An Optimal Voting Rule for Multilateral Financial Institutions

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March 11, 2010

Abstract

Governance in multilateral financial institutions is based on a quota system, and usually modeled after the one used in the International Monetary Fund. Among other functions, quotas affect contributions to the institutions and voting rights. I derive an optimal voting rule for these institutions, and show conditions for it to be a weighted voting rule, as determined by the quota system. The model predicts that a country's votes be calculated from three components. First, a weighted sum of trade flows with other members with weights proportional to a) the probabilities of each trade partner suffering a balance of payments crisis (for the IMF), or b) the probabilities of each trade partner experiencing a potential increase in TFP requiring outside financing (for development banks). Second, the ratio of the country's GDP PPP to GDP, and finally the ratio of total labor in efficiency units to total population. The model shows how the total level of resources of multilateral institutions should evolve relative to world trade. For the case of the IMF, it predicts that voting thresholds should increase with the importance of moral hazard inefficiencies associated with lending.

1 Introduction

A few years ago the International Monetary Fund seemed to be heading for irrelevance, its lending unnecessary in a world flush with private capital, and its advise despised by most of its members. The sudden crunch in private capital flows after the collapse of Lehman Brothers in September 2008 was a powerful reminder of the importance of an international lender of last resort, and interest in the IMF revived. In March 2009 the G20 decided to boost its capital, promising an extra 500 billion USD, to triple its lending

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capacity. Since developing countries perceive that IMF governance gives rich countries disproportionate power, they will likely continue to self-insure by building up huge foreign-exchange reserves. Therefore, the big emerging economies in the G20 made clear that they want to see detailed proposals on changes in voting power within the IMF, and they want the governance reform process speeded up.

The aim of this paper is to characterize the optimal voting rule for the IMF, thus providing a normative benchmark that might help in framing the discussion on governance reform. The model used serves several purposes. First, it proves that weighted voting, as determined by the quota system used since the foundation of the IMF in 1944, is indeed optimal. This is a fundamental result since it narrows the dimensions of the political discussion to the method used to calculate members' quotas. Thus, more complex proposals to aggregate members' preferences, as double majority systems, can be disregarded.[1]

Second, a formula to calculate members' optimal voting weights is derived. It differs significantly from the one currently used, the difference exacerbated by the evolution of the international financial system after the collapse of the Bretton Woods Agreement in 1973. This increased divergence from desirable governance likely explains the widespread criticism of the quota system of recent decades.[2] Although the recent return to a single formula to calculate quotas increases governance transparency, the representativeness and efficiency of the IMF will continue to be undermined as long as calculations are not consistent with an efficiency criterion.

Third, the model allows the determination of *total* quotas. This provides an objective measure of the volume of financial resources that the IMF should manage, and how this should relate to the conditions of the world economy. In particular the model rationalizes why industrial country quotas decreased by more than 50% relative to world exports between 1970 and 1975 and remained stable afterwards.

Modeling the IMF as an institution in which each member has a representative who votes on behalf of the citizens of her country, I adapt Barberà and Jackson (2006) model of optimal voting rules for a heterogeneous union. Votes at the IMF take place over two alternatives: whether or not to bailout a member country in crisis.[3] Therefore the status quo is not to provide the bailout, and the bailout produces a change that affects members' welfare. An optimal voting rule seeks to maximize a welfare function that takes into account the utilities of all citizens represented in the IMF. I show that, under general conditions, an optimal voting rule consists of a weight for each country's vote and a threshold that indicates how large the total weight of votes cast in favor of a bailout must be in order to implement the policy.

When making a decision for a particular vote, members of the IMF weight the benefits and costs that a bailout would have on their citizens. Benefits are assumed to relate to

[1] See, for example, O'Neill and Peleg (2000) and Rapkin and Strand (2006).

[2] Bird and Rowlands (2005) argue that the post-Bretton Woods IMF makes the use of quotas for the simultaneous determination of contributions, access and voting rights untenable.

[3] The formalization generalizes in a straightforward way to cases in which a number of member countries face a crisis simultaneously and the decision is whether to assist all of them, a subset of them, or none. Although the IMF performs other roles besides lending, as long as they do not involve making large disbursements of scarce resources, they could be resolved by a different voting mechanism and are therefore irrelevant to the problem studied in this paper.

trade linkages, and costs are assumed to originate from capital injections into the IMF.[4] When benefits outweigh costs a country will vote in favor of a bailout, and when the costs are larger than the benefits, the country will oppose a bailout. Votes are weighted by the ex-ante average intensity of preferences of citizens with respect to the choice they make. If the tally of weighted votes in favor of the bailout is larger than the one against it the bailout is approved. Thus, votes are determined by the "externalities" that a given bailout has on the welfare of other members, and financial aid is approved only if the average positive effects dominate the negative ones.

To determine how a crisis abroad affects the welfare of a member country's citizens, I use a variant of the Eaton and Kortum (2002) Ricardian model of trade to estimate the benefits accruing through trade linkages. Providing bailouts increases the likelihood that members would need to provide the IMF with capital injections in the future. Members that are experiencing tight fiscal positions, or a conflict of interest with their domestic political agenda, might oppose the bailout in order to avoid making contributions.

Two institutional arrangements are considered, whether the county requesting a bailout is allowed to cast a vote or not. The first case is optimal ex post as it incorporates the welfare of the country experiencing the crisis, and therefore benefiting from the bailout. Nevertheless it might be the case that from an ex ante perspective, and taking into account how countries' policies affect their probability of suffering a crisis, it might be optimal to exclude the country requesting a bailout from the subsequent vote.[5]

The model predicts that quotas should be determined by three components. First, the weighted sum of a country's trade volume with the rest of the members of the organization, with weights related to the probabilities of each trade partner suffering a crisis. Second, the ratio of the country's GDP converted at PPP exchange rates to GDP converted at market exchange rates. And finally, the country's average years of schooling. Two polar cases of interest are a world where every country has the same probability of experiencing a crisis, and a world where a subgroup of countries do not experience shocks, while the rest face the same risk. The first case serves as an approximation to the situation between the creation of the IMF and the breakdown of the Bretton-Woods fixed exchange rates regime in the seventies. The second case reflects the current situation with a divide between net creditors and net debtors members. An extension that allows the possibility that financial aid from the IMF has moral hazard effects on members policies indicates that the threshold of votes needed to approve a bailout should be increasing in the severity of the moral hazard problem.

The results derived can be extended to study optimal governance in regional development banks. In this case it would not be that the bank supplies assistance to a country suffering a sudden crisis, but determining which public investments opportunities to fund. Thus the same formulation for trade linkages can be applied, with regional bank members evaluating their private trade gains with the potential receiver of a loan.

In recent years there have been many reform proposals that focused on different aspects

[4]Benefits could also relate to capital flows. Since bilateral capital and trade flows are highly correlated there is not much loss of generality in restricting attention to trade. See Lane and Milesi-Ferretti (2006).

[5]Rules for decision making when the IMF has to borrow funds restrict the country requesting financial assistance from the vote. See International Monetary Fund (2007).

of IMF governance. Buira (2005), among others, calls for using PPP measures of GDP in the current quota formulas as a way to increase participation of developing countries and thus improve the Fund's "legitimacy". This suggestion was the subject of heated debate, and eventually incorporated in the reform proposal approved in 2008. Vaubel (2005) identifies as a problem the separation between the ultimate principals and the IMF executive directors, Woods (2005) calls for an increase in accountability, something that Frey and Stutzer (2006) suggest can be achieved through citizen participation in decision making. To the best of my knowledge, none of these, and other, reform proposals has considered an efficiency criterion to determine an optimal voting rule for the IMF.

The remainder of the paper is structured as follows. Section 2 summarizes the history and salient features of the IMF governance, including a reform of the quota system approved in April 2008. Section 3 describes the model, and compares quota distribution in the two polar cases described above. Section 4 discusses the effect of moral hazard. Section 5 concludes.

2 The IMF

The IMF is a financial institution founded in 1944 with the main purpose of assisting members facing temporary balance of payments problems. From an initial membership of 44 states, today almost all the countries in the world participate in it. Members of the IMF do not have equal power. They contribute a quota subscription of financial resources, and this quota is the basis for determining voting power. Historically, quota allocations have been based mainly on economic size and external trade volume.

The role of the IMF has changed since the fall of the system which has brought the organization into life: the Bretton Woods Agreement. Before 1973, all members were more or less equally likely to request financial assistance. For example, from 1947 to 1967 industrial countries represented almost 70% of the total amount of resources withdrawn, close to the amount of resources they had contributed to the IMF. After the liberation of the world exchange rate regime in 1973, the main users of the IMF resources became emerging economies in Africa and Latin America with balance of payments crisis. This has widened the already divergent preferences between more or less developed countries on policy issues, and the disagreement over how this preferences are to be aggregated into collective decisions, i.e. about the methods to calculate quota allocations. In response to this concern, and also in the face of mounting criticism from academics and policymakers, the IMF embarked in September 2005 on a large-scale program of modernization. Salient among its objectives was governance reform, including adjusting quota shares to "reflect better the relative weight of members in the world economy". And in April 2008, a reform proposal representing a step in this direction was approved.

