the right hand side of the first order condition for $\pi$ and pull towards the $\pi^* = 1$ corner solution. Uncertainty in the urban sector, on the other hand, will have a negative effect on $\pi^*$ because it will decrease the right hand side of the the first order condition for $\pi$ and thus pull towards the $\pi^* = 0$ corner solution.

In the following, I assume that there is no covariant uncertainty between second period income from children in the urban sector and children in the agricultural sector. This allows me to simplify the problem by normalising uncertainty about income from the agricultural sector to zero, and thus solely focus on the effect of uncertainty of urban income on the optimal proportion of children in formal schooling. Going back to the first order condition for $\pi$, equation (4), this means concentrating on the covariance term, which can reduce the right-hand side of the first order condition and thus reduce the optimal $\pi^*$. That is, focusing on the somewhat more relevant question of what can result in an optimal $\pi^*$ below 1, rather than what can result in an optimal $\pi^*$ above 0.

This is not to say that there is no uncertainty in the agricultural sector, but rather that uncertainty associated with income transfers from distant migrant children in the urban sector is higher. These migrant children may face higher income levels, but also relatively more variation, since the urban labour market entails a risk of unemployment, which is not present among subsistence farmers in the agricultural sector. Furthermore, parents may also perceive the size and the frequency of income transfers from urban migrant children to be more uncertain compared to the daily support and in-kind assistance from home children engaged in local agricultural sector. The uncertainty, that parents face about income transfers from migrant

\[\text{The uncertainty could thus also, in effect, be an intergenerational agency problem between parents and} \]
children in urban sector is modelled as a simple mean-preserving spread. Each migrant child can either get a good (typically formal sector) job or not, where the probability of a good draw in the urban labour market is given by \( p = 0.5 \). Migrant children in good jobs have an urban income of \( y^b_2 = \mu + \varepsilon \), whereas migrant children without good jobs have an urban income of \( y^b_2 = \mu - \varepsilon \). This means that second period urban income is given by

\[
y^b_2 = \begin{cases} 
\mu + \varepsilon & \text{w.p. } p = 0.5 \\
\mu - \varepsilon & \text{w.p. } (1 - p) = 0.5 
\end{cases}
\]

The mean and the variance for each child in the urban sector is \( E(y^b_2) = \mu \) and \( \text{Var}(y^b_2) = \varepsilon^2 \). Given this specification of uncertainty, the first order condition for \( \pi \) rewrites (4) as

\[
N(e^b - e^a)U'(c_1) = N^{1-\alpha}(\mu - y^a_2)EU'(c_2) + \text{cov}(N^{1-\alpha}y^b_2, U'(c_2)) - 0
\]

where the specification of the covariance term will depend on the degree of risk correlation in the urban labour market outcome. The expected total income transfers from all the \( \pi N \) children, which have gone to the urban sector, is simply \( E(\pi N^{1-\alpha}y^b_2) = \pi N^{1-\alpha}\mu \), independent of the degree of risk correlation among migrant siblings. But the variance of their expected total income, \( \text{Var}(\pi N^{1-\alpha}y^b_2) \) and the covariance above, \( \text{cov}(N^{1-\alpha}y^b_2, U'(c_2)) \) will both depend on the degree of risk correlation in urban income.

I consider the two extremes where income transfers from siblings in urban employment are either perfectly correlated or uncorrelated. Reality is likely to lie somewhere in between. When there is perfect risk correlation among siblings in urban employment, all siblings will either have a good draw and then their income transfers will amount to \( \pi N^{1-\alpha}(\mu + \varepsilon) \), or they will all have a bad draw and then their income transfers will amount to \( \pi N^{1-\alpha}(\mu - \varepsilon) \), hence the variance is \( \text{Var}(\pi N^{1-\alpha}y^b_2) = \pi^2 N^2 2^{1-2\alpha} \varepsilon^2 \). When there is no risk correlation among siblings, they all face the same urban labour market lottery irrespective of the labour market outcomes of their siblings. The variance under no risk correlation is thus smaller and depends on the binomial coefficient \( \binom{\pi N}{i} \), where \( i \) denotes the number of successful siblings in the urban labour market (i.e. those where \( y^b_2 = \mu + \varepsilon \)) and \( \pi N \) is the total number of siblings in the urban sector in the second period, \( \text{Var}(\pi N^{1-\alpha}y^b_2) = N^{1-\alpha} \sum_{i=0}^{\pi N} \binom{\pi N}{i} \frac{1}{2\pi N^2} (i\varepsilon - (\pi N - i)\varepsilon)^2 = \pi N^{1-\alpha} \varepsilon^2 \).

As long as uncertainty in the agricultural sector and the urban sector do not covary, house-migrant children. Their degree of success is harder to monitor and lack of family control increases with the distance. Social sanctions are often mentioned as effective means in overcoming such agency problems and thereby helping to reduce at least one source of future uncertainty. Lassen and Lilleør (2008) analyse the effect of such sanctions on the demand for formal schooling.

\[\text{I do not explicitly consider a mortality risk of young adults as in Estevan and Baland (2007). However, the model could easily be extended to include such risk, but if mortality risk is exogenous to choice of education, it would simply just add a higher level of uncertainty in both the agricultural and urban sector. The qualitative findings of the model would not change.}\]
holds will have an incentive to diversify their human capital investments to reduce future risk exposure. If the need for diversification is strong enough, this will have a negative impact on the proportion of children sent to school in the optimal human capital portfolio of the household.

3.2 Specification of preferences

The choice of preference structure and degree of risk aversion is crucial for the model predictions. In the following, analytical results are derived for the quadratic utility function to allow for risk aversion without prudence. Prudence is introduced later, first by introducing a very small cubic term in the quadratic utility function, and second simply by looking at a standard CRRA utility function, which incorporates both risk aversion and prudence. Analytically, a model with quadratic preferences is much more tractable than CRRA preferences, making it possible to arrive at an analytical solution for \( \pi \) and to look at its derivatives. Numerically, however, there is no difference in tractability, and, CRRA preferences are likely to be a more realistic preference structure. An additional benefit of CRRA preferences is that only one parameter needs to be determined exogenously, the relative degree of risk aversion, \( \gamma \). The model is calibrated for all three types of preferences in section 4, but the reported results will be mainly on the model predictions based on CRRA preferences.

3.2.1 Quadratic utility

It seems plausible to expect households in developing countries to be both risk averse and prudent. However, to keep these two matters apart and to ensure that results are not driven by prudence in the preference structure, but only by risk aversion, assume for now that the utility function is quadratic and thus that the third derivative is zero, i.e. no prudence. This implies that there is certainty equivalence in the marginal utility, \( E(U'(c_t)) = U'(E(c_t)) \), since marginal utility is linear in \( c_t \). Define

\[
U(c_t) = M c_t - \frac{1}{2} \gamma c_t^2
\]

for both periods. \( M \) is the bliss point of maximum consumption. So utility increases in \( c_t \), \( U'(c_t) = M - \gamma c_t > 0 \), but at a decreasing rate, \( U''(c_t) = -\gamma < 0 \) and \( U'''(c_t) = 0 \). It should be noted that the quadratic utility function does not belong to the class of CRRA or CARA utility functions, but has the rather awkward feature of increasing absolute risk aversion, when the consumption level increases. I will return to this below.

Given the quadratic utility function, the first order condition for \( s \) simply rewrites as

\[
M - \gamma c_1 = M - \gamma E c_2
\]
so the perfect credit market ensures that consumption in period 1 equals the expected consumption in period 2. From this it is also clear that in this simple model, endogenous \( N \) would result in an infinite number of children in each household as long as second period earnings are higher than first period education expenditures. Thus, since the choice of schooling is the focus of this analysis, and not the fertility choice, \( N \) is modelled as an exogenous variable.

The first order condition for the proportion of children in schooling, \( \pi \) under perfect risk correlation becomes

\[
N(e^b - e^a)(M - \gamma c_1) = N^{1-\alpha}(\mu - y_2^a)(M - \gamma Ec_2) - \gamma \pi N^{2-2\alpha} \varepsilon^2
\]

and the equivalent equation under no risk correlation among siblings in second period urban income is given by

\[
N(e^b - e^a)(M - \gamma c_1) = N^{1-\alpha}(\mu - y_2^a)(M - \gamma Ec_2) - \gamma \pi N^{1-2\alpha} \varepsilon^2
\]

Thus, only the covariance terms differ for these first order conditions for \( \pi \). Under perfect risk correlation \( \text{cov}(Ny_2^b, U'(c_2)) = -\gamma \pi N^{2-2\alpha} \varepsilon^2 \), and under no risk correlation \( \text{cov}(Ny_2^b, U'(c_2)) = -\gamma \pi N^{1-2\alpha} \varepsilon^2 \), see appendix A1.

