Minsky's financial instability hypothesis

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Abstract

My talk at the Department of Economics, University of Copenhagen, 9 April 2010, will be based on my own work - mainly from the 1990s - and that of a former student, Soon Ryoo.

This handout contains

– a short paper on "The Financial Instability Hypothesis" by Hyman Minsky for the Handbook of Radical Political Economy

– my own paper entitled "On the modelling of systemic financial fragility"

– Soon Ryoo's article on "Long waves and short cycles in a model of endogenous financial fragility" which has been accepted for publication in JEBO.

The Financial Instability Hypothesis

by

Hyman P. Minsky*

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*The Jerome Levy Economics Institute of Bard College

Prepared for Handbook of Radical Political Economy, edited by Philip Arestis and Malcolm Sawyer, Edward Elgar: Aldershot, 1993.

The financial instability hypothesis has both empirical and theoretical aspects. The readily observed empirical aspect is that, from time to time, capitalist economies exhibit inflations and debt deflations which seem to have the potential to spin out of control. In such processes the economic system's reactions to a movement of the economy amplify the movement--inflation feeds upon inflation and debt-deflation feeds upon debt-deflation. Government interventions aimed to contain the deterioration seem to have been inept in some of the historical crises. These historical episodes are evidence supporting the view that the economy does not always conform to the classic precepts of Smith and Walras: they implied that the economy can best be understood by assuming that it is constantly an equilibrium seeking and sustaining system.

The classic description of a debt deflation was offered by Irving Fisher (1933) and that of a self-sustaining disequilibrating processes by Charles Kindleberger (1978). Martin Wolfson (1986) not only presents a compilation of data on the emergence of financial relations conducive to financial instability, but also examines various financial crisis theories of business cycles.

As economic theory, the financial instability hypothesis is an interpretation of the substance of Keynes's "General Theory". This interpretation places the General Theory in history. As the General Theory was written in the early 1930s, the great financial and real contraction of the United States and the other capitalist economies of that time was a part of the evidence the theory aimed to explain. The financial instability hypothesis also draws upon the credit view of money and finance by Joseph Schumpeter (1934, Ch.3) Key works for the financial instability hypothesis in the narrow sense are, of course, Hyman P. Minsky (1975, 1986).

The theoretical argument of the financial instability hypothesis starts from the characterization of the economy as a capitalist economy with expensive capital assets and a complex, sophisticated financial system. The economic problem is identified following Keynes as the "capital development of the economy," rather than the Knightian "allocation of given resources among alternative employments." The focus is on an accumulating capitalist economy that moves through real calendar time.

The capital development of a capitalist economy is accompanied by exchanges of present money for future money. The present money pays for resources that go into the production of investment output, whereas the future money is the "profits" which will accrue to the capital asset owning firms (as the capital assets are used in production). As a result of the process by which investment is financed, the control over items in the capital stock by producing units is financed by liabilities--these are commitments to pay money at dates specified or as conditions arise. For each economic unit, the liabilities on its balance sheet determine a time series of prior payment commitments, even as the assets generate a time series of conjectured cash receipts.

This structure was well stated by Keynes (1972) :

> There is a multitude of real assets in the world which constitutes our capital wealth - buildings, stocks of commodities, goods in the course of manufacture and of transport, and so forth. The nominal owners of these assets, however, have not infrequently borrowed money (Keynes' emphasis) in order to become possessed of them. To a corresponding extent the actual owners of wealth have claims, not on real assets, but on money. A considerable part of this financing takes place through the banking system, which interposes its guarantee between its depositors who lend it money, and its borrowing customers to whom it loans money wherewith to finance the purchase of real assets. The interposition of this veil of money between the real asset and the wealth owner is an especially marked characteristic of the modern world."(p.151)

This Keynes "veil of money" is different from the Quantity Theory of money "veil of money." The Quantity Theory "veil of money" has the trading exchanges in commodity markets be of goods for money and money for goods: therefore, the exchanges are really of goods for goods. The Keynes veil implies that money is connected with financing through time. A part of the financing of the economy can be structured as dated payment commitments in which banks are the central player. The money flows are first from depositors to banks and from banks to firms: then, at some later dates, from firms to banks and from banks to their depositors. Initially, the exchanges are for the financing of investment, and subsequently, the exchanges fulfill the prior commitments which are stated in the financing contract.

In a Keynes "veil of money" world, the flow of money to firms is a response to expectations of future profits, and the flow of money from firms is financed by profits that are realized. In the Keynes set up, the key economic exchanges take place as a result of negotiations between generic bankers and generic businessmen. The documents "on the table" in such negotiations detail the costs and profit expectations of the businessmen: businessmen interpret the numbers and the expectations as enthusiasts, bankers as skeptics.

Thus, in a capitalist economy the past, the present, and the future are linked not only by capital assets and labor force characteristics but also by financial relations. The key financial relationships link the creation and the ownership of capital assets to the structure of financial relations and changes in this structure. Institutional complexity may result in several layers of intermediation between the ultimate owners of the communities' wealth and the units that control and operate the communities' wealth.

Expectations of business profits determine both the flow of financing contracts to business and the market price of existing financing contracts. Profit realizations determine whether the commitments in financial contracts are fulfilled--whether financial assets perform as the pro formas indicated by the negotiations.

In the modern world, analyses of financial relations and their implications for system behavior cannot be restricted to the liability structure of businesses and the cash flows they entail. Households (by the way of their ability to borrow on credit cards for big ticket consumer goods such as automobiles, house purchases, and to carry financial assets), governments (with their large floating and funded debts), and international units (as a result of the internationalization of finance) have liability structures which the current performance of the economy either validates or invalidates.

An increasing complexity of the financial structure, in connection with a greater involvement of governments as refinancing agents for financial institutions as well as ordinary business firms (both of which are marked characteristics of the modern world), may make the system behave differently than in earlier eras. In particular, the much greater participation of national governments in assuring that finance does not degenerate as in the 1929-1933 period means that the down side vulnerability of aggregate profit flows has been much diminished. However, the same interventions may well induce a greater degree of upside (i.e. inflationary) bias to the economy.

In spite of the greater complexity of financial relations, the key determinant of system behavior remains the level of profits. The financial instability hypothesis incorporates the Kalecki (1965)-Levy (1983) view of profits, in which the structure of aggregate demand determines profits. In the skeletal model, with highly simplified consumption behavior by receivers of profit incomes and wages, in each period aggregate profits equal aggregate investment. In a more complex (though still highly abstract) structure, aggregate profits equal aggregate investment plus the government deficit. Expectations of profits depend upon investment in the future, and realized profits are determined by investment: thus, whether or not liabilities are validated depends upon investment. Investment takes place now because businessmen and their bankers expect investment to take place in the future.

The financial instability hypothesis, therefore, is a theory of the impact of debt on system behavior and also incorporates the manner in which debt is validated. In contrast to the orthodox Quantity Theory of money, the financial instability hypothesis takes banking seriously as a profit-seeking activity. Banks seek profits by financing activity and bankers. Like all entrepreneurs in a capitalist economy, bankers are aware that innovation assures profits. Thus, bankers (using the term generically for all intermediaries in finance), whether they be brokers or dealers, are merchants of debt who strive to innovate in the assets they acquire and the liabilities they market. This innovative characteristic of banking and finance invalidates the fundamental presupposition of the orthodox Quantity Theory of money to the effect that there is an unchanging "money" item whose velocity of circulation is sufficiently close to being constant: hence, changes in this money's supply have a linear proportional relation to a well defined price level.

Three distinct income-debt relations for economic units, which are labeled as hedge, speculative, and Ponzi finance, can be identified.

Hedge financing units are those which can fulfill all of their contractual payment obligations by their cash flows: the greater the weight of equity financing in the liability structure, the greater the likelihood that the unit is a hedge financing unit. Speculative finance units are units that can meet their payment commitments on "income account" on their liabilities, even as they cannot repay the principle out of income cash flows. Such units need to "roll over" their liabilities: (e.g. issue new debt to meet commitments on maturing debt). Governments with floating debts, corporations with floating issues of commercial paper, and banks are typically hedge units.

For Ponzi units, the cash flows from operations are not sufficient to fulfill either the repayment of principle or the interest due on outstanding debts by their cash flows from operations. Such units can sell assets or borrow. Borrowing to pay interest or selling assets to pay interest (and even dividends) on common stock lowers the equity of a unit, even as it increases liabilities and the prior commitment of future incomes. A unit that Ponzi finances lowers the margin of safety that it offers the holders of its debts.

It can be shown that if hedge financing dominates, then the economy may well be an equilibrium seeking and containing system. In contrast, the greater the weight of speculative and Ponzi finance, the greater the likelihood that the economy is a deviation amplifying system. The first theorem of the financial instability hypothesis is that the economy has financing regimes under which it is stable, and financing regimes in which it is unstable. The second theorem of the financial instability hypothesis is that over periods of prolonged prosperity, the economy transits from financial relations that make for a stable system to financial relations that make for an unstable system.

In particular, over a protracted period of good times, capitalist economies tend to move from a financial structure dominated by hedge finance units to a structure in which there is large weight to units engaged in speculative and Ponzi finance. Furthermore, if an economy with a sizeable body of speculative financial units is in an inflationary state, and the authorities attempt to exorcise inflation by monetary constraint, then speculative units will become Ponzi units and the net worth of previously Ponzi units will quickly evaporate. Consequently, units with cash flow shortfalls will be forced to try to make position by selling out position. This is likely to lead to a collapse of asset values.

The financial instability hypothesis is a model of a capitalist economy which does not rely upon exogenous shocks to generate business cycles of varying severity. The hypothesis holds that business cycles of history are compounded out of (i) the internal dynamics of capitalist economies, and (ii) the system of interventions and regulations that are designed to keep the economy operating within reasonable bounds.

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2. On the modelling of systemic financial fragility*

Peter Skott

The dramatic transformation of the world's financial systems in the post-war period and the signs of increasing financial fragility have led to a renewed interest in the interaction between real and financial factors. Two issues, in particular, have attracted attention. It is not clear, first, that increasing financial 'sophistication' brings any real benefits. On the contrary, it has been argued that the relatively traditional financial systems in Japan and continental Europe may have helped these economies to avoid the 'short-termism' which allegedly plagues the UK and the US. There are fears, secondly, that whatever other benefits may flow from financial innovation, it carries with it serious dangers of destabilization. The depression in the 1930s and the speculative boom that preceded it are seen as examples of what may happen, and the question, in Minsky's words, is 'Can "It" Happen Again?' (Minsky, 1963, 1982).

The purpose of this paper is limited. It does not address short-termism and related issues [1] and it only considers certain aspects of the stability question. A convincing analysis of the effects of financial change on the stability of the economy would require detailed institutional and historical studies which are far beyond any single paper. But the analysis also depends on an understanding of the main linkages between financial and real variables. These linkages are complex, and simplified theoretical models may be needed to articulate the links and to examine the logical coherence of existing theories of financial crisis.

Financial aspects have been virtually absent from standard theories of cyclical fluctuations. Keynesian multiplier–accelerator models are cast in real terms, monetarist and new monetarist theories focus on unanticipated changes in simple monetary aggregates (whose significance is called into question by financial innovation and the very existence of a sophisticated financial system), and 'real business cycle' theories – like general equilibrium models – find no role at all for monetary and financial issues. The interaction between real and

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financial factors – and the central importance of this interaction in the generation of economic fluctuations – has been stressed, however, by a number of writers including Marx and Schumpeter.[2] More recently, this interaction has been a constant theme in the writings of Hyman Minsky.[3]

Minsky's theory of financial instability is described briefly in Section 2. Section 3 offers a formalization of some aspects of the theory, and Sections 4 and 5 integrate financial and real factors, Section 4 using a discrete-time specification with linear saving and investment functions and Section 5 a continuous-time specification with non-linearities in the investment function. The final section summarizes the conclusions and speculates on the usefulness of formal modelling in this area.

SECTION 2: MINSKY ON SYSTEMIC FINANCIAL FRAGILITY

Minsky argues that financial fragility develops endogenously during periods of expansion. Financial fragility is systemic in that 'the development of a fragile financial structure results from the normal functioning of our economy' (Minsky, 1977, p.140). The argument is that

> [a]cceptable financing techniques are not technologically constrained; they depend upon the subjective preferences and views of bankers and businessmen about prospects. With the financial structure that ruled in the 1950s, it was correct for businessmen and bankers to increase short-term indebtedness. However, success breeds a disregard of the possibility of failure; the absence of serious financial difficulties over a substantial period leads to the development of a euphoric economy in which increasing short-term financing of long positions becomes a normal way of life. As a previous financial crisis recedes in time, it is quite natural for central bankers, government officials, businessmen, and even economists to believe that a new era has arrived.
> (Minsky, 1986, p.213)

This line of reasoning leads to the following two 'fundamental propositions of the financial instability hypothesis':

> 1. Capitalist market mechanisms cannot lead to a sustained, stable-price, full-employment equilibrium.
> 2. Serious business cycles are due to financial attributes that are essential to capitalism.
> (Minsky, 1986, p.173)

Minsky is careful to point out that 'even though business cycles and financial crises are unchanging attributes of capitalism, the actual path an economy traverses depends upon institutions, usages and policies' (Minsky, 1986, p.174). It should be noted, in particular, that while financial innovation, involving

changes in both financial practices and institutions, may be part of the process towards increasing fragility, institutional constraints and policy intervention are also essential to transform the explosive movements into bounded fluctuations. The resolution of financial crises and the construction of the foundations for renewed prosperity do not happen automatically, and since each crisis has its own peculiarities there are no proven remedies to be pulled down from the shelf. The length and depth of the crises are thus variable, and the financial instability hypothesis does not predict the emergence of regular and time-invariant cycles. But it is suggested that real and financial factors combine to produce persistent fluctuations, and I shall use the term 'cycle' as a shorthand for these – possibly quite irregular – fluctuations.[4]

Minsky's theories would seem to offer a promising perspective on post-war financial developments. The cautious and conservative behaviour of financial institutions in the early post-war period followed the débâcles of the inter-war period but, as the memory of past crises faded, these cautious patterns of behaviour came under increasing pressure. No doubt, the post-war changes in institutions as well as in the attitudes and practices of private and public decision makers can be explained away scholastically to fit neoclassical presuppositions about rationality, but Minsky's historical and institutionalist perspective seems much more fruitful.[5]

Accepting Minsky's basic approach, however, still leaves open a number of questions. The mere existence of endogenous change in the financial sphere and of feedback effects from financial to real variables does not necessarily imply a cyclical tendency,[6] and Minsky has provided no formal demonstration of how real and financial interactions lead to financial instability.[7] Recently, however, there has been a number of attempts at formalizing his ideas, the most influential being Taylor and O'Connell (1985).[8] Their model, as any simple formalization of a complex argument, necessarily has weaknesses as well as strengths,[9] and in this paper I shall follow the same approach as in Skott (1994) with respect to the modelling of endogenous changes in financial practices. There are some changes of terminology, but the main differences compared with this earlier paper concern the specification of the real side of the economy and the interaction between real and financial variables. In addition, the present paper explores both discrete- and continuous-time versions of the interaction.

SECTION 3: A FORMALIZATION OF INDUCED FINANCIAL CHANGE

According to the financial instability hypothesis there is a tendency during periods of expansion and financial tranquillity for all decision makers to adopt practices which are increasingly risky and which intensify the underlying fragility of the system. According to this hypothesis *changes* in financial behaviour, in 'fragility', are thus related to the current degree of 'tranquillity' in financial markets, and the essence of Minsky's cyclical argument can, I suggest, be represented in terms of two scalar indicators: one variable, F, describing the degree of financial fragility and a second, T, capturing the degree of financial tranquillity.[10]

The term fragility refers to the state of the financial system. The system is fragile if small disturbances – an unforeseen drop in income, for instance, or a rise in interest rates – make it difficult or impossible for a significant proportion of the financial units to meet their contractual obligations. An increase in the share of contractual commitments in expected cash-flow would therefore indicate a rise in fragility: it would make the financial unit more vulnerable to shocks. Since the failure of one unit will have repercussions on the financial position of its creditors, the fragility of the system depends on the complete network of financial commitments between firms, households, financial institutions and the public sector. This network in turn reflects the behaviour of private agents as well as the influence of economic policy and regulation. High leverage and adventurous Ponzi schemes may represent typical manifestations of a high degree of fragility, but the relaxation, for instance, of the constraints on the activities of the US S & Ls in the early 1980s also led to an increase in fragility (which subsequently reduced the tranquillity of the system).

The existence of a fragile financial system must be distinguished from the actual appearance of crises: a fragile system need not be characterized by a lack of 'tranquillity'. Financial commitments have been made on the basis of expectations that may or may not prove accurate, and tranquillity – the ability to meet these commitments – therefore depends on *realized* cash-flow as well as on the amount of contractual payments (including the possibility of rolling over the existing debt or obtaining new loans). Tranquillity reflects the outcome of this interaction between financial and real factors. It is at its lowest during a full-scale financial crisis involving the collapse of key financial institutions, but variations in the degree of tranquillity may also be observed during more normal times; default and bankruptcy rates, for instance, are among the obvious (inverse) indicators of tranquillity.

The focus on scalar indicators of fragility and tranquillity clearly implies a gross simplification. Fragility and tranquillity are multi-dimensional concepts, and financial innovation and institutional change imply that the precise definitions of these dimensions must depend on the historical and institutional context. The significance of a given debt–equity ratio, for instance, depends on the institutional environment (what are the relations between firms and their banks? who are the shareholders? etc.), and 'the rate of interest' may be of little importance if credit markets are characterized by (variable degrees of) quantity rationing.

This historical contingency implies that it may be misleading to focus an analysis of persistent tendencies on seemingly well-defined elements of the financial structure (e.g. 'the interest rate'). The focus on a single well-defined element of the financial system – and the exclusion of other aspects of the system – would need to be justified, and the constant evolution of the financial system makes it difficult to provide any such justification. Thus, it is preferable to cast the general analysis in terms of hybrid variables F and T.[11] These hybrid indicators can be given operational definitions for use in empirical work, but the definitions will be historically and institutionally contingent. Thus, Wolfson (1990) has constructed indicators of movements in the financial tranquillity and fragility of the US economy between 1946 and 1987 and used these hybrid indicators in econometric regressions which link tranquillity to various measures of fragility.[12] As a particularly simple empirical definition, it may be helpful to think of F as the ratio of interest payments to *normal* income and T as the ratio of interest payments to *actual* income.[13] These simplified definitions, however, should not be confused with the general concepts of fragility and tranquillity.

Returning to the theoretical argument, fragility and tranquillity evolve endogenously and, in accordance with Minsky's behavioural assumptions, it is assumed that changes in fragility depend on the degree of tranquillity: in the absence of financial difficulties agents adopt schemes of increasing optimism. By suitable choice of units this relation takes a simple linear form

$$\dot{F} = T \tag{2.1}$$

where the dot denotes the time derivative.

Financial difficulties develop as the result of an incongruence between the optimism which motivated financial arrangements and the actual outcome. If the source of potential instability is located primarily in the business sector – and the exposure of financial institutions to bankruptcies and defaults in this sector – then T should depend on financial fragility (inversely) and on realized profit rates (positively), i.e.[14]

$$T = T(F,r)\ ;\ T_F < 0,\ T_r > 0 \tag{2.2}$$

where r denotes the profit rate.[15] The real sector influences the financial movements through the variable r in Equation (2.2), and this dependence of T on r implies that even fluctuations whose genesis lies outside the financial sphere will have financial ramifications. These, in turn, may feed back on the real side, adding a financial element to the cycles: the profit rate, as we shall see, is determined by the interaction of real and financial variables.

Adopting a standard Kaleckian approach, I assume a constant mark-up. With a given fixed-coefficient technology the rate of profit then becomes proportional to the rate of utilization of capital

$$r = \pi\, u\, \sigma^* = \pi\, \sigma \tag{2.3}$$

where π, u, σ and σ^* denote the (constant) profit share, the utilization rate, the actual output capital ratio and the maximum, technically determined output capital ratio, respectively.

A classical distinction between the saving propensities out of wage and profit income would add nothing to the analysis since, by assumption, the profit share remains constant in this model. The following simple specification of the saving function is therefore adopted,

$$S/K = s\, \sigma \tag{2.4}$$

where s, representing the (marginal and average) propensity to consume, may be interpreted as a weighted average of the saving propensities out of wages and profits.

Most investment functions relate the rate of accumulation to utilization and profitability, but with the explicit introduction of financial aspects this standard specification can be augmented to allow for the influence of both fragility and tranquillity.[16] Appendix A sets out a simple model of the joint investment and finance decision. The model implies that both the utilization rate and financial tranquillity have a positive effect on accumulation. During 'normal times' an increase in fragility – in the laxity of financial practices – will also stimulate accumulation but this positive relation may be reversed in periods of crisis: low utilization rates and/or a low degree of tranquillity may imply that accumulation responds negatively to a rise in fragility. The investment function thus takes the following form

$$I/K = g(\sigma, F, T)\ ;\ g_\sigma > 0, g_F \gtrless 0,\ g_T > 0. \tag{2.5}$$

Ignoring depreciation and assuming that investment plans are always realized, Equation (2.5) also describes the actual rate of accumulation.

