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ABSTRACT

This paper proposes an explanation of the international home bias in equity based on ambiguity aversion. Doubts imply an additional hedging motif driven by the interaction between real exchange rate risk and ambiguity aversion. What matters is the long-run as opposed to the short-run risk. Domestic equity is a good hedge with respect to long-run real exchange rate risk even when bonds are traded. The higher is the degree of ambiguity aversion, the stronger is the home bias. We identify the degree of ambiguity aversion with detection error probabilities and show that our framework is able to explain a large share of the observed US home bias, as well as other stylized facts on US cross-border asset holdings.

Without doubts, a standard open-economy macroeconomic model would be unsuccessful along all these dimensions.

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1 Introduction

The lack of international diversification in equity portfolios is one of the most persistent observations in international finance. Investors hold a large share of their wealth in domestic securities, more than what would be dictated by the share of these securities in the world market. This is known as the "the home-bias puzzle" (French and Poterba, 1991, Tesar and Werner, 1995).

We address this puzzle by introducing doubts in a standard open-economy model along the lines of Hansen and Sargent (2005, 2007).[1] Agents are endowed with a reference probability distribution which they do not trust. They express their doubts by surrounding the reference probability distribution with a set of nearby distributions. They fear this possible model misspecification and end up following the worst distribution in the set of surrounding distributions. This environment can account for an undiversified portfolio and home-bias in assets holdings. Our explanation relies on the interaction between natural asymmetries, characterizing open economies, and model uncertainty. Both features are necessary for our result. In particular, we stress the importance of fluctuations in the real exchange rate as the key asymmetry driving our results.[2]

Agents enter financial markets with different appetites for state-contingent wealth, and asset trading provides a way to reduce such differences. In a standard model, the appetite for wealth is driven by the marginal utility of consumption and indeed asset trading helps to reduce the idiosyncratic movements in the marginal utilities across countries. With model uncertainty, differences in the appetite for wealth can also be driven by differences in the beliefs. Ambiguity-averse agents are worried about bad news regarding their consumption profiles. Cross-country variations in beliefs translate into bad news about the cross-country differences in consumption growth. The real exchange rate is a source of fluctuations for relative consumption growth and an additional source of risk with respect to those driving the differences in the marginal utility of consumption.[3] Agents use portfolio holdings to hedge this risk and can bias their choices towards domestic asset. We provide empirical support for this result.

Hedging real exchange rate has been a popular argument for explaining asset home bias since the work of Adler and Dumas (1983). However, in their model, this channel follows from the desire of reducing the idiosyncrasies in the marginal utility of consumption across countries, while in our model it is driven by the wedge in the distorted beliefs. This is more than a subtlety. In their framework, to be able to account for some home bias, the risk-aversion coefficient should increase, at the cost of raising in a counterfactual way the mean of the risk free rate, the so-called risk-free rate puzzle (Weil, 1989). In our model, instead, the relevant parameter is the degree of ambiguity aversion – a measure of the size of the set of nearby probability distributions– which can increase without falling in the risk-free rate puzzle. The result of home bias depends on degrees of ambiguity aversion which are consistent with reasonable values of the detection error probability that agents face when confronting the reference and the subjective measures.

A more recent literature has de-emphasized hedging real exchange rate as a relevant channel to explain asset home bias (see van Wincoop and Warnock, 2006, 2008), because in the data the

[1] An incomplete list of papers that have tried to address the home-bias puzzle are: Benigno and Kucuk-Tuger (2008), Bottazzi et al. (1998), Coeurdacier (2005), Coeurdacier and Gourinchas (2009), Coeurdacier et al. (2007), Cole and Obstfeld (1991), Engel and Matsumoto (2006), Epstein and Miao (2003), Heathcote and Perri (2004), Kollmann (2006), Obstfeld and Rogoff (2001), Pavlova and Rigobon (2007), Uppal (1993).

[2] Deviations from purchasing power parity have been the subject of an extensive literature in open-economy macroeconomics, see Rogoff (1996).

[3] Even if the cross-country marginal utility of consumption might be also driven by real exchange rate fluctuations, this risk is not collinear to the one driving the cross-country differences in the beliefs. See Section 2 for details.

covariance between the real exchange rate and the excess return on foreign-versus-domestic equity is negligible, conditional on the excess return on bonds. Our model emphasizes the importance of long-run versus short-run real exchange rate risk. An important contribution, indeed, is to show that the long-run risk is empirically relevant, even conditional on other excess returns, and equities are a good hedge for this risk. This is key for our model to succeed in explaining the home-bias puzzle.[4]

On the contrary, we find that non-diversifiable labor-income risk is not sufficient to explain asset home bias, confirming previous results of Baxter and Jermann (1997) and in contrast to recent findings of Coeurdacier and Gourinchas (2009). As a consequence, standard open-economy macro models under rational expectations do not succeed in explaining the home-bias puzzle.

Finally, we show that ambiguity aversion is also able to reconcile the model with other stylized empirical facts on US cross-border holdings that the rational-expectations benchmark has difficulties in replicating. In particular, the US is a net creditor in equity instruments and a net debtor in bond instruments, its position in foreign-currency bonds is about balanced, whereas that in home-currency bonds is largely negative (Tille, 2005 and 2008).

The structure of this paper is the following. The next section gives the main intuition of the results and discusses more extensively the contribution of the paper with respect to related literature. Section 3 discusses the structure of model uncertainty. In Section 4 we present the model. We contrast the equilibrium portfolio allocation implied by the standard framework with rational expectations, in Section 5, with those implied by model uncertainty, in Section 6. Section 7 presents the empirical relevance of the model.

2 Intuition and related literature

In this section, we provide the intuition of the main results of the paper and discuss the comparison with related literature. A fundamental principle of finance, based on the arbitrage theory, is that households want to hold assets that pay well when needed. A measure of the appetite for wealth is given by the stochastic discount factor: the higher the stochastic discount factor in a particular state of nature, the higher the appetite for wealth of the households in that state. Therefore, when households have different stochastic discount factors, they might exploit trade in assets in order to hedge such differences. Indeed, when perfect risk-sharing is achieved, the wedge between the stochastic discount factors across agents is completely eliminated. In particular, for agents trading in international assets, perfect risk-sharing requires equalization of the stochastic discount factors evaluated in units of the same consumption index:

$$m_{t+1} = m^*_{t+1}\frac{q_t}{q_{t+1}},$$

where m_{t+1} is the stochastic discount factor for evaluating wealth in states of nature at time $t+1$ for the household of a generic country H, m^*_{t+1} the respective factor for the household

[4]Our approach differs from most of the existing literature which finds portfolio shares as a function of primitive parameters, like the risk-aversion coefficient, the share of traded goods, or the trade cost. This is clearly a desirable feature of general equilibrium models, but it has the drawback of hiding the hedging relationships based on observable variables that are at the root of the portfolio decisions. Van Wincoop and Warnock (2006, 2008) show that the covariances between the asset returns and the sources of risk implied by these models are often counterfactual: once data restrictions on asset prices are considered, these models fail to solve the portfolio home-bias puzzle. On the other hand, the few contributions focusing on the hedging relationships that underlie portfolio choices (such as Coeurdacier and Gourinchas, 2009 and van Wincoop and Warnock, 2006, 2008) typically use static models, which, by construction, neglect any possible source of long-run risk.

in country F and q_t is the real exchange rate. In a consumption-based model, the stochastic discount factors depend on the respective marginal utilities of consumption. In particular, in a quite general model and in a log-linear approximation, deviations from full risk-sharing can arise from three sources of risks:

$$\ln m_{t+1} + \Delta \ln q_{t+1} - \ln m^*_{t+1} \simeq m_e(\bar{\boldsymbol{\lambda}} - \bar{\boldsymbol{\lambda}}^{full})\mathbf{exr}_{t+1} + m_q \varepsilon_{q,t+1} + m_l \varepsilon_{l,t+1}. \tag{1}$$

In the equation above, $\mathbf{exr}_{t+1}$ represents the vector of excess returns – the first source of risk; $\varepsilon_{q,t+1}$ is a measure of the real exchange rate risk and $\varepsilon_{l,t+1}$ of cross-country non-diversifiable labor-income risk, expressed in units of the same consumption index; $\bar{\boldsymbol{\lambda}} - \bar{\boldsymbol{\lambda}}^{full}$ captures the deviations of the optimal steady-state portfolio from the one that achieves full diversification; finally, m_e, m_q and m_l depend on structural model parameters.[5] When there is no real exchange rate nor labor-income risk ($\varepsilon_{q,t+1} = \varepsilon_{l,t+1} = 0$ for all t), the optimal portfolio clearly implies full diversification: $\bar{\boldsymbol{\lambda}} = \bar{\boldsymbol{\lambda}}^{full}$. More generally, the real exchange rate risk is relevant only as long as the risk-aversion coefficient is different from the unitary value (otherwise $m_q = 0$). Departures from full diversification are optimal when excess returns display some covariance with the other two sources of risk.[6] Whichever channel is relevant and able to explain asset home bias is therefore a question of empirical evaluation.

Real exchange rate risk and non-diversifiable labor-income risk have been widely explored in the literature. However, neither of them is completely satisfactory. On the one hand, the covariance between real exchange rate risk and the excess return on equities has been found to be very small, conditional on the excess return on bonds. Accordingly, for this channel to be empirically relevant, the degree of risk aversion should rise, to increase m_q, but at the cost of increasing also the mean of the risk-free rate implied by the model – the so-called risk-free rate puzzle. On the other hand, the covariance between the labor-income risk and the excess return on equities can be weak or strong. In particular, we show that results might depend on the measure of labor-income risk used. Our model-based definition is not able to explain important departures from full diversification.

However, our explanation of the home-bias puzzle comes from a different source through model uncertainty and ambiguity aversion. Under model uncertainty agents in the two countries can act according to different subjective probabilities. The first important consequence implied by this environment is that perfect risk-sharing requires now the equalization of the stochastic discount factors evaluated under the same probability measure:

$$g_{t+1} m_{t+1} = g^*_{t+1} m^*_{t+1} \frac{q_t}{q_{t+1}}$$

where g_{t+1} and g^*_{t+1} are the changes of measure from the subjective distributions (which are country-specific) to the reference one (which is instead common across countries). Accordingly, agents face an additional hedging motif driven by the possible difference in the subjective probability measures.

Indeed, differences in the appetite for wealth now arise not only from possible differences in the marginal utilities of consumption across countries – first hedging motif, coming from (1) – but also from differences in the probabilities agents assign to states of nature. More specifically,

[5] In our model, we consider trade in two equities and two bonds (one for each country) and therefore three excess returns. The vector λ^{full} has three dimensions and implies zero bond holdings and an equal split of equity between countries. See Section 5 for details.

[6] When markets are incomplete, agents try to minimize the projection of the deviations from zero, described by equation (1), on the space spanned by the asset payoffs.

when agents are ambiguity averse, such subjective probabilities reflect the fear of model misspecification, and are related to revisions in the path of consumption growth. Consequently, the additional hedging motif is related to revisions in the relative cross-country consumption profile. Indeed, ambiguity-averse agents will assign a high subjective probability to those states of nature in which their consumption growth is lower than in the other country, and will therefore want to invest more in assets that pay well precisely in those states of nature. In a log-linear approximation, this additional hedging motif can be written as a linear function of the same three sources of risk identified before

$$\ln g_{t+1} - \ln g^*_{t+1} \simeq (\gamma - 1)[g_e(\bar{\boldsymbol{\lambda}} - \bar{\boldsymbol{\lambda}}^{full})\mathbf{exr}_{t+1} + g_q \varepsilon_{q,t+1} + g_l \varepsilon_{l,t+1}], \tag{2}$$

where $\gamma \geq 1$ denotes the degree of ambiguity aversion, which we assume to be equal across countries ($\gamma = \gamma^*$), and g_e, g_q and g_l are parameters. The optimal portfolio now seeks to "minimize" the sum of equations (1) and (2). In particular, with no real exchange rate nor labor-income risk, the optimal portfolio still implies full diversification, even with model uncertainty, and for any degree of ambiguity aversion γ. Moreover, if there is labor-income risk but no real exchange rate risk, the degree of ambiguity aversion still does not affect the optimal portfolio allocation. Indeed, we show that $g_l = m_l$ and $g_e = m_e$, and therefore that the two sources of risk behind the two hedging motifs are collinear, implying the same portfolio. On the contrary, g_q is always different from m_q; hence, as long as there are fluctuations in the real exchange rate, the degree of ambiguity aversion does affect the optimal portfolio allocation. This is particularly evident in the case of log-utility (which is assumed throughout the paper) in which $m_q = 0$ while $g_q = -1$. Now γ can increase to make the real exchange rate risk empirically more relevant without raising the mean of the risk-free rate.[7] Indeed, by calibrating γ using detection error probabilities, we show that reasonable values of ambiguity aversion are able to explain a substantial degree of home bias in US equity holdings.

There is a related literature on ambiguity and portfolio choices. The most related papers are Epstein and Miao (2003) and Uppal and Wang (2003). Epstein and Miao (2003) incorporate Knightian uncertainty in a multi-agent model by allowing for multiple priors. They explain home bias in asset holdings by assuming heterogeneity among agents concerning the degree of ambiguity about returns. They do not consider natural asymmetries coming from open-economy modeling and therefore $\varepsilon_{q,t+1} = \varepsilon_{l,t+1} = 0$ in their model. Instead, in our model, the implied under-diversification is not the result of an asymmetric attitude towards ambiguity across agents, since we assume $\gamma = \gamma^*$, but the consequence of natural open-economy asymmetries, which we show that are indeed relevant in the data. Uppal and Wang (2003), instead, analyzes a single-agent partial equilibrium model with trading in multiple assets, in which the agent has different degrees of ambiguity aversion across the return processes of the various assets. As in Epstein and Miao (2003), they too build key asymmetries in the degrees of ambiguity aversion, though with respect to different assets rather than across different agents. Uppal and Wang (2003) build on the framework proposed by Hansen and Sargent (2005, 2007), as we do, but to obtain tractability they adopt a non-innocuous modification of the original problem, as done also in Manhout (2004, 2006). This makes our model and theirs not comparable.[8] Our approach, instead, adheres completely to the methodology of Hansen and Sargent (2005, 2007). In this respect, a further methodological contribution of this paper is to derive a simple solution of non-linear robust-control problems through approximation methods.

[7] See in particular Barillas et al. (2006) on this point.

[8] Indeed, they transform a constant lagrange multiplier into a time-varying function of the value function to get a closed-form solution. Pathak (2002) describes the transformation employed by Maenhout (2004, 2006) and Uppal and Wang (2003) as poorly motivated and explains in details the unappealing consequences.

3 Model Uncertainty

We characterize model uncertainty as an environment in which agents are endowed with some probability distribution, but they are not sure that it is in fact the true data-generating one, and might instead act using a nearby distorted "subjective" probability distribution.