The first order conditions are given by two equations in two unknowns, \( s \) and \( \pi \), which can be solved for analytically. When there is perfect risk correlation among siblings in urban employment, the optimal educational allocation for the household in period one will be

\[
\pi^*_{\text{cor}} = \frac{\Delta (N^{\alpha}2M - \gamma(N^{\alpha}Y_1 + Ny_2^a - N^{1+\alpha}e^a))}{\gamma [N\Delta^2 + 2N\varepsilon^2]}
\]

where \( \Delta = (\mu - y_2^a) - N^{\alpha}(e^b - e^a) \). The corresponding choice under no risk correlation among urban employed siblings is

\[
\pi^*_{\text{uncor}} = \frac{\Delta (N^{\alpha}2M - \gamma(N^{\alpha}Y_1 + Ny_2^a - N^{1+\alpha}e^a))}{\gamma [N\Delta^2 + 2N\varepsilon^2]}
\]

If formal education is more costly than traditional education, but also sufficiently more profitable in expectation such that \( \Delta > 0 \), then \( \pi^* \) will always be positive, the question is if it will ever be less than unity. From equation (6) and (7), it is clear that \( \pi^*_{\text{cor}} < \pi^*_{\text{uncor}} \), the optimal allocation of children into formal education will always be lower when there is perfect risk correlation, compared to no risk correlation, among urban employed siblings. The optimal choice of savings will differ correspondingly, \( s^*_{\text{cor}} > s^*_{\text{uncor}} \). Only when there is no uncertainty, \( \varepsilon = 0 \), or only one child in the household, \( N = 1 \), will \( \pi^*_{\text{cor}} = \pi^*_{\text{uncor}} \). It should be noted that if \( \varepsilon = 0 \) and \( N = 1 \), then this model collapses to a standard model of human capital investment used in the child labour literature. Since there are no liquidity constraints or agency problems,
the model will always predict full school enrolment when there is no uncertainty, irrespective
of the number of children in the household as long as returns to formal education are higher
than returns to agricultural education that is as long as \(\Delta > 0\).

The real question of interest here is whether uncertainty alone is enough to drive \(\pi\) below
unity even under perfect credit markets. From the analytical solutions for \(\pi^*\), (6) and (7), it
is clear that an increase in uncertainty measured by \(\varepsilon\) or similarly an increase in the variance
of urban income, \(\varepsilon^2\), will always have a negative effect on the optimal proportion of children
in formal education, \(\pi^*\). Under perfect risk correlation among siblings in the urban labour
market, the derivate is given by

\[
\frac{\partial \pi^*_\text{cor}}{\partial \varepsilon^2} = \frac{-2N\Delta(N^\alpha Y_1 + N\varepsilon_2 - N^{1+\alpha}e^\alpha)}{\gamma[N\Delta^2 + 2N\varepsilon^2]^2} < 0
\]

and under no risk correlation by

\[
\frac{\partial \pi^*_\text{uncor}}{\partial \varepsilon^2} = \frac{-2\Delta(N^\alpha Y_1 + N\varepsilon_2 - N^{1+\alpha}e^\alpha))}{\gamma[N\Delta^2 + 2\varepsilon^2]^2} < 0
\]

However, although the partial derivative of \(\pi^*\) with respect to \(\varepsilon\) is clearly negative and stronger
under perfect risk correlation than in the uncorrelated case, it is uninformative about the size
of \(\varepsilon\) necessary for the model to predict an optimal \(\pi^*\) below unity. To answer such question,
numerical solutions are needed, for this see calibration results in section 4.

Another partial derivative of interest is the effect of belonging to a household with more
children, compared to one with less, on the optimal proportion of children in school, all else
equal. Given the portfolio approach in setting up the model, intuition says that the optimal
proportion of children in school should be reasonably constant for varying levels of \(N\) once \(N\)
is large enough to allow for some flexibility in the somewhat discrete \(\pi^*\). E.g. for \(N = 2\), \(\pi^*\)
can only take the following three values \([0, \frac{1}{2}, 1]\). Irrespective of the degree of risk correlation,
the derivatives cannot be signed, indicating either a non-monotonic relationship or simply a not
very strong relationship. The partial derivatives with respect to \(N\) is given by

\[
\frac{\partial \pi^*_\text{cor}}{\partial N} = \frac{\alpha \Delta'(2N^\alpha M - \gamma(N^\alpha Y_1 - N^{1+\alpha}e^\alpha)) - \Delta''\gamma N^2 \varepsilon_2 + \Delta \gamma N^{1+\alpha}e^\alpha}{\gamma N[N\Delta^2 + 2N\varepsilon^2]} - \frac{\Delta(2N^\alpha M - \gamma(N^\alpha Y_1 + N\varepsilon_2 - N^{1+\alpha}e^\alpha))((\Delta^2 - 2\alpha N^\alpha(e_b - e^\alpha)\Delta + 2\Delta^2)\leq 0}{\gamma[N\Delta^2 + 2N\varepsilon^2]^2}
\]

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under perfect risk correlation, and under no risk correlation by

\[ \frac{\partial \pi_{uncor}^*}{\partial N} = \frac{\alpha \Delta' (2N^\alpha M - \gamma (N^\alpha Y_1 - N^{1+\alpha} e^a)) + \Delta'' \gamma N y_2^a + \Delta \gamma N^{1+\alpha} e^a}{\gamma [N \Delta^2 + 2 \varepsilon^2]} \]

where both \( \Delta' = (\mu - y_2^a) - 2N^\alpha (e^b - e^a) \) and \( \Delta'' = (\mu - y_2^a) - (1 + \alpha)N^\alpha (e^b - e^a) \) are positive. These partial derivatives are of particular interest when compared to the ones produced by a similar model with liquidity constraints. Liquidity constraints are likely to create sibling rivalry over the limited resources, as suggested by the literature reviewed above, and one should expect a clear negative effect of coming from a household with more children compared to one with less when both households are liquidity constrained, see section 3.3.

Finally, the model can also easily be extended to show the recently much debated empirical result of non-monotonicity in income\(^{12}\). Since the model only applies to rural households, it is reasonable to assume that the earning abilities of children working in the agricultural sector in the second period are positively correlated with the income generated by their parents in the same sector in the first period. Such a positive relationship can be expected partly because parents transfer specific human capital to their children when educating them traditionally, and partly because children entering the agricultural sector would typically be endowed with parental farm land or other local land with similar characteristics and thus similar earning potentials, see Rosenzweig and Wolpin (1985). By simply defining second period agricultural income as a function of parental first period income, such that \( y_2^a = f(Y_1), f' > 0 \), non-monotonicity of proportion of children in school and parental first period income is generated. The partial derivative of \( \pi \) with respect to \( Y_1 \) becomes ambiguous.

\[ \frac{\partial \pi^*}{\partial Y_1} = \frac{-f'(Y_1)(2N^\alpha M - \gamma (N^\alpha Y_1 + N f(Y_1) - N^{1+\alpha} e^a) + \Delta \gamma N \Delta) - \gamma N^\alpha \Delta}{\gamma \Phi} + \frac{2N \Delta^2 (2N^\alpha M - \gamma (N^\alpha Y_1 + N f(Y_1) - N^{1+\alpha} e^a) f'(Y_1))}{\gamma \Phi^2} \leq 0 \]

where \( \Phi = N \Delta^2 + 2 \varepsilon^2 \) under perfect risk correlation and \( \Phi = N \Delta^2 + 2 \varepsilon^2 \) under no risk correlation.

The non-monotonicity result is rather intuitive. If the agricultural sector generates high levels of income, traditional education becomes a relatively more attractive alternative to formal education, which will shift \( \pi^* \) more towards zero and thus change the composition of the optimal household human capital portfolio away from schooling. This is particularly interesting in the

\(^{12}\)See Bhalotra (2002), Bhalotra and Heady (2003), Edmonds (2005) and Rogers and Swinnerton (2004), as well as section 2.2 for a discussion of these references.
case where liquidity constraints are binding, because the positive effect of higher parental income is then counterbalanced by the agricultural sector becoming relatively more profitable compared to the urban sector and thus generates an inverse U relationship between $\pi^*$ and $Y_1$, see section 3.3.

