Economic and Politico-Economic Equivalence of Fiscal Policies*

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Abstract

We extend "economic equivalence" results, like the Ricardian equivalence proposition, to the political sphere where policy is chosen sequentially. We derive conditions under which a policy regime (summarizing admissible policy choices in every period) and a state are "politico-economically equivalent" to another such pair, in the sense that both pairs give rise to the same equilibrium allocation. We apply the conditions in the context of politico-economic theories of government debt as a means to i) deliver intergenerational transfers or ii) smooth tax distortions. We find that certain politico-economic models of social security or variants thereof can be re-interpreted as novel politico-economic theories of debt while other models cannot, possibly explaining the political conflict surrounding social security reform. We also find that in environments with distorting taxes, economic equivalence relations between policies with different levels of debt do not extend to the political sphere.

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1 Introduction

Important results in public economics and macroeconomics establish equivalence classes of "economically equivalent" exogenous policies that support the same equilibrium allocation

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(conditional on an initial state). For example, in a simple model of household choice, policies relying on different combinations of consumption, capital-income and labor-income taxes form equivalence classes, and in the standard overlapping-generations model, pay-as-you-go social security policies are economically equivalent to certain policies relying on taxes and explicit government debt.

In politico-economic models, the primitives of the analysis include policy regimes which define the admissible policy instruments available to political decision makers, rather than policies which constitute equilibrium objects in these models. This raises the question whether equivalence classes over policy regimes can be defined and if so, how these equivalence classes relate to the economic equivalence classes defined over policies. An answer to this question has important policy implications. Consider for example proposals to "privatize" social security and debt finance the transition. From a narrow economic point of view, shifting from a pay-as-you-go financed social security regime to a regime with taxes and explicit government debt could be irrelevant because specific pay-as-you-go and tax-and-debt policies belong to the same economic equivalence class. From a politico-economic point of view, however, one would expect that such a regime change could in general alter the equilibrium allocation. In fact, this is what the observed disagreement among policy makers concerned with the regime change question suggests.

The objective of this paper is to extend the economic equivalence concept to the politico-economic sphere. We derive conditions under which a policy regime and a state (consisting of an economic state and a political state, the latter reflecting predetermined policy instruments and possibly non-fundamental state variables supporting trigger strategies) are "politico-economically equivalent" to another such pair in the sense that both pairs support politico-economic equilibria and both these equilibria correspond with the same allocation. And we consider several applications. While we focus on applications in models with public debt, the theoretical conditions we obtain are general in nature and apply in other contexts featuring an endogenous choice of policies.

Our theoretical results are derived within a dynamic framework with households, firms and a government with access to general taxes, transfers as well as debt. In a first step, we extend well-known neutrality propositions (e.g., Barro, 1974; Sargent, 1987; Rangel, 1997; Coleman, 2000; Ghiglino and Shell, 2000; Bassetto and Kocherlakota, 2004; Niepelt, 2005) and derive a general economic equivalence result. Using this result, we derive in a second step sufficient conditions for politico-economic equivalence of two policy regimes (conditional on the respective states).

Intuitively, these conditions specify requirements on the choice sets faced by political decision makers. These choice sets are constrained by the state on the one hand and the policy instruments under the control of political decision makers on the other. Accordingly, the first condition requires that state spaces must be comparable in the sense that states can unambiguously be related across policy regimes. Verifying this condition may not be immediate since policy instruments and commitment structures generally differ across regimes. The other two conditions which build on the first requirement concern the admissible policy instruments. The admissibility restrictions on those instruments in the "new" regime must be both sufficiently loose and sufficiently tight: Sufficiently loose for political decision makers in this new regime to be able to support competitive equilibria that political decision makers in the "initial" regime choose to implement; and sufficiently tight such that political decision makers in the new regime must not be able to support competitive equilibria that cannot be supported in the initial regime.

The politico-economic equivalence conditions we derive serve several purposes. In their general form, they constitute a useful tool for researchers interested in characterizing politico-economic equilibria. When high dimensional state and policy spaces render such a task difficult, the equivalence conditions can help by allowing to relate the equilibrium conditions of interest to their counterparts in a simpler setting that is easier to characterize. In the applications we consider, the politico-economic equivalence conditions help to identify factors that render government debt non-neutral from a political point of view. We consider two roles of government debt: As a means to deliver intergenerational transfers, and as a means to smooth tax distortions.

Regarding the former role, we start from the well-known fact that in overlapping generations economies, certain social security policies and debt policies are economically equivalent. Asking whether this equivalence extends to the political sphere, we contrast existing politico-economic models of social security (Cooley and Soares, 1999; Tabellini, 2000; Boldrin and Rustichini, 2000; Forni, 2005; Gonzalez-Eiras and Niepelt, 2008) with alternative models in which political decision makers may issue debt and choose the repayment rate on maturing debt. We show that certain politico-economic theories of social security that have been proposed in the literature may be re-interpreted as politico-economic theories of government debt, and our analysis therefore contributes to a small but growing literature on debt in politico-economic equilibrium (e.g., Battaglini and Coate, 2008; Díaz-Giménez, Giovanetti, Marimon and Teles, 2008; Yared, 2010; Song, Storesletten and Zilibotti, 2007; Niepelt, 2010).[1] Other theories cannot be re-interpreted in that way. By identifying factors that undermine politico-economic equivalence, our findings can help rationalize why interest groups might favor or oppose the privatization of social security although from a purely economic point of view, a regime change appears irrelevant. This might prove useful in constructing theories of social security reform.

More specifically, the analysis identifies three classes of overlapping generations models. First, a class characterized by minimal household heterogeneity and non distorting taxes in which politico-economic equivalence between social security and debt regimes holds for any political aggregation mechanism. Second, a class in which politico-economic equivalence may or may not hold, depending on the political aggregation mechanism in place. Equivalence may fail in this class because certain allocations are only implementable in the debt regime. Finally, a class of models with sufficient heterogeneity among households and without commitment. In models of this class, the debt ownership structure constitutes a non-trivial state variable and politico-economic equivalence generally fails because states cannot unambiguously be related across policy regimes.

Regarding the tax smoothing role of government debt, we start from Bassetto and Kocherlakota's (2004) observation that the timing of tax collections may be allocation neutral even if taxes are distorting, as long as taxes may be levied on contemporaneous

[1] Cukierman and Meltzer (1989) argue in a model with commitment that political decision makers are indifferent between social security and debt policies when the policy regime features both instruments and allows for lump sum taxes.

and lagged incomes.[2] We find that this economic equivalence result does not extend to the political sphere. A policy regime allowing for the taxation of current and lagged income generally is not politico-economically equivalent to a regime allowing for taxation of current income only.

The remainder of the paper is structured as follows. Section 2 lays out the model. In Sections 3 and 4, we derive the economic and politico-economic equivalence results, respectively. Section 5 contains the applications and Section 6 concludes with a discussion of the implications for social security reform and the applicability of our results to a wider class of policies.

2 The Economic Model

We consider an infinite-horizon deterministic, discrete-time economy with time indexed by $t = 0, 1, \ldots$.[3] The economy is inhabited by a government, firms (potentially including an external sector), and households.

Let $\mathcal{I}$ denote the set of households, and let $\mathcal{I}_t \subseteq \mathcal{I}$ denote the set of households alive in period t. For any household $i \in \mathcal{I}$, let i_1 and i_T denote the first period and last period of household i's lifetime, respectively.

There are L commodities in each period. Let $e_t^i \in \mathbb{R}_+^L$ and $x_t^i \in \mathbb{R}_+^L$ denote endowment and consumption vectors, respectively, of household i in period t and let $X_t^i \subset \mathbb{R}_+^L$ denote the household's consumption set. For example, e_t^i might include household i's time endowment, and x_t^i might include consumption of leisure or goods. Household preferences in period t are described by the function $\Omega_t^i : X_{i_1}^i \times \cdots \times X_{i_T}^i \to \mathbb{R}$ for all $i \in \mathcal{I}$ with $i_1 \leq t \leq i_T$. Let x_t denote the vector of consumption choices by all households $i \in \mathcal{I}_t$ in period t, and let x^i denote the consumption profile of household i over i's lifetime. Let b_t^i denote household i's holdings of maturing government debt in period t and let a_t^i denote household i's financial wealth net of government debt but including discounted firm profits, if they exist (all in terms of the numeraire). Household i's total financial wealth, f_t^i, is given by $f_t^i \equiv a_t^i + b_t^i z_t$ with z_t denoting the repayment rate on government debt, to be discussed below.

Let $\mathcal{J}$ denote the set of firms. Let $y_t^j \equiv (y_{1,t}^j, y_{2,t}^j) \in \mathbb{R}^{2L}$ denote a production plan of firm $j \in \mathcal{J}$ in period t and let $Y_t^j \subset \mathbb{R}^{2L}$ denote the production set of the firm in period t. A production plan y_t^j lists firm j's net input-outputs in period t (the first L elements, corresponding to $y_{1,t}^j$) as well as the resulting net input-outputs at the beginning of the following period (the second L elements, corresponding to $y_{2,t}^j$).[4] Given a predetermined $y_{2,t-1}^j$, firm j's sequence $(y_t^j, y_{t+1}^j, \ldots)$ constitutes a *production path* if $y_s^j \in Y_s^j$ for all $s \geq t$. This production path generates a sequence of net inputs-outputs of the L commodities

[2] Bassetto and Kocherlakota (2004) extend Barro's (1974) Ricardian (economic) equivalence result to environments with non-distorting taxes.

[3] The extension to the stochastic case is immediate. If the number of states in each period is finite, and with some adjustments to notation, t can alternatively be interpreted as indexing histories.

[4] See Mas-Colell, Whinston and Green (1995, ch. 20.C). For example, if the firm uses goods k and labor l to produce new goods k' (and if goods and labor are the only commodities), then $y_t^j = (-k, -l, k', 0)$.

equal to $\{y^j_{2,s-1} + y^j_{1,s}\}_{s \geq t}$.

Let q and r denote the pre-tax prices in the economy. In particular, the vector q_t denotes the period-t prices of the L commodities in terms of the numeraire; and $r_{t,s}$ denotes the period-t price of the numeraire in period s in terms of the numeraire in period t, that is, the inverse of the gross interest rate between periods t and s, $s \geq t$. The vector r_t denotes the term structure of interest rates in period t, and ${}_tr$ denotes the term structures of interest rates in periods preceding t.

It is useful to partition x^i as $x^i = ({}_tx^i, x^i_t, x^{it})$ with ${}_tx^i \equiv (x^i_{i_1}, x^i_{i_1+1}, \ldots, x^i_{t-1})$ and $x^{it} \equiv (x^i_{t+1}, x^i_{t+2}, \ldots, x^i_{i_T})$. Vectors defined over all households as well as the prices q are similarly partitioned. In particular, $x = ({}_tx, x_t, x^t)$ with ${}_tx \equiv (x_0, x_1, \ldots, x_{t-1})$ and $x^t \equiv (x_{t+1}, x_{t+2}, \ldots)$.

Let $g^i_t({}_tx^i, x^i_t, {}_tq, q_t, {}_tr, f^i_t)$ denote the tax imposed on household i in period t, defined in terms of the numeraire. Taxes may depend on the household's financial wealth as well as on its consumption choices and prices in the current or previous periods.[5] They may also depend on the household's endowments, but for ease of notation we do not list current or past endowments as arguments of the tax function. Let $g^i \equiv \{g^i_s(\cdot)\}^{i_T}_{s=i_1}$ and denote the profiles of tax functions across households by $g \equiv \{g^i\}_{i \in \mathcal{I}}$.[6] Examples of tax functions $g^i_t(\cdot)$ include labor income, consumption or capital income taxes. A proportional tax on contemporaneous labor income can be represented as $g^i_t(l^i_t, w_t) = \tau^w_t w_t l^i_t$ where τ^w_t, w_t, l^i_t denote the tax rate, wage, and labor supply (that is, the time endowment minus leisure consumption), respectively. A proportional tax on contemporaneous consumption can be represented as $g^i_t(c^i_t, q^c_t) = \tau^c_t q^c_t c^i_t$ with τ^c_t, q^c_t, c^i_t denoting the tax rate, price of the good, and consumption, respectively. A proportional tax on the net return on savings can be represented as $g^i_t({}_tr, f^i_t) = \tau^k_t f^i_t(1 - r_{t-1,t})$ with τ^k_t denoting the tax rate.

A government *policy* consists of a sequence of vectors of government purchases, profiles of tax functions imposed on firms, profiles of tax functions imposed on households, g, and sequences of government debt issuance and redemption (defined in terms of the numeraire). Let b_{t+1} denote the amount of government debt issued in period t and maturing in period $t+1$, and let v_t denote the price of debt at issuance.[7] The policies we consider differ with respect to the profiles of tax functions imposed on households, g, as well as the government debt policy (b, z), but not with respect to taxes imposed on firms or government purchases. To simplify the notation, we therefore assume that the latter two instruments are not employed at all. Accordingly, a policy p can be represented as $p = (g, (b, z))$. Let $p_{\geq t}$ denote the policy instruments applied under policy p in period t or later, $p_{\geq t} \equiv (g_{\geq t}, (b, z)_{\geq t}) \equiv \{\{g^i_s(\cdot)\}_{i \in \mathcal{I}_s}, (b_{s+1}, z_s)\}_{s \geq t}$.

Let $q_t \cdot (x^i_t - e^i_t)$ represent household i's net expenditure in period t before taxes and in terms of the numeraire. The *choice set* of household i as of period t, $\mathcal{B}^i_t(f^i_t, {}_tx^i, q, r, g^i)$,

[5] With sequential decision making of the form considered later, taxes cannot depend on future household choices.

[6] Tax functions imposed on households typically satisfy cross-section restrictions, e.g., $g^i_t(\cdot) = g^j_t(\cdot)$ for all $i, j \in \mathcal{I}$.

[7] For simplicity, we only consider short-term debt.

is given by[8]

$$\mathcal{B}^i_t(f^i_t, {}_t x^i, q, r, g^i) =$$
$$\left\{ (x^i_t, x^{it}) \middle| \begin{array}{l} (x^i_t, x^{it}) \in X^i_t \times \cdots \times X^i_{i_T}; \\ \sum_{s=t}^{i_T} r_{t,s} \left(q_s \cdot (x^i_s - e^i_s) + g^i_s({}_s x^i, x^i_s, {}_s q, q_s, {}_s r, f^i_s) \right) \le f^i_t; \\ f^i_{s+1} = \frac{r_{t,s}}{r_{t,s+1}} \left(f^i_s - q_s \cdot (x^i_s - e^i_s) - g^i_s({}_s x^i, x^i_s, {}_s q, q_s, {}_s r, f^i_s) \right), s \ge t \end{array} \right\}, \quad (1)$$

that is households may freely save or borrow. By convention, $(a^i_t, b^i_t) = (0, 0)$ for $i \in \mathcal{I}$ with $i_1 \ge t$.

