THE ROLE OF UNCERTAINTY AND INSURANCE IN ENDOGENOUS CLIMATE CHANGE*

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July 15, 2008

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Abstract

We investigate the role of uncertain, endogenous extreme events and insurance in a growth model and a fully-calibrated world simulation. We find that i) transparency of the insurance sector is the major determinant for the main variables, implying substantial policy opportunities; ii) the difference between the average Poisson path and the internalized full insurance one stems from the uncertainty; iii) precautionary beliefs on the frequency of extreme events lead to more sustainability; iv) technical change does not change the ordering of the paths but leads to a more sustainable future; v) a social security system which prices insurance fairly is preferable to an insurance industry which provides insurance with an overhead.

JEL Classification: Q5; O1.

Keywords: economic growth; climate change; insurance; integrated assessment; extreme events; catastrophes.

1 Introduction

The debate on global warming is one of the major issues which has protruded through society for the last 20 years. It has imposed never-seen challenges on the world's societies and on the research communities. From biology, psychology, over to geography, politics and economics - all disciplines join into the debate. Each of the disciplines has to fight its own problems, its own stumbling blocks and challenges. However, one question is clearly at the heart of all these disciplines: What surprises will come next? According to the Sigma database, the top 10 most costly hurricanes (at 2006 prices) in the US all happened during the past 5 years. The hurricane Katrina in 2005 alone left a damage of 86bn US dollars and nearly 2000 deaths. The 2002 floods in Europe left damages exceeding 23.6bn US dollars (Swiss Re [22]). A report by EM-DAT [12] concludes that approximately 255 million people were annually affected by extreme events between 1994 and 2003. Major surveys by reinsurance companies like Swiss Re or Munich Re all show a clear increases in the worldwide damages from extreme events. The EM-DAT database for disasters shows record-numbers for hydrological and meteorological disasters. The IPCC [17] and Stern report [25] both survey professional literature on climate change and have come to the conclusion that we are already starting to feel the impacts of climate change. With all these worrying news at hand, we have seen a new evolution of the climate change debate. This evolution is moving toward a combined view of mitigation coupled with adaptation as well as the role of insurance in a climate of change.

Surprisingly, the economic literature has hardly picked up on this debate. Extreme events barely figure in integrated assessment modeling and when they do, then in terms of a single event (Gjerde et al. [8]). Uncertainty is treated via small and frequent changes around a trend and are usually modeled through Brownian motions, via sensitivity analysis or Monte Carlo simulations (see Goodess et al. [11]). Similarly, there has been no role for the insurance company in integrated assessment modeling yet. The objective of this article is thus twofold.

Firstly, we introduce endogenous extreme events in a growth model where agents need to choose under the uncertainty of these events. Secondly, we wish to introduce a tentative analysis of insurance in a global model of climate change in order to understand the trade-off between consumption decisions, savings, abatement and insurance. The background model for insurance is Gollier [10]. For this we build an integrated assessment model (IAM) which we calibrate according to the current economic and climate system in order to provide an empirical counterpart to our theoretical analysis. Through this method we wish to understand the role of endogenous extreme events and insurance in a globally integrated world.

The potential role of the insurance company has been clearly formulated by Munich Re. "As well as preventing future emissions, we must also provide the financial means for adapting to those effects that can no longer be avoided. If we follow the Stern Review's line, this is an area where the insurance industry has a key part to play by providing insurance solutions to counter the financial losses." (Munich Re [21]) A similar statement by Swiss Re suggests that they wish to "minimize the impacts of its operations on the environment and society."

However, what do we actually know about the role of insurance? If agents can insure themselves, does this not simply mean that they act less precautionary, produce more and therefore increase climate change? If one of the main objectives of the reinsurance industry is to help reduce climate change, then how should it set its premia? How will the premia evolve given the expected evolution of damages? What is the role of transparency in the insurance industry?

For example, according to Munich Re [21] over 90% of all extreme events in 2006 were weather-related, and only 10% were of geological nature. Though the final answer has not been given yet, many reseachers believe that hydrological or meteorological extreme events are likely to be driven by climate change. Given this potential endogeneity, how should the insurance industry evolve? What kind of premia are better for the environment and which ones help to improve living conditions? What is the impact of an overhead on the take-out of insurance policies and the consumption and abatement decisions?

Our main results are as follows. We observe that the theoretical model analytically corresponds to the DICE model if we allow for a fair insurance premium which is fully internalized; transparency of the insurance sector is the most important determinant for the evolution of the main economic and environmental variables; different Poisson paths may lead to vastly different outcomes; the difference between an expected stochastic path and an internalized full insurance scenario is driven by uncertainty alone; precautionary beliefs on the frequency of extreme events are more in line with sustainability criteria; technical change does not lead to a qualitative change in the ordering of the paths but will be more in line with sustainability criteria; a social security system which prices insurance fairly would be preferable (in terms of global, realized welfare) to an insurance industry which provides insurance with an overhead.

The article is structured as follows. Section 2 provides an overview of the state of the art results on extreme events and climate change. Section 3 introduces and solves the theoretical model. Section 4 gives the integrated assessment model with computational experiments. Finally, Section 5 concludes.

2 Does Climate Change drive Extreme Events?

Our intention in this section is to provide an overview of the latest observations on climate change and to investigate whether there exists a link between climate change and extreme events.

It is becoming more and more common to use global temperature anomaly as a proxy for climate change. World temperature has increased by 0.6°C during the past 30 years and by 0.8°C during the last century, with stronger warming in higher northern latitudes. These changes drive higher sea levels, decreases in snow and glacier volumes, variations in precipitations, as well as changes in the frequencies of cold days and cold nights, frosts as well as hot days and hot nights. In addition we see a change in the storm tracks and in the intensity and frequency of meteorological events like hurricanes. Many authors believe that these changes are only the tip of a coming iceberg. For example, most SRES scenarios of the IPCC predict temperature increases of around 0.2°C *per decade* during the next century, leading to an increase in global temperature of between 2-4°C by 2100. Hansen et al. [13] show that the temperature today is only slightly less ($<$ 1°C) of its maximum during the past million years. It is thus clear that the predicted increases of 2-4°C may lead to unforeseeable changes in our current environment. For the moment, the expected impacts of sea level rises associated with a temperature increase of 2-4°C are 18-59cm (IPCC estimates) to several meters (Hansen et al. [13]) and much stronger impacts on storms as well as hot and cold spells, droughts, wildlife losses, contraction in glacier masses, species extinction, agricultural productivity, etc. We shall here take a closer look at some of the extreme events which are strongly related to the insurance industry, namely hurricanes, fires and lightning as well as floodings.

2.1 Storms

Up to now, the scientific community is rather uncertain about the consequences of temperature changes on hurricanes on a global scale. This comes about since there are strong regional changes, which are often simply averaged out in data analyzes. A recent contribution by Bengtsson et al. [1] concludes that "there are no indications in this study of more intense storms in the future climate, either in the Tropics or extratropics, but rather a minor reduction in the number of weaker storms. However, significant changes occur on a regional basis in the location and intensity of storm tracks." In contrast to this study, Webster et al. [26] find a doubling in the number of category 4 and 5 hurricanes for the past 30 years. Emanuel [5] suggests that the intensity of hurricanes has approximately doubled since 1950. Also, Mann and Emanuel [15] find a correlation between the amount of Atlantic tropical cyclones and sea-surface temperature. Similarly, a recent synthesis report by Swiss Re [24] finds that the global number and intensity of hurricanes has increased, which they believe to be causally linked to sea-surface temperature, which they again claim to be linked to global warming.

For the case of the US, Nordhaus [19] finds that changes in hurricane strength are strongly correlated with sea-surface temperature for North Atlantic tropical storms. His main conclusion is that tropical storms during the past 25 years increased both in frequency and strength, and that this is due to a warming in the sea-surface.

Though there might exist an impact of climate change on the number and intensity of hurricanes, this will not be important for economic growth modeling if these changes do not also translate into changes in the costs of extreme events. Additionally, changes in the probability and strength of extreme events can themselves have significant impacts on consumption, savings and insurance decisions. In this respect Nordhaus [19] estimates an OLS regression to assess whether there exists a relationship between normalized damages and maximum wind speed. His main conclusion is that the elasticity of damages from maximum wind speeds may be as high as 8, rather than the standardly-used square or cube.

2.2 Fires and lightning

We have reviewed data on the amount of wildland fires in the US from 1960 to 2006 and related this to Northern Temperature Anomaly. It is easily observable that wildland fires in the US are strongly correlated with Northern Temperature Anomaly. The correlation is 0.42 (significant at 5%) for the period 1960-1982 and 0.14 for 1984-2006.[1] We can conclude that warmer temperature bears a positive impact on the amount of wildland fires. Since most research predicts an increasing amount of warmer days and nights (IPCC, Stern report, etc.) we can certainly predict that the probability of wildfires is positively related to a warmer climate. In terms of economic development, the hazard of fires may not be underestimated. Swiss Re [23] estimates that during the past 25 years the loss due to fires as a percent of maximum annual losses is more than 20%. Again, given the predicted increase in warmer days and nights, we can therefore also expect an increase in annual losses from fires.

Another result concerns the amount of lightning-related claims. An analysis of Swiss Re [6] connects the amount of lightning-related claims to the monthly temperature. They find a concave relationship between the number of claims and temperature, and conclude that there is a 5-6% increase in lightning claims with each 1°F (0.6°C) rise in air temperature.

2.3 Floods

Data from the Center for Disease Control and Prevention suggests that the overall insurance claims from floods in the US during 1978 to 2007 amounts to $33,456,159,027$\$. These costs certainly understate the true costs, since not all property is insured. In addition, in the US roughly 74% of all deaths from meteorological and hydrological extreme events occurred from floods. We must therefore

[1] The data needs to be split in these two periods since the compilation method changed around 1983. The lower correlation for the later period should be interpreted as improvements in the firefighting methods.

view floods as a non-negligible impact on the economic system.

Beniston et al [2] simulate regional climate models for the EU and find that heavy winter and summer precipitation is going to increase in every part of the EU apart from the south[2]. This will certainly increase the probability of flash floods, one of the main causes of weather-related deaths. Easterling et al. [4] contains a short review of a number of articles which suggest that precipitation is going to increase in the east and south/east US, northern EU, Russia, China, India, South Africa and Australia. Again, it is believed that this change in precipitation will increase the probability of floods.