It should be noted that under quadratic preferences and no liquidity constraints, the direct effect of an income increase in $Y_1$ without considering the correlation with $y_a^2$ has, counterintuitively, a negative effect on $\pi$. Since $\pi^*$ is already at its optimum regardless of first period income, an income increase translates directly into a consumption increase and thus an increase in risk aversion. There is then an overall negative impact on investment in the risky compared to the risk free asset. This is, as mentioned above, a rather awkward feature of the quadratic utility function. Although quadratic preferences are more tractable analytically, they are less attractive because they lack the constant relative risk aversion characteristic over consumption. However, before turning to the more common class of CRRA utility functions, I will briefly analyse the effect of prudence on the optimal human capital portfolio of the household.

### 3.2.2 Cubic utility

The quadratic utility function was chosen to ensure that the existence of prudence is not in itself generating the results, and it will be shown below that the effects of prudence might be somewhat surprising. In order to be able to analyse the direct effects of prudence on the human capital investment decisions of the household, I will simply add a small cubic term to the quadratic utility function in equation (5). This introduces prudence, as the third derivative is now positive.

The cubic utility is given by

$$U(c_t) = M c_t - \frac{1}{2} \gamma c_t^2 + \frac{1}{6} \eta c_t^3$$  \hspace{1cm} (8)

Where the prudence parameter is $\eta$, which is very small and positive. Now $U'(c) = M - \gamma c + \frac{1}{2} \eta c^2 > 0$, $U''(c) = -\gamma + \eta c < 0$ (by assumption on the size of $\eta$), and the third derivative is positive and given by the prudence parameter, $U'''(c) = \eta > 0$. Notice that there is no longer certainty equivalence in the marginal utility due to the positive prudence parameter $EU''(c_2) > U'(Ec_2)$.\textsuperscript{13} This utility function is only well behaved for very small values of $\eta$, which is all that is needed for determining the effect of introducing prudence on the household proportion of children in school, $\pi$. This is simply given by the derivative of $\pi$ with respect to $\eta$ measured at $\eta = 0$, $\left. \frac{\partial \pi}{\partial \eta} \right|_{\eta=0}$. The optimal portfolio allocation $\pi^*$ and savings level $s^*$ under prudence are found by solving the two first order conditions. The maximisation problem is the same as above. Under perfect risk correlation in the labour market outcome among urban

\textsuperscript{13}See appendix A2.
siblings, the first order conditions with respect to \( s \) and \( \pi \), (3) and (4), are now

\[
M - \gamma c_1 + \frac{1}{2} \eta c_1^2 = M - \gamma E c_2 + \frac{1}{2} \eta (E c_2)^2 + \frac{1}{2} \eta (\pi N^{1-\alpha} \varepsilon)^2
\]

\[
N(e^b - e^a) U'(c_1) = N^{1-\alpha} (\mu - y^2_2) \left( U'(E c_2) + \frac{1}{2} \eta (\pi N^{1-\alpha} \varepsilon)^2 \right) - (\gamma - \eta (\Gamma + \mu)) \pi N^{2-2\alpha} \varepsilon^2
\]

respectively, where \( EU'(c_2) = M - \gamma E(c_2) + \frac{1}{2} \eta E(c_2)^2 + \frac{1}{2} \eta (\pi N^{1-\alpha} \varepsilon)^2 \).

And the corresponding first order conditions under no risk correlation are

\[
M - \gamma c_1 + \frac{1}{2} \eta c_1^2 = M - \gamma E(c_2) + \frac{1}{2} \eta E(c_2)^2 + \frac{1}{2} \eta \pi N^{1-2\alpha} \varepsilon^2
\]

\[
N(e^b - e^a) U'(c_1) = N^{1-\alpha} (\mu - y^2_2) \left( U'(E c_2) + \frac{1}{2} \eta \pi N^{1-2\alpha} \varepsilon^2 \right) - (\gamma - \eta (\Gamma + \mu)) \pi^2 N^{1-2\alpha} \varepsilon^2
\]

for \( s \) and \( \pi \), respectively, and \( EU'(c_2) = M - \gamma E(c_2) + \frac{1}{2} \eta E(c_2)^2 + \frac{1}{2} \eta \pi N^{1-2\alpha} \varepsilon^2 \). See appendix A2 for derivations. Again, this gives two equations, which can be solved for the two unknowns, \( s^* \) and \( \pi^* \).

It can then be shown, through implicit derivation of the analytical solutions for \( \pi^* \) with respect to \( \eta \) that introducing prudence will have a positive effect on the proportion of children sent to school, \( \frac{\partial \pi^*}{\partial \eta} \bigg|_{\eta=0} > 0 \). This may seem puzzling, since schooling is the more risky investment. However, by setting up the cubic utility function, risk aversion and prudence are two separate parameters. Prudence increases the preferences for precautionary savings and, somewhat surprisingly, at the same time \( \eta \) has a negative impact on the relative risk aversion. This can be seen from the specification of the degree of relative risk aversion under cubic preferences: \(-cU'''(c)/U'(c) = c(\gamma - \eta c)/(M - \gamma c + \frac{1}{2} \eta c^2)\). Introducing prudence thus makes it optimal for the household to reduce consumption today and postpone it for the future, which here results in allocating a larger proportion of children to the more costly and more risky type of education, schooling.

### 3.2.3 CRRA utility

The constant relative risk aversion (CRRA) utility functions are among the most commonly used utility functions. They allow for the presence of both risk aversion and prudence at the same time, and as the name indicates, the relative degree of risk aversion does not change as consumption levels increase, contrary to the quadratic utility function. It is therefore likely to be a more realistic preference structure. Especially so, when looking at poor households in developing countries. Analytically, however, the standard CRRA utility function is less tractable than the quadratic utility function. The comparison of the two sets of preferences will therefore be based on the calibration results, rather than on the analytical results.
The CRRA utility function used in the calibrations below is given by

\[
U(c_t) = \begin{cases} 
\frac{c_t^{1-\rho}}{1-\rho}, & \text{for } \rho \neq 1 \\
\ln(c_t), & \text{for } \rho = 1 
\end{cases}
\]

The constant relative risk aversion parameter is given by \( \rho = -cU''(c)/U'(c) \), where \( U'(c) = c^{-\rho} \) and \( U''(c) = -\rho c^{-\rho-1} \). Prudence is positive as can be seen from \( U'''(c) = \rho(\rho+1)c^{-\rho-2} > 0 \). The degree of relative prudence is also constant in consumption and given by \( \rho + 1 = -cU''''(c)/U'''(c) \). Thus, here it is not possible to separate out the effect of risk aversion from the effect of prudence, since they are both captured by \( \rho \).

### 3.3 Introducing liquidity constraints

The model described in section 3.1 with an unspecified preference structure differs fundamentally from most models on child labour and schooling by including both future uncertainty about returns to schooling, no liquidity constraints, no agency problems and \( N \) children. When comparing this to the, by now, benchmark model developed by Baland & Robinson (2000), this corresponds a situation, where uncertainty is added to their world of perfect capital markets and two-sided altruism. This differs from Pouliot (2005), who introduces uncertainty into the parallel world of one-child households with one-sided altruism, positive bequests and perfect capital markets, i.e. parents do not rely on their child for old-age support. As Pouliot, I find a clear negative effect of uncertainty on schooling. The effect is strengthened by the introduction of a liquidity constraint and even more so if agency problems are also introduced because this, in effect, simply just increases the amount of uncertainty.