The *economic state* in period t comprises several objects. First, the state variables resulting from production in the preceding period, $\{y^j_{2,t-1}\}_{j \in \mathcal{J}}$. Second, households' asset holdings. Third, past consumption choices and prices if these enter as arguments of tax functions. In the most general case, the economic state κ_t therefore is given by $\kappa_t \equiv (\{y^j_{2,t-1}\}_{j \in \mathcal{J}}, \{a^i_t, b^i_t\}_{i \in \mathcal{I}_t}, {}_t x, {}_t q, {}_t r)$.

Definition 1. A *competitive equilibrium* as of period t conditional on κ_t as well as $p_{\ge t}$, denoted by $\mathrm{CE}(\kappa_t, p_{\ge t})$ for short, consists of prices (q_t, q^t, r_t), household choices (x^i_t, x^{it}) for all $i \in \mathcal{I}$ with $i_T \ge t$, and production paths $\{\{y^j_s\}_{j \in \mathcal{J}}\}_{s \ge t}$ such that

i. household choices are optimal:

$$(x^i_t, x^{it}) \in \arg \max_{(\hat{x}^i_t, \hat{x}^{it}) \in \mathcal{B}^i_t(a^i_t + b^i_t z_t, {}_t x^i, q, r, g^i)} \Omega^i_t({}_t x^i, \hat{x}^i_t, \hat{x}^{it}) \text{ for all } i \in \mathcal{I} \text{ with } i_T \ge t;$$

ii. production paths are optimal:

$$y^j_s \in \arg \max_{\hat{y}^j_s \in Y^j_s} q_s \cdot \hat{y}^j_{1,s} + r_{s,s+1}(q_{s+1} \cdot \hat{y}^j_{2,s}) \text{ for all } j \in \mathcal{J}, s \ge t;$$

iii. the resource constraints are satisfied:

$$\sum_{i \in \mathcal{I}_s} (x^i_s - e^i_s) \le \sum_{j \in \mathcal{J}} (y^j_{2,s-1} + y^j_{1,s}) \text{ for all } s \ge t;$$

iv. government budget constraints are satisfied:

$$\sum_{i \in \mathcal{I}_s} g^i_s({}_s x^i, x^i_s, {}_s q, q_s, {}_s r, f^i_s) = b_s z_s - z_{s+1} \frac{r_{t,s+1}}{r_{t,s}} b_{s+1} \text{ for all } s \ge t, \quad (2)$$

where $b_t \equiv \sum_{i \in \mathcal{I}_t} b^i_t$, as well as the no-Ponzi-game condition.

Three remarks are in order. First, the government budget constraint (2) uses the fact that in equilibrium, government debt prices equal

$$v_s = z_{s+1} \frac{r_{t,s+1}}{r_{t,s}} \text{ for all } s \ge t.$$

[8]For ease of notation, we do not list the household's endowments as arguments of the choice set.

To simplify the notation, we do not include government debt prices in the definition of competitive equilibrium.

Second, the competitive equilibrium conditions together with the budget set (1) determine equilibrium net asset positions of households over time, $\{\{f_s^i\}_{i\in\mathcal{I}_s}\}_{s\geq t+1}$, but not the composition of households' portfolios, $\{\{a_s^i, b_s^i\}_{i\in\mathcal{I}_s}\}_{s\geq t+1}$, since the after-tax returns on all assets are identical in equilibrium.

Finally, note from the definition of $\mathcal{B}_t^i(\cdot)$ as well as from equation (2) that there exists a continuum of debt quantities and repayment rates corresponding to a given equilibrium allocation. In particular, consider the competitive equilibrium $\mathrm{CE}(\kappa_t, p_{\geq t})$ and let $\{\alpha_s\}_{s\geq t}$ be a sequence of strictly positive scalars. Then, the competitive equilibria $\mathrm{CE}(\kappa_t, p_{\geq t})$ and $\mathrm{CE}(\kappa_t', p_{\geq t}')$ coincide if the initial debt holdings $\{b_t^i\}_{i\in\mathcal{I}_t}$ in κ_t are replaced by $\{\alpha_t b_t^i\}_{i\in\mathcal{I}_t}$ in κ_t' and the sequences $\{b_{s+1}, z_s\}_{s\geq t}$ in $p_{\geq t}$ are replaced by the sequences $\{\alpha_{s+1}b_{s+1}, z_s/\alpha_s\}_{s\geq t}$ in $p_{\geq t}'$. We return to this issue later.

3 Economic Equivalence

This Section defines conditions under which pairs of policy instruments and state variables support the same competitive equilibrium. Let $\kappa_t^i \equiv (a_t^i, b_t^i, {}_tx^i, {}_tq, {}_tr)$ denote the *economic state of individual* i. Throughout the analysis, we disregard the possibility that a policy supports multiple competitive equilibria.[9]

Lemma 1. Suppose that two tax-profiles-cum-repayment-rate, $(g_{\geq t}^i, z_t)$ and $(g_{\geq t}^{i'}, z_t')$ respectively, give rise to the same choice set for household i as of period t, conditional on two economic states of individual i, κ_t^i and $\kappa_t^{i'}$ respectively:

$$\mathcal{B}_t^i(a_t^i + b_t^i z_t, {}_tx^i, q, r, g_{\geq t}^i) = \mathcal{B}_t^i(a_t^{i'} + b_t^{i'} z_t', {}_tx^{i'}, ({}_tq', q_t, q^t), ({}_tr', r_t), g_{\geq t}^{i'}). \tag{3}$$

Then, $(g_{\geq t}^i, z_t)$ together with κ_t^i supports the same household choices (x_t^i, x^{it}) as does $(g_{\geq t}^{i'}, z_t')$ together with $\kappa_t^{i'}$.

Proof. Immediate. □

Lemma 1 states a "partial equilibrium indifference" condition at the level of an individual household. For policy changes to be "irrelevant" in general equilibrium (as defined in Definition 2 below), this partial equilibrium condition must hold for all households. Moreover, it must be true that aggregate equilibrium conditions are equivalently satisfied under both policies. Assumption 1 and Proposition 1 below specify conditions under which this is the case.

Definition 2. An economic state and set of policy instruments, $(\kappa_t, p_{\geq t})$, is *economically equivalent* to another state and set of policy instruments, $(\kappa_t', p_{\geq t}')$, if

i. $(\kappa_t, p_{\geq t})$ supports a competitive equilibrium $\mathrm{CE}(\kappa_t, p_{\geq t})$;

[9] This case could easily be handled by introducing a sunspot variable whose realization, conditional on the policy choice, determines the equilibrium.

ii. $(\kappa'_t, p'_{\geq t})$ supports a competitive equilibrium $\text{CE}(\kappa'_t, p'_{\geq t})$;

iii. the two competitive equilibria are identical.

Assumption 1. Policy does not affect production sets.

Proposition 1. Consider an economic state and set of policy instruments, $(\kappa_t, p_{\geq t})$, that support a competitive equilibrium $\text{CE}(\kappa_t, p_{\geq t})$, and consider a new economic state and set of policy instruments, $(\kappa'_t, p'_{\geq t})$. Suppose that κ'_t and $p'_{\geq t}$ satisfy the following conditions:

i. predetermined net input-outputs coincide across economic states:

$$y^j_{2,t-1} = y^{j'}_{2,t-1} \text{ for all } j \in \mathcal{J};$$

ii. at equilibrium prices, $(\kappa_t, p_{\geq t})$ and $(\kappa'_t, p'_{\geq t})$ imply identical choice sets for all households as of period t:

$$(3) \text{ holds in period } t \text{ for all } i \in \mathcal{I} \text{ with } i_T \geq t; \tag{4}$$

iii. at equilibrium prices and quantities, κ'_t and $p'_{\geq t}$ satisfy the government budget constraints:

$$\sum_{i\in\mathcal{I}_s} g^{i'}_s \Big(({}_t x^{i'}, x^i_t, \ldots, x^i_{s-1}), x^i_s, ({}_t q', q_t, \ldots, q_{s-1}), q_s, ({}_t r', r_t, \ldots, r_{s-1}), f^{i'}_s \Big)$$
$$= b'_s z'_s - z'_{s+1} \frac{r_{t,s+1}}{r_{t,s}} b'_{s+1} \text{ for all } s \geq t, \tag{5}$$

where $b'_s \equiv \sum_{i\in\mathcal{I}_s} b^{i'}_s$.

Then, under Assumption 1, $(\kappa_t, p_{\geq t})$ is economically equivalent to $(\kappa'_t, p'_{\geq t})$.

Proof. Consider the situation with $(\kappa'_t, p'_{\geq t})$ and conjecture that prices (q_t, q^t, r_t) remain unchanged relative to $\text{CE}(\kappa_t, p_{\geq t})$.[10] Then, from Lemma 1 and condition (4), household choices remain optimal under $p'_{\geq t}$ and the first requirement of competitive equilibrium is satisfied. From Assumption 1, firm choices continue to constitute production paths. Since prices are unchanged, these choices also remain optimal such that the second requirement of competitive equilibrium is satisfied. Due to unchanged predetermined net input-outputs, the resource constraints continue to be satisfied and the third requirement of competitive equilibrium is met. From Equation (5), the new policy satisfies the government budget constraints and the fourth requirement of competitive equilibrium is satisfied.

We conclude that $(\kappa'_t, p'_{\geq t})$ supports a competitive equilibrium that coincides with $\text{CE}(\kappa_t, p_{\geq t})$. □

[10] Recall that debt prices are not included in the set of equilibrium objects.

In the equilibrium supported by the new policy, government debt continues to pay the required return since $v'_s = z'_{s+1}\frac{r_{t,s+1}}{r_{t,s}}$ for all $s \geq t$. As a consequence, households continue to be indifferent between holding government debt or privately issued debt. To finance their original consumption plans households adjust their savings in a period by the amount corresponding to the differential tax payments under the two policies. Summed over all households, demand for government debt therefore matches supply.

Proposition 1 summarizes the Ricardian equivalence result and related neutrality theorems discussed in the literature. Consider for example a model of a representative agent that values consumption and leisure, saves and borrows at the market interest rate, sells labor and capital services to firms, purchases consumption goods from firms, and lends funds to the government. This model satisfies Assumption 1, implying that tax collections and government deficits may be shifted over the entire horizon of the economy without affecting the equilibrium allocation as long as the choice set $\mathcal{B}_t(\cdot)$ of the representative household remains unchanged. In particular, with non-distorting taxes, economic equivalence of two fiscal policies (subject to initial states) only requires the present discounted value of taxes to be the same across policies; this is the standard Ricardian equivalence result (see Barro, 1974).

With overlapping generations of representative households rather than a single infinitely lived household, and under Assumption 1, Proposition 1 implies that a change in the timing of tax collections and deficits leaves the allocation unaffected as long as the choice set of each cohort remains unchanged. In particular, certain debt-and-tax policies are equivalent to pay-as-you-go financed transfer policies. Consider for example an environment with two-period lived overlapping generations. Let $p_{\geq t} = (g_{\geq t}, (\mathbf{0}, z)_{\geq t}) = ((\tau, \sigma)_{\geq t}, (\mathbf{0}, z)_{\geq t})$ be a policy without explicit government debt that supports the competitive equilibrium $\text{CE}(\kappa_t, p_{\geq t})$. Here, τ denotes social security contributions paid by workers and $-\sigma$ denotes benefits paid to retirees. Let $p'_{\geq t} = ((\tau', \sigma')_{\geq t}, (b', z')_{\geq t})$ be another policy with explicit government debt. Proposition 1 implies that $(\kappa'_t, p'_{\geq t})$ is economically equivalent to $(\kappa_t, p_{\geq t})$ if κ'_t and κ_t have the same predetermined net input-outputs and if (4) and (5) are satisfied. Letting $\mathcal{I}^r_t$ and $\mathcal{I}^w_t$ denote the sets of retirees and workers in period t, respectively, and dropping the arguments of tax functions, condition (4) requires

$$\sigma^i_t(\cdot) = \sigma^{i\prime}_t(\cdot) \text{ for all } x^i_t \in X^i_t, i \in \mathcal{I}^r_t$$

and

$$\tau^i_s(\cdot) - \frac{r_{t,s+1}}{r_{t,s}}\sigma^i_{s+1}(\cdot) = \tau^{i\prime}_s(\cdot) - \frac{r_{t,s+1}}{r_{t,s}}{\sigma^i_{s+1}}'(\cdot) \text{ for all } x^i \in X^i_s \times X^i_{s+1}, i \in \mathcal{I}^w_s, s \geq t.$$

Moreover, condition (5) requires

$$\sum_{i\in\mathcal{I}_s} \tau^{i\prime}_s(\cdot) + \sigma^{i\prime}_s(\cdot) = b'_s z'_s - z'_{s+1}\frac{r_{t,s+1}}{r_{t,s}}b'_{s+1}$$

for all $s \geq t$, where taxes are evaluated at equilibrium quantities and prices. Conditional on appropriately defined states κ_t and κ'_t, any pay-as-you-go financed social security policy without explicit debt, $((\tau, \sigma)_{\geq t}, (\mathbf{0}, z)_{\geq t})$, is therefore economically equivalent to policies relying on contributions, benefits, and explicit debt, $((\tau', \sigma')_{\geq t}, (b', z')_{\geq t})$ (see, e.g., Sargent (1987, ch. 8), Rangel (1997) and Niepelt (2005)). In particular, there exists such an equivalent policy that involves no benefits after period t.[11]

4 Politico-Economic Equivalence

Our aim in this Section is to establish conditions under which the economic equivalence result of Proposition 1 extends to situations where policy is sequentially chosen to maximize some objective function. More specifically, we are interested in the conditions under which economic equivalence of two policies (subject to appropriately defined initial states) implies that the second policy constitutes a politico-economic equilibrium if the first policy does so. In line with the maintained assumption that policy does not enter household preferences, we assume that the objective function maximized by political decision makers does not depend on policy choices.

Assumption 2. The political objective function in period t is given by $\Omega_t(x)$.

