

Efficient Emissions Reduction*

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Abstract

We propose a simple mechanism providing incentives to reduce harmful emissions to their efficient level without infracting upon productive efficiency. It employs a contest creating incentives among participating nations to simultaneously exert efficient productive and efficient abatement efforts. Participation in the most stylised form of the scheme is voluntary and individually rational. All rules are mutually agreeable and are unanimously adopted if proposed. The scheme balances its budget and requires no principal. The mechanism provides a benchmark result for the cost of the implementation of these desirable properties. In a more realistic setup which could potentially inform policy decisions, we discuss participation enforcement through punishment clauses, exclusive trade agreements, and environmental standards as effective means to discourage free-riding. (JEL *C7, D7, H4, Q5*. Keywords: *Climate policy, Contests, Efficiency*.)

1 Introduction

The disappointing 2009 Copenhagen Accord has highlighted the international impasse in preventing further global warming.¹ Yet immediate action seems to be called for: recent research reports rapid reductions in ice mass balance from both Greenland and Antarctica with a projected sea-level rise of one to two meters by 2100.² Since an estimated 160 million people live currently in locations less than one meter above sea level, the impact of this change on the world economy will be substantial.³ This paper studies and answers two questions arising in this context: *i*) How can incentives be provided to reduce harmful emissions to their socially efficient levels while not infracting upon productive efficiency? *ii*) What would this efficiency cost in terms of our model-GDP?

In our environment, there are two ways to reduce emissions: produce less or abate more. Our mechanism plays on these two aspects in order to achieve efficiency in both. Our answers to the

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¹ The UN draft decision is available at <http://unfccc.int/resource/docs/2009/cop15/eng/107.pdf>.

² See, for instance, Dasgupta, Laplante, Meisner, Wheeler, and Yan (2009) or Allison et al. (2009). Mitrovica, Gomez, and Clark (2009) predict less uniform sea level changes with a rise of up to six meters at some coastal sites in the northern hemisphere.

³ For instance, a recent analysis of migration induced through climate change is Kniveton, Smith, and Black (2012).

above two questions involve a stylised contest among nations, rewarding the countries with the highest abatement efforts with some share of global output. This contest is based on a relative ranking of all nations' abatement efforts. The precision with which this ranking is correct, i.e., the precision of mutual abatement monitoring, is one of the design parameters of our proposed mechanism. Since imperfect monitoring leaves an element of luck to winning, there is a lottery flavour to this contest which is crucial for the results of this paper.

The main type of emissions we have in mind are greenhouse gases. These are widely seen as the main contributing factor to global warming.⁴ Emitted by one country as inherent part of the productive process, they are disseminated around the globe regardless of where they were produced and, as such, present an externality. A reduction of emissions benefits all countries but the costs of such reductions are carried individually. This generates a classic free-rider problem in which each country would like the threat of global warming removed but none is ready to pay the cost.⁵ An example for how we think about productive and reductive (or abatement) efforts is the (simultaneous) investment decision into a power plant's generating capacity and emissions filters.⁶

The environmental literature employs three main approaches to overcome the inefficient emissions abatement problem: command-and-control regulation, quantity-oriented market approaches, and tax-or-pricing regimes. The approach adopted by the 187 signatories of the Kyoto protocol is the quantity-oriented market approach targeting a reduction based on developed countries' emissions in 1990.⁷ The treaty, however, failed to obtain ratification by major players including, most prominently, the United States. Moreover, the concern was expressed that developing countries might have ratified the treaty without the intention of keeping emissions in check. This mars the current emissions reduction reality with the dual frustrations of insufficient participation and diluted objectives.

The prevention of free-riding on the agreement, i.e., not joining the agreement but benefitting from the cleaner environment the agreement brings about without contributing, is crucial for any international environmental agreement. In the simplest negotiations setup where each nation decides simultaneously (and for good) whether or not to join the agreement, achieving participation does not pose problems in our symmetric model. In order to discourage free-riding also under a gradual negotiations policy we use (a combination of) two strategies. A punishment strategy where agreement members can deter free-riding on the agreement by switching to a pre-agreed and inefficient contract which leaves a deserter with less than the participation utility. Alternatively, we explore the ideas of exclusive trade agreements and environmental standards which exclude a deserter from a fraction of trade within the agreement. Both strategies discourage agreement desertion but the second is

⁴ The greenhouse effect is based on an increase of greenhouse gases such as CO₂. These have little effect on incoming sunlight but make the atmosphere more opaque at infrared wavelengths that radiate heat to space. This results in more heat being retained in the atmosphere and the planet to warm.