Most papers on child labour and schooling operate in a world with strong liquidity constraints. Shutting down the perfect credit market is a simple way of introducing such liquidity constraints in the human capital portfolio model above. By doing so, the model predictions become more directly comparable with the standard theories of child labour reviewed in section 2. In a world with no credit markets the households are faced with the following maximisation problem

\[
\max_{\pi} EW(c_1, c_2) = U(c_1) + EU(c_2)
\]

subject to the budget constraints for period 1 and period 2, respectively

\[
c_1 = Y_1 - (1 - \pi)Ne^a - \pi Ne^b
\]

\[
c_2 = N^{-\alpha}(1 - \pi)Ng^a + \pi Ng^b
\]

There is now one first order condition with one unknown, \( \pi \), the analytical solution for which
is
\[
\pi^*_{\text{cor}} = \frac{N^{1-\alpha} \Delta y (M - \gamma N^{1-\alpha} y_2^\alpha) - N \Delta e (M - \gamma (Y_1 - Ne^a))}{\gamma [N^{2-2\alpha} \Delta y^2 + N^2 \Delta e^2 + N^{2-2\alpha} \varepsilon^2]} 
\]
under perfect risk correlation in the urban labour market outcome among siblings and
\[
\pi^*_{\text{uncor}} = \frac{N^{1-\alpha} \Delta y (M - \gamma N^{1-\alpha} y_2^\alpha) - N \Delta e (M - \gamma (Y_1 - Ne^a))}{\gamma [N^{2-2\alpha} \Delta y^2 + N^2 \Delta e^2 + N^{2-2\alpha} \varepsilon^2]} 
\]
under no risk correlation. For both, $\Delta y = \mu - y_2^\alpha$ and $\Delta e = e^b - e^a$. From these analytical solutions it is clear that now the relative size of the marginal utility in period one compared to period two is important for determining the size of $\pi$. If marginal utility in period one is very high, the second term of the numerator is high, which in principle can run $\pi$ below zero if it is strong enough. The effect of uncertainty on $\pi^*$ (when $\pi^* > 0$) is now also stronger, especially if $N$ is high and for uncorrelated risk.

\[
\frac{\partial \pi^*_{\text{cor}}}{\partial \varepsilon^2} = - \frac{N^{3-3\alpha} \Delta y (M - \gamma N^{1-\alpha} y_2^\alpha) - N^3 \Delta e (M - \gamma Y_1 + \gamma Ne^a)}{\gamma [N^{2-2\alpha} \Delta y^2 + N^2 \Delta e^2 + N^{2-2\alpha} \varepsilon^2]^2} < 0 
\]
\[
\frac{\partial \pi^*_{\text{uncor}}}{\partial \varepsilon^2} = - \frac{N^{3-3\alpha} \Delta y (M - \gamma N^{1-\alpha} y_2^\alpha) - N^3 \Delta e (M - \gamma Y_1 + \gamma Ne^a)}{\gamma [N^{2-2\alpha} \Delta y^2 + N^2 \Delta e^2 + N^{2-2\alpha} \varepsilon^2]^2} < 0 
\]

The effects of fertility on the proportion of children in school are also altered. They are still ambiguous, but more likely to be negative than the corresponding derivatives under no liquidity constraints, especially so if $N$ is large or if $\alpha$ is close to 1 under no risk correlation among urban siblings. The two partial derivatives are now given by

\[
\frac{\partial \pi^*_{\text{cor}}}{\partial N} = \frac{\pi}{N} - \frac{(\gamma (1-\alpha) N^{1-2\alpha} (y_2^\alpha + 2\pi \Delta y) + \alpha N^{-\alpha} (M - \gamma N^{1-\alpha} y_2^\alpha) \Delta y \gamma [N^{2-2\alpha} \Delta y^2 + N^2 \Delta e^2 + N^{2-2\alpha} \varepsilon^2]} - \frac{\gamma N (e^a + 2\pi \Delta e) \Delta e + \gamma \pi N^{-2\alpha} (2 - 2\alpha) \varepsilon^2}{\gamma [N^{2-2\alpha} \Delta y^2 + N^2 \Delta e^2 + N^{2-2\alpha} \varepsilon^2]} \leq 0 
\]
\[
\frac{\partial \pi^*_{\text{uncor}}}{\partial N} = \frac{\pi}{N} - \frac{(\gamma (1-\alpha) N^{1-2\alpha} (y_2^\alpha + 2\pi \Delta y) + \alpha N^{-\alpha} (M - \gamma N^{1-\alpha} y_2^\alpha) \Delta y \gamma [N^{2-2\alpha} \Delta y^2 + N^2 \Delta e^2 + N^{2-2\alpha} \varepsilon^2]} - \frac{\gamma N (e^a + 2\pi \Delta e) \Delta e + \gamma \pi N^{-2\alpha} (1 - 2\alpha) \varepsilon^2}{\gamma [N^{2-2\alpha} \Delta y^2 + N^2 \Delta e^2 + N^{2-2\alpha} \varepsilon^2]} \leq 0 
\]

Finally, the non-monotonicity result with respect to parental income carries over to the situation with liquidity constraints. For the liquidity constrained household there is a clear direct positive effect of an increase in first period parental income

\[
\frac{\partial \pi^*_{\text{cor}}}{\partial Y_1} = \frac{N \Delta e}{N^{2-2\alpha} \Delta y^2 + N^2 \Delta e^2 + N^{2-2\alpha} \varepsilon^2} > 0 
\]
but the effect is counterbalanced by the negative effect of the corresponding increase in second period agricultural income when \( y_a^2 = f(Y_1) \), \( f' > 0 \), such that the overall effect of an increase in parental income becomes ambiguous

\[
\frac{\partial \pi^*_\text{uncor}}{\partial Y_1} = \frac{N \Delta e}{N^2 - 2\alpha \Delta y^2 + N^2 \Delta e^2 + N^1 - 2\alpha e^2} > 0
\]

both under perfect risk correlation and no risk correlation among siblings in the urban labour market.

In previous literature, the non-monotonicity in the relationship between schooling or child labour and income or even the lack of significance in the correlation is generally explained by either (i) a dramatic drop in the need for child labour as soon as the household is able to meet subsistence needs based purely on parental earnings, which generates strong non-linearities in the demand for child labour in the neighbourhood of the poverty line, Basu and Van (1998) and Edmonds (2005); (ii) missing or incomplete markets which can lead to the 'wealth paradox', when child labour has to compensate for incomplete labour markets as in Bhalotra and Heady (2003); (iii) or agency problems if parents cannot rely on getting the expected old-age support from their children because these consider the second period parental income too high to be in need of support, Rogers and Swinnerton (2004). All three explanations generate local non-monotonicities, while maintaining a global positively monotonic relationship between schooling and parental income.

In this paper, the non-monotonicity between income and schooling stems from the relative attractiveness of the agricultural sector compared to the urban sector, and from the assumption that there are no additional returns from formal compared to traditional education in the traditional agricultural sector. This generates global non-monotonicity with a positive effect of parental income on \( \pi^* \) for lower levels of \( Y_1 \) and a negative effect for higher levels of \( Y_1 \), since \( Y_1 \) and second period agricultural income \( y_a^2 \) are highly positively correlated.

4 Calibrations

Although one can find analytical solutions for the optimal proportion of formally educated children, \( \pi^* \) and show analytically that there is a negative effect of income dispersion or uncertainty, \( \frac{\partial \pi}{\partial \varepsilon} < 0 \), this does not indicate whether existing levels of uncertainty in urban income can actually result in less than full enrolment within a household. Only by calibrating the model,
using actual levels of school expenditures and income, is it possible to determine whether existing urban income dispersion, \( \text{Var}(y_y) = \varepsilon^2 \) is enough for the model to predict that at least one child will be educated traditionally and thus result in \( \pi^* < 1 \) even when there are no liquidity constraints. That is, whether existing levels of urban income uncertainty could potentially keep some children out of school purely due to future income diversification. Here it should be noted that, for calibration purposes, I am essentially equating uncertainty with income dispersion, and that the number of children in the calibration analysis is discrete.

In the following, there is a brief description of the data used and the assumptions made, when determining the size of the exogenous variables in the calibrations. In section 4.2, I show the results when calibrating the model from section 3 under quadratic, cubic and CRRA preferences. The focus is on how schooling, \( \pi \) react to future income uncertainty, \( \varepsilon \) when there are no liquidity constraints and no child labour; and on how the model derivatives with respect to \( N \) and \( Y_1 \) compare to the calibration results. These are important for future empirical testing of the model implications. In section 4.3, I introduce liquidity constraints and child labour and compare these effects to the effects of uncertainty on schooling when there are no child labour or liquidity constraints. The introduction of liquidity constraints and child labour is meant as an illustrative example of how the model captures the main components of the child labour literature, while allowing for the separate effects of uncertainty on school enrolment. Section 4.4 concludes.

### 4.1 Data

The model is calibrated using simple summary statistics from a large-scale nationwide household survey from Tanzania undertaken in 1994, the Human Resource and Development Survey (HRDS)\(^{14}\). It is a nationally representative survey of 5,000 households out of which more than half of the households have school-aged children. The HRDS data contains detailed information on individual household members, their educational status and current economic activity. At household level, it includes location, main source of income, detailed assets and expenditure information and, not least, schooling expenditures information. For calibration purposes only rural households with children of school-age are included, which results in a sample of 1982 households.

