Optimal income taxation with endogenous participation and search unemployment*

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March 13, 2009

Abstract

We characterize optimal redistributive taxation when individuals are heterogeneous in their skills and their values of non-market activities. Search-matching frictions on the labor markets create unemployment. Wages, labor demand and participation are endogenous. The government only observes wage levels. Under a Maximin objective, if the elasticity of participation decreases along the distribution of skills at the optimum, the average tax rate is increasing, marginal tax rates are positive everywhere, while wages, unemployment rates and participation rates are distorted downwards compared to their laissez-faire values. Under a general utilitarian objective, numerical simulations suggest that the downward distortions of wages and unemployment remain. However, the optimal policy then induces upward distortions of participation.

Keywords: Non-linear taxation; redistribution; adverse selection; random participation; unemployment; labor market frictions.

JEL codes: H21; H23; J64

*We thank for their comments participants at seminars at the Université Catholique de Louvain, CREST, Malaga, EPEE-Evry, Gains-Le Mans, CES-Paris 1, ERMES-Paris 2, the IZA-SOLE 2008 Transatlantic meeting, the 7th Journées Louis-André Gérard-Varet, the HetLab conference at the University of Konstanz, The Economics of Labor Income taxation workshop at IZA, Thema-Cergy Pontoise, AFSE 2008, EALE 2008 and T2M 2009, with a particular mention to Daron Acemoglu, Pierre Cahuc, Helmuth Cremer, Bruno Decreuse, Laurence Jacquet, Guy Laroque, Cecila Garcia-Peñalosa and Fabien Postel-Vinay. Mathias Hungerbühler was particularly helpful in providing suggestions and comments at various stages. Any errors are ours. This research has been funded by the Belgian Program on Interuniversity Poles of Attraction (P6/07 Economic Policy and Finance in the Global Economy: Equilibrium Analysis and Social Evaluation) initiated by the Belgian State, Prime Minister's Office, Science Policy Programming. Simulations programs are available at http://www.crest.fr/ckfinder/userfiles/files/Pageperso/elehmann/LPVsimulations.zip

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I Introduction

In the literature on optimal redistributive taxation initiated by Mirrlees (1971), non-employment, if any, is synonymous with non-participation. The importance of participation decisions is not debatable. However, according to Mirrlees (1999), "another desire is to have a model in which unemployment [in our words, "non-employment"] can arise and persist for reasons other than a preference for leisure". Along this view, it is important to recognize that some people remain jobless despite they do search for a job at the market wage. To account for this fact, one should depart from the assumption of walrasian labor markets. Our contribution is to characterize the optimal redistribution policy in a framework where wages, employment, (involuntary) unemployment and (voluntary) non participation are endogenously affected by taxation on labor income. We argue that the optimal redistributive taxation distorts wages and unemployment downwards. However, participation should not necessarily be distorted downwards since negative marginal tax rates can be optimal for the low skilled workers.

As it is standard in the optimal tax literature, we assume that the government is only able to condition taxation on wages. Our economy is made of a continuum of skill-specific labor markets. On each of them, we introduce matching frictions à la Mortensen and Pissarides (1999). This setting is particularly attractive because both labor supply (along the participation/extensive margin) and labor demand determine the equilibrium levels of employment. In our model, taxes are distorsive via the participation margin and the wage-cum-labor demand margin. Concerning participation, we assume that whatever their skill level, individuals differ in their value of remaining out of the labor force.[1] A higher level of taxes reduces the skill-specific value of participation, thereby inducing some individuals to leave the labor force. Labor demand is affected by taxation through wage formation. In various wage-setting models, the equilibrium gross wage maximizes an objective that is increasing in the after-tax (net) wage and decreasing in the pre-tax (gross) wage. For instance, a higher pre-tax wage reduces the labor demand in a monopoly union model while it reduces firms' profit in Nash bargaining models. Since an increase in tax progressivity renders a higher pre-tax wage less attractive to workers, a lower pre-tax wage is substituted for a lower after-tax wage.[2] This wage moderation effect of tax progressivity stimulates labor demand and reduces unemployment. In order to be as general as possible, we deal with wage-formation in a reduced form way that is consistent with those properties.

[1] Because of this additional unobserved heterogeneity, the government has to solve an adverse selection problem with « random participation » à la Rochet and Stole (2002).

[2] It is worth noting that this mechanism also holds in the textbook competitive labor supply framework. There, the after-tax wage equals consumption and a higher pre-tax wage is obtained thanks to more effort. Hence, solving the consumption/leisure tradeoff amounts to maximize an objective that is increasing in the net wage and decreasing in the gross wage. For simplicity, we ignore labor supply responses along the intensive margin in our model.

As it is standard in the optimal taxation literature, we stick to the welfarist view according to which the government's objective depends on utility levels. Moreover, in order to focus sharply on redistributive issues, we assume that the economy without taxes (*laissez faire*) is efficient (in the Benthamite sense). We first show that when the government has a Maximin (Rawlsian) objective and the elasticities of participation are decreasing along the skill distribution, optimal marginal tax rates are positive everywhere and optimal average tax rates are increasing. The reason is that a more progressive tax schedule increases the level of tax at the top of the skill distribution where participation decisions are less elastic and decreases the level of tax where participation reacts more strongly to the tax pressure. Since redistribution lowers participants' expected surplus, the participation rate is lower at the optimum than at the *laissez faire*. However, a more progressive tax schedule distorts wages and unemployment rates downwards.

The paper also suggests that the downward distortions of wages and unemployment rates are still optimal under a general utilitarian criterion. Unemployment has now two additional effects on social welfare (associated with the wage-cum-labor demand margin). First, since income net of taxes and transfers has to be higher in employment than in non-employment (to induce participation), unemployment *per se* causes a loss in social welfare. Second, because some participants to the labor market are eventually unemployed, enhancing participation increases earnings inequalities, which has a detrimental effect on social welfare. However, it can no longer be proved that optimal marginal tax rates are positive. Pushing down wages to stimulate labor demand has a negative impact on welfare, in particular at the low end of the skill distribution. This gives a rationale for providing a higher transfer to low-skilled workers than to the non-employed (which we henceforth define as an EITC). Thus, upward distortions of the low skilled individuals' participation rates can be optimal.

To illustrate how our optimal tax formulas could be used for applied purposes, we calibrate our model on the US economy. In the Maximin case, it turns out that the optimal tax profile is well approximated by an assistance benefit tapered away at a high and nearly constant rate. If the government maximizes a Bergson-Samuelson social welfare function, the optimal tax profile is different with hump-shaped marginal tax rates. Moreover, an EITC is optimal.

A number of studies are related to our work. In the optimal taxation literature that follows Mirrlees (1971), the intensive margin (i.e. work effort) is the only source of deadweight losses. In this competitive labor market model, tax progressivity induces a downward distortion of work effort and thus of pre-tax wages. In our non-competitive model, tax progressivity reduces pre-tax wages and increases labor demand. Thus, the equity-efficiency trade-off in our non walrasian labor market framework is dramatically different from the one appearing in the Mirrleesian literature. Both mechanisms can account for the empirical fact that gross incomes decrease with marginal tax rates (Feldstein, 1995, Gruber and Saez, 2002). Whether this wage moderating effect of tax progressivity is due to a labor supply response along the intensive margin or to a non competitive wage formation remains an open empirical question. However, we believe that the mechanism on which our model is based might be crucial. On the one hand, Blundell and MacCurdy (1999) and Meghir and Phillips (2008) conclude that the labor supply responses along the intensive margin are empirically very small. On the other hand, Manning (1993) finds a significantly negative effect of tax progressivity on the UK unemployment rate (see also Sørensen 1997 and Røed and Strøm 2002).

There is now growing evidence that the extensive margin (i.e. participation decisions) matters a lot. Diamond (1980) and Choné and Laroque (2005) have studied optimal income taxation when individuals' decisions are limited to a dichotomic choice about whether to work or not. The optimum trades off the equity gain of a higher level of tax against the efficiency loss of a lower level of participation. However, gross incomes are not distorted in these models because of a competitive labor market and exogenous productivity levels. Saez (2002) has proposed a model of optimal taxation where both extensive and intensive margins of the labor supply are present. He shows that the optimal tax schedule heavily depends on the comparison between the elasticities of participation decisions with respect to tax levels and of earnings with respect to marginal tax rates. Our model emphasizes that the monotonicity of the elasticities of participation is also important.

Some papers have made a distinction between unemployment and non-participation. Boadway *et alii* (2003) study redistribution when unemployment is endogenous and generated by matching frictions or efficiency wages. The government's information set is different from ours because they assume that it observes productivities and can distinguish among the various types of non-employed. Boone and Bovenberg (2004) depart from the standard model of nonlinear income taxation *à la* Mirrlees (1971) by adding a job-search margin that is the single determinant of the unemployment risk. As in our model, the government cannot verify job search. However, in their model, the cost of participation is homogeneous in the population and the unemployment risk does not depend on wages nor on taxation. In Boone and Bovenberg (2006), the framework is similar but since the government observes employed workers' skill, taxation is skill-specific. Their focus is on the respective roles of the assistance benefit and of in-work benefits in redistributing income while ours is on redistributive taxation when the government observes only wages.

Closely related to the current paper, Hungerbühler et alii (2006), henceforth HLPV, proposes an optimal income tax model with unobservable worker's ability and where unemployment is endogenous due to matching frictions. The main difference concerns the costs of participation. In HLPV, they are introduced in a minimalist way since they take a unique value whatever the skill level. Consequently, every agent above (below) an endogenous threshold of skill participates (does not participate). The elasticity of participation is thus infinite at the threshold and zero above. To take care of participation decisions in a much more general and realistic way, we let the opportunity cost of participation vary within and between skill levels. This has important consequences. First, labor demand distortions at the bottom of the skill distribution are radically different. In HLPV, the highest distortions along the wage-cum-labor demand margin appear for the least skilled participating agents. In our model, these distortions are lower for the least skilled since the tax schedule now trades off the benefit of distorting the labor demand upwards and the induced reduction in participation to the labor market. In particular, labor demand is not distorted at the bottom under a Maximin objective. Second, while most of the other results of HLPV can be retrieved under a Maximin criterion, this is no longer true under the general social welfare function. In particular, HLPV shows that marginal tax rates are positive everywhere while our more general treatment of participation decisions is compatible with negative marginal rates and EITC for the low skilled.

The paper is organized as follows. The model and fiscal incidence are presented in the next section. Section III characterizes the Maximin optimum. Section IV presents the optimality conditions under the general utilitarian criterion. Section V explains how we calibrate the model and presents numerical simulations of optimal tax schedules. Finally, Section VI concludes.

II The model

As usual in the optimal non linear tax literature that follows Mirrlees (1971), we consider a static framework where the government is averse to inequality. For simplicity we assume risk-neutral agents with homogeneous tastes. In our model, the sources of differences in earnings are threefold. First, individuals are endowed with different levels of productivity (or skill) denoted by a. The distribution of skills admits a continuous density function $f(.)$ on a support $[a_0, a_1]$, with $0 < a_0 < a_1 \leq +\infty$. The size of the population is normalized to 1. Second, whatever their skill, some people choose to stay out of the labor force while some others do participate to the labor market. To account for this fact, we assume that individuals of a given skill differ in their individual-specific gain χ of remaining out of the labor force. We call χ the value of non-market activities. Third, among those who participate to the labor market, some fail to be recruited and become unemployed. This "involuntary" unemployment is due to matching frictions *à la* Mortensen and Pissarides (1999) and Pissarides (2000). A worker of skill a produces a units of output if and only if she is employed in a type a job, otherwise her production is nil. This assumption of perfect-segmentation is made for tractability and seems more realistic than the polar one of a unique labor market for all skill levels. The timing of events is the following:

1. The government commits to an untaxed assistance benefit b and a tax function $T(.)$ that only depends on the (gross) wage w.

2. For each skill level a, firms choose the number of job vacancies they open. Creating a vacancy of type a costs $\kappa(a)$. Individuals of type (a, χ) decide whether they participate to the labor market of type a.

3. On each labor market, the matching process determines the number of filled jobs. An individual of type (a, χ) who chooses to participate renounces χ.[3] All participants of skill a are alike during the matching process. We henceforth call these individuals participants of type a for short. Each employed worker supplies an exogenous amount of labor normalized to 1. So, earnings and (gross) wages are equal among workers of the same skill level.

4. Each worker of skill a produces a units of goods, receives a wage $w = w_a$ and pays taxes. Taxes finance the assistance benefit b and an exogenous amount of public expenditures $E \geq 0$. Agents consume.

We assume that the government does neither observe individuals' types (a, χ) nor the job-search and matching processes.[4] It only observes workers' gross wages w_a and is unable to distinguish among the *non-employed* individuals those who have searched for a job but failed to find one (the *unemployed*) from the *non participants.* Moreover, as our model is static, the government is unable to infer the type of a jobless individual from her past earnings. Therefore, the government is constrained to give the same level of assistance benefit b to all non-employed individuals, whatever their type (a, χ) or their participation decisions. An individual of type (a, χ) can decide to remain out of the labor force, in which case her utility equals $b+\chi$. Otherwise, she finds a job with an endogenous probability ℓ_a and gets a net-of-tax wage $w_a - T(w_a)$ or she becomes unemployed with probability $1 - \ell_a$ and gets the assistance benefit b.[5]

II.1 Participation decisions

To participate, an individual of type (a, χ) should expect an income, $\ell_a (w_a - T(w_a)) + (1 - \ell_a) b$, higher than in case of non participation, $b + \chi$. Let

$$\Sigma_a \stackrel{\text{def}}{\equiv} \ell_a (w_a - T(w_a) - b)$$

[3] According to Krueger and Mueller (2008), Time use surveys "suggest that the unemployed spend considerably more time searching for a new job than do individuals who are classified as employed or out of the labor force. [...These] results [can be interpreted] as evidence that the conventional labor force categories represent meaningfully different states and behavior patterns" (p.13).

[4] The government is therefore unable to infer the skill of workers from the screening of job applicants made by firms. So, the tax schedule cannot be skill-specific. Moreover, we do not consider the possibility that redistribution could also be based on observable characteristics related to skills (see Akerlof, 1978).