A paper by Mudelsee et al [18] suggests a causal relationship from precipitation to flood occurrences in eastern Germany. Similarly, Brázdil et al. [3] attributes the higher frequency of floods in Eastern Europe to changes in precipitation. These results are supported by Kunkel et al [14], Glaser and Stangl [9], Fowler and Hennessy [7] and many more.

We therefore observe that a number of articles from a variety of research fields suggests that economic activity (as well as natural cycles) either already has or is likely to impact the strength and number of storms, the extend of precipitation and therefore the possibility of floods, as well as increase the overall temperature which is strongly related to the occurrence of fires and lightnings.

2.4 An Analysis of Disaster Data

Whereas most articles cited above deal with geographical data of some sort, we are here interested in the occurrence of extreme events or disasters. We wish to understand the causal forces that actually drive the number of disasters on a global scale. The data which we use comes from EM-DAT and Swiss Re for the disasters, GISS NASA for the temperature, UN data and Geohive for population

[2]This is a widely-accepted result, see e.g. Easterling et al. [4] or IPCC [17].

data, UN for the data on urbanization and IMF for the GDP data.

The disaster data from the EM-DAT dataset spans annual observations from 1900-2006. In the EM-DAT dataset, an event is called a catastrophe if at least one of the following holds: 10 or more people reported killed; 100 people reported affected; declaration of a state of emergency; and call for international assistance. Although there are many definitions of a disaster, we view this definition to be a particularly useful one since it does not include a measure of economic costs.

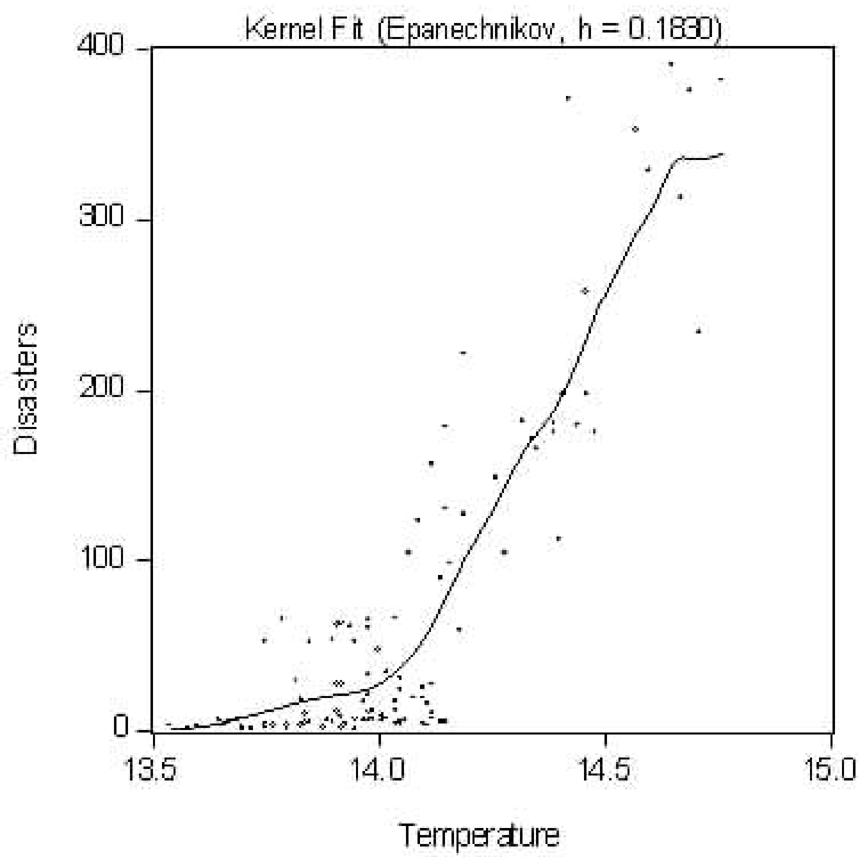

Figure 1: Weather-related Disasters and Global Temperature (1900-2005)
Source: Temperature from GISS NASA and disasters from EM-DAT.

A simple correlation between hydro-and meteorological disasters and global temperature anomaly would give a correlation coefficient of $\rho = 0.84$, suggesting at least a high correlation between disasters and temperature anomaly. One would therefore be inclined to suggest that hotter years coincide with globally more disasters and cooler years with globally fewer ones. Figure 1 plots the Kernel estimate of hydro- and meteorological disasters and the global temperature. The Kernel estimator belongs to the class of non-parametric density estimators and allows one to estimate the probability density function of a random variable. The method locally fits constants to the data via weighted least squares, and the closer the observations to the chosen point the more influence they have on the regression estimates. According to GISS NASA, the best estimate for absolute global mean temperature for 1951-1980 is 14°C. If we therefore take 14°C as the global mean temperature, then we observe that deviations below this mean temperature lead to insignificant changes in disasters. However, once the temperature increases above the average one, we see that the Kernel Estimator suggests a strong relation between both variables. This relationship is even stronger if we compare the Kernel estimates of the average southern temperature with that of the average northern temperature. It suggests that changes in southern temperature might be a more important driving force of disasters, which is also suggested by the various IPCC reports.[3]

As suggested by Munich Re [21], "since 1950, there has been a longterm upward trend in the number of events and the amount of economic and insured losses." However, and rightly so, a simple conclusion like this has been challenged by Pielke and Landsea [20] who suggest that these calculations do not take population changes, inflation and increased wealth into account. On the contrary, if these are taken into account, then they estimate that the costs of hurricanes in the US have decreased. That it is indeed wrong to simply look at the amount of disasters can be shown when looking at the data for ge-

[3] In the two Kernel plots one can see a kink at the end upper end of the estimate. This comes from too few observations at the higher temperature range.

ological disasters (data from EM-DAT). When one compares the number of geological disasters with that for hydrological disasters worldwide, then both types increase over time. However, the number of geological disasters should be rather constant over time. Any significant change in geological disasters should be mainly due to changes in population size, changes in reporting of disasters and better global coverage. Since the disasters are not defined over wealth, and if we control for population changes, then we are able to take care of all the criticisms of Pielke and Landsea.

We run a Kernel estimate using the Gaussian kernel controlling for population changes[4]. Therefore, our test becomes a semi-parametric estimate. Due to the semi-parametric estimate standard hypothesis

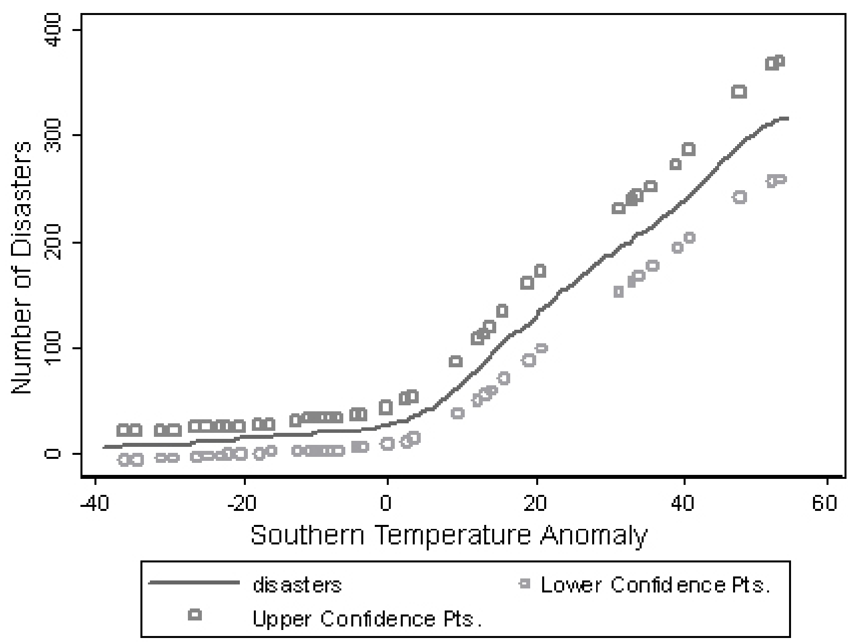

Figure 2: Weather-related Disasters and Southern Temperature Anomaly controlling for population. Source: Temperature from GISS NASA and disasters from EM-DAT.

testing won't work. However, one can calculate point-wise confidence intervals, like it is done for

[4]We are very grateful to Eric Strobl for help with this Figure.

Figure 2. It seems that Pielke and Landsea's criticism does not hold globally. There is still a significant positive correlation between disasters and global temperature.

We also use standard OLS-estimation in order to analyze a possible relationship between disasters and temperature. Since there is a structural break in the temperature series around 1945 and since we believe that the disaster data before that date is somewhat incomplete we shall only use the data from 1945 onwards. We find that both disasters and temperature contain a unit root (via an augmented Dickey-Fuller test). We therefore test for cointegration between disasters and temperature and can conclude that both variables are cointegrated, CI(1,1) (using MacKinnon's approach, at 1% critical value). In this case, the original coefficients are super-consistent and, given our sample size, should be close to the true coefficients. We then run the following regression.

$$\begin{array}{lccc} \text{Disasters} = & \gamma_1 & + & \gamma_2 \times \text{Temperature} \\ & -5142.459^{***} & & 371.3141^{***} \end{array}$$

It seems that indeed temperature exerts a positive effect upon the number of disasters. However, in this cointegration analysis we did not include one of Pielke and Landsea's criticism, namely the changes in population. We shall do so in the following adjusted ADL model. We start with an ECM model and then rewrite it slightly. Our initial relationship is

$$g(L)\text{dis}_t = -\alpha(\text{dis}_{t-1} - \beta_1 - \beta_2 \text{temp}_{t-1}) + g(L)D(\text{temp}_t) + g(L)D(\text{pop}_t) + u_t \qquad (1)$$

where $g(L)$ are lag operators, D is the difference operator, dis_t is disasters at time t, temp_t is temperature at time t and pop_t refers to population at time t measured in billions (data from UN). We then estimate

$$g(L)\text{dis}_t = \alpha\beta_1 - \alpha(\text{dis}_{t-1} - \text{temp}_{t-1}) + \alpha(\beta_2 - 1)\text{temp}_{t-1} + g(L)D(\text{temp}_t) + g(L)D(\text{pop}_t) + u_t \quad (2)$$