SECTION 4: A DISCRETE-TIME MODEL WITH FULL OUTPUT ADJUSTMENT

Most Keynesian models focus on short-run equilibria in which output has adjusted to equalize desired saving and investment. Unless one is willing to assume that output reacts infinitely fast so that disequilibria between ex ante saving and investment will never arise, it seems most appropriate to analyse the long-term dynamics of such models in terms of discrete-time sequences of short-run equilibria, the unit period being long enough to allow the economy to attain the short-run equilibrium.[17]

Adopting a linear version of the investment function[18]

$$I/K = a\sigma + bF + cT + d \tag{2.5a}$$

and assuming that T is additively separable and linear in σ,

$$T = A\sigma + f(F), \tag{2.2a}$$

the equilibrium value of σ is given by[19]

$$\sigma = (bF + cf(F) + d)/(s - a - cA). \tag{2.6}$$

Equation (2.6) relates the short-run equilibrium value of the output capital ratio to the predetermined value of financial fragility, and the evolution of the system can be found by combining (2.6) with a discrete-time version of Equation (2.1),

$$F_{t+1} - F_t = T_t \tag{2.1a}$$

Using (2.2a) and (2.6), Equation (2.1a) implies that

$$\begin{aligned} F_{t+1} &= F_t + A(bF_t + cf(F_t) + d)/(s - a - cA) + f(F_t) \\ &= \alpha_1 F_t + \beta_1 f(F_t) + \gamma_1 \end{aligned} \tag{2.7}$$

where

$$\begin{aligned} \alpha_1 &= 1 + Ab/(s - a - cA) > 0 \\ \beta_1 &= 1 + Ac/(s - a - cA) > 0 \\ \gamma_1 &= Ad/(s - a - cA) \end{aligned}$$

The properties of the dynamic system described by (2.7) depend on the function f. A linear specification of f, for instance, implies a linear difference equation with limited and well-known dynamic properties. Linearity, however, is a special case, and even simple non-linear specifications may generate a range of qualitatively different outcomes.

Although the nature of the two financial variables makes it difficult to have a clear intuition about the functional form of the relation between T and F,[20] it may be possible to provide some justification for a concave specification of f. Thus, it can be argued – and indeed it seems to be implicit in Minsky's argument – that tranquillity is insensitive to variations in financial practices as long as a reasonable margin of safety is maintained. Relaxations of financial constraints that have a given impact on accumulation may therefore be expected to have a larger effect on tranquillity in a fragile than in a robust financial environment. Marginal changes in leverage, for instance, will have little effect on the bankruptcy risk of well-consolidated firms while a high-debt firm whose contractual commitments cover a high proportion of expected earnings can be seriously affected.

A quadratic specification of f represents a particularly simple non-linearity of this kind[21]

$$f(F) = BF - CF^2 + D \; ; \; C > 0 \tag{2.8}$$

It then follows that

$$F_{t+1} = \alpha_2 F_t + \beta_2 F_t^{\,2} + \gamma_2 \tag{2.9}$$

with

$$\alpha_2 = \alpha_1 + \beta_1 B$$
$$\beta_2 = \beta_1 c < 0$$
$$\gamma_2 = \gamma_1 + \beta_1 D$$

This dynamic equation, as is well-known, may generate an extremely rich menu of outcomes, including chaotic movements. More generally, if the f-function in Equation (2.7) is concave and if either

a. Equation (2.7) has two stationary solutions, $F^* < F^{**}$, and

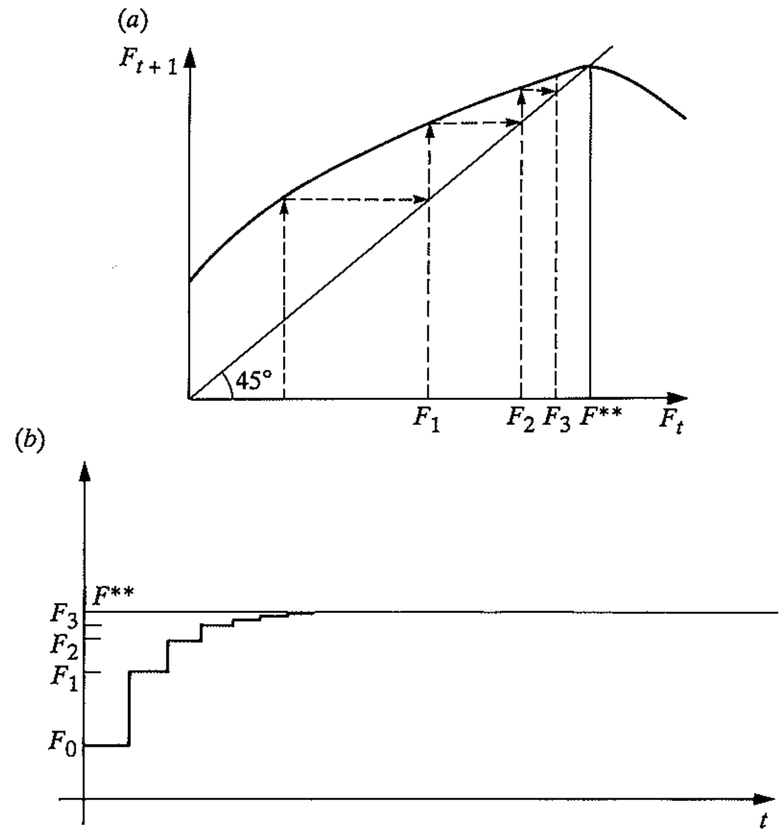
   i. at $F_t = F^{**}$ we have $dF_{t+1}/dF_t < -1$, and
   ii. $F_{t+1}(F^{max}) > F^*$ where $F^{max} = \max F_{t+1}(F_t)$[22]

or

b. Equation (2.7) has a unique solution F^* and

   i at $F_t = F^*$ we have $dF_{t+1}/dF_t < -1$, and
   ii. the function linking F_{t+1} to F_t has a global maximum

Figure 2.1 Stable equilibrium

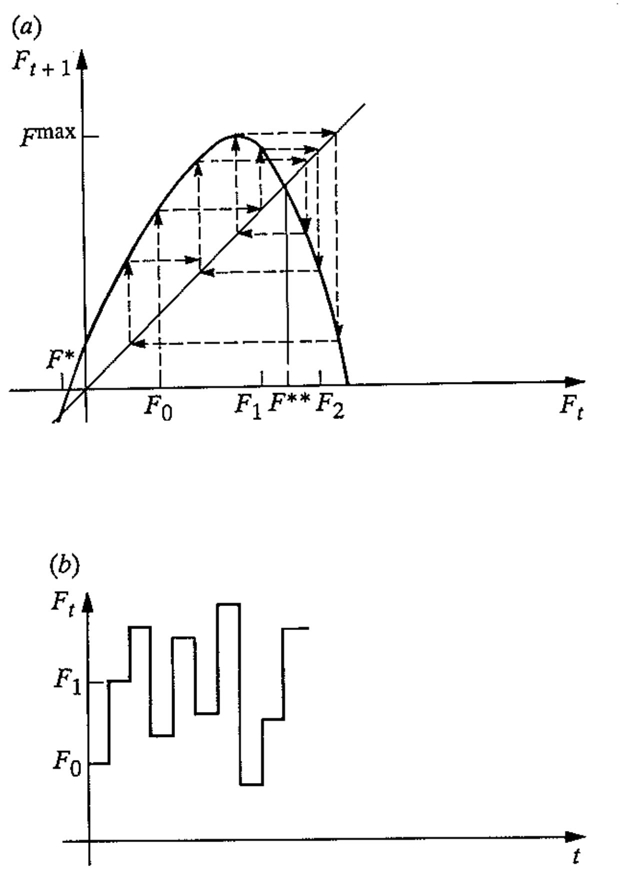


Note: $\frac{dF_{t+1}}{dF_t} > -1$ at $F_t = F^{**}$

then either chaotic movements or asymptotically regular cycles will result.[23] Figures 2.1–2.3 illustrate some of the possibilities.

The model has augmented a standard specification of the investment and saving functions by allowing financial fragility and tranquillity to influence the desired rate of accumulation. The resulting short-run equilibrium condition for the product market determines the rate of utilization as a function of the financial variables F and T. It was assumed that tranquillity, in turn, is determined by utilization and fragility, and – using the equilibrium condition – this assumption implies a reduced-form equation linking the contemporaneous values of T and F.

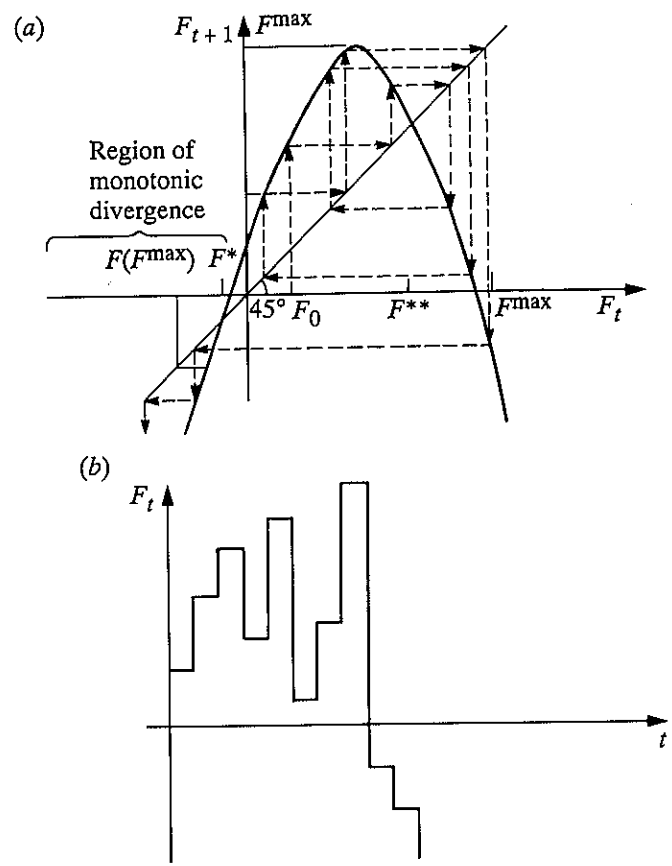
Figure 2.2 *Perpetual fluctuation*



Note: $\frac{dF_{t+1}}{dF_t} < -1$ at $F_t = F^{**}$ (local instability) and $F(F^{max}) > F^*$ (boundless).

Minsky's assumption that *changes* in financial practices – in fragility – depend on current levels of tranquillity transforms the contemporaneous equilibrium relation between F and T into a first order difference equation. Non-linearities in this equation imply that, despite its simplicity, the model may be capable of producing a wide range of dynamic outcomes. The chaotic possibilities, in particular, with perpetual but irregular fluctuations, would seem to substantiate, or at least conform with, Minsky's vision. Although the assump-

Figure 2.3 *Cumulative downward divergence*



Note: Local instability and $F(F^{max}) < F^*$ leading to cumulative downward divergence

tions behind these outcomes may not look implausible, it should be noted that non-linearities in the financial variables have played a central role in the model. The nature of the financial variables, however, makes it difficult to provide a compelling justification of any particular functional form, and one may therefore feel unhappy about introducing critical non-linearities in terms of these variables.

Additional questions can be raised with respect to the use of a discrete-time framework. The possibility of local instability through over-shooting is essential to the generation of chaotic patterns. A direct re-specification of Equation (2.9) in continuous time would imply convergence to a steady-state equilibrium, and the rigidities of the discrete-time framework thus carry a heavy weight. A separate but related question concerns the Keynesian short-run equilibrium assumption. Especially if a continuous-time framework is adopted, it would seem unreasonable to exclude deviations from Keynesian short-run equilibrium, and the model thus requires further modification: a re-specification in continuous time but with explicit allowance for short-run disequilibrium adjustment may be desirable.

SECTION 5: A KALDOR-MINSKY MODEL IN CONTINUOUS TIME

Kaldor's classic model of the trade cycle, Kaldor (1940), provides a useful starting point for a re-specification along these lines. Kaldor introduced non-linearities in the saving and investment functions and assumed that both the level of output and the size of the capital stock influence saving and investment. The result, he argued, could be perpetual trade cycle fluctuations in output and employment. The model was formalized by Chang and Smyth (1971) but as shown in Skott (1989, chapter 3) the cyclical result depends on restrictive and implausible assumptions: if, as seems reasonable, the saving and investment functions are homogeneous of degree one in output and capital, then the economy converges to a steady growth path.

Algebraically, a general version of the Kaldor model can be described by the following equations :

$$S = S(Y,K) \tag{2.10}$$
$$I = I(Y,K) \tag{2.11}$$
$$\dot{Y} = \lambda\,(I - S) \tag{2.12}$$
$$\dot{K} = I \tag{2.13}$$

Assuming linear homogeneity of (2.10) and (2.11), this system of equations can be reduced to a one-dimensional differential equation

$$\begin{aligned}\hat{\sigma} = \hat{Y} - \hat{K} &= \lambda/Y\,(I - S) - I/K = \lambda\,[I(1,1/\sigma) - S(1,1/\sigma)] - I(\sigma,1) \\ &= \lambda/\sigma\,(g(\sigma) - h(\sigma)) - g(\sigma)\end{aligned} \tag{2.14}$$

where $\sigma = Y/K$ is the output capital ratio and g and h are now defined by $g(\sigma) = I(\sigma,1) = I/K$ and $h(\sigma) = S(\sigma,1) = S/K$.

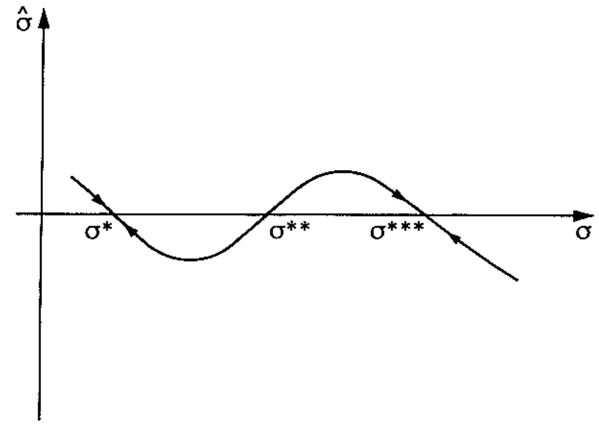
Non-linearities may imply the existence of multiple equilibria and the long-term outcome will then depend on the initial value of σ. But Equation (2.14) implies that asymptotically σ will converge to some equilibrium value σ^* as indicated in Figure 2.4.[24] This steady growth implication no longer holds when

the model is augmented by the introduction of financial variables in the investment function. Retaining the assumption of linear homogeneity in Y and K, this extension of Equation (2.11) yields

$$I/K = g(\sigma, F, T)\,;\ g_\sigma > 0,\ g_F \lesseqgtr 0,\ gT > 0 \tag{2.11a}$$

which is, in fact, identical to Equation (2.5).

Figure 2.4 Typical phase diagram for the Kaldor model with fast adjustment speed. Asymptotically σ approaches σ^* (σ^{***}) if $\sigma_0 < \sigma^{**}$ ($\sigma_0 > \sigma^{**}$)



Equations (2.10), (2.11a) and (2.12)–(2.13) can now be combined with the financial Equations (2.1)–(2.3), and the resulting, reduced-form, dynamic system can be written

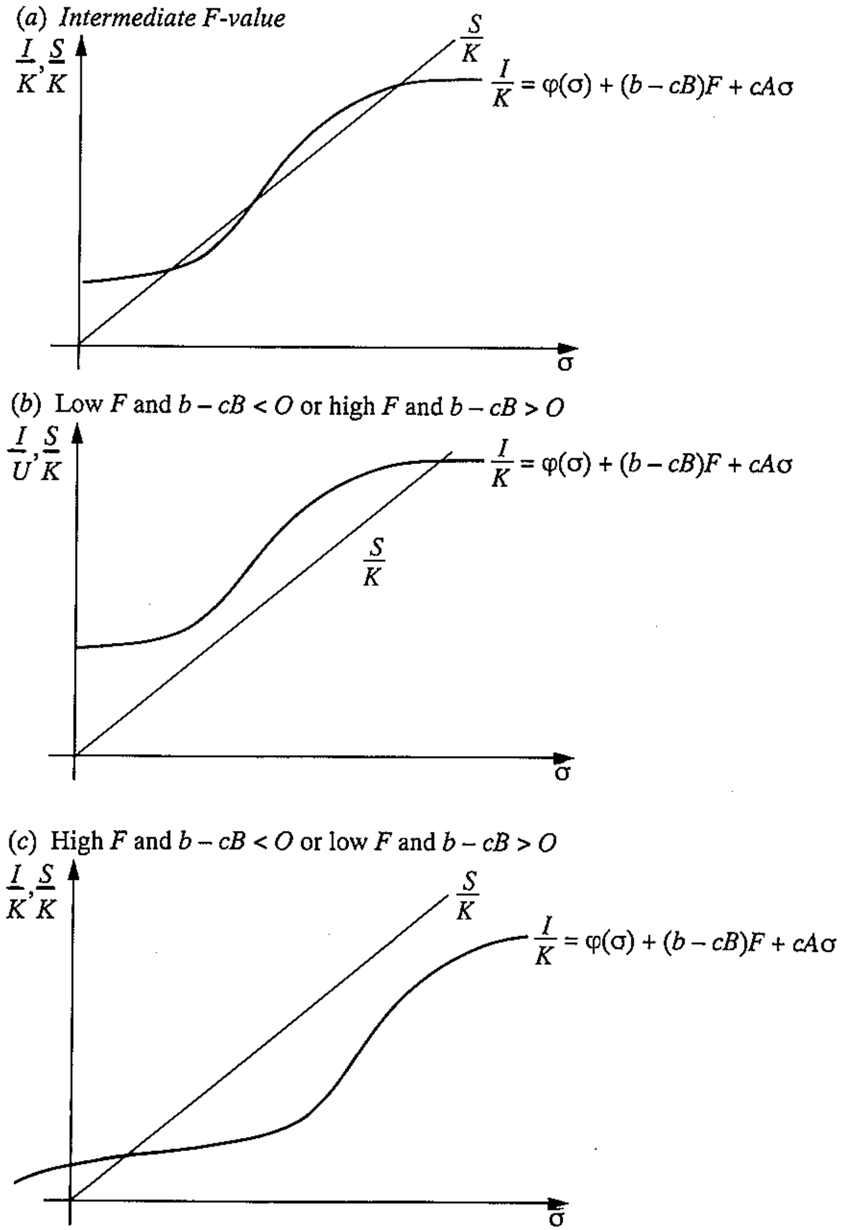
$$\dot{F} = T(F,\pi\sigma) \tag{2.15}$$
$$\hat{\sigma} = \lambda/\sigma\,[g(\sigma,F,T(F,\pi\sigma)) - h(\sigma)] - g(\sigma,F,T(F,\pi\sigma)) \tag{2.16}$$

The properties of this system depend on the functional forms and, in accordance with the earlier discussion of non-linearities, it is assumed that the financial variables F and T enter the investment function in a linear way and that T is linear in both F and σ.

$$T = A\,\sigma - B\,F \tag{2.2b}$$

$$I/K = \phi(\sigma) + b\,F + c\,T \tag{2.5b}$$

Figure 2.5 Saving and investment functions for the Kaldor–Minsky model



Following Kaldor, however, it is assumed that investment is an *s*-shaped function of σ for any given values of *F* and *T*, i.e. that ϕ is *s*-shaped. Non-linearities in the saving function of the kind envisaged by Kaldor would not change the qualitative analysis, and the linear specification in Equation (2.4) is adopted. Figure 2.5 illustrates these assumptions.