Consider a generic state of nature s_t at time t and define s^t as the history $s^t \equiv [s_t, s_{t-1}, ..., s_0]$. Let agents be endowed with $\pi(s^t)$ as the "approximating" or "reference" probability measure on histories s^t. Decision-makers may seek a different probability measure, a "subjective" one, denoted by $\widetilde{\pi}(s^t)$ which is absolutely continuous with respect to the "approximating" measure. Absolute continuity is obtained by using the Radon-Nykodym derivative.[9] First, the two probability measures agree on which events have zero probability. Second, there exists a non-negative martingale $G(s^t)$ with the property

$$E(G_t) \equiv \sum_{s^t} G(s^t)\pi(s^t) = 1 \tag{3}$$

such that, for a generic random variable $X(s^t)$,

$$\widetilde{E}(X_t) \equiv \sum_{s^t} \widetilde{\pi}(s^t)X(s^t) = \sum_{s^t} G(s^t)\pi(s^t)X(s^t) \equiv E(G_t X_t) \tag{4}$$

in which $E(\cdot)$ and $\widetilde{E}(\cdot)$ denote the expectation operators under the "approximating" and "subjective" probability measures, respectively. Specifically, $G(s^t)$ is a probability measure, equivalent to the ratio $\widetilde{\pi}(s^t)/\pi(s^t)$, that allows a change of measure from the "approximating" to the "subjective" measure.

Moreover, since G_t is a martingale, we can define its increment $g(s_{t+1}|s^t)$ as

$$g(s_{t+1}|s^t) \equiv \frac{G(s^{t+1})}{G(s^t)},$$

with the property $E_t g_{t+1} = 1$. It follows that $g(s_{t+1}|s^t)$ is equivalent to the likelihood ratio $\widetilde{\pi}(s_{t+1}|s^t)/\pi(s_{t+1}|s^t)$, and acts as a change of measure in conditional probabilities. High values of $g(s_{t+1}|s^t)$ imply that the decision-makers assign a higher subjective probability to state s_{t+1} conditional on history s^t.

For each random variable X_{t+1}, therefore, the martingale increment g_{t+1} defines a mapping between the conditional expectations under the two measures:

$$\widetilde{E}_t(X_{t+1}) = E_t(g_{t+1}X_{t+1}), \tag{5}$$

in which $E_t(\cdot)$ and $\widetilde{E}_t(\cdot)$ denote the conditional-expectation operators.

As in Hansen and Sargent (2005), we use conditional relative entropy as a measure of the divergence between the "approximating" and "subjective" probabilities,

$$E_t(g_{t+1} \ln g_{t+1}),$$

which approximately measures the variance of the distortions in the beliefs. When there are in fact no distortions this measure is zero: in this case, indeed, $g(s_{t+1}|s^t) = 1$ for each s_{t+1}. In particular, since we are going to work with a dynamic model, in what follows, it is more

[9] This way of constructing subjective probability measures is borrowed from Hansen and Sargent (2005, 2007).

appropriate to exploit the discounted version of conditional relative entropy discussed in Hansen and Sargent (2005)

$$\eta_{t_0} \equiv E_{t_0} \left\{ \sum_{t=t_0}^{\infty} \beta^{t-t_0} G_t E_t (g_{t+1} \ln g_{t+1}) \right\}, \tag{6}$$

where $0 < \beta < 1$. A high value of entropy can be interpreted as a very large divergence between the "subjective" and the "approximating" beliefs. On the contrary a low value of entropy implies beliefs that are not too distorted or dissimilar from the reference model.

4 Model

We consider a model with two countries, denoted domestic (H) and foreign (F), each populated by a representative agent. Representative agents supply a fixed amount of labor. In each country, there is a continuum of firms producing a continuum of goods in a market characterized by monopolistic competition. All goods are traded. Households enjoy consumption of both domestic and foreign goods and can trade in a set of financial assets. Specifically, there are four assets traded in the international markets: two risk-free nominal bonds, denominated in each currency, and two equity assets, representing claims on the dividends of domestic and foreign firms, respectively. In their consumption and portfolio decisions, households mistrust the reference probability distribution and therefore compute expectations according to the subjective one.

The representative agent in the domestic economy maximizes utility given by

$$\widetilde{E}_{t_0} \left\{ \sum_{t=t_0}^{\infty} \beta^{t-t_0} \ln c_t \right\} \tag{7}$$

where β, with $0 < \beta < 1$, is the intertemporal discount factor and $\widetilde{E}_{t_0}(\cdot)$ is the time$-t_0$ expectation operator taken with respect to the distorted probability measure. As discussed in the previous section, this distorted measure is absolutely continuous with respect to the "reference" measure and satisfies property (5). The expected utility can be written also in terms of the "reference" distribution as

$$\widetilde{E}_{t_0} \left\{ \sum_{t=t_0}^{\infty} \beta^{t-t_0} \ln c_t \right\} = E_{t_0} \left\{ \sum_{t=t_0}^{\infty} \beta^{t-t_0} G_t \ln c_t \right\}$$

where we have normalized $G_{t_0} = 1$. The representative agent in the other country has similar preferences but a possibly different subjective probability measure and therefore a different expectation operator $\widetilde{E}^*_{t_0}(\cdot)$.

The utility flow is logarithmic in the consumption index c. The latter is a CES aggregator of domestic (c_H) and imported (c_F) goods:

$$c \equiv \left[n^{\frac{1}{\vartheta}} (c_H)^{\frac{\vartheta-1}{\vartheta}} + (1-n)^{\frac{1}{\vartheta}} (c_F)^{\frac{\vartheta-1}{\vartheta}} \right]^{\frac{\vartheta}{\vartheta-1}},$$

in which n, with $0 < n < 1$, is the weight given to the consumption of domestic goods and ϑ, with $\vartheta > 0$, is the intratemporal elasticity of substitution between domestic and foreign goods. The consumption sub-indexes c_H and c_F are Dixit-Stiglitz aggregators of the continuum of differentiated goods produced in country H and F, respectively:

$$c_H \equiv \left[\int_0^1 c(h)^{\frac{\sigma_t-1}{\sigma_t}} dh \right]^{\frac{\sigma_t}{\sigma_t-1}} \qquad c_F \equiv \left[\int_0^1 c(f)^{\frac{\sigma_t-1}{\sigma_t}} df \right]^{\frac{\sigma_t}{\sigma_t-1}},$$

where σ_t is the time-varying elasticity of substitution across the continuum of measure one of goods produced in each country, with $\sigma_t > 1$, for all t. The appropriate consumption-based price indices expressed in units of the domestic currency are defined as

$$P \equiv \left[n(P_H)^{1-\vartheta} + (1-n)\,(P_F)^{1-\vartheta}\right]^{\frac{1}{1-\vartheta}}, \tag{8}$$

with

$$P_H \equiv \left[\int_0^1 p(h)^{1-\sigma_t} dh\right]^{\frac{1}{1-\sigma_t}} \qquad\qquad P_F \equiv \left[\int_0^1 p(f)^{1-\sigma_t} df\right]^{\frac{1}{1-\sigma_t}}.$$

A similar structure of preferences holds for the foreign agent marked with the appropriate asterisks. In particular the weight n^* in the foreign consumption index might not be equal to n. In the case in which $n > n^*$, home-bias in consumption arises. More generally when $n \neq n^*$, there are deviations from purchasing-power parity and fluctuations in the real exchange rate, even if the law-of-one price holds for each traded good. In our model, this is a possible source of risk to be hedged through portfolio choices. In particular, we will show that this channel, together with model uncertainty, can explain home bias in asset holdings.[10]

In each country, there is a continuum of firms of measure one producing the goods in a monopolistic-competitive market. A domestic firm of type h has a constant-return-to-scale production technology $y_t(h) = Z_t^{\phi} l_t^{1-\phi}$ where Z_t is a natural resource available in the country and l_t denotes labor which is employed at the wage rate W_t; ϕ is a parameter with $0 < \phi \leq 1$. When $\phi = 1$, the model collapses to an endowment economy.

Prices are set without frictions and the law-of-one price holds. Equilibrium implies that prices are equalized across all firms within a country and set as a time-varying markup $\mu_t \equiv \sigma_t/[(\sigma_t - 1)(1-\phi)] > 1$ over nominal marginal costs

$$P_{H,t} = \mu_t \frac{W_t l_t}{y_{H,t}},$$

implying that the wage payments are inversely related to the mark-up:

$$W_t l_t = \frac{P_{H,t} y_{H,t}}{\mu_t}.$$

Firms make profits and distribute them in the form of dividends. The aggregate dividends in the domestic economy are given by

$$D_{H,t} = P_{H,t} y_{H,t} - W_t l_t = \frac{(\mu_t - 1)}{\mu_t} P_{H,t} y_{H,t},$$

which displays a positive correlation between dividends and the mark-up. The existence of non-diversifiable labor income is another source of risk to be hedged through portfolio choices.[11] We indeed introduce mark-up shocks to allow for a possible negative correlation between labor income and equity returns.[12] However, we will show in our empirical analysis that in general non-diversifiable labor income risk is not strong enough to explain home bias in asset holdings.

[10]Notice that we could have alternatively modeled variations in the real exchange rate through deviations from the law-of-one price without affecting our conclusions.

[11]When $\phi = 1$ we are in a pure endowment economy, in which all income is diversifiable. In this case μ_t goes to infinity.

[12]Mark-up shocks can fall in the category of redistributive shocks, discussed by Coeurdacier et al. (2007) and Coeurdacier and Gourinchas (2009).

The market of foreign goods works in a similar way with the appropriate modifications.

There are two equity markets – one for each country – with shares that are traded internationally. The market prices for equity shares in local currency are $V_{H,t}$ and $V^*_{F,t}$ for the domestic and foreign country, respectively. Households can also trade in two risk-free nominal bonds, denominated in units of the two currencies. The flow-budget constraint of the domestic agent is

$$B_{H,t} + S_t B_{F,t} + x_{H,t} V_{H,t} + x_{F,t} S_t V^*_{F,t} \leq R_{H,t} B_{H,t-1} + S_t R^*_{F,t} B^*_{F,t-1} \\ + x_{H,t-1}(V_{H,t} + D_{H,t}) + x_{F,t-1} S_t (V^*_{F,t} + D^*_{F,t}) + W_t l_t - P_t c_t \quad (9)$$

in which $B_{H,t}$ and $B_{F,t}$ are the amounts of one-period nominal bonds, in units of the two currencies, held at time t; $R_{H,t}$ and $R^*_{F,t}$ are the risk-free returns from period $t-1$ to period t, in the respective currencies; $x_{H,t}$ and $x_{F,t}$ are the shares of the domestic and foreign equity, respectively, held by the domestic agent. Finally S_t is the nominal exchange rate, defined as the price of foreign currency in terms of domestic currency. The flow-budget constraint (9) can be written in a more compact form and in real terms – in units of the domestic consumption index – as

$$a_t = r_{p,t} a_{t-1} + \xi_t - c_t, \quad (10)$$

where we have defined

$$a_t \equiv \frac{B_{H,t} + S_t B_{F,t} + x_{H,t} V_{H,t} + x_{F,t} S_t V^*_{F,t}}{P_t}$$

and

$$r_{p,t} = \alpha_{H,t-1} r_{H,t} + \alpha_{F,t-1} r^*_{F,t} \frac{q_t}{q_{t-1}} + \alpha^e_{H,t-1} r^e_{H,t} + \alpha^e_{F,t-1} r^{e*}_{F,t} \frac{q_t}{q_{t-1}}.$$

In the definition above, $\alpha_{H,t}$, $\alpha_{F,t}$, $\alpha^e_{H,t}$, $\alpha^e_{F,t}$ represent the shares of wealth that the domestic agent invests in the domestic bond, foreign bond, domestic equity and foreign equity, respectively, satisfying the following restriction:

$$\alpha_{H,t} + \alpha_{F,t} + \alpha^e_{H,t} + \alpha^e_{F,t} = 1. \quad (11)$$

Moreover $r_{H,t}$, $r^*_{F,t}$, $r^e_{H,t}$ and $r^{e*}_{F,t}$ are the respective real returns.[13] The variable ξ_t denotes non-diversifiable real labor income, defined as $\xi_t \equiv W_t l_t / P_t$, and q_t is the real exchange rate defined as $q_t \equiv S_t P^*_t / P_t$.

The domestic agent's optimization problem is to choose consumption and the portfolio allocations to maximize (7) under the flow-budget constraint (10) and appropriate no-Ponzi game conditions.

4.1 Optimality conditions

The optimality condition with respect to consumption implies an orthogonality condition, in expectation, between the real stochastic discount factor and the real portfolio return

$$\widetilde{E}_t(m_{t+1} r_{p,t+1}) = 1, \quad (12)$$

in which m_{t+1} is the real stochastic discount factor defined as

$$m_{t+1} \equiv \beta \frac{c_t}{c_{t+1}}. \quad (13)$$

[13] Lower-case variables denote the real counterpart of the respective upper-case variable. See the appendix for details on the derivations and definitions.

A similar condition applies to the foreign economy:

$$\widetilde{E}_t^*(m_{t+1}^* r_{p,t+1}^*) = 1, \tag{14}$$

where the foreign stochastic discount factor is defined as

$$m_{t+1}^* \equiv \beta \frac{c_t^*}{c_{t+1}^*}. \tag{15}$$

The optimality conditions with respect to the portfolio allocation imply a set of four restrictions for each agent, one for each asset, given by:

$$\widetilde{E}_t\left(m_{t+1} r_{H,t+1}\right) = 1, \qquad \widetilde{E}_t^*\left(m_{t+1}^* r_{H,t+1} \frac{q_t}{q_{t+1}}\right) = 1, \tag{16}$$

$$\widetilde{E}_t\left(m_{t+1} r_{F,t+1}^* \frac{q_{t+1}}{q_t}\right) = 1, \qquad \widetilde{E}_t^*\left(m_{t+1}^* r_{F,t+1}^*\right) = 1, \tag{17}$$

$$\widetilde{E}_t\left(m_{t+1} r_{H,t+1}^e\right) = 1, \qquad \widetilde{E}_t^*\left(m_{t+1}^* r_{H,t+1}^e \frac{q_t}{q_{t+1}}\right) = 1, \tag{18}$$

$$\widetilde{E}_t\left(m_{t+1} r_{F,t+1}^{e*} \frac{q_{t+1}}{q_t}\right) = 1, \qquad \widetilde{E}_t^*\left(m_{t+1}^* r_{F,t+1}^{e*}\right) = 1. \tag{19}$$

Equilibrium in the goods market requires the production of each good to be equal to world consumption

$$y_{H,t} = c_{H,t} + c_{H,t}^* \qquad y_{F,t}^* = c_{F,t} + c_{F,t}^*.$$

The labor markets are in equilibrium at the exogenously supplied quantities of labor

$$l_t = \bar{l}_t \qquad l_t^* = \bar{l}_t^*.$$

Bonds are in zero-net supply worldwide

$$B_{H,t} + B_{H,t}^* = B_{F,t} + B_{F,t}^* = 0.$$

Equity shares sum to one

$$x_{H,t} + x_{H,t}^* = x_{F,t} + x_{F,t}^* = 1.$$

Given the path of the stochastic disturbances $\{\bar{l}_t,\ \bar{l}_t^*,\ Z_t,\ Z_t^*,\ \mu_t,\ \mu_t^*\}$, an equilibrium is an allocation of quantities $\{c_t, c_{H,t},\ c_{F,t},\ c_t^*,\ c_{H,t}^*,\ c_{F,t}^*,\ \alpha_{H,t},\ \alpha_{H,t},\ \alpha_{H,t}^e,\ \alpha_{F,t}^e, \alpha_{H,t}^*,\ \alpha_{H,t}^*,\ \alpha_{H,t}^{*e},\ \alpha_{F,t}^{*e},$ $a_t,\ a_t^*\}$ and prices $\{r_{H,t},\ r_{F,t}^*,\ r_{H,t}^e,\ r_{F,t}^{e*},\ q_t,\ P_{H,t}/P_{F,t}, W_t/P_t,\ W_t^*/P_t^*\}$ such that each agent's consumption, portfolio shares and wealth are optimal given prices, and goods, labor, and asset markets are in equilibrium.