Decisions in the IMF are made by weighted majority of votes. The power structure is organized in the following way: the Board of Governors, which possesses all the powers of the Fund, is composed by representatives from all member countries. Each country initially received 250 basic votes plus one additional vote for each hundred-thousand Special Drawing Rights (SDRs) that it possesses. The basic votes were a compromise

solution intended to reconcile the principle of sovereign equality with the fact of wide power asymmetries among members. The ratio of basic votes to total votes increased first, as new countries joined the IMF, reaching a historic high of 15.6% in 1958. Total quota increases thereof made this ratio decrease to roughly 2%, while the quota reform approved in April 2008 increased basic votes and introduced a mechanism to stabilize their number at 5.5% of total votes.[6] While initially there was a single formula for the calculation of quotas, in the early 1960s a complex multi-formula method was devised to determine quotas on the basis of GDP, exports and imports, variability of export receipts and reserves. While this allowed differing weights for the calculation of quotas for developed and developing countries, there was a significant loss of transparency. For a while ten formulas were used, later to be replaced by five, a system that was in place up to the reform of 2008 that returned to a single formula. There is no explicit rationale for using these and not other variables, nor for the weights attached to them in the mentioned formulas. In fact it appears that the formulas, as well as actual quotas, which in some cases diverge significantly from the calculated ones, are biased to produce a political outcome close to the one desired by the most powerful members of the IMF.[7] One of the objectives of the reform of 2008 is to increase the credibility of the IMF by increasing the transparency of the quota determination process, and by realigning actual quotas with calculated ones.

The Board of Governors can delegate certain decisions to the Board of Executive Directors, which is composed of one representative from each of the five members of the Fund having the largest quotas plus 19 other representatives, some of whom represent a certain subgroup of countries. Thus, each Executive Director has the number of votes equal to the sum of votes of the countries it represents. In this way, when the Board of Directors vote, there is at the beginning a first meeting in which each subgroup of countries determines how their representative will vote. Then, the Board of Executive Directors meets and cast their vote. There are different majority rules, including two supermajority provisions, and their use depends on the issue being discussed at the moment. A 70% majority, is required for issues of procedure (decisions involving matters of policy and operations) and an 85% majority, is required for issues of substance (for example, constitutional revisions or changes in quotas). An important observation of these majority rules and voting system is that the United States is the only country that retains a veto power since it possesses more than 15% of total quotas.

The quota system serves several functions, which creates the possibility of conflict, as pointed by Bird and Rowlands (2005). A member's quota defines four aspects of the relationship between the member country and the IMF: first, the amount of financial resources that members must contribute to the Fund; second, the amount of resources that they can draw from the IMF; third, their voting power in institutional decision making, and finally, the members' share of SDR allocations. After the breakdown of the Bretton Woods system of exchanges rates the IMF found it impossible to attain these multiple objectives with this single instrument. More developed capital markets made countries'

[6] In subsection 2.1 I will review in more detail the quota reform approved in April 2008.

[7] Mikesell (1944) acknowledges that the original Bretton Woods formula used to assign quotas among the first 44 members of the IMF were built with the objective to match a desired outcome.

request of financial assistance substantially larger than the quota structure had foreseen, as capital account imbalances grew significantly. And the IMF became bifurcated with the distinction of two types of members: rich country net lenders, and poor country net borrowers. This increased the conflict on using the quotas as a means of determining simultaneously contributions and access.

Related to the question of how are quotas determined, since its foundation, the IMF recognized that as it was going to make large disbursements of scarce financial resources, their decisions would have to be legally binding rather than merely advisory. More egalitarian decision methods, say a one-country, one-vote rule, would not be acceptable to the major powers that contribute the bulk of the IMF's resources. Accordingly, a scheme was devised by which each national member of the IMF has a quota that roughly equates its voting power to its financial subscription to the organization.[8]

General Quota Reviews are typically undertaken at five-year intervals with the objective of adjusting to changes in members' relative position in the world economy, as well as to accommodate new members. Each quota increase is divided at the discretion of the Board into an equiproportional and selective components. The former is akin to an expansion of capital, simply extending proportionally the existing quotas, while the latter tends to shift the new quotas towards the calculated ones. Since historically the equiproportional component has averaged 70% of the quota increases, there has been a significant status quo in the distribution of power in the IMF.

2.1 Recent Governance Reforms

In March 2008 the Executive Board presented a reform proposal of its quota system that a month later was approved by the Board of Governors. The proposal was the final product of extensive discussions at the Executive Board along the guidelines set in the Fund's Annual Meeting in Singapore in September 2006, and aimed at realigning members quota shares with their relative economic weight. The participation and voice of low-income countries was enhanced through a substantial increase in the number of basic votes, and a mechanism that will maintain the ratio of basic votes to total votes constant in the future. Furthermore, the IMF will seek to make quotas and voting shares more responsive to changes in economic realities in future General Quota Reviews.

A salient characteristic of the reform is that quotas are once again calculated using a simple formula. This improves the transparency of governance at the IMF and helps in better reflecting the members' relative position in the global economy. In coming up with this formula the Board has taken into account a number of restrictions, which include the multiplicity of roles that quotas have, that they be based on available data, and that they be politically feasible. The new formula proposed includes four economic variables, GDP, openness, variability and reserves, expressed in shares of global totals. The weighted average is then compressed in order to reduce dispersion in calculated quota shares,

$$ICQ = (0.5Y + 0.3CC + 0.15V + 0.05R)^k$$

[8]The presence of basic votes introduces a wedge between financial subscription and voting power, which is significant mostly for less developed countries.

were ICQ is the intermediate calculated quota share, Y is a weighted average of GDP converted at market exchange rates and PPP exchange rates averaged over a three year period. The weight on market-based GDP is 60%. CC is the annual average of the sum of current payments and current receipts for a five year period. V is the variability of current receipts and net capital flows, measured as a standard deviation from the centered three-year trend over a thirteen year period. R is the average over a year of official reserves, and $k = 0.95$ is a compression factor. Calculated quotas are obtained after rescaling the sum of intermediate calculated quota shares to 100.

The computation of GDP both at market rates and at PPP rates reflects a compromise between the position of developing countries which supported the later as a better estimate of the relative volume of goods and services produce by their economies, and the position of developed countries which see market rate GDP as the relevant indicator, particularly as a measure of the contributive power of members. The compression factor is an artifact to moderate the dispersion of calculated quotas. Given the limitation of the new quota formula in enhancing the voice of emerging economies, a tripling of basic votes was decided, a measure that increases vote share beyond quota share for less developed nations. The inclusion of PPP GDP and the compression factor were the more controversial aspects of the reform proposal and the Executive Board has decided to include them in the formula for a period of 20 years. At the end of this period the argument for retaining these components would be reviewed. The quota formula only calculates the relative quota of IMF members. Total quota determination continues to be discretionally decided at General Quota Reviews.

3 Model

There are n countries in the IMF, which are heterogeneous in terms of population, wealth, and integration to the world economy. Country i has a population of N_i citizens, all of whom derive the same utility from per capita consumption of final goods, c_i. It is known that, with some probability, a subset of the member countries will suffer a negative balance of payments shock, in which case a decision will have to be made on whether to financially assist the affected country, or countries. For ease of exposition I will assume that only one country, j, might experience a balance of payments shock, and later show that the analysis extends to a multi-country crisis. A state of the world is then a description of members' preferences on whether to assist or not country j. Without loss of generality utilities can be normalized to zero if the status quo prevails and no assistance is provided, and preferences are then denoted by a vector $\vec{u}(j) \in \mathcal{R}^n$ with element $u(j)_i \equiv u_{ij}$ being the utility of a representative agent in country i if country j is bailed out (thus $u_{ij} \equiv c_i^{jb} - c_i^{jn}$ were superscripts differentiate consumption when country j is bailed out or not).

After a shock takes place, each country's representative will decide to vote for a bailout or not, based on whether the utility of a bailout is positive or negative for that country's citizens. Thus the representative's voting behavior can be represented by a function $h_i : \mathcal{R} \longrightarrow \{b, n\}$, which maps the preferences of citizens into a vote. The notation $h_i(u_{ij}) = b$ indicates that the representative of country i votes in favor of a bailout. This

indicates that $u_{ij} > 0$, and equivalently a no bail out vote, $h_i(u_{ij}) = n$, indicates that $u_{ij} < 0$.