[5] Our model can easily be extended to include a skill-specific fixed cost of working.

denote the expected surplus of a participant of type a. Let $G(a,.)$ be the cumulative distribution of the value of non-market activities, conditional on the skill level, that is

$$G(a,\Sigma) \stackrel{\text{def}}{\equiv} \Pr[\chi \leq \Sigma \,|a]$$

Then, the participation rate among individuals of skill a equals $G(a,\Sigma_a)$ and hence the number of participants of type a equals $U_a = G(a,\Sigma_a) f(a)$. We denote the continuous conditional density of the value of non-market activities by $g(a,\Sigma)$. This density is supposed positive on an interval whose lower bound is 0. Note that the characteristics a and χ can be independent or not. We define

$$\pi_a \stackrel{\text{def}}{\equiv} \frac{\Sigma_a \cdot g(a,\Sigma_a)}{G(a,\Sigma_a)} \tag{1}$$

the elasticity of the participation rate with respect to Σ, at $\Sigma = \Sigma_a$. This elasticity is in general both endogenous and skill-dependent. Note that π_a also equals the elasticity of the participation rate of agents of skill a with respect to $w_a - T(w_a) - b$ when ℓ_a is fixed. The empirical literature typically estimates the latter elasticity.

II.2 Labor demand

On the labor market of skill a, creating a vacancy costs $\kappa(a) > 0$. This cost includes the investment in equipment and the screening of applicants. Only a fraction of vacancies finds a suitable worker to recruit. Following the matching literature (Mortensen and Pissarides 1999, Pissarides 2000 and Rogerson *et alii* 2005), we assume that the number of filled positions is a function $H(a,V_a,U_a)$ of the numbers V_a of vacancies and U_a of job-seekers. Contrary to HLPV, we do not limit the analysis to constant-returns-to-scale Cobb-Douglas matching functions but impose the following less restrictive assumptions:

Assumption 1 *The matching function $H(a,.,.)$ on the labor market of skill a is twice-continuously differentiable on $[a_0,a_1] \times \mathbb{R}_+^2$, is increasing in both U_a and V_a, exhibits constant returns to scale in (U_a,V_a), verifies $H(a,V_a,0) = H(a,0,U_a) = 0$, and $H(a,V_a,U_a) < \min(V_a,U_a)$.*

Define tightness θ_a as the ratio V_a/U_a. The probability that a vacancy is filled equals $q(a,\theta_a) \stackrel{\text{def}}{\equiv} H(a,1,1/\theta_a) = H(a,V_a,U_a)/V_a$. Due to search-matching externalities, the job-filling probability decreases with the number of vacancies and increases with the number of job-seekers. Because of constant returns to scale, only tightness matters and $q(a,\theta_a)$ is a decreasing function of θ_a. Symmetrically, the probability that a job-seeker finds a job is an increasing function of tightness $\theta_a q(a,\theta_a) = H(a,\theta_a,1) = H(a,V_a,U_a)/U_a$.[6] Firms and individuals being atomistic, they take tightness θ_a as given.

[6]The functions $q(a,\theta_a)$ and $\theta_a q(a,\theta_a)$ are defined from $[a_0,a_1] \times \mathbb{R}_+^*$ to $(0,1)$.

When a firm creates a vacancy of type a, it fills it with probability $q(a,\theta_a)$. Then, its profit at stage 4 equals $a - w_a$. Therefore, its expected profit at stage 2 equals $q(a,\theta_a)(a-w_a) - \kappa(a)$. Firms create vacancies until the free-entry condition $q(a,\theta_a)(a-w_a) = \kappa(a)$ is met. This pins down the value of tightness θ_a and in turn the probability of finding a job through

$$L(a,w_a) \stackrel{\text{def}}{\equiv} q^{-1}\left(a, \frac{\kappa(a)}{a-w_a}\right) \cdot \frac{\kappa(a)}{a-w_a} \tag{2}$$

where $q^{-1}(a,.)$ denotes the inverse function of $\theta \mapsto q(a,\theta)$, holding a constant.

In equilibrium, one has $\ell_a = L(a,w_a)$ and

$$\Sigma_a = L(a,w_a)(w_a - T(w_a) - b) \tag{3}$$

From the assumptions made on the matching function, $L(.,.)$ is twice-continuously differentiable and admits values within $(0,1)$. As the wage increases, firms get lower profit on each filled vacancy, fewer vacancies are created and tightness decreases. This explains why $\partial L/\partial w_a < 0$. Moreover, due to the constant-returns-to-scale assumption, the probability of being employed depends only on skill and wage levels and not on the number of participants. If for a given wage, there are twice more participants, the free-entry condition leads to twice more vacancies, so the level of employment is twice higher and the employment probability is unaffected. This property is in accordance with the empirical evidence that the size of the labor force has no lasting effect on group-specific unemployment rates. Finally, because labor markets are perfectly segmented by skill, the probability that a participant of type a finds a job depends only on the wage level w_a and not on wages in other segments of the labor market. The following Lemma, which is proved in Appendix A, implies that the labor demand function $L(.,.)$ can be taken as a structural primitive without loss of generality. The rest of the paper therefore uses this function.

Lemma 1 *Let $L(a,w_a)$ be a twice-continously differentiable labor demand function such that*

- $\partial L/\partial w_a < 0$ *and,*
- *for each a, there exists $\overline{w}_a$ such that $L(a,w_a) > 0$ if and only if $w_a < \overline{w}_a$.*

  Then, there exists a unique matching technology $H(a,.,.)$ and a vacancy cost function $\kappa(a)$ that generates $L(.,.)$ through (2). Moreover, $H(.,.,.)$ verifies Assumption 1.

II.3 The wage setting

We focus on redistribution and consider a setting such that the role of taxation is only to redistribute income (as in Mirrlees) and not to restore efficiency.[7] For this purpose, we consider

[7] Boone and Bovenberg (2002) studies how nonlinear taxation can restore efficiency when the Hosios condition is not fulfilled. Hungerbühler and Lehmann (2009) extends HLPV by relaxing the Hosios assumption.

a wage-setting mechanism that maximizes the sum of utility levels in the absence of taxes and benefits. To obtain this property, the matching literature typically assumes that wages are the outcome of Nash bargaining and that the workers' bargaining power equals the elasticity of the matching function with respect to unemployment (see Hosios 1990). This assumption is only meaningful if the elasticity of the matching function is constant and exogenous. With a Cobb-Douglas matching function $H(a, U_a, V_a) = A\ (U_a)^{\gamma} (V_a)^{1-\gamma}$, Equation (2) implies that $L(a,w) = A^{1/\gamma} ((a-w)/\kappa(a))^{((1-\gamma)/\gamma)}$. Then, Nash bargaining under the Hosios condition leads to a wage level that solves (see HLPV):[8]

$$w_a = \arg\max_{w} \quad L(a,w) \cdot (w - T(w) - b) \tag{4}$$

When the matching function is not of the Cobb-Douglas form, we assume that (4) still holds. So, $\Sigma_a = \max_{w} \quad L(a,w) \cdot (w - T(w) - b)$ and the equilibrium wage maximizes the skill-specific participation rates given the tax/benefit system.

Different wage-setting mechanisms can provide microfoundations for (4). The Competitive Search Equilibrium introduced by Moen (1997) and Shimer (1996) leads to this property. Another possibility is to assume that a skill-specific utilitarian monopoly union selects the wage w_a after individuals' participation decisions but before firms' decisions about vacancy creation (see Mortensen and Pissarides, 1999).

II.4 The equilibrium

The objective in (4) multiplies the employment probability by the difference between the net incomes in employment and in unemployment. We call this latter difference the *ex-post surplus* and denote it $x \stackrel{\text{def}}{\equiv} w - T(w) - b$. It subtracts an "employment tax", $T(w)+b$, from the earnings w.

For a given function $w \mapsto T(w) + b$, the equilibrium allocation is recursively defined. The wage-setting equations (4) determine wages w_a and in turn $x_a = w_a - T(w_a) - b$. The labor demand functions (2) determine the skill-specific employment probability $\ell_a = L(a, w_a)$ and unemployment rates $1 - L(a, w_a)$. Then, from (3), expected surpluses equal $\Sigma_a = \ell_a x_a$, so the participation rates are given by $G(a, \Sigma_a)$ and the employment rates equal $L(a, w_a)\, G(a, \Sigma_a)$. For each additional worker of type a, the government collects taxes $T(w_a)$ and saves the assistance benefit b. Since $E \geq 0$ is the exogenous amount of public expenditures, the government's budget constraint defines the level of b:

$$b = \int_{a_0}^{a_1} (T(w_a) + b) \cdot L(a, w_a) \cdot G(a, \Sigma_a) \cdot f(a)\, da - E \tag{5}$$

[8] If different wage levels solve (4), then we make the tie-breaking assumption that the wage level preferred by the government will be selected. See also the discussion in Mirrlees (1971, footnotes 2 and 3 pages 177). Moreover, we do not consider the case where there does not exist any solution w for which $w - T(w) - b > 0$.

II.5 The *laissez faire*

The *laissez faire* is defined as the economy without tax and benefit. According to (4), the equilibrium level of wage maximizes $L(a,w)\cdot w$. To ensure that program (4) is well-behaved at the *laissez faire*, we assume that for any (a,w),

$$\frac{\partial^2 \log L}{\partial w \cdot \partial \log w}(a,w) < 0 \tag{6}$$

We henceforth denote w_a^{LF} the wage at the *laissez faire*. To guarantee the reasonable property that w_a^{LF} increases with the level of skill, we further assume that for any (a,w):

$$\frac{\partial^2 \log L}{\partial a \partial w}(a,w) > 0 \tag{7}$$

As a matter of illustration, when the matching function is CES $H(a,U,V) = \left(U^{-\rho}+V^{-\rho}\right)^{-\frac{1}{\rho}}$ with $\rho > 0$,[9] then, Equation (2) implies that the labor demand function is:

$$L(a,w) = \left[1 - \left(\frac{a-w}{\kappa(a)}\right)^{-\rho}\right]^{\frac{1}{\rho}}$$

It can be checked that condition (6) is always verified and that condition (7) holds true if the vacancy cost increases less than proportionally or decreases with the skill level a (so that $a\ \dot{\kappa}(a)/\kappa(a) \leq 1$).[10] These conditions are standard in the matching literature (see Pissarides, 2000).

Appendix B verifies that, when the exogenous amount of public expenditures E is nil, the *laissez-faire* economy maximizes the Benthamite objective, which equals the sum of utility levels. Because of our wage-setting mechanism (4), wages at the laissez faire maximize "efficiency" (i.e. maximize the Benthamite criterion). Note that participation decisions are then also efficient.

II.6 Fiscal incidence

We now reintroduce the tax/benefit system and explain how tax reforms affect the equilibrium. In our setting, the influence of the tax and benefit system comes through the profile of the relationship between the ex-post surplus $x = w - T(w) - b$ and earnings w. Because of the multiplicative form of (4), what actually matters is how $\log x$ varies with $\log w$. When $T(.)$ is differentiable, the first-order condition[11] of Program (4) writes:

$$-\frac{\partial \log L}{\partial \log w}(a, w_a) = \eta(w_a) \tag{8}$$

[9] The positive value of ρ ensures that $H(a,U,0) = H(a,0,V) = 0$ and $H(a,U,V) < \min(V,U)$.

[10] In addition, one has then $\partial L/\partial a > 0$.

[11] The solution to (4), if any, necessarily lies in $(-\infty, a-\kappa(a))$. Since $L(a, a-\kappa(a)) = 0$, $w = a-\kappa(a)$ does not solve (4). From a theoretical viewpoint, the wage can be negative whenever $T(.)$ is negative enough to keep some agents of type a participating to the labor market (i.e. $w - T(w) > b$). Hence the solution to (4) is necessarily interior. In the rest of the paper, we focus on positive wage levels. Since $\partial \log L/\partial w < 0$, $\eta(w)$ has to be positive. As the expected surplus is positive, so is $w - T(w) - b$. Hence, the marginal tax rate $T'(w)$ has to be lower than 1.

where

$$\eta(w) \stackrel{\text{def}}{\equiv} \frac{1 - T'(w)}{1 - \frac{T(w)+b}{w}} = \frac{\partial \log(w - T(w) - b)}{\partial \log w} \tag{9}$$

When the wage increases by one percent, the term $\partial \log L/\partial \log w$ measures the relative decrease in the employment probability, while $\eta(w)$ measures the relative increase in the ex-post surplus. In equilibrium, Equation (8) requires that these two relative changes sum to zero. Notice that in our setting the profile of $\eta(w)$ gathers all the information about the profile of the tax/benefit system needed to fix the equilibrium wage. Figure 1 displays indifference expected surplus curves. The equation of these indifference curves can be written as $\log x =$ constant $- \log L(a, w)$. From (2) and (6), these curves are increasing and convex. The solution to Program (4) then consists in choosing the highest indifference curve taking the relationship between $\log x$ and $\log w$ into account. In case of differentiability, this amounts to choosing the highest indifference curve tangent to the $\log w \mapsto \log x = \log(w - T(w) - b)$ schedule. The first-order condition (8) combined with (9) expresses this tangency condition.

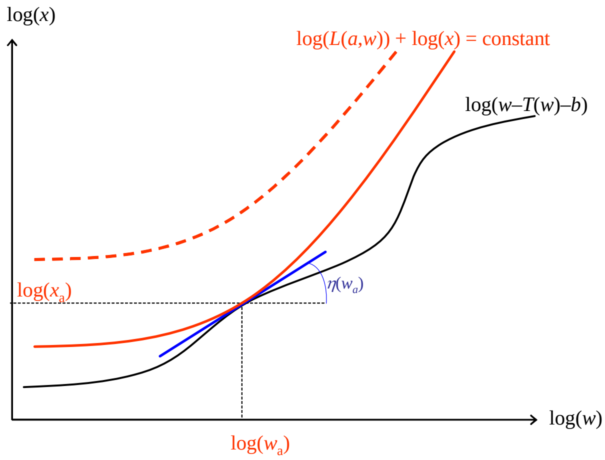

Figure 1: The choice of the wage for a type a match.