If the coefficient $\alpha \neq 0$ then the series temperature and disasters have a long-run relationship. Our estimates are given in the following Table. This suggests a positive long-run relationship between

Table 1: Regression results

Variable	Coefficient	Probability
Intercept	-1495.955	0.001
dis_{t-1}-temp_{t-1}	-0.21646	0.0051
temp_{t-1}	106.3124	0.0011
D(temp_t)	50.17324	0.0346
D(temp_{t-1})	-85.43442	0.0005
D(pop_t)	276.1428	0.158
R-sq	0.281	
adj R-sq	0.217	
DW-stat	2.305	
F-stat	4.377	

disasters and temperature (since the coefficient is negative), but also strong short-run effects from changes in temperature. The regression results do not change when one drops population changes, which are insignificant at the 10% level. That population changes are insignificant is somewhat surprising, since the world population increased by nearly three times between 1945 and 2006. One would have therefore expected an increase in the amount of disasters being noticed and reported. This could however be simply due to population density deepening rather than a widening. By this we mean the fact that the world population did not spread out further after 1945 but became more condensed. For example, the UN World Urbanization Prospects report suggests that the world urbanisation rate rose from 29% in 1950 to 49% in 2005. However, an interaction term consisting of the urbanization rate with population and introducing this into equation (2) instead of the population variable is insignificant, too. Similarly, one might suggest that richer people will be able to cover themselves from disasters or move into regions where there are fewer ones. Yet again, introducing changes in world GDP (data from IMF) prove to be an insignificant explanation for changes in disasters. It seems that temperature is the only variable which is able to consistently account for changes in disasters[5].

One should however take these results with a pinch of salt. The data is highly aggregated across space and disaster types. Some regions became hotter and others cooler. Finally, the data from EM-DAT is known to be incomplete and may consist of some faults. Nevertheless, it is surprising how consistently significant temperature is in the regression.

Armed with these results we now proceed to analyze the hazard of endogenous, uncertain extreme events in a Ramsey-type growth model.

3 The Theoretical Model

The setup of the model follows a standard Ramsey-type growth problem, where an infinitely-lived agent maximizes his uncertain stream of utility subject to consumption, savings, abatement and insurance penetration. Our model here extends the work by Gollier [10] by allowing for abatement, endogenous number and size of extreme events and a climate change sector. Capital increases with production and is reduced by constant depreciation ($\delta_k > 0$), consumption ($c(t)$), abatement activity ($a(t)$), the size of the insurance premium ($P(t)$), the percent of coverage ($1 \geq 1 - \gamma(t) \geq 0$), and an extreme event. The extreme event is modeled according to a Poisson shock. When $q(t) = 0$ no shock occurs, whereas if $q(t) = 1$, then an extreme event reduces capital stock by $\gamma(t)\psi(T(t))k(t)$. The expected number of extreme events in any point in time is an endogenous, increasing function of temperature and given by $\lambda(T(t)) > 0$. The percent of capital destroyed during an extreme event is

[5] We also run a VECM to assess whether disasters bear any impact on temperature. We find that temperature is able to explain disasters but disasters are not able to explain temperature changes. This thus confirms our result above and our main intuition.

given by the function $\psi(T(t))$, which too is increasing in temperature. When there is full insurance, thus $\gamma(t) = 0$, then the agent has full insurance coverage and will not be (financially) affected by the extreme event. Finally, temperature is increased by productive activity and decreases due to a natural regeneration. In our model, abatement can reduce emissions, but it cannot affect temperature directly. One should therefore interpret abatement as an investment in greener technology.

The agent thus solves the following problem:

$$V(k_0, T_0) = \max_{\{c(t),\gamma(t),a(t)\}} E_{t_0}\left\{ \int_{t_0}^{\infty} u(c(t), T(t)) e^{-\rho(t-t_0)} dt \right\} \tag{3}$$

subject to

$$\begin{aligned} dk(t) &= \{f(k(t)) - c(t) - a(t) - (1-\gamma(t))P(t) - \delta_k k(t)\}dt - \gamma(t_-)\psi(T(t_-))k(t_-)dq(t), && (4)\\ dT(t) &= \{g(k(t), a(t)) - \delta_T T(t)\}dt, && (5)\\ &\text{and} \quad k(0), T(0) \quad \text{given.} \end{aligned}$$

The interpretation of T as an argument in the utility function can be that this is a short-hand writing for $u(c, \lambda(T))$ and means that the agent's utility is decreasing in the probability of an accident. If T is interpreted as the stock of CO_2, then another interpretation could simply be amenity value for the environment or other non-market damages. We assume that capital follows a cádlág process, such that $k(t)$ is continuous from the right having left limits. The left limits are given by $\lim_{s\uparrow t} k(s) = k(t_-)$. Intuitively, if an extreme event occurs, then the size of the jump depends on the amount of capital just before the jump. In the subsequent part we skip this extra piece of notation but hope the reader keeps this in mind.

We assume the following functional forms.

Assumption 1 *The utility function* $u : \mathbb{R}_+^2 \to \mathbb{R}_+$ *is at least twice continuously differentiable, concave in both arguments, with* $u_1(c,T) > 0$, $u_2(c,T) < 0$, *and* $\lim_{c\to 0} u'(c) = \infty$.

Assumption 2 *The production function* $f : \mathbb{R}_+ \to \mathbb{R}_+$ *follows* $f(k) \geq 0$, $f(0) = 0$, $f'(k) > 0$, $f''(k) < 0$, *with* $\lim_{k\to 0} f'(k) = \infty$, *and* $\lim_{k\to\infty} f'(k) = 0$.

Assumption 3 *The percent of capital stock destroyed has the following functional form.* $\psi(T) > 0$, $\psi'(T) > 0$.

The assumptions on $\psi(T)$ come from the previous section.

Assumption 4 *The extreme event is modeled via a Poisson process. The endogenous probability of an extreme event follows* $\lambda(T) > 0$, $\lambda'(T) > 0$.

Assumption 5 *Temperature increases concavely with capital* $g_1(k,a) > 0$, $g_{11}(k,a) < 0$, *and decreases concavely with abatement activity* $g_2(k,a) < 0$, $g_{22}(k,a) < 0$.

These last assumptions on temperature accumulation are standard in the integrated assessment literature.

The Bellman equation of this control problem is given by

$$\rho V(k_t, T_t) = \max_{\{c_t, a_t, \gamma_t\}} \left\{ u(c_t) + \frac{1}{dt} E_t dV(k_t, T_t) \right\},$$

and $V(k_t, T_t)$ refers to the optimized utility functional. This equation suggests that the return of having k_t and T_t, denoted in indirect utility terms, should at any point in time be equal to the instantaneous felicity as well as the expected change in the future indirect utility stream. Making use of the Change of Variable formula, which is the equivalent of Itô's Lemma but holds for Poisson processes, and taking the expectation we arrive at the following Hamiltonian- Bellman-Jacobian equation.

$$\begin{aligned}\rho V(k_t, T_t) &= \max_{\{c_t, a_t, \gamma_t\}} \Big\{ u(c_t) + \big(f(k_t) - c_t - a_t - (1-\gamma_t)P_t - \delta_k k_t\big) V_k(k_t, T_t) \\ &\quad + \big(g(k_t, a_t) - \delta_T T_t\big) V_T + \lambda(T_t)\big[V\big(k_t - \gamma_t \psi(T_t) k_t, T_t\big) - V(k_t, T_t)\big]\Big\}. \end{aligned} \tag{6}$$

The part of the Bellman equation with the squared brackets includes the adjustment through extreme event. The term $V\big(k_t - \gamma_t\psi(T_t)k_t, T_t\big) - V(k_t, T_t)$ describes the total cost of the extreme event, since it refers to the difference in indirect utility after a jump occurred minus the indirect utility without a jump. This difference is multiplied by $\lambda(T)$ to transform it in expected value terms.

The dynamic system after optimization is completely characterized by the following system of equations:

$$\begin{aligned} -\frac{u''(c)}{u'(c)} dc &= \Big\{ f'(k) - \delta_k + \frac{g_k}{g_a} - \rho + \frac{u_{cT}}{u_c}\big(g(k,a) - \delta_T T\big) \\ &\quad + \lambda(T)\bigg(\frac{u'(\tilde{c})}{u'(c)}\big(1 - \gamma\psi(T)\big) - 1\bigg)\Big\} dt - \Big\{\frac{u'(\tilde{c})}{u'(c)} - 1\Big\} dq(t), \end{aligned} \tag{7}$$

$$\begin{aligned} \frac{g_{aa}}{g_a} da &= \Big\{ g_a \frac{u_T}{u_c} - f'(k) + \delta_k - \delta_T - \frac{g_k}{g_a} - \frac{g_{ak}}{g_a}\Big[f(k) - c - a - (1-\gamma)P - \delta_k k\Big] \\ &\quad + \lambda'(T)\Big[\frac{\tilde{V} - V}{V_T}\Big] + \lambda(T)\Big[\frac{\tilde{V}_T}{V_T} - \frac{\tilde{V}_k}{V_T}\gamma\psi'(T)k - \frac{u'(\tilde{c})}{u'(c)}(1 - \gamma\psi(T))\Big]\Big\} dt \\ &\quad + \Big\{\frac{u'(\tilde{c})}{u'(c)} - \frac{\tilde{V}_T}{V_T} - \frac{\tilde{g}_a}{g_a} + 1\Big\} dq(t) \end{aligned} \tag{8}$$

$$dk = \{f(k) - c - a - (1-\gamma)P - \delta_k k\} dt - \gamma\psi(T) k dq(t), \tag{9}$$

$$dT = \{g(k,a) - \delta_T T\} dt, \tag{10}$$

$$\frac{u'(\tilde{c})}{u'(c)} = \frac{P}{\lambda(T)\psi(T)k}, \tag{11}$$

where $\tilde{x}$ refers to a variable after a jump occurred and time subscripts are submitted for simplicity.