In a second stage, the votes of the representatives are aggregated according to a voting rule. Let $v : \mathcal{R}^n \longrightarrow \{0, \frac{1}{2}, 1\}$ denote the outcome of this two-stage voting procedure as a function of the state of the world, $\vec{u}(j)$. Here $v(\vec{u}(j)) = 1$ indicates that a bailout is approved, $v(\vec{u}(j)) = 0$ means that country j will not be assisted, and $v(\vec{u}(j)) = \frac{1}{2}$ denotes a tie that will be resolved by the toss of a coin.

Let an efficient voting rule be one that maximizes the expected social welfare function among the class of feasible voting rules.[9] The social welfare function is given by the expected total utility, giving equal weight to any citizen of the IMF, independent of the country of residence. Therefore I will consider voting rules that maximize the following welfare function:

$$E\left[\sum_i v(\vec{u}(j))N_i u_{ij}\right]$$

were the expectation is taken over the distribution of balance of payments shocks, given by $\mu(\cdot)$, affecting any subgroup of the countries in the IMF. It will be initially assumed that the probability and severity of a crisis in country j, is independent of the policies it follows, i.e. that $v(\cdot)$ has no effect on $\mu(\cdot)$. I will later lift this assumption, and study how the optimal rule is determined when the expectation of a bailout affects domestic policies and thus the probability distribution of balance of payments shocks.

We start by characterizing the first best outcome, i.e. the case in which the underlying preferences, u_{ij} are publicly observed (or perfectly inferred from the state of nature). In this case it is trivial to see that the bailout decision rule should be,[10]

$$v^E(u) = \begin{cases} 1 & \text{if} \quad \sum_i N_i u_{ij} > 0, \\ 0 & \text{if} \quad \sum_i N_i u_{ij} < 0, \\ \frac{1}{2} & \text{if} \quad \sum_i N_i u_{ij} = 0. \end{cases} \tag{1}$$

Of more interest is the case in which the intensity of preferences for a given choice are private information of each country. For this case consider the following voting rule, proposed by Barberà and Jackson (2006) in a similar context where they study the optimal voting rule in a union where citizens of member countries differ on their preferences over policy. For each country, and for each possible state, two weights are assigned, one when the country votes in favor of bailing out country j, and another for votes against this. For the former we have,

$$\omega_{ij}^b = N_i E[u_{ij} | u_{ij} > 0, j].$$

Therefore the weight assigned to country i is proportional to the total expected welfare of its citizens when a bail-out of country j is indeed their preferred policy. Similarly, the

[9] These are voting rules that depend only on the information obtained from the votes of the representatives.

[10] Since there is no private information in this case there is no need for a vote to take place. Note the implicit assumption, made here and henceforth, that a decision is made even if a subset of countries oppose it. This assumes either that there is an external enforcement mechanism or if that decision-making in the IMF satisfies a self-enforcement constraint. See Maggi and Morelli (2006) for an analysis of self-enforcing voting in international organizations.

weight assigned to country i when it votes against the bail-out is given by,

$$\omega_{ij}^n = -N_i E[u_{ij}|u_{ij} < 0, j].$$

The efficient voting rule $v^E(u)$ is then defined by,

$$v^E(\vec{u}(j)) = \begin{cases} 1 & \text{if} \quad \sum_{i:h_i(\vec{u}(j))=b} \omega_{ij}^b > \sum_{i:h_i(\vec{u}(j))=n} \omega_{ij}^n, \\ 0 & \text{if} \quad \sum_{i:h_i(\vec{u}(j))=b} \omega_{ij}^b < \sum_{i:h_i(\vec{u}(j))=n} \omega_{ij}^n, \\ \frac{1}{2} & \text{if} \quad \sum_{i:h_i(\vec{u}(j))=b} \omega_{ij}^b = \sum_{i:h_i(\vec{u}(j))=n} \omega_{ij}^n. \end{cases}$$

Proposition 1. If preferences are independent across countries (meaning that one country's utility for a given alternative does not depend on the full profile of votes of the rest of the countries), then a voting rule is efficient if and only if it is equivalent up to ties to v^E.

This result is Theorem 1 in Barberà and Jackson (2006). It could be possible that for political reasons the voting rule can not be made contingent on the identity of the crisis country. Ex-ante some potential members of the IMF might feel unfairly treated by such a tailor-made governance structure and would decide not to join the organization. While modeling the determinants of IMF membership is beyond the scope of this paper, I will consider an additional constraint on the optimization problem, mainly that the voting rules can not be contingent on the state of the world. If we redefine the above voting rule correspondingly we get that each country is assigned the following weights when voting in favor or against a bail-out, irrespective of who is in need of financial assistance,

$$\omega_i^b = N_i E[u_{ij}|u_{ij} > 0], \tag{2}$$
$$\omega_i^n = N_i E[u_{ij}|u_{ij} < 0], \tag{3}$$

where now expectations are taken ex-ante over the joint probability distribution of the likelihood and severity of a crisis in country j and over the effect this has on preferences of citizens in country i.

I will also consider the case that the country that requires assistance from the IMF is not allowed to vote on the bailout decision.[11] This restriction might be justified when the expectation of a bailout might induce reckless behavior in (some) members which increases their likelihood of suffering a crisis. This institutional arrangement modifies the previous weights when a country votes in favor a bailout[12]

$$\omega_i^b = N_i E[u_{ij}|u_{ij} > 0, j \neq i]. \tag{4}$$

[11] Rules for decision making when the IMF has to borrow funds to increase its resources beyond those generated by quotas (General Arrangement to Borrow, and New Arrangement to Borrow) restrict the country requesting financial assistance from the vote. See International Monetary Fund (2007).

[12] In this case it is simpler to assume that the objective function to be maximized ex ante is

$$E\left[\sum_i v(\vec{u}(j)) N_i u_{ij}(1 - I_{i,j})\right],$$

where $I_{i,j}$ is an indicator that takes the value 1 when $i = j$, and make a correction on the optimal threshold that incorporates the average impact of a bailout on countries requesting it.

The weight when a country votes against a bailout need not be modified since a country requesting a bailout would always benefit from it. For either arrangement the efficient voting rule is given by (1) replacing the corresponding weights in the formulation of v^E. As we are now considering a non-contingent voting rule, chosen ex-ante under a veil of ignorance about the future state of nature, we can assume w.l.o.g. that ex-post the intensity of preferences, u_{ij}, are publicly observed. This simplifies the calculation of the weights ω_i^b and ω_i^n.

Under either of these arrangements, weights are affected by the intensity of preferences inside a country for the alternatives, as captured by the values of u_{ij}. Thus countries that on average care more intensely about a bailout decision should be given more weight than countries that are less affected by the outcome. In their work, Barberà and Jackson (2006) consider an abstract decision and therefore have no reason for heterogeneity among members' *intensity* of preferences. They thus give every citizen of the union the same possible utilities, of $+1$ or -1.[13] Given the nature of the problem studied here I enrich the characterization of countries' preferences and the extent to which a shock in country j affects a representative citizen in country i.

I will assume all countries experience a positive effect from bailing out country j, due to trade linkages, and a negative effect due to the eventual need to raise funds to provide the IMF with working capital in the event a bailout is approved. I model trade linkages using Alvarez and Lucas (2007) variation of the Eaton and Kortum (2002) Ricardian model of trade with a probabilistic representation of intermediate goods' production efficiencies.[14] Alvarez and Lucas (2007) have two types of goods produced in each country, with Cobb-Douglas production technologies. There is a single final good, c, which is not traded and is the only good valued by consumers. There are also a continuum of intermediate goods affecting production symmetrically through a Dixit-Stiglitz aggregate that are tradeable. Countries differ in the total factor productivity levels in the production of these intermediates, and this is the driving force behind trade. Both papers are able to estimate bilateral trade flows, showing that these take the form of a gravity equation with geographic barriers and importers' price levels creating trade frictions. Of particular interest to this application, they also estimate welfare gains of trade liberalizations. I will use an approximation of Alvarez and Lucas (2007) to estimate the welfare gains to country i of having the IMF give financial assistance to country j.[15]

Alvarez and Lucas (2007) consider equilibrium in a world of n countries with balanced trade. Each country is characterized by its total labor endownment, L_i,[16] and a parameter, λ_i, which determines its productivity across a range of intermediate goods. Total factor productivity for each of the intermediate goods is random, and it is assumed that its

[13] The model of Barberà and Jackson (2006) has structure on how preferences for alternatives are distributed *inside* each country. I abstract from this complication by assuming that citizens of a given country have the same preferences with respect to the bailout decision.

[14] In Alvarez and Lucas (2007), and in Eaton and Kortum (2002) with mobile labor, an increase in foreign technology always benefit every country in the world.

[15] Alvarez and Lucas (2007) solve for a number of approximations of the theory and find that predictions about wages, trade flows, and welfare gains are close to those found in the general model.