Although we have written a general equilibrium model, in the next section we show that we do not really need to solve the entire model to understand the determinants of the portfolio allocation. Instead, we can isolate a block of the general-equilibrium conditions to determine the portfolio shares $\{\alpha_{H,t},\ \alpha_{H,t},\ \alpha_{H,t}^e,\ \alpha_{F,t}^e, \alpha_{H,t}^*,\ \alpha_{H,t}^*,\ \alpha_{H,t}^{*e},\ \alpha_{F,t}^{*e}\}$ by taking as given the path of returns $\{r_{H,t},\ r_{F,t}^*,\ r_{H,t}^e,\ r_{F,t}^{e*}\}$, the real exchange rate q_t and the processes of non-diversifiable labor incomes $\{\xi_t, \xi_t^*\}$. The optimal portfolio allocation, therefore, depends on the co-movements between these sources of risk. Moreover, since all these variables are observable, we can use data restrictions to evaluate the co-movements and the empirical relevance of the model.[14]

[14]Empirical restrictions of this kind should apply to any general equilibrium model of international portfolio allocation. Indeed, recent papers in the literature characterize portfolio allocations in terms of primitive parameters or shocks, yet without considering such empirical restrictions. Along this dimension, they would be less successful. See van Wincoop and Warnock (2006, 2008) and Coeurdacier and Gourinchas (2009) for a related argument and for models that are instead evaluated under data restrictions.

5 The benchmark case of no model uncertainty

When there is no model uncertainty, investors fully trust the "reference" probability distribution to be the true one, so that "approximating" and "subjective" measures coincide. For a generic random variable X_{t+1}, it follows that $\widetilde{E}_t X_{t+1} = \widetilde{E}_t^* X_{t+1} = E_t X_{t+1}$.

Accordingly, we can characterize the equilibrium portfolio allocation combining equations (16)–(19) to obtain the following set of orthogonality conditions:

$$E_t\left[\left(m_{t+1} - m^*_{t+1}\frac{q_t}{q_{t+1}}\right)\left(r^e_{H,t+1} - r_{H,t+1}\right)\right] = 0 \tag{20}$$

$$E_t\left[\left(m_{t+1} - m^*_{t+1}\frac{q_t}{q_{t+1}}\right)\left(r^{e*}_{F,t+1}\frac{q_{t+1}}{q_t} - r^e_{H,t+1}\right)\right] = 0 \tag{21}$$

$$E_t\left[\left(m_{t+1} - m^*_{t+1}\frac{q_t}{q_{t+1}}\right)\left(r^*_{F,t+1}\frac{q_{t+1}}{q_t} - r_{H,t+1}\right)\right] = 0. \tag{22}$$

The above conditions require the cross-country difference in the real stochastic discount factors – evaluated in terms of domestic consumption – to be orthogonal to three relevant excess returns: the domestic equity premium, the excess return on foreign versus domestic equity, and the excess return on foreign versus domestic bonds. Indeed, conditions (20), (21) and (22) and restriction (11) are sufficient to characterize the equilibrium portfolio allocation.

Given the assumption of incomplete markets, we cannot solve for the optimal portfolio allocation in non-linear closed form.[15] However, we can still derive many insights by using the approximation methods developed by Devereux and Sutherland (2006) and Tille and van Wincoop (2006). As a first step, we solve for the paths of consumption and wealth, given returns and the steady-state portfolio shares, using a *first-order* approximation of the Euler equations and the budget constraints. In particular, letting variables with hats denote log-deviations from the steady state and variables with upper-bars the steady-state level, this yields to

$$\hat{m}_{t+1} + \Delta\hat{q}_{t+1} - \hat{m}^*_{t+1} = -(\Delta\hat{c}^R_{t+1} - \Delta\hat{q}_{t+1}) = -\frac{(1-\beta)}{\beta s_c}\bar{\boldsymbol{\lambda}}'\mathbf{exr}_{t+1} - \frac{s_\xi}{s_c}\varepsilon_{l,t+1}, \tag{23}$$

where s_ξ is the steady-state ratio between non-traded income and financial wealth, which is common across countries and given by $s_\xi \equiv \bar{\xi}/\bar{a}$, and s_c is the steady-state ratio between consumption and financial wealth and such that $s_c = (1-\beta)/\beta + s_\xi$. Moreover, $\bar{\boldsymbol{\lambda}} \equiv \bar{\boldsymbol{\alpha}} - \bar{\boldsymbol{\alpha}}^*$ captures the cross-country difference in optimal portfolios and is given by[16]

$$\bar{\boldsymbol{\lambda}} = \begin{bmatrix} 2(\bar{\alpha}^e_H + \bar{\alpha}^e_F) - 2 \\ 2\bar{\alpha}^e_F - 1 \\ 2\bar{\alpha}_F \end{bmatrix}, \tag{24}$$

and the vector of excess returns **exr** is defined as

$$\mathbf{exr}_t \equiv \begin{bmatrix} \hat{r}^e_{H,t} - \hat{r}_{H,t} \\ \hat{r}^{e*}_{F,t} + \Delta\hat{q}_t - \hat{r}^e_{H,y} \\ \hat{r}^*_{F,t} + \Delta\hat{q}_t - \hat{r}_{H,t} \end{bmatrix}.$$

[15] Van Wincoop and Warnock (2006, 2008) work with a closed-form solution, but in a partial-equilibrium two-period model. Coeurdacier and Gourinchas (2009), Coeurdacier et al. (2007), Heathcote and Perri (2004), Kollman (2006) obtain closed-form solutions by assuming that markets are locally complete.

[16] It can be shown that in a symmetric steady state in which $\bar{A} = \bar{S}\bar{A}^*$, $\bar{\alpha}^{e*}_i = 1 - \bar{\alpha}^e_i$ and $\bar{\alpha}^*_i = -\bar{\alpha}_i$, for $i = H, F$.

The definition of $\bar{\boldsymbol{\lambda}}$ implies that full diversification is achieved when $\bar{\boldsymbol{\lambda}} = \bar{\boldsymbol{\lambda}}^{full} = 0$. Equation (23) is the equivalent of equation (1) and describes the first hedging motif underlying portfolio choices, related to differences in the marginal utilities of consumption. It shows that under log utility the cross-country differences in the stochastic discount factors arise from two sources: fluctuations in the excess returns and cross-country variations in labor income. In particular, $\varepsilon_{l,t+1}$ represents the news at time $t+1$ in the growth path of the cross-country non-diversifiable labor incomes (in units of the domestic consumption index)

$$\varepsilon_{l,t+1} = \sum_{j=0}^{\infty} \beta^k [E_{t+1}(\Delta \hat{\xi}^R_{t+1+k} - \Delta \hat{q}_{t+1+k}) - E_t(\Delta \hat{\xi}^R_{t+1+k} - \Delta \hat{q}_{t+1+k})], \tag{25}$$

which captures the source of labor-income risk relevant to our model.

As a second step, we use equation (23) and a *second-order* approximation of the orthogonality conditions (20)–(22) to determine the steady-state portfolio shares as a function of prices, returns and non-diversifiable labor income.[17] The optimal steady-state portfolio shares satisfy the following equation

$$\bar{\boldsymbol{\lambda}} = -s_\xi \frac{\beta}{1-\beta} \boldsymbol{\Sigma}_t^{-1} E_t(\mathbf{exr}_{t+1} \cdot \varepsilon_{l,t+1}), \tag{26}$$

where $\boldsymbol{\Sigma}_t$ is the time$-t$ conditional variance-covariance matrix of the vector of excess returns $\mathbf{exr}_{t+1}$.

When $s_\xi = 0$, all income risk is tradeable. As argued in Section 2, the optimal portfolio implies full diversification: $\bar{\boldsymbol{\lambda}} = 0$ and accordingly $\bar{\alpha}^e_i = \bar{\alpha}^{*e}_i = 1/2$ and $\bar{\alpha}_i = \bar{\alpha}^*_i = 0$, for $i = H, F$. When there is non-diversifiable income, instead, the model implies a departure from full diversification that depends on the covariances between labor-income risk and the excess returns.

The set of conditions in (26) can be written in a simpler form as

$$\bar{\alpha}^e_H + \bar{\alpha}^e_F = 1 - \frac{s_\xi}{2} \frac{\beta}{1-\beta} \frac{cov_t(\varepsilon_{l,t+1}, \hat{r}^e_{H,t+1} - \hat{r}_{H,t+1} | exr^{ib}_{t+1}, exr^{ie}_{t+1})}{var_t(\hat{r}^e_{H,t+1} - \hat{r}_{H,t+1} | exr^{ib}_{t+1}, exr^{ie}_{t+1})} \tag{27}$$

$$\bar{\alpha}^e_F = \frac{1}{2} - \frac{s_\xi}{2} \frac{\beta}{1-\beta} \frac{cov_t(\varepsilon_{l,t+1}, \hat{r}^{e*}_{F,t+1} + \Delta \hat{q}_{t+1} - \hat{r}^e_{H,t+1} | exr^{ib}_{t+1}, exr^{de}_{t+1})}{var_t(\hat{r}^{e*}_{F,t+1} + \Delta \hat{q}_{t+1} - \hat{r}^e_{H,t+1} | exr^{ib}_{t+1}, exr^{de}_{t+1})} \tag{28}$$

$$\bar{\alpha}_F = - \frac{s_\xi}{2} \frac{\beta}{1-\beta} \frac{cov_t(\varepsilon_{l,t+1}, \hat{r}^{*}_{F,t+1} + \Delta \hat{q}_{t+1} - \hat{r}_{H,t+1} | exr^{de}_{t+1}, exr^{ie}_{t+1})}{var_t(\hat{r}^{*}_{F,t+1} + \Delta \hat{q}_{t+1} - \hat{r}_{H,t+1} | exr^{de}_{t+1}, exr^{ie}_{t+1})}, \tag{29}$$

in which variances and covariances are conditional on the other excess returns and previous-period information. We denote with exr^{de}, exr^{ie} and exr^{ib} the excess returns on domestic equity, international equity, and international bonds, respectively:

$$\begin{aligned} exr^{de}_{t+1} &\equiv \hat{r}^e_{H,t+1} - \hat{r}_{H,t+1}, \\ exr^{ie}_{t+1} &\equiv \hat{r}^{e*}_{F,t+1} + \Delta \hat{q}_{t+1} - \hat{r}^e_{H,t+1}, \\ exr^{ib}_{t+1} &\equiv \hat{r}^{*}_{F,t+1} + \Delta \hat{q}_{t+1} - \hat{r}_{H,t+1}. \end{aligned}$$

Using (27) to (29) together with (11), we are able to determine the split of wealth across the different assets. In particular, equation (27) determines the share of financial wealth invested in

[17] Details of the derivation are shown in the appendix.

the equity market relative to the bond market: when $s_\xi \neq 0$ and $\varepsilon_{l,t+1}$ co-varies positively with the excess return of domestic equity over domestic bonds, domestic agents will take an overall long position in the bond markets ($\bar{\alpha}_H + \bar{\alpha}_F > 0$). In this case, indeed, in the face of a bad shock to labor income, domestic bonds pay relatively better than equities: bonds are a better hedge with respect to labor-income risk.

Equation (28), instead, determines the diversification between domestic and foreign equities. In particular, to obtain home bias in equity, the excess return on international equity should co-vary positively with the surprises in the cross-country differential in the growth of non-diversifiable labor income. In this case, the return on domestic equity will increase, relative to that on foreign equity, when indeed domestic agents receive a bad shock regarding their labor income. This makes domestic equity a better hedge against labor-income risk relative to foreign equity and points toward explaining the home bias in equity holdings.

Finally, equation (29) describes the position taken in the foreign bond market and as a consequence in the domestic bond market, given the overall position implied by (27). When the covariance between $\varepsilon_{l,t+1}$ and the excess return of the foreign bond with respect to the domestic bond is positive, then foreign bonds do not pay well when needed. In this case the domestic agent would like to take a short position in the foreign bond market ($\bar{\alpha}_F < 0$).[18]

Although simpler versions of (27) and (29) have been treated in the literature, to our knowledge, this is the first complete analysis in a dynamic general equilibrium model with incomplete markets. Results of the previous literature are nested in the above framework. When there is only trading in equities, the relevant condition for determining the portfolio allocation collapses to

$$\bar{\alpha}_F^e = \frac{1}{2} - \frac{s_\xi}{2}\frac{\beta}{1-\beta}\frac{cov_t(\varepsilon_{l,t+1}, \hat{r}^{e*}_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}^e_{H,t+1})}{var_t(\hat{r}^{e*}_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}^e_{H,t+1})}. \tag{30}$$

Covariances are no longer conditional on the other excess returns, but only on the information set at time–t. There is home bias in equity holdings when home equity is a good hedge with respect to non-diversifiable income risk, i.e. when $cov_t(\varepsilon_{l,t+1}, exr^{ie}_{t+1}) > 0$.

A popular argument for international diversification being worse is the neoclassical model of Baxter and Jermann (1997) in which labor income and dividends are correlated. In this case, the above covariance would be negative, implying even larger holdings of foreign assets. Heathcote and Perri (2004), instead, show a case in which the correlation can become positive in a model with capital accumulation and home bias in consumption preferences. Furthermore, Coeurdacier and Gourinchas (2009) discuss several theoretical cases that can rationalize a positive covariance and then imply home-bias in equity.[19] In the case of our general-equilibrium model, Section 3 shows that the covariance can be positive or negative depending on the relative strength of the mark-up shocks. Conditional on a positive mark-up shock, profits and dividends increase, whereas labor income decreases. This might imply a negative correlation between labor income and the return on domestic equity. Therefore, the domestic agent would hold more of its own assets to hedge against labor-income risk.

6 Portfolio Choices under Model Uncertainty

Under rational expectations, agents form expectations trusting the "reference" probability measure. On the contrary, with model uncertainty, they surround the "reference" probability distri-

[18]Note that this does not necessarily imply a long position in the domestic bond market. Indeed, the overall position depends on equation (27), as previously discussed.

[19]See also Coeurdacier et al. (2007) and Engel and Matsumoto (2006).

bution with a set of nearby distributions and act according to a distorted probability measure. In particular, the "subjective" conditional expectations, as shown in (5), are linked to the "reference" conditional expectations through the martingale increments g and g^*, for country H and F respectively. Accordingly, we can write conditions (20)–(22) as

$$
\begin{aligned}
E_t\left[\left(g_{t+1}m_{t+1} - g^*_{t+1}m^*_{t+1}\frac{q_t}{q_{t+1}}\right)\left(r^e_{H,t+1} - r_{H,t+1}\right)\right] &= 0\\
E_t\left[\left(g_{t+1}m_{t+1} - g^*_{t+1}m^*_{t+1}\frac{q_t}{q_{t+1}}\right)\left(r^{e*}_{F,t+1}\frac{q_{t+1}}{q_t} - r^e_{H,t+1}\right)\right] &= 0\\
E_t\left[\left(g_{t+1}m_{t+1} - g^*_{t+1}m^*_{t+1}\frac{q_t}{q_{t+1}}\right)\left(r^*_{F,t+1}\frac{q_{t+1}}{q_t} - r_{H,t+1}\right)\right] &= 0.
\end{aligned}
$$

This set of equations implies the three restrictions needed to determine the portfolio allocation. In a second-order approximation, and in compact form, they read as

$$
E_t\left[(\Delta\hat{c}^R_{t+1} - \Delta\hat{q}_{t+1} - \hat{g}^R_{t+1})\cdot \mathbf{exr}_{t+1}\right] = 0, \tag{31}
$$

where a superscript R denotes the difference between the domestic and the foreign variable. The restriction above shows that the optimal portfolio allocation is going to be affected by the factor $\hat{g}^R_{t+1}$, which measures the cross-country difference between the subjective distortions and the approximating probability distribution. We now enrich our set of assumptions to endogenize the way in which these distortions arise.