Without loss of generality, we focus on the case where political decisions are taken in every period.[12] We assume that political decision makers act in the beginning of a period, before the private sector. Let p_t denote the *policy choice* by political decision makers in period t that is, the choice of values for the policy instruments in $p_{\geq t}$ that are under the control of political decision makers in period t; let p^t denote the *continuation policy choice* by subsequent political decision makers; and let ${}_tp$ denote the choices by policy makers in preceding periods (potentially including some initial values for policy instruments). A policy p can then be partitioned as $p = ({}_tp, p_t, p^t)$.

Let $\mathcal{P}_t$ denote the set of *admissible* policy choices in period t. The restrictions embedded in $\mathcal{P}_t$ specify the policy instruments under the control of political decision makers in period t (and thus, the degree of commitment) as well as restrictions on the numerical values of those instruments. A *policy regime* is defined by $\mathcal{P} \equiv \prod_t \mathcal{P}_t$ which can be partitioned as $\mathcal{P} = ({}_t\mathcal{P}, \mathcal{P}_t, \mathcal{P}^t)$. The *policy space* in period t, $\mathcal{Q}_t$, is defined as the superset of $\mathcal{P}_t$ that results if restrictions on the numerical values of the policy instruments in $\mathcal{P}_t$ are dropped. The policy space $\mathcal{Q} \equiv \prod_t \mathcal{Q}_t$ can be partitioned as $\mathcal{Q} = ({}_t\mathcal{Q}, \mathcal{Q}_t, \mathcal{Q}^t)$.

Recall from the discussion after Definition 1 that, if government debt constitutes a policy instrument, there exists a continuum of debt quantities and repayment rates corresponding with a given set of prices, taxes, household choices and production paths. Without loss of generality, we eliminate this indeterminacy when debt is a policy instrument by fixing government debt per household or per retiree (or otherwise suitably normalized) at some strictly positive exogenous sequence, $\{\bar{b}_t\}_{t\geq 0} > 0$. Adopting this

[11]According to some authors, pay-as-you-go financed social security policies are *not* equivalent to policies relying on contributions, benefits, and debt (see, e.g., Birkeland and Prescott, 2007). These authors reach this different conclusion because they restrict the set of available policy instruments such that condition (4) cannot be met.

[12]If political decisions are taken once and for all, economic equivalence trivially extends to the political sphere. If political decisions are taken more than once, but not in every period, then the analysis in the text applies if periods are redefined appropriately.

normalization does not constrain the effective choice set of policy makers—with $\bar{b}_t > 0$, the amount of resources transferred to bond holders can be controlled by the choice of repayment rate—nor does it constrain the ownership structure of government debt and thus, the relative exposure of different groups of households to public debt.

If political decision makers can commit to certain future policy instruments then some of the policy instruments employed from period t onwards, $p_{\geq t}$, are predetermined. We denote such predetermined policy instruments by ${}_tp_{\geq t}$, implying $p_{\geq t} = ({}_tp_{\geq t}, p_t, p^t)$. The predetermined policy instruments ${}_tp_{\geq t}$ include *committed tax functions*, $\{g_t^{ic}(\cdot)\}_{i \in \mathcal{I}_t}$, that are imposed on households in period t but are chosen by political decision makers in an earlier period, and they include the repayment rate on contemporaneously maturing debt, z_t, if political decision makers can issue debt and commit to repay.

Let μ_t denote the set of state variables in period t that determine (together with $\mathcal{P}$) which competitive equilibria are implementable. The elements contained in μ_t are fully described by κ_t and ${}_tp_{\geq t}$ but they differ depending on the policy regime in place. First, with explicit government debt but without commitment to debt repayment, $\mu_t = (\kappa_t, {}_tp_{\geq t}) = (\kappa_t, \{\{g_s^{ic}(\cdot)\}_{i \in \mathcal{I}_s}\}_{s \geq t})$. In this case, the asset ownership structures $\{a_t^i, b_t^i\}_{i \in \mathcal{I}_t}$ (contained in κ_t) are separately included in μ_t because they determine the extent to which political decision makers may affect the relative wealth positions of households by choosing the repayment rate z_t. Second, with explicit government debt and with commitment to debt repayment, political decision makers cannot affect the financial wealth of households ex post implying $\mu_t = (\kappa_t \setminus \{a_t^i, b_t^i\}_{i \in \mathcal{I}_t}, \{f_t^i\}_{i \in \mathcal{I}_t}, \{\{g_s^{ic}(\cdot)\}_{i \in \mathcal{I}_s}\}_{s \geq t})$. Finally, in a setup without government debt, $\mu_t = (\kappa_t \setminus \{b_t^i\}_{i \in \mathcal{I}_t}, \{\{g_s^{ic}(\cdot)\}_{i \in \mathcal{I}_s}\}_{s \geq t})$ and b_t^i (as well as $\bar{b}_t$) equals zero for all $i \in \mathcal{I}_t$ and all t.[13]

By construction, the state variables contained in μ_t and the policy choices $p^{t-1} = (p_t, p^t)$ completely characterize a competitive equilibrium. With some slight abuse of notation, we can therefore denote a competitive equilibrium by $\mathrm{CE}(\mu_t, p^{t-1})$ rather than $\mathrm{CE}(\kappa_t, p_{\geq t})$. Accordingly, we modify Definition 2 of economic equivalence. For convenience, we also extend the Definition of economic equivalence to sets:

Definition 3. A state and sequence of policy choices, (μ_t, p^{t-1}), is *economically equivalent* to another state and sequence of policy choices, (μ_t', p'^{t-1}), if

i. (μ_t, p^{t-1}) supports a competitive equilibrium $\mathrm{CE}(\mu_t, p^{t-1})$;

ii. (μ_t', p'^{t-1}) supports a competitive equilibrium $\mathrm{CE}(\mu_t', p'^{t-1})$;

iii. the two competitive equilibria are identical.

[13]Leaving physical state variables aside, one might expect the state to summarize the cumulative restrictions on households' budget sets as implied by policy choices in previous periods. These restrictions would be given by the present value of those tax functions that are predetermined. This view is not correct, for two reasons. First, the economic equilibrium conditions do not only include the net present value of the predetermined tax functions (and the physical state variables) but also the contemporaneous predetermined tax functions since these enter into the government's dynamic budget constraint. Second, and more importantly, absent commitment to debt repayment, the ownership structure of debt also enters the state (although it is in general not under the control of previous governments).

A state and a set of sequences of policy choices, $(\mu_t, \{p^{t-1}\})$, is *economically equivalent* to another state and set of sequences of policy choices, $(\mu'_t, \{p'^{t-1}\})$, if

i. for every $p^{t-1} \in \{p^{t-1}\}$ such that (μ_t, p^{t-1}) supports a competitive equilibrium, there exists a $p'^{t-1} \in \{p'^{t-1}\}$ such that (μ_t, p^{t-1}) is economically equivalent to (μ'_t, p'^{t-1});

ii. for every $p'^{t-1} \in \{p'^{t-1}\}$ such that (μ'_t, p'^{t-1}) supports a competitive equilibrium, there exists a $p^{t-1} \in \{p^{t-1}\}$ such that (μ_t, p^{t-1}) is economically equivalent to (μ'_t, p'^{t-1}).

An admissible continuation policy choice $p^t \in \mathcal{P}^t$ is *feasible* conditional on μ_{t+1} if p^t supports a competitive equilibrium $\mathrm{CE}(\mu_{t+1}, p^t)$. Let $\mathcal{P}^t(\mu_{t+1}) \subseteq \mathcal{P}^t$ denote the set of admissible and feasible continuation policy choices conditional on μ_{t+1}. An admissible policy choice $p_t \in \mathcal{P}_t$ is feasible conditional on μ_t if there exists an admissible continuation policy $p^t \in \mathcal{P}^t$ such that $p^{t-1} = (p_t, p^t)$ supports a competitive equilibrium $\mathrm{CE}(\mu_t, p^{t-1})$. Let $\mathcal{P}_t(\mu_t) \subseteq \mathcal{P}_t$ denote the set of admissible and feasible policy choices conditional on μ_t. Every admissible and feasible continuation policy choice at time 0, $p^{-1} = (p_0, p^0) \in \mathcal{P}^{-1}(\mu_0)$, and the allocation it supports correspond with a sequence of the state, $\{\mu_t\}_{t\geq 0}$. This sequence need not be unique.[14] Let $\mathcal{M}_t$ denote the set of values that the state may take in period t across all such admissible and feasible continuation policy choices.

4.1 Fundamental State Variables

Sequential decision making implies that policy choices in period t are functions of the economy's history. We assume that this history is only relevant insofar as it constrains the set of competitive equilibria as of period t conditional on μ_t that can be supported by admissible continuation policies. (Below, we will relax this assumption and consider an enlarged state space allowing for trigger strategies.) Accordingly, the state in the program of political decision makers in period t is given by μ_t, and the *policy function* $p_t(\cdot)$ is a mapping from $\mathcal{M}_t$ into $\bigcup_{\mu_t \in \mathcal{M}_t} \mathcal{P}_t(\mu_t) \subseteq \mathcal{P}_t$. Similarly, a *continuation policy function* $p^t(\cdot)$ is a mapping from $\mathcal{M}_{t+1}$ into $\bigcup_{\mu_{t+1} \in \mathcal{M}_{t+1}} \mathcal{P}^t(\mu_{t+1}) \subseteq \mathcal{P}^t$. To streamline notation, we define continuation policy functions not only for $t \geq 0$ but also for $t = -1$.

We are now ready to state the definition of politico-economic equilibrium.

Definition 4. A *politico-economic equilibrium* as of period t conditional on $\mu_t \in \mathcal{M}_t$ as well as policy regime $\mathcal{P}$, denoted as $\mathrm{PEE}(\mu_t, \mathcal{P})$ for short, consists of a sequence of policy functions $\{p_s(\cdot)\}_{s\geq t}$, a sequence of continuation policy functions $\{p^s(\cdot)\}_{s\geq t-1}$, policy choices $p^{\star t-1}$, prices $(q_t^\star, q^{\star t}, r_t^\star)$, household choices $(x_t^\star, x^{\star t})$, and production paths $\{\{y_s^{j\star}\}_{j\in\mathcal{J}}\}_{s\geq t}$ such that

i. policy functions are optimal subject to continuation policy functions:

$$p_s(\mu_s) \in \arg \max_{p_s \in \mathcal{P}_s(\mu_s)} \Omega_s({}_s x, x_s, x^s) \text{ s.t. } p^s = p^s(\mu_{s+1}) \text{ for all } \mu_s \in \mathcal{M}_s, s \geq t,$$

where (x_s, x^s) and μ_{s+1} correspond with $\mathrm{CE}(\mu_s, (p_s, p^s))$;

[14] The sequence is not unique if the cross section of asset holdings is indeterminate as might be the case in a policy regime with debt and without commitment to debt repayment.

ii. continuation policy functions are consistent with actual policy choices:

$$p^{s-1}(\mu_s) = (p_s(\mu_s), p^s(\mu_{s+1})) \text{ for all } \mu_s \in \mathcal{M}_s, s \geq t,$$

where μ_{s+1} corresponds with $\mathrm{CE}(\mu_s, p^{s-1}(\mu_s))$;

iii. equilibrium policy choices $p^{\star t-1}$ are generated by the continuation policy function:

$$p^{\star t-1} = p^{t-1}(\mu_t);$$

iv. $(q_t^\star, q^{\star t}, r_t^\star)$, $(x_t^\star, x^{\star t})$ and $\{\{y_s^{j\star}\}_{j\in\mathcal{J}}\}_{s\geq t}$ constitute $\mathrm{CE}(\mu_t, p^{\star t-1})$.

Returning to the motivating question, consider an "initial" policy regime $\mathcal{P}$ with associated politico-economic equilibrium $\mathrm{PEE}(\mu_0, \mathcal{P})$, and a "new" policy regime $\mathcal{P}'$. We are interested in conditions that, if satisfied, guarantee politico-economic equivalence as specified in the following Definition:

Definition 5. A state and policy regime, $(\mu_t, \mathcal{P})$, is *politico-economically equivalent* to another state and policy regime, $(\mu_t', \mathcal{P}')$, if

i. $(\mu_t, \mathcal{P})$ supports a politico-economic equilibrium $\mathrm{PEE}(\mu_t, \mathcal{P})$ with policy choices $p^{\star t-1}$;

ii. $(\mu_t', \mathcal{P}')$ supports a politico-economic equilibrium $\mathrm{PEE}(\mu_t', \mathcal{P}')$ with policy choices $p^{'\star t-1}$;

iii. $(\mu_t, p^{\star t-1})$ is economically equivalent to $(\mu_t', p^{'\star t-1})$.

Note that politico-economic equivalence is defined with respect to pairs of a state and policy regime whereas economic equivalence was defined with respect to pairs of a state and policy. This difference arises because policy is exogenous in competitive equilibrium but endogenous (and shaped by the policy regime) in politico-economic equilibrium.

A sufficient condition for politico-economic equivalence is that the choice set of political decision makers in the new regime satisfies two requirements. On the one hand, this choice set must be sufficiently large in the sense that political decision makers in the new regime can support those competitive equilibria that political decision makers in the initial regime find optimal to implement, on or off the equilibrium path. On the other hand, the choice set in the new regime must not be too large. In particular, political decision makers in the new regime must not be able to support competitive equilibria that cannot be supported in the initial regime. If both requirements are satisfied, then political decision makers in the new regime will implement policies that support the same competitive equilibrium as in the initial regime.

Within the set of competitive equilibria that can be supported by the economic state and the admissible policy instruments, the choice set of political decision makers is constrained by the political state, the admissibility restrictions on the policy instruments under their own control, and the equilibrium behavior of subsequent political decision makers. Accordingly, our politico-economic equivalence conditions impose cross-regime restrictions on the state and the policy spaces. Condition 1 stipulates that states can unambiguously be related to each other across regimes, and Conditions 2 and 3 stipulate that the admissibility restrictions on policy instruments render the choice set in the new regime sufficiently large but not too large.

First, we define a relation between states in different regimes.

Definition 6. For a state $\mu_t \in \mathcal{M}_t$ in the policy regime $\mathcal{P}$, an *associated state* μ'_t in the policy regime $\mathcal{P}'$ satisfies

i. ${}_tp'_{\geq t}$ is part of a ${}_tp' \in {}_t\mathcal{Q}'$;

ii. there exists a $p'^{t-1} \in \mathcal{Q}'^{t-1}$ such that $(\mu_t, p^{t-1}(\mu_t))$ is economically equivalent to (μ'_t, p'^{t-1}).