[16] This will be the product of total population and a function of the level of human capital.

inverse, x, is distributed according to

$$x_i \sim exp(\lambda_i).$$

These draws are amplified in percentage terms by the parameter θ (assumed to be common across countries), such that $x^{-\theta}$ has a Frechet distribution. Equilibrium is obtained by imposing trade balance for the n countries in the world, i.e. equality of demand for intermediate goods in country i with aggregate demand for intermediates produced in country i. For simplicity, I will work with the special case when there are no tariffs on trade. For this case the equilibrium condition can be expressed as

$$L_i w_i = \sum_{k=1}^{n} L_k w_k D_{ki}, \tag{5}$$

where w_i is the wage rate, and D_{ki} is the fraction of per capita spending in country k of intermediate goods produced in country i,

$$\begin{aligned} D_{ki} &= (AB)^{-1/\theta} \left(\frac{w_i^{\beta} p_{mi}(w)^{1-\beta}}{p_{mk}(w)\kappa_{ki}} \right)^{-1/\theta} \lambda_i, \\ p_{mi}(w) &= AB \left(\sum_{k=1}^{n} \left(\frac{w_k^{\beta} p_{mk}(w)^{1-\beta}}{\kappa_{ik}} \right)^{-1/\theta} \lambda_k \right)^{-\theta}, \end{aligned}$$

where p_{mi} is the price index of tradeables in country i, $\kappa_{ki} \leq 1$ measures transportation costs as units of goods shipped from i to k arriving in k per unit of goods shipped. Finally β is the share of labor in the production of intermediates, and A and B are functions of the parameters.[17]

I model a crisis in country j as a decrease in its TFP parameter: $\lambda_j(1-\epsilon_j)$ where ϵ_j measures the magnitude of the crisis in relative terms. The effect of bailing out j is then found by solving the above system of equations and comparing welfare levels $c_i = \frac{w_i}{p_i}$ with and without the shock, where $p_i = \alpha^{-\alpha}(1-\alpha)^{-1+\alpha} w_i^{\alpha} p_{mi}^{1-\alpha}$ is the price level of final goods, and α is the share of labor in the production of final goods. This presumes that the IMF intervention can restore pre-crisis condition. More realistically the effect of IMF intervention is proportional to this welfare differential.[18] Since this can only be carried out numerically, I make an approximation which consists in estimating the effects that this productivity shock has on wages and prices in country i keeping wages and prices in all other countries $k \neq i, j$ unaffected. Performing this approximation leads to the

[17] $A = A(\theta, \eta) = \left[\int_o^{\infty} e^{-z} z^{\theta(1-\eta)} dz\right]^{1/(1-\eta)}$, and $B = \beta^{-\beta}(1-\beta)^{-1+\beta}$, where η measures the elasticity of substitution in forming the tradeables aggregate and does not play a role in the equilibrium conditions.

[18] An alternative formulation would start from Waugh (2009) that includes capital in Eaton and Kortum (2002), and having a crisis as a reduction in working capital in country j. In this case the IMF can lend enough resources to restore pre-crisis conditions.

following expression for the positive effects of the bailout (see Appendix for details),

$$N_i u_{ij}^+ = G\frac{N_i \frac{w_i}{p_i}}{L_i w_i}\left[(1-\beta)^2 L_j w_j D_{ji} + \beta L_i w_i D_{ij}\right]\epsilon_j, \tag{6}$$

$$N_i u_{ij}^+ = G\frac{N_i}{L_i}\frac{1}{p_i}\left[(1-\beta)^2 M_{ji} + \beta M_{ij}\right]\epsilon_j, \tag{7}$$

where $G \equiv \frac{(\frac{1-\alpha}{\alpha})^\alpha}{\beta(1-\beta)(2-\beta)^2}$ and M_{ij} are total imports of country i from country j. Following Eaton and Kortum (2002) I take total units of labor in efficiency units to be given by total population times e^{gH}, with H average years of schooling and g the return to education. Approximating p_i by the ratio of GDP to GDP PPP, and making $\beta = (1-\beta)^2$ we have,

$$N_i u_{ij}^+ = \beta G\frac{GDP\ PPP_i}{GDP_i}e^{-gH_i}\left[M_{ji} + M_{ij}\right]\epsilon_j. \tag{8}$$

Thus positive effects of bailing out a country are proportional to bilateral trade with this country, and the constant of proportionality is country specific and depends on the ratio of GDP PPP to GDP and on average years of schooling.[19] The correction introduced by the ratio of GDP PPP to GDP is due to the presence of trade frictions that lead to a violation of the law of one price, while the presence of years of schooling relates to welfare being weighted by population while output is determined by efficiency units of labor.[20] Note that given that the positive effects are proportional to bilateral trade flows, in a world with trade frictions a bailout would have more support among neighbors of the country requesting assistance.

Since giving financial assistance uses scarce resources this might reduce the probability that the IMF would be able to bailout another country experiencing a balance of payments crisis in the future. If we assume the probability of having resources to provide a future bailout is given by $1 - \xi\epsilon_j GDP_j$, country i would vote in favor of bailing out country j when,

$$u_{ij} = N_i u_{ij}^+ + \delta(1-\xi\epsilon_j GDP_j)\omega_i^* - \delta\omega_i^* > 0, \tag{9}$$

$$u_{ij} = \beta G\frac{GDP\ PPP_i}{GDP_i}e^{-gH_i}\epsilon_j GDP_j\left(\frac{M_{ji}+M_{ij}}{GDP_j} - \delta\xi E\left[\left[M_{ki}+M_{ik}\right]\epsilon_k\right]\right) > 0,$$

where δ is a discount factor, and $\omega_i^* \equiv E[N_i u_{ij}^+] = Prob[u_{ij} > 0]E[N_i u_{ij}^+ | u_{ij} > 0] + Prob[u_{ij} < 0]E[N_i u_{ij}^+ | u_{ij} < 0]$ is the period benefit to country i of being a member of the IMF. The first thing to notice is that a country is biased towards voting in favor of bailing out countries with which the ratio of bilateral trade to GDP, $\frac{M_{ij}+M_{ji}}{GDP_j}$, is high. Globally this favors the bailout of countries that are relatively more open, and given the presence of trade frictions, country i would vote to bail out countries in its geographical

[19] Following Eaton and Kortum (2002), I will take $g = 0.06$ which Bils and Klenow (2000) suggest is a conservative estimate.

[20] Solving the Eaton and Kortum (2002) model with frictionless trade leads to an expression of u_{ij}^+ that depends only on trade flows. A more thorough specification of the relation between N_i and L_i should include a correction based on dependency ratios.

neighborhood. Second, with this specifications for u_{ij}^+, preferences with respect to the bailout decision, given by (9), are indeed independent across countries and Proposition 1 holds. The calculation of weights ω_i^b, and ω_i^n fully characterizes the optimal voting rule for the IMF,

$$\begin{aligned}\omega_i^b &= N_i E[u_{ij}^+|u_{ij}>0] - \delta\xi\omega_i^* E[\epsilon_j GDP_j|u_{ij}>0] = N_i E[u_{ij}^+|u_{ij}>0] - \delta\xi\omega_i^*\Phi,\\ \omega_i^n &= -N_i E[u_{ij}^+|u_{ij}<0] + \delta\xi\omega_i^* E[\epsilon_j GDP_j|u_{ij}<0] = -N_i E[u_{ij}^+|u_{ij}<0] + \delta\xi\omega_i^*\Phi,\end{aligned}$$

where we assume $E[\epsilon_j GDP_j|u_{ij}>0] = E[\epsilon_j GDP_j|u_{ij}<0] = E[\epsilon_j GDP_j] \equiv \Phi$, i.e. the expected size of a bailout package is independent of whether country i is voting in favor or against this bailout. This is a reasonable assumption given that we assume that the reduction in the probability that the IMF will be able to provide a bailout in the future is linear in the size of the bailout package affects, therefore the scale will affect the intensities of preferences in favor or against a bailout but not the sign of u_{ij} (see (9)).[21] Recalling the definition of ω_i^* we get,

$$\begin{aligned}\omega_i^b &= N_i E[u_{ij}^+|u_{ij}>0](1-\delta\xi\Phi Prob[u_{ij}>0]) - N_i E[u_{ij}^+|u_{ij}<0]\delta\xi\Phi Prob[u_{ij}<0],\\ \omega_i^n &= -N_i E[u_{ij}^+|u_{ij}<0](1-\delta\xi\Phi Prob[u_{ij}<0]) + N_i E[u_{ij}^+|u_{ij}>0]\delta\xi\Phi Prob[u_{ij}>0].\end{aligned}$$

These formulas, together with (8), completely characterize the optimal voting rule. This will be a weighted voting rule when the ratio between weights ω_i^n and ω_i^b is the same across countries, i.e. if all countries have a common bias given by $\frac{\omega_i^n}{\omega_i^b} \equiv \gamma$.[22] This requires

$$\frac{\omega_i^n}{\omega_i^b} = \frac{\delta\xi\Phi Prob[u_{ij}>0]\frac{E[u_{ij}^+|u_{ij}>0]}{E[u_{ij}^+|u_{ij}<0]} - (1-\delta\xi\Phi Prob[u_{ij}<0])}{(1-\delta\xi\Phi Prob[u_{ij}>0])\frac{E[u_{ij}^+|u_{ij}>0]}{E[u_{ij}^+|u_{ij}<0]} - \delta\xi\Phi Prob[u_{ij}<0]} = \gamma \quad \forall i.$$