---

\(^{14}\)The Tanzanian Human Resource and Development Survey (HRDS) is a nationally representative survey from 1994 of 5,000 households. The survey was a joint effort undertaken by the Department of Economics of the University of Dar es Salaam, the Government of Tanzania, and the World Bank, and was funded by the World Bank, the Government of Japan, and the British Overseas Development Agency. For more information or access to the data see [www.worldbank.org/lsms](http://www.worldbank.org/lsms)
Table 1. Summary statistics of HRDS variables and their model equivalents.

<table>
<thead>
<tr>
<th>HRDS variable</th>
<th>HRDS data</th>
<th>normalised</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>AE daily HH expenditure, urban sector</td>
<td>mean 1.84</td>
<td>2.42</td>
<td>$y^b_2$</td>
</tr>
<tr>
<td></td>
<td>s.d. 2.02</td>
<td>1.99</td>
<td>$\varepsilon$</td>
</tr>
<tr>
<td>AE daily HH expenditure, agri sector</td>
<td>mean 0.76</td>
<td>1</td>
<td>$Y_1 = y^a_2$</td>
</tr>
<tr>
<td></td>
<td>s.d. 0.51</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Annual school expenditure, cluster mean</td>
<td>mean 5.96</td>
<td>0.02</td>
<td>$e^b$</td>
</tr>
<tr>
<td>Total number of children in HH</td>
<td>mean 3.91</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td>Proportion of children in/through school</td>
<td>mean 0.63</td>
<td>$\pi$</td>
<td></td>
</tr>
</tbody>
</table>

# observations 1982

Data source: HRDS data. Note, $y^b_2$ is the household expenditure among urban households, where the main source of income is urban. $y^a_2$ is the household expenditure among rural households, where the main source of income is agricultural. All expenditure amounts are in USD. An exchange rate of 1 USD = 455 Tsh is used. AE is short for adult equivalent.

The model is thus calibrated for the average rural household is school-aged children in 1994 Tanzania. Calibrating the model using data driven numerical values is helpful in determining the relative levels of exogenous variables.

Rural and urban income levels are proxied by the adult equivalent household expenditure levels for households in rural and urban areas, respectively. Expenditure measures in the data include values of home production. Agricultural income, $Y_1$ and $y^a_2$ are assumed to be of the same size, and expected future urban income, $E(y^b_2) = \mu$ is simply set to current adult equivalent expenditure levels of urban households whose main income source is also urban.

The educational expenditure associated with schooling, $e^b$ is directly given in the data as the cluster average of primary school expenditures. Since the model is set up for rural households, the mean for rural clusters is used. The educational expenditure associated with traditional agricultural education is not observable. If $e^a$ is negative, it can be thought of as the opportunity costs of time children spend in school, and thus as a measure of income generated by child labour. If $e^a$ is positive, it can be thought of as the opportunity costs of parents’ time spent supervising the children in traditional education. When calibrating the model with no child labour and no liquidity constraints, I simply proxy $e^a$ by half of the costs associated with formal schooling. Traditional education is then cheaper than formal education, but also less profitable.

Agricultural income levels in the two periods are normalised to unity, $Y_1 = y^a_2 = 1$ with zero standard deviation. This results in $E(y^b_2) = \mu = 1.84/0.76 = 2.42$ and $\varepsilon = s.d.(y^b_2) = \sqrt{(2.02^2 - 0.51^2)/0.76} = 2.24$.\(^{15}\) The actual cost of schooling in rural areas is very low and only

\[^{15}\]The expenditure standard deviation among urban households is very high due to a long right hand side tail in the expenditure distribution. Alternatively, I therefore cap $\varepsilon$ at the value of $\mu$, such that the urban uncertainty is an uncertainty which either drives income in zero or doubles it, i.e. $\varepsilon^b = 2.42 - 0.51/0.76 = 1.75$
2% of household expenditures, thus \( e^b = 0.02 \) and \( e^a = 0.01 \). These schooling expenditures do not include indirect costs of schooling, such as distance, and should therefore be seen as a lower bound. They do, however, include uniform costs. It should be noted that all of these amounts are measured in USD and adult equivalent terms.

When calibrating the model, I primarily allow \( \varepsilon \) and \( N \) to vary. The urban income dispersion or uncertainty, \( \varepsilon \) runs in the \([0; 2.4]\) interval with steps of 0.1. Thus the degree of uncertainty can run roughly from 0 to 100 per cent of average income level. The number of children, \( N \) is allowed to be 1, 2, 4, or 6 children, i.e. the model is calibrated for discrete numbers of children only and \( \pi \) can therefore also only take a limited number of values. \( N = 1 \) is included to allow comparisons with the standard models of child labour and schooling in the literature. According to the summary statistics in table 1, rural households have an average almost 4 children. The schooling rate among the 7-17 year olds in rural areas was 63% in 1994 (as opposed to 66% at national level). Unless mentioned otherwise, \( \alpha = 0.95 \). I choose a high \( \alpha \) in order to make first and second period income levels comparable and to avoid strong consumption smoothing mechanisms. The effect of changing \( \alpha \) is shown below.

As in the analytical set-up, the model is calibrated with two choice variables, \( \pi \) and \( s \), which are chosen to maximise the household utility function (1) given the budget constraints (2). The calibration results for \( \pi^* \) will show how large the dispersion in urban income, \( \varepsilon^2 \), has to be for the model to produce realistic enrolment rates under the three different types of preferences.

### 4.2 Preference structures

#### 4.2.1 Quadratic utility

In order to calibrate the model for the quadratic utility function, it is necessary to specify the preference parameters parameters, \( M \) and \( \gamma \). In a world of no consumption smoothing, first period consumption would be below 1, whereas expected second period consumption would be around 2 if all children are sent to school and \( \alpha \) is close to 1. For these levels, \( M = 7 \) and the risk aversion parameter, \( \gamma = 2 \) ensure that marginal utilities of the two periods are positive given the allowed variations in income.

The results for the optimal portfolio choice of the proportion of children in school, \( \pi^* \) are summarised in figure 0 for the case of no risk correlation and perfect risk correlation in siblings urban labour market outcome and for the specific case of \( N = 4 \). Figure 0 is meant as an introduction to the following figures and therefore includes data points. Uncertainty measured by \( \varepsilon \) is on the X-axis, the optimal proportion of children in school, \( \pi \) is on the Y-axis. The left panel shows the effect of uncertainty on the optimal proportion of children in school, when there is no correlation among migrant siblings’ urban income risk. The right panel show the effect of uncertainty, when there is perfect correlation among siblings’ urban income risk. When
uncertainty is perfectly uncorrelated (left panel), the model calibrations predict full enrolment
\((\pi = 1)\), given the parameter specifications, as long as \(\varepsilon \leq 2.3\). Remember, everything is
discrete. Thus, when epsilon jumps to \(\varepsilon = 2.4, \pi = 0.75\) meaning that the household now
chooses only to educate 3 out of 4 children formally, i.e. one child is educated traditionally. In
the right panel, less uncertainty is needed before it is optimal for the household to only send
3 out of 4 children to school. Already for \(\varepsilon = 1.7, \pi = 0.75\). As epsilon increases, the optimal
proportion of children in school drops, but in a discrete manner. For \(\varepsilon \geq 2.1\), only 2 out of 4
children are sent to school.

[Figure 0]

Thus, as it was shown analytically above, there is a clear negative effect of \(\varepsilon\) on \(\pi\). The
important information is, however, that the negative effect of uncertainty is present in the
neighbourhood of the actual level of urban income spread, that is for \(\varepsilon = 2.24\). As expected,
the effect is stronger under perfect risk correlation compared to no risk correlation. Figure 0 is a
representation of the average household without any liquidity constraints or immediate returns
to child labour. The negative effects of uncertainty on the optimal proportion of children
in school is purely driven by the need for risk diversification and thus future income source
diversification. When there is perfect risk correlation among siblings in their urban labour
market outcomes, the only source of risk diversification is between the agricultural and the
urban sector. On the other hand, when uncertainty about the urban labour market lottery
is perfectly uncorrelated across siblings, the risk diversification can happen both between the
agricultural and the urban sector, and among the migrant children in the urban sector, the
negative effect of uncertainty is therefore substantially reduced.

In figure 1, I allow for different household sizes by letting the total number of children \(N\)
equal 1, 2, 4 or 6. It is clear that no matter how many children the household has, if parents
face no uncertainty about the future income of their children \((\varepsilon = 0)\), then they will always
educate all of their children irrespective of \(N\). This is an obvious implication of the fact that
there are no liquidity constraints.