We can easily interpret this system describing the way preferences and technical possibilities work together. Starting with equation (11) we know this defines the optimal level of insurance cover for the agent. If the premium is fair and therefore equal to the expected damages, then we obtain $\tilde{c} = c$ and no jump will occur. If there is an overhead on the premium, then $u'(\tilde{c}) > u'(c)$ which implies consumption after an extreme event is lower than before an extreme event. The larger the overhead on the premium the bigger the jump of consumption.

Equation (7) refers to the optimal consumption choice of the agent. The part in the first curly brackets explains the way the agent chooses if there is no extreme event but taking the possibility of an extreme event into account. This term is a standard Ramsey-Keynes component, where the agent will choose to increase consumption if the benefits of producing more outweigh the costs of capital depreciation, time preference, and it also involves a valuation of how changes in temperature affect future marginal utilities. In addition, the term $\lambda(T)\left(\frac{u'(\tilde{c})}{u'(c)}\left(1-\gamma\psi(T)\right)-1\right)$ is new to the literature, and it can be positive or negative. If we assume that the currently exogenously given insurance premium sufficiently exceeds the expected damage, then we know that if an extreme event occurs, consumption will decrease and therefore $u'(\tilde{c}) > u'(c)$. Hence, if the jump is large enough, then this term will have a positive effect on consumption growth. We interpret this as precautionary consumption. Since the agent knows that in the future his consumption might be reduced through an extreme event, he prefers to increase his current consumption in order to fall back to some average level later. However, this term needn't always be positive. If insurance cover is low and the percent of capital stock which gets destroyed large, then the overall term might turn negative, leading to a reduction in consumption in favor of either abatement activity, insurance or precautionary capital accumulation. It therefore becomes easily clear that the theoretical model leaves us with an insurance dilemma: Whether the agent increases or decreases his future consumption will depend on the relative strength of the precautionary consumption versus the precautionary savings effect. The last term in the curly brackets refers to the adjustment in case an extreme event occurs. Since the ratio $u'(\tilde{c})/u'(c) \geq 1$ we know that the jump in case an extreme event occurs will be negative. Indeed, the size of the jump can be found for the case of an interior solution in $\gamma \in (0,1)$, CRRA utility and a standard insurance contract with an overhead on an otherwise fair premium. In that case we know from equation (11) that $u'(\tilde{c})/u'(c) = \phi$, where $\phi > 1$ gives the overhead. Thus the jump in consumption will be determined by the level of consumption before the jump, the size of the overhead and the intertemporal elasticity of substitution (IES). It will be given by $\tilde{c} = \phi^{-1/\sigma}c$, where $\sigma > 0$ is the IES. We can easily calculate that $\frac{d\tilde{c}}{d\sigma} > 0$ (taking given the effect of σ on c), implying a stronger consumption smoothing the larger the intertemporal elasticity of substitution. This also allows us to obtain a condition for the precautionary consumption versus savings question. Given the previous assumptions, precautionary consumption is positive if $\gamma\psi < (\phi - 1)/\phi$, whereas precautionary savings are positive otherwise. Clearly, the more capital gets destroyed the more incentive will be for precautionary savings. On the converse, the larger the overhead the more likely is precautionary consumption.[6] Surprisingly, one would therefore expect more precautionary consumption if agents insure less.

The equation (8) gives the amount of optimal abatement. The terms in the first line are the standard ones describing trade-offs between abatement versus capital for their relative effectiveness on temperature, capital accumulation and future costs. The first term in the second line describes the cost of a marginal increase in the probability of an extreme event on the future stream of utilities. The larger the marginal impact of temperature on the probability of extreme events and the higher the costs of an extreme event in terms of utility foregone, the stronger will the agent increase abatement. The second term in the second line gives the amount of precautionary abatement. Precautionary abatement is positive if, in case of an extreme event, we expect a lot of capital to be destroyed and if the impact

[6] Since $\gamma\psi < 1$ is bounded by $\psi < 1$ for $\gamma \rightarrow 1$ whereas $(\phi - 1)/\phi \rightarrow 1$ for increasing ϕ.

of changes in temperature on the percent of capital which gets destroyed is very large. Precautionary abatement can be negative though, too. This will be the case if precautionary consumption is very big. This trade-off obviously comes from the capital constraint. The third line of equation (8) gives the impact on abatement in case of an extreme event. Abatement growth itself will respond to an extreme event with either an upward or a downward jump. It will jump upwards if consumption falls significantly after a jump, but it will decline if the marginal impact of temperature on indirect utility after a jump is much higher than before a jump.

In general, this system is not analytically tractable but it allows us to see certain dilemmas which would normally not have been observed or taken into account. We shall now impose assumptions which allow us to find solutions to a few special cases of this cumbersome dynamic system. They provide us with tractable benchmarks which we shall also later use in the computational experiments.[7]

3.1 The insurance industry

Our intention now is to understand the role of an insurance company in this framework. For this we shall start with a strong assumption: We assume that the insurance company behaves like a risk-neutral firm in a perfectly competitive market. We are well-aware of the two major implications of this assumption. Firstly, we do not account for a possible default of the insurance sector. Though this is a viable threat for small insurance companies, we do not believe that this is likely to occur for the global insurance sector. If the premia are chosen with a certain foresight that reflects the actual probability and size of extreme events, then the probability of a global default of the insurance industry is likely to be small. Furthermore, certain risks which smaller insurance companies deem

[7] In general it is not our objective to solve for the dynamics of these systems. However, it can be shown that a steady state exists (in the certainty case). Solving for the dynamics around steady state will however give conditions which do not help us in interpreting the results.

too risky are transferred to reinsurance companies that can control these excess risks much better. Secondly, with this assumption we abstract from the possibility that the insurance sector accumulates capital which it can invest in a capital market.[8] The implications of the insurance industry in our model could therefore be described as follows. An agent will be able to obtain insurance but in his decisions of insurance he will not include the possibility of default in the insurance industry (since the probability of default is zero). If the insurance industry is able to receive more capital during certain periods than it has to pay out, then this capital will not be invested in the capital market but stays perfectly liquid in order to pay for future claims later. With this in mind we can turn to the problem of the insurance sector.

We know that the expected insurance claims at any point in time are $\lambda(T(t))\psi(T(t))k(t)(1-\gamma(t))$, whereas the insurance premia obtained are $(1-\gamma(t))P(t)$. There may exist transaction costs or operational costs, represented by a fraction $\phi > 0$ of the insurance claims. The expected profits are therefore

$$E(\Pi(t)) = (1-\gamma(t))P(t) - (1+\phi)\lambda(T(t))\psi(T(t))k(t)(1-\gamma(t)).$$

The zero-profit condition then implies that we obtain $P(t) = (1+\phi)\lambda(T(t))\psi(T(t))k(t)$. Then, $(1+\phi)$ represents overhead charges on the premium. If the transaction costs or operational costs are negligible, then the premium will be equal to the expected costs from an extreme event and thus provides a fair insurance cover.

[8] One could argue that the insurance industry, and especially re-insurance companies, are non-negligible players on the world capital market. However, allowing for a positive profit would raise questions like: To whom belongs this capital? Are these companies then risk-neutral or do they reflect the preferences of their owners? This would obviously give rise to similarly strong assumptions. Finally, a representative insurance company would have to withdraw capital from the capital market in equal amount to the losses from an extreme event, which thus would be equivalent to no insurance at all.

3.2 The case of naive insurance

Assuming no overhead for the moment[9] such that $\phi = 0$, we find that any risk-averse agent will choose $\gamma(t) = 0$. This implies that our dynamic system reduces to

$$-\frac{u''(c)}{u'(c)}\dot{c} = f'(k) - \rho - \delta_k + \frac{g_k}{g_a} + \frac{u_{cT}}{u_c}\dot{T}, \tag{12}$$

$$-\frac{g_{aa}}{g_a}\dot{a} = -g_a\frac{u_T}{u_c} - \delta_k + \delta_T + f'(k) + \frac{g_k}{g_a} + \frac{g_{ak}}{g_a}\dot{k} \tag{13}$$

$$\dot{k} = f(k) - c - a - \lambda(T)\psi(T)k - \delta_k k, \tag{14}$$

$$\dot{T} = g(k,a) - \delta_T T. \tag{15}$$

Our result is that under a fair insurance premium the only effect which the extreme event might have is that the agent behaves as if he has a capital stock which is reduced by the expected damage of extreme events (his payment to the insurance industry). Not surprisingly, his consumption and abatement decisions are not affected by any precautionary decision process. Most interestingly, since the agent takes the evolution of the insurance premium as given, he will not take the impact of his decisions on the probability and size of extreme events into account. For example, one could very well imagine that this case corresponds to the way a small agent would act who believes that his decisions have no influence on the evolution of the premium. Possible changes in the premium do not even figure in his abatement decisions. We interpret this case as that of a *naive insurance*, and it is here where the role of the insurance industry becomes dominant. If the insurance industry does not inform the agents of their influence on the evolution of the insurance premium, then this should have drastic consequences for consumption and savings decisions. Since the agent does not control for the effect of temperature on the premium, one would expect that temperature increases drastically in the

[9] We investigate the implication of an overhead in the next section.

naive insurance case, which should lead to large reductions in future capital stocks. We shall confirm this in a subsequent section via computational experiments.

As we have seen, the transparency of the insurance industry may play a significant role for the evolution of climatic and economic variables. Of course, if the insurance industry feels that future environmental and economic conditions are important, then it could improve upon this situation. Apart from setting the size of the premium and thereby influencing the decisions of the agent, the insurance company can inform the agent about the evolution of the premium given his production choices. We shall analyze this case now.

3.3 The case of an internalized insurance premium

Imagine that the agent knows that he will get a fair premium and will therefore take up full insurance. Furthermore assume that the insurance company clearly shows all agents how their economic decisions impact the premium which they have to pay. Then the agents will incorporate the changing costs of the insurance premium into their decision taking. The problem then writes as follows:

$$V(k_0, T_0) = \max_{\{c(t),a(t)\}} \int_{t_0}^{\infty} u(c(t), T(t)) e^{-\rho(t-t_0)} dt \tag{16}$$

subject to

$$\dot{k}(t) = f(k(t)) - c(t) - a(t) - \lambda(T(t))\psi(T(t))k(t) - \delta_k k(t), \tag{17}$$

$$\dot{T}(t) = g(k(t), a(t)) - \delta_T T(t). \tag{18}$$

A rather surprising observation is that this is observationally equivalent to a standard integrated assessment model like DICE. The DICE model is therefore equivalent to our model if we have a fair premium and thus full insurance plus an agent who takes the evolution of this premium into account. One could rephrase this slightly. Assuming one wishes to use the DICE model to analyze extreme events without altering its structure. Then we can conclude that the model will produce acceptable results only under full insurance and the internalization of the evolution of the premium.