Sufficient conditions for equality of these ratios across countries are that $Prob[u_{ij}>0]$, and $\frac{E[u_{ij}^+|u_{ij}>0]}{E[u_{ij}^+|u_{ij}<0]}$, be independent of country i. The first condition is that every country in the IMF be ex-ante equally likely to vote in favor of a bailout. The second condition requires that the distribution of $\frac{M_{ij}+M_{ji}}{E[(M_{ik}+M_{ki})\epsilon_k]}$ be the same across countries. We will assume these conditions to be satisfied. Under this assumption, the ratio $\frac{\omega_i^*}{\omega_i^b}$ will also be constant across countries. This allows to characterize the optimal voting rule as assigning weights

$$\omega_i^* = \beta G \frac{GDP\ PPP_i}{GDP_i} e^{-gH_i} \sum_j \mu(j)(M_{ji}+M_{ij}),$$

to each country, and a threshold of $\frac{\gamma}{1+\gamma}\sum_{i=1}^n \omega_i^*$ of votes to approve a bailout. Given that weights are derived from the expected welfare impact of a bailout on member countries,

[21] Alternatively if we assume that the size of a bailout package is not related to the size of the shock experienced by country j then countries experiencing larger shocks are more likely to be bailed out, and we no longer can assume $E[\epsilon_j GDP_j|u_{ij}>0] = E[\epsilon_j GDP_j|u_{ij}<0]$. This only complicates the expressions derived, but does not affect the results.

[22] See Corollary 1 in Barberà and Jackson (2006).

they also relate to the absolute level of capital contributions that each country should make to the IMF (i.e. each member's contributions should be proportional to the benefits from belonging to the IMF). Therefore the model also helps to determine the total level of working capital that the IMF should have, a decision that has so far been discretionally determined at General Quota Reviews.

To use this formula to calculate quotas we need to estimate the likelihood that country j suffers a balance of payments crisis. A way of estimating this would be to use sovereign spreads, or credit ratings, for those countries that have issued public debt, assigning a common (high) value to those poor countries that have not been able to issue debt, and a value of zero to rich countries having no outstanding debt issues. Before doing this it is illustrative to consider how the above formula for quotas looks like in two polar cases, a world with equal exposure to balance of payments risk, and a North-South world where some countries become independent of IMF assistance.

3.1 Developments in the International Financial System

Between its creation and the collapse of the Bretton Woods system of fixed exchange rates, the IMF worked as a credit union where most members were seen as being equally likely to require temporary financial assistance.[23] In a few years after the collapse of the Bretton Woods system the IMF became bifurcated with industrialized countries no longer demanding loan agreements.[24] Effectively there emerged two types of members: rich country net lenders, and poor country net borrowers.

The post-Bretton Woods IMF found it impossible to attain the multiple objectives of the quota system with this single instrument. In particular the close connection between quotas and borrowing limits was de facto scrapped. With more developed global capital markets countries with current account deficits could avoid crises, at least for a while, by relying on private financing. However, when these alternative sources dried up, countries were forced to ask the IMF for substantially more financial assistance than the quota structure had foreseen.

At the same time, the optimality of the quota system to determine voting shares is unaffected by the collapse of the fixed exchange rates arrangement. This can be seen from the above formulas where all the changes are absorbed by the probability distribution of balance of payments shocks that require a bailout, $\mu(\cdot)$. To get an idea of what drives the allocation of voting rights we compare now the optimum voting weights in a symmetric world where every country has the same probability of suffering a balance of payments shock, with a "North-South" world where a subgroup of countries no longer is affected by balance of payments shocks (or if affected, does not need IMF assistance).

In the symmetric case $\mu(\cdot) = \mu$ for all countries, i.e. the product of the probability of a shock times its relative size is constant, and independent of country j. This makes the

[23]Between 1947 and 1967 industrialized nations accounted for almost 70% of the total amount of resources withdrawn from the IMF.

[24]With the recent noted exception of Iceland, that received financial assistance in the aftermath of the global meltdown in 2008.

optimal voting weights to be,

$$\omega_i^* = 2\mu\beta G \frac{GDP\ PPP_i}{GDP_i} e^{-gH_i} GDP_i.$$

Thus we see that quotas are proportional to GDP PPP, and adjusted by years of schooling. This expression would change to the following in a North-South world,

$$\omega_i^* = \mu\beta G \frac{GDP\ PPP_i}{GDP_i} e^{-gH_i} \left[M_{iS} + X_{iS} + \left(GDP_i - \frac{M_i + X_i}{2} \right) \mathbf{1}_{i \in S} \right].$$

where M_{iS} and X_{iS} refer to trade volume of country i with all South countries, i.e. countries that are subject to balance of payments shocks (and still assuming $\mu(\cdot) = \mu$ for all South countries), and $\mathbf{1}_{i \in S}$ is an indicator function that takes the value of 1 if country i is in the South. Note that in this calculation we have to include domestic absorption of those developing countries that are exposed to balance of payments shocks, while for developed countries only trade with South countries is considered. Before analyzing how the collapse of Bretton Woods distributed optimal voting weights across the world, the previous formulas help to rationalize the marked decrease in IMF resources relative to world aggregates that took place in the seventies. For example, industrial country quotas decreased by more than 50% relative to world exports between 1970 and 1975 and has remained stable afterwards. Splitting the countries in the world between North and South on the basis of their use of IMF resources in the past three decades and considering their GDP per capita, I found from aggregation of bilateral trade flows for the period 2002-7 that a weighted average of trade with South countries corresponds to 50% of total world trade.[25] According to the model this decrease was optimal since it reflected the decrease in expected welfare gains from having the IMF in place (weights w_i^b and w_i^n reflect the expected intensity of preferences for or against a bailout, and are affected when a subset of countries is no longer demanding financial assistance).

Table 1 has actual and calculated quotas for the 19 countries in the G20 (i.e. excluding the EU). For comparison I include the results that would obtain in the symmetric case where every country has the same probability of facing a balance of payments shock. As can be seen in Table 1, an application of the derived quota formula leads to a small reduction in the influence of the richest industrial countries. But this effect is not driven by the pattern of trade between North and South countries. It is produced by the adjustment in trade flows given by the ratio of GDP PPP to GDP, and years of schooling (which increases the voice of relatively poor countries that have lower levels of human capital). A move from total trade volume to trade with South nations reduces the influence of East Asian countries (including China) that trade mostly with developed nations, and Canada and Mexico (and some Central American countries) that trade mostly with the US.[26] And it increases the vote shares in Latin American and Africa.

From the formula for optimal voting weights we see that the total weights, and correspondingly the working capital of the IMF should respond to developments in the volume

[25]The average is weighted by GDP, the unweighted average is slightly higher.

[26]This might explain why East Asian countries appear to be underrepresented in the IMF. See Rapkin and Strand (2003).

and distribution of trade, changes in the relative wealth of nations, education attainment, and likelihood of balance of payment shocks, $\mu(\cdot)$. The threshold of votes needed to approve a bailout is unaffected by these developments, and should only change with the severity of fiscal problems, $\bar{\phi}$, and the average discount factor $E[\delta(j)]$. If fiscal problems are less of a concern to countries, then the threshold of votes should be reduced (making a bailout more likely to be approved). The same should happen in calm periods when few countries request IMF assistance. With respect to the possibility of a multi-country crisis, the model assumes that the negative effects of a bailout are independent of the characteristics of the country being bailed out. Thus the negative effects would be the same independently of how many countries are assisted by the IMF. Therefore the criteria to decide whether to help the countries suffering the balance of payments shock is the same as if all of them were lumped into a single country. Notice how this gives incentives to small countries to distort domestic policy with the objective of increasing the correlation of their balance of payments shocks with those of bigger economies, thus increasing the chances of a bailout when one is needed.

4 Moral Hazard

Suppose now that the probability that a country experiences a crisis depends on its domestic policies. Furthermore the ruler of country i receives a private benefit of $B > 0$ if she does not exert effort into pursuing "good" policies that reduce the likelihood of a crisis. Suppose further that when a country experiences a crisis this has a cost $C_i > 0$ for the ruler if there is no bailout from the IMF, and $\emptyset C_i$ $(0 < \emptyset < 1)$ if there is bailout. Then a ruler will decide whether to exert effort or not based on a private cost benefit analysis, taking into consideration the likelihood of a bailout as implied from the voting rule. Denoting $\Delta\mu$ the increase in the likelihood of experiencing a crisis if effort is not exerted, the ruler chooses to adopt "good" policies if

$$B < \Delta\mu \left[(1 - s_i)C_i + s_i \emptyset C_i\right], \tag{10}$$

where s_i is the probability of being bailed out by the IMF. With our specification of preferences (9) this is either zero or one, since a country knows exactly which other members of the IMF would be voting in favor of its bailout (remember that only the intensity of preferences is affected by the magnitude of the shock, not the sign of u_{ji}).