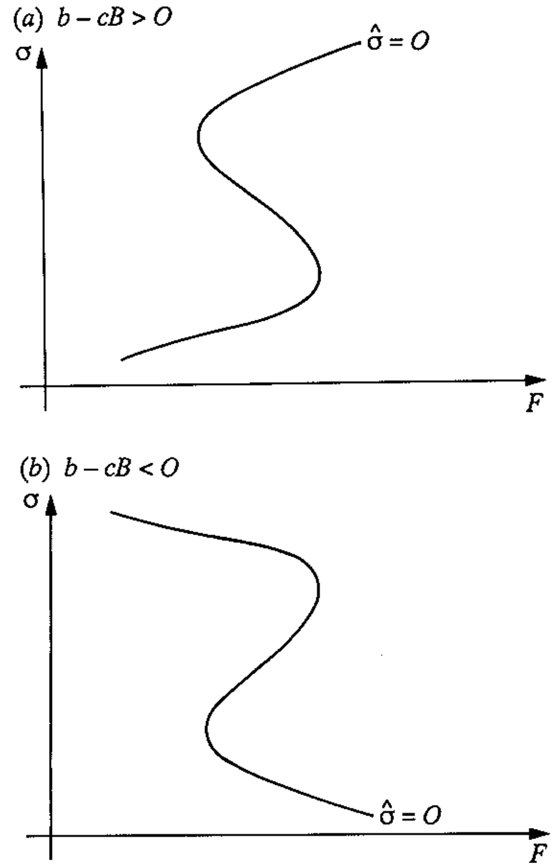
With these special assumptions, Equations (2.1), (2.2b), (2.3),(2.4), (2.5b) and (2.12)–(2.13) imply that Equations (2.15)–(2.16) take the form

$$\dot{F} = T = A\,\sigma - B\,F \qquad (2.15a)$$

$$\hat{\sigma} = \lambda/\sigma\,[\phi(\sigma) + b\,F + c\,T - s\sigma] - \phi(\sigma) - b\,F - c\,T \qquad (2.16a)$$

$$= \lambda/\sigma\,[\phi(\sigma) + (b - cB)F + (cA - s)\sigma] - \phi(\sigma) - (b - cB)F - cA\,\sigma$$

Figure 2.6 $\hat{\sigma} = 0$ curves for the Kaldor–Minsky model



For 'large' values[25] of the adjustment parameter λ the assumed s-shape of $\phi(.)$ and the existence – for intermediate values of F – of three equilibria with $I = S$ imply that the $\hat{\sigma} = 0$ locus has the shapes indicated in Figures 2.6a and 2.6b for $b - cB > 0$ and $b - cB < 0$ respectively.

Superimposing the positively sloped $\dot{F} = 0$ locus on 2.6a, it is readily seen that when $b - cB > 0$ the economy either converges to an equilibrium (F^*, σ^*) or exhibits unbounded divergence. Multiple equilibria may exist and the asymptotic behaviour may thus depend on initial conditions, but there are no cyclical tendencies in this case. Figure 2.7a depicts the possible scenarios.

When $b - cB < 0$, however, persistent cyclical movements become possible. In this case there is either one or three equilibria as shown in Figure 2.7b. The possibility of limit cycles arises when there is a unique equilibrium and when this equilibrium is located on the upward sloping part of the $\hat{\sigma} = 0$ locus as in Figure 2.7biii. A high adjustment speed for output makes this equilibrium locally asymptotically unstable, and relatively weak conditions will ensure the boundedness of all trajectories. The Poincaré-Bendixson Theorem then implies the existence of a limit cycle.

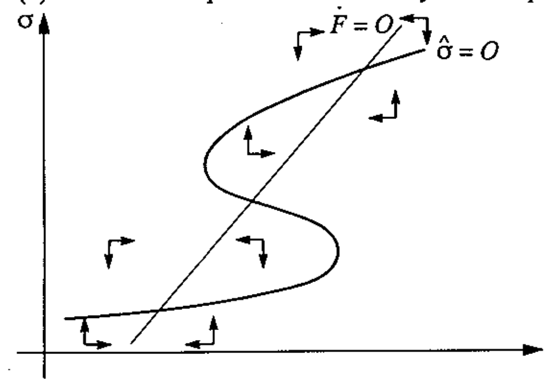
An extreme case with $\lambda \to \infty$ is illustrated in Figure 2.8. The output capital ratio becomes a 'fast' variable and starting from an arbitrary point, A, the economy jumps (almost) vertically onto the $\hat{\sigma} = 0$ locus. The dynamics of the 'slow' variable, F, takes the economy up along the $\hat{\sigma} = 0$ locus until, at point C, a 'catastrophe' occurs and the economy jumps to point D. Fragility is now increasing and the system moves down along the upper arm of the $\hat{\sigma} = 0$ locus. At E there is a new 'catastrophe' when the economy tumbles over the cliff hitting B on the low arm of the locus. Perpetual cycles along $BCDE$ now ensue.

The economic intuition behind the results is straightforward. If fragility had been constant there would have been convergence to steady growth as in the simple Kaldor model: depending on the value of F and the initial rate of utilization, the economy would converge to a point on one of the downwards sloping segments of the $\hat{\sigma} = 0$ locus in Figure 2.8. Along these segments, however, fragility is non-constant. At an equilibrium north-west of E there is a high degree of tranquillity (profitability is good relative to the level of financial commitments), and agents are lured into more risky financial practices: there is a movement down towards the critical point E. Analogously, equilibria south-east of C are characterized by financial trouble (profitability is low relative to commitments) and agents are led to adopt more cautious practices, that is move towards point C.

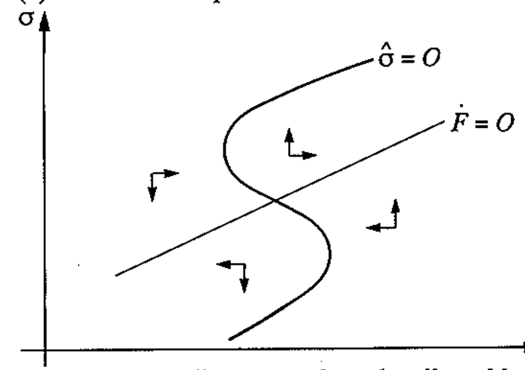
The augmented Kaldor–Minsky model may thus generate endogenous cycles, and these cycles are caused by the interaction between the investment behaviour and the financial structure. The non-linearities are located exclusively in the real side of the model and relate to observable variables (the accumulation rate and the rate of capital utilization), but the simple non-linear Kaldor model

Figure 2.7 Dynamics in the Kaldor–Minsky model

(*a*) i *One saddle point and two locally stable equilibria*



(*a*) ii *One saddle point*



(*a*) iii *One saddle point and one locally stable equilibrium*

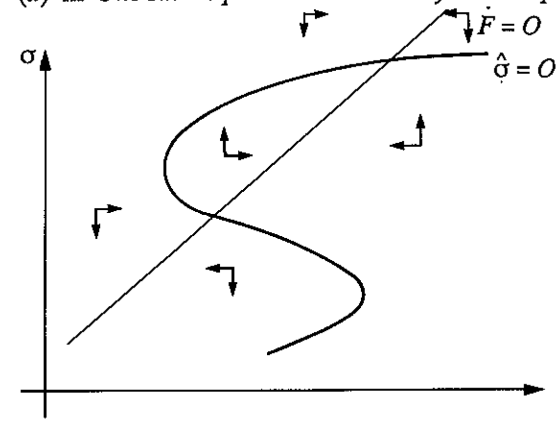
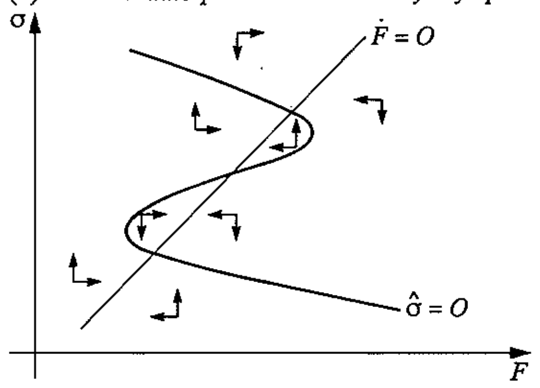
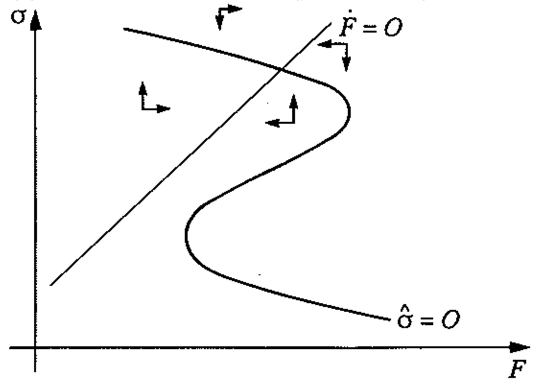


Figure 2.7 Dynamics in the Kaldor–Minsky model continued

(*b*) i *One saddle point and two locally asymptotically stable equilibria*



(*b*) ii *One locally stable equilibrium (globally stable under weak conditions)*



(*b*) iii *One equilibrium locally unstable for high values of λ*

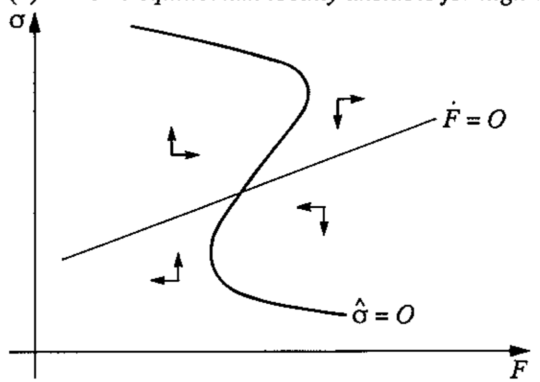
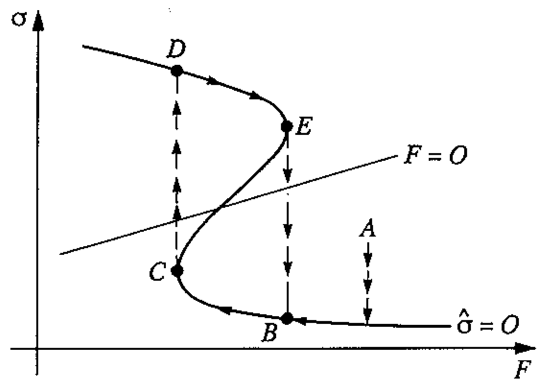


Figure 2.8 'Catastrophes' and perpetual cycles in the Kaldor–Minsky model



generates a one-dimensional differential equation which exhibits convergence to steady growth; the cycles therefore arise out of the interaction with the financial variables. Minsky's hypothesis concerning a dynamic relation between changes in fragility and the level of tranquillity increases the dimension of the reduced form dynamic system, and this increase allows cyclical fluctuations to emerge.

Minsky's argument appears to have been substantiated. But it should be noted that cycles is only one possible outcome of the Kaldor–Minsky model. The conditions for cycles may not seem unreasonable, but they are not really compelling either. Minsky's verbal argument as well as the existence of upper limits on the output capital ratio suggest that increasing fragility, ultimately, cannot be offset by improved utilization: at least from some point, an increase in fragility will affect tranquillity adversely, even when derived effects on investment are taken into account.[26] With a linear specification of the functional relation between T and F this implies that $b - cB$ in Equation (2.16a) must be negative, and the noncyclical cases in Figure 2.7a can be excluded. But it is difficult to see convincing arguments in favour of the cyclical configuration in Figure 2.7biii rather than the non-cyclical possibilities in 2.7bi and 2.7bii.[27]

SECTION 6: CONCLUDING COMMENTS

Minsky's 'financial instability hypothesis' suggests that steady growth is unattainable in a capitalist market economy: the interaction between financial factors and long-term investment makes the system prone to crises and generates an irregular pattern of fluctuations. In this paper I have tried to formalize Minsky's basic argument by adding financial considerations to standard macroeconomic models.

It has been shown that under certain conditions these extended models may indeed yield fluctuations of the kind suggested by Minsky. Financial factors may destabilize an otherwise stable system and produce cycles or, in some cases, a complete breakdown of the economy. In this sense the analysis supports some of Minsky's claims. But the analysis and the simplifications that were needed to make it tractable also raise a number of questions.

The use of hybrid financial variables like fragility and tranquillity has already been discussed, and despite all problems I believe that this formalization does capture Minsky's argument. But should one attempt to formalize a theory that relies on such vague and nebulous terms? Does it make sense to set up precise and rigorous models to describe the evolution of the degree of financial fragility? Many economists undoubtedly would give negative answers to these questions. In my view they are wrong.

There are two aspects to consider. One may reject Minsky's own analysis of the financial instability hypothesis on the grounds that measurement problems make concepts like fragility and tranquillity too imprecise. This argument, however, would be tantamount to a rejection of most, if not all, attempts to ask broad questions about the evolution of the economy. The use of vague and abstract concepts cannot be eschewed if one wants to address such issues. Secondly, one may be willing to engage in this kind of investigation, but question the usefulness of formal modelling. The analysis of financial instability in the present paper demonstrates the falsity of this position: it is not just defensible but in many cases essential to employ techniques of formal modelling.

Without formalization it is virtually impossible to check the logic of Minsky's argument: the implications of the interaction between financial and real factors depend on the specification of the dynamic structure, the functional forms and the parameter values, aspects which just cannot be adequately considered without formalization. The very sensitivity of our results to variations in the assumptions is a powerful argument in favour of, rather than against, formal modelling. It is also, of course, a reminder that formal modelling can never be sufficient. The specification of a model must be guided by historical and institutional information as well as statistical analysis.

Leaving aside the methodological issues surrounding the definition of the financial variables, the models demonstrate that cycles may result from the interaction between real and financial elements. But this result clearly does not imply that the models provide a good explanation of fluctuations in the real economy.

One may question, firstly, the appropriateness of those non-financial models which have been used as starting points for the analysis. It has been assumed that without the introduction of financial elements the real side of the economy would be stable. This assumption appears to be in accordance with Minsky's argument (see Crotty, 1990) but other approaches would dispute the stability of the real side: a Harrodian perspective, for instance, would emphasize destabilizing accelerator effects and Marxists may point to the destabilizing dynamics of distributional conflict. For present purposes, however, it is sufficient to note that the specification of the real side of the economy also shapes the conclusions and that this specification can be questioned.

Secondly, and perhaps more importantly, the models in this paper present a highly selective picture, leaving out many features that contribute significantly to the observed patterns of fluctuation. The absence of spatial elements (open economy issues) and of a public sector represent obvious simplifications but, from a theoretical perspective, the omission of labour market considerations seems more questionable. Even as a first approximation it would seem unwise for an explanation of cyclical fluctuations to exclude the influence of labour market conditions and, in particular, the strengthening of workers vis-à-vis capital at near full employment. This influence is central to Marxian theory (Goodwin, 1967, is the classic formalization), it can be combined with Keynesian questions of effective demand (Skott, 1989, 1989a, 1992) and mainstream economic theory views the natural rate of unemployment (or NAIRU) as the centre around which fluctuations may take place. Other influences apart from the labour market have been emphasized by Bowles et al. (1986) as well as by the French regulation school and neo-Schumpeterian writers.

Ideally these different forces should be analysed together since the interaction between the mechanisms makes it invalid to suppose that the results of partial models can be simply added together. But the integration of different theories and mechanisms within a common framework presents formidable technical difficulties. This paper has therefore focused on a particular mechanism, but the restrictiveness of the implicit exclusion or separability assumptions should be borne in mind in the interpretation of the results. The models in this paper should not be seen as direct attempts at explaining the observed pattern of cyclical fluctuations in existing capitalist economies. Rather, they represent an effort to examine the logic of Minsky's theory of fluctuations as the consequence of systemic financial fragility.

NOTES

1. Recent discussions of short termism and the possible advantages of Japanese or German institutions include Cosh, Hughes and Singh (1990) and Mayer (1988). See Auerbach and Skott (1992) for a discussion of these issues in a broader context of corporate planning.
2. See Crotty (1985) for a discussion of the role of financial factors in Marx's theory of crisis and Wolfson (1986) for a broader survey of financial theories of fluctuations in economic activity.
3. See e.g. Minsky (1964, 1975, 1977, 1982, 1986).

4. An upturn, a resolution of the crisis, is not even assured; a prolonged crisis may lead to fundamental changes in the economic system.
5. See Auerbach (1989, chapter 7), Auerbach and Skott (1992) and Skott (1994) for further discussion of these issues.
6. There also appears to be some ambiguity in Minsky's work with respect to the average periodicity of the cycles. The changes in financial fragility are sometimes described as following a long wave pattern while elsewhere a short-run trade cycle is implied.
7. Early contributions based on accelerator-multiplier models (Minsky, 1957, 1959) do not seem to capture the argument.
8. See also Lavoie (1986/87), Semmler (1987), Semmler and Franke (1991), Delli Gatti and Gallegati (1990) and Sethi (1992). Other recent work on the cyclical interaction of real and financial factors includes Jarsulic (1989), Skott (1989, chapter 7) and Dutt (1992).
9. Taylor and O'Connell base their model on the dynamic interaction between two variables, the interest rate and the 'state of confidence'. The latter variable is defined as the difference between the expected future return on capital and the current profit rate, and it is assumed that the *change* in confidence depends inversely on the current level of the interest rate. The rationale of this assumption is not clear, and it seems strange that expected profitability should evolve without any feedback from realized outcomes. If, for instance, the interest rate were maintained at a constant, low level then profit expectations would be *ever-increasing* and with a constant profit rate, expectations would diverge further and further from actual outcomes.

   The choice of the interest rate as a key variable can also be questioned. Exegetical support for the use of interest rates to describe the financial environment can be found in Minsky's work (e.g. Minsky, 1986, p.214) but movements in interest rates would seem to be a poor indicator of those multi-dimensional changes in financial practices and institutions which Minsky himself has described in great detail.
10. Skott (1994) considers the hybrid financial variables 'laxity' and 'trouble'. Financial laxity corresponds to fragility in the present paper and financial trouble is inversely related to tranquillity. The change of terminology may clarify the links with Minsky's analysis.
11. Formalizations of Minsky's work which focus on orthodox economic variables can in many cases be seen as special cases of this general framework. Taylor and O'Connell (1985), for instance, use 'the interest rate' and the 'state of confidence' as their key financial variables and assume that changes in confidence depend inversely on the interest rate. Their confidence variable corresponds to fragility (fragile financial arrangements are based on 'confident' forecasts) while the rate of interest acts as an (inverse) indicator of tranquillity. But there are radical differences in the way these financial variables are linked to the rest of the model.
12. Wolfson does not use the term tranquillity but his general concept of 'financial instability' is inversely related to our definition of tranquillity.
13. If firms rather than households are the main source of instability then F might therefore be measured as

$$F = iB/(r^*K)$$

   where i, B and K denote the interest rate, the level of debt and the capital stock and where r^* represents the normal profit rate: the rate of profit that will be achieved if demand conditions allow firms to get a normal mark-up on a level of output corresponding to normal capacity utilization.

   An analogous measure of tranquillity – indicating the actual ability to meet contractual obligations – may be given by

$$T = T\,(iB/rK)\ ;\ T' < O$$

   where r is the actual profit rate.
14. Underlying this specification is some distributional or probabilistic assumption. The average profit rate and degree of fragility hide a diversity of individual experience. This dispersion around the mean is not modelled explicitly, and implicitly it is therefore assumed that the average outcome

is a sufficient statistic for the proportion of agents who are unable to fulfil their obligations and hence for the degree of tranquillity.
15. In the present model utilization, employment and household income move together in the short term. Equation (2.2) therefore may also capture the influence of the household sector on financial tranquillity.
16. The financial variables may also influence saving. This complication of the model would not affect the results significantly, and it has therefore been omitted.
17. But see Skott (1989, chapter 4) for a critical discussion of this procedure.
18. This linear specification excludes the possibility of a sign reversal of g_F as σ or T changes. The non-linearity suggested by the example in Appendix A would strengthen the argument (below) for strict concavity of the relation between F_{t+1} and F_t
19. The standard stability assumption in Keynesian short-run models is that $dS/dY > dI/dY$ or, in the present model, $s > a + cA$. It is assumed that this condition is satisfied.
20. Neither 'tranquillity' nor 'fragility' possesses a clear and well-defined unit of measurement, but this does not imply that the functional form of f is undetermined and that it can be chosen arbitrarily: units of measurement have already been chosen through the specification of equation (2.5a). This equation relates the observable value of the accumulation rate to the values of F and T, and the functional form of the equation and the size of the parameters thus implicitly define cardinal measures of both F and T.
21. For F-values below $B/2C$ this quadratic specification violates the sign assumption in Equation (2.2) on the partial derivative of T with respect to F. The quadratic specification therefore presupposes that the relevant range of F-values is above this critical value.
22. Condition (i) combined with the existence of two stationary solutions and the concavity of the f-function ensures the existence of the global maximum, F^{max}
23. See Baumol and Benhabib (1989) for a discussion of chaotic systems with economic applications.
24. The non-linearities suggested by Kaldor imply that $0 < \sigma^* < \infty$ but other specifications may entail the possibility of cumulative divergence, i.e. $\sigma^* = \infty$ or $\sigma^* = 0$.
25. The focus in standard macroeconomics on short-run equilibria implicitly relies on a very high adjustment speed: it is assumed that the adjustment towards equilibrium is so fast that changes in the capital stock can be ignored. This standard assumption implies that λ is likely to be 'large' in the required sense. If λ fails to satisfy this condition, the $\hat{\sigma} = 0$ locus will be monotonic and the unique equilibrium will be either a saddlepoint or a stable node/focus. See Skott (1989, chapter 6) or Skott (1989a) for a more detailed analysis of the implication of the standard approach for the output adjustment speed.
26. This argument would be reinforced by the possible sign reversal of g_F.
27. It should be noted, though, that the asymptotically stable equilibria in Figures 2.7bi and, 2.7bii may show damped fluctuations; these equilibria can be either nodes or focuses.