The set of states in the policy regime $\mathcal{P}'$ that are associated with $\mu_t \in \mathcal{M}_t$ is denoted $\tilde{\mathcal{M}}'_t(\mu_t)$.

The first part of Definition 6 requires that the predetermined policy instruments lie in the policy space of the new regime. Similarly, the continuation policy choice p'^{t-1} in the second requirement in Definition 6 is constrained by the policy space $\mathcal{Q}'$ and not by the policy regime $\mathcal{P}'$. That is, while the policy choices must contain policy instruments available in the new regime, the numerical values of these instruments do not need to satisfy the admissibility restrictions present in the new regime. If $\tilde{\mathcal{M}}'_t(\mu_t)$ is empty for some μ_t, then the policy instruments in the new policy regime are not flexible enough to support the equilibrium allocation given μ_t in the initial policy regime, even disregarding numerical restrictions on the instruments.

Condition 1. The following holds true for all t:

i. $\mathcal{M}'_t \subseteq \cup_{\mu_t \in \mathcal{M}_t} \tilde{\mathcal{M}}'_t(\mu_t)$;

ii. $\tilde{\mathcal{M}}'_t(\mu_t) \neq \emptyset$ for all $\mu_t \in \mathcal{M}_t$;

iii. if $\mu_t, \hat{\mu}_t \in \mathcal{M}_t$ and $\mathrm{CE}(\mu_t, p^{t-1}(\mu_t)) \neq \mathrm{CE}(\hat{\mu}_t, p^{t-1}(\hat{\mu}_t))$ then $\tilde{\mathcal{M}}'_t(\mu_t) \cap \tilde{\mathcal{M}}'_t(\hat{\mu}_t) = \emptyset$.

The first part of Condition 1 requires that every state in the new policy regime can be associated with a state in the initial regime, and the second part requires that for every state in the initial regime, there is a state in the new regime that can be associated with it. The third part of the Condition requires that a state in the new policy regime can be associated with more than one state in the initial regime only if the latter induce identical competitive equilibria.

We can then define an *equivalent continuation policy function* $\tilde{p}'^{t-1}(\cdot)$ that maps the state μ'_t which is associated with μ_t into a continuation policy choice $\tilde{p}'^{t-1}(\mu'_t) \in \mathcal{Q}'^{t-1}$ such that $(\mu_t, p^{t-1}(\mu_t))$ is economically equivalent to $(\mu'_t, \tilde{p}'^{t-1}(\mu'_t))$. Similarly, we can define an *equivalent policy function* $\tilde{p}'_t(\cdot)$ that maps the state μ'_t into a policy choice $\tilde{p}'_t(\mu'_t) \in \mathcal{Q}'_t$ that corresponds to the time-t component of $\tilde{p}'^{t-1}(\mu'_t)$. Both functions have domain $\cup_{\mu_t \in \mathcal{M}_t} \tilde{\mathcal{M}}'_t(\mu_t)$. If policy instruments in the new policy regime are redundant then the equivalent continuation policy function and the equivalent policy function generally are correspondences rather than functions. For simplicity, we disregard this possibility when stating the following conditions.

Condition 2 formalizes the requirement that the choice set of political decision makers in the new regime be sufficiently large:

Condition 2. The following holds true for all $\mu'_t \in \mathcal{M}'_t$ and all t:

i. $\tilde{p}'_t(\mu'_t) \in \mathcal{P}'_t$.

Condition 3 formalizes the requirement that the choice set not be too large. It stipulates that every competitive equilibrium supported by μ'_t, an admissible period-t policy choice in the new policy regime and the economically equivalent continuation policy function, can also be supported in the initial regime:

Condition 3. The following holds true for all $\mu'_t \in \mathcal{M}'_t$ and all t, where $\mu'_t \in \tilde{\mathcal{M}}'_t(\mu_t)$, $\mu_t \in \mathcal{M}_t$:

i. If there exists a $p'_t \in \mathcal{P}'_t$ such that $(\mu'_t, (p'_t, \tilde{p}^{'t}(\mu'_{t+1})))$ supports the competitive equilibrium $\text{CE}(\mu'_t, (p'_t, \tilde{p}^{'t}(\mu'_{t+1})))$ corresponding with μ'_{t+1}, then there exists a $p_t \in \mathcal{P}_t$ such that $(\mu_t, (p_t, p^t(\mu_{t+1})))$ is economically equivalent to $(\mu'_t, (p'_t, \tilde{p}^{'t}(\mu'_{t+1})))$ where μ_{t+1} corresponds with the competitive equilibrium $\text{CE}(\mu_t, (p_t, p^t(\mu_{t+1})))$.

Note that under Condition 1, the μ_{t+1} and μ'_{t+1} in Condition 3 satisfy $\mu'_{t+1} \in \tilde{\mathcal{M}}'_{t+1}(\mu_{t+1})$ because economic equivalence of $(\mu_t, (p_t, p^t(\mu_{t+1})))$ and $(\mu'_t, (p'_t, \tilde{p}^{'t}(\mu'_{t+1})))$ with $p'_t \in \mathcal{P}'_t$ implies ${}_{t+1}p' \in {}_{t+1}\mathcal{Q}'$ as well as economic equivalence of $(\mu_{t+1}, p^t(\mu_{t+1}))$ and $(\mu'_{t+1}, \tilde{p}^{'t}(\mu'_{t+1}))$.

We can now state the politico-economic equivalence result:

Proposition 2. Consider a state and policy regime, $(\mu_0, \mathcal{P})$ with $\mu_0 \in \mathcal{M}_0$, that support a politico-economic equilibrium $\text{PEE}(\mu_0, \mathcal{P})$, and consider a new state and policy regime, $(\mu'_0, \mathcal{P}')$ with $\mu'_0 \in \tilde{\mathcal{M}}'_0(\mu_0)$. Suppose that Conditions 1–3 are satisfied. Then, under Assumptions 1–2, $(\mu_0, \mathcal{P})$ is politico-economically equivalent to $(\mu'_0, \mathcal{P}')$.

Proof. We show that there exists a politico-economic equilibrium in the new regime that consists of the policy and continuation policy functions $\{\tilde{p}'_t(\cdot), \tilde{p}^{'t-1}(\cdot)\}_{t\geq 0}$, the policy choices $p^{'\star -1} \equiv \tilde{p}^{'-1}(\mu'_0)$, and the same prices, household choices and production paths as in $\text{PEE}(\mu_0, \mathcal{P})$.

Conjecture that in the new regime in period t, political decision makers as well as the private sector expect future policy choices to be determined according to the continuation policy function $\tilde{p}^{'t}(\cdot)$. (From Condition 1, this function is well defined over the domain $\mathcal{M}'_{t+1}$). We claim that, under this conjecture, the policy function in the new regime is given by $\tilde{p}'_t(\cdot)$. To verify the claim by contradiction, suppose instead that the policy function is given by another function, $\pi'_t(\cdot)$ say, such that for some $\mu'_t \in \mathcal{M}'_t$ with $\mu'_t \in \tilde{\mathcal{M}}'_t(\mu_t)$, $\mu_t \in \mathcal{M}_t$, the allocation in $\text{CE}(\mu'_t, (\pi'_t(\mu'_t), \tilde{p}^{'t}(\mu'_{t+1})))$ is strictly preferred over the allocation in $\text{CE}(\mu'_t, (\tilde{p}'_t(\mu'_t), \tilde{p}^{'t}(\mu'_{t+1})))$ (where μ'_{t+1} corresponds with the respective equilibrium) and $\pi'_t(\mu'_t) \in \mathcal{P}'_t$. From Condition 3, there exists an admissible policy choice $\pi_t \in \mathcal{P}_t$ in the initial regime such that $(\mu'_t, (\pi'_t(\mu'_t), \tilde{p}'^t(\mu'_{t+1})))$ is economically equivalent to $(\mu_t, (\pi_t, p^t(\mu_{t+1})))$ (where μ_{t+1} corresponds with the latter equilibrium). By definition of the policy function, $\mathrm{CE}(\mu_t, p^{t-1}(\mu_t))$ is preferred (at least weakly) over $\mathrm{CE}(\mu_t, (\pi_t, p^t(\mu_{t+1})))$. From Assumption 2, political decision makers in the new regime share this preference. From Condition 2, political decision makers in the new regime can support the former equilibrium by choosing $\tilde{p}'_t(\mu'_t)$ rather than $\pi'_t(\mu'_t)$. This establishes the desired contradiction and thus, verifies the claim.

We conclude that for all $\mu'_t \in \mathcal{M}'_t$ and all t, political decision makers in the new regime implement policy choices according to the policy function $\tilde{p}'_t(\cdot)$ if agents expect the continuation policy function $\tilde{p}'^t(\cdot)$. We show next that such expectations are consistent with equilibrium. As noted earlier, $\mu'_t \in \tilde{\mathcal{M}}'_t(\mu_t)$ as well as economic equivalence of $(\mu_t, p^{t-1}(\mu_t))$ and $(\mu'_t, (\tilde{p}'_t(\mu'_t), \tilde{p}'^t(\mu'_{t+1})))$ with $p_t(\mu_t) \in \mathcal{P}_t$ and $\tilde{p}'_t(\mu'_t) \in \mathcal{P}'_t$ implies that $\mu'_{t+1} \in \tilde{\mathcal{M}}'_{t+1}(\mu_{t+1})$. By induction, the above argument for period t therefore extends to subsequent periods and the conjectured expected continuation policy functions are consistent with the policy functions governing actual policy choices. Accordingly, the functions $\tilde{p}'_t(\cdot)$ and $\tilde{p}'^t(\cdot)$ satisfy the conditions of politico-economic equilibrium.

Economic equivalence of $(\mu'_0, (\tilde{p}'_0(\mu'_0), \tilde{p}'^0(\mu'_1)))$ and $(\mu_0, (p_0(\mu_0), p^0(\mu_1)))$ (where μ'_1 and μ_1 correspond with the respective equilibria) implies that the equilibrium policy choices in the new policy regime support the same competitive equilibrium as in the old policy regime. The result then follows. □

Conditions 1–3 are sufficient for politico-economic equivalence but not all three conditions are necessary. More specifically, while failure of Condition 2 necessarily undermines politico-economic equivalence (since it implies that equivalent continuation policy functions in the new regime are not admissible) the same does not hold true with respect to Conditions 1 and 3. If the latter two conditions are violated then politico-economic equivalence cannot be guaranteed but cannot be ruled out either. Failure of Condition 1 implies that a one-to-one relation between states cannot be established and thus, that equivalent continuation policy functions cannot be defined. While our strategy to prove equivalence then cannot be pursued, equivalence nevertheless may hold. Failure of Condition 3 implies that some allocations may only be implementable in the new regime such that the choice set of political decision makers in the new regime is not a subset of the choice set in the initial regime. Equivalence still may hold since the *equilibrium* allocation in the new regime may be implementable in the initial regime as well.

4.2 Non-Fundamental State Variables

Proposition 2 can be extended to accommodate trigger strategies sustained by non-fundamental state variables. Let ξ_t represent such a non-fundamental state variable and let $\mathcal{S}_t \subseteq \mathcal{P}^{t-1}$ denote a "proposed policy" or "suggested policy" that political decision makers in period t are confronted with. A proposed policy contains a single p^{t-1} if it prescribes a unique admissible policy choice in each period. Otherwise, the proposed policy contains multiple p^{t-1}. The non-fundamental state variable takes the value one if political decision makers in earlier periods implemented policies consistent with the proposed policies they were confronted with, and zero otherwise.

In addition to the state variables contained in μ_t, political decision makers in period t inherit a proposed policy $\mathcal{S}_t$ as well as the non-fundamental state variable ξ_t. Given ξ_t and $\mathcal{S}_t$, the policy choice p_t determines ξ_{t+1} according to the law of motion

$$\xi_0 = 1 \text{ and } \xi_{t+1} = \begin{cases} 1 & \text{if } \xi_t = 1 \text{ and } \exists p^t \in \mathcal{P}^t : (p_t, p^t) \in \mathcal{S}_t \\ 0 & \text{otherwise} \end{cases}, \quad t \geq 0. \tag{6}$$

Depending on the institutional setup, political decision makers in period t may choose to modify the policy suggested to successive political decision makers. Let $\mathbf{S}_{t+1}(\mathcal{S}_t)$ denote the set of proposed policies that political decision makers can choose from if they themselves are confronted with the proposed policy $\mathcal{S}_t$. Moreover, let $\langle \mathcal{S}_t \rangle \subseteq \mathcal{P}^t$ denote the policy choices contained in the set $\mathcal{S}_t$ that apply in period $t+1$ or later, $\langle \mathcal{S}_t \rangle = \{p^t | \ \exists p_t \in \mathcal{P}_t : (p_t, p^t) \in \mathcal{S}_t\}$. If political decision makers in period t may not modify future suggestions embedded in the proposed policy, then the choice set $\mathbf{S}_{t+1}(\mathcal{S}_t)$ contains a single element, $\langle \mathcal{S}_t \rangle$, and subsequent political decision makers are confronted with essentially the same suggestion as contemporaneous ones, $\mathcal{S}_{t+1} = \langle \mathcal{S}_t \rangle$. If, in contrast, the institutional setup allows political decision makers in period t to choose among a set of proposed policies, then $\mathbf{S}_{t+1}(\mathcal{S}_t)$ is a subset of the power set $2^{\langle \mathcal{S}_t \rangle}$, that is, it contains several elements each of which is a subset of $\langle \mathcal{S}_t \rangle$. For example, if political decision makers in period t may suggest a particular value for the policy choice in the subsequent period then $\mathbf{S}_{t+1}(\mathcal{S}_t) = \{\{p^t \in \langle \mathcal{S}_t \rangle | \ p_{t+1} = p_{t+1}^{sug}\}\}_{p_{t+1}^{sug} \in \mathcal{P}_{t+1}}$.

A trigger strategy is defined by $\mathbf{S} \equiv (\mathcal{S}_0, \mathbf{S}_1(\cdot), \mathbf{S}_2(\cdot), \mathbf{S}_3(\cdot), \ldots)$. Let $\mathbf{S}_{t+1}$ denote the set of proposed policies in period $t+1$ that can be generated under the trigger strategy. This set is defined recursively as $\mathbf{S}_{t+1} \equiv \bigcup_{\mathcal{S}_t \in \mathbf{S}_t} \mathbf{S}_{t+1}(\mathcal{S}_t)$ with $\mathbf{S}_0 = \mathcal{S}_0$.