[Figure 1]

However, as uncertainty increases, there are clear portfolio effects in households with more
than one child. For \(N = 1\) there is no difference between being in the world of perfectly
correlated or uncorrelated \(\varepsilon\)'s. This is natural, since the correlation is between migrant siblings
in urban areas. Comparing the two panels of figure 1 also gives an indication of the importance
of allowing for sibling dependence in the portfolio model. Assuming that the human capital
investment decision of each child in the household is made independently of all of his/her
siblings (which corresponds to the \(N = 1\) case) and then just adding over the total number of
children in the household will yield very different predictions from a model, where such sibling dependence is taken into account, say for $N = 4$, in particularly so for correlated $\varepsilon$'s.

### 4.2.2 Cubic utility

Calibrating the cubic utility function as opposed to the quadratic is simply done by substituting the utility function in (5) with the one in (8) using the same parameter values as above, $M = 7, \gamma = 2$ and now allowing the prudence parameter to vary at low values, $\eta = [0.1; 0.4]$, to ensure that $U''(c)$ will always be negative. The results are as expected. Introducing prudence has a positive impact on the optimal proportion of children sent to school $\pi^*$, which is mostly evident from the case of perfect risk correlation in the urban labour market outcomes, see figure 2 for $N = 4$.

![Figure 2](image)

Figure 2 shows that for uncertainty levels of $\varepsilon = 2$ and a prudence parameter $\eta < 0.3$, households will educate 1 out of 4 children traditionally ($\pi = 0.75$) if there is perfect correlation among siblings in the urban labour market, whereas they will educate all children formally ($\pi = 1$) if the urban labour market draws are perfectly uncorrelated over migrant children. For $\eta \geq 0.3$ and $\varepsilon \leq 2$, all four children are sent to school. Compared to the quadratic preferences, slightly higher levels of uncertainty is now necessary for it to be optimal for the household to keep at least one child at home for traditional education. Formal education is simply a better savings strategy than traditional education.

### 4.2.3 CRRA utility

Deciding on the parameter values for the quadratic and cubic preferences is somewhat arbitrary in the sense that they are sensitive to the level of consumption and are chosen to ensure that marginal utilities in both period one and period two are non-negative. The remaining results are therefore all based on CRRA preferences. The value of the relative risk aversion parameter of $\rho$ is allowed to vary and all calibrations are done for $\rho = 1, 2$ and $3$, although the results reported in the text below are for $\rho = 2$. See appendix A3 for all CRRA calibration results. In general, the larger $\rho$ is, the more sensitive $\pi$ is to changes in the exogenous variables and increasing the relative risk aversion has the expected effect of shifting the graphs downwards and thus reducing the optimal proportion of children sent to school. Looking at the graphs, there are indications that the chosen preference parameters of the quadratic and cubic utility functions most closely resemble the case of log utility and $\rho = 1$.

![Figure 3](image)
Figure 3 corresponds to figure 1 above, now based on CRRA preferences with $\rho = 2$. First, as for the case of quadratic utility, households will always send all their children to school if there is no uncertainty. Second, as the level of uncertainty about future urban income increases, the need for risk diversification gets stronger and the optimal human capital portfolio shifts towards traditional education for one or more children. Under CRRA preferences, the model predicts that the average household with 4 children will educate at least one child traditionally if the dispersion in urban income $\varepsilon > 1.5$ under perfect risk correlation in the urban labour market. More uncertainty is needed when the urban labour market draws are perfectly uncorrelated across migrant siblings, only when $\varepsilon > 2.1$ will the household need to diversify income sources not only within the urban sector, but also between the urban and the agricultural sector. Again, the adjusted observed spread in urban income, $\varepsilon = 1.75$, lies well within the span of these two extremes. Third, the portfolio effects of having more than one child are now more pronounced compared to quadratic utility, higher $N$ and thus higher consumption levels no longer results in higher risk risk aversion as it is the case under quadrati preferences. There are clear positive portfolio effects of belonging to households with more children compared to less when the urban labour markets draws are perfectly uncorrelated, more children makes it possible to increase the diversification of the urban income risk reducing the need for the agricultural sector in achieving the optimal risk diversification. The results are more ambiguous when there is perfect correlation in the urban labour markets draws.

The important thing to notice here is that existing levels of uncertainty can indeed result in parents only sending some, but not all children to school. This negative effect on the optimal human capital portfolio allocation is surprisingly large, taking the perfect credit markets into consideration. Even for moderate levels of uncertainty, which match the actual income spread among urban households, and without any liquidity constraint or child labour, the model is able to predict an interval of optimal school enrolment rates within which the actual enrolment rate of $\pi = 0.63$ lies. For the average household, the pure effect of uncertainty is thus so strong that actual school enrolment rates could, in principle, be explained solely by the existence of uncertainty. Hence, the roots of child labour and lack of schooling need not lie solely with incomplete credit markets and poverty, but could also be caused by the fact that rural households are not only concerned with securing their current, but also their future old-age income.

The calibration of this simple human capital portfolio model thus shows that realistic levels of uncertainty about future income of children can indeed have a negative impact on the optimal proportion of children in school within the household, even under no liquidity constraints and only future returns to children engaged in traditional education. This central implication of the model relies upon the assumptions of parents depending on their children for old age security, of no covariant risk between urban and agricultural income, as well as on
the sectoral divide in returns to formal and traditional education. Assumptions which might not be standard in the child labour and schooling literature, but which each have substantial support in other literatures, all reviewed above.

4.3 Introducing child labour and liquidity constraints

Literature on child labour and schooling focuses on explaining the existence of child labour and lack of schooling as consequences of ex-post risk coping mechanisms when households are faced with negative income shocks and of the inability of parents to borrow against the future returns of schooling of the children. That is, they assume liquidity constraints and immediate net returns to children working in the traditional agricultural sector as opposed to future returns. In the following, I allow for both. Child labour thus still carries an element of education in the sense that there are returns to learning-by-doing and \( y_d > -e^a \). By introducing both liquidity constraints and child labour, I am able to compare the model predictions under uncertainty (\( \varepsilon > 0 \)) and sibling dependence (\( N > 1 \)) with those of standard child labour models under no uncertainty (\( \varepsilon = 0 \)) and one-child households (\( N = 1 \)), as well as with the two recent papers where uncertainty has been introduced into one-child households.

In figure 4, simple liquidity constraints have been introduced in the portfolio model above under CRRA preferences. Households can now save, but they can no longer borrow on the credit market, \( s \geq 0 \). Figure A3 in appendix A3 shows the corresponding figures under different degrees of relative risk aversion. Comparing figure 3 and 4 (as well as figures A1 and A3), it easily shows that - given the numerical values for the average household, where costs of schooling are relatively low and returns are 1.5 times larger than in the agricultural sector - the introduction of a liquidity constraint has virtually no effect\(^{16}\). Only once immediate returns to child labour are also introduced such that one child in the agricultural sector generates exactly enough income to cover the schooling expenses of a sibling \( e^a = -e^b \); is there a clear negative effect.

[Figure 4 & 5]

The introduction of child labour as an immediate return to traditional education generates a possibility of transferring income from period two to period one via the human capital market, given the incompleteness of the financial capital market. This does not seem to be necessary for households with 4 children or less, but for households with 6 children it is now optimal to always educate one child traditionally, even when there is no uncertainty. Comparing the isolated effect of uncertainty in figure 3 with the isolated effect of liquidity constraints and

\(^{16}\)In chapter 4 of this thesis, the same model is calibrated using numerical values from a different data set where costs of schooling is slightly higher and returns are lower, and there are more children in the average household. This results in more markedly effects of introducing liquidity constraints.
child labour for $\varepsilon = 0$ in figure 5, it is clear that uncertainty has a negative effect on the optimal choice of education of all children, whereas the constraint and child labour effects only really dominate in households with more children than the average $N = 4$. This emphasises the importance of allowing for $N$ children, rather than just one child. Assuming that the optimal solution for one child carries through for all $N$ children of the same household is clearly not correct, regardless of the degree of uncertainty. Under no uncertainty, even if the immediate returns to child labour were of the same size as current parental income or future returns to traditional education, i.e. $-e^a = Y_1 = y_2^a = 1$, the optimal solution for the one child would still be schooling, unless future returns are discounted enough to drop below current returns. As uncertainty about future urban income increases, the importance of allowing for some degree of sibling dependency is clear from the portfolio effects implied by differences in fertility. These portfolio effects seem even more pronounced in figure 5, compared to figure 4.