We consider the sophisticated agents of the robust-control theory of Hansen and Sargent (2005, 2007). These agents fear model misspecification, and seek decision rules that are robust to it. Following Hansen and Sargent (2005, 2007), we can regard such robust-decision-making process as a two-player game between the representative household and an "evil" agent. The household surrounds the reference model with a set of alternative distributions, in which he/she believes the true one lies. The "evil" agent will, then, choose the most unfavorable distribution in this set, and the household will act accordingly. To choose the worst-case distribution, the "evil" agent seeks to minimize the utility of the decision-maker under an entropy constraint of the form similar to (6). The latter defines the size of the set of alternative models, and imposes a bound on the allowed divergence between the distorted and the approximating measures. In a more formal way, $\{g_t\}$ is chosen to minimize

$$
E_{t_0}\left\{\sum_{t=t_0}^{\infty}\beta^{t-t_0}G_t\ln c_t\right\},
$$

under the entropy constraint

$$
E_{t_0}\left\{\sum_{t=t_0}^{\infty}\beta^{t-t_0}G_t\beta E_t(g_{t+1}\ln g_{t+1})\right\} \leq K,
$$

and the restrictions given by the martingale assumption on G_t:

$$
G_{t+1} = g_{t+1}G_t \tag{32}
$$

$$
E_t g_{t+1} = 1. \tag{33}
$$

The parameter K in the entropy constraint imposes an upper-bound on the divergence between the distorted and the approximating beliefs. The higher K, the more afraid of misspecification the agent is, because a higher K allows the "evil" agent to choose larger distortions.

Hansen and Sargent (2005) propose an alternative formulation of this problem in which the entropy constraint is added to the utility of the agent to form a modified objective function

$$E_{t_0}\left\{\sum_{t=t_0}^{\infty}\beta^{t-t_0}G_t\ln c_t\right\}+\theta E_{t_0}\left\{\sum_{t=t_0}^{\infty}\beta^{t-t_0}G_t\beta E_t(g_{t+1}\ln g_{t+1})\right\}, \tag{34}$$

where $\theta > 0$ is a penalty parameter on discounted entropy.

The problem of the "evil" agent becomes that of choosing the path $\{g_t\}$ to minimize (34) under the constraints (32) and (33). Higher values of θ imply less fear of model misspecification, because the "evil" agent is penalized more by raising entropy when minimizing the utility of the decision-maker. When θ goes to infinity, the optimal choice of the "evil" agent is to set $g_{t+1} = 1$ at all times, meaning that the optimal distortion is zero: the rational expectations equilibrium is nested as a special case.

The problem of the decision-maker is instead that of choosing sequences for consumption and portfolio shares to maximize (34) taking into account the minimizing action of the evil agent. As discussed in the literature, among others by Barillas et al (2006), it can be shown that the solution of the inner minimization problem implies a transformation of the original utility function (34) into a non-expected recursive utility function of the form[20]

$$v_t = c_t^{1-\beta}\left([E_t(v_{t+1})^{1-\gamma}]^{\frac{1}{1-\gamma}}\right)^{\beta}, \tag{35}$$

where the parameter γ is the following monotonic transformation of θ:

$$\gamma = 1+\frac{1}{(1-\beta)\theta}. \tag{36}$$

The equilibrium real stochastic discount factor implied by (35) – evaluated under the reference probability measure – is

$$g_{t+1}m_{t+1} = \beta\frac{c_t}{c_{t+1}}\left(\frac{v_{t+1}^{1-\gamma}}{[E_t(v_{t+1})^{1-\gamma}]}\right),$$

and the optimal distortion is therefore

$$g_{t+1} = \left(\frac{v_{t+1}^{1-\gamma}}{[E_t(v_{t+1})^{1-\gamma}]}\right).$$

Notice that in (35) we can scale continuation values by consumption to get

$$\frac{v_t}{c_t} = \left[E_t\left(\frac{v_{t+1}}{c_{t+1}}\frac{c_{t+1}}{c_t}\right)^{1-\gamma}\right]^{\frac{\beta}{1-\gamma}},$$

showing that g_{t+1} can be related to the current and future consumption path. Indeed, in a first-order approximation – which suffices to evaluate (31) – we can write:

$$\hat{g}_{t+1} = -(\gamma-1)\sum_{k=0}^{\infty}\beta^k\left[E_{t+1}\Delta\hat{c}_{t+1+k}-E_t\Delta\hat{c}_{t+1+k}\right],$$

[20]Barillas et al (2006) discuss the relation between fear of model misspecification and the class of risk-adjusted preferences described in Kreps and Porteus (1978) and Epstein and Zin (1989). See also Strzalecki (2009) for an analysis on how models of ambiguity aversion imply different preferences for the timing resolution of uncertainty.

Notice also that the framework with model uncertainty is observationally equivalent to a model with preference shocks (see Pavlova and Rigobon, 2007). Importantly, however, in our context they are endogenous.

in which $\hat{g}_{t+1}$ increases when the agent fears bad news with respect to the consumption-growth profile. Hence, the worst-case scenario takes the form of downward revisions in current and future consumption growth.[21]

Recall that $g(s_{t+1}|s^t)$ is equivalent to the ratio between the "subjective" and "approximating" probabilities, $\widetilde{\pi}(s_{t+1}|s^t)/\pi(s_{t+1}|s^t)$. Higher values of $g(s_{t+1}|s^t)$ implies that the agent is assigning a higher probability to those states of nature where there are bad news on the consumption-growth profile. When $g(s_{t+1}|s^t)$ increases, the appetite for receiving additional wealth increases as well. In this case, the agent would like to hold assets that pay well when there are indeed bad news on the consumption-growth profile.

The above derivations apply also to the foreign agent. In the symmetric case in which $\gamma = \gamma^*$, we can show that the optimal relative distortion depends negatively on the surprises in the consumption-growth differential across countries:

$$\hat{g}_{t+1}^R = -(\gamma-1)\sum_{k=0}^{\infty}\beta^k\left[E_{t+1}\Delta\hat{c}_{t+1+k}^R - E_t\Delta\hat{c}_{t+1+k}^R\right]$$
$$= -(\gamma-1)\left[\frac{(1-\beta)}{\beta s_c}\overline{\boldsymbol{\lambda}}'\mathbf{exr}_{t+1} + \frac{s_\xi}{s_c}\varepsilon_{l,t+1} + \varepsilon_{q,t+1}\right], \quad (37)$$

where the second equality substitutes for the relative consumption growth using a first-order approximation of the Euler equations and the budget constraints. Moreover, $\varepsilon_{q,t+1}$ is defined as the time-t surprises in the real exchange rate growth:

$$\varepsilon_{q,t+1} \equiv \sum_{k=0}^{\infty}\beta^k\left[E_{t+1}\Delta\hat{q}_{t+1+k} - E_t\Delta\hat{q}_{t+1+k}\right]. \quad (38)$$

With model uncertainty and fears of model misspecification, the cross-country difference in the appetite for wealth is given by the sum of (23) – capturing the first hedging motif described by the wedge between the marginal utilities of consumption – and (37) – capturing the additional hedging motif given by the wedge between the subjective distortions. Accordingly, equation (37) corresponds to equation (2).

The second hedging motif, captured by the variable $\hat{g}_{t+1}^R$ and given by the revisions in the expectation regarding the future path of relative consumption growth, is driven by three sources of risk: the fluctuations in the excess returns, the labor-income risk and the fluctuations in the real exchange rate. As argued in Section 2, the coefficients related to excess returns and labor-income risk are the same for the two hedging motifs: $m_e = g_e = -(1-\beta)/\beta s_c$ and $m_l = g_l = -s_\xi/s_c$. As a consequence, absent real exchange rate risk, the optimal portfolio allocation driven by ambiguity aversion is the same as the one driven by the benchmark model with no uncertainty. The degree of ambiguity aversion γ is irrelevant in this case.

On the contrary, the coefficient on real exchange rate risk is different across the two components: $m_q = 0$ while $g_q = -1$. As a consequence, under log-utility, ambiguity aversion implies an additional term driving the optimal steady-state portfolio shares, compared to equation (26):[22]

$$\overline{\boldsymbol{\lambda}} = -s_\xi\frac{\beta}{1-\beta}\boldsymbol{\Sigma}_t^{-1}E_t(\mathbf{exr}_{t+1}\cdot\varepsilon_{l,t+1}) - s_c\frac{(\gamma-1)}{\gamma}\frac{\beta}{1-\beta}\boldsymbol{\Sigma}_t^{-1}E_t(\mathbf{exr}_{t+1}\cdot\varepsilon_{q,t+1}), \quad (39)$$

which depends on the covariances between the excess returns and the surprises in the real exchange rate, and on the degree of ambiguity aversion. The higher γ is, the more averse to model

[21] Hansen et al. (2008) show how to derive g_{t+1} as a closed-form solution including risk-premia terms, which, however, are not important in our approximation for computing the steady-state portfolio shares.

[22] Refer to the Appendix for details.

uncertainty the investors are, the more important this additional component is. When investors are not concerned about model uncertainty (i.e. as $\theta \to \infty$) then γ is equal to 1: beliefs are not distorted and (39) coincides with (26).

In particular, equation (39) implies the following condition:

$$\bar{\alpha}_F^e = \frac{1}{2} - \frac{1}{2}\frac{\beta}{1-\beta}s_\xi \frac{cov_t(\varepsilon_{l,t+1}, \hat{r}_{F,t+1}^{e*} + \Delta\hat{q}_{t+1} - \hat{r}_{H,t+1}^{e}|exr_{t+1}^{ib}, exr_{t+1}^{de})}{var_t(\hat{r}_{F,t+1}^{e*} + \Delta\hat{q}_{t+1} - \hat{r}_{H,t+1}^{e}|exr_{t+1}^{ib}, exr_{t+1}^{de})}$$
$$- \frac{1}{2}\frac{(\gamma-1)}{\gamma}\frac{\beta}{1-\beta}s_c \frac{cov_t(\varepsilon_{q,t+1}, \hat{r}_{F,t+1}^{e*} + \Delta\hat{q}_{t+1} - \hat{r}_{H,t+1}^{e}|exr_{t+1}^{ib}, exr_{t+1}^{de})}{var_t(\hat{r}_{F,t+1}^{e*} + \Delta\hat{q}_{t+1} - \hat{r}_{H,t+1}^{e}|exr_{t+1}^{ib}, exr_{t+1}^{de})}. \quad (40)$$

On top of equation (28), agents would like to hold more domestic equities if their return is high when the real exchange rate is expected to appreciate. This requires that $\varepsilon_{q,t+1}$ co-varies positively with the excess returns on foreign-versus-domestic equity, $\hat{r}_{F,t+1}^{e*} + \Delta\hat{q}_{t+1} - \hat{r}_{H,t+1}^{e}$, conditional on the other excess returns. As the fear of model misspecification increases, then, this additional hedging motif matters more for determining home bias in international portfolio choice.

There is an important distinction to underline at this point. As anticipated in Section 2, the additional component in equation (39), capturing the hedge against real exchange rate risk, would also be present in a standard model with non-distorted beliefs and non-unitary risk aversion, as in Adler and Dumas (1983). In that case, indeed, it would be

$$m_e = -\rho\frac{1-\beta}{\beta s_c} \qquad m_l = -\rho\frac{s_\xi}{s_c} \qquad m_q = \rho - 1,$$

with ρ measuring the degree of risk aversion, and $\hat{g}_{t+1}^R = 0$ at all times. Hence, the second component in equation (39), with ρ substituting for γ, would also apply, but for a completely different reason. What would matter is the impact of the real exchange rate in driving apart the real marginal utilities of consumption across the two countries. Agents would like to trade assets to hedge these differences, but not under log utility.[23] In our model, instead, fluctuations in the real exchange rate affect the optimal portfolio allocation because of the distortions in the beliefs (hedging motif (2)), which is relevant even under log-utility.

Moreover, in the standard case, ρ would represent the risk-aversion coefficient and, at the same time, the inverse of the intertemporal elasticity of substitution. By raising risk aversion, to make the second component larger, the intertemporal elasticity of substitution would be lowered and the implied risk-free rate would increase in a counterfactual way. In our model, instead, the intertemporal elasticity of substitution is tied to one (a value close to recent empirical estimates, as discussed in Vissing-Jœrgensen and Attanasio, 2003) whereas the parameter γ – now capturing the degree of ambiguity aversion – can be larger than one, thus increasing the importance of the second component without affecting the mean of the risk-free rate, as shown in Barillas et al (2006).

Furthermore, equation (38) shows that what matters is not only the risk of an immediate variation in the real exchange rate, but also the risk of future ones. As β gets close to one, only the *long-run risk* remains relevant. In this case, indeed, $\varepsilon_{q,t+1}$ becomes proportional to the revisions in the conditional expectations of the long-run real exchange rate:

$$\varepsilon_{q,t+1} \cong E_{t+1}\hat{q}_\infty - E_t\hat{q}_\infty.$$

[23] Under log-utility, indeed, substitution and income effects cancel each other out and relative-inflation risk does not imply differences in the marginal utility of consumption across countries once evaluated in the same units of consumption goods.

This is a further novel implication of our model. Indeed, the existing literature that analyzes the hedging motifs underlying portfolio choices in international macro models, by building on static frameworks, necessarily focuses only on short-run real exchange rate risk. In the next Section we will show that the difference between short- and long-run real exchange rate risk can be empirically relevant when evaluating the implications for international portfolio allocation.

7 Empirical Evidence

One of the appealing features of the theoretical model presented in the previous section is that it derives clear implications about the second moments of variables that are directly observable. These implications can therefore be tested empirically without further assumptions on the empirical counterparts of our theoretical variables.

7.1 Data

To evaluate the implications of equations (26) and (39), we collect and use quarterly data for the G7 Countries, over the sample 1980q1-2007q4. We consider the US as the Home country and the aggregation of the remaining G7 countries as the Foreign country.[24]

We define the CPI index for the foreign country, expressed in USD, as

$$S_t P_t^* = \sum_i \omega_{i,t} S_{i,t} P_{i,t},$$

in which $P_{i,t}$ is the CPI in local currency for country i, $S_{i,t}$ is the bilateral nominal exchange rate between the local currency in country i and the dollar (US dollars for one unit of local currency), and $\omega_{i,t}$ is the actual time-t GDP-weight of country i relative to the aggregation of the G6 countries:[25]

$$\omega_{i,t} = \frac{GDP_{i,t}}{\sum_i GDP_{i,t}}.$$

Accordingly, the real exchange rate between the US and the G6 countries is simply computed as:

$$\hat{q}_t = \log\left(\frac{S_t P_t^*}{P_t}\right) = \log\left(\frac{\sum_i \omega_{i,t} S_{i,t} P_{i,t}}{P_t}\right),$$

where P_t is the CPI index for the US.