A policy regime with trigger strategies is defined by $(\mathcal{P}, \mathbf{S})$. We denote the state in such a policy regime by $\phi_t \equiv (\mu_t, \xi_t, \mathcal{S}_t)$. (Both ξ_t and $\mathcal{S}_t$ become irrelevant state variables if $\xi_t = 0$.) The policy functions $p_t(\cdot)$ and $\mathcal{S}_{t+1}(\cdot)$ map $\Phi_t \equiv \mathcal{M}_t \times \{0, 1\} \times \mathbf{S}_t$ into $\cup_{\mu_t \in \mathcal{M}_t} \mathcal{P}_t(\mu_t) \subseteq \mathcal{P}_t$ and $\mathbf{S}_{t+1}$, respectively, and the continuation policy function $p^t(\cdot)$ maps Φ_{t+1} into $\cup_{\mu_{t+1} \in \mathcal{M}_{t+1}} \mathcal{P}^t(\mu_{t+1}) \subseteq \mathcal{P}^t$. (There is no need to specify the continuation proposed policy function.)

Definition 7. A *politico-economic equilibrium with trigger strategies* as of period t conditional on $\phi_t \in \Phi_t$ as well as policy regime $(\mathcal{P}, \mathbf{S})$, denoted as PEET$(\phi_t, \mathcal{P}, \mathbf{S})$ for short, consists of a sequence of policy functions $\{p_s(\cdot)\}_{s \geq t}$, a sequence of continuation policy functions $\{p^s(\cdot)\}_{s \geq t-1}$, a sequence of proposed policy functions $\{\mathcal{S}_s(\cdot)\}_{s \geq t+1}$, policy choices $p^{\star t-1}$, prices $(q_t^\star, q^{\star t}, r_t^\star)$, household choices $(x_t^\star, x^{\star t})$, and production paths $\{\{y_s^{j\star}\}_{j \in \mathcal{J}}\}_{s \geq t}$ such that

i. policy and proposed policy functions are optimal subject to continuation policy functions:

$$(p_s, \mathcal{S}_{s+1})(\phi_s) \in \arg \max_{p_s \in \mathcal{P}_s(\mu_s), \mathcal{S}_{s+1} \in \mathbf{S}_{s+1}(\mathcal{S}_s)} \Omega_s({}_s x, x_s, x^s) \text{ s.t. } p^s = p^s(\phi_{s+1}) \text{ for all } \phi_s \in \Phi_s, s \geq t,$$

where (x_s, x^s) and μ_{s+1} correspond with CE$(\mu_s, (p_s, p^s))$ and ξ_{s+1} follows from (6);

ii. continuation policy functions are consistent with actual policy choices:

$$p^{s-1}(\phi_s) = (p_s(\phi_s), p^s(\mu_{s+1}, \xi_{s+1}, \mathcal{S}_{s+1}(\phi_s))) \text{ for all } \phi_s \in \Phi_s, s \geq t,$$

where μ_{s+1} corresponds with $\mathrm{CE}(\mu_s, p^{s-1}(\phi_s))$ and ξ_{s+1} follows from (6);

iii. equilibrium policy choices $p^{\star t-1}$ are generated by the continuation policy function:

$$p^{\star t-1} = p^{t-1}(\phi_t);$$

iv. $(q_t^\star, q^{\star t}, r_t^\star)$, $(x_t^\star, x^{\star t})$ and $\{\{y_s^{j\star}\}_{j \in \mathcal{J}}\}_{s \geq t}$ constitute $\mathrm{CE}(\mu_t, p^{\star t-1})$.

In Appendix A.1, we extend Definitions 5–6 and Conditions 1–3 to accommodate the state variables ξ_t and $\mathcal{S}_t$ as well as the proposed policy functions $\mathcal{S}_t(\cdot)$. Besides the fact that the institutional environment now is characterized by both $\mathcal{P}$ and $\mathcal{S}$, the state is larger and political decision makers choose both policy instruments and a proposed policy, the extended definitions and conditions differ from the original ones in a straightforward manner. First, associated-ness of states additionally requires that $\xi_t = \xi_t'$ and that proposed policies are economically equivalent across regimes (conditional on μ_t, μ_t'). Secondly, for the choice set of political decision makers in the new regime to be sufficiently large, both the equivalent policy and the equivalent proposed policy have to be admissible in the new regime. And finally, for the choice set of political decision makers in the new regime not to be too large, the requirement specified in Condition 3 must be met both for the policy choice and the proposed policy choice in the initial regime. Proposition 3 in Appendix A.1 extends the politico-economic equivalence result for pairs $(\mu_0, \mathcal{P})$ and $(\mu_0', \mathcal{P}')$ in Proposition 2 to a result for pairs $(\phi_0, \mathcal{P}, \mathbf{S})$ and $(\phi_0', \mathcal{P}', \mathbf{S}')$.

For a pair of associated states, $\phi_t \in \Phi_t$ and $\phi_t' \in \tilde{\Phi}_t'(\phi_t)$, the admissible proposed policies are economically equivalent across regimes if for each $\mathcal{S}_{t+1} \in \mathbf{S}_t(\mathcal{S}_t)$ there exists a $\mathcal{S}_{t+1}' \in \mathbf{S}_t'(\mathcal{S}_t')$ such that $(\mu_t, \mathcal{S}_{t+1})$ is economically equivalent to $(\mu_t', \mathcal{S}_{t+1}')$, and vice versa. Two trigger strategies $\mathbf{S}$ and $\mathbf{S}'$ then are economically equivalent (conditional on associated initial states) if for all possible subsequent pairs of associated states the admissible proposed policies are economically equivalent. If it is known that two trigger strategies are economically equivalent (for example because one trigger strategy was constructed to be economically equivalent to the other), then the politico-economic equivalence conditions essentially reduce to the conditions that must be satisfied in the case without trigger strategies.

5 Applications

We now show how the theoretical framework developed above can be put to work. We consider two types of environments where public debt plays a central role. First, an environment with overlapping generations where debt repayment transfers resources between cohorts, parallel to social security benefits. And second, an environment where debt serves to smooth tax distortions.

In addition to the notation introduced in the previous sections, and unless otherwise noted, we let w_t, l_t and l_t^i denote the wage, labor supply of the representative worker, and labor supply of type i (or household i) in period t, respectively; k_t the capital stock per worker; $\mathcal{I}_t^w$ and $\mathcal{I}_t^r$ the set of workers and retirees, respectively; $g_t^w(\cdot)$ and $g_t^r(\cdot)$ tax functions imposed on workers and retirees, respectively; and ν the gross population growth rate.

5.1 Debt Repayment as Transfer

We start by contrasting politico-economic theories of social security on the one hand and debt on the other. Based on our theoretical results, we show that certain politico-economic theories of social security that have been proposed in the literature may be re-interpreted as politico-economic theories of government debt. Beyond these novel theories, the analysis generates three general insights. First, it identifies an important class of economic environments in which politico-economic equivalence between social security and debt regimes holds robustly (that is, independently of particular political aggregation mechanisms). Second, it shows how the equivalence conditions can be employed to develop novel theories even in those environments in which politico-economic equivalence may not generally be guaranteed. And finally, it proves that with a sufficient degree of heterogeneity among households and absent commitment, the conditions for politico-economic equivalence generally are undermined unless certain exogenous restrictions are imposed.

5.1.1 Robust Politico-Economic Equivalence

We start by characterizing a baseline setup in which politico-economic equivalence of a social security and a debt regime is guaranteed for arbitrary political aggregation mechanisms. The economy is inhabited by two-period lived overlapping generations that are homogeneous within cohorts; the number of young relative to old households is denoted ν_t. Young households inelastically supply labor and production is neoclassical. In a first step, we analyze the case without commitment and trigger strategies. The state then only includes the capital stock, k_t.

A social security regime is characterized by a labor income tax levied at rate τ_t and funding transfers to retirees.[15] An alternative policy regime with debt is characterized by the repayment rate z_t', the exogenous debt stock per retiree $\bar{b}_t' > 0$ and labor income taxes levied at rate τ_t'. (To streamline notation, we do not distinguish between debt repayment in periods $t \geq 1$ and "debt repayment" to retirees in the initial period who did not purchase the debt but simply receive a transfer.) Since retirees are homogeneous within a cohort, the cross section of debt holdings among living households is fully characterized by $\bar{b}_t'$.

[15]With inelastic labor supply and within-cohort homogeneity, labor income taxes are equivalent to lump sum taxes. Below, when introducing tax distortions and within-cohort heterogeneity, this is no longer the case. We specify labor income rather than lump sum taxes already at this point to render the different setups more easily comparable.

Table 1 summarizes the two policy regimes. The social security regime is characterized on top of the left column, the debt regime on top of the right column. The lower part of the Table summarizes the economic-equivalence cross-regime restrictions implied by Proposition 1. These “EE restrictions” require in each period identical capital stocks and government cash flows across regimes, and for each cohort identical present values of tax payments across regimes.

$g_t^w(w_t) = \tau_t w_t,\ t \geq 0$	$g_t^{w'}(w'_t) = \tau'_t w'_t,\ t \geq 0$
$g_t^r(w_t) = -\nu_t \tau_t w_t,\ t \geq 0$	$g_0^{r'} = -\bar{b}'_0 z'_0;\ \ g_t^{r'} = 0,\ t \geq 1$
$\mathcal{P}_t = \{\tau_t \in \mathbb{R}_+\},\ t \geq 0$	$\mathcal{P}'_t = \{(\tau'_t, z'_t) \in \mathbb{R} \times \mathbb{R}_+\},\ t \geq 0$
$\mathbf{S} = \emptyset$	$\mathbf{S}' = \emptyset$
$\mu_t = k_t,\ t \geq 0$	$\mu'_t = (k'_t, \bar{b}'_t),\ t \geq 0$
EE restrictions, μ'_t	$k'_t = k_t$ ($\bar{b}'_t$ exogenous)
p'^{t-1}	$z'_s = \tau_s \nu_s w_s / \bar{b}'_s,\ \ s \geq t$
	$\tau'_s = \tau_s - \frac{r_{t,s+1}}{r_{t,s}} \frac{\tau_{s+1}\nu_{s+1}w_{s+1}}{w_s},\ \ s \geq t$

Table 1: Setup with robust politico-economic equivalence.

To assess politico-economic equivalence note first (from the EE restrictions in Table 1) that any state $k_t \in \mathcal{M}_t$ and admissible continuation policy sequence τ^{t-1} in the social security regime (not only the equilibrium continuation policy sequence $\tau^{t-1}(k_t)$) is economically equivalent to the same state and to continuation policy sequences $(\tau'^{t-1}, z'^{t-1}) \in \mathcal{Q}'^{t-1}$ in the debt regime. Associated states therefore satisfy $k'_t = k_t$ for all $k_t \in \mathcal{M}_t$. In fact, the corresponding continuation policy sequences are admissible in the debt regime, $(\tau'^{t-1}, z'^{t-1}) \in \mathcal{P}'^{t-1}$. As long as $k'_0 = k_0$, any state that may result under some feasible policy sequence in the social security regime therefore may also result under a feasible policy sequence in the debt regime, $\mathcal{M}_t \subseteq \mathcal{M}'_t$. A parallel argument relating states across the two regimes in the opposite direction establishes that $\mathcal{M}'_t \subseteq \mathcal{M}_t$. We conclude that if $k'_0 = k_0$, then $\mathcal{M}_t = \mathcal{M}'_t$ and Condition 1 is satisfied regardless of whether the initial regime is the social security regime or the debt regime.

Condition 2 is satisfied as well regardless of the initial regime. This follows immediately from the fact that equivalent continuation policy sequences are admissible, both in the debt regime and the social security regime, as argued above. In fact, a stronger condition than Condition 2 is satisfied because the equivalent continuation policy sequences of arbitrary admissible policy sequences (not only the equilibrium continuation policy sequence) are admissible, and this holds true regardless of the initial regime. But this stricter version of Condition 2 for the debt regime as the initial regime is equivalent to Condition 3 for the social security regime, and vice versa. As long as $k'_0 = k_0$, Conditions 1–3 then are all satisfied and politico-economic equivalence is guaranteed. Note that this conclusion does not rely on assumptions about the political aggregator function. In the baseline setup, politico-economic equivalence therefore is guaranteed for any political aggregator function. Essentially, this generality follows from the fact that the EE restrictions can be satisfied for all admissible rather than just the equilibrium continuation policy sequences.

Forni (2005) analyzes the baseline setup under the assumption that a median voter is politically decisive. He shows that, for some parameter constellations, an equilibrium with self-fulfilling expectations may exist in which strictly positive social security tax rates are sustained. Contemporaneous political decision makers support strictly positive taxes if they expect future social security benefits to be a decreasing function of the capital stock.[16] From the above discussion, we can immediately conclude that the social security regime in Forni's (2005) model is politico-economically equivalent (conditional on some initial capital stock) to a debt regime.

The general equivalence result for the baseline setup extends to the case with one-period, symmetric commitment. In this case, the state in the social security regime is given by $\mu_t = (k_t, \tau_t)$ and in the debt regime by $\mu'_t = (k'_t, z'_t)$. The EE restrictions in Table 1 continue to hold, with the exception that τ_t and z'_t are part of the respective states rather than the continuation policy sequences from period $t-1$ onwards. With this qualification, and as long as $k'_0 = k_0$ and $z'_0 = \tau_0 \nu_0 w_0 / \bar{b}'_0$, all arguments establishing the validity of Conditions 1–3 in the case without commitment extend to the situation with one-period, symmetric commitment. Politico-economic equivalence therefore is guaranteed for any political aggregator function.

The general equivalence result for the baseline setup also extends to the case with trigger strategies. In a social security regime with trigger strategy $\mathbf{S}$, political decision makers in period t are confronted with a suggested policy $\mathcal{S}_t$ and choose the suggested policy $\mathcal{S}_{t+1}$. Politico-economic equivalence in this setup is guaranteed if $k'_0 = k_0$ and $\xi'_0 = \xi_0$ and if the trigger strategy in the social security regime, $\mathbf{S}$, is economically equivalent to the trigger strategy in the debt regime, $\mathbf{S}'$. In the absence of a priori restrictions on the latter trigger strategy, this requirement can easily be satisfied by constructing an appropriate $\mathbf{S}'$ based on the EE restrictions in Table 1 which associate a $\phi'_t = (\mu'_t, \xi'_t, \mathcal{S}'_t)$ to each $\phi_t \in \Phi_t$.