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APPENDIX A

For simplicity, assume that there are only two sources of finance, retained earnings and bank loans, and that a constant price level makes it unnecessary to distinguish between nominal and real values. Firms' financing constraint is then given by

$$I = \Pi - iB - D + \dot{B} \qquad (2.A1)$$

where I is investment, Π total profits, B the existing debt, $\dot{B}$ the net change in debt (new loan finance), i the interest rate and D the amount of dividends. Dividing through by K, the financing constraint (2.1) can be rewritten

$$g = \pi\sigma - iB/K - \mathrm{d} + B/K\,\hat{B} \qquad (2.A2)$$

where $d = D/K$ is the ratio of dividends to the value of the capital stock.

In addition to this accounting constraint firms may face limits on the amount of bank borrowing they can obtain, and this credit constraint may take the form of quantity rationing rather than variations in interest rates (e.g. Stiglitz and Weiss, 1981). To simplify, it is assumed that the interest rate is kept constant and that banks set their credit ceilings in terms of a maximum ratio of debt to capital, B/K. With constant values of i, π and σ^*, there is a one-to-one correspondence between B/K and $iB/\pi\sigma^*K$, and the ceiling could therefore be expressed, alternatively, as an upper limit on the ratio of interest payments to normal profits, i.e. it represents a limit on fragility as defined in endnote 12.

If F denotes the credit ceiling, the constraint can be written

$$B/K \leq F \qquad (2.A3)$$

but the ceiling does not remain constant over time. Bankers revise their views on prudent behaviour in the light of experience: high default rates lead to a downwards revision while low default rates encourage a relaxation of lending practices. Assuming that default rates depend on the average ratio of interest payments to actual profits, the evolution of F thus depends on (the simple, endnote 12 definition of) tranquillity

$$\dot{F} = T \qquad (2.A4)$$

Firms make their investment and finance decisions subject to the constraints (A1)–(A4), and the analysis is particularly simple if it is assumed that only the rate of accumulation and the ratio of dividends to capital enter the objective function V,

$$V = V(g,d)\,;\ V_g > 0,\ V_d > 0 \qquad (2.A5)$$

[1]

In this case it will always be optimal for firms to increase their bank loans, and the bank-imposed credit ceiling will be binding. This implies that the constraints can be simplified. Using (2.A3) and (2.A4), Equation (2.A2) can be rewritten

$$g = \pi\sigma - iF - d + F(\hat{F} + g) \qquad (2.A6)$$

or

$$(1 - F)g + d = \pi\sigma - iF + T \qquad (2.A7)$$

The maximization of (2.A5) subject to (2.A7) implies that

$$g = g(\sigma, F, T) \qquad (2.A8)$$

and if g and d are 'normal' goods, it is readily seen that

$$g_\sigma > 0,\ g_T > 0 \qquad (2.A9)$$

An increase in F, on the other hand, has substitution effects (favouring a shift from d to g) as well as income effects which will be positive (negative) if $g > i$ ($g < i$). The sign of g_F therefore depends on the initial values of (σ, F, T), and for a given F

$$g_F \begin{cases} > 0 \text{ for 'high' values of } \sigma \text{ and } T \\ < 0 \text{ for 'low' values of } \sigma \text{ and } T \end{cases} \qquad (2.A10)$$

If $V(g,d)$ is Cobb–Douglas, for instance,

$$V = g^\rho d^{1-\rho} \qquad (2.A11)$$

then the optimal solution is

1. A positive derivative V_g may be rationalized by a managerial concern with growth rather than profitability. Alternatively, managers who aim to maximize profits may be constrained by shortages of finance from reaching what they consider the most profitable growth path, thus leaving V_g positive in the relevant range.
A management preference for high dividends may appear paradoxical. It is often assumed that managers prefer retained earnings over dividends, but in so far as the desire for retained earnings is derived from the growth objective this standard assumption is irrelevant for the partial derivative. If shareholders favour high dividends and if managers' job security depends on the goodwill of shareholders then V_d will be positive.

$$g = \rho\,(\pi\sigma - iF + T)/(1 - F) = \rho i\,[(\pi\sigma + T)/i - F]/(1 - F) \qquad (2.A12)$$

and

$$g_F \gtrless 0 \text{ for } \pi\sigma + T \gtrless i \qquad (2.A13)$$

If the movements in fragility are bounded then the average value of $T = \dot{F}$ will be zero, and the long-run viability of capitalism requires that the profit rate exceeds the interest rate (at least on average). One would therefore expect g_F to be positive in 'normal times', but the sign may be reversed in periods of crisis (when σ and/or T are low).

So far it has been assumed that V only depends on g and d, but the objective function is likely to include additional arguments. High utilization rates (a shortage of capital capacity) will strengthen the growth objective, and tranquillity may also exert a direct and positive influence on 'animal spirits'.

These effects, however, would merely reinforce the conclusions in (2.A9) that $g_\sigma > 0$ and $g_T > 0$ and leave the results for g_F unchanged.

A direct influence of indebtedness on managerial 'utility' may also seem reasonable. As long as the bank-imposed constraint remains binding, this extension of the analysis need not affect the conclusions, and it should be noted that a binding loan constraint may be compatible with the existence of unused lines of credit. Firms may need a cushion of liquidity, and this cushion must be subtracted from the total loan ceiling to get the amount that is available to finance investment. The analysis, furthermore, does not require that *all* firms are constrained; it is sufficient that some (non-negligible) proportion of firms face a binding constraint.

Long waves and short cycles in a model of endogenous financial fragility

Abstract

This paper presents a stock-flow consistent macroeconomic model in which financial fragility in firm and household sectors evolves endogenously through the interaction between real and financial sectors. Changes in firms' and households' financial practices produce long waves. The Hopf bifurcation theorem is applied to clarify the conditions for the existence of limit cycles, and simulations illustrate stable limit cycles. The long waves are characterized by periodic economic crises following long expansions. Short cycles, generated by the interaction between effective demand and labor market dynamics, fluctuate around the long waves.

Key words. cycles, long waves, financial fragility, stock-flow consistency

JEL classification. E12, E32, E44

1. Introduction

Financial crisis hit the U.S and world economy in 2008. Giant financial institutions have collapsed. Stock markets have tumbled, and exchange rates are in turmoil. Governments and central banks around the world have responded by implementing bailout plans for troubled financial institutions and cutting interest rates to contain the financial panic, and expansionary fiscal packages are being pushed through to prop up aggregate demand. Hyman Minsky's Financial Instability Hypothesis offers an interesting perspective on these developments, which came after a long period of financial deregulation, rapid securitization and the development of a range of new financial instruments and markets.[1]

According to Minsky's financial instability hypothesis, a capitalist economy cannot lead to a sustained full employment equilibrium and serious business

[1] Wray (2008), Cynamon and Fazzari (2008) and Crotty (2009), among others, provide perspectives on how shaky are the foundations of these 'sophisticated' developments in financial markets.

cycles are unavoidable due to the unstable nature of capitalist finance (Minsky, 1986, 173). An initially robust financial system is endogenously turned into a fragile system as a prolonged period of good years induces firms and bankers to take riskier financial practices. During expansions, an investment boom generates a profit boom but this induces investors and banks to adopt more speculative financial arrangements. This is typically reflected in rising debt finance, which eventually turns out to be unsustainable because the rising debt changes cash flow relations and leads to various types of financial distress. Minsky suggests that this kind of endogenous change in financial fragility can generate debt-driven long expansions followed by deep depressions (Minsky 1964, 1995). In Minsky's theory of long waves, short cycles fluctuate around the long waves produced by endogenous changes in financial structure. Thus, the distinction between short cycles and long waves is an important characteristic of Minsky's cycle theory.

In spite of difficulties inherent in the formalization of Minsky's theories, Minsky's financial instability hypothesis has inspired a number of researchers to model the dynamic interaction between real and financial sectors. Taylor and O'Connell (1985), Foley (1986), Semmler (1987), Jarsulic (1989), Delli Gatti and Gallegati (1990), Skott (1994), Dutt (1995), Keen (1995) and Flaschel et al. (1998, Ch.12) are early contributions. Recent studies include Setterfield (2004), Nasica and Raybaut (2005), Lima and Meirelles (2007), and Fazzari et al. (2008).

This paper presents a stock-flow consistent model where firms' and households' financial practices evolve endogenously through the interaction between real and financial sectors. The interaction between changes in firms' and households' financial practices produces long waves. The resulting long waves are characterized by periodic economic crises following long expansions. Short cycles, generated by the interaction between effective demand and labor market dynamics, fluctuate around the long waves.

Compared to the previous literature, this paper has three distinct features:

First, the model in this paper is stock-flow consistent.[2] Financial stocks are explicitly introduced and their implications for income and financial flows are carefully modeled. In particular, unlike the previous studies listed above, capital gains from holding stocks are not assumed away and enter the definition

[2] See Skott (1981), Godley and Cripps (1983) and Taylor (1985) for early introductions of explicit stock-flow relations in a post-Keynesian / structuralist context. Simulation exercises based on the stock-flow consistent framework have been flourishing since Lavoie and Godley (2001-2).

of the rate of return on equity. The rate of return on equity defined in this way provides a basis of households' portfolio decision. Firms' and households' financial decisions jointly determine stock prices and the rate of return on equity in equilibrium. Thus, stock markets receive a careful treatment in this model and play a central role in producing cycles.

Second, this paper pays attention to both firms' and households' financial decisions. Minsky's own account of financial instability tends to privilege the firm sector as a source of fragility.[3] Most previous studies follow this tradition and tend to neglect the role of households' financial decisions in creating instability and cycles. Some of the previous studies, including Taylor and O'Connell (1985), Delli Gatti and Gallegati (1990), and Flaschel et al. (1998, Ch.12), do not suffer from this kind of limitation but analyze households' portfolio decision as well. However, their neglect of the role of capital gains in households' portfolio decision makes it difficult to analyze the implication of households' financial decisions and stock market behavior for instability and cycles. In contrast to these models, the model in this paper analyzes both households' and firms' financial decisions. Capital gains and stock markets are considered explicitly in a stock-flow consistent framework. The interactions between households and firms turn out to be critical to the behavior of the system. The model consists of two subsystems: firms' debt dynamics and households' portfolio dynamics. One interesting result of our analysis is that two stable subsystems can be combined to produce instability and cycles in the whole system (See section 3). Thus, the resulting instability and cycles are genuinely attributed to the interaction between sectors rather than characteristics of one particular sector.

Lastly, existing Minskian models do not distinguish long waves from short cycles and the periodicity of cycles in those models is ambiguous. Our model is explicit in this matter. It produces two distinct cycles: long waves and short cycles. Long waves are produced by the interaction between firms' and households' financial decisions, while short cycles are generated by the interaction between effective demand and labor market dynamics. The key idea underlying Minsky's financial instability hypothesis is that firms', bankers', and households' financial practices change endogenously. In the real world characterized by complexity and uncertainty, agents' financial practices are largely affected by norms and conventions, which include borrowing and lending standards as

[3]Minsky's neglect of the household sector is explained by his observation that "[H]ousehold debt-financing of consumption is almost always hedge financing." (1982, p. 32) This position, however, has been challenged by some Minskian explanations of the sub-prime mortgage crisis. (e.g. Wray(2008) and Kregel (2008))

well as portfolio investors' attitude to risks and uncertainty. Changes in these norms and conventions take time and tend to exhibit inertia. The long-term trend in these elements would not be greatly disturbed by ups and downs during a course of short-run business cycles.[4] Thus we interpret Minsky's financial instability hypothesis as a basis of long waves rather than a theory of short run business cycles.[5] Some of Minsky's own writings support our interpretation. For instance, Minsky argues that (i) "The more severe depressions of history occur after a period of good economic performance, with only minor cycles disturbing a generally expanding economy."(Minsky, 1995, p.85); (ii) the "mechanism which has generated the long swings centers around the cumulative changes in financial variables that take place over the long-swing expansions and contractions."(Minsky, 1964).

To the best of our knowledge, our model is the first to integrate an analysis of Minskian long waves with that of short cycles.

The analysis of the implications of financial behavior for instability and cycles in this paper complements a previous study on financialization and finance-led growth in Skott and Ryoo (2008) where the emphasis is on the effects of changes in financial behavior on long-run steady growth path with little attention to questions of stability and fluctuations.

The rest of the paper is structured as follows. Section 2 sets up a stock-flow consistent model. Section 3 analyzes how the interaction between firms' and households' financial practices produces long waves. Section 4 briefly introduces a model of short cycles into the current context. Section 5 combines our model of long waves with the short-cycle model and provides simulation results. Section 6 examines some alternative specifications. Section 7, finally, offers some concluding remarks.

2. Model

This section presents a model. Firms make decisions concerning accumulation, financing, and pricing/output; households make consumption and portfolio decisions; banks accept deposits and make loans. It is assumed that there are

[4]As pointed out by a referee, 'financial behaviour in Minsky is clearly based on borrowing and lending norms, and norms (like all institutions) are relatively inert and hence slow to evolve. On this basis, it is surely more plausible to think that the drama of the financial instability hypothesis is more likely to play itself out over the course of a long wave rather than a single business cycle.'

[5]It is surprising that Minsky's theory of long waves has received little attention not only by mainstream but also by heterodox economists. Palley (2009) recently called for understanding Minsky's theory through the lens of long term swings.

only two types of financial assets - equity and bank deposits - and banks are the only financial institution. It is assumed that the available labor force grows at a constant rate[6] and long run growth is constrained by the availability of labor.

2.1. Firms

2.1.1. The finance constraint

Firms have three sources of funds in our framework: profits, new issue of equity and debt finance. Using these funds, firms make investments in real capital, pay out dividends and make interest payments. Algebraically,

$$pI + Div + iM = \Pi + v\dot{N} + \dot{M} \tag{1}$$

where I, Π, Div, M, and N are real gross investment, gross profits, dividends, bank loans and the number of shares, respectively. Bank loans carry the nominal interest rate (i). p represents the price of investment goods as well as the general price of output in this one-sector model. All shares are assumed to have the same price v.[7]

We assume that firms' dividend payout is determined as a constant fraction of profits net of depreciation and real interest payments. The dividend payout rate is denoted as $1 - s_f$ and, consequently, s_f represents firms' retention rate. Thus, we have

$$Div = (1 - s_f)(\Pi - \delta pK - rM) \tag{2}$$

where K and δ are real capital stock and the rate of depreciation of real capital. r represents the real interest rate, $r = i - \hat{p}$, where $\hat{p}$ is the inflation rate. Lavoie and Godley (2001-2002) and Dos Santos and Zezza (2007), among others, use the specifications similar to (2) regarding firms' retention policy. The real interest rate, rather than, the norminal rate, enters in the specification of dividend payments, (2). Using the real interest rate in equation (2) may be justified if firms treat the capital gain on existing debt from inflation ($= \hat{p}M$) as a source of profit.[8] Apart from the plausibility of this justification, specification (2) helps our analysis avoid possible complications due to the effect of inflation. Equation (2), in conjunction with the assumption of exogenous real interest rate (see section 2.2 below), makes dividend payments unaffected by a change

[6]We assume that there is no technical progress but the model can easily accommodate Harrod neutral technical progress

[7]A dot over a variable refers to a time derivative ($\dot{y} = dy/dt$).

[8]This interpretation is provided by an anonymous referee.

in the inflation rate. This kind of inflation neutrality ceases to hold if the real interest rate is replaced by the nominal rate in (2).[9]

New equity issue can be represented by the growth of the number of shares ($\hat{N}$) or by the share of investment financed by new issues denoted as x. Skott (1989) and Foley and Taylor (2004) use the former and Lavoie and Godley (2001-2002) the latter. Two measures, however, are related to each other in the following manner. It should be noted that x (and $\hat{N}$) is not treated as a constant parameter in this paper.

$$vN\hat{N} = xpI \tag{3}$$

Substituting (2) into (1), we get

$$pI - \delta pK = s_f(\Pi - \delta pK - rM) + vN\hat{N} + M(\hat{M} - \hat{p}) \tag{4}$$

Scaling by the value of capital stock (pK), we have[10]

$$\hat{K} \equiv g = s_f(\pi u\sigma - \delta - rm) + x(g + \delta) + \dot{m} + gm \tag{5}$$

where π, u, and m are the profit share ($\pi \equiv \frac{\Pi}{pY}$), the utilization rate ($u \equiv \frac{Y}{Y_F}$, Y_F is full capacity output) and the debt-capital ratio ($m \equiv \frac{M}{pK}$). The technical output/capital ratio, σ ($\equiv \frac{Y_F}{K}$), is assumed to be fixed. Equation (5) has a straightforward interpretation: firms' investment (g) is financed by three sources: retained earnings, $s_f(\pi u\sigma - \delta - rm)$, new equity issue, $x(g + \delta)$ and bank loans, $\dot{m} + gm$. Given this finance constraint, firms' financial behavior is characterized by s_f, x (or $\hat{N}$) and m. Most theories treat the rates of firms' retention and equity issue as parameters and debt finance as an accommodating variable (Skott 1989, Lavoie and Godley 2001-2002 and Dos Santos and Zezza 2007). This paper assumes that the retention rate (s_f) is exogenous as in the above literature but both the rate of equity issue (x or $\hat{N}$) and the leverage ratio m are endogenous. However, our way of treating equity finance and debt finance is not symmetric.

Debt finance evolves through endogenous changes in firms' and banks' financial practices which are directly influenced by the relationship between firms'

[9]In Fazzari et al.(2008), inflation plays a crucial role in generating a turning point of a cycle: an investment boom leads to tightening labor market and increasing wage inflation. The resulting price inflation raises the nominal interest rate, given the assumption that the real rate is fixed. The increase in the nominal rate squeezes firms' cash flow, which constrains firms' investment. Thus the inflation-cash flow-investment nexus is the key element of their money non-neutrality result.

[10]Equation (5) is obtained by dividing equation (4) by pK and then applying equation (3) and some definitions ($\frac{I}{K} - \delta \equiv g$, $\frac{\Pi}{pK} \equiv \pi u\sigma$, $\frac{M}{pK} \equiv m$, and $\hat{M} - \hat{p} \equiv \hat{m} + \hat{K}$).

profitability and leverage ratio (see section 2.1.2 below). With debt finance determined in this way, equity finance (x) serves as a buffer in the sense that once the other sources of finance $-$ the retention and debt finance policies $-$ and investment plans are determined, equity issues fill the gap between the funds needed for the investment plans and the funds available from retained earnings and bank loans. In this regard, equity finance is seen as a residual of firms' financing constraint.

Formally, for a given set of parameters s_f, σ, δ and r, the trajectories of endogenous variables g, π, u, m and $\dot{m}$ determine the required ratio of equity finance to gross investment:

$$x = \frac{g - s_f(\pi u \sigma - \delta - rm) - \dot{m} - gm}{g + \delta} \tag{6}$$

Our assumption that x is a residual suggests that firms cannot control the share of investment financed by equity issues. In the present model, the trajectory of x is determined by a number of parameters including those describing household consumption/portfolio behavior and banks' loan supply decision. Firms' desire to issue or buy back equities inconsistent with the trajectory of x implied by the underlying parameters will be frustrated in the equity market.

Our assumption regarding equity finance implies that x is treated as a fast variable in our dynamical system, while the other methods of finance are modeled as an exogenous variable (s_f) or a state variable (m). As Figure 1 shows, in the U.S., the share of investment financed by equity issues - x - has substantially changed over time. The movement in the ratio appears to be very flexible. This was even more prominent when there were significant stock buybacks, i.e. the rate of net issue of equity was negative ($x < 0$). For instance, the share of fixed investment financed by equity issues was nearly zero in 1982 but reached -42% in 1985. It then bounced back to a positive rate, 4.3% in 1991, and hit the historical low, -71.5% in 2007. Firms have extensively used stock buybacks as a distributional mechanism since the 1980s, which, in our opinion, tends to increase the flexibility of movements in the equity finance variable.

[Figure 1 about here]

Increasing stock buybacks, in parallel to the reduction in the retention rate in the past decades, have received growing attention in the so-called financialization literature. Many studies on this issue have suggested that there have been structural changes in firms' management and financial strategy in favor of shareholders. Most formal analyses of this subject have examined steady state implications of changes in firms' retention and equity finance policies, assuming these changes in the policies can be represented by parametric shifts in the corresponding exogenous variables (s_f or x).[11] Our specification of equity finance as an endogenous variable provides another interesting interpretation of increasing stock buybacks. Equation (9) shows that increases in profitability ($\pi u\sigma - \delta - rm$) and debt finance ($\dot{m} + gm$) reduce the value of x, since they tend to relax firms' budget constraints, other things equal. Given this relation, the observed shareholder value oriented management such as increasing stock buybacks may be a consequence of a prolonged period of a debt driven profit boom. The present model, in fact, produces a result in which a long upswing driven by rising firms' debt finance and a stock market boom is accompanied by a substantial decline in x.