Analogously, we compute nominal labor income in US dollars for the Foreign country as:

$$S_t W_t^* \bar{l}_t^* = \sum_i \omega_{i,t} S_{i,t} W_{i,t} \bar{l}_{i,t},$$

[24] In particular, we use data on aggregate nominal compensation of employees, from the OECD Quarterly National Accounts (**OCOS02B, where ** is the two-letter country code), the Consumer Price Indexes from the IFS database (**I64..F), nominal returns on short-term treasury bills from the IFS database (**I60C..), nominal National Price and Gross Return indexes on the domestic stock market, from MSCI Barra (MS****L), in local currency, and bilateral nominal exchange rate vis-à-vis the USD, constructed using the domestic stock-price indexes in USD, from the MSCI Barra (MS****$). Moving from the monthly National Price and Gross Return indexes from MSCI database, we construct series for the quarterly nominal returns on equity ($R^e_{i,t}$) following Campbell (1999).

[25] To check for robustness, we repeated the analysis using average GDP-weights as an alternative aggregation methodology, as in Coeurdacier and Gourinchas (2009), and using both aggregate and per-capita levels for the quantity variables. None of our results is significantly affected.

Table 1: Some Data Statistics (Annual rates)

	$\mu(\cdot)$	$\sigma(\cdot)$	$\rho(\cdot)$	$\rho(\cdot,\Delta\hat{\xi}^R-\Delta\hat{q})$	$\rho(\cdot,\Delta\hat{q})$
$\Delta\hat{\xi}^R-\Delta\hat{q}$	0.773	13.051	0.024	1.000	−0.438
$\Delta\hat{q}$	0.165	11.348	0.175	−0.438	1.000
$\hat{r}_F^{e*}+\Delta\hat{q}-\hat{r}_H^e$	0.699	13.535	0.108	−0.530	0.436
$\hat{r}_F^{*}+\Delta\hat{q}-\hat{r}_H$	0.984	10.718	0.030	−0.919	0.722
$\hat{r}_H^e-\hat{r}_H$	6.350	15.850	−0.004	−0.027	−0.139

Note: means and standard deviations are in percentage points

in which we measure $W_{i,t}\bar{l}_{i,t}$ using data on aggregate nominal compensation of employees in country i. Accordingly, relative labor income in units of US dollars is the log difference between the aggregate nominal compensation in the US and in the other G7 countries:

$$\log\left(\frac{W_t\bar{l}_t}{S_tW_t^*\bar{l}_t^*}\right)=\log\left(\frac{W_t\bar{l}_t}{P_t}\frac{P_t}{S_tP_t^*}\frac{P_t^*}{W_t^*\bar{l}_t^*}\right)=\log\left(\frac{\xi_t}{q_t\xi_t^*}\right)=\hat{\xi}_t^R-\hat{q}_t.$$

Given nominal quarterly returns on the stock market, defined by $R_{i,t}^e$ for each country i and R_t^e for the US, and nominal quarterly returns on bonds, defined by $R_{i,t}$ for each country i and R_t for the US, we can obtain the real returns as $r_{i,t}\equiv R_{i,t}P_{i,t-1}/P_{i,t}$ and $r_{i,t}^e\equiv R_{i,t}^eP_{i,t-1}/P_{i,t}$ for each country i and for the US. Using those, we construct the three excess returns of interest as:

$$exr_t^{de}\equiv\hat{r}_{H,t}^e-\hat{r}_{H,t}=\log\left(\frac{r_t^e}{r_t}\right)$$

$$exr_t^{ie}\equiv\hat{r}_{F,t}^{e*}+\Delta\hat{q}_t-\hat{r}_{H,t}^e=\log\left(\frac{\sum_i\omega_{i,t}r_{i,t}^e\frac{q_{i,t}}{q_{i,t-1}}}{r_t^e}\right)$$

$$exr_t^{ib}\equiv\hat{r}_{F,t}^{*}+\Delta\hat{q}_t-\hat{r}_{H,t}=\log\left(\frac{\sum_i\omega_{i,t}r_{i,t}\frac{q_{i,t}}{q_{i,t-1}}}{r_t}\right).$$

Table 1 reports some summary statistics for the variables of interest. We report the average level $\mu(\cdot)$ and the standard deviation $\sigma(\cdot)$, both annualized and in percentage points, the serial correlation coefficient $\rho(\cdot)$ and the correlation with the growth rate in relative labor income $\rho(\cdot,\Delta\hat{\xi}^R-\Delta\hat{q})$ and in the real exchange rate $\rho(\cdot,\Delta\hat{q})$. These simple correlations already suggest that domestic equity seems a poor hedge against labor income risk, relative to foreign stocks, while both domestic equity and domestic bonds seem somewhat useful in providing the right co-movement to hedge against real exchange rate fluctuations. In the next sections we will refine and articulate these results.

In order to evaluate the optimal portfolio allocation implied by our model, we need to calibrate the steady-state consumption-to-financial wealth ratio, s_c. To this end, we use the average financial wealth-to-disposable income ratio for the US computed by Bertaut (2002), and the average consumption-to-disposable income ratio for the US, computed using data on personal consumption of non-durable goods and personal disposable income. The former, on a quarterly frequency, amounts to about 20, while the latter to around .3: by using these numbers we get a calibrated consumption-to-wealth ratio $s_c=.3/20=.015$. We calibrate the quarterly time discount factor following Tallarini (2000) and Barillas et al (2006): $\beta=.995$. Using the value of s_c obtained above, we derive the model-consistent steady-state value of the labor income-to-financial wealth ratio, by using $s_\xi=s_c-(1-\beta)/\beta=.01$.

7.2 The statistical model

We define the data vector $\mathbf{y}_t \equiv \left[\Delta\hat{\xi}_t,\, \Delta\hat{\xi}^*_t,\, \Delta\hat{q}_t,\, exr^{de}_t,\, exr^{ie}_t,\, exr^{ib}_t,\, \hat{r}_{H,t},\, \mathbf{x}_t\right]'$ and estimate the following VAR(1) model

$$\mathbf{y}_t = \boldsymbol{\mu} + \mathbf{A}\mathbf{y}_{t-1} + \mathbf{e}_t, \tag{41}$$

in which $\mathbf{e}_t$ is distributed as a multivariate normal with zero mean and variance-covariance matrix $\mathbf{\Omega}$.[26] In the data vector $\mathbf{y}$ we also include a series of additional controls, collected into the vector $\mathbf{x}$, which might be useful in describing the dynamic path of the variables of interest. In practice, $\mathbf{x}$ includes the growth rate of relative GDP, the slope of the US yield curve, the international excess return on ten-year government bonds and the growth rate in the US trade balance.[27] Using the above, we can evaluate the theoretical implications of our framework, and relate the results to existing literature.[28]

7.3 Asset structure

In our empirical analysis, we study three alternative cases concerning the asset structure. The first case assumes that the only asset available for international trade is equity (henceforth *Asset Menu I*) as in Baxter and Jermann (1997). In the second case we allow for trade in both equity and bonds, but restrict the latter to an overall balanced position (*Asset Menu II*), as in van Wincoop and Warnock (2006, 2008) and Coeurdacier and Gourinchas (2009); in this case, therefore, a long position on domestic bonds necessarily implies a short position of equal magnitude on foreign ones. Finally, we introduce a third asset structure, which is the general case in which bonds and equity are both available, and leveraged positions between risky and riskless assets are also allowed for (*Asset Menu III*); as a consequence, in this case the relative positions in home and foreign bonds are not directly related.

7.4 The role of labor-income risk

In this section we evaluate whether labor-income risk can explain the home-bias puzzle in equity holdings. In particular, this is the first component in equation (39) and depends on the positive covariance between the present discounted value of domestic-versus-foreign labor income and the excess return of foreign-versus-domestic equity. This hedging motif, which arises even under no model uncertainty, has been emphasized by several studies without reaching a clear consensus. Baxter and Jermann (1997) show that when equity is the only asset that can be traded internationally, the presence of non-diversifiable income risk actually implies a foreign-equity bias. On the other hand, Bottazzi et al. (1996) and more recently Julliard (2003) and Coeurdacier and Gourinchas (2009) bring evidence supporting the view that hedging against labor-income risk

[26] The length of the VAR is chosen optimally using the Schwarz's Bayesian Criterion for each estimation, and turns out to be always 1.

[27] Gourinchas and Rey (2007) show that the net-export growth rate is a useful predictor for portfolio returns at long horizons, while the other variables are among the forecasting variables commonly used for predicting asset returns and labor income. See also Campbell (1996).

[28] For what concerns the statistical model, as a robustness check, we estimated three alternative specifications. The first specification is the minimal requirement to describe the model economy and include only data on labor income and the excess return on foreign equity. The second and third specifications augment the first one by introducing data on the residual excess returns. Moreover, for each of the specifications above, we also varied the informational content of the data-vector by adding the real exchange rate, in changes, and the auxiliary regressors included in vector $\mathbf{x}$. In the text we report results for the extensive specification only, since results are robust to the other alternatives. The full set of results is available upon request.

Table 2: The empirical role of Labor-Income Risk

	Asset Menu I	Asset Menu II	Asset Menu III
Conditional covariance-to-variance ratios of LIR with selected excess returns			
$\hat{r}^{e*}_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}^{e}_{H,t+1}$	**−0.524**	**0.016**	**0.040**
$\hat{r}^{*}_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}_{H,t+1}$	—	**−1.116**	**−1.129**
$\hat{r}^{e}_{F,t+1} - \hat{r}_{H,t+1}$	—	—	**0.062**
Optimal Portfolio Allocation under Rational Expectations			
$\bar{\alpha}^e_F$	**1.020**	**0.484**	**0.460**
$\bar{\alpha}^e_H$	**−0.020**	**0.516**	**0.479**
$\bar{\alpha}_F$	—	**1.108**	**1.121**
$\bar{\alpha}^e_H + \bar{\alpha}^e_F$	**1.000**	**1.000**	**0.939**
$\bar{\alpha}_H + \bar{\alpha}_F$	—	**0.000**	**0.061**
$\bar{\alpha}^e_F + \bar{\alpha}_F$	**1.020**	**1.592**	**1.580**

Note: LIR denotes Labor-Income Risk; *Asset Menu I*: equities only; *Asset Menu II*: equities and balanced bonds; *Asset Menu III*: general model with equities and bonds. $\bar{\alpha}^e_F$ denotes the share of wealth invested in foreign equity; $\bar{\alpha}^e_H$ denotes the share of wealth invested in domestic equity; $\bar{\alpha}_F$ denotes the share of wealth invested in foreign bonds; $\bar{\alpha}^e_H + \bar{\alpha}^e_F$ measures the overall share of wealth invested in equity assets; $\bar{\alpha}_H + \bar{\alpha}_F$ measures the overall share of wealth invested in debt instruments; $\bar{\alpha}^e_F + \bar{\alpha}_F$ measures the overall share of wealth invested in foreign assets.

can explain some degree of home-bias in equity holdings. Heathcote and Perri (2004) and Coeurdacier and Gourinchas (2009), moreover, discuss some theoretical examples that can produce the required co-movements to explain home-bias.

We analyze this interaction in the context of our dynamic model. Using the output of the VAR, we construct the surprises in the path of relative labor income across countries, and compute the time-t conditional moments that we need. For comparisons with existing literature, however, we also compute the unconditional covariance-to-variance ratios, obtained through straightforward OLS projection of the surprises in relative labor income on the excess returns of the assets available for trade.[29] In the simple case of *Asset Menu I* (only equities) we obtain:

$$\varepsilon_{l,t+1} = \underset{(0.081)}{-\mathbf{0.479}} \cdot (\hat{r}^{e*}_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}^{e}_{H,t+1}) + u_{l,t+1},$$

where the standard error is reported in parenthesis. The unconditional covariance-to-variance ratio therefore is negative, statistically significant and economically rather large. The implication is that hedging labor-income risk does not produce home-bias in equity, but rather implies a *foreign*-equity bias. This result on the one hand supports Baxter and Jermann (1997), and on the other hand weakens the argument of Heathcote and Perri (2007).

In Table 2 we report the time-t conditional covariance-to-variance ratios related to labor-income risk, and the portfolio allocation implied by our theoretical model under the assumption that there is no real exchange rate risk. These results, moreover, are consistent with the benchmark case of no model uncertainty. In particular, the second column of Table 2 shows that the finding of Baxter and Jermann (1997), i.e. that the portfolio diversification puzzle is even worse than expected, is confirmed even when we move to the evaluation of the time-t conditional moments, consistently with equation (30).

This result has recently been challenged by Coeurdacier and Gourinchas (2009), who point out that, once also riskless bonds are traded, variances and covariances should be computed

[29] The unconditional covariance-to-variance ratios would be appropriate in our case if the process $\mathbf{y}_t$ were in fact a multivariate white noise. Our data, however, do not support this representation.

Table 3: Unconditional covariance-variance ratios

Loadings of:	Coeurdacier–Gourinchas (2009)*	Benigno–Nisticò (2009)**
$\hat{r}^{e*}_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}^{e}_{H,t+1}$	**0.260** (0.070)	**−0.027** (0.067)
$\hat{r}^{*}_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}_{H,t+1}$	**−1.170** (0.110)	**−0.982** (0.084)

Note: standard errors in parentheses. * Dependent variable is $\hat{r}^{w}_{t+1} - E_t\hat{r}^{w}_{t+1}$. ** Dependent variable is $\varepsilon_{l,t+1}$.

conditional on the other asset returns. Their claim is that, with the appropriate conditioning, the previous result would be overturned, and their empirical findings indeed support this claim.

We repeat their analysis of Coeurdacier and Gourinchas (2009) within our dynamic framework, where the asset structure assumed therein corresponds to our *Asset Menu II*. Specifically, Table 3 reports the result of an OLS projection of labor-income risk on the two relevant excess returns, and contrasts our findings with theirs.[30] In the second column we report the findings of Coeurdacier and Gourinchas (2009) which show that, conditioning on the excess return on foreign-versus-domestic bonds, there is a positive covariance between the excess return on foreign-versus-domestic equity and non-diversifiable labor-income risk, whereas the unconditional covariance is instead negative. They conclude that the results of Baxter and Jermann (1997) are driven by their particular asset structure, and do not hold when bonds are included. In the third column, we report our estimation's results, which show instead a negative (though insignificant) conditional covariance-to-variance ratio.

The difference between the two results can be explained by the different approach to measuring labor-income risk. Coeurdacier and Gourinchas (2009) use the unexpected component of the (home relative to foreign) return-to-labor, which they construct as

$$\hat{r}^{w}_{t+1} - E_t\hat{r}^{w}_{t+1} = \sum_{k=0}^{\infty}\rho^k(E_{t+1}-E_t)(\Delta\hat{\xi}^{R}_{t+1+k} - \Delta\hat{q}_{t+1+k}) - \sum_{k=1}^{\infty}\rho^k(E_{t+1}-E_t)(\hat{r}^{e}_{H,t+1+k} - \Delta\hat{q}_{t+1+k} - \hat{r}^{e*}_{F,t+1+k}), \quad (42)$$

where $\rho \equiv 1 - s_c$ is a constant of linearization that depends on the average consumption-to-wealth ratio. It is worth noticing that this measure is not directly implied by their model, which is static, but rather it is borrowed from Campbell (1996). Two important assumptions underlie this formulation, which are critical to distinguish their approach from ours. First, it is assumed that there exists a market for domestically tradeable claims on the stream of future labor-income flows, which implies that the return on labor is computed in analogy to the return on the financial assets, using the log-linear approximation of Campbell and Shiller (1988).[31] Second, the expected relative return on domestic non-financial wealth is equated to the expected excess return on domestic-versus-foreign equities. This is a strong assumption, as also discussed by Campbell (1996), and explains why the first term on the second line of equation (42) arises.