The main conclusion to take from these calibration results is that although the combination of child labour and liquidity constraints can have negative effects on the optimal proportion of children in school, these effects are strengthened partly by the introduction of $N > 1$ children, and partly by the existence of uncertainty $\varepsilon > 0$, which also in itself has strong negative effects on the optimal human capital portfolio. While the existing explanations in the literature for low enrolment rates into primary schools are focussed on the inability of parents to meet the direct and indirect costs of schooling and the role of children in ex-post risk coping mechanisms, the calibrations show that the ex-ante risk diversification strategies of a household may be at least equally important for the human capital investment decisions of the household. The introduction of uncertainty into a simple human capital portfolio model, which allows for a joint schooling decision of children in a household thus offers an alternative and complementary explanation to why it may be optimal for parents not to send all of their children to school, even if they can afford to do so.

In addition, the portfolio model offers a simple explanation for a non-monotonic relationship between child labour, schooling and income. The difference in returns between the agricultural sector and the urban sector generates global non-monotonicity, as discussed above. This is obvious from figure 6, which shows the effect on different income levels $Y_1 = [0.5; 3]$ on the optimal human capital portfolio $\pi^*$ for the average household with $N = 4$ under liquidity constrainst and with immediate returns to child labour. For the very low levels of (agricultural) income there is a positive effect of income increases on $\pi^*$ driven by the fact that the household is constrained and income increases allow households to allocate more children to the most profitable educational alternative, schooling. However, if the first period parental income is very high, so is the expected second period agricultural income and thus the relative returns to traditional education compared to formal education increase, making traditional education relatively more attractive. It is therefore optimal for the household to educate some children
traditionally. This shift toward traditional education happens earlier the higher the level of uncertainty in the urban sector relative to the agricultural sector, which here is normalised to be risk free. This provides an alternative explanation for the mixed empirical evidence with respect to income, schooling and child labour.

Finally, it should be noted that there is one parameter in the calibration, which has not yet been discussed, $\alpha$. This determines the fraction of income that each child shares with his/her parents in the second period. When $\alpha = 0$, children share all of their income with parents, when $\alpha = 1$ children share $1/Nth$ of their income with parents. In all of the calibrations above $\alpha = 0.95$ and thus children share slightly more than $1/Nth$ of their income with parents, such that parents in the second period in total receives slightly more than one full income. This number is, of course, chosen arbitrarily. From the three panels of figure A7, it shows that the effect of changes in $\alpha$ are fairly small when there is no immediate return to child labour, but large and negative as $\alpha$ approaches zero and there are immediate returns to child labour. This effect is purely a result of consumption smoothing. For very low $\alpha$, parental income in the second period can be more than $N$ times the current first period income and the only possibility of transferring resources from the second period to the first period is to shift children from formal education to traditional education, which now generates not only future but also immediate returns. Thus for low levels of $\alpha$, the negative effects of the combination of liquidity constraints and child labour are strengthened.

5 Conclusions and Policy Implications

In this paper I asked the question of whether future income uncertainty can result in households not educating all their children formally as an optimal risk diversification strategy to secure old-age subsistence of parents. To answer the question I develop a simple portfolio model of human capital investment of all children in a household. The model differs from most models of child labour and schooling by analysing the human capital investment decisions from the broader perspective of a rural household, allowing for future income uncertainty and considering both the old-age dependency of parents on children and the sibling dependency. When focusing on the human capital investment decisions of all children, it becomes obvious that several factors can influence such the joint decision. The basis for the model and its assumptions build on insights from different strands of literature with the aim of incorporating the variety of factors, which could be of importance. The emphasis is placed on ex-ante, rather than ex-post, risk diversification as a means of income smoothing, on the strong sectoral divide between the agricultural and urban sector and the dichotomy in the returns to specific versus general human
capital, on the role of children as old-age security assets of their parents, and on the dependency that this creates among siblings because educational choices are not made independently for each child, but rather as a joint decision over siblings giving natural rise to sibling differences, which is not in any way driven by heterogeneity or adverse economic conditions.

It is straightforward to show analytically that uncertainty about future income transfers from children, which in essence is uncertainty about returns to the human capital investments, has a negative effect on investments in the most uncertain type of human capital, here schooling. This result hinges upon the assumption of a sectoral divide in returns to formal and traditional education for which there is ample evidence in the literature, e.g. Rosenzweig (1995), Foster and Rosenzweig (1996) and Fafchamps and Wahba (2006).

The analytical result is, however, a qualitative finding and it does not indicate whether actual levels of uncertainty have any effect on the optimal proportion of children in school. The actual level of uncertainty could in principle be too low for the household to consider it worth giving up income in return for less risk exposure. The model is therefore calibrated using data driven numerical values and a variety of difference preference specifications. I find that moderate levels of uncertainty, based on the spread of income observed in data, is enough uncertainty for the average household choose a suboptimal human capital portfolio allocation of their children compared to a situation of no uncertainty. The need for risk diversification can thus result in parents only sending some, but not all, children to school. The negative effect of uncertainty is surprisingly large. Comparing the isolated effect of uncertainty with the isolated effect of liquidity constraints and child labour, it is clear that uncertainty influence the optimal choice of education of all children, whereas the constraint and child labour effects only really dominate in households with more children than the average \( N = 4 \). Although fairly robust to the choice of preference parameters, these results are based on simple moments taken from the data. The logical next step is therefore to find empirical implications of the model, which can be estimated and tested on a full data set.

However, based on the findings of the model calibrations, it does seem safe to conclude that future income uncertainty can indeed result in less than full school enrolment among siblings of a household. The focus on ex-ante income smoothing adds a new perspective to the child labour debate, which has previously been centered around the need for ex-post consumption smoothing for the liquidity constrained household. It also has direct implications for educational policies aimed at ensuring full enrolment, since lack of enrolment might not only be a matter of costs of schooling, but also of content. If the dichotomy in the educational system force parents to diversify human capital investments of their young children between traditional agricultural education and modern formal schooling in order to achieve future income source diversification, then an obvious policy implication is to increase the returns of formal schooling in the agricultural sector. This can be done either by shifting part of the traditional educa-
tion, currently undertaken by parents, into the formal schooling system, thus teaching children specific agricultural skills along with more general skills, such as writing and algebra; or by modernising the agricultural sector to create 'learning opportunities' and thus increase returns to formal schooling in the agricultural sector, see Foster and Rosenzweig (1996). Households are likely still to diversify future income sources, but it need no longer be a diversification decision taken at an early stage of human capital investments.
References


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6 Figures

Figure 0. Quadratic preferences ($M = 7, \gamma = 2$), effect of uncertainty, $\varepsilon$ on proportion of children in school, $\pi^*$

Figure 1. Quadratic preferences ($M = 7, \gamma = 2$), effect of uncertainty, $\varepsilon$ on proportion of children in school, $\pi^*$ over number of children in the household, $N$.

Figure 2. Cubic preferences \((M = 7, \gamma = 2)\), effect of uncertainty, \(\varepsilon\) on proportion of children in school, \(\pi^*\) over different degrees of prudence, \(\eta\) and for fixed \(N = 4\).

Figure 3. CRRA preferences \((\rho = 2)\), effect of uncertainty, \(\varepsilon\) on proportion of children in school, \(\pi\) over number of children in the household, \(N\) - under no liquidity constraints and no child labour.
Figure 4. CRRA preferences ($\rho = 2$), effect of uncertainty, $\varepsilon$ on proportion of children in school, $\pi$ over number of children in the household, $N$  
- under liquidity constraints and no child labour

Figure 5. CRRA preferences ($\rho = 2$), effect of uncertainty, $\varepsilon$ on proportion of children in school, $\pi$ over number of children in the household, $N$  
- under liquidity constraints and child labour
Figure 6. CRRA preferences ($\rho = 2$), effect of agricultural income $Y_1$ on proportion of children in school, $\pi$ for $N = 4$

- under liquidity constraints and child labour
7 Appendix A1

The covariance term \( \text{cov}(N^{1-\alpha}y^b_2, U'(c_2)) \) differ depending on whether there is perfect risk correlation or no risk correlation between the second period urban labour market outcome of siblings. Under perfect risk correlation and quadratic preferences, the covariance term is given by

\[
cov(N^{1-\alpha}y^b_2, U'(c_2)) = E[(N^{1-\alpha}y^b_2 - N^{1-\alpha}\mu)(U'(c_2) - EU'(c_2))] = \frac{1}{2}(N^{1-\alpha}(\mu + \varepsilon) - N^{1-\alpha}\mu)((M - \gamma(N^{-\alpha}(1 - \pi)Ny^a_2 + \pi N(\mu + \varepsilon) + s)}
\]

\[+ \frac{1}{2}(N^{1-\alpha}(\mu - \varepsilon) - N^{1-\alpha}\mu)((M - \gamma(N^{-\alpha}(1 - \pi)Ny^a_2 + \pi N(\mu - \varepsilon) + s)}
\]