For comparative static purposes, consider for a while the average tax rate $T(w_a)/w_a$, the assistance benefit ratio b/w_a and the marginal tax rate $T'(w_a)$ as parameters. So, $\eta(w_a)$ is provisionally a parameter, too. Under Condition (6), Equations (8) and (9) imply that the equilibrium wage w_a (thereby the unemployment rate $1-L(a, w_a)$) increases with the average tax rate and the assistance benefit ratio and decreases with the marginal tax rate. These properties are standard in the equilibrium unemployment literature. They hold under monopoly unions (Hersoug 1984), right-to-manage bargaining (Lockwood and Manning 1993), efficiency wages with continuous effort (Pisauro 1991) or matching models with Nash bargaining (Pissarides 1998). Sørensen (1997) and Røed and Strøm (2002) provide some empirical evidence in favor of the wage-moderating effect of higher marginal tax rates. In addition, Manning (1993) finds that higher marginal tax rates lower unemployment in the UK.

From Equation (8), the average tax rate, the assistance benefit ratio and the marginal tax rate affect the equilibrium wage only through changes in the slope η of the $\log w \mapsto \log x$ function. To see why, imagine a tax reform such that participants of type a face a steeper $\log w \mapsto \log x$ tax function. A relative rise in the wage induces now a higher relative gain in the ex-post surplus x. Still, the relative loss in the employment probability is unchanged. Consequently, the rise in η induces an increase in the equilibrium wage w_a that substitutes ex-post surplus for employment probability. This is reminiscent of the *substitution effect* in a competitive framework with adjustments along the intensive margin. There, a lower marginal tax rate raises the net hourly wage and leads to a substitution toward consumption and away from leisure time. Returning to our setting, Equation (8) indicates that for a given slope of the $\log w \mapsto \log x$ function, the level of this function does not affect the equilibrium wage. In this specific sense, there is no *income effect* of the tax schedule on wages in our model.

In the general case where η is a function of the wage, a change in this slope produces a direct change in wage levels. This in turn creates a second change in η which produces a further change in the wage. To clarify this circular process[12] and to prepare the analysis developed in Sections III and IV in terms of a small tax reform, imagine that the slope $\eta(w)$ of the $\log w \mapsto \log x$ relationship exogenously increases by a small amount $\tilde{\eta}$. Let us rewrite the first-order condition (8) as $\mathcal{W}(w_a, a, 0) = 0$, where:

$$\mathcal{W}(w, a, \tilde{\eta}) \stackrel{\text{def}}{\equiv} \frac{\partial \log L}{\partial \log w}(a, w) + \eta(w) + \tilde{\eta} \tag{10}$$

The second-order condition of (4) writes $\mathcal{W}'_w(w_a, a, 0) \leq 0$ where

$$\mathcal{W}'_w(w_a, a, \tilde{\eta}) = \frac{\partial^2 \log L(a, w_a)}{\partial w \cdot \partial \log w} + \eta'(w_a) \tag{11}$$

This second-order condition states that at the equilibrium wage w_a, the $\log w \mapsto \log x$ relationship depicted in Figure 1 has to be either concave or less convex than the indifference expected surplus curves.[13]

Consider now how the equilibrium wage w_a is influenced by small changes in the parameter $\tilde{\eta}$ and in the type a. Whenever the second-order condition of (4) is a strict inequality and the maximum of (4) is globally unique, we can apply the implicit function theorem on $\mathcal{W}(w_a, a, \tilde{\eta}) = 0$. We then obtain the elasticity ε_a of the equilibrium wage w_a with respect to a small local change in $\tilde{\eta}$ around a given $\log w \mapsto \log x$ function. We also obtain the elasticity α_a of the

[12] Which is also present in the literature following Mirrlees (see Saez 2001).

[13] When this condition is not verified over an interval, the earnings function $a \mapsto w_a$ is discontinuous.

equilibrium wage w_a with respect to the skill level a along the same $\log w \mapsto \log x$ function:

$$\varepsilon_a \quad \stackrel{\text{def}}{\equiv} \frac{\eta(w_a)}{w_a} \cdot \frac{\partial w_a}{\partial \tilde{\eta}} = -\frac{\eta(w_a)}{w_a \cdot \mathcal{W}'_w(w_a, a, 0)} > 0 \tag{12a}$$

$$\alpha_a \quad \stackrel{\text{def}}{\equiv} \frac{a}{w_a}\frac{\partial w_a}{\partial a} = -\frac{a}{w_a \cdot \mathcal{W}'_w(w_a, a, 0)} \cdot \frac{\partial^2 \log L}{\partial a \partial w}(a, w_a) > 0 \tag{12b}$$

These elasticities are in general endogenous and in particular they depend on the curvature term $\eta'(w_a)$ in $\mathcal{W}'_w$. This is because a change in wage Δw_a, that is either caused by a change in $\tilde{\eta}$ or in a, induces a change in $\eta(w_a)$ that equals $\eta'(w_a)\Delta w_a$ and a further change in the wage. This is at the origin of a circular process captured by the term $\eta'(w_a)$ in $\mathcal{W}'_w$. However, as will be clear in Sections III and IV, only the ratio ε_a/α_a enters the optimality conditions and this ratio does not depend on $\eta'(w_a)$ but only on a and w_a. The positive signs of ε_a and α_a follow from the strict second-order condition $\mathcal{W}'_w < 0$ and from (7).

In addition to its effect on wage and unemployment through $\eta(.)$, taxation also influences participation decisions. To isolate this effect, consider a tax reform that rises $\log(w - T(w) - b)$ by a constant amount for all w so that $\eta(w)$ is kept unchanged. Such a tax reform does neither change the wage level, nor the employment probability. However, the employment tax $T(w_a) + b$ is reduced and hence the surplus Σ_a an agent of type a can expect from participation increases. Therefore, such a reform increases the participation rate $G(a, \Sigma_a)$, thereby the employment rate $L(a, w_a) \cdot G(a, \Sigma_a)$. The magnitude of this behavioral response is captured by the elasticity π_a defined in (1). In sum, the income effect affects the participation margin and not the wage-cum-labor demand margin.

II.7 Government's objective and incentive constraints

The government cares about inequalities measured in terms of the net income that accrues to agents according to their status on the labor market. Section III is devoted to the Maximin (Rawlsian) objective. There, the government values only the utility of the least well-off. Unemployed individuals are the least well-off because they get b, which is always lower than the workers' and non participants' utility levels, which are respectively equal to $w - T(w)$ and $b + \chi$. Section IV considers the following general Bergson-Samuelson social welfare function:

$$\int_{a_0}^{a_1} \left\{ \left[L(a, w_a)\Phi(w_a - T(w_a)) + (1 - L(a, w_a))\Phi(b)\right] G(a, \Sigma_a) + \int_{\Sigma_a}^{+\infty} \Phi(b + \chi)\, g(a, \chi)\, d\chi \right\} f(a)\, da \tag{13}$$

where $\Phi'(.) > 0 > \Phi''(.)$. The "pure" (Benthamite) utilitarian case sums the utility levels of all individuals and corresponds to the case where $\Phi(.)$ is linear. The stronger the concavity of $\Phi(.)$, the more averse to inequality is the government.

The government does not observe the productivity of each job but only the wage negotiated by each worker-firm pair. So, it aims at maximizing its objective subject to the budget constraint

(5) and the choices made by the agents. Since a worker-firm pair maximizes an objective $L(a,w)(w - T(w) - b)$ that is increasing in the ex-post surplus $x = w - T(w) - b$ and decreasing in gross wages, the government's self-selection problem can be viewed as one where worker-firms pairs of skill a are agents with an objective $L(a,w)\ x$. Therefore, according to the *taxation principle* (Hammond 1979, Rochet 1985 and Guesnerie 1995), the set of allocations induced by a tax/benefit system $\{T(.), b\}$ through the wage-setting equations (4) corresponds to the set of incentive-compatible allocations $\left(b, \{w_a, x_a, \Sigma_a\}_{a\in[a_0,a_1]}\right)$ that verify:

$$\forall \left(a, a'\right) \in [a_0, a_1]^2 \qquad \Sigma_a \equiv L(a, w_a)\ x_a \geq L(a, w_{a'})\ x_{a'} \tag{14}$$

This condition expresses that a firm-worker pair of type a chooses the bundle (w_a, x_a) rather than any other bundle $(w_{a'}, x_{a'})$ designed for firm-worker pairs of another type a'. From (7), the strict single-crossing condition holds. Hence, (14) is equivalent to the envelope condition associated to (4)

$$\dot{\Sigma}_a = \Sigma_a \cdot \frac{\partial \log L}{\partial a}(a, w_a) \tag{15}$$

and the monotonicity requirement that the wage w_a is a nondecreasing function of the skill level a. Following Mirrlees (1971), it is much more convenient to solve the government's problem in terms of allocations.[14] Contrary to HLPV, and in the spirit of Saez (2001), we will express the optimality conditions in terms of behavioral elasticities that can be easily interpreted for applied purposes.

III The Maximin case

Let $h_a = L(a, w_a)\ G(a, \Sigma_a)\ f(a)$ denote the (endogenous) mass of workers of skill a. We obtain (see Appendix C):

Proposition 1 *For any skill level $a \in [a_0, a_1]$, the maximin-optimal tax schedule verifies:*

$$\frac{1-\eta(w_a)}{\eta(w_a)} \cdot \frac{\varepsilon_a}{\alpha_a} \cdot w_a \cdot a \cdot h_a = Z_a \qquad \textit{and} \qquad Z_{a_0} = 0 \tag{16a}$$

$$Z_a = \int_a^{a_1} \left[x_t - \pi_t\left(T(w_t) + b\right)\right] h_t \cdot dt, \tag{16b}$$

where $T(w_t) + b = w_t - x_t$ and since $\eta(w) = \partial \log(w - T(w) - b) / \partial \log w$, x_t verifies:

$$\forall t, u \qquad \log x_t = \log x_u + \int_{w_u}^{w_t} \eta(w)\ d\log w$$

[14] We assume the existence of an optimal allocation $a \mapsto (w_a, x_a)$ that is continuous, differentiable and increasing. Existence and continuity are usual regularity assumptions (see e.g. Mirrlees 1971, 1976 or Guesnerie and Laffont 1984). The monotonicity assumption means that we rule out bunching. We verify in the simulations that the monotonicity requirement is verified along the optimum. The differentiability assumption is made only for convenience. It implies that the tax schedule $T(.)$ is almost everywhere differentiable in the wage.

In Proposition 1, the elasticities π_a of the participation rate, ε_a of the wage with respect to η and α_a of the wage with respect to the skill level a are respectively given by (1), (12a) and (12b) along the optimal allocation. Moreover, w_a is determined by the wage-setting condition (8).

III.1 Intuitive proof of Proposition 1

The resolution in terms of incentive-compatible allocations enables a rigorous derivation. However, this method does not provide much economic intuition. So, we propose here an intuitive proof in the spirit of Saez (2001). Recall that in our model, it is much more convenient to think of the tax schedule as a function that associates the log of the ex-post surplus to the log of the wage. We consider the effect of the following small tax reform around the optimum depicted in Figure 2. The slope $\eta(w)$ of $\log w \mapsto \log x$ is marginally increased by $\tilde{\eta} = \Delta\eta$ for wages in the small interval $[w_a - \delta w, w_a]$.[15] We take $\Delta\eta$ sufficiently small compared to δw, so that bunching or gaps in the wage distribution around $w_a - \delta w$ or w_a induced by the tax reform can be neglected. This reform has two effects on the government's objective (5). There is first a *tax level* effect that concerns individuals of skill t above a. Those of them who are employed thus receive a wage w_t above w_a. Second, there is *a wage response* effect. It takes place for those whose wages lie in the $[w_a - \delta w, w_a]$ interval.

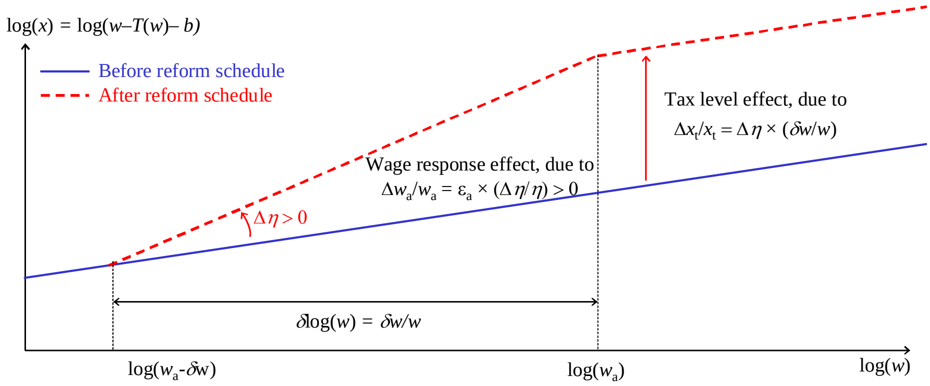

Figure 2: The tax reform

[15] The reasoning below will be entirely developed in terms of this local change in η. For the reader interested by the implementation of such a reform, the small local increase $\Delta\eta$ would be the result of a small decline in the marginal tax rate, the level of the average employment tax being kept locally constant. Above w_a, the induced reduction in the employment tax should be compensated for by an appropriate reduction of the marginal tax rate to keep η unchanged.

Due to:	**Wage response effect** $\Delta w_a/w_a = \varepsilon_a\ (\Delta\eta/\eta)$	**Tax Level effect** $\Delta x_t/x_t = \Delta\eta\ (\delta w/w)$
Mechanical Component	$T(w_a)$ increases by $T'(w_a)\ \Delta w_a$	$T(w_t)$ decreases by $\Delta T(w_t) = -\Delta x_t$
Behavioral component	Labor demand is reduced by $\Delta\ell_a/\ell_a = -\eta(w_a)(\Delta w_a/w_a)$	Participation rates increase by: $\Delta G_t/G_t = \pi_t\ (\Delta x_t/x_t)$

Table 1: Summary of the different components of the effects of the tax reform

The tax level effect

Consider skill levels t above a. Since $\eta(.)$ is unchanged around w_t, the equilibrium wage w_t is unaffected by the tax reform, and so is the employment probability $L(t, w_t)$. From (9), the tax reform increases the ex-post surplus $x_t = w_t - T(w_t) - b$ by

$$\frac{\Delta x_t}{x_t} = \Delta\eta \cdot \frac{\delta w}{w}$$

(see Figure 2). The consequence of this rise of (the log of) the ex-post surplus can be decomposed into a *mechanical component* and a *behavioral component* through a change in the participation decisions (see Table 1).