With this definition, it follows that the return-to-labor is likely to be positively related, by construction, with the excess return on foreign-versus-domestic equity.

[30] Note that we have defined the excess returns as foreign-versus-domestic returns, the opposite of Coeurdacier and Gourinchas (2009). Accordingly, for comparison, in Table 3 we report their results multiplied by -1.

[31] Furthermore, it seems odd to assume that there are tradeable claims on human capital which are traded domestically and not internationally.

We do not make either of the assumptions above. Instead, in our framework, the relevant measure of non-diversifiable labor risk is directly implied by the theoretical model, and corresponds to the revision in the present-discounted value of cross-country labor income $\varepsilon_{l,t+1}$, as shown by equation (25).[32] It is worth noticing that our measure of labor-income risk is instead similar to those used by Shiller (1995) and Baxter and Jermann (1997), which coincide with the first summation on the right-hand-side of (42).[33] Using this definition, we find that domestic equity is not a good hedge, reinforcing Baxter and Jermann's (1997) result even if we condition on bond returns.

To derive the equilibrium portfolio allocations implied by our theoretical model, we compute the relevant covariance-to-variance ratios conditioning them also on the information set available at time$-t$, and report the results in the second column of Table 2. The covariance between labor-income risk and the excess return on equities becomes of the right sign, but it is quantitatively negligible, and it does not imply a substantial degree of home-bias. On this respect, therefore, our results again contrast with Coeurdacier and Gourinchas (2009). However, we share the finding that agents should go long in foreign bonds and short in domestic ones, with the counterfactual implication that almost 160% of domestic wealth is allocated to foreign assets.

The third column of Table 2 displays the results for *Asset Menu III*, and shows that allowing for leveraged positions between equity and riskless assets does not change the results. Indeed, the covariance of labor-income risk with the domestic equity premium is positive, implying an overall long position in the international bond market ($\bar{\alpha}^e_H + \bar{\alpha}^e_F < 1$). This further exacerbates the inability of labor-income risk alone to support home-bias in equity: even though less than half of the steady-state wealth is allocated to foreign equities, the share allocated to domestic ones is also smaller than 50%.

7.5 The role of real exchange rate risk

In the above section we showed that there is no support for the view that domestic equity is a good hedge against non-diversifiable labor-income risk to explain the home-bias in US equity holdings. We now move to analyze the portfolio implications of model uncertainty where the fear of model misspecification translates into long-run real exchange rate risk.

The role of hedging real exchange rate fluctuations as an explanation for the home-bias puzzle has been recently questioned by van Wincoop and Warnock (2006, 2008) and Coeurdacier and Gourinchas (2009). Their main argument is based on the evidence that the covariance between real exchange rate changes and the excess return on foreign-versus-domestic equity becomes negligible once this covariance is taken conditional on other returns, like the excess return on riskless bonds. This observation comes from the results of a simple OLS regression between one-period ahead changes in the real exchange rate and the vector of excess returns

$$\Delta\hat{q}_{t+1} = \kappa_q + \boldsymbol{\psi}'_q \mathbf{exr}_{t+1} + u_{q,t+1}, \tag{43}$$

reported in Table 4. While the loading of the excess returns on foreign equity is significant and positive if equity is the only tradeable asset, once the vector of excess returns is augmented to include also the excess return on foreign-versus-domestic bonds, the covariance-to-variance ratio between the real exchange rate and the excess return on equity becomes negligible.

[32] Note that in a first-order approximation (which is all is needed to evaluate the orthogonality conditions and derive the portfolio allocation) expected excess returns are always zero, so the last terms in (42) would drop even if we did make the two assumptions discussed above.

[33] Indeed, the only difference between (25) and the measure in Shiller (1995) and Baxter and Jermann (1997) is the discount parameter: while they use $\rho \equiv 1 - s_c$, we use the time discount factor β. Numerically, however, they are also very close to each other.

Table 4: Loadings of excess returns on real exchange rate depreciations

Loadings of:	Asset Menu I	Asset Menu II	Asset Menu III
$\hat{r}^{e*}_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}^{e}_{H,t+1}$	**0.365** (0.072)	**0.021** (0.068)	**−0.026** (0.071)
$\hat{r}^{*}_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}_{H,t+1}$	– –	**0.747** (0.086)	**0.781** (0.086)
$\hat{r}^{e}_{H,t+1} - \hat{r}_{H,t+1}$	– –	– –	**−0.098** (0.048)

Note: standard errors in parentheses. Dependent variable is $\Delta\hat{q}_{t+1}$.

In a rational-expectation model such small covariances (provided they are of the right sign) would require an unreasonably large degree of risk aversion to justify the hedging role of domestic equities, which would then open room for other puzzles, like the aforementioned risk-free rate puzzle.

Instead, our dynamic model with distorted beliefs gives a new role to real exchange rate risk: what matters is not only the current real-exchange-rate risk but also the revisions in the entire future expected path of the real exchange rate. What is relevant, therefore, is not so much the role of equity to hedge against *short-run* real exchange rate risk, but rather its hedging properties against *long-run* fluctuations.

To study whether shifting from a short-run to a long-run perspective affects the hedging properties of equity with respect to real exchange rate risk, we start writing equation (38) in terms of levels instead of growth rates:

$$\varepsilon_{q,t+1} \equiv \sum_{k=0}^{\infty} \beta^k \left[E_{t+1}\Delta\hat{q}_{t+1+k} - E_t\Delta\hat{q}_{t+1+k}\right] = (1-\beta)\sum_{k=0}^{\infty} \beta^k \left[E_{t+1}\hat{q}_{t+1+k} - E_t\hat{q}_{t+1+k}\right]. \quad (44)$$

By looking at different terms in the summation above, we can investigate the co-movement between asset returns and surprises in the real exchange rate path, at different time horizons. In particular, we can evaluate whether the hedging properties of equity and bonds change when

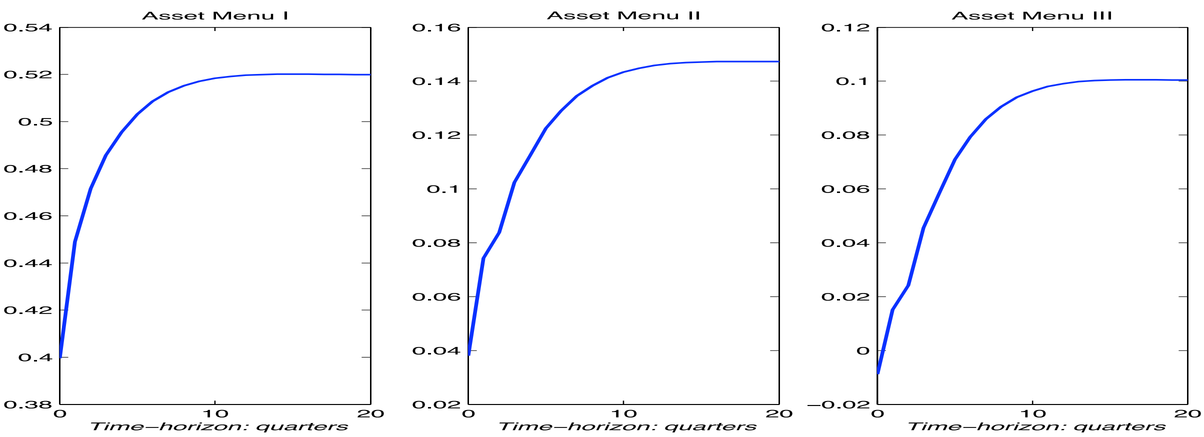

Figure 1: The covariance-to-variance ratio between $\Delta E_{t+1}\hat{q}_{t+1+k}$ and exr^{ie}_{t+1}, for increasing k (horizontal axis). *Asset Menu I*: equities only. *Asset Menu II*: equities and balanced bonds. *Asset Menu III*: general model with equities and bonds.

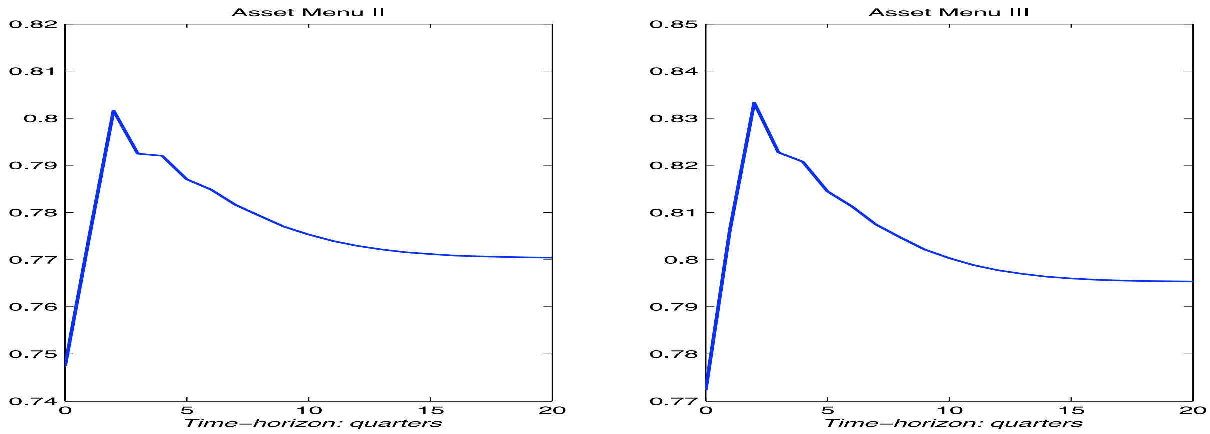

Figure 2: The covariance-to-variance ratio between $\Delta E_{t+1}\hat{q}_{t+1+k}$ and exr^{ib}_{t+1}, for increasing k (horizontal axis). Asset Menu II: equities and balanced bonds. Asset Menu III: general model with equities and bonds.

the risk to be hedged is farther away in the future, as opposed to very soon.

To this end, given our estimated model (41), we compute the time$-t+1$ news about the real exchange rate k periods ahead, given by $\Delta E_{t+1}\hat{q}_{t+1+k}$, in which $\Delta E_{t+1}(\cdot) \equiv E_{t+1}(\cdot) - E_t(\cdot)$ denotes the time-$t+1$ revisions in conditional expectations, and in particular the news about the long-run component $\Delta E_{t+1}\hat{q}_{\infty}$. Moreover, for each time-horizon we also evaluate the covariance-to-variance ratios with respect to all excess returns of interest, conditional on time-t information and on the residual asset space, given by $\mathbf{\Sigma}_t^{-1}E_t(\Delta E_{t+1}\hat{q}_{t+1+k} \cdot \mathbf{exr}_{t+1})$. Hence, Figure 1 plots the covariance-to-variance ratios of the news in the real exchange rate path with the excess return on foreign-versus-domestic equity, against the time-horizon k, for the three asset structures that we consider. Figure 2 does the same for the excess return on foreign-versus-domestic bonds, for the two asset menus which include bonds (*II* and *III*).

The first point in each plot, i.e. when $k = 0$, corresponds to the covariance-to-variance ratio of a static model, in which only the short-run risk matters. Moving from the left to the right panel of Figure 1, the first point drops from about .4 to virtually zero, implying that the hedging power of equity against real exchange rate risk fades away, when we condition on other excess returns and in particular on bonds. This is the core of the results in van Wincoop and Warnock (2006) and Coeurdacier and Gourinchas (2009).

However, at longer horizons the hedging properties of equity sharply improve, even when we condition on other excess returns. Figure 2, instead, shows that the hedging properties of bonds are only marginally affected. We view this evidence as suggesting that domestic equity can have a relatively more important role in hedging the real exchange rate risk at longer horizons in order to explain the international home-bias puzzle.[34]

7.6 The role of model uncertainty

We now merge the results and discussions of the previous sections and present the empirical implications of our framework through equation (39).

Table 5 reports the covariance-to-variance ratios of the two sources of risk implied by our model (labor-income and real exchange rate) with the relevant excess returns, for the three asset

[34] A recent literature documents the quantitatively substantial implications of long-run risk for asset valuation, in the context of non-expected utility frameworks. See, among others, Hansen et al. (2008), who also provide an interpretation related to model uncertainty.

menus that we consider. In particular, the empirical co-movements with respect to the international excess return on equity reveals that domestic equity – relative to foreign – is qualitatively useful to hedge both risks, as long as riskless bonds are available for trade;[35] however, they also imply that the role of domestic equity as a hedge against real exchange rate risk is at least twice as important as its role to hedge against labor-income risk (almost ten times more important under *Asset Menu II*). Co-movements with exr^{ib} imply that domestic bonds – relative to foreign – represent always a useful hedge against real exchange rate risk (positive ratio), but a bad hedge against labor-income risk (negative ratio). In the general case of *Asset Menu III*, moreover, the top-panel of Table 5 implies that debt instruments are a relatively better hedge against labor-income risk (positive ratio), while equity assets are relatively better to hedge real exchange rate risk (negative ratio).

Table 5: The empirical role of Real Exchange Rate Risk

	Asset Menu I	Asset Menu II	Asset Menu III
Conditional covariance-to-variance ratios of LIR with selected excess returns			
$\hat{r}^{e*}_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}^{e}_{H,t+1}$	**−0.524**	**0.016**	**0.040**
$\hat{r}^{*}_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}_{H,t+1}$	—	**−1.116**	**−1.129**
$\hat{r}^{e}_{F,t+1} - \hat{r}_{H,t+1}$	—	—	**0.062**
Conditional covariance-to-variance ratios of RERR with selected excess returns			
$\hat{r}^{e*}_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}^{e}_{H,t+1}$	**0.518**	**0.145**	**0.098**
$\hat{r}^{*}_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}_{H,t+1}$	—	**0.771**	**0.796**
$\hat{r}^{e}_{F,t+1} - \hat{r}_{H,t+1}$	—	—	**−0.117**

Note: LIR denotes Labor-Income Risk; RERR denotes Real Exchange Rate Risk; *Asset Menu I*: equities only; *Asset Menu II*: equities and balanced bonds; *Asset Menu III*: general model with equities and bonds.

Figure 3 shows the implications of these empirical co-movements for the optimal portfolio allocation, for increasing degrees of ambiguity aversion and for the three asset menus considered. In particular, the first point of each line captures the case of no-model uncertainty ($\gamma = 1$) in which the optimal portfolio allocation is driven only by labor-income risk – the first component of the right-hand-side of (39). As the degree of ambiguity aversion γ increases, instead, the second component of (39), related to real exchange rate risk, becomes progressively more important.