Boldrin and Rustichini (2000) analyze the baseline setup with a trigger strategy under the assumption that a young median voter is politically decisive. They assume that political decision makers are confronted with a suggested policy $\mathcal{S}_t$ consisting of a set of continuation policies τ^{t-1} with τ_t fixed according to the suggestion made by political decision makers in period $t-1$. In turn, political decision makers in period t choose an updated suggested policy $\mathcal{S}_{t+1} = \{\tau^t \in \langle \mathcal{S}_t \rangle | \tau_{t+1} = \tau^{\text{sug}}_{t+1}\}$ that is characterized by a particular proposal for the tax rate in the subsequent period, τ^{sug}_{t+1}. Boldrin and Rustichini (2000) show that this trigger strategy provides sufficiently strong incentives for political decision makers to support equilibria with strictly positive social security transfers. From the above discussion, we can immediately conclude that the social security regime in Boldrin and Rustichini's (2000) model is politico-economically equivalent to the debt regime (conditional on some initial capital stock and $\xi'_0 = \xi_0$) if the trigger strategy in the debt regime is appropriately specified. In particular, equivalence is guaranteed if for each pair of associated states ϕ_t and ϕ'_t the trigger strategy in the debt regime satisfies

[16] Forni (2005) considers the case where the initial capital stock evolves within a certain range of parameter-dependent values. See Gonzalez-Eiras (2011) for a general characterization of equilibrium.

$\mathcal{S}'_{t+1} = \{(\tau'^t, z'^t) \in \langle \mathcal{S}'_t \rangle | z'_{t+1} = z'^{\text{sug}}_{t+1}\}$ where z'^{sug}_{t+1} is part of the continuation policy sequence that is economically equivalent to the continuation policy sequence containing τ^{sug}_{t+1}.

5.1.2 Fragile Politico-Economic Equivalence

In the setup with robust politico-economic equivalence, the equivalent continuation policy sequences of arbitrary admissible policy sequences are themselves admissible. As a consequence, Conditions 2 and 3 are satisfied independently of a specific political aggregator function. With extensions to the baseline setup, in contrast, the equivalent continuation policy sequences of only some but not all admissible policy sequences may themselves be admissible, and Conditions 2 or 3 may therefore only be satisfied for specific political aggregator functions. We refer to this situation as "fragile" politico-economic equivalence.

For a simple but important extension of the baseline setup with fragile politico-economic equivalence, consider the case with elastic labor supply. Maintaining the assumption of proportional labor income taxes, economic equivalence now requires that the marginal tax rate for each cohort be identical across regimes (in addition to the restrictions of identical capital stocks, government cash flows, and lifetime tax burdens across regimes). In general, to satisfy this requirement necessitates two tax instruments. Table 2 summarizes a social-security regime on the left-hand side and a debt regime on the right-hand side with two such instruments. The admissibility restrictions on these instruments rule out lump-sum taxes. The first tax, raised at rate τ_t or τ'_t, transfers resources from workers to retirees (by means of social security benefits or debt repayment). The second one, raised at rate θ_t or θ'_t, is a purely distorting tax whose proceeds are redistributed lump-sum among workers. The EE restrictions capture the restrictions on households' budget sets across the two regimes.

$g^w_t(w_t, l^i_t, l_t) = \tau_t w_t l^i_t + \theta_t w_t(l^i_t - l_t),\ t \geq 0$	$g^{w'}_t(w'_t, l^{i'}_t, l'_t) = \tau'_t w'_t l^{i'}_t + \theta'_t w'_t(l^{i'}_t - l'_t),\ t \geq 0$
$g^r_t(w_t, l_t) = -\nu_t \tau_t w_t l_t,\ t \geq 0$	$g^{r'}_0 = -\bar{b}'_0 z'_0;\ \ g^{r'}_t = 0,\ t \geq 1$
$\mathcal{P}_t = \{(\tau_t, \theta_t) \in \mathbb{R}^2_+\},\ t \geq 0$	$\mathcal{P}'_t = \{(\tau'_t, \theta'_t, z'_t) \in \mathbb{R} \times \mathbb{R}^2_+\},\ t \geq 0$
$\mathbf{S} = \emptyset$	$\mathbf{S}' = \emptyset$
$\mu_t = k_t,\ t \geq 0$	$\mu'_t = (k'_t, \bar{b}'_t),\ t \geq 0$
EE restrictions, μ'_t p'^{t-1}	$k'_t = k_t$ ($\bar{b}'_t$ exogenous)
	$z'_s = \tau_s \nu_s w_s l_s / \bar{b}'_s,\ s \geq t$
	$\tau'_s = \tau_s - \frac{r_{t,s+1}}{r_{t,s}} \frac{\tau_{s+1}\nu_{s+1}w_{s+1}l_{s+1}}{w_s l_s},\ s \geq t$
	$\theta'_s = \theta_s + \frac{r_{t,s+1}}{r_{t,s}} \frac{\tau_{s+1}\nu_{s+1}w_{s+1}l_{s+1}}{w_s l_s},\ s \geq t$

Table 2: Setup with fragile politico-economic equivalence.

Note that every possible state in either of the two regimes is associated with a unique state in the other regime, i.e. $\tilde{\mathcal{M}}'_t(k_t) = k_t$ and $\tilde{\mathcal{M}}_t(k'_t) = k'_t$. Moreover, in either of the two regimes the set of possible states in period t—the set of possible capital stocks k_t that can be attained by admissible and feasible policies—ranges from zero (the capital stock subject to confiscatory taxation) to a maximum value, $\bar{k}_t(k_0)$ or $\bar{k}_t(k_0')$. Since the latter results in the absence of any taxation, we have $\bar{k}_t(k_0) = \bar{k}_t(k_0')$ as long as $k_0 = k_0'$. As a consequence, $\mathcal{M}_t = \mathcal{M}_t'$ and Condition 1 is satisfied, regardless of whether the initial regime is the social security regime or the debt regime.

Consider next Condition 2. For any admissible policy sequence (and thus, for the equilibrium policy sequence under any political aggregator function) in the social security regime the equivalent policy sequence in the debt regime is admissible as well because $\tau_s, \theta_s \geq 0$ for all $s \geq t$ implies that $z_s', \theta_s' \geq 0$ and $\tau_s' \in \mathbb{R}$ for all $s \geq t$. Condition 2 therefore holds for any political aggregator function if the initial regime is the social security regime.

In contrast, this is not the case if the initial regime is the debt regime. There exist admissible and feasible policy sequences in the debt regime whose equivalent policy sequences are not admissible in the social security regime. To see this, consider an admissible and feasible policy choice in the debt regime, $p_t' = (\tau_t', \theta_t', z_t')$, that supports wages and labor supplies, $w_s', l_s', s \geq t$, (conditional on k_t' and the continuation policy function $p'^t(\cdot)$). If the political aggregator function implies $z_{t+1}' > 0$ under the continuation policy function $p'^t(\cdot)$, then one feasible policy choice p_t' involves contemporaneous total tax rate $\tau_t' + \theta_t' = 0$ and contemporaneous debt repayment $z_t' > 0$ (which can be financed out of new debt issues because $z_{t+1}' > 0$). From the EE restrictions, the economically equivalent policy in the social security regime then satisfies $\theta_t = \tau_t' + \theta_t' - z_t'\bar{b}_t'/(\nu_t w_t' l_t') < 0$, which is not admissible. Condition 2 therefore does not hold for every political aggregator function when the initial regime is the debt regime.

This has direct implications for the validity of Condition 3. If the initial regime is the social security regime then this condition does not hold for every political aggregator function. For example, suppose that the equilibrium continuation policy function in the social security regime specifies a positive tax rate τ_{t+1} (depending on the political aggregator function, this is clearly possible). From the EE restrictions, this translates into a positive debt repayment rate z_{t+1}' under the equivalent continuation policy function in the debt regime. The previous reasoning then applies; certain allocations can be implemented in the debt regime but cannot be supported in the social security regime. Condition 3 therefore does not hold in this example.

Gonzalez-Eiras and Niepelt (2008) analyze the setup with endogenous labor supply and social security under the assumption that preferences are aggregated through probabilistic voting. They show that strictly positive social security transfers are sustained in politico-economic equilibrium. As argued above, these transfers translate into positive debt repayment rates under the equivalent continuation policy function in the debt regime and imply that politico-economic equivalence cannot be guaranteed.[17]

[17] In fact, politico-economic equivalence fails in Gonzalez-Eiras and Niepelt's (2008) model. In their model, the tax rate θ_t sometimes is in a corner. Without the non-negativity constraint on θ_t, a different policy would be implemented and thus, a different allocation supported. This different allocation would also be supported in the debt regime where the admissibility restrictions are less tight. Since the equilibrium allocation supported in the debt regime (or the social security regime with relaxed admissibility

The opposite conclusion follows in the setup with endogenous labor supply and social security if one assumes that a median voter is politically decisive and that this median voter is a young household. Restricting attention to (the limit of) a finite horizon economy, the equilibrium social security tax rate then satisfies $\tau_s = 0$ in all periods $s \geq t$. In this case, politico-economic equivalence therefore is guaranteed.[18]

It is frequently argued that pre-funding of social security (a shift from a social security regime to a debt regime) improves outcomes by reducing labor supply distortions. This argument relies on the assumption, which often remains implicit, that certain competitive equilibria may be supported by admissible debt policies but not by admissible social security policies, that is, the argument presupposes violations of *economic* equivalence.[19] Our conclusion regarding the failure of *politico-economic* equivalence differs from that standard argument but is related. According to our conclusion, political decision makers in a debt regime have larger choice sets. This has two implications. First, if the restriction to smaller choice sets is binding, then political decision makers may implement more distorting policies in a social security regime than in a debt regime. Second, this may generate political support for a regime change towards pre-funding.

We have seen that in a setup with endogenous labor supply and social security, a politico-economic equilibrium may not necessarily be re-interpreted as an equilibrium in a debt regime. Nevertheless, a researcher wishing to characterize the politico-economic equilibrium in a debt regime may be able to rely on our equivalence result. In particular, a simple strategy to that end consists in first characterizing the politico-economic equilibrium in a social security regime subject to relaxed admissibility restrictions, $(\tau_t, \theta_t) \in \mathbb{R}_+ \times \mathbb{R}$ rather than $(\tau_t, \theta_t) \in \mathbb{R}^2_+$. If the policy sequences in the debt regime that are economically equivalent to the equilibrium policy sequences in the relaxed social security regime are admissible (such that Condition 2 is satisfied), then politico-economic equivalence holds[20] and the equilibrium allocation characterized in the relaxed social security regime also constitutes the equilibrium allocation in the debt regime.

5.1.3 Breakdown of Politico-Economic Equivalence

If heterogeneity may be reflected in a non-trivial debt ownership structure, and absent commitment to the repayment rate, politico-economic equivalence generally fails. Consider an environment with debt where households within a cohort are non-representative or where households live for more than two periods. The debt ownership structure then is

restrictions) cannot be supported in the social security regime with the original admissibility restrictions, Condition 3 specifically is violated for the equilibrium policy under the debt regime. This implies that politico-economic equivalence fails.

[18] From the EE restrictions, zero tax rates τ_s imply $\tilde{p}'^t(k'_{t+1}) \equiv (\tilde{\tau}'^t(k'_{t+1}), \tilde{\theta}'^t(k'_{t+1}), \tilde{z}'^t(k'_{t+1})) = (\mathbf{0}, \theta^t(k_{t+1}), \mathbf{0})$ where $k'_{t+1} = k_{t+1}$. With future debt repayment rates at zero, no funds can be raised from debt issuance and any feasible policy in the debt regime must finance contemporaneous debt repayment out of current social security taxes, $z'_t = \tau'_t \nu_t w'_t l'_t / \bar{b}'_t$. Since z'_t must be non-negative, the tax rate $\tau'_t \geq 0$. As a consequence, the economically equivalent policy choice in the social security regime satisfies $\tau_t = \tau'_t \geq 0$ and $\theta_t = \theta'_t \geq 0$ which does not violate any admissibility restriction.

[19] See Feldstein and Liebman (2002) for an overview over the literature and Rangel (1997) for an insightful critical analysis of this argument.

[20] Condition 3 necessarily holds as well in this case.

endogenous (in contrast to a setup with homogeneous, two-period lived households) and without commitment, it constitutes a state variable because it determines the extent to which a change in the repayment rate affects the wealth distribution.[21] In such an environment, the set of implementable policies thus varies with an endogenous state variable that is not present in a social security regime. Evidently, this discrepancy generally would undermine Condition 3. More fundamentally, it undermines Condition 1.

To see how an endogenous, non-trivial debt ownership structure undermines Condition 1 in the absence of commitment, consider a state $\mu_t = \{a_t^i\}_{i \in \mathcal{I}_t}$ in a social security regime.[22] This state is associated with the state $\mu_t' = \{a_t^{i'}, b_t^{i'}\}_{i \in \mathcal{I}_t}$ in a debt regime if there exists an admissible continuation debt policy $p^{'t-1}$ such that the following conditions are satisfied:

i. identical capital stocks: $\int_{i \in \mathcal{I}_t} a_t^i \, di = \int_{i \in \mathcal{I}_t} a_t^{i'} \, di$;

ii. identical budget sets: $a_t^i - \mathrm{NTF}_t^i(\cdot; \mu_t, p^{t-1}(\mu_t)) = a_t^{i'} + b_t^{i'} z_t' - \mathrm{NTF}_t^i(\cdot; \mu_t', p^{'t-1})$ for all $i \in \mathcal{I}_s, s \geq t$;

iii. debt market clearing: $\int_{i \in \mathcal{I}_t} (b_t^{i'} - \bar{b}_t') \, di = 0$.

Here, $\mathrm{NTF}_t^i(\cdot; \mu_t, p^{t-1})$ denotes the "net tax function" for household $i \in \mathcal{I}_s, s \geq t$, in period t. This net tax function gives the present value of taxes net of transfers of household i as a function of i's choices in period t and later; it is parameterized by the state (which includes asset holdings), the continuation policy as well as prices and interest rates which in turn depend on the state and the continuation policy through the equilibrium allocation.