Writing the Hamiltonian from the above equations leads to

$$\mathcal{H} = u(c,T) + \mu\big[f(k) - c - a - \lambda(T)\psi(T)k - \delta_k\big] + \xi\big[g(k,a) - \delta_T T\big]. \tag{19}$$

We can derive the following system of equations which characterize the dynamics:

$$\dot{c} = c\sigma(c)\left[f'(k) - \lambda(T)\psi(T) - \rho + \frac{g_k}{g_a} - \delta_k + \frac{u_{cT}}{u_c}\dot{T}\right], \tag{20}$$

$$\dot{a} = a\theta(a)\left[f'(k) - \lambda(T)\psi(T) - \delta_k + \delta_T + \frac{g_k}{g_a}\right. \tag{21}$$

$$\left. - \frac{u_T}{u_c}g_a + g_a\big(\lambda'(T)\psi(T) + \lambda(T)\psi'(T)\big)k + \frac{g_{ak}}{g_a}\dot{k}\right], \tag{22}$$

$$\dot{k} = f(k) - c - a - \lambda(T)\psi(T)k - \delta_k k, \tag{23}$$

$$\dot{T} = g(k,a) - \delta_T T, \tag{24}$$

where we define $\sigma(c) = -u_c/(cu_{cc})$ and $\theta(a) = -g_a/(ag_{aa})$. Two new effects can be singled out. Firstly, the direct effect of the premium's size. If $\lambda(T)\psi(T)$ is large, meaning that many events happen and they destroy a significant part of the capital stock, then this term may lead to a decrease in consumption and abatement growth. Both may be optimally reduced since stronger damage leads to a lower global capital stock which does not allow to continue consumption and abatement at the previous levels. The other new term only affects abatement and it relates to the direct impact of abatement on the change in the expected value of an extreme event through a change in temperature.

The larger the marginal effect of abatement on temperature or the larger the effect of temperature on the expected costs of the extreme event, the more will be invested by the agent into reducing temperature. This term does not show up in the accumulation for consumption since consumption affects temperature or the expected costs of extreme events only indirectly. In the computational experiments we shall study the magnitude of this result in a properly calibrated model.

4 Computational Experiments

Though we are able to understand the role of uncertainty and the insurance sector already through the analytical model, it is certainly worthwhile to investigate how important the quantitative differences in the various scenarios are for a properly calibrated model of the world economy. We shall therefore translate the analytical model into a computable version. This requires some minor modifications. In particular we need to specify all functions involved and to calibrate the parameters. In the end, our intention is to complement our previous analysis in a quantitative way to think about the economics of extreme weather events.

4.1 Specifications

The instantaneous utility function u is assumed CRRA, $u(c(t), T(t)) = \frac{(\theta(\Delta T(t))c(t))^{1-\sigma}}{1-\sigma}$. The risk aversion parameter σ is set equal to 2, $\Delta T(t)$ measures temperature relative to the initial atmospheric temperature (ie. $\Delta T(t) = T(t) - T(0)$). Climate change directly affects utility, which is captured by a consumption loss factor $\theta(\Delta T(t))$. These non–market damages (Manne et al. [16]) are assumed quadratic in climate change perturbation ΔT, $\theta(T(t)) = 1 - \min\left\{1, \frac{\Delta T(t)^2}{\Omega_U}\right\}$, where Ω_U is calibrated such that doubling atmospheric carbon relative to pre–industrial levels accounts for a loss of 1.5 per cent in consumption-equivalent utility.

For the climate system we adopt the structure of the CWS model. This climate module is similar to that used in the new DICE 2007 model by Nordhaus but calibrated to fit the IPCC scenarios slightly better. It is sufficiently non-linear to approximate the results of even larger climate models like those used in the IPCC scenarios. Carbon dioxide emissions $e(t)$ accumulate in the atmosphere according to

$$M_A(t+1) = (1+\tau_{11})M_A(t) + \tau_{21}M_U(t) + e(t),$$

where M_A and M_U denote atmospheric carbon concentrations in the atmosphere and upper ocean, respectively. For parameters τ_{11} and τ_{21}, see Table 2.[10]

Upper ocean carbon concentrations depend upon concentrations in both the lower ocean M_L and the atmosphere M_A,

$$M_U(t+1) = (1+\tau_{22})M_U(t) + \tau_{12}M_A(t) + \tau_{32}M_L(t).$$

Lower ocean concentrations evolve according to

$$M_L(t+1) = (1+\tau_{33})M_L(t) + \tau_{23}M_U(t).$$

Forcing f_o is given by $f_o(t) = 4.03\frac{\log(M_A(0)/590)}{\log(2)}$. Ocean temperature T_O and atmospheric (surface) temperature T follow

$$T(t+1) = .2562\; T(t) + \tau_4 f_o(t) + \tau_5 T_O(t)$$

and $T_O(t+1) = \tau_6 T(t)$. Carbon emissions $e(t)$ arise from production $y(t)$ in fixed proportions. They can be abated, for example by substituting solar energy for oil. Abatement activities are summarized

[10] We are particularly grateful to Philippe Marbaix for new calibrations of the parameters of the climate module as well as to Johan Eyckmans for the code.

Table 2: Climate System Parameters

Parameter	Value
τ_{11}	-0.033384
τ_{21}	0.027607
τ_{23}	0.011496
τ_{33}	-0.000422
τ_4	1.7
τ_5	0.794
τ_6	0.03609

through $m(t) \in [0, 1]$ which gives the share of abated emissions in total emissions.

Production of $y(t)$ is Cobb–Douglas type with labor and capital as inputs. The output elasticity of capital is assumed 0.36. Labor endowment is exogenously given and price–inelastically supplied. Output is spent on consumption, investment, insurance and abatametent, with $a(t) = .3m(t)^2 y(t)$ being the abatement costs. A complete decarbonization of the economy ($m(t) = 1$) would take thirty per cent of the GDP in addition to current energy spending.

Capital accumulates according to

$$K(t+1) = \big(1 - (1 - \gamma(t))\Psi(T(t)\big)dq(t))(1 - \delta_K)K(t) + i(t)$$

where

$$\Psi(T(t)) = 0.05 + \min\left\{1, \frac{\Delta T(t)^2}{\Omega_K}\right\}.$$

Here, γ is the percent insured (in contrast to the theoretical model). The damage parameter Ω_K is calibrated such that an extreme event at double carbon concentration relative to the pre–industrial level (equivalent to a rise in temperature of 2.5 C) will destroy forty per cent of the total capital stock.

Finally, the discount rate on instantaneous utility ρ is assumed 0.03, the arrival rate of extreme events is fixed at $\lambda = .2$, and capital depreciation is $\delta_k = .05$.

Simulations are carried out with GAMS/CONOPT3. Time is taken as discrete with a one year time–step. The model is solved by nonlinear programming. To cope with the curse of dimensionality, we take only a reduced scenario sample into account. By sensitivity analysis we observe that samples of 20 randomly drawn scenarios give sufficiently robust results. The scenarios are sampled such that an agent observing these scenarios would conclude from statistical inference that he faces a Poisson process with the given arrival rate $\lambda = .2$.

At any point in time, the agent maximizes expected utility by looking 70 years ahead. We assume that starting in the base year 2005, the agent decides about consumption, investments and abatement up to 2070, contingent upon the unfolding sequence of extreme weather events. After reaching and carrying out insurance, abatement and investment decisions, an extreme weather event either does occur or not. In 2006, the agent revises his original state–contingent policies, looking forward till 2071. Again by sensitivity analysis we found out that this time horizon is sufficiently long to ensure time–consistency.

The purpose of the computational experiments is to answer specific questions which the theoretical model is ambiguous or silent about. These questions are: What is the implication of uncertainty and insurance for policy decisions? What is the role of transparency in the insurance sector? What happens if agents hold false beliefs about the risk of extreme events? what is the role of technical change? Who should provide insurance?

These questions naturally define several scenarios. From the previous section we can single out three broad scenario types.

- *Full internalized insurance* transfers all risks to the insurance company, but agents now take into account that they endogenously determine the insurance premium by their abatement activities. This is similar to a deterministic case with fully internalized climate change like it is the case of DICE or MERGE models. The insurance premium is fair. This will be our reference scenario. We shall call this "full insurance" for simplicity.

- *Stochastic* means that agents can not insure at all. They are fully exposed to extreme weather events. Agents anticipate their influence on the future climate.

- *Naive insurance*, meaning agents can insure again extreme weather events but in contrast to the full insurance scenario they consider the insurance premium as exogenous. Indeed, this case would also correspond to the competitive market outcome where agents themselves do not perceive an impact of their decisions on the insurance premia.

As should be clear, the variance of the fully stochastic paths tends to infinity over time due to the Poisson process. Relying therefore on only a selection of paths might lead to a biased result since one might have picked up very unlikely paths with either many or very few events. One way to avoid this is to plot a reference path, which we call the average stochastic path. This corresponds to an averaging over a large number of stochastic paths and is equivalent to taking the conditional expectations at each point in time. Our procedure for this is explained in the Appendix.

Our intention is to first explain the basic results from the simulations and then apply these to specific questions of interest. We plot two fully stochastic paths (Jump1 and Jump2), the average stochastic path (AvgStoch) and the case of full insurance with (FairI) and without (Naive) the internalized premium.

$>$ Figure 4 to Figure 6 about here $<$

We can observe several important results. Firstly, the difference between the average stochastic path and the full insurance path stems solely from the uncertainty. Under fair insurance the agent insures fully and therefore pays a premium equal to the expected damages in each period. In that case the agent transfers all the uncertainty to the insurance industry. He will abate because he knows that his decisions have an impact on future premium payments. In the average stochastic case the agent cannot insure, solves under uncertainty but in each period the expected damages actually arise, wherefore the agent will bear costs which are equivalent to those under the full insurance case. He will abate more though which is solely due to the uncertainty.