The top-left panel of Figure 3 displays the share of wealth allocated to domestic equity ($\bar{\alpha}^e_H$). As the degree of ambiguity aversion rises, $\bar{\alpha}^e_H$ sharply increases, regardless of the specific asset structure (up to 70% for *Asset Menus I* and *II* and up to 80% for *Asset Menu III*). In particular, in the general case of *Asset Menu III*, when $\gamma = 10$ the share of wealth allocated to domestic equity reaches about 77%, explaining therefore a large proportion of the home-bias found in the data, and in contrast with the 48% of the case with no model uncertainty. The top-right panel shows the overall share of wealth that the domestic agent invests in the equity market. *Asset Menu III* is the only case in which leveraged positions between different kinds of securities are allowed for. Indeed, as the degree of ambiguity aversion increases, the domestic agent takes a short position in the overall bond portfolio to invest more in equities: the component driven by hedging real exchange rate risk in (39) dominates, and implies that domestic equities pay relatively better than domestic bonds precisely when an unexpected appreciation of the real exchange rate occurs. In particular, for $\gamma = 10$, the overall share of wealth allocated to equity

[35] Indeed, in the case of *Asset Menu I*, domestic equity are a better hedge against real exchange rate risk (positive ratio) and a worse hedge against labor-income risk (negative ratio), relative to foreign equity.

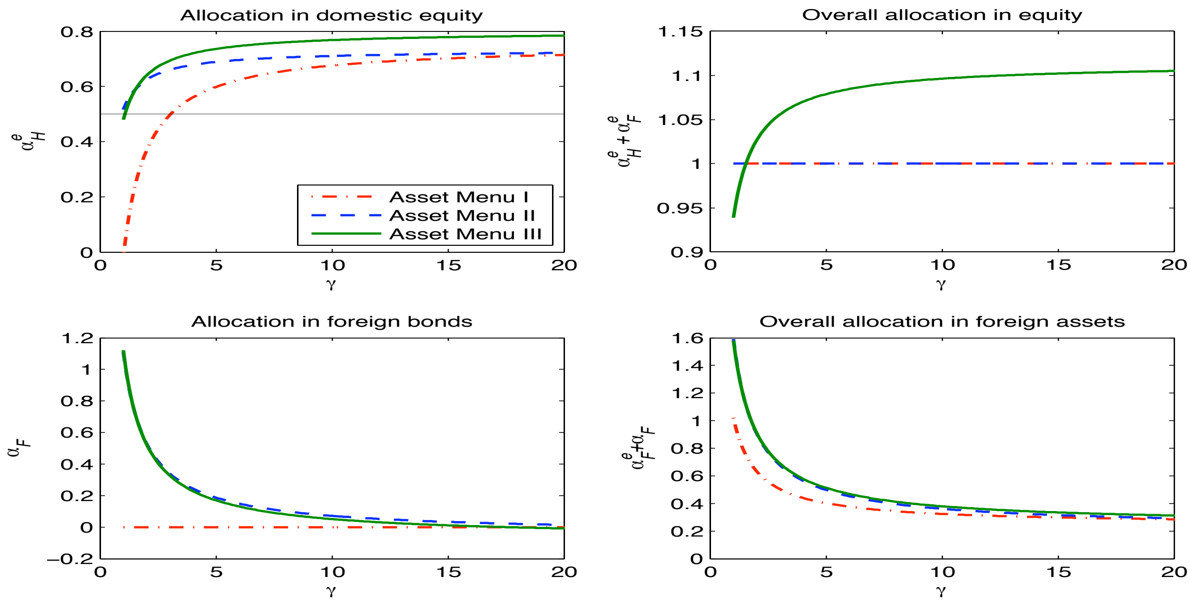

Figure 3: Optimal portfolio allocation: the effect of increasing degrees of concern about model misspecification. Asset Menu I: equities only. Asset Menu II: equities and balanced bonds. Asset Menu III: general model with equities and bonds.

$(\bar{\alpha}_H^e+\bar{\alpha}_F^e)$ increases to about 110%. This leveraged position in favor of equity assets also explains why the optimal degree of home-bias in the case of *Asset Menu III* is larger than in the other two cases. The bottom-left panel of Figure 3 shows the share invested in foreign bonds. When $\gamma = 1$, foreign bonds are a good hedge against labor-income risk (relative to domestic bonds), and the agent invests virtually all of his/her wealth in foreign bonds: this implication is counterfactual. Instead, foreign bonds are not a good hedge with respect to long-run real exchange rate risk and indeed agents would like to take a short position in this respect. As γ increases, therefore, the bad-hedge component with respect to real exchange rate risk becomes more important, and the share of wealth allocated to foreign bonds decrease to about 5% when $\gamma = 10$.

The bottom-right panel of Figure 3 shows the overall share invested in foreign assets. When $\gamma = 1$, there is a large foreign bias in asset holdings (mainly bonds) which is financed by shortening the domestic bond: foreign bonds are, indeed, a good hedge with respect to labor-income risk, as previously discussed. Both foreign bonds and equity are, however, a bad hedge with respect to long-run real exchange rate risk; accordingly, when γ rises, the overall wealth invested in foreign assets decreases substantially (38% when $\gamma = 10$, in the case of *Asset Menu III*).

Moreover, in the general case of *Asset Menu III,* we can evaluate the ability of the model to replicate other stylized facts that are receiving increasing attention by the empirical literature. Tille (2005, 2008), for example, reports a detailed breakdown of the composition of US foreign assets and liabilities, and documents four basic features: 1) the US is a large net creditor in equity instruments and 2) a net debtor in bond instruments; 3) the net position on foreign-currency bonds is about balanced, while 4) the position in bonds denominated in US dollars is largely negative. In our framework, the net-foreign asset position in equities is given by $NFE = \bar{\alpha}_F^e + \bar{\alpha}_H^e - 1$ and the one in bonds by $NFB = \bar{\alpha}_F + \bar{\alpha}_H$, in which $\bar{\alpha}_F$ and $\bar{\alpha}_H$ capture country H's position on bonds denominated in foreign and domestic currency, respectively.[36]

[36]Indeed, the steady-state net-foreign asset position (as a share of steady-state domestic wealth) is given by

As shown by the first points in Figure 3, labor-income risk alone is not useful along this dimension: the first component, indeed, implies an overall long position in the international bond market, making the US a net debtor in equity assets ($\bar{\alpha}^e_H + \bar{\alpha}^e_F < 1$), and a largely long position on foreign-currency bonds ($\bar{\alpha}_F \simeq 1.1$). On the contrary, real exchange rate risk and fear of model misspecification are able to reconcile the model with all the stylized facts on US cross-border holdings documented by Tille (2005, 2008). Indeed, rising degrees of ambiguity aversion imply a creditor position in equity assets ($\bar{\alpha}^e_H + \bar{\alpha}^e_F > 1$) and a debtor position in debt instruments ($\bar{\alpha}_H + \bar{\alpha}_F < 0$) – top-right panel. Moreover, the position in foreign-currency bonds shrinks progressively towards a balanced position ($\bar{\alpha}_F \simeq 0$), while the one in home-currency bonds remains negative ($\bar{\alpha}_H < 0$).

7.7 Calibrating γ using detection error probabilities

This section describes how to appropriately calibrate γ as a parameter capturing the concern about model misspecification. We follow Anderson et al. (2003) and Hansen and Sargent (2007) in using detection error probabilities. Let us call model A the approximating model and model $B(\gamma)$ the worst-case model associated with a particular γ. Agents start with the belief that the models are equally likely. That is, they assign 50% prior probability to each model. After having seen T observations, they can perform a likelihood ratio test for distinguishing the two models. Under the hypothesis that model A is correct, we denote with $p_A(\gamma)$ the probability that a likelihood ratio test would instead falsely say that model $B(\gamma)$ generated the data. Conversely, we denote with $p_B(\gamma)$ the probability that a likelihood ratio test would falsely say that model A generated the data, when in fact model $B(\gamma)$ is correct. The detection error probability, then, is the weighted average of $p_A(\gamma)$ and $p_B(\gamma)$ with the weights given by the prior probabilities:

$$p(\gamma) = \frac{1}{2}\Big(p_A(\gamma) + p_B(\gamma)\Big).$$

The detection error probability is a decreasing function of γ, since a larger γ (and therefore a smaller θ) implies a lower penalization upon relaxing the entropy constraint in equation (34). Indeed, the higher γ the wider is the entropy ball inside which the consumer allows the evil agent to choose the worst-case distortion, and therefore the more afraid of misspecification the consumer is. Accordingly, higher values of γ imply a larger divergence between the worst-case model and the approximating one, and is therefore less probable that the likelihood-ratio test will favor the wrong model. When $\gamma = 1$, on the contrary, the two models are equivalent and $p(\gamma)$ is therefore equal to 1/2.

It is important to notice that the mapping between γ and $p(\gamma)$, is model-specific and varies in different contexts. This is why the plausibility of a given value of γ, as a measure of the concern about model misspecification, should be appropriately determined in terms of the detection error probability that it implies, which can instead be regarded as a context-invariant measure.

In our context the approximating model is given by the VAR in (41):

$$\mathbf{y}_t = \boldsymbol{\mu} + \mathbf{A}\mathbf{y}_{t-1} + \mathbf{e}_t$$

where $\mathbf{e}_t$ is distributed as a multivariate normal with zero mean and variance-covariance matrix $\boldsymbol{\Omega}$. In the Appendix we show that the worst-case models, associated with specific values of γ and γ^*, imply a distortion in the mean of the VAR, and take the form

$$\mathbf{y}_t = \boldsymbol{\mu} + \mathbf{w}(\gamma) + \mathbf{A}\mathbf{y}_{t-1} + \mathbf{e}_t \tag{45}$$

$NFA \equiv \bar{\alpha}^e_F + \bar{\alpha}_F - \bar{\alpha}^{e*}_H - \bar{\alpha}^*_H$, and the two components are $NFE \equiv \bar{\alpha}^e_F - \bar{\alpha}^{e*}_H$ and $NFB \equiv \bar{\alpha}_F - \bar{\alpha}^*_H$. The equations in the text use the properties of the symmetric steady state in which $\bar{a} = \bar{q}\bar{a}^*$: $\bar{\alpha}^{e*}_H = 1 - \bar{\alpha}^e_H$ and $\bar{\alpha}^*_H = -\bar{\alpha}_H$.

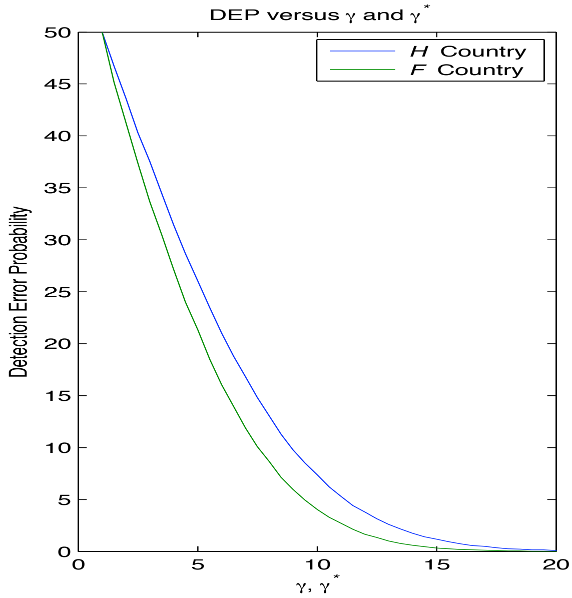

Figure 4: Detection Error Probabilities (DEP) versus fear of model misspecification (γ and γ^*, left panel) and versus discounted conditional relative entropy (η and η^*, right panel).

for consumers in country H and

$$\mathbf{y}_t = \boldsymbol{\mu} + \mathbf{w}^*(\gamma^*) + \mathbf{A}\mathbf{y}_{t-1} + \mathbf{e}_t \tag{46}$$

for those in country F, where $\mathbf{w}(\gamma)$ and $\mathbf{w}^*(\gamma^*)$ are the optimal distortions in the mean.

We simulated 100,000 samples, each of size 112 observations (corresponding to the sample 1980q1-2007q4 that we use in the VAR estimation), and computed the detection error probabilities associated with the approximating and the worst-case models, by varying the parameters γ and γ^*. The results are displayed in Figure 4.

The left panel of Figure 4 shows the detection error probabilities, $p(\gamma)$ and $p(\gamma^*)$, plotted against γ and γ^*. We follow Anderson et al. (2003), Maenhout (2006) and Barillas et al. (2006), and consider alternative models whose detection error probabilities are around 10 per cent as "difficult to detect". Figure 3 has shown that values of γ or γ^* between 5 and 10 are sufficient to get the most of the model fit in terms of home-bias in equity and other empirical evidence on cross-border holdings. Figure 4 then shows that values of γ and γ^* between 7 and 10 are still associated with detection error probabilities around 0.10.

The degree of ambiguity aversion needed to explain the empirical facts is therefore consistent with conservative values of the detection error probabilities, thus validating the empirical relevance of the model's implications. Given that the left panel shows that for similar detection error probability γ and γ^* are very close, we can also conclude that the assumption $\gamma = \gamma^*$ is generally innocuous.[37]

The right panel of Figure 4 plots the detection error probabilities against the discounted conditional relative entropy defined in (6), which in our case is time-invariant and equals

$$\eta(\gamma) = .5\frac{\beta}{1-\beta}\mathbf{w}(\gamma)'\boldsymbol{\Omega}^{-1}\mathbf{w}(\gamma)$$

[37]At the threshold value of 10%, the values for γ and γ^* are, respectively, 9 and 7.5.

for agents in country H and

$$\eta^*(\gamma^*) = .5\frac{\beta}{1-\beta}\mathbf{w}^*(\gamma^*)'\mathbf{\Omega}^{-1}\mathbf{w}^*(\gamma^*)$$

for those in country F. This panel reveals that for each value of detection error probability, the discounted entropies are the same for the two agents, in further support of the view that a bound on detection error probabilities, rather than a given value for γ, appropriately defines a context-invariant measure of concern about model uncertainty.

8 Conclusions

The observation that international investors hold a disproportionate share of their wealth in domestic rather than foreign assets is one of the most persistent fact in international finance. This is named the international home-bias puzzle, that the literature has been dealing with for a couple of decades.

This paper develops a dynamic general equilibrium model of portfolio and consumption choices, with incomplete markets and distorted beliefs. Households might use a "subjective" probability distribution that is generally different from the "approximating" one and make robust optimal choices against model uncertainty. This framework assigns a new role to real exchange rate risk for portfolio allocation even in a model with log utility. Importantly, moreover, what matters is not only the short-run risk but also and foremost the long-run risk of real exchange rate fluctuations.

Within this framework we characterize optimal portfolio allocations in terms of covariances between measurable sources of risk to be hedged (non-diversifiable labor-income risk and real exchange rate risk) and a vector of cross-country excess returns, and evaluate their empirical relevance using financial and macro data on the G7 countries.

Our results suggest that, contrary to what claimed in recent related contributions, hedging non-diversifiable labor-income risk is not sufficient to account for the lack of international portfolio diversification. Indeed, in a setting in which equity is the only available asset, correlations in financial data support a large foreign-equity bias, as in Baxter and Jermann (1997). Adding further assets does not help in identifying a clear role for this risk in explaining the home-bias puzzle, once the former is measured in a model-consistent way. On the other hand, a "plausible" concern about model misspecification is able to generate a substantial equilibrium home bias in equity holdings, and allows to match other empirical facts regarding the US cross-border holdings. We evaluate the "plausibility" of the concern for model uncertainty by resorting to detection error probabilities, which measure how easily the competing models can be told apart using a finite amount of data.