Suppose that the state μ_t in a social security regime is associated with some state $\mu_t^{'1}$ in a debt regime. (If no such state $\mu_t^{'1}$ exists, then Condition 1 ii. is violated and we do not need to proceed further.) Suppose further that another state $\hat{\mu}_t$ in the social security regime—different from μ_t but with the same capital stock as μ_t (that is, $\int_{i \in \mathcal{I}_t} a_t^i \, di = \int_{i \in \mathcal{I}_t} \hat{a}_t^i \, di$ and $\mathrm{CE}(\mu_t, p^{t-1}(\mu_t)) \neq \mathrm{CE}(\hat{\mu}_t, p^{t-1}(\hat{\mu}_t))$)—is associated with some other state $\mu_t^{'2}$ in the debt regime. (Dito.) The following conditions then hold:

i. identical capital stocks across $\mu_t, \hat{\mu}_t, \mu_t^{'1}, \mu_t^{'2}$ as well as debt market clearing under $\mu_t^{'1}, \mu_t^{'2}$;

ii. $a_t^i - \mathrm{NTF}_t^i(\cdot; \mu_t, p^{t-1}(\mu_t)) = a_t^{i'1} + b_t^{i'1} z_t^{'1} - \mathrm{NTF}_t^i(\cdot; \mu_t^{'1}, p^{'1,t-1})$ for all $i \in \mathcal{I}_s, s \geq t$, and some admissible $p^{'1,t-1}$ (and thus $z_t^{'1}$);

iii. $\hat{a}_t^i - \mathrm{NTF}_t^i(\cdot; \hat{\mu}_t, p^{t-1}(\hat{\mu}_t)) = a_t^{i'2} + b_t^{i'2} z_t^{'2} - \mathrm{NTF}_t^i(\cdot; \mu_t^{'2}, p^{'2,t-1})$ for all $i \in \mathcal{I}_s, s \geq t$, and some admissible $p^{'2,t-1}$ (and thus $z_t^{'2}$).

[21] With commitment to the repayment rate, debt holdings do not constitute a separate state variable (see the discussion on page 11). Cukierman and Meltzer (1989) analyze a model with social security and debt where political decision makers can commit to policy instruments one period in advance. They show that voters are indifferent between using debt or social security as instruments for intergenerational redistribution.

[22] The state does not separately include the capital stock since the latter equals aggregate private asset holdings.

Letting $e_t^{i'1} \equiv a_t^{i'1} + b_t^{i'1} z_t^{'1}$ and $e_t^{i'2} \equiv a_t^{i'2} + b_t^{i'2} z_t^{'2}$, consider the $\mu_t' \equiv \{a_t^{i'}, b_t^{i'}\}_{i \in \mathcal{I}_t}$ satisfying

$$a_t^{i'} = \frac{e_t^{i'2} z_t^{'1} - e_t^{i'1} z_t^{'2}}{z_t^{'1} - z_t^{'2}} \text{ and } b_t^{i'} = \frac{e_t^{i'1} - e_t^{i'2}}{z_t^{'1} - z_t^{'2}}.$$

Generically, such a μ_t' exists.[23] Moreover, it satisfies

$$\left.\begin{array}{l} a_t^{i'} + b_t^{i'} z_t^{'1} = a_t^{i'1} + b_t^{i'1} z_t^{'1} \\ a_t^{i'} + b_t^{i'} z_t^{'2} = a_t^{i'2} + b_t^{i'2} z_t^{'2} \end{array}\right\} \text{ for all } i \in \mathcal{I}_s, s \geq t,$$

as well as $\int_{i \in \mathcal{I}_t} a_t^{i'} \, di = \int_{i \in \mathcal{I}_t} a_t^{i'1} \, di$ and $\int_{i \in \mathcal{I}_t} (b_t^{i'} - \bar{b}_t') \, di = 0$. The capital stock in state μ_t' therefore corresponds with the capital stock in state $\mu_t^{'1}$ (or in state $\mu_t^{'2}$); debt markets clear in state μ_t'; each household's financial wealth under $(\mu_t', p^{'1,t-1})$ corresponds to its financial wealth under $(\mu_t^{'1}, p^{'1,t-1})$; and each household's financial wealth under $(\mu_t', p^{'2,t-1})$ corresponds to its financial wealth under $(\mu_t^{'2}, p^{'2,t-1})$. State μ_t' therefore is associated with both μ_t and $\hat{\mu}_t$. We conclude that Condition 1 iii. necessarily is violated as soon as Condition 1 ii. is satisfied (such that a $\mu_t^{'1}$ and $\mu_t^{'2}$ exist) and $z_t^{'1} \neq z_t^{'2}$ (which holds generically). Clearly, this negative result may be overturned if exogenous restrictions on the ownership structure of debt are imposed.[24]

The possibility of an endogenous, non-trivial debt ownership structure arises in the environment considered by Tabellini (2000). He analyzes a two-period lived overlapping generations economy with inelastic labor supply, exogenous average labor productivity w_t, heterogeneous time endowments among young households, and no capital nor government debt. Household heterogeneity renders the social security system with its tax on labor income τ_t redistributive.[25] There is no commitment and no trigger strategy. Tabellini (2000) shows that, in a median voter framework with weak intergenerational altruism, a coalition of poor young and old households may sustain a social security system whose size increases with the degree of inequality, but decreases with the rate of population growth ν.

Cooley and Soares (1999) analyze a four-period lived overlapping generations economy with capital accumulation, within-cohort homogeneity and a pay-as-you-go financed social security system with proportional taxes levied on the labor income of workers (households during their first three periods of life) and distributed in a lump-sum fashion among retirees (households during their last period of life). Cooley and Soares (1999) analyze the politico-economic equilibrium under the assumption that the median voter in the initial

[23] The μ_t' exists if $z_t^{'1} \neq z_t^{'2}$. If the Jacobian of the system of equations relating $(\mu_t, p^{t-1}(\mu_t))$ to $(\mu_t^{'1}, p^{'1,t-1})$ is of full rank then variations in μ_t (e.g., to $\hat{\mu}_t$) generically result in a change of $z_t^{'1}$ (e.g., to $z_t^{'2}$). See, for example, Mas-Colell et al. (1995, p. 593).

[24] For example, one may restrict debt issuance to be symmetric across certain types of households, or targeted to some but not others, and impose that secondary markets be closed.

[25] Tabellini (2000) assumes proportional taxes levied on the young and a lump-sum benefit paid to the old. To be of relevance for our discussion, Tabellini's (2000) model must be extended to allow for linear rather than proportional taxes, due to *economic* equivalence considerations. For in a social security regime with proportional taxes and lump-sum benefits lifetime taxes of a household are a linear function of income during young age. Replicating households' budget sets in a debt regime (without old-age benefits) thus requires a linear tax function.

period chooses a tax rate that serves as time-invariant proposed social security tax rate in all subsequent periods. Successive median voters only choose between implementing the proposed tax rate or dismantling the social security system forever. Numerically solving a calibrated version of their model, Cooley and Soares (1999) find that the median voter is of age two (out of four) and sustains positive intergenerational transfers.

For the general reasons discussed above, politico-economic equivalence between a social security and debt regime fails in Tabellini's (2000) and Cooley and Soares's (1999) environments. To satisfy Condition 1 and possibly guarantee equivalence, debt holdings could be restricted to be symmetric across retirees (in the former model) or to be targeted to workers in their last period before retirement (in the latter). But even if debt could be issued in accordance with these restrictions, secondary markets could easily compromise those efforts as government promises would tend to be reallocated to the politically most influential investors (Broner, Martin and Ventura, 2010). In general, this would undermine Condition 1.

5.2 Debt as Tax Smoothing Device

We have seen in the previous subsection that in certain environments, economic equivalence of social security and debt policies extends to the political sphere in the sense that social security and debt regimes are politico-economically equivalent. It is natural to ask whether a similar result holds when debt serves a purpose other than transferring resources across groups. One such alternative purpose concerns the role of debt as a tax smoothing device (Barro, 1979; Lucas and Stokey, 1983). An important economic equivalence result in environments with distorting taxes states that tax-and-debt policies that differ with respect to the timing but not the present value of tax collections can be economically equivalent (Bassetto and Kocherlakota, 2004). We ask whether this economic equivalence result in environments with distorting taxes extends to the political sphere and find that this is not the case.[26]

For simplicity, we first abstract from capital accumulation and focus on tax-and-debt policies that tax labor income at a proportional rate. As Bassetto and Kocherlakota (2004) show, variations in the timing of tax collections and the associated debt path need not alter the equilibrium allocation even if taxes are distorting as long as taxes on lagged labor income are admissible. Consider for example the case where labor income in period t might either be taxed at rate $\tau_{t,t}$ in period t or both at rate $\tau'_{t,t}$ in period t and at rate $\tau'_{t,t+1}$ in period $t+1$. If $\tau_{t,t} = \tau'_{t,t} + r_{t,t+1}\tau'_{t,t+1}$, switching from the former to the latter tax policy changes the timing of tax collections and the level of debt but does not alter effective marginal or average tax rates on period t labor income. A policy change of this kind therefore preserves households' budget sets and the equilibrium allocation. In

[26] We consider the case with and without commitment. Absent commitment, debt does not only serve as a tax smoothing device but may also redistribute wealth.

general, economic equivalence requires

$$\left.\begin{array}{l}\tau_{s,s} = \tau'_{s,s} + \frac{r_{t,s+1}}{r_{t,s}}\tau'_{s,s+1} \\ z_t b_t^i = z'_t b_t^{i'} - \tau'_{t-1,t} l_{t-1}^{i'} \\ z_s \text{ satisfies government DBC}\end{array}\right\} \text{ for all } i \in \mathcal{I},\ s \geq t,$$

where l_t^i denotes labor supply of household i.

Consider now the situation with sequential decision making, and suppose first that governments cannot commit to debt repayment. The initial policy regime with contemporaneous taxes only is characterized by $\mathcal{P}_t = \{(\tau_{t,t}, z_t) \in \mathbb{R} \times \mathbb{R}_+\}$ and the new policy regime with contemporaneous and lagged taxes by $\mathcal{P}'_t = \{(\tau'_{t,t}, \tau'_{t-1,t}, z'_t) \in \mathbb{R}^2 \times \mathbb{R}_+\}$. In the initial regime, $\mu_t = \{b_t^i\}_{i \in \mathcal{I}}$ while in the new regime, the state is composed of debt holdings as well as lagged labor supply, $\mu'_t = \{b_t^{i'}, l_{t-1}^{i'}\}_{i \in \mathcal{I}}$. (For simplicity, wages are exogenous and not included in the state.)

Condition 1 iii. fails in this environment because two different states in the initial regime, $\mu_t = \{b_t^i\}_{i \in \mathcal{I}}$ and $\hat{\mu}_t = \{\hat{b}_t^i\}_{i \in \mathcal{I}}$ say, can be associated with one and the same state in the new regime. This can be shown by following the same strategy as in subsubsection 5.1.3. If μ_t is associated with some state $\mu_t^{'1}$ and $\hat{\mu}_t$ with some state $\mu_t^{'2}$, then the state $\mu'_t = \{b_t^{i'}, l_{t-1}^{i'}\}_{i \in \mathcal{I}}$ in the new regime satisfying

$$\left.\begin{array}{l}-z_t^{'1} b_t^{i'1} + \tau_{t-1,t}^{'1} l_{t-1}^{i'1} = -z_t^{'1} b_t^{i'} + \tau_{t-1,t}^{'1} l_{t-1}^{i'} \\ -z_t^{'2} b_t^{i'2} + \tau_{t-1,t}^{'2} l_{t-1}^{i'2} = -z_t^{'2} b_t^{i'} + \tau_{t-1,t}^{'2} l_{t-1}^{i'}\end{array}\right\} \text{ for all } i \in \mathcal{I} \tag{7}$$

is associated with both μ_t and $\hat{\mu}_t$ since each household's financial wealth net of lump sum taxes under $(\mu'_t, p^{'1,t-1})$ and $(\mu_t^{'1}, p^{'1,t-1})$ coincide and the same holds true for financial wealth net of lump sum taxes under $(\mu'_t, p^{'2,t-1})$ and $(\mu_t^{'2}, p^{'2,t-1})$. Politico-economic equivalence therefore is not guaranteed.[27]

Suppose next that governments can commit to debt repayment such that the state is given by $\{z_t b_t^i\}_{i \in \mathcal{I}}$ in the initial regime and by $\{z'_t b_t^{i'}, l_{t-1}^{i'}\}_{i \in \mathcal{I}}$ in the new regime. The admissibility restrictions then change to $\mathcal{P}_t = \{(\tau_{t,t}, z_{t+1}) \in \mathbb{R} \times \mathbb{R}_+\}$ and $\mathcal{P}'_t = \{(\tau'_{t,t}, \tau'_{t-1,t}, z'_{t+1}) \in \mathbb{R}^2 \times \mathbb{R}_+\}$ but Condition 1 iii. continues to be violated. Following a parallel argument as above, a μ'_t can be constructed that satisfies equation (7) in slightly modified form, with $-z_t^{'1} b_t^{i'}$ and $-z_t^{'2} b_t^{i'}$ on the right-hand side of the equations being replaced by $-z'_t b_t^{i'}$ since the repayment rate now is part of the state.

Note that our simplifying assumptions according to which taxes are proportional and the economy does not feature capital are not restrictive as they do not affect the previous arguments. Even if both regimes allowed for a lump sum tax, Condition 1 iii. would still be violated. (Equation (7) would feature a constant on both sides of the equations

[27] Under the assumption of a representative agent (which is of little relevance in the politico-economic context), the state in the initial regime could only take one value, rendering it impossible for a state in the new regime to be associated with more than one state in the initial regime. Condition 1 iii. would therefore be satisfied. Condition 2 would also be satisfied since allocations supported by the policy instruments $(\tau_{s,s}, z_s)$ in the initial regime could be supported in the new regime as well by letting $(\tau'_{s,s}, \tau'_{s-1,s}, z'_s) = (\tau_{s,s}, 0, z_s)$. In contrast, Condition 3 would be violated since in the new regime, negative net transfers could be implemented while this is not possible in the initial regime (where it would require $z_t < 0$). Politico-economic equivalence therefore could still not be guaranteed.

in that case.) The crucial factor undermining Condition 1 iii. is that the tax on lagged income is both non distorting at the time it is levied and a function of a tax base that varies across households. This generates the exchangeability of debt holdings and the tax base which lies at the source of the violation of Condition 1 iii. We conclude that politico-economic equivalence generically fails in environments of the type considered by Bassetto and Kocherlakota (2004). Battaglini and Coate (2008) and Yared (2010) contain politico-economic models of government debt in environments with tax distortions.[28] Our results indicate that the equilibria in these models cannot be re-interpreted as equilibria in models where taxes are additionally raised on lagged income.