There appears to be no reason to believe that the retention rate s_f remains constant over time. The retention rate has gradually changed in the U.S. economy. It was 75% in 1952 and had increased until it reached 88% in 1979. The retention rate has fallen to about 70% in the past three decades (Skott and Ryoo, 2008). This gradual pattern of the changes in the retention rate over the long period may be best captured by modeling s_f as a state variable along with other key state variables such as firm debt ratio and household portfolio composition. For instance, firms' profit-interest ratio, the key determinant of firms' liability structure (see section 2.1.2), may also affect firms' desired retention rate by changing their perception of the margin of safety. Thus firms' high profitability relative to payment commitments may motivate them not only to raise debt finance but also to pay out more dividends to shareholders.[12] These kinds of laxer financial practices induced by strong profitability tend to stimulate aggregate demand and may contribute to the mechanism of a long expansion because an increase in dividend income tend to raise consumption through its direct effect on household income as well as its indirect effect on household stock market wealth.[13] In this setting, the two key developments

[11] See Skott and Ryoo (2008) for the related literature and a critical analysis of macroeconomic implications of these developments.

[12] This line of reasoning can be formalized as the following dynamic equation: $\dot{s}_f = \psi\left(s_f^*\left(\frac{\rho T}{rm}\right) - s_f\right)$ where $\psi'(\cdot) > 0$, $\psi(0) = 0$ and $s_f^{*\prime}(\cdot) < 0$. $s_f^*(\cdot)$ is the desired retention rate. This equation represents the sluggish adjustment of firms' retention policy. The present model, along with this dynamic equation, can generate the paths of s_f and x declining during a long expansion (our simulation results are available upon request).

[13] Minsky acknowledged this kind of mechanism in the following remark: "During a run of good times, the well-being of share owners improves because dividends to share ownership increases and share prices rise to reflect both the higher earnings and optimistic prospects. The rise in stockholder's wealth leads to increased consumption by dividend receivers, which

associated with financialization, falling s_f and x, represent merely a phase of a long cycle of endogenous changes in financial practices, as briefly suggested in Skott and Ryoo (2008). Although the endogeneity of the retention rate would produce interesting results, we leave out this extension for the future research. s_f will be assumed to be constant throughout this paper.

2.1.2. Endogenous changes in firms' liability structure

Endogenous changes in firms' liability structure, which are captured by changes in firms' debt-capital ratio (m), are central in this paper, and a Minskian perspective suggests that the debt-capital ratio evolves according to sustained changes in firms' profitability relative to their payment obligations on debt. Changes in profitability that are perceived as highly temporary have only limited effects on desired leverage. I, therefore, distinguish cyclical movements in profitability from the trend in average profitability and assume that changes in liability structure are determined by the trend of profitability.[14]

The perception of strong profitability relative to payment commitments during good years, Minsky argues, induces bankers and businessmen to adopt riskier financial practices which typically results in increases in the leverage ratio. Following Minsky's idea (Minsky, 1982, 1986), we assume that changes in the ratio of profit to debt service commitments drive changes in the debt structure. Formally,

$$\dot{m} = \tau\left(\frac{\rho_T}{rm}\right); \quad \tau'(\cdot) > 0 \tag{7}$$

where ρ_T represents the *trend* rate of profit[15]and τ is an increasing function. During a period of good years when the level of profit is sufficiently high compared to interest payment obligations, firms' and bankers' optimism, reinforced by their success, tends to make them adopt riskier financial arrangements which involve higher leverage ratios. Moreover, a high profit level compared to debt servicing is typically associated with a low probability of default which helps bankers maintain their optimism. The opposite is true when the ratio of profit to interest payments is low. Firms' failure to repay debt obligations - defaults and bankruptcies in the firm sector - put financial institutions linked to those firms in trouble as well. This situation, which is often manifested in a system-wide credit crunch, tends to force firms and bankers to reduce firms' indebtedness.

leads to a further rise in profits. This relation between profits and consumption financed by profit income is one factor making for upward instability." (Minsky, 1986, 152)

[14]See section 3.1 for more discussion.

[15]A definition of the trend rate of profit will be provided in section 3.

2.1.3. Accumulation

In general, capital accumulation is affected by several factors including profitability, utilization, Tobin's q, the level of internal cash flows, the real interest rates and the debt ratio, but there is no consensus among theorists concerning the sensitivity of firms' accumulation behavior to changes in the various arguments. This paper follows a Harrodian perspective in which capacity utilization has foremost importance in firms' accumulation behavior (Harrod, 1939). The perspective assumes that firms have a desired rate of utilization. In the short run, the actual rate of utilization may deviate from the desired rate since firms' demand expectations are not always met and capital stocks slowly adjust. If the actual rate exceeds the desired rate, firms will accelerate accumulation to increase their productive capacity and if the actual rate is smaller than the desired rate, they will slow down accumulation to reduce the undesired reserve of excess productive capacity. However, in the long run, it is not reasonable to assume that the actual rate can persistently deviate from the desired rate because capital stocks can flexibly adjust to maintain the desired rate. This perspective naturally distinguishes the short-run accumulation function from the long-run accumulation function.[16]

[Figure 2 about here]

A simple version of the long-run accumulation function can be written as

$$u = u^* \tag{8}$$

where u^* is an exogenously given desired rate of utilization. (8) represents the idea that in the long run, the utilization rate must be at what firms want it to be and capital accumulation is perfectly elastic so as to maintain the desired rate. The strict exogeneity of the desired rate in (8) may exaggerate reality but tries to capture mild variations of the utilization rate in the long-run. For instance, Figure 2 (a) and (b) plot the rate of capacity utilization in the U.S. for the industrial sector and the manufacturing sector, respectively. The Hodrick-Prescott filtered series (dotted lines) are added to capture the long-run variations in the utilization rate. The figures show that the degree of capacity utilization is subject to significant short-run variations but exhibits only mild variations around 80% in the long-run.

In this paper, we use the long run accumulation function (8) to analyze long waves: as long as we are interested in cycles over a fairly long period of time,

[16] This Harrodian perspective is elaborated in Skott (1989, 2008a, 2008b) in greater detail.

the assumption that the actual utilization rate is on average at the desired rate is a reasonable approximation.

Note that the long run accumulation function (8) leaves the growth rate of capital, g, undetermined. The long-run average of g, however, will be approximately equal to the natural rate of growth, n, if the economy fluctuates around a steady growth path with a constant employment rate. As section 4 will show, the system of short cycles in the present model indeed produces the fluctuations of g around n. Thus in the analysis of long waves, g is approximated by its long run average n and the anlaysis of short cycles in section 4 will provide a justification of this procedure,

In the analysis of short cycles, $u = u^*$ will not be a reasonable assumption any longer and it will be replaced by a short-run accumulation function (see section 4).

2.2. Banks

It should be noted that equation (7) represents both bankers' and firms' financial practices. In other words, equation (7) is a reduced form of bank-firm interactions[17] regarding the determination of firms' liability structure. Thus bankers play important roles in shaping firms' financial structure in this model.

Banks' role in the determination of firms' debt structure has system-wide implications as well. For a given profit-interest ratio, equation (7) determines the trajectory of the debt-capital ratio m. At any moment, the amount of loans supplied to firms will be $M = mpK$. It is assumed that neither households nor firms hold cash, the loan and deposit rates are equal and there are no costs involved in banking. With these assumptions, the amount of loans to the firm sector must equal the total deposits of the household sector.

$$M = M^H \tag{9}$$

where M^H represents households' deposit holdings. Thus deposits are generated endogenously through banks' loan making process. Deposits created in this way affect households' wealth, thereby changing the level of effective demand (See section 2.3 below).

[17]Banks and firms may map the profit-interest ratio to the debt ratio in a different manner. For instance, banks' willingness to lend, on the one hand, may be captured by $\dot{m}_B = \tau_B\left(\frac{\rho_T}{rm}\right)$ where $\tau'_B(\cdot) > 0$ and $\dot{m}_B$ represents changes in firms' leverage allowed by bankers. Firms' loan demand, on the other hand, may be represented by $\dot{m}_F = \tau_F\left(\frac{\rho_T}{rm}\right)$ where $\tau'_F(\cdot) > 0$ and $\dot{m}_F$ refers to changes in firms' leverage implied by firms' loan demand. If the actual movement of the debt-capital ratio is assumed to be a non-decreasing function of $\tau_B(\cdot)$ and $\tau_F(\cdot)$, the $\tau(\cdot)$ function can be defined as $\dot{m} = T\left(\tau_B\left(\frac{\rho_T}{rm}\right), \tau_F\left(\frac{\rho_T}{rm}\right)\right) \equiv \tau\left(\frac{\rho_T}{rm}\right)$ with $T_1 \geq 0$ and $T_2 \geq 0$. A special case is obtained if the T-function is chosen as a lower envelope of $\tau_B(\cdot)$ and $\tau_F(\cdot)$.

Banks' adjustment of the volume of loan supply during the course of cycles may have implications for their pricing behavior regarding interest rates. For instance, banks may have a tendency to raise loan interest rates as increases in the volume of loans raise the probability of default risks. At the same time, financial innovations may offset this tendency by making the supply of finance more elastic.[18] This consideration is likely more important in the long run than in the short run. Monetary authority's responses add more complications to these developments. Its concern about inflation may or may not be dominated by the development of its own euphoric expectations.

Precise modeling of banks' pricing behavior, however, is beyond the scope of this paper. For the sake of simplicity, we assume that banks effectively control the real interest rate r. While the actual movements of interest rates are affected by financial market conditions as well as various institutional changes and policy responses, the assumption of perfectly elastic loan supply at a given interest rate appears to fit well with the focus of this paper on the endogenous adjustment of the size of bank balance sheet especially in the longer run.

2.3. Households

Households receive wage income, dividends in return for their stock holdings and interest income. Thus, household real disposable income denoted as Y^H is given as: $Y^H = \frac{W+Div+rM^H}{p}$.

Households hold stocks and deposits and household wealth is denoted as NW^H, where $NW^H = \frac{vN^H+M^H}{p}$. Although the possibility of negative M^H cannot be excluded,[19] this paper only concerns the case in which M^H turns out to be positive. In other words, the household sector as a whole is in a net credit position against the rest of the economy. This does not exclude the possibility in which *some* households are in a debtor position, but any such debt is assumed to be netted out for the household sector as a whole.[20]

Based on their income and wealth, households make consumption and portfolio decisions. We adopt a conventional specification of consumption function.

[18] "During periods of tranquil expansion, profit-seeking financial institutions invent and reinvent "new" forms of money, substitutes for money in portfolios, and financial techniques for various types of activity: financial innovation is a characteristic of our economy in good times." (Minsky, 1986, 178)

[19] In this case, the absolute value of M^H represents households' net indebtedness against the rest of the economy.

[20] To introduce the implications of household debt, the model may have to be extended to allow heterogeneity among households.

(e.g. Ando and Modigliani, 1963)

$$C = C(Y^H, NW^H);\ C_{Y^H} > 0\ , C_{NW^H} > 0 \tag{10}$$

For simplification, we assume that the function takes a linear form. We then have, after normalizing by capital stock and simple manipulations,

$$\frac{C}{K} = c_1[u\sigma - \delta - s_f(\pi u\sigma - \delta - rm)] + c_2 q \tag{11}$$

where $u\sigma - \delta - s_f(\pi u\sigma - \delta - rm)$ is household income scaled by capital stock and Tobin's q captures household wealth. c_1 and c_2 are household propensities to consume out of income and wealth. Note that the expression of household income, $u\sigma - \delta - s_f(\pi u\sigma - \delta - rm)$, implies that an increase in interest *raises* household income, other things equal. A dollar of interest increases household income by the same amount directly but decreases dividend income indirectly by $100 \times (1 - s_f)$ cents since it decreases firms' net profits. The net effect will be an increase in household income by $100 \times s_f$ cents. If the real interest rate is constant as in this paper, an increase in the debt ratio (m) tends to stimulate consumption demand by raising household income.

In addition to consumption/saving decisions, households make portfolio decisions. We denote the equity-deposit ratio as α, where $\alpha \equiv \frac{vN^H}{M^H}$.

We assume that the composition of households' portfolio is affected by their views on stock market performance. Applying a Minskian hypothesis to household behavior, it is assumed that during good years, households tend to hold a greater proportion of financial assets in the form of riskier assets. In our two-asset framework, equity represents a risky asset and deposits a safe asset. Thus, a rise in fragility during good years is captured by a rise in α. We introduce a new variable z to represent the degree of households' optimism about stock markets. We can normalize the variable z so that $z = 0$ corresponds to the state where households' perception of tranquility is neutral and there is no change in α. Given this framework, the evolution of α is determined by an increasing function of z.

$$\dot{\alpha} = \zeta(z);\quad \zeta(0) = 0,\quad \zeta'(z) > 0 \tag{12}$$

The next question is what determines households' views about stock markets, z. It is natural to assume that household portfolio decisions, the division of their wealth into stocks and deposits, will be affected by the difference between the rates of return on stocks and deposits.

Our specification of the process in which households form their views on stock markets emphasizes historical elements in financial markets. Thus, the past trajectories of rates of return on assets matter in the formation of z. In addition to the history of rates of return, the history of household portfolio decisions (α's) may affect current households' views on stock markets if current household portfolio decisions are largely influenced by their habits and conventions. As a crude approximation of this perception formation process, the following exponential decay specification is introduced:

$$z = \int_{-\infty}^{t} \exp\left[-\lambda(t-\nu)\right]\kappa\left(r_{\nu}^{e} - r, \alpha_{\nu}\right) d\nu \tag{13}$$

where r^e is the real rate of return on equity, $\kappa_{r^e} \equiv \frac{\partial \kappa(r^e - r, \alpha)}{\partial r^e} > 0$ and $\kappa_{\alpha} \equiv \frac{\partial \kappa(r^e - r, \alpha)}{\partial \alpha} < 0$. In expression (13), $\kappa\left(r_{\nu}^{e} - r, \alpha_{\nu}\right)$ represents the information regarding the state of asset markets at time ν. The higher the rate of return on equity relative to the deposit rate of interest, the more optimistic households' view on stock markets becomes ($\kappa_{r^e} > 0$). However, other things equal, a higher proportion of their financial wealth in the form of stock holdings (high α) tempers the desire of further increases in equity holdings, i.e. $\kappa_{\alpha} < 0$.

Information on asset markets at different times enters in the formation of z with different weights. The term, $\exp\left[-\lambda(t-\nu)\right]$, represents these weights, implying that a more remote past receives a smaller weight in the formation of households' perception of tranquility. Thus, λ may be seen as the rate of loss of relevance or loss of memory of past events. The higher λ, the more quickly eroded is the relevance of past events.[21] [22]

Differentiation of (14) with respect to t yields the following differential equa-

[21] As pointed out by a referee, equation (13) implies that the weights on the history of α are the same as those on $r^e - r$. This implicit assumption is not reasonable, but (13) can be modified to allow different weights on the history of α and $r^e - r$ in an additively separable form. This modification increases the dimension of the resulting dynamical system, making the qualitative analysis more cumbersome. The author, however, found that with introducing the different weights an even wider range of parameter values successfully generate the cyclical pattern proposed in this paper. This result is not surprising because the dimension of the parameter set increases along with the change in the specification.

[22] If the history of α does not matter for household portfolio decisions, (12) and (13) may be modified as follows:

$$\dot{\alpha} = \zeta(\alpha^* - \alpha) \tag{12a}$$

$$\alpha^* = \int_{-\infty}^{t} \exp\left[-\lambda(t-\nu)\right]\bar{\kappa}\left(r_{\nu}^{e} - r\right) d\nu \tag{13a}$$

where $\bar{\kappa}'(\cdot) > 0$ and α^* is the desired equity-deposit ratio. (13a) tells us that households' desired portfolio is determined by the trajectory of the difference between the rates of return on equity and deposit. This desired ratio may not be instantaneously attained so that the adjustment of the actual to the desired ratio takes time. (12a) represents this kind of lagged adjustment of the actual equity-deposit ratio toward the desired ratio. In spite of different interpretations, the two specifications, (12)-(13) and (12a)-(13a), are qualitatively similar. To see this, let $z \equiv \alpha^* - \alpha$. Then $\dot{z} = \dot{\alpha}^* - \dot{\alpha}$. Differentiating (13a) with respect to t, we have tion:

$$\dot{z} = \kappa\left(r^e - r, \alpha\right) - \lambda z \tag{14}$$

Two dynamic equations (12) and (14), along with the equation describing the evolution of firms' liability structure, (7), are essential building blocks for our model of long waves. To proceed, we need to see how the rate of return on equity, r^e, is determined. r^e is defined as follows:

$$r^e \equiv \frac{Div + \Gamma}{vN^H} = \frac{(1 - s_f)(\Pi - \delta pK - rM) + (\hat{v} - \hat{p})vN^H}{vN^H} \tag{15}$$

where Γ is capital gains adjusted for inflation ($\Gamma \equiv (\hat{v} - \hat{p})vN^H$).

The rate of return on equity is determined by stock market equilibrium. Stock market equilibrium requires that the number of shares supplied by firms equals that of shares held by households, $N = N^H$, which implies $\dot{N} = \dot{N}^H$ in terms of the change in the number of shares. Firms issue new shares whenever retained earnings and bank loans fall short of the funds needed to carry their investment plans. Thus firms' finance constraint (1) implies that:

$$\dot{N} = \frac{1}{v}[pI + Div + iM - \Pi - \dot{M}] \tag{16}$$

Simple algebra shows that capital gains can be expressed as follows:

$$\Gamma = (\hat{v} - \hat{p})vN^H = (\hat{\alpha} + \hat{m} + \hat{K})vN^H - v\dot{N}^H \tag{17}$$

$(\hat{\alpha} + \hat{m} + \hat{K})vN^H$ represents the total increase in the real value of stock market wealth[23] but some of the increase is attributed to the increase in the number of shares ($= v\dot{N}^H$). To get the measure of capital gains, the latter should be deducted from the total increase.

Using $N = N^H$, substituting (16) in (17) and plugging this result in (15), we get the new expression for r^e:

$$r^e = \frac{\Pi - iM + \dot{M} + (\hat{\alpha} + \hat{m} + \hat{K})vN^H - pI}{vN^H} \tag{18}$$

$\dot{\alpha}^* = \bar{\kappa}(r^e) - \lambda\alpha^* = \bar{\kappa}(r^e) - \lambda(\alpha + z)$. Therefore, we can rewrite (12a) and (13a) to:

$$\dot{\alpha} = \zeta(z) \tag{12b}$$

$$\dot{z} = \bar{\kappa}(r^e) - \lambda\alpha - \zeta(z) - \lambda z \tag{14a}$$

With (12b)-(14a), the qualitative analysis of the existence of a limit cycle is more complicated than the case in the main text. To guarantee the existence of a limit cycle by way of the Hopf bifurcation, more assumptions about the higher order derivatives of the underlying functions are required.

[23] Note that $\hat{\alpha} + \hat{m} + \hat{K} = \hat{v} + \hat{N} - \hat{p}$.

Normalizing by pK, we get the expression for r^e as a function of π, u, m, $\dot{m}$, α and $\dot{\alpha}$:

$$r^e = \frac{\pi u\sigma - \delta - rm + (1+\alpha)[\dot{m} + mg)] + \dot{\alpha}m - g}{\alpha m} \quad (19)$$
$$\equiv r^e(\pi, u, g, m, \alpha, \dot{m}, \dot{\alpha}) \quad (20)$$

Substituting this expression in the dynamic equation (14), we have:

$$\dot{z} = \kappa \left[r^e(\pi, u, g, m, \alpha, \dot{m}, \dot{\alpha}) - r, \alpha\right] - \lambda z \quad (21)$$

(21) shows that households' views of tranquility are affected by a number of variables and the relationship is complex. We consider several cases according to the property of (21) in section 3.