Under no risk correlation, it is given by

\[
cov(N^{1-\alpha}y^b_2, U'(c_2)) = E[(N^{1-\alpha}y^b_2 - N^{1-\alpha}\mu)(U'(c_2) - EU'(c_2))] = E[N^{1-\alpha}(y^b_2 - \mu)(-\gamma\pi N^{1-\alpha}(y^b_2 - \mu))]
\]

\[= -\gamma N^{-2\alpha}E[\{N(y^b_2 - \mu)\{\pi N(y^b_2 - \mu)\}]
\]

\[= -\gamma N^{-2\alpha}\sum_{i=0}^{\pi N} \binom{\pi N}{i} \frac{1}{2\pi N} \{[(1 - \pi)N + i]\varepsilon - [N - i]\varepsilon\} \{i\varepsilon - [\pi N - i]\varepsilon\}
\]

\[= -\gamma\pi N^{1-2\alpha}\varepsilon^2
\]
8 Appendix A2

Deriving first order conditions under the cubic utility function. The first order condition for savings, $s$ under perfect risk correlation among siblings in urban labour market is

$$U'(c_1) = EU'(c_2)$$
$$M - \gamma c_1 + \frac{1}{2} \eta c_1^2 = M - \gamma Ec_2 + \frac{1}{2} \eta E(c_2^2)$$
$$M - \gamma c_1 + \frac{1}{2} \eta c_1^2 = M - \gamma Ec_2 + \frac{1}{2} \eta E([((1 - \pi)N^{1-\alpha}y_2^b + \pi N^{1-\alpha}y_2 + s)^2])$$
$$M - \gamma c_1 + \frac{1}{2} \eta c_1^2 = M - \gamma Ec_2 + \frac{1}{2} \eta (Ec_2)^2 + \frac{1}{2} \eta (\pi N^{1-\alpha} \varepsilon)^2$$

and under no risk correlation is

$$U'(c_1) = EU'(c_2)$$
$$M - \gamma c_1 + \frac{1}{2} \eta c_1^2 = M - \gamma Ec_2 + \frac{1}{2} \eta E(c_2^2)$$
$$M - \gamma c_1 + \frac{1}{2} \eta c_1^2 = M - \gamma Ec_2 + \frac{1}{2} \eta \sum_{i=0}^{\pi N} \left( \frac{\pi N}{i} \right) \frac{1}{2iN} [Ec_2 + N^{-\alpha}((\pi N - i)\varepsilon + i\varepsilon)]^2$$
$$M - \gamma c_1 + \frac{1}{2} \eta c_1^2 = M - \gamma Ec_2 + \frac{1}{2} \eta (Ec_2)^2 + \frac{1}{2} \eta \pi N^{-2\alpha} \varepsilon^2$$

The covariance term in the first order condition for the proportion of children in formal education, $\pi$ under perfect risk correlation among siblings in urban labour market is then

$$\text{cov}(N^{1-\alpha}y_2^b, U'(c_2)) = E[(N^{1-\alpha}y_2^b - N^{1-\alpha} \mu)(U'(c_2) - EU'(c_2))]$$
$$= E[N^{1-\alpha}(y_2^b - \mu)(-\gamma(c_2 - Ec_2) + \frac{1}{2} \eta (c_2^2 - E(c_2)^2 - (\pi N^{1-\alpha} \varepsilon)^2))]$$
$$= E[N^{1-\alpha}(y_2^b - \mu)(-\gamma \pi N^{1-\alpha}(y_2^b - \mu) + \frac{1}{2} \eta ((\pi N^{1-\alpha}y_2^b)^2 + (\pi N^{1-\alpha} \mu)^2 + 2\Gamma \pi N^{1-\alpha}(y_2^b - \mu)^2 - (\pi N^{1-\alpha} \varepsilon)^2))]$$
$$= (-\gamma + \eta (\Gamma + \mu))\pi N^{2-2\alpha} \varepsilon^2$$

where $\Gamma = (1 - \pi)N^{1-\alpha}y_2^b + s$. The first order condition for $\pi$ under perfect risk correlation is then given by

$$N(e^b - e^\alpha)U'(c_1) = N^{1-\alpha}(\mu - y_2^b)EU'(c_2) + \text{cov}[N^{1-\alpha}y_2^b, U'(c_2)]$$
$$N(e^b - e^\alpha)U'(c_1) = N^{1-\alpha}(\mu - y_2^b)(U'(Ec_2) + \frac{1}{2} \eta (\pi N^{1-\alpha} \varepsilon)^2) - (\gamma - \eta (\Gamma + \mu))\pi N^{2-2\alpha} \varepsilon^2$$

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while under no risk correlation the covariance is

\[
\text{cov}(N^{1-\alpha}y_b^2, U'(c_2)) = E[(N^{1-\alpha}y_b^2 - N^{1-\alpha}\mu)(U'(c_2) - EU'(c_2))]
\]

\[
= E[N^{1-\alpha}(y_b^2 - \mu)(-\gamma(c_2 - Ec_2) + \frac{1}{2}\eta(c_2^2 - E(c_2)^2 - \pi N^{1-2\alpha}\varepsilon^2))]
\]

\[
= E[N^{1-\alpha}(y_b^2 - \mu)(-\gamma\pi N^{1-\alpha}(y_b^2 - \mu)
+ \frac{1}{2}\eta((\pi N^{1-\alpha}y_b^2)^2 - (\pi N^{1-\alpha}\mu)^2 + 2\Gamma\pi N^{1-\alpha}(y_b^2 - \mu) - \pi N^{1-2\alpha}\varepsilon^2))]
\]

\[
= \frac{\pi N}{\sum_{i=0}^{\pi N}} \left( \frac{\pi N}{i} \right) \frac{1}{2\pi N} [\pi N^{-2\alpha}(-\gamma + \eta(\Gamma + \mu) ((N - i)\varepsilon - ((1 - \pi)N + i)\varepsilon)^2)
+ \frac{1}{2}\eta\pi N^{-3\alpha}((\pi N - i)\varepsilon - ((1 - \pi)N + i)\varepsilon)^3 - ((N - i)^2\varepsilon^3 + ((1 - \pi)N + i)^2(-\varepsilon^3))]
\]

\[
= (-\gamma + \eta(\Gamma + \mu)) \pi^2 N^{1-2\alpha}\varepsilon^2
\]

where \( \Gamma = (1 - \pi)N^{1-\alpha}y_b^2 + s \). The first order condition for \( \pi \) under no risk correlation is then given by

\[
N(e^b - e^a)U'(c_1) = N^{1-\alpha}(\mu - y_b^2)EU'(c_2) + \text{cov}[N^{1-\alpha}y_b^2, U'(c_2)]
\]

\[
N(e^b - e^a)U'(c_1) = N^{1-\alpha}(\mu - y_b^2) \left( U'(Ec_2) + \frac{1}{2}\eta\pi N^{1-2\alpha}\varepsilon^2 \right) - (\gamma - \eta(\Gamma + \mu)) \pi^2 N^{1-2\alpha}\varepsilon^2
\]
9 Appendix A3: CRRA figures

Figure A1. Effect of uncertainty $\varepsilon$ on $\pi^*$ under no liquidity constraints and no child labour.
Figure A2. Effect of agricultural income $Y_1$ on $\pi^*$ under no liquidity constraints and no child labour.
Figure A3. Effect of uncertainty $\varepsilon$ on $\pi^*$ under liquidity constraints and no child labour.
Figure A4. Effect of agricultural income $Y_1$ on $\pi^*$ under liquidity constraints and no child labour.
Figure A5. Effect of uncertainty $\varepsilon$ on $\pi^*$ under liquidity constraints and child labour.
Figure A6. Effect of agricultural income $Y_1$ on $\pi^*$ under liquidity constraints and child labour.
Figure A7 Effect of changes in $\alpha$ on $\pi^*$
- under no liquidity constraints and no child labour

- under liquidity constraints and no child labour

- under liquidity constraints and child labour