6 Conclusions

We have derived general conditions for economic and politico-economic equivalence. We have applied these conditions in the context of well-known models in the literature with the aim to understand why a change of policy regime might matter in politico-economic equilibrium even if policies in these regimes are equivalent from a purely economic point of view.

Exploiting economic equivalence relations, our sufficient conditions for politico-economic equivalence rely on an intuitive comparison of choice sets faced by political decision makers. As these choice sets are constrained by the state on the one hand and the policy instruments under the control of political decision makers on the other, the politico-economic equivalence conditions impose restrictions on the state and policy spaces across policy regimes.

The equivalence conditions provide a powerful tool to analyze politico-economic models of fiscal policy. When applied to an environment with overlapping generations, they identify classes of models with different equivalence properties as far as social security and debt regimes are concerned. In one such class—characterized by minimal household heterogeneity and non distorting taxes—politico-economic equivalence between social security and debt regimes holds independently of particular political aggregation mechanisms. In another class—characterized by sufficient heterogeneity among households and no commitment—the cross-regime state space restrictions are violated and social security and debt regimes generally are not politico-economically equivalent. In a third class, politico-economic equivalence of social security and debt regimes may or may not hold, depending on the political aggregation mechanism in place. In this class, the cross-regime state space restrictions are satisfied but differentially tight admissibility restrictions on policy instruments may undermine equivalence.

These results establish that certain politico-economic theories of social security that have been proposed in the literature may be re-interpreted as politico-economic theories of government debt. Moreover, by identifying factors that undermine politico-economic equivalence, the results can help rationalize why interest groups might favor or oppose the privatization of social security even if from a purely economic point of view, such a

[28]Our analysis can easily accommodate endogenous (or exogenous) government spending as featured in these models.

regime change appears irrelevant. We expect this to prove useful in constructing theories of social security reform, as we intend to do in future research. Finally, the results show that although certain social security regimes in existing models are not politico-economically equivalent to debt regimes, suitably modified versions of the social security regimes can be used to easily characterize politico-economic equilibria in debt regimes.[29]

When applied to environments with tax distortions, our results make clear that an important economic equivalence relation does not extend to the political sphere. From a purely economic point of view, the net present value of distorting taxes on households' budget sets determines the equilibrium allocation and the exact timing of tax collections often is irrelevant. From a politico-economic point of view, in contrast, timing considerations are crucial.

The applicability of our equivalence conditions extends beyond the particular environments we considered and it is not confined to the realm of fiscal policy. Before the background of an appropriately defined equivalence class of policies—be they fiscal, monetary or other—the conditions may be applied to any model featuring an endogenous choice of such policies.

A Appendix

A.1 Non-Fundamental State Variables

In the presence of trigger strategies, the definitions of politico-economic equivalence and associated states need slight adjustment (cf. Definitions 5 and 6):

Definition 8. A state and policy regime with trigger strategies, $(\phi_t, \mathcal{P}, \mathbf{S})$, is *politico-economically equivalent* to another state and policy regime with trigger strategies, $(\phi'_t, \mathcal{P}', \mathbf{S}')$, if

i. $(\phi_t, \mathcal{P}, \mathbf{S})$ supports a politico-economic equilibrium with trigger strategies $\text{PEET}(\phi_t, \mathcal{P}, \mathbf{S})$ with policy choices $p^{\star t-1}$;

ii. $(\phi'_t, \mathcal{P}', \mathbf{S}')$ supports a politico-economic equilibrium with trigger strategies $\text{PEET}(\phi'_t, \mathcal{P}', \mathbf{S}')$ with policy choices $p^{'\star t-1}$;

iii. $(\mu_t, p^{\star t-1})$ is economically equivalent to $(\mu'_t, p^{'\star t-1})$.

Definition 9. For a state $\phi_t \in \Phi_t$ in the policy regime with trigger strategies $(\mathcal{P}, \mathbf{S})$, an *associated state* ϕ'_t in the policy regime with trigger strategies $(\mathcal{P}', \mathbf{S}')$ satisfies

i. ${}_tp'_{\geq t}$ is part of a ${}_tp' \in {}_t\mathcal{Q}'$;

ii. there exists a $p^{'t-1} \in \mathcal{Q}^{'t-1}$ such that $(\mu_t, p^{t-1}(\phi_t))$ is economically equivalent to $(\mu'_t, p^{'t-1})$;

[29] In future work, we intend to pursue this strategy using Gonzalez-Eiras and Niepelt's (2008) model of social security dynamics (with suitably adjusted admissibility restrictions) to understand the dynamics of government debt in politico-economic equilibrium.

iii. $\xi'_t = \xi_t$;

iv. $\mathcal{S}'_t \subseteq \mathcal{Q}'^{t-1}$;

v. $(\mu_t, \mathcal{S}_t)$ is economically equivalent to $(\mu'_t, \mathcal{S}'_t)$.

The set of states in the policy regime with trigger strategies $(\mathcal{P}', \mathbf{S}')$ that are associated with $\phi_t \in \Phi_t$ is denoted $\tilde{\Phi}'_t(\phi_t)$.

The first two requirements in Definition 9 parallel the requirements in Definition 6. The third requirement postulates that the state variables summarizing adherence to proposed policies in the past be identical across regimes. The final two requirements guarantee that the proposed policies allow for the same set of competitive equilibria across regimes. If $\tilde{\Phi}'_t(\phi_t)$ is empty for some ϕ_t, then the policy instruments or the trigger strategy in the new policy regime are not flexible enough to support the equilibrium allocation given μ_t in the initial policy regime, even disregarding numerical restrictions on the instruments.

Conditions 1–3 may now be adjusted accordingly.

Condition 4. The following holds true for all t:

i. $\Phi'_t \subseteq \cup_{\phi_t \in \Phi_t} \tilde{\Phi}'_t(\phi_t)$;

ii. $\tilde{\Phi}'_t(\phi_t) \neq \emptyset$ for all $\phi_t \in \Phi_t$;

iii. if $\phi_t \equiv (\mu_t, \xi_t, \mathcal{S}_t), \hat{\phi}_t \equiv (\hat{\mu}_t, \hat{\xi}_t, \hat{\mathcal{S}}_t) \in \Phi_t$ and $\text{CE}(\mu_t, p^{t-1}(\phi_t)) \neq \text{CE}(\hat{\mu}_t, p^{t-1}(\hat{\phi}_t))$ then $\tilde{\Phi}'_t(\phi_t) \cap \tilde{\Phi}'_t(\hat{\phi}_t) = \emptyset$.

If Condition 4 is satisfied, we can again define an equivalent continuation policy function $\tilde{p}'^{t-1}(\cdot)$ and equivalent policy function $\tilde{p}'_t(\cdot)$ that map the state ϕ'_t into a continuation policy choice $\tilde{p}'^{t-1}(\phi'_t) \in \mathcal{Q}'^{t-1}$ and a policy choice $\tilde{p}'_t(\phi'_t) \in \mathcal{Q}'_t$, respectively. Both functions have domain $\cup_{\phi_t \in \Phi_t} \tilde{\Phi}'_t(\phi_t)$. Moreover, we can define an *equivalent proposed policy function* $\tilde{\mathcal{S}}'_{t+1}(\cdot)$ that maps the state $\phi'_t \in \cup_{\phi_t \in \Phi_t} \tilde{\Phi}'_t(\phi_t)$ into a proposed policy $\tilde{\mathcal{S}}'_{t+1}(\phi'_t) \subseteq \mathcal{Q}'^t$ by letting $\tilde{\mathcal{S}}'_{t+1}(\phi'_t)$ be the set of continuation policies p'^t in the new policy space such that $(\mu_t, \mathcal{S}_{t+1}(\phi_t))$ is economically equivalent to $(\mu'_t, \tilde{\mathcal{S}}'_{t+1}(\phi'_t))$ with $\phi'_t \in \tilde{\Phi}'_t(\phi_t)$. As before, we disregard the possibility that the equivalent continuation policy function or the equivalent policy function are correspondences.

Condition 5 formalizes the requirement that the choice set of political decision makers in the new regime be sufficiently large. Relative to Condition 2, Condition 5 adds the requirement that the equivalent proposed policy be admissible in the new policy regime:

Condition 5. The following holds true for all $\phi'_t \in \Phi'_t$ and all t:

i. $\tilde{p}'_t(\phi'_t) \in \mathcal{P}'_t$ and $\tilde{\mathcal{S}}'_{t+1}(\phi'_t) \in \mathbf{S}'_{t+1}(\mathcal{S}'_t)$.

Condition 6 formalizes the requirement that the choice set not be too large:

Condition 6. The following holds true for all $\phi'_t \in \Phi'_t$ and all t, where $\phi'_t \in \tilde{\Phi}'_t(\phi_t), \phi_t \in \Phi_t$:

i. If there exists a $p'_t \in \mathcal{P}'_t$ and $\mathcal{S}'_{t+1} \in \mathbf{S}'_{t+1}(\mathcal{S}'_t)$ such that $(\mu'_t, (p'_t, \tilde{p}'^t(\phi'_{t+1})))$ supports the competitive equilibrium $\mathrm{CE}(\mu'_t, (p'_t, \tilde{p}'^t(\phi'_{t+1})))$ corresponding with $\phi'_{t+1} = (\mu'_{t+1}, \xi'_{t+1}, \mathcal{S}'_{t+1})$, then there exists a $p_t \in \mathcal{P}_t$ and $\mathcal{S}_{t+1} \in \mathbf{S}_{t+1}(\mathcal{S}_t)$ such that

   (i) $(\mu_t, (p_t, p^t(\phi_{t+1})))$ is economically equivalent to $(\mu'_t, (p'_t, \tilde{p}'^t(\phi'_{t+1})))$ and

   (ii) $(\mu_{t+1}, \mathcal{S}_{t+1})$ is economically equivalent to $(\mu'_{t+1}, \mathcal{S}'_{t+1})$

   where $\phi_{t+1} = (\mu_{t+1}, \xi_{t+1}, \mathcal{S}_{t+1})$ corresponds with the competitive equilibrium $\mathrm{CE}(\mu_t, (p_t, p^t(\phi_{t+1})))$.

Note that under Condition 4, the ϕ_{t+1} and ϕ'_{t+1} in Condition 6 satisfy $\phi'_{t+1} \in \tilde{\Phi}'_{t+1}(\phi_{t+1})$, because of three facts. First, economic equivalence of $(\mu_t, (p_t, p^t(\phi_{t+1})))$ and $(\mu'_t, (p'_t, \tilde{p}'^t(\phi'_{t+1})))$ with $p'_t \in \mathcal{P}'_t$ implies ${}_{t+1}p' \in {}_{t+1}\mathcal{Q}'$ as well as economic equivalence of $(\mu_{t+1}, p^t(\phi_{t+1}))$ and $(\mu'_{t+1}, \tilde{p}'^t(\phi'_{t+1}))$, in parallel to the case without trigger strategies. Second, $\xi_{t+1} = \xi'_{t+1}$ because of $\phi'_t \in \tilde{\Phi}'_t(\phi_t)$ and of economic equivalence of $(\mu_t, (p_t, p^t(\phi_{t+1})))$ and $(\mu'_t, (p'_t, \tilde{p}'^t(\phi'_{t+1})))$ on the one hand and $(\mu_t, \mathcal{S}_t)$ and $(\mu'_t, \mathcal{S}'_t)$ on the other. Third, $(\mu_{t+1}, \mathcal{S}_{t+1})$ is economically equivalent to $(\mu'_{t+1}, \mathcal{S}'_{t+1})$ by Condition 6 (ii).

We can now state the extended equivalence result:

Proposition 3. Consider a state and policy regime with trigger strategies, $(\phi_0, \mathcal{P}, \mathbf{S})$ with $\phi_0 \in \Phi_0$, that support a politico-economic equilibrium with trigger strategies $\mathrm{PEET}(\phi_0, \mathcal{P}, \mathbf{S})$, and consider a new state and policy regime with trigger strategies, $(\phi'_0, \mathcal{P}', \mathbf{S}')$ with $\phi'_0 \in \tilde{\Phi}'_0(\phi_0)$. Suppose that Conditions 4–6 are satisfied. Then, under Assumptions 1–2, $(\phi_0, \mathcal{P}, \mathbf{S})$ is politico-economically equivalent to $(\phi'_0, \mathcal{P}', \mathbf{S}')$.

Proof. We show that there exists a politico-economic equilibrium in the new regime that consists of the policy, continuation policy and proposed policy functions $\{\tilde{p}'_t(\cdot), \tilde{p}'^{t-1}(\cdot), \tilde{\mathcal{S}}'_{t+1}(\cdot)\}_{t\geq 0}$, the policy choices $p'^{\star-1} \equiv \tilde{p}'^{-1}(\phi'_0)$, and the same prices, household choices and production paths as in $\mathrm{PEET}(\mu_0, \mathcal{P}, \mathbf{S})$.

The logic of the proof follows the one of Proposition 2. Assumption 2 and Conditions 4–6 imply that, for all $\phi'_t \in \Phi'_t$ and all t, political decision makers in the new regime implement policy and proposed policy choices according to the policy and proposed policy functions $\tilde{p}'_t(\cdot)$ and $\tilde{\mathcal{S}}'_{t+1}(\cdot)$ respectively, if agents expect the continuation policy function $\tilde{p}'^t(\cdot)$. Such expectations are consistent with equilibrium. For, as noted above, $\phi'_t \in \Phi'_t(\phi_t)$ and implementation of equivalent (proposed) policies implies $\phi'_{t+1} \in \Phi'_{t+1}(\phi_{t+1})$. By induction, the continuation policy functions are consistent with the policy functions governing actual policy choices. Accordingly, the functions $\tilde{p}'_t(\cdot)$, $\tilde{p}'^t(\cdot)$ and $\tilde{\mathcal{S}}'_{t+1}(\cdot)$ satisfy the conditions of politico-economic equilibrium.

Economic equivalence of $(\mu'_0, (\tilde{p}'_0(\phi'_0), \tilde{p}'^0(\phi'_1)))$ and $(\mu_0, (p_0(\phi_0), p^0(\phi_1)))$ (where ϕ'_1 and ϕ_1 correspond with the respective equilibria) implies that the equilibrium policy choices in the new policy regime support the same competitive equilibrium as in the old policy regime. The result then follows. □

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