The rise in x_t corresponds to a reduction in the employment tax level $T(w_t) + b$ such that $\Delta(T(w_t) + b) = -x_t \cdot \Delta\eta \cdot (\delta w/w)$. Since there are h_t workers of type t, the mechanical component of the tax level effect at skill level t equals:

$$-x_t \cdot h_t \cdot \Delta\eta \cdot \frac{\delta w}{w} \tag{17}$$

Consider now the participation decisions of individuals of skill t above a. From (3), since their employment probability is unchanged, their expected surplus increases by the same relative amount $\Delta\Sigma_t/\Sigma_t = \Delta\eta \cdot (\delta w/w)$ as their ex-post surplus x_t. According to (1) the number of employed individuals of type t thus increases by $\pi_t \cdot h_t \cdot \Delta\eta \cdot (\delta w/w)$. For each of these additional employed individuals, the government receives $T(w_t) + b$ additional employment taxes. Hence, the behavioral component of the tax level effect at skill level t equals:

$$\pi_t \cdot (T(w_t) + b) \cdot h_t \cdot \Delta\eta \cdot \frac{\delta w}{w} \tag{18}$$

From (16b), the sum of the mechanical and behavioral components over all skill levels t above a gives *the tax level effect*. It equals $-Z_a \cdot \Delta\eta \cdot (\delta w/w)$.

The wage response effect

This effect concerns individuals whose skill level is such that their wage in case of employment lies in the interval $[w_a - \delta w, w_a]$. Let $[a - \delta a, a]$ be the corresponding interval of the skill distribution. From (12b), one has

$$\delta a = \frac{a}{\alpha_a} \cdot \frac{\delta w}{w} \tag{19}$$

Therefore, the number of agents concerned by this effect is $(a/\alpha_a) f(a)(\delta w/w)$.

Due to the small tax reform, those employed face a more increasing $\log w \mapsto \log x$ tax schedule. The tax reform thus induces a wage increase Δw_a that substitutes ex-post surplus for employment probability. From (12a), one has

$$\frac{\Delta w_a}{w_a} = \frac{\varepsilon_a}{\eta(w_a)} \cdot \Delta\eta \tag{20}$$

Since the equilibrium wage maximizes participants' ex-post surplus Σ_a, the tax reform has only a second-order effect on Σ_a and thereby on the participation rate of these individuals. The wage response effect can be decomposed into a *mechanical component* and a *behavioral component* through a change in the labor demand decisions (see Table 1).

The wage increase Δw_a changes the employment tax paid by $T'(w_a) \cdot \Delta w_a$. From (9), one gets $1 - T'(w_a) = x_a \cdot \eta(w_a)/w_a$, so

$$\Delta(T(w_a) + b) = T'(w_a) \cdot \Delta w_a = [(1 - \eta(w_a)) w_a + \eta(w_a)(T(w_a) + b)] \frac{\Delta w_a}{w_a} \tag{21}$$

Multiplying the last term by the number of employed individuals h_a gives the mechanical component of the wage response effect.

The wage increase Δw_a also induces a reduction in the employment probability $L(a, w_a)$. Given (8), the fraction of employed among participants is decreased by:

$$\Delta L(a, w_a) = -\eta(w_a) \frac{\Delta w_a}{w_a} L(a, w_a) \tag{22}$$

When an additional participant of type a finds a job, the government levies additional taxes $T(w_a)$ and saves b. Multiplying the employment tax $T(w_a) + b$ by $\Delta\ell_a$ times the number of participants $G(a, \Sigma_a) f(a) \delta a$ gives the behavioral component of the wage response effect. The sum of these two components equals

$$\Delta[(T(w_a) + b) \cdot L(a, w_a)] \cdot G(a, \Sigma_a) \cdot f(a) \cdot \delta a = (1 - \eta(w_a)) w_a \cdot h_a \cdot \frac{\Delta w_a}{w_a} \cdot \delta a$$

Given (19), (20) and the last expression, the total *wage response* effect on the interval $[w_a - \delta w, w_a]$ equals

$$\frac{1 - \eta(w_a)}{\eta(w_a)} \cdot \frac{\varepsilon_a}{\alpha_a} \cdot a \cdot w_a \cdot h_a \cdot \Delta\eta \cdot \frac{\delta w}{w} \tag{23}$$

The wage response effect can be either positive or negative. From Subsection II.5, recall that the *laissez-faire* value of the wage is efficient. If $\eta(w_a) < 1$, (resp. $\eta(w_a) > 1$) the wage is below (above) its *laissez-faire* value, hence it is inefficiently low (high). Adding the wage response and the tax level effects gives (16a) in Proposition 1.

To obtain $Z_{a_0} = 0$ in (16a), consider a tax reform that rises $\log(w - T(w) - b)$ by a constant amount for all w, so that $\eta(w)$ is kept unchanged. This reform is implemented by increasing the level of Σ_{a_0} and thus the level of Σ_a for all a (from (15), the rise in Σ_a has to be a proportional rise). Such a tax reform induces an effect that is proportional to Z_{a_0} and no wage response effect. At the optimum, such a marginal reform should not have a first-order impact on the government's objective. This implies that the sum of all mechanical and behavioral effects has to be nil i.e. that $Z_{a_0} = 0$.

III.2 Instructive cases

To better understand the implications of our optimal tax formula, we now consider its implications when additional restrictions are imposed. Given the literature, a natural starting point is the case where wages are exogenously fixed ($\varepsilon_a = 0$). Then, we return to the case where wages are endogenous but impose some constraints on the elasticities of participation.

No wage response effect

We provisionally assume that marginal tax reforms do not change the employment probabilities ℓ_a. However, wages still increase exogenously with the skill (i.e. α_a remains positive). This case corresponds to the model with only extensive margin responses of labor supply considered by Diamond (1980), Saez (2002) and Choné and Laroque (2005).[16] Intuitively, as wages do not react to changes in taxes, the solution is given by putting to zero the sum of the mechanical (17) and behavioral (18) components of the tax level effect. This has to be true for all levels of skill. Consequently, from (16a) and (16b), $x_a - \pi_a (T(w_a) + b) = 0$ whatever the skill a. Therefore, at the optimum, the employment tax (respectively, the employment surplus) verifies:

$$\frac{T(w_a) + b}{w_a} = \frac{1}{1 + \pi_a} \qquad \Leftrightarrow \qquad \frac{x_a}{w_a} = \frac{\pi_a}{1 + \pi_a} \tag{24}$$

These relationships are implicit ones when π_a depends on the expected surplus. The optimal employment tax rate only depends on the behavioral response (through π_a) and not on the distribution of skills. In Figure 1, the optimal allocation $\log x_a$ is necessarily below the 45 degree line at a distance given by $\mid \log(\pi_a/(1 + \pi_a)) \mid$. In accordance with Saez (2002) in the Maximin case, the employment tax $T(w_a) + b$ is positive i.e. there is no EITC. Where the participation rate is more elastic, the behavioral component matters more. Therefore, the optimal ex-post surplus has to be higher to induce participation (a necessary condition to collect taxes to finance b).

Constant elasticity of participation

We now investigate under which condition the tax schedule described by Equation (24) is optimal when wages are responsive to taxation ($\varepsilon_a > 0$). This tax schedule induces that the

[16] However here, as in Boone and Bovenberg (2004, 2006), participants face a positive but exogenous probability to be "involuntarily" unemployed.

aggregate tax level effect Z_a equals 0 everywhere along the skill distribution (See Equation 16b). Therefore, the wage response effect has to be nil everywhere. So, according to (16a), the slope η of the $\log w \mapsto \log x$ function has to equal 1 everywhere. Therefore, from (9), the ratio x_a/w_a has to be constant. This is consistent with (24) only when the elasticity of participation π_a is the same for all skill levels at the optimum.

Reciprocally, assume that the elasticity of participation is constant and consider the tax policy defined by an employment tax $T(w) + b$ that equals $w/(1+\pi)$ for all wage levels w. In this case, the mechanical (17) and behavioral (18) components of the tax level effect sum to 0 at each skill level. Moreover, from (9), this policy induces $\eta(w)$ to be constant and equal to 1, so wages are not distorted and the wage response effect is nil everywhere. Therefore, this policy satisfies the conditions in Proposition 1.

Decreasing elasticity of participation

We think that the assumption of a constant elasticity of participation is not plausible. This judgment is based on empirical evidence that suggests a decreasing profile with a (see the empirical evidence in Juhn et alii, 1991, Immervoll et alii, 2007 and Meghir and Phillips, 2008). Of course, the profile of π_a at the optimum can be different from the one observed in the current economy. Still, the two following examples suggest that the slopes of π_a in the current economy and at the optimum might have the same signs.

The first example specifies

$$G(a, \Sigma) = A(a) \cdot \Sigma^{\pi_a} \qquad \text{with} \qquad A(a) > 0 \text{ and } \pi_a > 0 \tag{25}$$

Then, provided that $\Sigma_a \leq (A(a))^{-1/\pi_a}$, the participation rates remain within $(0,1)$ and the elasticity of participation is exogenous and equals π_a.

The second example is based on the following three assumptions. Firstly, the value of non market activities χ is distributed indepentently of the skill level a. Thus, $G(.,.)$ does not depend on a. Secondly, the distribution of χ is such that $\Sigma \mapsto \Sigma g(\Sigma)/G(\Sigma)$ is a decreasing function. This is for instance the case if χ follows the exponentional distribution $G(\Sigma) = 1 - \exp(-\sigma_1 \Sigma)$ or the Pareto distribution $G(\Sigma) = 1 - \sigma_0 \Sigma^{-\sigma_1}$, both with $\sigma_0 > 0$ and $\sigma_1 > 0$. Thirdly, for a given wage level, the employment probability increases in the skill level (See Footnote 10). Then, since $\dot{\Sigma}_a > 0$ from (15), $\pi_a = \Sigma_a g(\Sigma_a)/G(\Sigma_a)$ is decreasing in the skill level a in the current economy and at the optimum.

When the elasticity of participation is decreasing in the skill level along the optimum, we find:

Proposition 2 *If everywhere along the Maximin optimum one has $\dot{\pi}_a < 0$, then*

i) *$w_a < w_a^{LF}$ and $L(a, w_a) > L\left(a, w_a^{LF}\right)$ for all a in (a_0, a_1), while $w_{a_0} = w_{a_0}^{LF}$, $L(a_0, w_{a_0}) = L\left(a_0, w_{a_0}^{LF}\right)$, $w_{a_1} = w_{a_1}^{LF}$ and $L(a_1, w_{a_1}) = L\left(a, w_{a_1}^{LF}\right)$.*

ii) *Compared to the laissez faire, the participation rates are distorted downwards.*

iii) *The average tax rate $T(w)/w$ is an increasing function of the wage and the marginal tax rates $T'(w)$ are positive everywhere. The in-work benefit (if any) at the bottom-end of the distribution is lower than the assistance benefit $-T(w_{a_0}) < b$.*

This Proposition is proved in Appendix D. Its intuition is illustrated in Figure 3. This Figure depicts the ratio of the ex-post surplus over the wage, x_a/w_a, as a function of the level of skill. In the absence of wage responses, as we have seen above, the optimum implements a policy such that x_a/w_a is equal to $\pi_a/(1+\pi_a)$ and hence the tax level effect is nil. The dashed decreasing curve $\pi_a/(1+\pi_a)$ in Figure 3 illustrates this profile in the current context where $\dot{\pi}_a < 0$. However, when wages are responsive to taxation (i.e. when $\varepsilon_a > 0$), implementing this policy means that $x_a = w_a - T(w_a) - b$ increases less than proportionally in the wage w_a, so $\eta(w_a) < 1$. Hence, wages are distorted downwards. The optimum trades off the distortions along the wage response effect and along the tax level effect. Since an optimization along the wage response effect corresponds to a flat curve, the optimal policy corresponds to the one illustrated by the solid curve in Figure 3. Thus, the solid curve remains decreasing, which induces that wages and unemployment are distorted downwards for all interior skill levels (point *i)* of the Proposition).

Whether participation rates are distorted upwards or downwards compared to the *laissez faire,* depend on whether the expected surplus is higher along the optimum Σ_a or along the *laissez faire* $w_a^{\mathrm{LF}} L\left(a, w_a^{\mathrm{LF}}\right)$. Let us write Σ_a as $w_a L(a, w_a) \cdot (x_a/w_a)$. First, since wages are distorted, $w_a L(a, w_a)$ is lower at the optimum compared to the *laissez faire.* Second, as illustrated by Figure 3 the ratio x_a/w_a reaches its highest value along the optimum for the lowest skill level. Moreover, x_{a_0}/w_{a_0} is lower at the optimum with wage response effect compared to the optimum without wage response effect. These two features hold because the optimum with wage response effect trades off distortions along the tax level effect and along the wage response effect. Finally, along the optimum without wage response effect, x_{a_0}/w_{a_0} is lower than 1 because the government has a Maximin objective (see 24). Hence, $x_a/w_a < 1$, which finally gives point *ii*) of the Proposition.

Moreover, as $x_a/w_a < 1$, one has $T(w) + b > 0$ for all wage levels. So, transfers for (low income) workers are never higher than for the jobless: There is no EITC in the words of Saez (2002, p. 1055). Furthermore, since x/w is decreasing, $(T(w)+b)/w$ is increasing in wages, hence average tax rates are increasing in wages, too. Finally, since $(T(w)+b)/w$ is positive everywhere and marginal tax rates are higher than this ratio (because $\eta < 1$), marginal tax rates are positive everywhere, including at the boundaries of the skill distribution (Point *iii)* of the Proposition).