2.4. Goods market equilibrium

The equilibrium condition for the goods market is that $\frac{C}{K} + \frac{I}{K} = \frac{Y}{K}$, and the definition of q implies that $q = (1+\alpha)m$. Using these expressions, the equilibrium condition for the goods market can be written as:

$$c_1[u\sigma - \delta - s_f(\pi u\sigma - \delta - rm)] + c_2(1+\alpha)m + g + \delta = u\sigma \quad (22)$$

We take the profit share (π) as endogenous and the equilibrium value of π can be found for given values of u, g, m and α. Explicitly, we have:

$$\pi = \frac{g - (1-c_1)(u\sigma - \delta) + c_2(1+\alpha)m + c_1 s_f(\delta + rm)}{c_1 s_f u\sigma} \quad (23)$$
$$\equiv \pi(u, g, m, \alpha) \quad (24)$$

As u, g, m and α evolve over time, the profit share changes as well. The Harrodian investment function adopted in this paper emphasizes a high sensitivity of investment to changes in the utilization rate. Specifically, it assumes that investment is more sensitive than saving to variations in the utilization rate. This Harrodian assumption has an implication for the effect of changes in utilization on profitability: utilization has a positive effect on the profit share and the magnitude will be quantitatively large.[24] The large effect of changes in utilization on the profit share plays an important role in generating short cycles. (See section 4)

[24]If $\frac{\partial(I/K)}{\partial u} > (1-c_1)\sigma + c_1 s_f \pi\sigma = \frac{\partial(S/K)}{\partial u}$, then $\frac{\partial \pi}{\partial u} = \frac{\frac{\partial(I/K)}{\partial u} - (1-c_1)\sigma - c_1 s_f \pi\sigma}{c_1 s_f u\sigma} > 0$.

It is also readily seen that changes in the debt ratio and the equity-deposit ratio positively affect the profit share. Increases in the debt ratio or the equity-deposit ratio raise consumption demand though changes in disposable income or wealth, thereby increasing the profit share.[25]

2.5. Summary of the system of long waves

The model contains a number of behavioral relations. It may be useful to summarize the key equations. The system of long waves is a three dimensional dynamical system (7), (12) and (14), which consists of three state variables, m (firms' liability structure: the debt-capital ratio), α (household portfolio composition: the equity-deposit ratio) and z (households' confidence in stock markets).

$$\dot{m} = \tau\left(\frac{\rho_T}{rm}\right), \quad \tau'(\cdot) > 0 \tag{7}$$

$$\dot{\alpha} = \zeta(z), \quad \zeta' > 0 \tag{13}$$

$$\dot{z} = \kappa\left(r^e - r, \alpha\right) - \lambda z, \quad \kappa_{r^e} > 0; \quad \kappa_\alpha < 0 \tag{15}$$

As long as the other two endogenous variables, ρ_T (the trend rate of corporate profitability) and r^e (the rate of return on equity), are determined as a function of the state variables, the above dynamical system is closed.

Goods market equilibrium determines the current rate of profit as a function of u, m, and α. The trend rate of profit, ρ_T, is determined as a function of m and α after adding the long run accumulation function $u = u^*$ (See section 3.1 below). ρ_T is increasing in m and α.

$$\rho_T = \rho_T(m, \alpha); \quad \rho_{T m} > 0, \quad \rho_{T \alpha} > 0 \tag{24A}$$

The rate of return on equity is determined by stock markets.

$$r^e = r^e(\pi(u^*, n, m, \alpha), u^*, n, m, \alpha, \dot{m}, \dot{\alpha}) \tag{20A}$$

where (20) is evaluated at $u = u^*$ and $g = n$. The rate of return on equity responds to various arguments and its behavior appears to be complex. In any case, for any given values of m, α, and z, r^e is determined.

[25] $\frac{\partial \pi}{\partial m} = \frac{c_1 s_f r + c_2(1+\alpha)}{c_1 s_f u \sigma} > 0$ and $\frac{\partial \pi}{\partial \alpha} = \frac{c_2 m}{c_1 s_f u \sigma} > 0$

3. Long Waves

This section shows how endogenous changes in firms' and households' financial practices generate long waves. Our model of long waves consists of two subsystems: one describes changes in firms' liability structure and the other specifies changes in households' portfolio composition. Section 3.1 analyzes the evolution of firms' liability structure, assuming households' portfolio composition is frozen. Section 3.2 examines households' portfolio dynamics, given the assumption that firms' liability structure does not change. Section 3.3 combines two subsystems and shows how long waves emerge from the interaction between two subsystems.

3.1. Long-Run Debt Dynamics

This section analyzes the long-run evolution of firms' debt structure. For convenience, we reproduce equation (7).

$$\dot{m} = \tau\left(\frac{\rho_T}{rm}\right) \quad \text{where} \quad \tau'(\cdot) > 0 \tag{7}$$

Regarding the shape of τ in (7), Minsky's discussion suggests that the prosperity during tranquil years tends to induce firms and bankers to *gradually* raise the leverage ratio; the rise in the leverage ratio, however, cannot be sustained because it worsens the profit/interest relation. Minsky points out that the financial system is prone to crises as the ratio of profit to interest traverses a critical level (Minsky, 1995). The resulting systemic crisis may prompt a rapid de-leveraging process. To capture this idea, we assume that $\tau'(\cdot)$ takes relatively small positive values within a narrow bound when $\frac{\rho_T}{rm}$ is above a threshold level (good years), whereas it takes relatively large negative values when $\frac{\rho_T}{rm}$ is below the threshold level (bad years). When falling profit/interest ratio passes through the threshold level, $\dot{m}$ sharply falls reflecting a rapid de-leveraging process. Thus, $\tau'(\cdot)$ is likely to be very large when $\frac{\rho_T}{rm} = \tau^{-1}(0)$. Figure 3 reflects this assumption.

[Figure 3 about here]

As briefly discussed in section 2.1.2, we use the trend rate of profit ρ_T as a basis of the evolution of firms' liability structure. Behind equation (7) is the idea that firms' liability structure evolves endogenously over time and that the key determinant of the evolution is firms' and banks' perception of tranquility. The level of firms' profit relative to payment commitments on liabilities is an indicator of firms' performance and solvency status. Movements of the profit rate in general include both trend and cyclical components. It seems reasonable to assume that the long-run evolution of firms' liability structure is primarily determined by the trend of the profit rate rather than the current profit rate.[26]

The driving force of the short-run cyclical movements in the current profit rate is changes in capacity utilization while the desired rate, u^*, provides a good approximation of the long-run average of actual rates of utilization. Thus setting the utilization rate at the desired rate, the short-run cyclical component in the profit rate is effectively eliminated. In addition, the long-run average of g can be approximated by n if the growth rate of capital fluctuates around the natural rate. We then have:

$$\begin{aligned} \rho_T &= \pi(u^*, n, m, \alpha)u^*\sigma \\ &= \frac{n - (1 - c_1)(u^*\sigma - \delta) + c_2(1 + \alpha)m + c_1 s_f(\delta + rm)}{c_1 s_f} \end{aligned} \tag{25}$$

The trend rate of profit defined as (25) depends positively on the debt-capital ratio m and the equity-deposit ratio α ($\frac{\partial \rho_T}{\partial m} > 0$ and $\frac{\partial \rho_T}{\partial \alpha} > 0$). The profit-interest ratio, the key determinant of the liability structure, is written as

$$\frac{\rho_T}{rm} = \frac{n - (1 - c_1)(u^*\sigma - \delta) + c_2(1 + \alpha)m + c_1 s_f(\delta + rm)}{c_1 s_f rm} \tag{26}$$

(26) implies that for a given value of α, the profit-interest ratio is uniquely determined by the debt-capital ratio m. Note that an increase in m raises the numerator (profits) as well as the denominator (interest payments) of this ratio. Minsky's implicit assumption that a rising debt ratio causes the profit-interest ratio to deteriorate is satisfied only if the numerator rises slowly relative to the denominator as m increases. Formally, the latter condition requires $\frac{\partial \rho_T}{\partial m} < \frac{\rho_T}{m}$: the level of profits generated by a marginal increase in debt, due to the expansionary effect on aggregate demand of debt, falls short of the current profit-debt ratio. In our linear specification of consumption function, this condition will hold if the 'autonomous' component of profits - the part of profits which is independent of variations in m - is positive.[27] Thus Minsky's implicit assumption is met if a sufficient level of demand, which is not entirely explained by the

[26] This perspective is in line with Minsky's statement that "[T]he inherited debt reflects the history of the economy, which includes a period in the not too distant past in which the economy did not do well. Acceptable liability structures are based on some margin of safety so that expected cash flows, even in periods when the economy is not doing well, will cover contractual debt payments"(Minsky, 1982, 65).

[27] This proposition in the linear case can be generalized to any consumption function that is homogenous of degree one with respect to household income and wealth. If consumption function violates this homogeneity assumption, then the positiveness of 'autonomous' profits

positive effect of debt on demand, exists so that firms can make positive profits even when $m = 0$.[28] In the present model, the condition can be written in terms of parameters:

$$n - (1 - c_1)(u^*\sigma - \delta) + c_1 s_f \delta > 0 \tag{27}$$

Condition (27) may or may not be satisfied for plausible parameter values.[29] For instance, if household marginal propensity consume c_1 is relatively low, the inequality in (2) can be reversed. However, we will assume that condition (27) holds in order to keep track of the dynamic implications of the Minsky's assumption of the inverse relationship between the profit-interest ratio and the debt ratio.[30]

Using (7) and (26), $\dot{m}$ can be written as a function of m and α.

$$\dot{m} = \tau \left(\frac{n - (1 - c_1)(u^*\sigma - \delta) + c_2(1 + \alpha)m + c_1 s_f(\delta + rm)}{c_1 s_f rm} \right) \equiv \mathrm{F}(\underset{-}{m}, \underset{+}{\alpha}) \tag{28}$$

(28), along with the condition (27), implies that for any value of α, (i) F is decreasing in m, (ii) there exists a unique value of the debt ratio $m^*(\alpha)$ such that if $m = m^*(\alpha)$, $\dot{m} = 0$, and (iii) $m^*(\alpha)$ depends positively on α, i.e. $m^{*\prime}(\alpha) > 0$. By setting $\dot{m}$ to zero and solving for m, we obtain the algebraic expression for $m^*(\alpha)$:

$$m^*(\alpha) \equiv \frac{n - (1 - c_1)(u^*\sigma - \delta) + c_1 s_f \delta}{[\tau^{-1}(0) - 1]c_1 s_f r - c_2(1 + \alpha)} \tag{29}$$

It is straightforward from properties (i), (ii) and (iii) that (assuming α constant) our dynamic specification of Minsky's financial instability hypothesis implies that firms' debt structure monotonically converges to a stable fixed point m^*. The intuition is simple. When the actual debt ratio (m) is lower than $m^*(\alpha)$, the corresponding profit-interest ratio is greater than the threshold level at which the debt ratio does not change. This will induce firms to raise the

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does not guarantee the inverse relationship between m and $\frac{\rho T}{rm}$. In particular, if the expansionary effect of the debt ratio on profitability is excessively strong at a high level of m due to the strong nonlinearity of consumption function, then the increase in m may increase the profit-interest ratio.

[28] The case in which $m = 0$ is just hypothetical especially in the present model where the existence of debt is the basis of endogenous money creation. $m = 0$ amounts to the assumption of non-monetary economy in the present context.

[29] To illustrate, suppose that $n = 0.03$, $u^* = 0.8$, $\sigma = 0.5$, $\delta = 0.1$ and $s_f = 0.75$. Given these parameter values, condition (27) requires that $c_1 > 0.72$, which may or may not be met in practice.

[30] The implications of the violation of (27) are relatively lucid. For a given value of α, the debt ratio will increase or decrease forever depending on the initial condition. The adjustment of α tends to amplify this kind of unstable dynamics, which most likely yields exploding trajectories.

debt ratio. The same kind of event will happen as long as $m < m^*(\alpha)$: m will eventually converge to $m^*(\alpha)$. The opposite will happen when the debt ratio is greater than the critical level ($m > m^*(\alpha)$).

Given assumption (28), a *stable* dynamics is inevitable in a one-dimensional continuous time framework. Moving from a continuous to a discrete time framework may change the picture so that firms' debt dynamics alone can produce long-run cyclical movements. In this paper, however, we explore another avenue toward long waves by integrating firms' debt dynamics into households' portfolio dynamics.

3.2. Household Portfolio Dynamics

The other subsystem of our model of long waves, which describes households' portfolio dynamics, consists of two dynamic equations:

$$\dot{\alpha} = \zeta(z) \tag{12}$$

$$\dot{z} = \kappa\left(r^e - r, \alpha\right) - \lambda z \tag{14}$$

Analogously to the analysis of firms' debt dynamics, we are interested in the long-run evolution of household portfolio decisions and, to simplify the analysis we abstract from the effect of short-run variations in capacity utilization. The rate of return on equity evaluated at $u = u^*$ equals

$$r^e|_{u=u^*} = \frac{\rho_T(m,\alpha) - \delta - rm + (1+\alpha)[\mathrm{F}(m,\alpha) + mn] + \zeta(z)m - n}{\alpha m} \tag{30}$$

Given this expression for r^e, equation (14) becomes

$$\dot{z} = \kappa\left(r^e|_{u=u^*} - r, \alpha\right) - \lambda z \equiv \mathrm{G}(m,\alpha,z) \tag{31}$$

(12), (28), and (31) constitute a three-dimensional dynamical system. To better understand the mechanics of this three dimensional system, let us take a look at the subsystem (12) and (31), assuming that m is fixed. By differentiating (31) with respect to α and z, the effects of α and z on $\dot{z}$ are given by:

$$\mathrm{G}_\alpha = \kappa_{r^e}\frac{\partial r^e}{\partial \alpha} + \kappa_\alpha \lesseqgtr 0 \tag{32}$$

$$\mathrm{G}_z = \kappa_{r^e}\frac{\partial r^e}{\partial z} - \lambda = \kappa_{r^e}\frac{\zeta'}{\alpha} - \lambda \lesseqgtr 0 \tag{33}$$

The effect of changes in α on z, G_α in (32), is decomposed into two parts. First, changes in α affect the rate of return on equity, which influences households' views on stock markets, $\kappa_{r^e}\frac{\partial r^e}{\partial \alpha}$. The effect of an increase in α on r^e, $\frac{\partial r^e}{\partial \alpha}$, can be negative or positive in the steady state. Second, an increase in α mitigates the desire for further increases in equity holdings ($\kappa_\alpha < 0$). Thus, the overall effect depends on the precise magnitude of these two effects.

The effect of z on $\dot{z}$ is also unclear. On the one hand, an increase in households' optimism about stock markets accelerates stock holdings, which raises capital gains and the rate of return on equity. The increase in r^e reinforces their optimism ($\kappa_{r^e} \frac{\partial r^e}{\partial z} > 0$). On the other hand, the degree of optimism will erode at a speed of λ, holding r^e and α constant. Thus, the net effect is ambiguous.

Let J^H be the Jacobian matrix evaluated at the fixed point of (12) and (31). The ambiguity of the signs of G_α and G_z yields four cases. Table 1 summarizes it.

Table 1: Classifying Fixed Points

	$\mathrm{G}_z < 0$	$\mathrm{G}_z > 0$
$\mathrm{G}_\alpha < 0$	**Case I** Stable $\mathrm{Tr}(\mathrm{J^H}) < 0$ and $\mathrm{Det}(\mathrm{J^H}) > 0$	**Case II** Unstable $\mathrm{Tr}(\mathrm{J^H}) > 0$ and $\mathrm{Det}(\mathrm{J^H}) > 0$
$\mathrm{G}_\alpha > 0$	**Case III** Saddle $\mathrm{Tr}(\mathrm{J^H}) < 0$ and $\mathrm{Det}(\mathrm{J^H}) < 0$	**Case IV** Saddle $\mathrm{Tr}(\mathrm{J^H}) > 0$ and $\mathrm{Det}(\mathrm{J^H}) < 0$

A locally stable steady state in the subsystem is obtained when G_z and G_α are both negative (Case I). In this case, λ is large relative to $\kappa_{r^e} \frac{\partial r^e}{\partial z}$, and $\kappa_{r^e} \frac{\partial r^e}{\partial \alpha}$ is negative or, if positive, relatively small compared to the absolute value of κ_α. Thus, to get a locally stable steady state for households' portfolio dynamics, the positive effect of changes in α and z on $\dot{z}$ via the rate of return on equity needs to remain relatively small in the neighborhood of the steady state.

Moving from Case I, as λ gets smaller than $\kappa_{r^e} \frac{\partial r^e}{\partial z}$ ($\mathrm{G}_z > 0$), keeping the condition $\mathrm{G}_\alpha < 0$, the steady state becomes locally unstable, yielding Case II. In this case, a high optimism further boosts households' optimistic views on stock markets, creating destabilizing forces. The locally unstable steady state, along with nonlinearities of (12) and (31), can produce limit cycles as long as λ is not too small. Thus, in this case, households' portfolio dynamics alone can generate persistent long waves.

If $\mathrm{G}_\alpha > 0$, i.e. $\kappa_{r^e} \frac{\partial r^e}{\partial \alpha}$ is larger than $|\kappa_\alpha|$, then the fixed point of the households' portfolio dynamics becomes a saddle point, regardless of the sign of G_z (Case III and IV). In both Case III and IV, a high level of equity holdings creates increasing optimism ($\mathrm{G}_\alpha > 0$), making the steady state a saddle point. However, Case IV is distinguished from Case III because it is an exceptional case:

it turns out that the destabilizing force in Case IV is too strong to produce a limit cycle for the three dimensional full system ((12), (28), and (31)), whereas, in all other three cases I, II, and III, an appropriate choice of parameter values can produce a limit cycle for the full system. The next section analyzes the full system of long waves.

3.3. Full Dynamics: Long Waves

We now put together firms' debt and households' portfolio dynamics and obtain the following three dimensional dynamical system:

$$\dot{m} = \mathrm{F}(m, \alpha) \tag{28}$$

$$\dot{\alpha} = \zeta(z) \tag{13}$$

$$\dot{z} = \mathrm{G}(m, \alpha, z) \tag{31}$$

Let us first consider the Jacobian matrix of the system evaluated in the steady state.

$$\mathbf{J} = \begin{bmatrix} \mathrm{F}_m & \mathrm{F}_\alpha & 0 \\ 0 & 0 & \zeta' \\ \mathrm{G}_m & \mathrm{G}_\alpha & \mathrm{G}_z \end{bmatrix} = \begin{bmatrix} - & + & 0 \\ 0 & 0 & + \\ - & +/- & +/- \end{bmatrix} \tag{34}$$

G_α and G_z are ambiguously signed but the partial derivative of G with respect to m is likely to be negative:

$$\mathrm{G}_m = \kappa_{r^e} \frac{\partial r^e}{\partial m} \tag{35}$$

where

$$\frac{\partial r^e}{\partial m} = \frac{\left[\frac{\partial \rho_T}{\partial m} m - \rho_T\right] + (1+\alpha) m \mathrm{F}_m + n + \delta}{\alpha m^2} \tag{36}$$

in the steady state. The sign of (36) may appear to be indeterminate: while $\frac{\partial \rho_T}{\partial m} m - \rho_T$ is negative due to assumption (27) and $(1+\alpha)m\mathrm{F}_m$ is negative since $\mathrm{F}_m < 0$, $n + \delta$ is positive. The discussion of the shape of $\tau(\cdot)$ in section 3.1, however, suggests that F_m is large in magnitude at the steady state growth path.[31] Thus, at the steady state, the negative terms in the numerator in (36) dominate, and the rate of return on equity will decrease as firms' indebtedness increases in the neighborhood of the steady state. Thus, we have $\mathrm{G}_m = \kappa_{r^e} \frac{\partial r^e}{\partial m} < 0$.

[31] If $\tau'(\cdot)$ is large at $\frac{\rho_T}{rm} = \tau^{-1}(0)$, the derivative of $\mathrm{F}(m, \alpha)$ with respect to m is strongly negative at $m = m^*(\alpha)$, i.e. $|\mathrm{F}_m|$ is large. In a limiting case where the de-leveraging process is instantaneous at $m^*(\alpha)$, $\mathrm{F}_m \to -\infty$.