⁵ One may advocate the view that some countries could climatically benefit from warming. Russian President Vladimir Putin, for instance, is reported to have said that climate change might be good for his country as people would no longer need to buy fur coats (Reuters, 2-April-07). The impact on the world economy and consequences in terms of migration, however, make us pessimistic about the likelihood of emerging net beneficiaries.

⁶ See, for instance, the investments of Brandon Shores generation station in emissions reduction documented in Maryland Department of Natural Resources (2007). Other well-publicised examples include design tradeoffs between engine thrust and emissions in Boeing's 787 Dreamliner or Airbus Industry's A380 aircrafts.

⁷ For details, see Barrett (1998) and Nordhaus (2006).

more useful in encouraging the participation of individual nations after the agreement is formed by a core group. In reality, of course, reaching agreement on and committing to the (structuring of the) necessary transfers and the exact specification of the contest still presents a formidable task.⁸

1.1 Related literature

The idea behind our efficiency result stems from Gershkov, Li, and Schweinzer (2009) who analyse the efficient effort choice in team-partnership problems.⁹ They construct a contest mechanism where total partnership output is used as a remuneration pool for the partners. The intuition is that, through a relative ranking of efforts, the highest-effort partner expects the biggest share of joint output. Therefore, incentives are generated to exert high efforts which offset the free-riding problem inherent in the partnership. Contrasting to our paper, their partnership setup has only one effort dimension and, consequently, there is only one free-riding problem. The emissions reduction problem analysed in the present paper has two such problems: overproduction of harmful emissions and underprovision of reductive efforts. Moreover, the mechanism in Gershkov, Li, and Schweinzer (2009) is only capable of implementing efficiency along a single dimension while ours needs to balance two effort dimensions.

The most directly related studies of the public goods provision problem relating to contests that we are aware of are Morgan (2000), Goeree, Maasland, Onderstal, and Turner (2005), and Giebe and Schweinzer (2011). Morgan (2000) studies a lottery mechanism which uses proceeds obtained from ticket sales for the provision of a public good. Contrasting with our analysis, he is neither concerned with designing a mechanism to implement exact efficiency nor with balancing the mechanism's budget. Using a fixed precision contest, he obtains the result that contests are unable to implement exact efficiency. Goeree, Maasland, Onderstal, and Turner (2005) derive the optimal fund-raising mechanism among the class of all-pay auctions. Again, they are not concerned with efficiency but with the best way of raising funds in a charity context. Giebe and Schweinzer (2011) explore the possibility of the efficient provision of a public good through non-distortionary taxation of a private good which is linked to a lottery. By fine tuning the tax with the lottery, they are able to get efficient consumption levels for both private and public goods. This is in the same spirit as the present analysis, but their individual public good contribution is only a function of the private good consumption and not at the individual's discretion. Moreover, our environmental model needs to balance two dimensions of inefficiency: excessive production and insufficient reduction of emissions. This is impossible in their single-dimensional model where tailoring only private goods

⁸ As we view the main purpose of this paper as implementing efficiency we only confront a stylized version of the participation problem. For a discussion of some of the involved difficulties see, for instance, Wagner (2002) or Liverman (2009). For a recent critique from the political science point of view see Biermann, Pattberg, van Asselt, and Zelli (2009) and the references therein. For a solution to the participation problem in a framework similar to this paper, see Bos and Schweinzer (2012) who introduce income uncertainty in order to derive an insurance property of the redistributive contest mechanism developed in this paper.

⁹ The idea that in many circumstances efficient efforts can be induced by awarding a prize on the basis of a rank order among competitors' efforts is due to Lazear and Rosen (1981). This insight has found numerous applications and extensions, for instance in the work of Green and Stokey (1983), Nalebuff and Stiglitz (1983), Dixit (1987), Moldovanu and Sela (2001), or Siegel (2009). For a detailed survey of the contests literature see the comprehensive Konrad (2008).

consumption can lead to efficiency. In a paper not relating to contests, Buchholz, Cornes, and Rübbelke (2011) study the existence of equilibria in an aggregative game on which they impose the efficient allocation. In particular, they analyse a matching mechanism in which agents cross-subsidise each other in order to achieve efficiency and study the question which income distributions can be compatible with voluntary and efficient provision of the public good. Contrasting the present analysis, the authors do not explore the design of explicit mechanisms capable of providing incentives for the implementation of this efficient matching equilibrium.