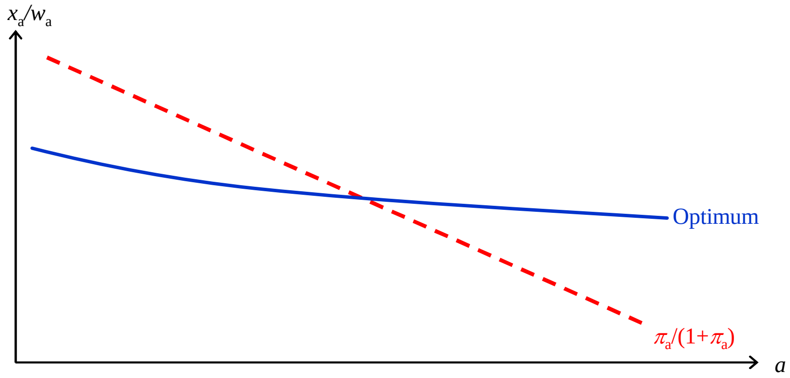

Figure 3: Intuition of Proposition 2

Most of the properties put forward in Proposition 2 are in line with HLPV. There, the value χ of non market activities is identical for all types. Therefore, a unique threshold level of skill separates nonparticipants from participants. The elasticity of participation is thus infinite at the threshold and then nil for all participating types, which is a very specific decreasing $a \mapsto \pi_a$ relationship. In this sense, under a Maximin criterion, the results of HPLV are generalized. However, an exception concerns the distorsion of the labor demand of the low skilled agents. Our transversality condition $Z_{a_0} = 0$ implies that the lowest level of expected surplus Σ_{a_0} is optimized. Our model is thus characterized by a no-distorsion at the bottom of the skill distribution property (provided that there is no bunching, a case that never occurs in our numerical experiments). Since $\eta(w_{a_0}) = 1$, the wage-cum-labor demand margin is not distorted for the lowest participating skill. This result is in contrast with HLPV. There, the government can not optimize along the surplus of the lowest participating skill level because of binding participation constraints. Thus, the governement distorts the labor demand margin at the bottom of the participating skill level distribution.

Moreover, Proposition 2 is in contrast to the literature initiated by Mirrlees (1971). In this framework, optimal marginal tax rates are positive whenever the government values redistribution (see e.g. the discussion in Choné and Laroque 2007). Therefore, labor supply, thereby the volume of labor used, are distorted downwards while in our case the volume of labor among participants is distorted upwards. However, Point ii) reduces this contrast. In our model, participation is distorted downwards. Consequently, the net effect on aggregate employment is ambiguous.

Finally, the property according to which employment tax rates are always positive is also obtained in the models of Saez (2002) and Choné and Laroque (2005) where participation margins are central. Saez (2002) however emphasizes that this result only holds under a Maximin criterion. With a more general objective, he finds that the optimal income tax schedule is typically characterized by a negative employment tax at the bottom provided that labor supply responses along the extensive margin are high enough compared to responses along the intensive margin.

IV The general utilitarian case

Under the general social objective (13), Appendix E shows that the optimum verifies (recall that $\ell_a = L(a, w_a)$ and $h_a = \ell_a \; G(a, \Sigma_a) \; f(a)$):

Proposition 3 *For any skill level $a \in [a_0, a_1]$, the optimal tax schedule verifies:*

$$\left(\frac{1 - \eta(w_a)}{\eta(w_a)} \cdot w_a - \frac{\Phi(w_a - T(w_a)) - \Phi(b) - x_a \cdot \Phi'(w_a - T(w_a))}{\lambda} \right) \cdot \frac{\varepsilon_a}{\alpha_a} \cdot a \cdot h_a = Z_a \tag{26a}$$

$$Z_{a_0} = 0 \tag{26b}$$

$$\textit{where} \qquad Z_a = \int_a^{a_1} \left\{ \left(1 - \frac{\Phi'(w_t - T(w_t))}{\lambda} \right) x_t - \pi_t \left[T(w_t) + b + \Xi_t \right] \right\} h_t \cdot dt \tag{26c}$$

$$\textit{and} \qquad \Xi_t = \frac{\ell_t \cdot \Phi(w_t - T(w_t)) + (1 - \ell_t)\, \Phi(b) - \Phi(b + \Sigma_t)}{\lambda \cdot \ell_t}, \tag{26d}$$

in which the positive Lagrange multiplier associated to the budget constraint (5), λ, verifies

$$\lambda = \int_{a_0}^{a_1} \left\{ \ell_a G(.)\, \Phi'(w_a - T(w_a)) + (1 - \ell_a)\, G(.)\, \Phi'(b) + \int_{\Sigma_a}^{+\infty} \Phi'(b + \chi)\, g(a, \chi)\, d\chi \right\} f(a)\, da \tag{27}$$

We now explain how to extend the intuitive proof of Section III. Equation (27) defines the marginal social value of public funds, λ. It is obtained by a unit increase in E financed by a unit decrease in b holding $w \mapsto w - T(w) - b$ constant. Next, we consider again the small tax reform depicted in Figure 2. This tax reform has a tax level effect and a wage response effect, each of them being decomposed into mechanical and behavioral components (see Table 1). In the Maximin case, these components only capture the impact on the least well-off (i.e. on additional tax receipts to finance the assistance benefit b). Now, the government also values how the utility levels of all other economic agents are affected by the tax reform. To make the formula comparable, we divide these additional impacts by λ, so as to express them in terms of the value of public funds. For each component, we now examine how the various components are changed.

Tax level effect

The rise in the ex-post surplus x_t increases the social welfare of the corresponding workers by $\Phi'(w_t - T(w_t))/\lambda$. Adding this welfare gain to the loss in tax receipts, the mechanical component of the tax level effect at skill level t equals

$$-\left(1 - \frac{\Phi'(w_t - T(w_t))}{\lambda}\right) \cdot x_t \cdot h_t \cdot \Delta\eta \cdot \frac{\delta w}{w} \tag{28}$$

instead of (17). The integral of relation (28) over the skill distribution above a corresponds to the "*between-skill*" motive of redistribution. Since λ averages marginal social welfare over the whole population and Φ is concave, the term in parentheses is positive for most workers. This means that the rise in x_t is in general detrimental to the government's objective. This might however not be true for workers with sufficiently low earnings. In this case, the government would increase the ex-post surplus with respect to the laissez faire for these workers. In opposition to the case where the government has a Maximin objective, this would imply a rise in the participation rate of the less skilled workers.

As far as the behavioral component is concerned, consider individuals of type t who are induced to participate by the tax reform. Their expected utility levels only change by a second-order amount. However, this change in participation decisions increases inequalities because participants' income is different whether they get a job or not. The inequality-averse government values this by $(\ell_t \cdot \Phi(w_t - T(w_t)) + (1 - \ell_t)\Phi(b) - \Phi(b + \Sigma_t))/\lambda$, which equals $\ell_t \cdot \Xi_t$ (by Definition (26d)) and is negative. So, the behavioral component of the tax level effect at skill level t equals

$$\pi_t \{T(w_t) + b + \Xi_t\} \cdot h_t \cdot \Delta\eta \cdot \frac{\delta w}{w} \tag{29}$$

instead of (18). From (26c), the sum of the mechanical and behavioral components over all skill levels t above a equals $-\Delta\eta \cdot (\delta w/w) \cdot Z_a$. It is hard to draw clear conclusions about the value of Z_a. Still, two opposite effects are specific to the general utilitarian case. Compared to the Maximin, raising the ex-post surplus for skills above a is now less detrimental for the social welfare in terms of the mechanical component but the welfare gain of additional participants is less important because of the negative induced impact of increased inequalities on social welfare (the negative Ξ_t term).

Wage response effect

In addition to its impact on b through the tax receipts (described in (21) and (22)), the wage response effect has also a direct influence on social welfare through a change in the expected social welfare of participants of type a, $\ell_a\Phi(w_a - T(w_a)) + (1 - \ell_a)\Phi(b)$. Holding b constant, a mechanical and a behavioral component should again be distinguished.

The wage increase Δw_a rises $\Phi\left(w_a - T\left(w_a\right)\right)$ by the marginal social welfare $\Phi'\left(w_a - T\left(w_a\right)\right)$ times the small increase in the post-tax wage $\left(1 - T'\left(w_a\right)\right)\Delta w_a$. Using (9), the additional mechanical component expressed in terms of the value of public funds equals:

$$\frac{\Phi'\left(w_a - T\left(w_a\right)\right)}{\lambda} \cdot x_a \cdot \eta\left(w_a\right) \cdot h_a \cdot \frac{\Delta w_a}{w_a} \cdot \delta a$$

This component has a positive effect on social welfare. However, the rise in the wage lowers the employment probability ℓ_a by $\Delta \ell_a = -\eta\left(w_a\right) \cdot \left(\Delta w_a / w_a\right) \cdot \ell_a$. Each additional unemployed individual decreases social welfare by $\Phi\left(w_a - T\left(w_a\right)\right) - \Phi\left(b\right)$. Hence, using (8), the additional behavioral component equals

$$-\frac{\Phi\left(w_a - T\left(w_a\right)\right) - \Phi\left(b\right)}{\lambda} \cdot \eta\left(w_a\right) \cdot h_a \cdot \frac{\Delta w_a}{w_a} \cdot \delta a$$

Adding these two components, then using (19) and (20), we get the welfare consequence of the wage response effect

$$-\frac{\Phi\left(w_a - T\left(w_a\right)\right) - \Phi\left(b\right) - x_a \cdot \Phi'\left(w_a - T\left(w_a\right)\right)}{\lambda} \cdot \frac{\varepsilon_a}{\alpha_a} \cdot a \cdot h_a \cdot \delta a \tag{30}$$

The welfare consequence of the wage response effect is negative because it increases inequalities among participants. This is first due to the fact that for a given number of unemployed, the ex-post surplus of each employee increases. Secondly, for a given employee surplus, the number of unemployed increases. The wage response effect implies a "*within-skill*" motive of redistribution that attenuates the will of the government to mitigate the between-skill inequalities. Thus, this effect pushes optimal wages downwards to reduce inequalities among participants and to lower unemployment.

By adding (30) to the impact (23) of the wage response effect on the level of the assistance benefit b, one obtains the left-hand side of (26a) times $\Delta\eta \cdot (\delta w / w)$.

This intuitive proof of Proposition 3 has highlighted that (search) unemployment has two effects on social welfare that cannot be recognized if the wage-cum-labor demand margin is ignored. First, unemployment *per se* is a source of loss in social welfare which calls for downward wage distortions. This is captured by the negative sign of (30). Second, because the fate of participants is not employment for sure, policies that enhance participation have a detrimental induced effect on inequality, as captured by the negative term Ξ_t in (26d).

To gain further insight into the role of Ξ_t, consider the particular case where wages are not responsive to taxation ($\varepsilon_a = 0$ everywhere). Then, the tax level effect has to be nil everywhere at the optimum. From (26c), whatever the skill t, the employment tax should verify:

$$\frac{T(w_t) + b}{w_t - T(w_t) - b} = \frac{1}{\pi_t}\left(1 - \frac{\Phi'\left(w_t - T\left(w_t\right)\right)}{\lambda}\right) - \frac{\Xi_t}{w_t - T(w_t) - b} \tag{31}$$

If Ξ_t was zero, Formula (31) would be identical to Expression (4) in Saez (2002). Then, if the welfare of low skilled workers is highly valued by the government, i.e. if their ability and thus wage is sufficiently low (i.e. such that $\Phi'(w_t - T(w_t))/\lambda > 1$), the employment tax $T(w_t) + b$ should be negative, meaning that transfers for low income workers, $-T(w_t)$, are higher than for the jobless. Now because of unemployment, inequalities between the agents induced to participate by this policy are increased (since Ξ_t is negative). This reduces the willingness of the government to redistribute to low income workers.

When wages are responsive to taxation, the only analytical result in the general utilitarian case concerns wage distortions at both extremes of the skill distribution. There, as in the Maximin case, the tax level effect is nil. Nevertheless, there is a reason to choose an inefficient wage level. This is because unemployment reduces social welfare. To mitigate this effect, it is worth distorting wages downwards at both extremes of the skill distribution.

Concerning the robustness of Proposition 2 obtained under a Maximin objective, we cannot say whether nor when the two new terms in (28) and (29) change the sign of the tax level effect. We can nevertheless make the following conjectures in line with this proposition. As far as point i) is concerned, the government has now an additional incentive to reduce wages and stimulate labor demand since the welfare impact of the wage response effect (30) is negative. However, pushing wages downwards obviously reduces social welfare, and the more so as one moves towards the low-end of the wage distribution. Therefore, compensating transfers for low-skilled workers are expected. Numerical simulations are needed to throw some light on these conjectures.

Before presenting the simulations, it is worth considering an alternative specification to the government's objective (13). First, this allows to better understand the role of government's aversion towards inequality among participants of the same skill level. Appendix F shows that when the government is only concerned by redistribution between (a, χ) groups but not between the unemployed and the employed of the type, the term Ξ_t in (29) and the term given by (30) disappear from the optimal tax formula. The optimal tax schedule now verifies (16a) and

$$Z_a = \int_a^{a_1} \left\{ \left(1 - \frac{\Phi'(w_t - T(w_t))}{\lambda}\right) x_t - \pi_t \left[T(w_t) + b\right] \right\} h_t \cdot dt \tag{32a}$$

$$\lambda = \int_{a_0}^{a_1} \left\{ G(.)\, \Phi'(\ell_a (w_a - T(w_a)) + (1 - \ell_a) b) + \int_{\Sigma_a}^{+\infty} \Phi'(b + \chi)\, g(a, \chi)\, d\chi \right\} f(a)\, da \tag{32b}$$

instead of Equations (26a)-(27). Therefore, only the mechanical component of the tax level effect is changed with respect to our Maximin case (relation (28) replaces relation (17)). As already mentioned, this component implies a reduction of the "between skill motive of redistribution" compared to the Maximin case. Thus, the extent of redistribution and the distortions along the wage-cum-labor demand margin are reduced compared to the Maximin case.