We are interested in the conditions under which the system exhibits limit cycle behavior. As sections 3.1 and 3.2 showed, the specification of firms' financial decisions, (28), leads to asymptotically stable dynamics, whereas households' portfolio dynamics ((12) and (31)) produces several cases in Table 1. Our analytic result suggests that if households' portfolio dynamics are neither strongly stabilizing nor strongly destabilizing, our baseline system of (12), (28) and (31) tends to generate limit cycles. Our analysis of limit cycles is based on the Hopf bifurcation theorem. The Hopf bifurcation occurs if the nature of the system experiences the transition from stable fixed point to stable cycle as we gradually change a parameter value of a dynamical system (Medio, 1992, section 2.7). we will use λ as the parameter for the analysis of bifurcation.[32] Proposition 1[33] provides the main results of our analysis of long waves:

Proposition 1. *Consider the three dimensional system of (11), (28) and (31) and the Jacobian matrix (35) where the partial derivatives are taken at the steady state values. Let*

$$b \equiv \frac{(|\mathrm{F}_m|^2 - \zeta'\mathrm{G}_\alpha) - \sqrt{(|\mathrm{F}_m|^2 - \zeta'\mathrm{G}_\alpha)^2 + 4\zeta'|\mathrm{F}_m||\mathrm{G}_m|\mathrm{F}_\alpha}}{2|\mathrm{F}_m|} < 0$$

(I) (***Case I and Case II***) *Suppose that* $\mathrm{G}_z < \min\left\{|\mathrm{F}_m|, \frac{\zeta'|\mathrm{G}_\alpha|}{|\mathrm{F}_m|}\right\}$[34] *and* $\mathrm{G}_\alpha < 0$. *Then a Hopf bifurcation occurs at* $\lambda = \lambda^* \equiv \kappa_{r^e}\frac{\partial r^e}{\partial z} + |b|$. *As* λ *falls passing through* λ^*, *the system with a stable steady state loses its stability, giving rise to a limit cycle.*

(II) (***Case III***) *Suppose that* $\mathrm{G}_z < 0$ *and* $0 < \mathrm{G}_\alpha < \min\left\{\frac{|\mathrm{F}_m||\mathrm{G}_z|}{\zeta'}, \frac{\mathrm{F}_\alpha|\mathrm{G}_m|}{|\mathrm{F}_m|}\right\}$. *Then a Hopf bifurcation occurs at* $\lambda = \lambda^* \equiv \kappa_{r^e}\frac{\partial r^e}{\partial z} + |b|$. *As* λ *falls passing through* λ^*, *the system with a stable steady state loses its stability, giving rise to a limit cycle.*

(III) (***Case IV***) *Suppose that* $\mathrm{G}_\alpha > 0$ *and* $\mathrm{G}_z > 0$. *Then the steady state is unstable. There exists no limit cycle by way of Hopf bifurcation.*

[Figure 4 about here]

[32] λ is particularly useful for the analysis not only because it is of obvious behavioral importance but also because it provides analytic tractability due to the fact that changes in λ do not affect steady state values.

[33] The proof of Proposition I is found in Appendix A but the proof is concerned about only the existence of a limit cycle. The computation of the coefficient that shows whether the limit cycle is stable is very complicated and hard to interpret. Therefore, we extensively use simulation exercises to observe the stability of cycles.

[34] Note that Case I automatically satisfies the second condition since $\mathrm{G}_z < 0$ in Case I.

Part (I) in the proposition suggests that the existence of a limit cycle requires at least three conditions: first, the mitigation effect of a high proportion of equity holdings on increasing optimism ($|\kappa_\alpha|$) is sufficiently large so that $\mathrm{G}_\alpha < 0$[35]; second, households' optimistic or pessimistic view of stock markets is not excessively persistent ($\mathrm{G}_z < \min\left\{|\mathrm{F}_m|, \frac{\zeta'|\mathrm{G}_\alpha|}{|\mathrm{F}_m|}\right\}$); third, the rate of loss of relevance of past events (λ) should not be too large ($\lambda < \lambda^*$).[36] The second and third conditions imply that for the existence of a limit cycle, λ should be of appropriate magnitude:

$$\kappa_{r^e}\frac{\partial r^e}{\partial z} - \min\left\{|\mathrm{F}_m|, \frac{\zeta'|\mathrm{G}_\alpha|}{|\mathrm{F}_m|}\right\} < \lambda < \kappa_{r^e}\frac{\partial r^e}{\partial z} + |b| \tag{37}$$

All of these conditions imply that to get a limit cycle, households' portfolio dynamics should be neither strongly stabilizing nor strongly destabilizing.

One interesting aspect of Part (I) in Proposition I is that the interaction between two *stable* subsystems - firms' debt and households' portfolio dynamics - can generate an unstable steady state and a limit cycle (Case I). Thus, in this case, the source of the resulting long waves lies purely in the *interaction* between both firm and household sectors. Figure 4 depicts the emergence of a limit cycle in this case in a three dimensional space. Figure 5 shows the trajectories of the debt-capital ratio and the equity-deposit ratio in this case.

[Figure 5 about here]

The debt-capital ratio and the equity-deposit ratio steadily increase during a long boom.[37] This expansion, however, is followed by a sharp fall in m and α, which have significant negative impacts on effective demand and trigger an abrupt downturn in the real sector (See section 4 below).

Part (I) also covers Case II where the subsystem of households' portfolio dynamics is unstable. As shown in 3.2, in Case II, portfolio dynamics alone can create a limit cycle. Part (I) in the proposition suggests that the system can still have a limit cycle when the portfolio dynamics is combined with firms' debt dynamics. Then what is the implication of introducing the debt dynamics into portfolio dynamics? The qualitative analysis does not tell much about

[35]Or the positive effect of changes in α on $\dot{z}$ via its effect on the rate of return on equity should not be too large.

[36]If λ exceeds λ^*, then the system will be stabilized.

[37]The functions and parameter values for this simulation, which are also used for the simulation in section 5, are found in Appendix B. A sufficiently long period of time (from $t = 0$ to $t = 30000$) is taken in all simulation exercises in this paper.

the answer to this question. Numerical experiments, however, provide a case in which the amplitude and period of long waves get significantly larger as we move from the 2D subsystem of portfolio dynamics to the full 3D system.

Part (II) in the proposition concerns Case III where the household portfolio subsystem yields a saddle point steady state. Thus, this part of Proposition 1 shows how stabilizing debt dynamics and households' portfolio dynamics with saddle property are combined to produce a limit cycle. Not surprisingly, not all saddle cases can generate a limit cycle. First, the destabilizing effect that makes the fixed point in the 2D household subsystem saddle − the magnitude of G_α − should be mild: $\mathrm{G}_\alpha < \min\left\{\frac{|\mathrm{F}_m||\mathrm{G}_z|}{\zeta'}, \frac{\mathrm{F}_\alpha|\mathrm{G}_m|}{|\mathrm{F}_m|}\right\}$. Second, G_z should be negative. If it is positive ($\mathrm{G}_z > 0$), the condition for the saddle point, $\mathrm{G}_\alpha > 0$, eliminates the possibility of the emergence of a limit cycle *a la* the Hopf bifurcation. Proposition 1-(III) makes this point. Intuitively, if *both* $\mathrm{G}_\alpha > 0$ and $\mathrm{G}_z > 0$ (Case IV), the portfolio dynamics in the household sector is excessively destabilizing in the sense that stabilizing forces in firms' debt dynamics cannot contain such a strong destabilizing effect.

[Figure 6 about here]

To understand the mechanism behind the long waves, it is illuminating to compare the full system with the subsystem of debt dynamics. As seen in section 3.1, with households' portfolio composition (α) fixed, the debt-capital ratio (m) monotonically converges to its steady state value $m^*(\alpha)$. The main reason for this convergence was the inverse relation between m and $\frac{\rho_T}{rm}$: a rising debt-capital ratio causes firms' profit-interest ratio to deteriorate for any given α. However, once households' portfolio composition evolves endogenously, this kind of strict inverse relationship breaks down because changes in α also affect $\frac{\rho_T}{rm}$.

Figure 6 illustrates this point, where the horizontal dotted line represents the threshold level ($= \tau^{-1}(0)$) of the profit-interest ratio that makes $\dot{m}$ zero. In the area above the horizontal line, the debt-capital ratio increases and in the area below the line, it decreases. With α held fixed, the movement along the curve AB is not possible since for any given α, a rise in m is incompatible with a rise in $\frac{\rho_T}{rm}$. However, increases in α fueled by households' optimism during an expansion have a positive effect on the profit-interest ratio by raising aggregate demand. Thus, from A to B, the economy experiences increases in both α and m.[38] However, households' optimistic views on stock markets eventually fade

[38]The positive effect of the rise in α on the profit-interest ratio dominates the negative effect

as both m and α increase. As a result, the negative effect of a rise in the debt ratio starts to be dominant at some point and the profit-interest ratio begins falling (point B). Because the profit-interest ratio is still above the threshold level, the debt ratio keeps increasing and the profit-interest ratio falls along the curve BC. When the profit-interest ratio passes through point C, the debt-capital ratio starts to fall. When the economy reaches point A, a new cycle begins.

[Figure 7 about here]

Figure 7 depicts the same story from a slightly different angle. The solid line plots a trajectory of the actual debt-capital ratio over time and the dotted line a trajectory of the desired debt ratio ($m^* \equiv m^*(\alpha)$ in (29)). For a given value of α, the debt dynamics, (28), implies that the actual debt ratio m tends to gravitate toward the desired ratio $m^*(\alpha)$. However, when α changes, the desired ratio becomes a *moving* target of the actual ratio. From this view, a period of expansion (contraction) is the time when the actual ratio is below (above) the desired ratio, i.e. $m < m^*$ ($m > m^*$) and consequently the actual debt ratio is increasing (decreasing). In words, a stock market boom (rising α) tends to raise the tolerable level of the debt-capital ratio which the actual ratio is chasing. When the relation between m and m^* is reversed, a long downturn begins (See point C in Figure 7). The endogeneity of the desired debt ratio (or the acceptable liability structure) plays a pivotal role in Minsky's explanation of boom and bust cycles.

As Minsky put it, "[B]orrowing and lending take place on the basis of margins of safety."(Minsky, 1982, 74) The ratio of gross profits to cash payment obligations on debts is "the fundamental margin of safety." The profit-interest ratio, $\frac{\rho_T}{rm}$, in the present model represents this fundamental margin of safety, and the τ-function the relation between the liability structure and the margin of safety. The nonlinearity of the τ-function plays a crucial role in producing the asymmetric pattern of long waves. During good times, the economy operates to the right of $\tau^{-1}(0)$ in Figure 3, where the debt ratio increases due to the sufficient margin of safety. Slow increases in the debt ratio tend to erode the margin of safety but the asset market boom more than offsets this negative effect initially. Thus $\frac{\rho_T}{rm}$ increases and the economy moves further to the right. Since m changes much more slowly in the region to the right of $\tau^{-1}(0)$ than

of the rise in m and consequently the profit-interest ratio also increases during this period.

to the left of it, the economy will stay in that region much longer than in the other. As the stock market boom subsides and begins to fall, the margin of safety starts to decline quickly since the stock market development reinforces the negative effect of rising debt ratio on the margin of safety. As the margin of safety is eroded and traverses the barrier given by $\tau^{-1}(0)$, the systemic crisis can occur along with a detrimental de-leveraging process. The asymmetric shape of the τ-function represents this kind of rapid de-leveraging mechanism: $\dot{m}$ is very large when $\frac{\rho\tau}{rm} < \tau^{-1}(0)$. The margin of safety would be recovered quickly due to the sharp decline in m if the stock market condition remained tranquil. The reality is that stock market crashes exacerbate the problem of the malfunctioning banking system. It will take some time until the economy reaches the turning point, bypassing the point of $\tau^{-1}(0)$ from the left to the right. Thus our τ function shows how the fundemantal margin of safety and firms' liability structure interacts to creat a Minskian boom-bust cycle.

4. A Model of Short Cycles

The model of long waves in section 3.3 can be combined with a model of short cycles. In our analysis of long waves, the degree of capacity utilization is set at its long run average. However, when it comes to short cycles, the utilization rate can deviate from the desired rate due to falsified demand expectations and slow adjustment of capital stocks. Thus we introduce the following short-run accumulation function:

$$\hat{K} \equiv g = \phi(u - u^{*}); \quad \phi'(\cdot) \gg 0, \quad \phi(0) = n \tag{38}$$

The strong positive effect of utilization on accumulation in (38) embodies the Harrodian accelerator principle. This specification of the short-run accumulation as well as the long-run accumulation in (8) is clearly an oversimplification since it leaves out other determinants of investment. For instance, it does not capture the direct impact on accumulation of financial variables such as cash flow and asset prices which are highly emphasized by Minsky (1975, 1982, 1986) and Tobin (1969), as well as current New Keynesian economics (Fazzari et al.(1988) and Bernanke, Gertler and Gilchrist (1996), among others).

The direct financial effects on investment play an important role in the existing Minskian models. For instance, Taylor and O'Connell (1985) assumes that investment depends on the demand price of capital assets, following Minksy's two-price system approach. Delli Gatti and Gallegati (1990) and Fazzari et al. (2008) both assume that investment depends on cash flow in a nonlinear way. Skott (1994) suggests that investment depends on hybrid variables such as 'fragility' and 'tranquility' which reflect financial conditions underlying investment decisions.

It is worth noting that the direct impact of financial variables on accumulation, however, is not necessary to generate long waves in this paper. The key mechanism leading to long waves in this model is the effect of financial variables on aggregate demand. In the baseline model, this demand effect of financial variables works primarily through households' consumption demand. However, equation (8) and (38) can be easily extended to accommodate the direct effect of financial variables on investment without affecting major results of this study, and one possible extension will be considered in section 6.2, where we will show that the direct effect of financial variables on accumulation strengthens our main results.

By plugging (38) into (23), we derive the profit share that ensures goods market equilibrium, which depends positively on u, m, and α.

$$\pi = \pi(\underset{+}{u}, \underset{+}{m}, \underset{+}{\alpha}) \tag{39}$$

Regarding firms' pricing/output decisions, this paper adopts a Marshallian approach elaborated in Skott (1989). The Keynesian literature often assumes that prices are sticky while output adjusts instantaneously and costlessly to absorb demand shocks but the Marshallian approach assumes the opposite. Output does not adjust instantaneously due to a production lag and substantial adjustment costs.[39] In this framework, fast adjustments in prices and the profit share establish product market equilibrium for a given level of output. In a continuous-time setting, sluggish output adjustment can be approximated by assuming that output is predetermined at each moment and that firms choose the rate of growth of output, rather than the level of output. Then output growth is determined by comparing the costs and benefits involved in the output adjustment which in turn are determined by the labor market conditions and the profit signal in the goods market, respectively. Thus we can formulate:

$$\hat{Y} = h(\pi, e);\ h_\pi > 0,\ h_e < 0 \tag{40}$$

where e is the employment rate. A higher profitability induces firms to expand output more rapidly whereas the tightened labor market gives firms negative

[39] For instance, increases in production and employment require substantial search, hiring and training costs. Hiring or layout costs include not only explicit costs but also hidden costs such as a deterioration in industrial relations and morale.

incentives to expand production.[40] Assuming a fixed-coefficient Leontief technology, $Y = \min\{\sigma K, \nu L\}$, the employment rate can be expressed as: $e = \frac{Y/\nu}{\bar{L}}$, where ν is constant labor productivity and $\bar{L}$ is available labor force which exponentially grows at a constant natural rate n. From this definition,

$$\hat{e} = \hat{Y} - n \tag{41}$$

The definition of u yields:

$$\hat{u} = \hat{Y} - \hat{K} \tag{42}$$

Putting together (38) - (42), we get the following system of short cycles.

$$\hat{u} = h(\pi(\underset{+}{u}, \underset{+}{m}, \underset{+}{\alpha}), \underset{-}{e}) - \phi(\underset{+}{u} - u^*) \tag{43}$$

$$\hat{e} = h(\pi(\underset{+}{u}, \underset{+}{m}, \underset{+}{\alpha}), \underset{-}{e}) - n \tag{44}$$

When m and α are fixed, the system of (43) and (44) exhibits essentially the same dynamic properties as Skott (1989). As Skott shows, under plausible assumptions, the system of (43) and (44) ensures the existence of a steady growth equilibrium and the steady state is locally asymptotically unstable unless the negative effect of employment on output expansion is implausibly large. Once the boundedness of the trajectories is proved, the system (43) and (44) will generate a limit cycle *a la* the Poincare-Bendixson theorem (See Skott 1989, Appendix 6C for the proof).

For plausible values of parameters, the trajectory of g produced by (43) and (44) fluctuates around the natural rate n. In addition, the path of the utilization rate u fluctuates around the desired rate, u^*.[41] These aspects of the system of short cycles deserve attention because they justify our use of u^* and n as long-run averages of u and g in the system of long waves, respectively, in section 3.

Harrod (1939) defines the warranted growth rate as the ratio of the average saving rate to the desired capital-output ratio. In the present model, Harrod's warranted growth rate can be defined as $\frac{S}{K}$ evaluated at $u = u^*$:

$$g_w \equiv \left.\frac{S}{K}\right|_{u=u^*} = u^*\sigma - c_1[u^*\sigma - \delta - s_f(\pi u^*\sigma - \delta - rm)] - c_2 q \tag{45}$$

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[40] For more details about the behavioral foundation of (39), see Skott (1989, Ch.4).

[41] This result, the fluctuations of u around u^*, requires the calibration of the accumulation function (38) so that $\phi(0) = n$. If $\phi(0) \neq n$, the fixed point of u would be different from u^* and u would oscillate around that value. This case, though logically possible, is hardly interesting since the persistent deviations of the average value of u from u^* would deprive the desired rate of utilization (u^*) of any economic content.

(45) shows that the warranted growth rate depends on the profit share. Variations in the profit share make possible the adjustment of the warranted rate to the natural rate. The adjustment of the employment rate, as implied by (44), brings output growth in line with the natural rate.

5. Putting the pieces together: Long Waves and Short Cycles

This section puts all of the elements together in order to integrate long waves with short cycles and presents our simulation results.[42] Our full model of long waves and short cycles is a five dimensional dynamical system that consists of (12), (28), (31), (43), and (44). We have seen that (11), (28), and (31) provide a model of long waves, whereas (42) and (43) generate a mechanism of short cycles. [43]

[Figure 8 about here]

As seen in section 4, if m and α are fixed, (43) and (44) produce a limit cycle under plausible conditions. It can be shown that the resulting limit cycle exhibits a clockwise movement in e-u space, or alternatively, in e-π space. Figure 8 (a) presents an example of the limit cycle on the e-π space. The system of (11), (28) and (31), however, generates long waves of the debt-capital ratio (m) and the equity-deposit ratio (α), which are represented in Figure 5. As m and α change endogenously, the limit cycle in Figure 8 (a) breaks down and the clockwise movement of e and π spirals up to the northeast or down to the southwest, depending on the direction of changes in m and α. Figure 8 (b) illustrates this. The upward spiral from A to B represents a long expansion driven by increases in the debt-capital ratio and the equity-deposit ratio, whereas the downward spiral from B to A an economic downturn prompted by sharp decreases in m and α.

[Figure 9 about here]

During each long expansion, the profit share exhibits a strong upward movement with mild cyclical fluctuations around the trend (Figure 9 (a)). The similar

[42] Parameter values and functions used for this simulation are available in Appendix B. The simulation in this section is based on Case I in Table 1. Simulation results in other cases are available upon request.

[43] By using (25) as our definition of trend profitability based on $u = u^*$, the system of long waves becomes independent of that of short cycles, while the latter depends on the former. Issues regarding the relation between long and short cycles will be discussed in section 6.1.

pattern characterizes the movements in the profit rates (Figure 9 (b)). During crises, the rate of profit net of depreciation and interest payment ($\pi u\sigma - \delta - rm$) tumbles even to negative rates. Changes in the debt structure have large impacts on the real sector performance through its effect on profitability. This is prominently shown in the behavior of the employment rate (Figure 9 (c)). Figure 9 (d) depicts a trajectory of the rate of return on equity. During long booms, the rate of return on equity is strong and sound on average but during crises, it suddenly drops to significantly negative rates.

[Figure 10 about here]

Figure 10 (b) shows the growth rate of output where the Hodrick-Prescott filtered trend is added.[44] A financial sector induced crisis triggers a deep recession in the real sector which is reflected in the negative growth rates during periodic deep downturns. Capacity utilization and capital accumulation follow the pattern similar to that of output growth(Figure 10 (a) and (c)). Figure 10 (d), finally, plots the ratio of consumption to household income. The series follows the basic long waves/short cycles patten as shown in the profit share and the employment rate but the movement in the consumption/income ratio is noticeably smooth compared to other simulated series.[45]

6. Alternative specifications

6.1. Direct effect of financial variables on investment

This subsection introduces the direct effect of financial variables on investment. In our Harrodian framework, this can be achieved by assuming desired utilization depends on financial variables. Financial variables may affect firms' desired capital stock by changing the cost of finance. Two financial variables are of interest: Tobin's q and cash flow, denoted as c. An increase in q tends to reduce the cost of finance, thereby increasing firms' desired capital stock. This kind of traditional cost of capital channel may be captured by the inverse relationship between desired utilization and q. In imperfect capital markets, the level of cash flow may also affect the cost of finance because internal funds

[44] The filtered series is only for illustrative purpose since it simply smoothes the original series and it does not adequately capture asymmetric features and structural breaks in the original series.

[45] The long run behavior of consumption is closely related to the movement in household net worth to income ratio: $\frac{C}{Y^H} = \frac{c_1 Y^H + c_2 NW^H}{Y^H} = c_1 + c_2 \frac{NW^H}{Y^H}$ where $\frac{NW^H}{Y^H} = \frac{(1+\alpha)m}{u\sigma - s_f(\pi u\sigma - \delta - rm)}$.

are cheaper than external finance due to the existence of external finance premium or the financial accelerator (Bernanke, Gertler, and Gilchrist,1996). Thus imperfect capital markets may yield the inverse relationship between desired utilizatin and cash flow. Based on these considerations, the long-run accumulation function (8) is rewritten as

$$u = u^*(q, c), \quad u_q^* \equiv \frac{\partial u^*}{\partial q} < 0, \quad u_c^* \equiv \frac{\partial u^*}{\partial c} < 0, \tag{46}$$

and the short-run accumulation function (38) is correspondingly modified to

$$g = \phi\left(u - u^*(q, c)\right), \quad \phi'(\cdot) > 0, \quad \phi(0) = n \tag{47}$$

where $c = s_f(\pi u\sigma - \delta - rm)$ and $q = (1 + \alpha)m$.