Unrelated to contest mechanisms, Gerber and Wichardt (2009) develop a mechanism that results in the efficient provision of a public good. In their mechanism, countries first commit to paying a fixed fee and then choose their abatement effort. Once a country is committed, its utility maximising strategy is to provide efficient abatement effort. More specifically, a global fund is created with the collected fixed fees and redistributed among countries according to their public good provision levels. If the public good is provided inefficiently, a country is punished by losing the committed funds contained in the global fund. In an intertemporal setup, Gersbach and Winkler (2007) design a global refunding scheme and analyse its potential for mitigating climate change. As we show in the discussion of our benchmark efficiency case, such a tax-based system cannot implement efficiency in general. In an emissions permit trading model, Gersbach and Winkler (2011) develop a model designed to limit free-riding in the form of countries allocating inefficiently many permits. In their model, part of the permits are allocated for free, whereas part is allocated through an auction. The money raised through the auction goes into a global fund and is used to reimburse participating countries according to some previously negotiated share. If a country issues too many permits, then the auction leads to a low permit price, whereas if the country emits few permits, the price of the permits goes up and so does the revenue from their reimbursed share of the global fund. Although we share important ideas with all three papers, neither paper incorporates a negotiation stage or models productive efforts and therefore cannot consider the issue of efficiently balancing polluting overproduction with reductive efforts.

In the environmental literature, our team setup is vindicated by the universally accepted property of international environmental agreements (IEA) to be self-enforcing. Indeed, there is no supranational principal to enforce such arrangements between countries. Nevertheless, self-enforcement is typically not achieved in the IEA literature.¹⁰ The main contributions have found that IEA are either unlikely to consist of many participants, or if they do, are similarly unlikely to produce substantial benefits. In particular, Diamantoudi and Sartzetakis (2006) show that no more than four countries will find it profitable to form a coalition regardless of the number of countries participating in the negotiations. Kolstad (2007) demonstrates that the size of IEAs decrease as uncertainty grows. Besides, the outcome of non-cooperative coalition formation games depends on the specific membership rules imposed on an IEA. For example, Carraro, Marchiori, and Oreffice (2009) show that the introduction of a minimum participation rule increases the number of signatories.¹¹ In a framework similar to this paper, Bos and Schweinzer (2012) introduce stochastic income in order to derive

¹⁰ See Barrett (1994), Barrett (2003), Finus (2008), and Guesnerie and Tulkens (2009) for the main results and further discussion.

¹¹ Chander and Tulkens (2006) show that, typically, the involved contracts are not renegotiation proof.

an insurance property of the redistributive contest mechanism. They show that, for sufficiently income-shock-averse players, this motive can assure participation.

Barrett (2006) studies an alternative to the Kyoto protocol in proposing a system of two treaties, one promoting cooperative 'breakthrough' R&D investments and the other encouraging collective adoption of new technologies emerging from this R&D activity. This solution improves participation but may not be cost-effective. A recent paper which does achieve efficient resource allocations in some cases is Ogawa and Wildasin (2009). They use increased local taxation to encourage the migration of polluters (and pollution) to neighbouring constituencies to achieve efficiency. Recent contributions to the literature on IEA-membership dynamics include Rubio and Ulph (2007), Breton, Sbragia, and Zaccour (2010) and Harstad (2010). Particularly referring to climate change agreements, Harstad (2010) shows how short-term agreements may have adverse effects on countries' investments in green technology. Indeed, as Buchholz and Konrad (1994) and Beccherle and Tirole (2010) point out, anticipating negotiations can decrease the level of R&D and green investments.¹²

Our key contribution is to show that a contest can implement efficiency in a specific environmental model along both productive and reductive effort dimensions. We provide a benchmark result showing which model-resources it takes to implement this first-best solution under the following objectives: absence of principal, efficiency in both production and abatement efforts, no distortions, and budget-balancing. This is a theoretical result. Even if the highly stylised mechanism we discuss may seem unrealistic and difficult to implement directly, our analysis delivers new and significant insights on the available options, on the cost of abatement, and on enforcement policies. Nevertheless, we argue that our proposed mechanism requires fewer institutions for collecting the funds, paying out prizes, and gathering information about relative performance than the main competitors described above. Agreement should be easier to reach as countries need to agree only on the three components of the proposed mechanism: how much to contribute, the contest prizes and the monitoring technology.