Secondly, the individual stochastic paths can vary significantly. For example, we plot two randomly-drawn stochastic paths and they clearly grow vastly different. This confirms our previous suggestion that it can be misleading to make policy analysis based on only one randomly-drawn path and supports our arguments for using the average stochastic path. It is also difficult to see whether precautionary

Realized Welfare without technical change				
FairIns	AvgStoch	Naive	Jump1	Jump2
-567	-587	-743	-535	-584

consumption or precautionary savings prevail in the stochastic jump paths. However, if one looks closer at the consumption paths for the two jumps, then one notices that both paths tend to increase rapidly after having been driven down by consecutive shocks. We would therefore conclude that precautionary consumption prevails in our analysis. Indeed, based on the theoretical model we would only expect this if the jumps in consumption after an extreme event occurred are sufficiently big in comparison to the capital loss from the event itself.

The last path is the naive insurance case which is our most pessimistic case. This scenario is equivalent to a non-transparent or non-internalized insurance policy, but also equivalent to a competitive economy case where an agent believes himself to be sufficiently small in order not to bear any impact on the insurance policy. The only abatement done is directed toward the non-market damages since the agent does not take his impact on the insurance premium into account. This leads to large initial increases in investment allowing for high consumption in the subsequent periods, followed by a fast worsening of the climate feedback. In our simulation, these increases in temperature lead to a large amount of extreme events for the agent which after around 25 years implies massive destructions of capital stock, leading to a non-sustainable evolution of consumption. In terms of realized welfare, the naive insurer does the worst of all.

We shall use our results now to answer some policy-oriented questions.

4.2 What is the role of transparency in the insurance sector?

One way to understand the potential role of the insurance company in environmental and economic policy is by investigating the importance of transparency. A fully transparent insurance policy would be equivalent to the fair insurance scenario whereas an opaque policy would be depicted by the naive insurance case. Indeed, as already discussed above, the difference between the two scenarios in terms of realized welfare, in terms of the impact on the environment and in terms of consumption is dramatic.

The preceding point cannot be emphasized too much. Transparency of how oneself affects the premium is a vital ingredient for sustainable consumption and improves global welfare. Indeed, one would expect this opens up a potential role for regulatory efforts. Obviously, one would wish for regulation directed toward a stronger information revelation. If the insurance companies inform exactly how they set their insurance premium then it is clear that agents will incorporate this into their decision plans.[11] Obviously this will lead to tighter competition on the insurance market, with the effect that overheads will decrease and policy uptakes increase. Indeed, as we show later under the analysis of partial insurance, this is likely to increase global welfare. In addition, one would wish regulatory efforts directed toward a reduction of asymmetric information. Insurance companies generally do not inform the public about the expected extreme events in their area even though disaster hot spots are perfectly well-known to the insurance industry. It is hardly possible to obtain any data from the insurance sector due to fear of competitive pressure. It is here where regulatory efforts are most useful.

4.3 Are precautionary beliefs welfare-improving?

We have previously argued that information on the evolution of the insurance premium as well as on the probability and size of extreme events are important ingredients for welfare-improving outcomes. We also noticed that information on how the policy maker can influence the evolution of the insurance premium is one of the key determinants of a sustainable path of the main economic and environmental variables. We did not yet investigate the effect of a false belief on the probability of extreme events.

Our main question here is the following: If the policy maker does not know the true λ, what would be the effect of under-estimating or over-estimating this parameter? In other words, is precaution preferable to a possible under-estimation of events? It is not possible to compare expected welfare levels here (due to the different probabilities attached), but one can ask: Is precaution[12] preferable in terms of realized welfare or in terms of some sustainability measure?

[11] In a decentralized version one would assume that agents who are aware of their impact on the insurance would act in a cooperative way, through e.g. voting, in order to reduce climate change.

[12] By precaution we here mean that, given a policy maker is faced with a possible distribution of λ's, he would choose a high one.

$>$ Figure 10 to Figure 12 about here $<$

We shall firstly look at the optimal decisions from a discounted utilitarian perspective. Figures 10 to 12 show the optimal paths of consumption, temperature and abatement for the case of an under-estimation of the amount of extreme events ($\lambda = 0.1$), for a correct belief ($\lambda = 0.2$) and for an over-estimation of the number of events ($\lambda = 0.3$). If an agent under-estimates the amount of extreme events, then he will undertake little abatement, leading to significant increases in emissions and thus temperature. Since he under-estimates the true number of events he puts little significance to the environment and therefore consumes much more than if he had correct beliefs. However, due the strengthening of climate change, we estimate strong increases in extreme events which reduce the capital stock and eventually lead to a decrease in consumption below the level of correct beliefs. The scenario with precautionary beliefs, or an over-expectation of extreme events ($\lambda = 0.3$) behaves in exactly the opposite way, strong abatement activity, low overall temperature and increasing consumption levels in the future. Therefore, from a discounted utilitarian perspective, we find that average realized welfare is higher if the agent under-estimates the amount of extreme events than if he over-estimates or correctly estimates the amount of extreme events. This can be explained through the inertia in the climate system and discounting. Clearly, if one were to adhere to the discounting

Average realized welfare with false beliefs		
lambda=0.1	lambda=0.2 (correct)	lambda=0.3
-515	-587	-597

utilitarian perspective then it might be advisable to place more emphasis on consumption now and simply give fewer weight to the climate system (at least from a realized welfare perspective). Indeed, this could be one of the main reasons to question the use of the discounted utilitarian criterion in climate analysis altogether. The path which is the most inequitable one in terms of utility, consumption or environmental quality is the one which gives the highest realized utility. On the other hand, the precautionary beliefs give the lowest realized utility but fare best in terms of realized welfare and sustainable consumption. To answer our question, we find that precautionary beliefs maximize future welfare, whereas current welfare is evidently maximized by just doing the opposite. To put it more bluntly: An under-estimation of extreme events is equivalent to choosing the Damocles sword, whereas an over-estimation of extreme events is equivalent to choosing a safety net.

4.4 Implication of Technical Progress

We assume that technical change (TFP) comes as Harrod-neutral technical progress in the production function with a growth rate of 1.5% per period for 70 years, and zero thereafter. As expected, this simply allows for higher consumption, abatement, but also initially leads to a higher stock of atmospheric carbon and thus temperature. Since there is a higher income later, more money can be spent on abatement in later periods, which thus drives temperature down below the case of no technical change. Indeed, increases in TFP allow for abatement levels which lead to a nearly carbon free production. It is again the possibility to insure fully which implies a lower abatement effort and therefore stronger climate change. This might imply the need to trade off several policy targets, depending on how one would define sustainability or welfare. Clearly, a stronger emphasis on the environment would imply a preference towards no insurance possibility, since then agents would abate more due to the higher uncertainty. On the other hand, a pure welfarist would only look at the realized welfare levels and prefer a world of fairly-priced insurance.

$>$ Figure 7 to Figure 9 about here $<$

Finally, there are no qualitative changes in the order of our previous results, and neither are there significant changes in the distances of the optimal paths. We can however observe that consumption in this case is non-decreasing on all paths except the naive insurance path. Nevertheless, in the naive insurance path the consumption does not any longer drop below current levels, which gives a slight hope that technical progress helps in avoiding the violation of e.g. the basic needs criterion. Finally, again just like in the previous cases, the realized welfare of the naive scenario under technical progress is by far the worst.

Realized Welfare with technical change				
FairIns	AvgStoch	Jump1	Jump2	Naive
-469	-471	-476	-441	-743

4.5 Who should provide the insurance?

It should be clear that for many regions the occurrence of extreme events has significant impacts on the people's welfare and quality of life. The possibility to obtain insurance should therefore be seen as one of the major means of making those people at least partly independent of the uncertainty surrounding the part of their lives which is affected by these extreme events.

$>$ Figure 13 to Figure 15 about here $<$

However, we also know that many insurance companies work locally, in smaller units. Under those circumstances one would expect higher overhead charges on the premia. Furthermore, the risk which the smaller insurance companies do not wish to bear are delegated to the reinsurance industry, which again implies larger overhead costs for the final demander of insurance. Though this minimizes the risk of default, it could potentially imply that risk pooling is not as efficient as if it were provided by an institutionalized, governmental agency. It would also imply that insurance provided in a type of social security system where the government aims at full insurance could potentially be welfare-improving.

We investigate this claim by analyzing the effects of overheads on the insurance premium which lead to unfair premia and therefore to partial insurance. If an insurance policy which provides insurance at a fair premium provides higher welfare than one which is supplied at an unfair premium, then this would indeed be giving support for a social security system which provides insurance at a fair premium.[13] We look at several policies. FairIns, which implies full insurance at any time and would correspond to the social security system; PartialInsur1025, PartialInsur105, PartialInsur11 implying an overhead of 2.5%, 5% and 10% respectively; AvgStoch, which would be equivalent to a policy with such a high premium that noone insures. We find that indeed the fair-priced insurance leads to the highest global welfare, supporting the view that a social security system might fare better than a private insurance industry which demands premia with an overhead. However, we do not find that smaller premia necessarily lead to higher welfare. Indeed, we find the contrary. Under a small overhead (here 2.5%), we notice that agents insure themselves fully after only a few periods.[14] This however implies that they have to pay more than the expected costs of the extreme events, which therefore implies a lower global welfare. Furthermore, we find that overheads which lead to partial insurance still fare better in terms of realized welfare than no insurance at all or small overheads which lead to full insurance.

Realized Welfare under unfair insurance				
FairIns	PartialInsur1025	PartialInsur105	PartialInsur11	AvgStoch
-567	-586	-572	-573	-587

[13] In the least one could also say it provides some food for thought for regulation of the insurance industry with respect to the premia that they demand.

[14] See Gollier [10] for the theoretical argument.

5 Conclusion

In this article we investigate the role of uncertainty and the insurance industry in the economics of endogenous extreme events. We deal with three major questions. Firstly, is there an empirical relationship between extreme events and economic activity? For this we review a part of the geographical and meteorological literature in order to see whether different disciplines support this point of view. We observe that in general one can conclude a relationship between hydrological and meteorological events and temperature. We then take data from a disaster database and analyze the major determinants of disasters. A priori one would believe that population, urbanization, GDP and temperature should be a good set of variables which potentially explain the changes in disasters worldwide. From these variables, the only one which is robust and consistently significant is temperature.