Second, this alternative social welfare criterion, also used in HLPV, allows to emphasize a difference with respect to this paper. When $\varepsilon_a = 0$, since the term Ξ_t is equal to 0, one retrieves again Expression (4) of Saez (2002). Thus, the employment tax might also be negative for the lowest skill level agents. Heterogenous costs of participation might thus remove the property of positive marginal tax rates found by HLPV. Thus, the difference in the results with HLPV are not due to the objective function but to our more general treatment of the cost of participation.

V Simulations

To illustrate how our optimal tax formulae could be used for applied purposes, this section proposes a calibration of our model based on the US economy. This enables us to compute optimal income tax schedules that provide some numerical feel of the policy implications of our analysis. As the underlying model remains stylized in several dimensions the following simulation results should only be considered as illustrative.

V.1 Calibration

To avoid the complexity of interrelated participation decisions within families, we only consider single adults in the US.[17] In addition to the function $\Phi(.)$ for the Bergson-Samuelson criterion, the structural primitives of the model are the density function of skills $f(a)$, the cumulated density function of non-market activities $G(a, \chi)$ and the labor demand function $L(.,.)$ (or the matching function $H(a, V, U)$ and vacancies costs $\kappa(a)$ as explained in Section II.2). In order to control the behavioral responses, we do not calibrate the parameters of the matching function nor the $a \mapsto \kappa(a)$ function but specify the labor demand function. We take

$$\log L(a, w) = B(a) - \varepsilon \left(\frac{w}{c \cdot a}\right)^{\frac{1}{\varepsilon}}$$

Under this specification, the first-order condition (8) for the wage-setting program implies:

$$w_a = c \cdot a \cdot (\eta(w_a))^{\varepsilon} \tag{33}$$

Hence, along a tax schedule where $\log w \mapsto \log(w - T(w) - b)$ is linear, the behavioral elasticities ε_a and α_a are exogenous.

Next, we roughly approximate the tax system that is applied to single adults without children by a linear function $T(w) = \tau \cdot w + \tau_0$ with $\tau = 25\%$ and $\tau_0 = -3000$. The selection of a value of b for the current economy determines whether $\eta(w)$ is lower or larger than 1, and, consequently, whether wages (and thus unemployment) are distorted upwards or downwards. As a benchmark

[17] These are "primary individuals", i.e. persons without children living alone or in households with adults who are not their relatives. They are older than 16 and younger than 66.

and to be consistent with our theoretical analysis where taxes are used only to redistribute income, we assume that wages are efficient in the current economy, so we take $b = -\tau_0 = 3000$. Since η is then constant, the elasticity α_a of the wage with respect to the skill equals 1 in the current economy (see 12b), as it would be the case in a perfectly competitive economy. Moreover ε equals the elasticity of the wage with respect to η in the current economy (12a). This elasticity also equals the *compensated* elasticity of wage with respect to $1-T'$.[18] Following Gruber and Saez (2002), estimates of the latter elasticity would lie between 0.2 and 0.4. We take a conservative value $\varepsilon = 0.1$ in the benchmark calibration and conduct a sensitivity analysis where $\varepsilon = 0.2$. We set c to 2/3, so that in the current economy, total wage income represents two third of the total production. Finally, we use (33) and the distribution of weekly earnings of the Current Population Survey of May 2007 to approximate a distribution of skills among employed workers. Reexpressing variables in annual terms, the range of skills is [\$3,900; \$218,400].[19] Using a quadratic Kernel with a bandwidth of \$63,800 we get an approximation of $L(a)\, G(a, \Sigma_a)\, f(a)$ in the current economy which is depicted by the lowest curve in Figure 4.

To be able to infer relevant properties of the $a \mapsto \pi_a$ relationship from observed elasticities, we adopt the simplest specification of the cumulative distribution of non-market activities, namely (25). So, the elasticity of participation varies exogenously with the level of skill. Because, to our knowledge, the empirical literature does not provide any information about the concavity of the function $a \mapsto \pi_a$, we assume the following simple declining profile $\pi_a = (\pi_{a_0} - \pi_{a_1})\left(\frac{a_1 - a}{a_1 - a_0}\right)^3 + \pi_{a_1}$. We set the elasticity at the bottom, π_{a_0}, to 0.4 and the elasticity at the top, π_{a_1}, to 0.2 in the benchmark calibration and conduct sensitivity analysis. These elasticities are in line with the evidence summarized by Immervoll *et alii* (2007) and Meghir and Phillips (2008).

We adjust the scale parameters $B(a)$ of the labor demand function and $A(a)$ of (25) to generate some realistic properties of skill-specific unemployment and participation rates along our approximation of the current economy. The profile of unemployment (resp. participation) rates in the current economy is calibrated by a decreasing (increasing) function of a:[20]

$$1 - \ell_a = 0.035 + \left(\frac{a_1 - a}{a_1 - a_0}\right)^4 0.045 \qquad \text{and} \qquad \mathcal{G}_a = 0.31\left(1 - \left(\frac{a_1 - a}{a_1 - a_0}\right)^6\right) + 0.58$$

In our approximation of the current economy, the mean unemployment rate is 5.1%, the mean participation rate equals 80.3% and the mean elasticity of the participation rate equals 0.29. Figure 4 depicts the calibrated skill distribution $f(a)$,[21] the distribution of skill among participants in the current economy $\mathcal{G}_a f(a)$ and the distribution of skills among employed individuals

[18] For any *compensated* change in marginal tax rates, one has $\Delta\eta = \frac{\Delta(1-T')}{1-T'} \frac{1-T'}{1-(T+b)/w} = \frac{\Delta(1-T')}{1-T'} \cdot \eta$.

[19] The data are collected for wage and salary workers. We ignore weekly earnings below 50\$, which corresponds to the lowest 1.2% of the earnings distribution.

[20] By approximating a by the educational attainment, our unemployment and participation rates for the whole population, for a_0 and a_1 are in line with the CPS data in June 2007.

[21] $f(a)$ is deducted from the above approximation of $\ell_a \mathcal{G}_a f(a)$.and the calibrations of functions ℓ_a and $\mathcal{G}_a$.

$\ell_a \mathcal{G}_a f(a)$. We compute the level of exogenous public expenditures E from the government's bud-

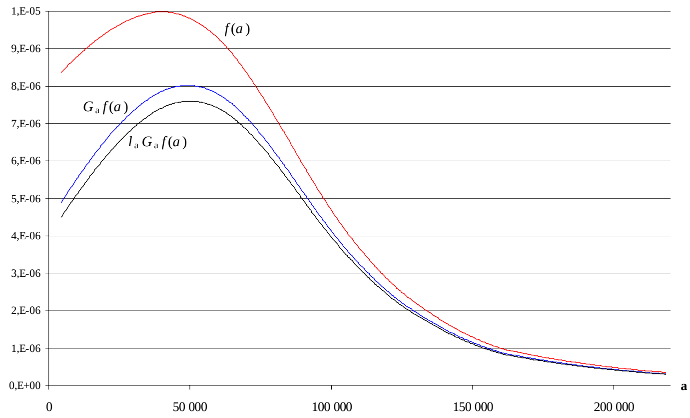

Figure 4: Densities $f(a)$, $\mathcal{G}_a f(a)$ and $\ell_a \mathcal{G}_a f(a)$ in the current economy.

get constraint (5). This leads to an amount $E = \$5,636$ per capita. In the Bergson-Samuelson utilitarian case, we take $\Phi(y) = (y+E)^{1-\sigma}/(1-\sigma)$, with $\sigma = 0.2$ in the benchmark. The exogenous public expenditures finances a public good that generates social utility that is considered as a perfect substitute to private consumption under this specification.

V.2 The results in the benchmark

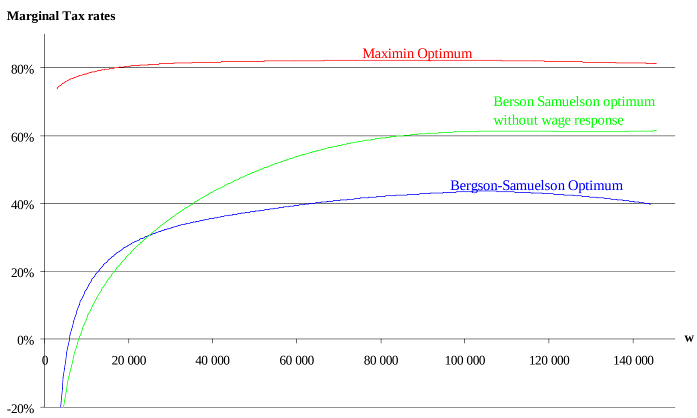

Figure 5: Marginal Tax Rates under the benchmark calibration

Marginal tax rates are drawn in Figure 5. Under the Maximin, redistribution takes the form of a Negative Income Tax (NIT) in the following sense: An assistance benefit close to $\$14,198$ is taxed away at a high, and in this case nearly constant, marginal tax rate close to 80%. This figure appears to be very high. However, this result is close to Saez (2001) and is due to the extreme aversion to inequality of a maximin government. With the more general utilitarian criterion, the well-being of workers, in particular the low-paid ones, enters the scene. This changes dramatically the form and the level of marginal tax rates. At the bottom of the wage distribution, the marginal tax rate is negative and then sharply increases to about 40%. The tax schedule has now the basic features of an EITC-type taxation. In particular, the level of b equals $\$1,015$ per year, while there is an in-work benefit at the bottom whose level is substantially higher since $T(w_{a_0}) = -\$3,167$. The wage of the low skilled agents is actually very low and the governement values highly their welfare. This explains why it compensates them by setting a negative employment tax. Moreover, the assistance benefit and marginal tax rates are much lower with respect to the Maximin case because the motive of redistribution for the non-employed is attenuated (the welfare of all agents being now valued) and the within skill motive of redistribution is now present.

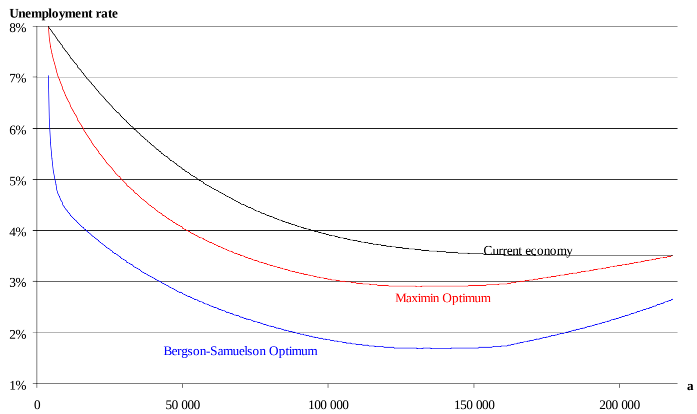

Figure 6: Unemployment under the benchmark calibration

To illustrate Part i) of Proposition 1, let us compare the actual profile of unemployment rates and the optimal ones under the Maximin and Bergson-Samuelson criteria (Figure 6). The actual unemployment rate turns out to be too high from a Maximin perspective. However, the actual unemployment rate is optimal at both ends of the skill distribution since there is no bunching. This result is in contrast to HLPV where the unemployment distortions were maximal for the lowest participating type. Again, the result of no-distorsion at the bottom is the consequence of the additional heterogeneity introduced in the present paper. Here, the highest unemployment distortions appear for the middle of the skill distribution. This explains why the optimal unemployment rate is U-shaped. From the general utilitarian viewpoint, the unemployment rate should even decrease further with respect to the actual one, confirming the importance of the welfare impact of the wage response effect (30). This impact in stronger for the less skilled because their welfare is highly valued by the government. Their unemployment rate is thus strongly distorted downwards.

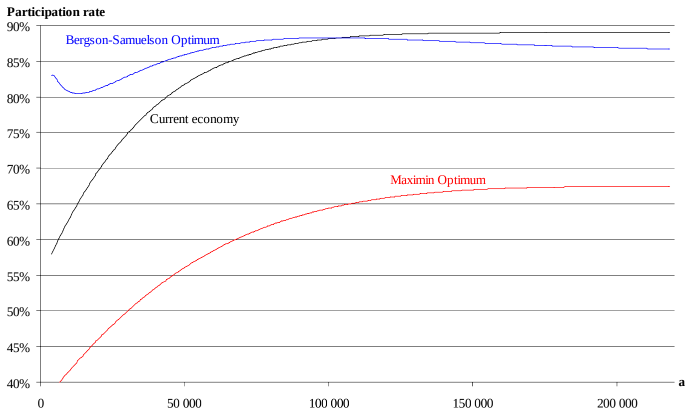

Figure 7: Participation rates under the benchmark calibration.

As an illustration of Part *ii*) of Proposition 1, Figure 7 shows that a Maximin government would accept a sharp decline in participation rates. In order to finance an important assistance benefit, the government has to set high tax levels that decrease the ex-post surplus. Under the more general utilitarian objective, optimal participation rates are higher for low skilled workers and lower for high skilled workers. In order to reduce the unemployment rates of the low skilled, the governement distorts highly their wages downwards. However, it compensates them by fixing a negative employment tax. Their expected surplus is thus increased, thereby pushing up their participation rates. Since unemployment rates are lower and participation rates are higher at the bottom of the skill distribution, the tax-schedule is designed to boost low-skill employment.

V.3 Sensitivity analysis

Figure 5 depicts the optimal tax schedule if the reaction of wages to taxation is ignored ($\varepsilon = 0$). Compared to our benchmark where $\varepsilon = 0.1$, the optimal profile is notably different. In particular, the marginal tax rates are lower at the low-end of the wage distribution and higher for more skilled agents since, by assumption, they do not have any effect on wages and unemployment . So, this figure illustrates that taking into account or ignoring the wage-cum-labor demand margin has substantial quantitative implications. Still, the assistance benefit and the tax reimbursement at the bottom are close to those in the benchmark (so that the property $T(w_{a_0}) + b < 0$ still holds).

If the sensitivity of wages to taxation is raised from $\varepsilon = 0.1$ towards $\varepsilon = 0.2$, the wage response effects are reinforced. The Maximin optimum therefore implements a tax schedule where the function $w \mapsto x(w)/w$ vary less (i.e. the solid curve of Figure 3 becomes flatter) so as to prevent too important distortions along the wage-cum-labor demand margin. The tax schedule becomes closer to a linear one, marginal tax rates vary less. The simulations displayed in Figure 8 show that this also happens along the Bergson-Samuelson optimum.