The methodological contribution of the paper goes beyond the analysis of the home-bias puzzle. The class of preferences that we suggest, in fact, produces a perturbation of the equilibrium stochastic discount factor which decouples the attitudes towards intertemporal substitution with those towards risk and ambiguity, and can prove useful in addressing other failures of standard preference specifications along the asset-price dimension.[38] Indeed, it has been shown, in closed-economy settings, that disentangling the elasticity of intertemporal substitution from the degree of risk aversion helps in accounting for the equity premium puzzle. Once we open the economy to international trade in assets, there are additional puzzling features of financial data, among which the international equity- and bond-premia puzzles and the Backus-Smith anomaly are

[38] See for a discussion Backus et al. (2004).

notable examples.[39] All these stylized facts imply restrictions on the stochastic discount factor that standard preferences cannot meet at the same time, and that might be all reconnected to some common misspecification.[40] The modification of the stochastic discount factor that our preference specification implies is a promising tool to correct this misspecification and build macro models whose predictions are closer to the empirical implications of financial data.

[39] Barillas et al. (2006) discusses the implications of model uncertainty for the equity premium puzzle; Piazzesi and Schneider (2006) studies the slope of the yield curve with Epstein-Zin preferences. Ilut (2008) studies how ambiguity aversion can help explain the uncovered-interest-rate puzzle.

[40] All excess-return puzzles, for example, imply "high" lower bounds on the volatility of the equilibrium stochastic discount factor, as discussed for the equity premium by Hansen and Jagannathan (1991).

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Appendix

A Some Useful Definitions

To get equation (10), we have defined

$$A_t \equiv B_{H,t} + S_t B_{F,t} + x_{H,t} V_{H,t} + x_{F,t} S_t V^*_{F,t},$$

and

$$R_{p,t} \equiv \alpha_{H,t-1} R_{H,t} + \alpha_{F,t-1} R^*_{F,t} \frac{S_t}{S_{t-1}} + \alpha^e_{H,t-1} R^e_{H,t} + \alpha^e_{F,t-1} R^{e*}_{F,t} \frac{S_t}{S_{t-1}},$$

with

$$R^e_{H,t} \equiv \frac{V_{H,t} + D_{H,t}}{V_{H,t-1}},$$
$$R^{e*}_{F,t} \equiv \frac{V^*_{F,t} + D^*_{F,t}}{V^*_{F,t-1}},$$

and

$$B_{H,t} \equiv \alpha_{H,t} A_t,$$
$$S_t B_{F,t} \equiv \alpha_{F,t} A_t,$$
$$x_{H,t} V_{H,t} \equiv \alpha^e_{H,t} A_t,$$
$$x_{F,t} S_t V^*_{F,t} = \alpha^e_{F,t} A_t,$$

and analogously for the foreign country:

$$B^*_{H,t} \equiv \alpha^*_{H,t} A^*_t S_t,$$
$$B^*_{F,t} \equiv \alpha^*_{F,t} A^*_t,$$
$$x^*_{H,t} V_{H,t} \equiv \alpha^{e*}_{H,t} A^*_t S_t,$$
$$x^*_{F,t} V^*_{F,t} = \alpha^{e*}_{F,t} A^*_t.$$

B Derivation of equation (26)

In what follows, a variable with an "upper-bar" denotes the symmetric steady state and a "hat" denotes the log-deviation with respect to such steady state. A first-order approximation of the Euler conditions (12) and (14) implies

$$E_t \Delta \hat{c}_{t+1} = E_t \hat{r}_{p,t+1}, \tag{B.1}$$
$$E_t \Delta \hat{c}^*_{t+1} = E_t \hat{r}^*_{p,t+1}. \tag{B.2}$$

In particular, the portfolio returns can be approximated to first order as

$$\hat{r}_{p,t+1} = \hat{r}_{H,t+1} + \bar{\boldsymbol{\alpha}}' \mathbf{exr}_{t+1},$$

$$\hat{r}^*_{p,t+1} = \hat{r}_{H,t+1} + \bar{\boldsymbol{\alpha}}^{*\prime} \mathbf{exr}_{t+1} - \Delta \hat{q}_{t+1},$$

where we have defined

$$\bar{\boldsymbol{\alpha}} \equiv \begin{bmatrix} \bar{\alpha}_F \\ \bar{\alpha}^e_H + \bar{\alpha}^e_F \\ \bar{\alpha}^e_F \end{bmatrix} \qquad \bar{\boldsymbol{\alpha}}^* \equiv \begin{bmatrix} \bar{\alpha}^*_F \\ \bar{\alpha}^{*e}_H + \bar{\alpha}^{*e}_F \\ \bar{\alpha}^{*e}_F \end{bmatrix}, \tag{B.3}$$

and the vector of excess returns as

$$\mathbf{exr}_t \equiv \begin{bmatrix} \hat{r}^*_{F,t} + \Delta \hat{q}_t - \hat{r}_{H,t} \\ \hat{r}^e_{H,t} - \hat{r}_{H,t} \\ \hat{r}^{e*}_{F,t} + \Delta \hat{q}_t - \hat{r}^e_{H,t} \end{bmatrix}.$$

In a first-order approximation, the no-arbitrage conditions imply that excess returns have zero conditional means, $E_t \mathbf{exr}_{t+1} = 0$. It follows, using equations (B.1) and (B.2), that the cross-country differential in the expected consumption growth depends on the expected depreciation in the real exchange rate

$$E_t \Delta \hat{c}^R_{t+1} = E_t \Delta \hat{q}_{t+1}, \tag{B.4}$$

where an upper-script R denotes the difference between the domestic and foreign variables.

A first-order approximation of the flow budget constraint (10) together with the budget constraint of the foreign agent implies

$$\beta \hat{a}_t^R = \hat{a}_{t-1}^R + \bar{\boldsymbol{\lambda}}' \mathbf{exr}_t + \Delta \hat{q}_t + \beta s_\xi \hat{\xi}_t^R - \beta s_c \hat{c}_t^R, \tag{B.5}$$

where s_ξ is the steady-state ratio between non-traded income and financial wealth, given by $s_\xi \equiv \bar{\xi}/\bar{a}$, which is equal in the two countries; s_c is the steady-state ratio between consumption and financial wealth and such that $s_c = (1-\beta)/\beta + s_\xi$. Moreover, the vector $\bar{\boldsymbol{\lambda}}$ is defined as

$$\bar{\boldsymbol{\lambda}} \equiv \begin{bmatrix} 2\bar{\alpha}_F \\ 2(\bar{\alpha}_H^e + \bar{\alpha}_F^e) - 2 \\ 2\bar{\alpha}_F^e - 1 \end{bmatrix}. \tag{B.6}$$

The set of difference equations (B.4) and (B.5) can be solved forward to obtain relative consumption and relative wealth ($\hat{c}_t^R$, ${}_t^R$) as a function of the states (${}_{t-1}^R$, $\hat{q}_{t-1}$) and the processes of excess returns, relative non-diversifiable income and the real exchange rate $\{\mathbf{exr}_t, \hat{\xi}_t^R, \hat{q}_t\}$. In particular, we obtain

$$(\hat{c}_t^R - \hat{q}_t) = \frac{(1-\beta)}{\beta s_c}(\hat{a}_{t-1}^R - \hat{q}_{t-1}) + \frac{(1-\beta)}{\beta s_c}\bar{\boldsymbol{\lambda}}' \mathbf{exr}_t + \frac{(1-\beta)s_\xi}{s_c} E_t \sum_{T=t}^{\infty} \beta^{T-t}(\hat{\xi}_T^R - \hat{q}_T), \tag{B.7}$$

$$(\hat{a}_t^R - \hat{q}_t) = (\hat{a}_{t-1}^R - \hat{q}_{t-1}) + \bar{\boldsymbol{\lambda}}' \mathbf{exr}_t + s_\xi(\hat{\xi}_t^R - \hat{q}_t) - (1-\beta)s_\xi E_t \sum_{T=t}^{\infty} \beta^{T-t}(\hat{\xi}_T^R - \hat{q}_T). \tag{B.8}$$

We determine the portfolio shares by using a second-order approximation of the moment conditions (20)–(22). In particular we just need three restrictions to determine the vector $\bar{\boldsymbol{\lambda}}$:

$$E_t\left[(\Delta \hat{c}_{t+1}^R - \Delta \hat{q}_{t+1})(\hat{r}_{H,t+1}^e - \hat{r}_{H,t+1})\right] = 0,$$

$$E_t\left[(\Delta \hat{c}_{t+1}^R - \Delta \hat{q}_{t+1})(\hat{r}_{F,t+1}^{e*} + \Delta \hat{q}_{t+1} - \hat{r}_{H,t+1}^e)\right] = 0.$$

$$E_t\left[(\Delta \hat{c}_{t+1}^R - \Delta \hat{q}_{t+1})(\hat{r}_{F,t+1}^{*} + \Delta \hat{q}_{t+1} - \hat{r}_{H,t+1})\right] = 0,$$

We can now use equations (B.7)–(B.8) in the conditions above and solve for the steady-state vector of portfolio shares and obtain equation (26).

C Derivation of equation (39)

Under model uncertainty, it is still true that (B.4) and (B.5) still hold and can be used to write (37) as

$$\hat{g}_{t+1}^R = -(\gamma-1)\frac{(1-\beta)}{\beta s_c}\bar{\boldsymbol{\lambda}}' \mathbf{exr}_{t+1} - (\gamma-1)\varepsilon_{q,t+1} - (\gamma-1)\frac{s_\xi}{s_c}\varepsilon_{l,t+1},$$

where we have defined the time-t surprises in the real exchange rate growth as

$$\varepsilon_{q,t+1} \equiv \sum_{j=0}^{\infty} \beta^j \left[E_{t+1}\Delta \hat{q}_{t+1+j} - E_t \Delta \hat{q}_{t+1+j}\right]. \tag{C.9}$$

Therefore, the left-hand side of the orthogonality condition (31) can be written as

$$(\Delta \hat{c}_{t+1}^R - \Delta \hat{q}_{t+1} - \hat{g}_{t+1}^R) = \gamma \frac{(1-\beta)}{\beta s_c}\bar{\boldsymbol{\lambda}}' \mathbf{exr}_{t+1} + (\gamma-1)\varepsilon_{q,t+1} + \gamma \frac{s_\xi}{s_c}\varepsilon_{l,t+1},$$

from which it follows that (31) implies equation (39).

D Derivation of equations (45)–(46)

In the approximating model

$$\mathbf{y}_t = \boldsymbol{\mu} + \mathbf{A}\mathbf{y}_{t-1} + \mathbf{e}_t,$$

the vector of shocks $\mathbf{e}_t$ is distributed as a multivariate normal with zero mean and variance-covariance matrix $\boldsymbol{\Omega}$. Accordingly, the probability density of $\mathbf{e}_t$, denoted by $f(\mathbf{e}_t)$, is proportional to

$$\exp\left(-\frac{1}{2}\mathbf{e}_t' \boldsymbol{\Omega}^{-1} \mathbf{e}_t\right).$$

The approximating and the worst-case models are linked through the martingale increments g and g^* for the agents of country H and F, respectively. We showed in Section 5 that in a first-order approximation g and g^* are related to the revisions in the expected future path of the respective consumption growth:

$$\hat{g}_t = -(\gamma - 1)\sum_{k=0}^{\infty}\beta^k\Big(E_t\Delta\hat{c}_{t+k} - E_{t-1}\Delta\hat{c}_{t+k}\Big)$$
$$\hat{g}_t^* = -(\gamma^* - 1)\sum_{k=0}^{\infty}\beta^k\Big(E_t\Delta\hat{c}^*_{t+k} - E_{t-1}\Delta\hat{c}^*_{t+k}\Big),$$

in which we are allowing for different γ and γ^*.

Using equations (B.1)–(B.2) and a first-order approximation of the flow-budget constraint (10), we can solve for the growth rate of domestic consumption, as a function of steady-state portfolio shares, asset returns and labor income. It follows that we can write g and g^* as linear combinations of the VAR innovations:

$$\hat{g}_t = -(\gamma - 1)\mathbf{z}(\gamma)'\mathbf{e}_t$$
$$\hat{g}_t^* = -(\gamma^* - 1)\mathbf{z}^*(\gamma^*)'\mathbf{e}_t,$$

in which vectors $\mathbf{z}$ and $\mathbf{z}^*$ depend on γ and γ^* through the steady-state portfolio shares. Indeed, simple algebra shows that

$$\mathbf{z}(\gamma) \equiv \frac{1-\beta}{\beta s_c}\boldsymbol{\iota}_{exr}\bar{\boldsymbol{\alpha}}(\gamma) + \mathbf{H}'\boldsymbol{\iota}_r + \frac{s_\xi}{s_c}\mathbf{H}'(\boldsymbol{\iota}_\xi - \boldsymbol{\iota}_r)$$

for country H, and

$$\mathbf{z}^*(\gamma^*) \equiv \frac{1-\beta}{\beta s_c}\boldsymbol{\iota}_{exr}\bar{\boldsymbol{\alpha}}^*(\gamma^*) + \mathbf{H}'(\boldsymbol{\iota}_r - \boldsymbol{\iota}_q) + \frac{s_\xi}{s_c}\mathbf{H}'(\boldsymbol{\iota}_\xi + \boldsymbol{\iota}_q - \boldsymbol{\iota}_r),$$

for country F, in which $\bar{\boldsymbol{\alpha}}$ and $\bar{\boldsymbol{\alpha}}^*$ are defined in (B.3), and $\mathbf{H} \equiv (\mathbf{I} - \beta\mathbf{A})^{-1}$.

It follows that the probability distribution of the distorted model for the agent in country H, denoted by $\tilde{f}(\mathbf{e}_t)$, is given by

$$\tilde{f}(\mathbf{e}_t) \equiv f(\mathbf{e}_t)\cdot g_t \propto \exp\left(-\frac{1}{2}\mathbf{e}_t'\boldsymbol{\Omega}^{-1}\mathbf{e}_t\right)\exp\Big(-(\gamma-1)\mathbf{z}(\gamma)'\mathbf{e}_t\Big).$$

Completing the square finally allows us to write $\tilde{f}(\mathbf{e}_t)$ as

$$\tilde{f}(\mathbf{e}_t) \propto \exp\left(-\frac{1}{2}\Big(\mathbf{e}_t - \mathbf{w}(\gamma)\Big)'\boldsymbol{\Omega}^{-1}\Big(\mathbf{e}_t - \mathbf{w}(\gamma)\Big)\right),$$

in which $\mathbf{w}(\gamma) \equiv -(\gamma-1)\boldsymbol{\Omega}\mathbf{z}(\gamma)$ is the mean distortion implied by the preference for robustness. Similarly, the distorted probability distribution function for the agent in country F, $\tilde{f}^*(\mathbf{e}_t)$, is given by

$$\tilde{f}^*(\mathbf{e}_t) \equiv f(\mathbf{e}_t)\cdot g_t^* \propto \exp\left(-\frac{1}{2}\Big(\mathbf{e}_t - \mathbf{w}^*(\gamma^*)\Big)'\boldsymbol{\Omega}^{-1}\Big(\mathbf{e}_t - \mathbf{w}^*(\gamma^*)\Big)\right),$$

in which $\mathbf{w}^*(\gamma^*) \equiv -(\gamma^*-1)\boldsymbol{\Omega}\mathbf{z}^*(\gamma^*)$.

Equations (45)–(46) directly follow.