If a large group of countries decides not to adopt "good" policies, then the IMF might find optimal to increase the threshold of votes required to approve a bailout. This reduces the number of countries that would be rescued, and as can be seen in (10), this has a disciplining effect, as the cost of shirking effort increases. The threshold should also be raised if there is an increases in moral hazard problems, as measured by B.

Note that members' weights need not be adjusted, as they do not affect their incentives to exert effort. Heterogeneity in members' preference with respect to domestic policy, reflected in differences in C_i, will likely result in a threshold that, while disciplining some countries, will still not be high enough to rule out moral hazard throughout the world.

5 Conclusions

I have derived a theoretical model for the optimal calculation of quotas in the IMF. Under general conditions a simple weighted voting rule is the efficient voting rule. Optimal weights are proportional to a weighted sum of a country's trade flows with the rest of the members with weights related to the probabilities of each trade partner suffering a crisis. Weights are also proportional to the ratio of GDP PPP to GDP reflecting trade frictions that affect domestic consumption price levels, and include a correction based on years of schooling since this determines labor measured in efficiency units. Introducing moral hazard in the model, the optimal response is to leave weights unchanged, but to increase the threshold of votes required to approve a bailout.

Given that weights are derived from the expected welfare impact of a bailout on countries, they also relate to the absolute level of capital contributions that each country should make to the IMF. Therefore the model also helps to determine the total level of resources that the IMF should have, a decision that has always been discretionally determined at General Quota Reviews.

The model predicts that changes in the international financial system, such as the breakdown of Bretton Woods fixed exchange rates arrangement, should be reflected in the distribution of voting shares, and in the total level of quotas, but does not affect the optimality of using a quota system for the governance of the IMF. In fact, the observed decrease in the ratio of IMF resources to world trade that took place between 1970 and 1975 closely matches the reduction in weighted trade flows in the derived quota formulas.

The model has implication for the reform proposals that have been presented to improve the legitimacy of the IMF (See for example Cottarelli (2005) and Rapkin and Strand (2006)). In particular under the assumptions of the model, there is no rationale for a double majority system as the "count and account" proposal of O'Neill and Peleg (2000). Further research will introduce intertemporal considerations when the voting rule is also used to decide whether or not to adjust quotas every period, after observing developments in members' economies (e.g. through changes in L_i and λ_i), but before the realization of balance of payments shocks.

As mentioned in the introduction, the results derived can be extended to study optimal governance in regional development banks. In this case the bank faces loan demands from several members to finance various public investments opportunities, and must decide which one to satisfy. Thus providing a loan can be modeled as an *increase* in aggregate TFP in the country receiving the loan, and regional bank members evaluate their private trade gains with the potential receiver of a loan when deciding how to vote.

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6 Appendix

6.1 Derivation of positive effects of a bailout

In Alvarez and Lucas (2007) equilibrium is obtained by imposing trade balance for the n countries in the world.

$$L_i w_i = \sum_{k=1}^{n} L_k w_k D_{ki}(w) \tag{11}$$

where L_i is the total units of labor in country i measured in efficiency units, w_i is the wage rate, and D_{ki} is the fraction of per capita spending in country k of intermediate goods produced in country i,

$$\begin{aligned} D_{ki}(w) &= (AB)^{-1/\theta} \left(\frac{w_i^{\beta} p_{mi}(w)^{1-\beta}}{p_{mk}(w)\kappa_{ki}} \right)^{-1/\theta} \lambda_i, \\ p_{mi}(w)^{-1/\theta} &= (AB)^{-1/\theta} \sum_{k=1}^{n} \left(\frac{w_k^{\beta} p_{mk}(w)^{1-\beta}}{\kappa_{ik}} \right)^{-1/\theta} \lambda_k, \end{aligned} \tag{12}$$

where θ is a parameter that measures the variability in productivity draws, λ_k is a technology parameter associated with the level of TFP in country k, p_{mi} is the price index of tradeables in country i, $\kappa_{ki} \leq 1$ measures transportation costs as units of goods shipped from i to k arriving in k per unit of goods shipped. Finally β is the share of labor in the production of intermediates, and A and B are functions of the parameters.[27] For convenience I use the following notation,

$$\begin{aligned} \psi_{ik} &= \left(\frac{w_k^{\beta} p_{mk}^{1-\beta}}{\kappa_{ik}} \right)^{-1/\theta} \lambda_k, \\ \psi_{ij} &= \left(\frac{w_j^{\beta} p_{mj}^{1-\beta}}{\kappa_{ij}} \right)^{-1/\theta} \lambda_j (1-\epsilon_j). \end{aligned}$$

Equations (11) and (12) can be seen as a system of $2n$ equations in $2n$ unknowns, w_i and p_{mi}. Once these are found, the price level of final goods, p_i can be calculated using the following relation,

$$p_i = \alpha^{-\alpha}(1-\alpha)^{-1+\alpha} w^{\alpha} p_{mi}^{1-\alpha}. \tag{13}$$

I model a crisis in country j as a decrease in its TFP parameter: $\lambda_j(1-\epsilon_j)$ where ϵ_j measures the magnitude of the crisis in relative terms. The effect of bailing out j is then found by solving the above system of equations and comparing welfare levels $c_i = \frac{w_i}{p_i}$ with and without the shock. To this effect I make an approximation which consists in estimating the effects that this productivity shock has on wages and prices in country i keeping wages and prices in all other countries $k \neq i, j$ unaffected. The first step is to

[27] $A = A(\theta, \eta) = \left[\int_o^{\infty} e^{-z} z^{\theta(1-\eta)} dz\right]^{1/(1-\eta)}$, and $B = \beta^{-\beta}(1-\beta)^{-1+\beta}$, where η measures the elasticity of substitution in forming the tradeables aggregate and does not play a role in the equilibrium conditions.

estimate the effect of a TFP shock on wages and prices in country j. Taking derivatives with respect to ϵ_j in (11) and (12) for the case $i = j$ we find,

$$\begin{aligned}\left[1 - D_{jj} - \sum_k \frac{L_k w_k}{L_j}\frac{\mathrm{d}\, D_{kj}}{\mathrm{d}\, w_j}\right]\frac{\mathrm{d}\, w_j}{\mathrm{d}\,\epsilon_j} + \left[-\sum_k \frac{L_k w_k}{L_j}\frac{\mathrm{d}\, D_{kj}}{\mathrm{d}\, p_{mj}}\right]\frac{\mathrm{d}\, p_{mj}}{\mathrm{d}\,\epsilon_j} &= 0\\ \left[-(AB)^{-1/\theta}\sum_k \frac{\mathrm{d}\,\psi_{jk}}{\mathrm{d}\, w_j}\right]\frac{\mathrm{d}\, w_j}{\mathrm{d}\,\epsilon_j} + \left[-(AB)^{-1/\theta}\sum_k \frac{\mathrm{d}\,\psi_{jk}}{\mathrm{d}\, p_{mj}} - \frac{1}{\theta}p_{mj}^{-(1+\theta)/\theta}\right]\frac{\mathrm{d}\, p_{mj}}{\mathrm{d}\,\epsilon_j}&\\ +(AB)^{-1/\theta}\psi_{jj}|_{\epsilon_j=0} &= 0\end{aligned}$$

Now to simplify, note that for the first equation, $\frac{\mathrm{d}\, D_{kj}}{\mathrm{d}\, w_j} = -\frac{\beta}{\theta}\frac{D_{kj}}{w_j}$, $\frac{\mathrm{d}\, D_{kj}}{\mathrm{d}\, p_{mj}} = -\frac{1-\beta}{\theta}\frac{D_{kj}}{p_{mj}}$ when $k \neq j$ and $\frac{\mathrm{d}\, D_{jj}}{\mathrm{d}\, p_{mj}} = -\frac{1-\beta}{\theta}\frac{D_{jj}}{p_{mj}} + \frac{1}{\theta}\frac{D_{jj}}{p_{mj}}$, and $\sum_k \frac{L_k w_k}{L_j w_j} D_{kj} = 1$. Dividing the second equation by $p_{mj}^{-1/\theta} = (AB)^{-1/\theta}\sum_k \psi_{jk}$, and using $\frac{\mathrm{d}\,\psi_{jk}}{\mathrm{d}\, w_j} = \frac{\mathrm{d}\,\psi_{jk}}{\mathrm{d}\, p_{mj}} = 0$ if $j \neq k$, $\frac{\mathrm{d}\,\psi_{jj}}{\mathrm{d}\, w_j} = -\frac{\beta}{\theta}\frac{\psi_{jj}}{w_j}$, $\frac{\mathrm{d}\,\psi_{jj}}{\mathrm{d}\, p_{mj}} = -\frac{1-\beta}{\theta}\frac{\psi_{jj}}{p_{mj}}$, and $\frac{\psi_{jj}}{\sum_k \psi_{jk}} = D_{jj}$,[28]

$$\begin{aligned}\left[1 - D_{jj} + \frac{\beta}{\theta}\right]\frac{\mathrm{d}\, w_j}{\mathrm{d}\,\epsilon_j} + \left[\frac{1-\beta-D_{jj}}{\theta}\frac{w_j}{p_{mj}}\right]\frac{\mathrm{d}\, p_{mj}}{\mathrm{d}\,\epsilon_j} &= 0\\ \left[\frac{\beta}{\theta}\frac{D_{jj}}{w_j}\right]\frac{\mathrm{d}\, w_j}{\mathrm{d}\,\epsilon_j} + \left[\frac{1-\beta}{\theta}\frac{D_{jj}}{p_{mj}} - \frac{1}{\theta}\frac{1}{p_{mj}}\right]\frac{\mathrm{d}\, p_{mj}}{\mathrm{d}\,\epsilon_j} + D_{jj} &= 0\end{aligned}$$