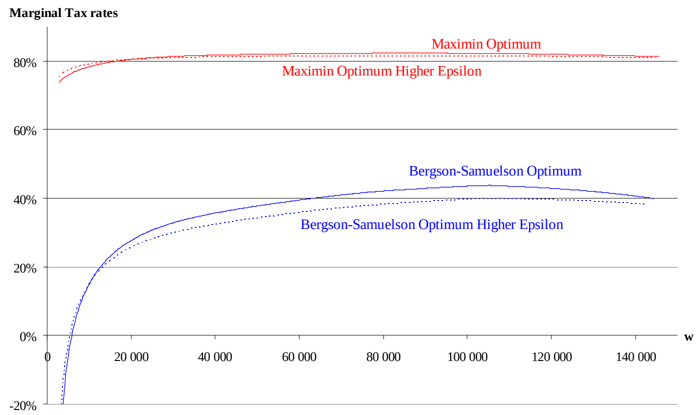

Figure 8: Dotted curves: ε equals 0.2 instead of 0.1 (solid curves).

The other sensitivity analyses we conduct concern the calibration of the elasticity of participation π_a. First, we decrease by a constant amount of 0.05 all the shape of $a \mapsto \pi_a$. In the Maximin case without wage response, Equation (24) implies that the government would choose higher tax levels as participation responds less, so the dashed curve in Figure 3 is shifted downwards. Consequently, in the presence of wage response, the solid curve shifts downwards too. Hence the Maximin optimum implements higher levels of $(T(w) + b)/w$ and therefore higher marginal tax rates. Figure 9 quantifies this mechanism. Once again, The Bergson-Samuelson optimum is affected in a similar way compared to the Maximin optimum.

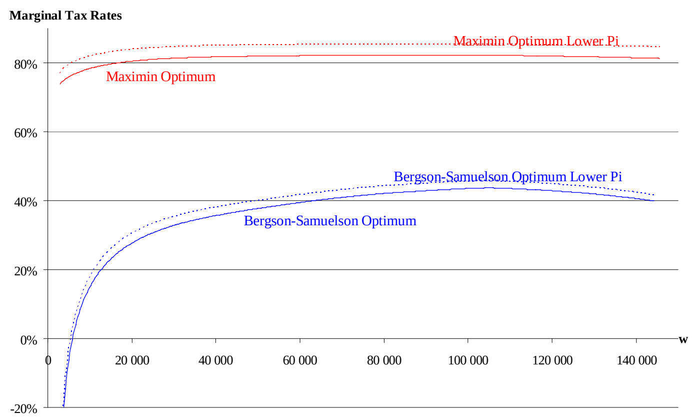

Figure 9: Dashed curves: (π_{a_0}, π_{a_1}) equals $(0.35; 0.15)$ instead of $(0.4; 0.2)$ (solid curves)

Last, we change the elasticities of participation so that the relationship $a \mapsto \pi_a$ is steeper while keeping the average elasticity in the current economy almost constant. For that purpose, we take $(\pi_{a_0}, \pi_{a_1}) = (0.48; 0.13)$ instead of $(0.4; 0.2)$. To understand the rise in marginal tax rates displayed by Figure 10, it is again convenient to come back to Figure 3. In the Maximin optimum without wage response, the government whishes to implement a tax schedule with a more decreasing $a \mapsto x_a/w_a$ function, so the dashed curve of Figure 3 becomes stepper. Hence, in the presence of wage responses, the distortions along the wage cum labor demand are reinforced and the solid curve of Figure 3 becomes stepper too. As a consequence, $\eta(w_a)$ are decreased and marginal tax rates are raised (see 9).

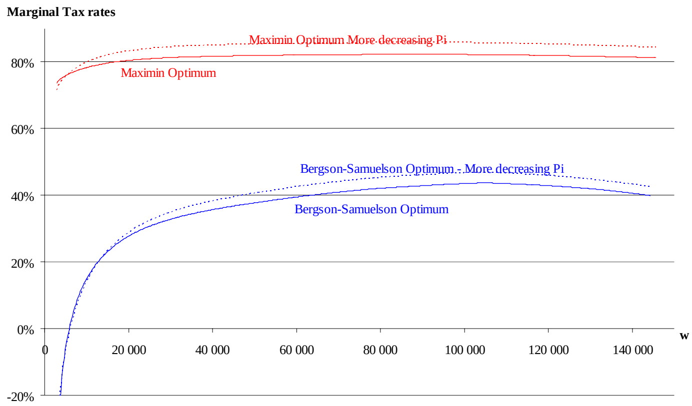

Figure 10: Dashed curves: (π_{a_0}, π_{a_1}) equals $(0.48; 0.13)$ instead of $(0.4; 0.2)$ (solid curves)

In all the simulation exercises, unemployment rates are even lower at the Bergson-Samuelson optimum than at the Maximin one. This confirms the importance of the welfare impact of the wage response effect (30). Participation rates are always higher at the Bergson-Samuelson optimum compared to the Maximin one. They remain lower than the current ones for high skill workers and higher for lower skill workers. Average tax rates are always increasing at the Bergson-Samuelson optimum.

Saez (2001) has simulated optimal marginal tax rates using the empirical distribution of income to compute the underlying distribution of skills, as we do for our model. He has found in a "Mirrlees-type model" that optimal marginal tax rates are U-shaped whereas we find a hump-shaped profile in all our simulations. Saez (2002) has proposed simulations of optimal tax rates at the bottom of the distribution with labor supply responses along both the extensive and the intensive margins. He has showed that an EITC can emerge if the government is not Maximin. Our numerical simulations are thus in line with Saez (2002) on this point and confirm an important difference with HLPV who treated the participation decisions in a crude way.

VI Conclusions

According to authors such as Immervoll et al (2007), the competitive framework is well suited for studying optimal income taxation because the introduction of imperfections would not deeply modify the equity-efficiency trade-off. By modelling jointly participation decisions, wage formation and labor demand in a frictional economy, we show on the contrary that this trade off is deeply modified. In the Maximin case, a set of clear-cut analytical properties are shown if the elasticity of participation decreases with the level of skill. Then at the optimum, the average tax rate is increasing, marginal tax rates are positive everywhere, while wages, unemployment rates and participation rates are distorted downwards compared to their laissez-faire values. These precise recommendations contrast with the small number of analytical properties derived in the literature following Mirrlees (1971).

When the government has a general utilitarian social welfare function, the equity-efficiency trade-off is more deeply affected by the wage-cum-labor demand margin. To induce participation, the net income of workers should be higher than the one of the non-employed. This creates an inequality that matters from a utilitarian perspective. Taxation should then promote wage moderation to reduce the detrimental effect of unemployment on social welfare. Moreover, the role of taxation on participation is more complex because some participants will not find a job. Therefore, stimulating participation through lower tax levels raises inequalities. Our numerical exercise shows that optimal unemployment rates are substantially distorted downwards and that an EITC can be optimal.

Finally, our paper contributes to the literature of self-selection models with random participation. The seminal paper of Rochet and Stole (2002) is developped in a context of nonlinear pricing. Our paper proposes a new method for signing the distorsions in a different framework. This method considers in a first step the tax function that minimizes distortions along the extensive margin by removing wage responses. In a second step, we show that the "full" optimum with wage responses is the result of a tradeoff between this tax function and a linear function for which wages and unemployment are not distorted.

The present model could be extended in different directions. First, a dynamic model would enable to introduce earning-related unemployment insurance. Hence, one can expect that a "dynamic optimal taxation" version (*à la* Golosov *et alii* 2003) of our model would deliver interesting insights about the optimal combination of unemployment insurance and taxation to redistribute income. Second, we abstract from any response of the labor supply along the intensive margin. Although we are confident that responses along the extensive margin are much more important, enriching our framework to include hours of work, in-work effort or educational effort belongs to our research agenda. Finally, in the real world, labor supply decisions are typically taken at the household level, not at the individual one.

Appendices

A Proof of Lemma 1

From the free-entry condition, tightness on the labor market is positive if and only if $a - w > \kappa(a)$. Hence, one must have $\kappa(a) = a - \overline{w}_a$. Let $\xi = \kappa(a) / (a - w_a)$, so that $w_a = a - (\kappa(a)/\xi)$. As w_a increases from $-\infty$ to $\overline{w}_a$, ξ increases from 0 to 1. Then, (2) can be rewritten $L\left(a, a - \frac{\kappa(a)}{\xi}\right) = \xi \cdot q^{-1}(a, \xi)$ and so one must have

$$q^{-1}(a, \xi) = \frac{L\left(a, a - \frac{\kappa(a)}{\xi}\right)}{\xi}$$

for each ξ in $(0, 1]$. For each a, since $L\left(a, a - \frac{\kappa(a)}{\xi}\right)$ is bounded, one has $\lim_{\xi \mapsto 0} q^{-1}(a, \xi) = +\infty$. Hence, $q^{-1}(a, .)$ defines a decreasing function from $[0, 1]$ onto $[0, +\infty]$. Inverting the last function, one retrieves $q(a, \theta)$ defined over $[a_0, a_1] \times \mathbb{R}_+$ onto $[0, 1]$. Then, the matching function $H(a, V, U) \equiv V \cdot q(a, V/U)$ is well defined over $[a_0, a_1] \times \mathbb{R}_+^2$ and exhibits constant returns to scales. Since $q(., .)$ is bounded above by 1, one obtains that $H(a, 0, U) = H(a, V, 0) = 0$ and $H(a, V, U) < V$. Moreover, since the elasticity in ξ of $L\left(a, a - \frac{\kappa(a)}{\xi}\right)$ is negative, the elasticity with respect to ξ of q^{-1} is lower than -1. Consequently, the elasticity of q with respect to θ must lie in between -1 and 0. Therefore $H(a, ., .)$ is increasing in both arguments. To finally show that $H(a, V, U) < U$, let us define $\theta = V/U$ and w by $(a - w)\, q(a, \theta) = \kappa(a)$. Then, one has that $H(a, V, U)/U = \theta q(a, \theta) = L(a, w) < 1$.

B Benthamite efficiency of the *laissez-faire* allocation

Let $\mathcal{U}$ be the Benthamite objective. Consider an equilibrium allocation. There are $G(a, \Sigma_a) f(a)$ participants of type a whose net income is $w_a - T(w_a)$ if they are employed and b otherwise, while non participants obtain $b + \chi$. So, the Benthamite objective writes:

$$\begin{aligned}
\mathcal{U} &= \int_{a_0}^{a_1} \left\{ (L(a, w_a)(w_a - T(.)) + (1 - L(a, w_a))b) \cdot G(a, \Sigma_a) + \int_{\Sigma_a}^{+\infty} (b + \chi) \cdot g(a, \chi) \cdot d\chi \right\} f(a) \cdot da \\
&= \int_{a_0}^{a_1} \left\{ (\Sigma_a + b) \cdot G(a, \Sigma_a) + \int_{\Sigma_a}^{+\infty} (b + \chi) \cdot g(a, \chi) \cdot d\chi \right\} f(a) \cdot da
\end{aligned}$$

where the second equality uses (3). Given the government's budget constraint (5), this objective can be rewritten when $E = 0$ as:

$$\mathcal{U} = \int_{a_0}^{a_1} \left\{ L(a, w_a) \cdot w_a \cdot G(a, \Sigma_a) + \int_{\Sigma_a}^{+\infty} \chi \cdot g(a, \chi) \cdot d\chi \right\} f(a) \cdot da$$

The Benthamite objective aggregates average earnings plus the value of non-market activities over the whole population, no matter how they are distributed. In this sense, the Benthamite criterion is an extreme case.

For each a and w, the function $\Sigma \mapsto L(a, w) \cdot w \cdot G(a, \Sigma) + \int_{\Sigma}^{+\infty} \delta \cdot g(a, \delta) \cdot d\delta$ reaches a unique maximum for $\Sigma = L(a, w) \cdot w$. Therefore, when we compare any allocation $a \mapsto (w_a, \Sigma_a)$

to the *laissez-faire* one, we get:

$$\begin{aligned}
\mathcal{U}^{\mathrm{LF}} &= \int_{a_0}^{a_1} \left\{ L\left(a, w_a^{\mathrm{LF}}\right) \cdot w_a^{\mathrm{LF}} \cdot G\left(a, \Sigma_a^{\mathrm{LF}}\right) + \int_{\Sigma_a^{\mathrm{LF}}}^{+\infty} \chi \cdot g\left(a, \chi\right) \cdot d\chi \right\} f\left(a\right) \cdot da \\
&\geq \int_{a_0}^{a_1} \left\{ L\left(a, w_a^{\mathrm{LF}}\right) \cdot w_a^{\mathrm{LF}} \cdot G\left(a, \Sigma_a\right) + \int_{\Sigma_a}^{+\infty} \chi \cdot g\left(a, \chi\right) \cdot d\chi \right\} f\left(a\right) \cdot da \\
&\geq \int_{a_0}^{a_1} \left\{ L\left(a, w_a\right) \cdot w_a \cdot G\left(a, \Sigma_a\right) + \int_{\Sigma_a}^{+\infty} \chi \cdot g\left(a, \chi\right) \cdot d\chi \right\} f\left(a\right) \cdot da = \mathcal{U}
\end{aligned}$$

The first inequality holds because $\Sigma_a^{\mathrm{LF}} = L\left(a, w_a^{\mathrm{LF}}\right) \cdot w_a^{\mathrm{LF}}$ at the *laissez faire*, according to (3). The second inequality holds because w_a^{LF} maximizes $w \mapsto L\left(a, w\right) \cdot w$.