Using equations, (11) and (48), the goods market equilibrium condition becomes:

$$c_1(u\sigma - \delta - c) + c_2 q + \phi\left(u - u^*(q, c)\right) + \delta = u\sigma \tag{48}$$

As long as $|u_c^*| \neq \frac{c_1}{\phi'}$, π can be written as a function of u, m and α with the aid of the implicit function theorem (Note that c is a function of π). In the new short-run investment specification, the expression for the Harrodian assumption - investment is more sensitive than saving to variations in the utilization rate - is slightly modified to

$$\frac{\partial(I/K)}{\partial u} = \phi' \cdot (1 + |u_c^*| s_f \pi\sigma) > (1 - c_1)\sigma + s_f \pi\sigma = \frac{\partial(S/K)}{\partial u} \tag{49}$$

Changes in the actual rate of utilization directly affect accumulation through the flexible accelerator mechanism. In addition, changes in actual utilization influence accumulation indirectly because they change the level of cash flow, which affect the desired utilization rate. The indirect effect reinforces the direct effect. Assuming the Harrodian condition (50), it can be shown that if the effect of cash flow on desired utilization is not too large, i.e. $|u_c^*| < \frac{c_1}{\phi'}$,[46] the profit share, π, is increasing in u, m and α.

$$\pi = \pi^*(u, m, \alpha), \quad \pi_u^* > 0, \quad \pi_m^* > 0, \quad \pi_\alpha^* > 0 \tag{50}$$

Furthermore, the examination of the partial derivatives of π with respect to its arguments reveals that the positive effects of u, m, and α on π are all

[46]This condition implies that saving is more sensitive than investment to variations in π. This condition is critical for the stability of (ultra) short-run product market equilibrium. Apart from the stability issue, the violation of this condition produces an empirically implausible result such as the profit share decreasing in capacity utilization.

stronger in the new investment specification than in the case of the constant rate of desired utilization. The main intuition for this result is that changes in u, m, and α have additional demand effects since increases in these variables tend to reduce desired utilization, thereby stimulating accumulation.

Trend profitability is obtained by substituting $u = u^*(q, c)$ into (49). We then have:

$$c_1(u^*(q,c)\sigma - \delta - c) + c_2 q + n + \delta = u^*(q,c)\sigma \tag{51}$$

where $c = s_f(\rho_T^* - \delta - rm)$ and $q = (1+\alpha)m$. If $|u_c^*| < \frac{c_1}{(1-c_1)\sigma}$,[47] then the trend rate of profit can be expressed as an increasing function of α and m:

$$\rho_T^* = \rho_T^*(m, \alpha), \quad \rho_{Tm}^* > 0, \quad \rho_{T\alpha}^* > 0 \tag{52}$$

It can be shown that as in the short run case, the new specification of long-run accumulation, (47), strengthens the expansionary effects of m and α on ρ_T in the original specification.

In this paper, the key characteristics of the trend rate of profit (25) and the actual profit share (24) are their positive dependence on m and α. It has been shown that these properties of these ρ_T- and π- functions remain unaffected by introducing the direct effect of financial variables on firms' accumulation behavior. Moreover, these expansionary effects get stronger as the direct financial effects on investment are allowed.

Simulation results are qualitatively similar to the baseline model but the new specification implies that desired capacity utilization gradually declines during a long boom and suddenly jumps as a crisis hits the economy. Not surprisingly, the deviation of the actual rate of utilization from the desired rate exhibits a pattern similar to that in the case of the fixed rate of desired utilization.

6.2. Relation between long waves and short cycles

In section 5, long waves were strictly separated from short cycles because trend profitability defined as (25) was not affected by changes in the actual utilization rate. This formal separability should not disguise the important role of short cycles in producing long waves. The long waves in the present model are generated based on the assumption that u^* and n provide a good approximation of the long run averages of actual movements in utilization and accumulation. This assumption can be justified only when the system of short cycles actually produces the fluctuations of u and g around u^* and n. The system of short

[47] Assuming the Harrodian assumption holds in the case of constant desired utilization, $|u_c^*| < \frac{c_1}{\phi'}$ implies $|u_c^*| < \frac{c_1}{(1-c_1)\sigma}$.

cycles in the present model does this job, thereby making firms' and bankers' long-term expectations consistent with the actual trajectories of the economy. If the system of short cycles fails to produce the fluctuations of u and g around u^* and n, this consistency requirement will not be satisfied. In sum, the system of short cycles is not subsidiary but it plays a pivotal role in providing firms and bankers with an anchor of their long-term expectations of corporate profitability.

The assumption that trend profitability is not affected by acutal utilization, however, appears to be strong. The formal separation of long waves from short cycles ceases to hold once changes in the actual capacity utilization rate affect trend profitability. Then the question is, how robust are the analytic results in the previous sections once this strict separation is relaxed?

Any reasonable definition of trend profitability requires a conceptual distinction between the long run and the short run components of actual movements in the profit rate. The actual profit rate is defined as $\rho \equiv \pi(u, m, \alpha)u\sigma$ where the definition of $\pi(u, m, \alpha)$ is given in (23). The actual rate of profit is affected by changes in financial practices (m or α) and changes in the utilization rate (u). The approach taken in the earlier part of this paper treats changes in u as entirely short-run cyclical factors while it considers changes in m and α as long-run factors. This was the basis of the definition of trend profitability in equation (25), but one may argue that changes in the utilization rate contain both trend and cyclical components. For instance, suppose that the trend utilization rate, denoted as u_T, follows an averaging process given by (54).

$$\dot{u}_T = \mu(u - u_T) \tag{53}$$

where μ represents the adjustment speed. This adjustment process yields an alternative measure of trend profitability, (ρ_T^A):

$$\rho_T^A \equiv \pi(u_T, m, \alpha)u_T\sigma \tag{54}$$

The constant desired utilization rate (u^*) is replaced by the trend utilization rate (u_T). If the long-run evolution of firms' liability structure is determined by ρ_T^A rather than ρ_T, then the system of long waves is no longer strictly separable from that of short cycles because the actual utilization rate affects the trend utilization rate, which in turn influences trend profitability in (55).

Analytic results depend on the value of μ. Two polar cases of (54) are of interest: (i) $\mu \to 0$ with $u_T = u^*$ and (ii) $\mu \to \infty$. It is readily seen that case (i) is equivalent to the approach taken in the earlier part of this paper, which leads to the strict separability of long waves from short cycles. Case (ii), $\mu \to \infty$, implies that the trend utilization rate instantaneously adjusts to the actual rate and thus trend profitability is always equal to the actual profit rate. Thus case (ii) undermines the conceptual distinction between trend and actual profitability.

If the value of μ is small enough, one can get a sufficiently smooth trend of capacity utilization and trend profitability given by (55). The analytic results based on the assumption that $u = u^*$ in the previous sections remain unaffected if μ is sufficiently small. The trend rate of utilization gradually adjusts itself toward actual utilization but the link between the trend rate of utilization and the desired rate will not break down because the interaction between the goods market and the labor market in the short cycle system will prevent actual utilization from diverging away from desired utilization indefinitely. Thus both u_T and u will fluctuate around u^*.

7. Conclusion

The U.S. economy is going through a deep recession triggered by the biggest financial crisis since the Great Depression. A Minskian perspective suggests that the explanation of this crisis should be found in endogenous changes in financial fragility.

This study has modeled a Minskian theory of long waves. The model clarifies the underlying mechanism of endogenous changes in financial fragility and the interaction between real and financial sectors. At a theoretical level, the study provides a promising way of integrating two types of instability principles: Minsky's Financial Instability Hypothesis and Harrod's Instability Principle. While both principles provide a source of cycles, they have distinct frequencies and amplitudes in this model. The Minskian instability hypothesis creates long waves and the Harrodian instability principle produces short cycles. The limit to the upward trend created by Minskian instability is imposed by financial crisis, while explosive trajectories implied by Harrodian instability are contained by stabilizing labor market dynamics.[48] When two principles are combined into a coherent stock-flow consistent framework, the proposed pattern of long waves and short cycles emerges.

A purely mathematical model of this kind may clarify the logic of interactions but clearly has many limitations. The depth of the current crisis and the time

[48] The following quote from Minsky (1995, 84) is suggestive: "As reasonable values of the parameters of the endogenous interactions lead to an explosive endogenous process, and as explosive expansions and contractions rarely occur, then constraints by devices such as the relative inelasticity of finance or an inelastic labor supply need to be imposed and be effective in generating what actually happens."

needed to initiate a new cycle depend on institutional and policy dimensions. Minsky devotes a large part of his analysis to the institutional and historical developments of financial markets and policy responses. Thus, the patterns of long waves are heavily affected by these elements. The full account of long waves and crises is possible only when one takes a serious look at these dimensions.

Disregarding the historical contingencies of actual movements, it may be useful to extend the model in a number of directions. First, it may be desirable to give a more detailed treatment to banks' behavior. The assumption of the fixed real interest rate may be too simple to characterize banks' pricing behavior over the course of cycles. The financial sector may have to be disaggregated to address the issues regarding securitization, a key aspect of the recent financial crisis. Bankers' perception of tranquility is possibly affected by their own profitability. Next, this paper did not explore the implications of households' indebtedness. Instead, it has focused on an increasing share of stocks (riskier asset) in households' financial wealth as an indicator of increasing fragility in the household sector. It would be interesting to see the effect of the introduction of the evolution of household debt into the model. Third, the proposed model is inflation neutral in the sense that the decisions on real quantities such as investment, consumption and output expansion are made with no reference to inflation and the banking sector holds the real interest rate at a constant level. In some account of Minskian ideas (e.g Fazzari et al., 2008), changes in the inflation rate play an important role. Finally, the assumption of a closed economy in this paper is another major limitation. Unfettered international capital flows, in contrast to the belief of its proponents, have created growing instability and global imbalances (Blecker, 1999). Several authors suggest that Minsky's theory can be extended to an international context (e.g. Wolfson, 2002, and Arestis and Glickman, 2002), but few attempts have been made to formalize the ideas and to propose precise mechanisms behind them. Addressing these issues is left for future research.

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Appendix A: Proof of Proposition 1

To prove the existence of a limit cycle for the system of (11), (28), and (31), we need to show that the Jacobian matrix (34) evaluated at ($m(\lambda)$, $\alpha(\lambda)$, $z(\lambda)$, λ), where ($m(\lambda)$, $\alpha(\lambda)$, $z(\lambda)$) is a fixed point of the system,[49] should have the following properties:

- The Jacobian matrix has a pair of complex conjugate eigenvalues $\beta(\lambda) \pm \theta(\lambda)i$ such that $\beta(\lambda^*) = 0$, $\theta(\lambda^*) \neq 0$, and $\beta'(\lambda^*) \neq 0$ and no other eigenvalues with zero real part exist at ($m(\lambda^*)$, $\alpha(\lambda^*)$, $z(\lambda^*)$, λ^*)

where λ^* is a Hopf bifurcation point.

To apply the above condition for the Hopf bifurcation to the current context, we will use the fact that the Jacobian matrix will have a negative real root and a pair of pure imaginary roots if and only if:

(R1) $\mathbf{Tr}(\mathbf{J}) = \mathrm{F}_m + \mathrm{G}_z < 0$

(R2) $\mathbf{J_1} + \mathbf{J_2} + \mathbf{J_3} = \mathrm{F}_m\mathrm{G}_z - \zeta' \cdot \mathrm{G}_\alpha > 0$

(R3) $\mathbf{Det}(\mathbf{J}) = -\zeta' \cdot (\mathrm{F}_m\mathrm{G}_\alpha - \mathrm{F}_\alpha\mathrm{G}_m) < 0$

(R4) $-\mathbf{Tr}(\mathbf{J})(\mathbf{J_1} + \mathbf{J_2} + \mathbf{J_3}) + \mathbf{Det}(\mathbf{J}) = -(\mathrm{F}_m + \mathrm{G}_z)(\mathrm{F}_m\mathrm{G}_z - \zeta' \cdot \mathrm{G}_\alpha) - \zeta' \cdot (\mathrm{F}_m\mathrm{G}_\alpha - \mathrm{F}_\alpha\mathrm{G}_m) = 0$

Let us denote the eigenvalues of the Jacobian matrix as $\mu(\lambda)$ and $\beta(\lambda) \pm \theta(\lambda)i$.

Proof of (I). Suppose that $\mathrm{G}_\alpha < 0$. Then (R3) is always met. In order to satisfy (R1) and (R2), we should have $\mathrm{G}_z < \min\left\{|\mathrm{F}_m|, \frac{\zeta'|\mathrm{G}_\alpha|}{|\mathrm{F}_m|}\right\}$. (R4) is quadratic in G_z. (R4) can be rewritten as:

$$a_1\mathrm{G}_z^2 + a_2\mathrm{G}_z + a_3 = 0 \tag{A1}$$

where

$$\begin{aligned} a_1 &\equiv -\mathrm{F}_m > 0 \\ a_2 &\equiv -(\mathrm{F}_m^2 - \zeta'\mathrm{G}_\alpha) \lesseqgtr 0 \\ a_3 &\equiv \zeta'\mathrm{F}_\alpha\mathrm{G}_m < 0 \end{aligned}$$

Solving (A1) for G_z, we obtain one negative and one positive real roots. Let us select the negative root[50], which is given as:

[49] Note that in our case the fixed point is independent of the value of λ.
[50] It can be shown that the positive root is irrelevant for the analysis.

$$b \equiv \frac{(|\mathrm{F}_m|^2 - \zeta'\mathrm{G}_\alpha) - \sqrt{(|\mathrm{F}_m|^2 - \zeta'\mathrm{G}_\alpha)^2 + 4\zeta'|\mathrm{F}_m||\mathrm{G}_m|\mathrm{F}_\alpha}}{2|\mathrm{F}_m|} < 0 \tag{A2}$$

Since $\mathrm{G}_z = \kappa_{r^e}\frac{\partial r^e}{\partial z} - \lambda$, the value of λ that satisfies (R4) is: $\lambda = \kappa_{r^e}\frac{\partial r^e}{\partial z} + |b|$. Let $\lambda^* \equiv \kappa_{r^e}\frac{\partial r^e}{\partial z} + |b|$. We have shown that if $\mathrm{G}_z < \min\left\{|\mathrm{F}_m|, \frac{\zeta'|\mathrm{G}_\alpha|}{|\mathrm{F}_m|}\right\}$ and $\lambda = \lambda^*$, then the Jacobian matrix has a negative real root and a pair of imaginary roots: $\mu(\lambda^*) < 0$, $\beta(\lambda^*) = 0$, and $\theta(\lambda^*) \neq 0$. To prove λ^* is indeed the bifurcation point, we still need to show that $\beta'(\lambda^*) \neq 0$. To prove $\beta'(\lambda^*) \neq 0$, let us use the following fact:

$$\begin{aligned}
&\mu(\lambda) + 2\beta(\lambda) = \mathrm{F}_m + \mathrm{G}_z \\
&2\mu(\lambda)\beta(\lambda) + \beta(\lambda)^2 + \theta(\lambda)^2 = \mathrm{F}_m\mathrm{G}_z - \zeta' \cdot \mathrm{G}_\alpha \\
&\mu(\lambda)[\beta(\lambda)^2 + \theta(\lambda)^2] = -\zeta' \cdot (\mathrm{F}_m\mathrm{G}_\alpha - \mathrm{F}_\alpha\mathrm{G}_m)
\end{aligned}$$

Totally differentiating both sides with respect to λ, we get

$$\begin{bmatrix} 1 & 2 & 0 \\ 2\beta(\lambda) & 2[\mu(\lambda) + \beta(\lambda)] & 2\theta(\lambda) \\ [\beta(\lambda)^2 + \theta(\lambda)^2] & 2\mu(\lambda)\beta(\lambda) & 2\mu(\lambda)\theta(\lambda) \end{bmatrix} \begin{bmatrix} \mu'(\lambda) \\ \beta'(\lambda) \\ \theta'(\lambda) \end{bmatrix} = \begin{bmatrix} -1 \\ |\mathrm{F}_m| \\ 0 \end{bmatrix} \tag{A3}$$

The right hand side of (A3) is obtained using the fact that $\frac{\partial \mathrm{G}_z}{\partial \lambda} = -1$ and λ does not affect all other partial derivatives than G_z. Evaluating (A3) at $\lambda = \lambda^*$, we have:

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 2\mu(\lambda^*) & 2\theta(\lambda^*) \\ \theta(\lambda^*)^2 & 0 & 2\mu(\lambda^*)\theta(\lambda^*) \end{bmatrix} \begin{bmatrix} \mu'(\lambda^*) \\ \beta'(\lambda^*) \\ \theta'(\lambda^*) \end{bmatrix} = \begin{bmatrix} -1 \\ |\mathrm{F}_m| \\ 0 \end{bmatrix}$$

Solving this for $\beta'(\lambda^*)$, we finally get:

$$\beta'(\lambda^*) = \frac{2\mu(\lambda^*)\theta(\lambda^*)|\mathrm{F}_m| - 2\theta(\lambda^*)^3}{4\mu(\lambda^*)^2\theta(\lambda^*) + 4\theta(\lambda^*)^3} < 0 \quad \text{since} \quad \mu(\lambda^*) < 0$$

Thus, $\beta'(\lambda^*)$ is strictly negative.

Proof of (II). Suppose that $\mathrm{G}_\alpha > 0$ and $\mathrm{G}_z < 0$. Then (R1) is always satisfied. To meet (R2) and (R3), we need $\mathrm{G}_\alpha < \min\left\{\frac{|\mathrm{F}_m||\mathrm{G}_z|}{\zeta'}, \frac{\mathrm{F}_\alpha|\mathrm{G}_m|}{|\mathrm{F}_m|}\right\}$. The rest of the proof is essentially the same as that of (I).

Proof of (III). Routh-Hurwitz necessary and sufficient conditions for the local stability of a three dimensional system are (R1), (R2) and (R3) with replacing the equality in (R4) by the inequality: $-\mathbf{Tr}(\mathbf{J})(\mathbf{J_1}+\mathbf{J_2}+\mathbf{J_3})+\mathbf{Det}(\mathbf{J}) > 0$. Suppose that $\mathrm{G}_\alpha > 0$ and $\mathrm{G}_z > 0$. Then (R2) is always violated and the fixed point is unstable. At the same time, since (R2) is not met, it is impossible to get a limit cycle *a la* the Hopf bifurcation.

Appendix B: Functions and Parameter Values in Simulation

$$g = \gamma_0 + \gamma_1 u \tag{B1}$$

$$\frac{I}{K} = g + \delta \tag{B2}$$

$$\hat{Y} = h(\pi, e) = h_0 + \frac{h_1}{1 + \exp[-h_2(\pi + h_3 \ln(h_4 - e) + h_5))]} \tag{B3}$$

$$\dot{m} = \tau\left(\frac{\rho_T}{rm}\right) = \tau_0 + \frac{\tau_1 - \tau_0}{1 + \exp[-\tau_2\left(\frac{\rho_T}{rm} - \tau_3\right)]} \tag{B4}$$

$$\text{where } \rho_T = \pi(u^*, m, \alpha)u^*\sigma \text{ and } u^* = \frac{1}{\gamma_1}(n - \gamma_0)$$

$$\dot{\alpha} = \zeta(z) = \zeta_0 + \frac{\zeta_1 - \zeta_0}{1 + \exp[-\zeta_2(z - \zeta_3)]} \tag{B5}$$

$$\dot{z} = \kappa\left(r^e|_{u=u^*} - r, \alpha\right) - \lambda z = \kappa_0 + \kappa_1(r^e|_{u=u^*} - r) - \kappa_2\alpha - \lambda z \tag{B6}$$

$$\text{where } r^e|_{u=u^*} = \frac{\rho_T - \delta - rm + (1+\alpha)(\dot{m} + mn) + \dot{\alpha}m - n}{\alpha m}.$$

Table 2: Parameter Values

γ_0	γ_1	h_0	h_1	h_2	h_3	h_4	h_5	σ
-0.93	1.2	-0.03	0.085	40	0.4	1.1	0.435	0.5
n	δ	s_f	r	c_1	c_2	κ_0	κ_1	κ_2
0.03	0.1	0.75	0.03	0.8	0.04	0.083	0.03	0.08
λ	ζ_0	ζ_1	ζ_2	ζ_3	τ_0	τ_1	τ_2	τ_3
0.04	-0.22	0.025	55	-0.069	-0.125	0.0055	25	8.4