We then develop a theoretical growth model where extreme events are endogenously determined but agents can insure themselves against these events. We find several important results. Firstly, we find a role for precautionary consumption but also one for precautionary savings. Which of the two prevails over the other critically depends on the percent of capital insured as well as how much of the capital the agent expects to lose. Secondly, we notice that the DICE model corresponds to our one in case of full insurance and if the policy maker internalizes the insurance premium. This is an important result since this allows us to define a benchmark and thus comparability to other results, and it also suggests the limitations of the DICE or MERGE model in dealing with extreme events or catastrophes. Thirdly, we find a significant role for transparency. The more transparent the insurance industry, or the more information it leaks as to how it sets its premium, the more sustainable will the economic and environmental system be.

We then develop an integrated assessment model to quantify the results of the analytical model. We calibrate this IAM to correspond to the economic and environmental system in 2005, and use similar functional forms as the DICE and MERGE model. In addition, we introduce endogenous, Poisson-driven shocks and an insurance sector.

We firstly want to know how relevant the uncertainty is for the policy maker's decisions. We find that if one constructs an average path from a large set of Poisson-driven uncertain paths, then the difference between the resulting average stochastic path and the full insurance path when the premium is internalized is driven solely by the uncertainty.

We then take a look at the role of transparency in the insurance sector. We observe drastic differences in the evolution of the premium in case the insurance industry provides full information, in comparison to when the policy maker does not internalize his effect on the evolution of the premium. If the premium is not internalized, then abatement effort is negligible, climate change will take drastic forms and consumption can even drop below current levels, which would violate any sustainability criterion. We suggest that transparency is the major determinant of the economic and environmental system, opening the possibility for important regulative possibilities.

We find that false beliefs really show the implication of the discounted utilitarian criterion. The highest realized welfare is obtained when the agent under-estimates the amount of disasters, which leads to strong climate change, high current consumption but an unsustainable consumption in the future. On the contrary, an over-estimation of the frequency of disasters leads to high abatement, little climate change and high consumption in the future. One would wonder whether our initial question, namely if precautionary beliefs are welfare-improving, should not be re-phrased into the old question: Whose welfare should we take care of?

In addition, we analyze the implication of technical change. Our main finding is that this does not change the qualitative results of the model. We however find that technical progress avoids decreasing consumption in all scenarios except the naive insurance. In that scenario we find that consumption will drop, but not below the current consumption levels. We therefore expect that technical change helps in avoiding the violation of basic needs criteria.

Our final contribution is in approaching the question of whether insurance of extreme events that affect a potentially significant proportion of the population should be left to the private sector. Our conclusion is that if the private sector demands an overhead due to its structure which a social insurance system may not due to possible advantages in risk pooling, then a social insurance system should be preferred from a welfarist point of view.

In terms of future research, we believe that the most useful improvements in this field will be in the insurance sector. Allowing for the default of the insurance industry will be a good extension, although it is questionable whether this will lead to qualitative changes in the results. In case of default, the insurance industry will most likely keep a certain cushion of capital which it will call upon in case of multiple consecutive events. We should therefore expect a slightly higher premium in the first periods, which implies an overhead on the premium, and our results from the unfair insurance premium should apply.

More interesting will be if one allows the insurance industry to undertake investments in the capital market. This will however require an unfair premium such that the insurance industry can gather capital which it can invest in the capital market. It will also imply stochastic capital markets and therefore uncertain returns to capital, implying further uncertainties in the agents budget constraint. And finally, one will have to deal with the allocation of the excess profits of the insurance industry.

One of the most important extensions of this work will be a regional approach like the RICE model but with a global insurance industry. One can then study how risk gets transferred from more risky to less risky regions and from higher income regions to lower income ones. We are currently extending our work in this direction.

Finally, we have hinted at the possibility that a social insurance system might be welfare-improving in comparison to a private insurance industry. One ought to look at this from different angles than ours, too. For example, one should look at this from a paternalistic point of view, or a redistributive one, and finally from the problems of market failure. Indeed, one could think about a social insurance system which might be able to internalize the externality of todays emissions imposed on future generations in the insurance premia now, which would somewhat adhere to the polluter pays principle.

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6 Appendix

6.1 The Average Stochastic Path

It should be clear that if one wishes to integrate a Poisson-driven sample path in CGE models, then any interpretation of one given sample path could be rather misleading since the variance of a random variable driven by a Poisson distribution tends to infinity over time. As we interpret the sample paths in the graphs from the CGE experiment based on their relative position to other paths, the probability of obtaining that one path shown, and thus its interpretative power, is very low. One could try to plot a large sample of Poisson paths, which is however extremely time-intensive and not very instructive. We therefore suggest to use an average Poisson path. Our idea is the following. The random variable $q(t)$ is defined over a suitable probability space $(\Omega, \mathcal{F}, P)$, where Ω is a set of possible events ω, $\mathcal{F}$ is a σ-algebra of subsets of Ω, and P is a probability measure. We assume that $q(t)$ is driven by a Poisson process and gives the number of events in the interval $[s, t]$. Just like before, we assume its parameter is given by $\lambda > 0$, which is its mean. However, we also know that its mean is given by $E(q) = \int_{\Omega} q(\omega) dP(\omega)$, where $\omega \in \Omega$. The expected amount of jumps in a short interval of time is therefore given by λ itself. If we therefore solve the optimization problem under uncertainty but in each period of time we draw the expected value of a jump, then we obtain a path which represents the average path out of all possible sample paths in that point in time. We shall call this path the average Poisson path. The interpretative power of the average Poisson path is obviously much higher than that of a randomly-drawn sample path. Figure 3 shows a sample of twelve stochastic scenarios (with jumps) and the resulting average stochastic scenario.