C Proof of Proposition 1

From (3), one gets that $\left(T\left(w_a\right) + b\right) L\left(a, w_a\right)$ equals $L\left(a, w_a\right) \cdot w_a - \Sigma_a$, so the budget constraint (5) can be rewritten as

$$b = \int_{a_0}^{a_1} \left[L\left(a, w_a\right) \cdot w_a - \Sigma_a\right] \cdot G\left(a, \Sigma_a\right) \cdot f\left(a\right) \cdot da - E$$

Let $\sigma_a = \log \Sigma_a$. We use optimal control by considering σ_a as the state variable and w_a as the control.

$$\begin{aligned}
&\max_{w_a, \sigma_a} \int_{a_0}^{a_1} \left[L\left(a, w_a\right) \cdot w_a - \exp \sigma_a\right] \cdot G\left(a, \exp \sigma_a\right) \cdot f\left(a\right) da \\
&s.t \quad : \quad \dot{\sigma}_a = \frac{\partial \log L}{\partial a}\left(a, w_a\right)
\end{aligned}$$

Let q_a be the multiplier associated to the equations of motion of σ_a and let $Z_a = -q_a$. The Hamiltonian writes

$$\mathcal{H}\left(w, \sigma, q, a\right) \stackrel{\mathrm{def}}{\equiv} \left[L\left(a, w\right) \cdot w - \exp \sigma\right] \cdot G\left(a, \exp \sigma\right) \cdot f\left(a\right) + q \cdot \frac{\partial \log L}{\partial a}\left(a, w\right)$$

Since we assume that a maximum exists where w_a is a continuous function of a (see footnote 14), there exists a continuously differentiable function $a \mapsto q_a$, such that the following first-order condition are verified:

$$0 = \frac{\partial \mathcal{H}}{\partial w} = \frac{\partial\left(L\left(a, w\right) \cdot w\right)}{\partial w}\left(a, w_a\right) \cdot G\left(a, \Sigma_a\right) \cdot f\left(a\right) + q_a \cdot \frac{\partial^2 \log L}{\partial a \partial w}\left(a, w_a\right) \tag{35a}$$

$$-\dot{q}_a = \frac{\partial \mathcal{H}}{\partial \sigma} = -\left\{G\left(a, \Sigma_a\right) - \left[L\left(a, w_a\right) \cdot w_a - \Sigma_a\right] \cdot g\left(a, \Sigma_a\right)\right\} \cdot \Sigma_a \cdot f\left(a\right) \tag{35b}$$

together with the transversality conditions $q_{a_0} = q_{a_1} = \mu_{a_0} = \mu_{a_1} = 0$. Using $q_{a_1} = 0$, $Z_a = -q_a$, one has $Z_a = \int_a^{a_1} \dot{q}_t \cdot dt$. Hence, (35b) with (1) gives (16b). The transversality condition $q_{a_0} = 0$ gives $Z_{a_0} = 0$ in (16a). From (8), one has

$$\frac{\partial\left(L\left(a, w\right) \cdot w\right)}{\partial w}\left(a, w_a\right) = \left(1 - \eta\left(w_a\right)\right) \cdot L\left(a, w_a\right) \cdot w_a \tag{36}$$

From (12a) and (12b) one obtains

$$\frac{\partial^2 \log L}{\partial a \partial w}(a, w_a) = \frac{\alpha_a}{\varepsilon_a} \cdot \frac{\eta(w_a)}{a} \tag{37}$$

Introducing these two last expressions into (35b) gives the first equality in (16a).

D Proof of Proposition 2

We first show that Z is positive on (a_0, a_1). From (16b), one has

$$\dot{Z}_a = \left(\frac{\pi_a}{1+\pi_a} - \frac{x_a}{w_a}\right)(1+\pi_a) \cdot w_a \cdot h_a \tag{38}$$

Assume by contradiction that Z is negative at some point. Since $a \mapsto Z_a$ is continuous, there exists an interval where Z remains negative. Given that $Z_{a_0} = Z_{a_1} = 0$, this implies the existence of an interval $[\underline{a}, \overline{a}]$ such that $Z_{\underline{a}} = Z_{\overline{a}} = 0$ and such that $Z_a \leq 0$ for all $a \in [\underline{a}, \overline{a}]$.

- Since $Z_{\underline{a}} = 0$ and Z_a is negative in the neighborhood on the right of $\underline{a}$, one has $\dot{Z}_{\underline{a}} \leq 0$. Given (38) this implies that:
$$\frac{\pi_{\underline{a}}}{1+\pi_{\underline{a}}} \leq \frac{x_{\underline{a}}}{w_{\underline{a}}}$$
- Since $Z_a \leq 0$, one has from (16a) that $\eta(w_a) \geq 1$ for all $a \in [\underline{a}, \overline{a}]$. Given (9), this implies that x_a/w_a is nondecreasing, so
$$\frac{x_{\underline{a}}}{w_{\underline{a}}} \leq \frac{x_{\overline{a}}}{w_{\overline{a}}}$$
- Since $Z_{\overline{a}} = 0$ and Z_a is negative in the neighborhood on the left of $\overline{a}$, one has $\dot{Z}_{\overline{a}} \geq 0$. Given (38) this implies that
$$\frac{x_{\overline{a}}}{w_{\overline{a}}} \leq \frac{\pi_{\overline{a}}}{1+\pi_{\overline{a}}}$$

These three inequalities leads to $\pi_{\overline{a}} \geq \pi_{\underline{a}}$, so one must have $\underline{a} = \overline{a}$ since $a \to \pi_a$ is decreasing. Hence, Z_a is nonnegative on (a_0, a_1) and can only be nil pointwise.

Next, assume by contradiction that there exists $a_2 \in (a_0, a_1)$ such that $Z_{a_2} = 0$. Since Z_a is everywhere nonnegative, a_2 is an interior minimum of Z_a, so $\dot{Z}_{a_2} = 0$, and from (38)

$$\frac{\pi_{a_2}}{1+\pi_{a_2}} = \frac{x_{a_2}}{w_{a_2}}$$

However since $Z_{a_2} = 0$, one has $\eta(w_{a_2}) = 1$ from (16a). Hence, from (9) and the differentiability of $a \mapsto w_a$, x_a/w_a admits a derivative with respect to a that is nil. Since Z_a can only be nil pointwise within (a_0, a_1), there exists a real a_3 in the neighborhood of a_2 such that $a_3 > a_2$ and $Z_{a_3} > 0$. According to the mean value theorem, there exists $a_4 \in (a_2, a_3)$ such that $\dot{Z}_{a_4} = (Z_{a_3} - Z_{a_2}) / (a_3 - a_2) > 0$. From (38), one obtains

$$\frac{\pi_{a_4}}{1+\pi_{a_4}} > \frac{x_{a_4}}{w_{a_4}}$$

Since a_4 is in the neighborhood of a_2 and $a \mapsto x_a/w_a$ has a zero derivative at a_2, then one has $(x_{a_4}/w_{a_4}) \simeq (x_{a_2}/w_{a_2})$ at a first-order approximation. However, $(\pi_{a_4}/(1+\pi_{a_4})) \simeq$

$(\pi_{a_2}/(1+\pi_{a_2})) + \left(\dot{\pi}_{a_2}/(1+\pi_{a_2})^2\right)(a_4 - a_2)$ at a first-order approximation. Hence, since since $\dot{\pi}_{a_2} < 0$, one must have

$$\frac{\pi_{a_4}}{1+\pi_{a_4}} < \frac{\pi_{a_2}}{1+\pi_{a_2}} = \frac{x_{a_2}}{w_{a_2}} \simeq \frac{x_{a_4}}{w_{a_4}}$$

which leads to the contradiction. Therefore, Z_a is positive everywhere within (a_0, a_1).

From (16a), one has $\eta(w_a) < 1$ for any $a \in (a_0, a_1)$, which has different implications.

i) For any $a \in (a_0, a_1)$, one has $\partial \log L / \partial w (a, w_a) > -1$ from (8). Moreover, at the *Laissez faire,* $\partial \log L/\partial w \left(a, w_a^{\mathrm{LF}}\right) = -1$ from (8) and (9). Hence, from (6) $w_a < w_a^{\mathrm{LF}}$ which means that optimal wages are distorted downwards. Furthermore, since $\partial L/\partial w(a,.) < 0$, one has $1 - L(a, w_a) < 1 - L\left(a, w_a^{\mathrm{LF}}\right)$ and unemployment rates are distorted downwards. Finally, $Z_{a_0} = Z_{a_1} = 0$ induces $w_{a_0} = w_{a_0}^{\mathrm{LF}}$, $L(a_0, w_{a_0}) = L\left(a_0, w_{a_0}^{\mathrm{LF}}\right)$, $w_{a_1} = w_{a_1}^{\mathrm{LF}}$ and $L(a_1, w_{a_1}) = L\left(a_1, w_{a_1}^{\mathrm{LF}}\right)$.

ii) Since $\eta(w_a) < 1$, x_a/w_a is nonincreasing in a, so it is maximized at a_0. Since $Z_{a_0} = 0$ and $Z_a > 0$ on (a_0, a_1), one must have $\dot{Z}_{a_0} \geq 0$. Therefore, $x_{a_0}/w_{a_0} \leq \pi_{a_0}/(1+\pi_{a_0}) < 1$. Hence for all a, $x_a < w_a$ and participation rates are distorted downwards.

iii) $x < w$ for all w implies that the employment tax rate $(T(w)+b)/w$ is always positive. Moreover, it is nondecreasing since $\eta(w) < 1$. So, the average tax rate $T(w)/w$ is increasing in wage w. Finally (9) and $\eta(w) \leq 1$ induces $T'(w) \geq (T(w)+b)/w$, so marginal tax rate are positive everywhere.

E Proof of Proposition 3

The proof of Proposition 3 extends the one of Proposition 1 in Appendix C. Let λ be the multiplier associated to the budget constraint. From (3), $w_a - T(w_a) = (\Sigma_a/L(a,w_a)) + b$, so the Hamiltonian becomes:

$$\mathcal{H}(w,\sigma,q,a,b,\lambda) \stackrel{\mathrm{def}}{\equiv} \left[L(a,w)\,\Phi\left(\frac{\exp\sigma}{L(a,w)} + b\right) + (1 - L(a,w)\,\Phi(b))\right] G(a,\exp\sigma)\cdot f(a)$$
$$+ \int_{\exp\sigma}^{+\infty} \Phi(b+\chi)\, g(a,\chi)\, f(a)\, d\chi + \lambda\left[L(a,w)\cdot w - \exp\sigma\right]\cdot G(a,\exp\sigma)\cdot f(a) + q\cdot\frac{\partial \log L}{\partial a}(a,w)$$

The first-order conditions now becomes, where we define $Z_a = -q_a/\lambda$

$$\begin{aligned}
0 \quad &= \quad \frac{1}{\lambda}\frac{\partial\mathcal{H}}{\partial w} = \left[\frac{\partial L(a,w)}{\partial w}(a,w_a)\cdot\frac{\Phi\left(\frac{\Sigma_a}{L(a,w_a)}+b\right) - \Phi(b) - \frac{\Sigma_a}{L(a,w_a)}\Phi'\left(\frac{\Sigma_a}{L(a,w_a)}+b\right)}{\lambda}\right.\\
&\quad\left. + \frac{\partial\left(L(a,w)\cdot w\right)}{\partial w}(a,w_a)\right]\cdot G(a,\Sigma_a)\cdot f(a) - Z_a\cdot\frac{\partial^2 \log L}{\partial a\partial w}(a,w_a)\\
\dot{Z}_a \quad &= \quad \frac{1}{\lambda}\frac{\partial\mathcal{H}}{\partial\sigma} = \left\{\frac{\Phi'\left(\frac{\Sigma_a}{L(a,w_a)}+b\right)}{\lambda} - G(a,\Sigma_a) + \left[L(a,w_a)\cdot w_a - \Sigma_a\right]\cdot g(a,\Sigma_a)\right.\\
&\quad\left. \frac{L(a,w_a)\,\Phi\left(\frac{\Sigma_a}{L(a,w_a)}+b\right) + (1-L(a,w_a))\,\Phi(b) - \Phi(b+\Sigma_a)}{\lambda}\cdot g(a,\Sigma_a)\right\}\cdot\Sigma_a\cdot f(a)
\end{aligned}$$

These two conditions with the transversality conditions $Z_{a_0} = Z_{a_1} = 0$, (36) and (37) give (26a) to (26d). Finally, the condition with respect to b is exactly (27).

F Optimal allocation under an alternative objective

When the government is concerned by redistribution between skill groups but not between the unemployed and the employed of each skill group, its objective writes

$$\int_{a_0}^{a_1} \{\Phi\left[L(a,w_a)(w_a - T(w_a)) + (1 - L(a,w_a))\, b\right] G(a,\Sigma_a) \tag{40}$$

$$\left. + \int_{\Sigma_a}^{+\infty} \Phi(b+\chi)\; g(a,\chi)\; d\chi \right\} f(a)\; da$$

Then, the Hamiltonian becomes:

$$\mathcal{H}(w,\sigma,q,a,b,\lambda) \stackrel{\text{def}}{\equiv} \left[\Phi(\exp\sigma + b)\right] G(a,\exp\sigma)\cdot f(a)$$

$$+ \int_{\exp\sigma}^{+\infty} \Phi(b+\chi)\, g(a,\chi)\, f(a)\, d\chi + \lambda\left[L(a,w)\cdot w - \exp\sigma\right]\cdot G(a,\exp\sigma)\cdot f(a) + q\cdot\frac{\partial \log L}{\partial a}(a,w)$$

The first-order conditions now becomes, where we define $Z_a = -q_a/\lambda$

$$\begin{aligned} 0 &= \frac{1}{\lambda}\frac{\partial\mathcal{H}}{\partial w} = \frac{\partial\left(L(a,w)\cdot w\right)}{\partial w}(a,w_a)\cdot G(a,\Sigma_a)\cdot f(a) - Z_a\cdot\frac{\partial^2\log L}{\partial a\partial w}(a,w_a) \\ \dot{Z}_a &= \frac{1}{\lambda}\frac{\partial\mathcal{H}}{\partial\sigma} = \left\{\frac{\Phi'\left(\frac{\Sigma_a}{L(a,w_a)} + b\right)}{\lambda} - G(a,\Sigma_a) + \left[L(a,w_a)\cdot w_a - \Sigma_a\right]\cdot g(a,\Sigma_a)\right\}\cdot\Sigma_a\cdot f(a) \end{aligned}$$

These two conditions with the transversality conditions $Z_{a_0} = Z_{a_1} = 0$, (36) and (37) give (16a), and (32a). Finally, the condition with respect to b is exactly (32b).

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