Multiplying the first equation by θ, and the second by θw_j we get

$$\begin{aligned}[\beta + \theta(1 - D_{jj})]\frac{\mathrm{d}\, w_j}{\mathrm{d}\,\epsilon_j} + (1 - \beta - D_{jj})\frac{w_j}{p_{mj}}\frac{\mathrm{d}\, p_{mj}}{\mathrm{d}\,\epsilon_j} &= 0\\ \beta D_{jj}\frac{\mathrm{d}\, w_j}{\mathrm{d}\,\epsilon_j} - (1 - (1-\beta)D_{jj})\frac{w_j}{p_{mj}}\frac{\mathrm{d}\, p_{mj}}{\mathrm{d}\,\epsilon_j} &= -\theta w_j D_{jj}\end{aligned}$$

Solving for the effects of ϵ_j on wages and prices in country j we find

$$\begin{aligned}\frac{\mathrm{d}\, w_j}{\mathrm{d}\,\epsilon_j} &= = \frac{1}{\Delta_j}\theta\frac{w_j^2}{p_{mj}}(1 - D_{jj}^2)\\ \frac{\mathrm{d}\, p_{mj}}{\mathrm{d}\,\epsilon_j} &= -\frac{1}{\Delta_j}\theta^2 w_j D_{jj}(1 - D_{jj})\\ \Delta_j &= -\frac{w_j}{p_{mj}}\left[\beta + \theta - \theta(2-\beta)D_{jj} + (\theta(1-\beta) - \beta)D_{jj}^2\right]\end{aligned}$$

In order to get closed form solutions for the weights ω_i^b, and ω_i^n, we need to make the following assumption

$$\theta = \frac{\beta}{1-\beta}$$

[28] In what follows I simplify notation such that $\psi_{jj}|_{\epsilon_j=0}$, and the same holds for all expression that feature a term λ_j (as D_{jj}).

This assumption means, for example, that if $\beta = 0.38$[29] then $\theta = 0.61$, a larger value than the ones used by Alvarez and Lucas in their calibrations (they considered the range $[0.1, 0.25]$), or the benchmark estimate in Waugh (2009), $\theta = 0.18$. This implies that

$$\begin{aligned}
\Delta_j &= -\frac{w_j}{p_{mj}}\beta\frac{2-\beta}{1-\beta}(1-D_{jj}) && (14)\\
\frac{\mathrm{d}\, w_j}{\mathrm{d}\,\epsilon_j} &= = -\frac{1}{2-\beta}w_j(1+D_{jj}) && (15)\\
\frac{\mathrm{d}\, p_{mj}}{\mathrm{d}\,\epsilon_j} &= \frac{\beta}{(1-\beta)(2-\beta)}p_{mj}D_{jj} && (16)
\end{aligned}$$

We can now approximate the effect of ϵ_j on wages and prices in country $i \neq j$ by considering both the direct effect and the indirect effect through w_j and p_{mj}. To do this we take derivatives with respect to ϵ_j in (11) and (12) we find,

$$\begin{aligned}
\left[1 - D_{ii} - \sum_k \frac{L_k w_k}{L_i}\frac{\mathrm{d}\, D_{ki}}{\mathrm{d}\, w_i}\right]\frac{\mathrm{d}\, w_i}{\mathrm{d}\,\epsilon_j} + \left[-\sum_k \frac{L_k w_k}{L_i}\frac{\mathrm{d}\, D_{ki}}{\mathrm{d}\, p_{mi}}\right]\frac{\mathrm{d}\, p_{mi}}{\mathrm{d}\,\epsilon_j}&\\
-\frac{L_j}{L_i}D_{ji}\frac{\mathrm{d}\, w_j}{\mathrm{d}\,\epsilon_j} - \frac{L_j w_j}{L_i}\frac{\mathrm{d}\, D_{ji}}{\mathrm{d}\, p_{mj}}\frac{\mathrm{d}\, p_{mj}}{\mathrm{d}\,\epsilon_j} &= 0\\
\left[-(AB)^{-1/\theta}\sum_k \frac{\mathrm{d}\,\psi_{ik}}{\mathrm{d}\, w_i}\right]\frac{\mathrm{d}\, w_i}{\mathrm{d}\,\epsilon_j} + \left[-(AB)^{-1/\theta}\sum_k \frac{\mathrm{d}\,\psi_{ik}}{\mathrm{d}\, p_{mi}} - \frac{1}{\theta}p_{mi}^{-(1+\theta)/\theta}\right]\frac{\mathrm{d}\, p_{mi}}{\mathrm{d}\,\epsilon_j}&\\
-(AB)^{-1/\theta}\frac{\mathrm{d}\,\psi_{ij}}{\mathrm{d}\, w_j}\frac{\mathrm{d}\, w_j}{\mathrm{d}\,\epsilon_j} - (AB)^{-1/\theta}\frac{\mathrm{d}\,\psi_{ij}}{\mathrm{d}\, p_{mj}}\frac{\mathrm{d}\, p_{mj}}{\mathrm{d}\,\epsilon_j} + (AB)^{-1/\theta}\psi_{ij}|_{\epsilon_j=0} &= 0
\end{aligned}$$

Simplifying these equations as done before and multiplying by θ and θw_i we get

$$\begin{aligned}
[\beta + \theta(1-D_{ii})]\frac{\mathrm{d}\, w_i}{\mathrm{d}\,\epsilon_j} + (1-\beta-D_{ii})\frac{w_i}{p_{mi}}\frac{\mathrm{d}\, p_{mi}}{\mathrm{d}\,\epsilon_j} &= \left[\frac{L_j}{L_i}\frac{\mathrm{d}\, w_j}{\mathrm{d}\,\epsilon_j} + \frac{L_j}{L_i}\frac{1}{\theta}\frac{w_j}{p_{mj}}\frac{\mathrm{d}\, p_{mj}}{\mathrm{d}\,\epsilon_j}\right]D_{ji}\\
\beta D_{ii}\frac{\mathrm{d}\, w_i}{\mathrm{d}\,\epsilon_j} - (1-(1-\beta)D_{ii})\frac{w_i}{p_{mi}}\frac{\mathrm{d}\, p_{mi}}{\mathrm{d}\,\epsilon_j} &= -\left[\theta w_i + \beta\frac{w_i}{w_j}\frac{\mathrm{d}\, w_j}{\mathrm{d}\,\epsilon_j} + (1-\beta)\frac{w_i}{p_{mj}}\frac{\mathrm{d}\, p_{mj}}{\mathrm{d}\,\epsilon_j}\right]D_{ij}
\end{aligned}$$

Notice the similar structure between this system of equations and the one solved for country j. Using the expressions (15) and (16) these equations simplify to

$$\begin{aligned}
[\beta + \theta(1-D_{ii})]\frac{\mathrm{d}\, w_i}{\mathrm{d}\,\epsilon_j} + (1-\beta-D_{ii})\frac{w_i}{p_{mi}}\frac{\mathrm{d}\, p_{mi}}{\mathrm{d}\,\epsilon_j} &= -\frac{L_j}{L_i}\frac{1}{2-\beta}w_j D_{ji}\\
\beta D_{ii}\frac{\mathrm{d}\, w_i}{\mathrm{d}\,\epsilon_j} - (1-(1-\beta)D_{ii})\frac{w_i}{p_{mi}}\frac{\mathrm{d}\, p_{mi}}{\mathrm{d}\,\epsilon_j} &= -\frac{\beta}{(2-\beta)(1-\beta)}w_i D_{ij}
\end{aligned}$$

[29] This is the ratio of value added in manufacturing to total value of production in the US for 1996-1999 according to BEA input-output data. See Alvarez and Lucas (2007).