6.2 Figures

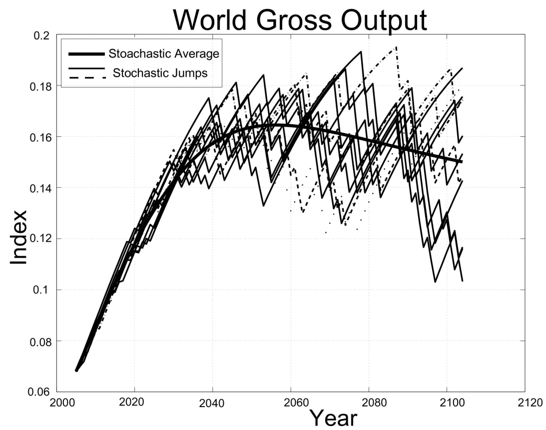

Figure 3: World Output

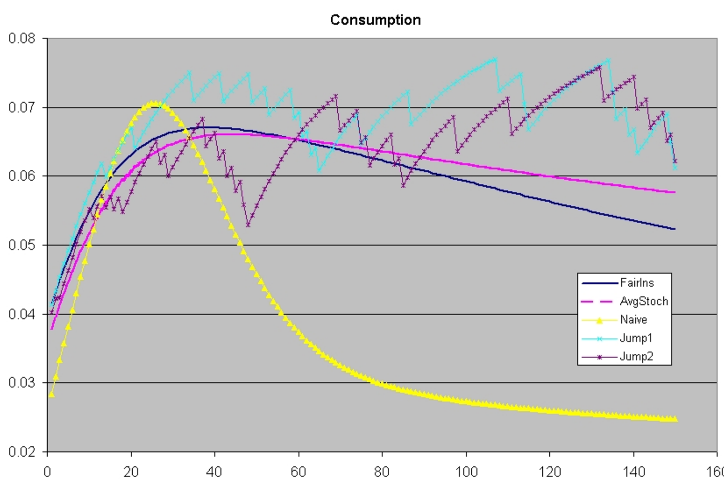

Figure 4: Consumption without technical change

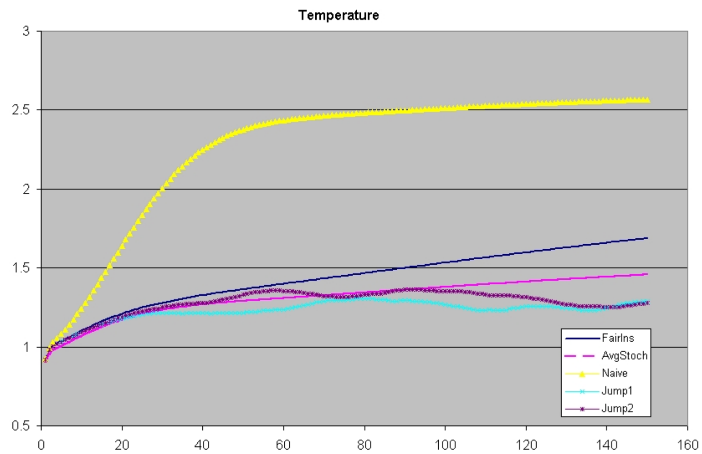

Figure 5: Temperature without technical change

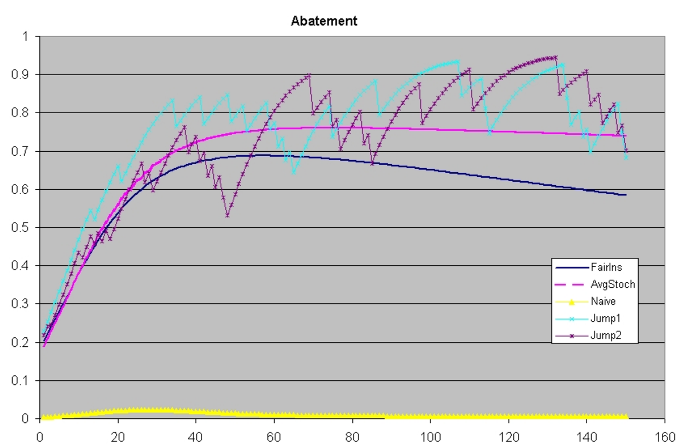

Figure 6: Abatement without technical change

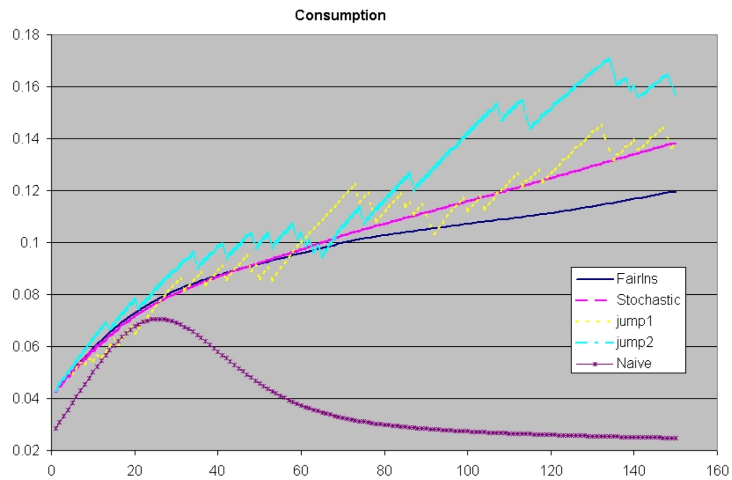

Figure 7: Consumption with technical change

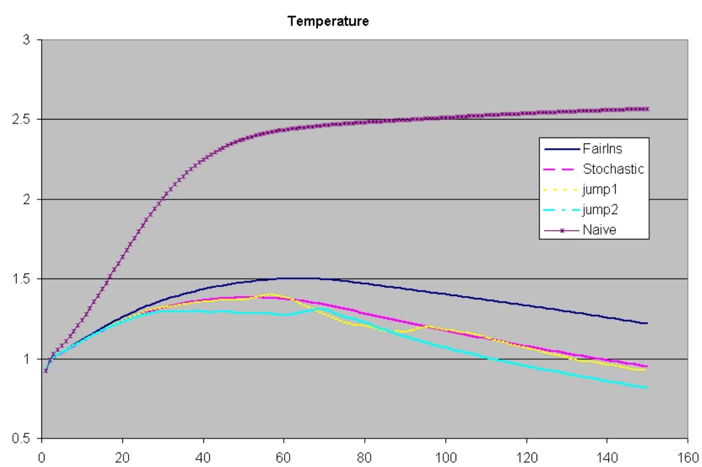

Figure 8: Temperature with technical change

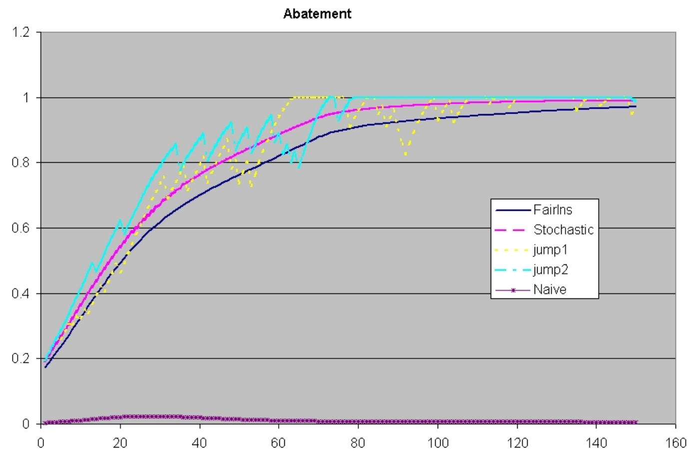

Figure 9: Abatement with technical change

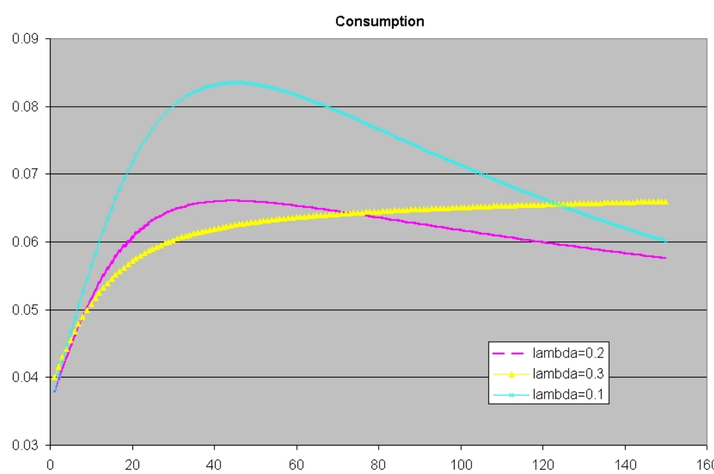

Figure 10: Consumption under false beliefs

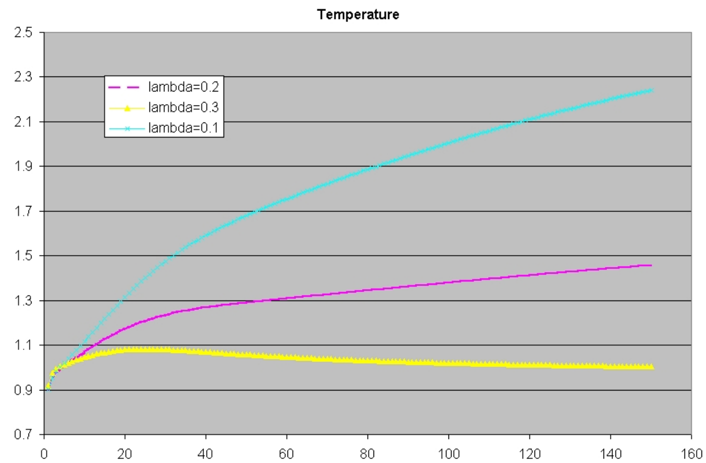

Figure 11: Temperature under false beliefs

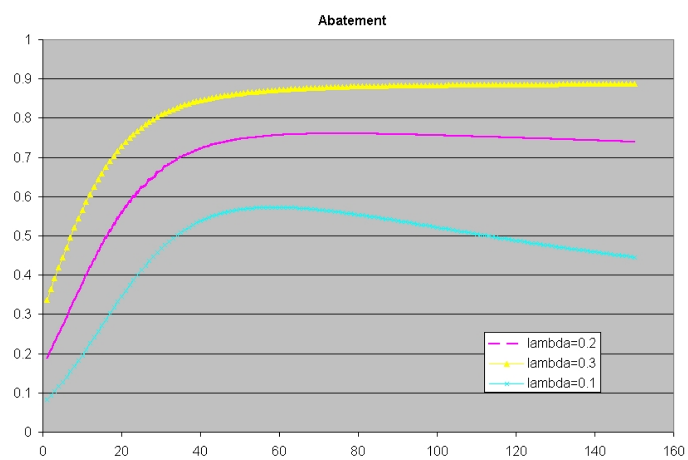

Figure 12: Abatement under false beliefs

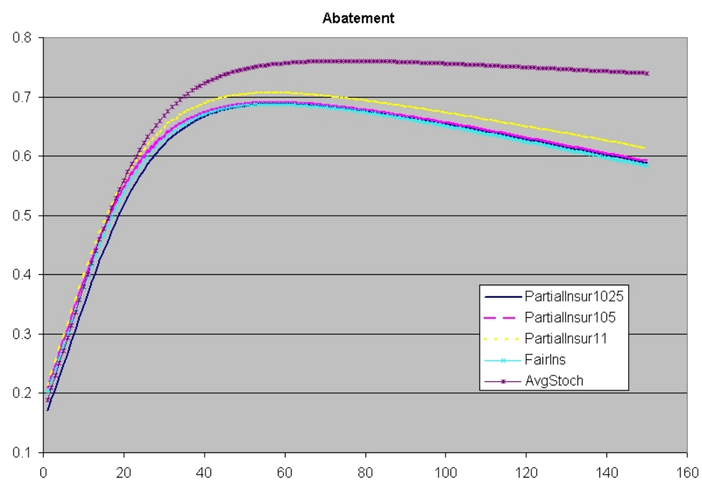

Figure 13: Abatement under partial insurance

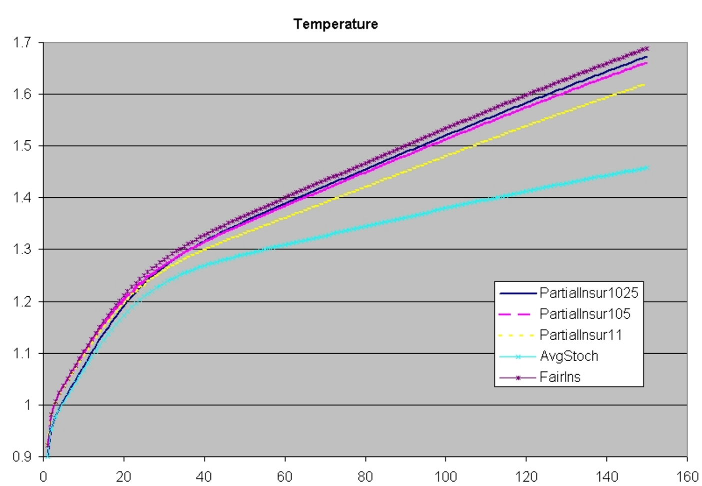

Figure 14: Temperature under partial insurance

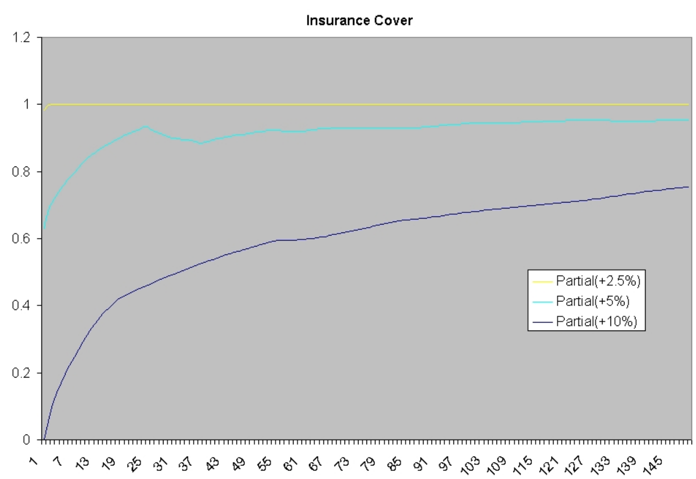

Figure 15: Insurance Cover under partial insurance