Solving for the effects of ϵ_j on wages and prices in country i we find

$$\begin{aligned}\frac{\mathrm{d}\, w_i}{\mathrm{d}\, \epsilon_j} &= \frac{1}{\Delta_i}\left[\frac{w_i}{p_{mi}}(1-(1-\beta)D_{ii})\frac{L_j}{L_i}\frac{1}{2-\beta}w_j D_{ji} + (1-\beta-D_{ii})\frac{w_i^2}{p_{mi}}\frac{\beta}{(2-\beta)(1-\beta)}D_{ij}\right]\\ \frac{\mathrm{d}\, p_{mi}}{\mathrm{d}\, \epsilon_j} &= \frac{1}{\Delta_i}\left[\beta D_{ii}\frac{L_j}{L_i}\frac{1}{2-\beta}w_j D_{ji} - (\beta+\theta(1-D_{ii}))\frac{\beta}{(2-\beta)(1-\beta)}w_i D_{ij}\right]\end{aligned}$$

We are now ready to estimate the approximate positive effect that a bailout of country j has on country i: $u_{ij}^+ \equiv -\frac{\mathrm{d}\,\frac{w_i}{p_i}}{\mathrm{d}\,\epsilon_j}$, using (13) and the assumption $\theta = \frac{\beta}{1-\beta}$ that lead to Δ_i given by (14) with j replaced by i. Again start with country j.

$$u_{jj}^+ = -(1-\alpha)(\frac{w_j}{p_{mj}})^{-\alpha}\frac{\mathrm{d}\,\frac{w_j}{p_{mj}}}{\mathrm{d}\,\epsilon_j}\epsilon_j = -(1-\alpha)(\frac{w_j}{p_{mj}})^{-\alpha}\left[\frac{1}{p_{mj}}\frac{\mathrm{d}\, w_j}{\mathrm{d}\,\epsilon_j} - \frac{w_j^2}{p_{mj}}\frac{\mathrm{d}\, p_{mj}}{\mathrm{d}\,\epsilon_j}\right]\epsilon_j.$$

Using (15) and (16)

$$u_{jj}^+ = (1-\alpha)(\frac{w_j}{p_{mj}})^{1-\alpha}\frac{(1-\beta+D_{jj})}{(2-\beta)(1-\beta)}\epsilon_j = \frac{(\frac{1-\alpha}{\alpha})^\alpha}{(2-\beta)(1-\beta)}(1-\beta+D_{jj})\frac{w_j}{p_j}\epsilon_j,$$

where in the last equality we used (13). Note that u_{jj}^+ is proportional to welfare. Using the same logic we find welfare effects on country i

$$u_{ij}^+ = (1-\alpha)(\frac{w_i}{p_{mi}})^{1-\alpha}\frac{1}{\beta(1-\beta)(2-\beta)^2}\frac{1}{L_i w_i}\left[(1-\beta)^2 L_j w_j D_{ji} + \beta L_i w_i D_{ij}\right]\epsilon_j.$$

Multiplying by population N_i and defining $G \equiv \frac{(\frac{1-\alpha}{\alpha})^\alpha}{\beta(1-\beta)(2-\beta)^2}$ we get the expression used for the calculation of quotas:

$$N_i u_{ij}^+ = G\frac{N_i\frac{w_i}{p_i}}{L_i w_i}\left[(1-\beta)^2 L_j w_j D_{ji} + \beta L_i w_i D_{ij}\right]\epsilon_j.$$

6.2 Alternative characterization of voting weights

Since giving financial assistance eventually requires that members make capital injections to the IMF, some countries might oppose to a bailout if they are experiencing a tight fiscal position, or if voting for a bailout creates a conflict with its domestic political agenda.[30] I will model this as a random cost independently distributed across countries, and independent of the balance of payments shocks that affects country j. The negative effects of a bailout then take the form

$$N_i u_{ij}^- = -\delta(j)\phi_i\omega_i^b$$

where ω_i^b is the weight that rule v^E assigns to a country when it votes in favor of a bailout, and here I make the conjecture that a country's quota, which also determines

[30] It is likely that the first concern is more relevant for voting decisions among poor and middle-income countries, while the latter might affect decision-making in high income countries.

capital contributions, is proportional to this weight. ϕ_i is the random cost associated to country i's fiscal position or domestic politics, and $\delta(j)$ is a discount factor that captures the possibility that the capital injection takes place in the future.[31] $\delta(j)$ may be an increasing function of the size of the loan required by country j, reflecting that approving a larger rescue package increases the probability of future capital injections more than approving a smaller one. For simplicity I will assume that: a) ϕ_i can only take two values,

$$\phi_i = \begin{cases} \bar{\phi} > \frac{1}{E[\delta(j)]} & \text{with probability } \pi_i, \\ 0 & \text{with probability } 1-\pi_i \end{cases}$$

and, b) when $\phi_i = \bar{\phi}$ the negative effect always dominates the positive one.[32] Note that with this specifications for u_{ij}^+ and u_{ij}^-, preferences with respect to the bailout decision, given by $u_{ij} = u_{ij}^+ - u_{ij}^-$, are indeed independent across countries and Proposition 1 holds. The calculation of weights ω_i^b, and ω_i^n fully characterizes the optimal voting rule for the IMF. We will start with ω_i^b and consider the case where country j is not allowed to vote (see (3)),

$$\begin{aligned} \omega_i^b &= N_i E[u_{ij}^+ + u_{ij}^- | u_{ij}^+ > u_{ij}^-, j \neq i] = N_i E[u_{ij}^+ | \phi_i = 0, j \neq i] = \\ &= \beta G \frac{GDP\ PPP_i}{GDP_i} e^{-gH_i} \sum_{j \neq i} \mu(j) \left(M_{ji} + M_{ij}\right) \end{aligned}$$

And ω_i^n is given by (see (2)),

$$\begin{aligned} \omega_i^n &= -N_i E[u_{ij}^+ + u_{ij}^- | u_{ij}^+ < u_{ij}^-] = -N_i E[u_{ij}^+ + u_{ij}^- | \phi_i = \bar{\phi}, j \neq i] = \left(E[\delta(j)|j \neq i]\bar{\phi} - 1\right) w_i^b \\ &= \left(E[\delta(j)|j \neq i]\bar{\phi} - 1\right) \beta G \frac{GDP\ PPP_i}{GDP_i} e^{-gH_i} \sum_{j \neq i} \mu(j) \left(M_{ji} + M_{ij}\right) \end{aligned}$$

Under the assumption that $E[\delta(j)|j \neq i] = E[\delta(j)]$, the ratio $\gamma \equiv \frac{\omega_i^n}{\omega_i^b} = E[\delta(j)]\bar{\phi} - 1$ is constant and independent of country i. γ measures a country's bias against a bailout, and the fact that in this case all countries have a common bias means that the efficient voting rule can be written as a weighted voting rule.[33] This rule assigns weights,

$$\omega_i^* = \omega_i^b$$

to each country, and a threshold of $\frac{E[\delta(j)]\bar{\phi}-1}{E[\delta(j)]\bar{\phi}} \sum_{i=1}^n \omega_i^*$ of votes to approve a bailout. Given that weights are derived from the expected welfare impact of a bailout on member countries, they also relate to the absolute level of capital contributions that each country should make to the IMF (i.e. each member's contributions should be proportional to the benefits from belonging to the IMF). Therefore the model also helps to determine the total level of working capital that the IMF should have, a decision that has so far been discretionally determined at General Quota Reviews.

[31] Since this is a static model this parameter only captures the expected lag between a bailout and a capital injection into the IMF. Thus a high $E[\delta(j)]$ reflects times of significant lending activity by the IMF, while a low $E[\delta(j)]$ signals periods of relative tranquility.

[32] Thus, $\delta(j)$ must satisfy: $\inf_j \delta(j)\bar{\phi}w_i^b > \sup_j N_i u_{ij}^+ \ \forall i$. This is a simplification that allows a simple calculation of weights w_i^b, and w_i^n.

[33] See Corollary 1 in Barberà and Jackson (2006).

Table 1: Quota Proposal - G20 countries

Country	Current	Symmetric	North-South
Argentina	0.888	0.576	0.886
Australia	1.358	0.610	0.523
Brazil	1.783	1.455	1.764
Canada	2.672	2.034	0.835
China	3.996	11.227	7.689
France	4.505	3.768	4.101
Germany	6.110	7.251	8.307
India	2.442	2.239	2.173
Indonesia	0.872	1.107	0.783
Italy	3.307	3.264	3.809
Japan	6.556	3.355	3.248
Korea	1.412	2.197	2.192
Mexico	1.521	2.319	0.905
Russia	2.494	2.447	3.224
Saudi Arabia	2.930	1.244	1.005
South Africa	0.784	0.583	0.584
Turkey	0.611	1.218	1.467
United Kingdom	4.505	2.690	2.696
United States	17.670	9.764	10.730