Following the model definition in section 2, we present the main idea of our mechanism through an illustrative example in section 3. Although stylised, this simple example conveys much of the intuition of the general results presented in section 4, many of which are, again, complemented by examples. Participation, the asymmetric case, an alternative family of success functions, exclusive trade agreements, and comparative statics are examined in section 5. All proofs can be found in the appendix.

¹² Many environmental papers employ contests to model lobbying activities; see, for example, Hurley and Shogren (1997), Heayes (1997), or, more recently, Kotchen and Salant (2011) and the references therein. The literature on environmental contest modelling of abatement incentives is, nevertheless, small. The only paper that we are aware of is Dijkstra (2007). He is interested in the time (in)consistency of environmental policy under imperfect government commitment and is not concerned with implementing efficiency.

2 The symmetric model

There is a set \mathcal{N} of $n \geq 2$ risk-neutral players. These players are symmetric in the basic model.¹³ Each player $i \in \mathcal{N}$ exerts efforts along two dimensions: productive effort $e_i \in [0, \infty]$ and reductive (abatement) effort $f_i \in [0, \infty]$. The reductive efforts need not, in principle, be verifiable.¹⁴ We denote the full effort vectors by $\mathbf{e} = e_1, \dots, e_n$ and $\mathbf{f} = f_1, \dots, f_n$, respectively. The effort costs $c_e(e_i)$ and $c_f(f_i)$ are assumed to be strictly convex and zero for zero effort. Productive efforts generate strictly concave individual gains of $y(e_i)$ and cause strictly convex global emissions of $m(\max\{0, \sum_h e_h - \sum_h f_h\})$ —only depending on the difference between global productive and reductive efforts—of which player i suffers a known share s_i .¹⁵ Emissions are seen as an externality, an inseparable by-product (or factor) of production. In the simplest interpretation, reductive efforts are modelled as ‘end of pipe’ because there is no productive element to f .¹⁶ We interpret $m(\cdot)$ as physical pollution and therefore assume that $\sum_h s_h = 1$ which introduces a public bad team problem.¹⁷

As means to alleviate this problem we introduce an incentive system based on a ranking of individual reductive efforts and award the top-ranked players prizes. The total prize pool (from which these prizes are taken) is defined as the sum of fraction $(1-\alpha)$ of each participant’s individual income or output $y(e_i)$. Thus, the incentive mechanism redistributes income and its budget balances by definition. The incentive mechanism awards $\beta^1 P$ to the winner, $\beta^2 P$ to the player coming second, and so on, with $\sum_h \beta^h = 1$.

We assume that some noisy (partial) ranking of the players’ reductive efforts is observe- and verifiable. We interpret this ranking technology as arising from the agreement’s monitoring of mutual abatement efforts. It is part of the mechanism the players need to agree on and gives player i ’s probability $p_i^h(\mathbf{f})$ of being awarded prize h as a function of the imperfectly monitored reductive efforts of all participants. We assume that $p_i(\mathbf{f})$ is strictly increasing in f_i , strictly decreasing in all other arguments, equal to $1/n$ for identical arguments, twice continuously differentiable, and zero for

¹³ Our main results apply to the simple symmetric setting. Subsection 4.3 generalises the model to the asymmetric case; its workings are illustrated in several examples in subsections 4.3 and 5.2.

¹⁴ We view the ranking probabilities as generated by some automaton or monitoring device (see also footnote 18). While agreement on this machine and verifiability of its readings are indispensable, the underlying efforts themselves need neither be observe- nor verifiable.

¹⁵ Requiring non-negative differences in the damage function $m(\cdot)$ ensures that reductive efforts cannot substitute productive efforts. Since this requirement is fulfilled for most of our analysis, we redefine $m := m(\max\{0, \sum_h e_h - \sum_h f_h\})$ and only make the non-negative argument explicit when necessary.

¹⁶ In general, we can think of $c(f_i)$ as the cost of moving from the status quo to the targeted level of GHG emissions. Stern (2006) argues similarly that the cost of stabilising GHG emission will be at about 1% of GDP per year compared to business-as-usual: abatement is seen as a cost to the productive process. In the non-separable case, productive abatement has a benefit of, so to speak, $\tilde{y}(f)$. But ignoring this benefit makes our problem harder to solve and therefore our model errs on the side of caution.

¹⁷ In principle, our results also apply to the more general case of $0 < S = \sum_h s_h < n$ in which the individual shares s_i could be interpreted as, e.g., perceived pollution. Depending on this precise interpretation the planner’s objective (1) may then change to $\sum_i (y(e_i) - c_e(f_i) - c_f(f_i)) - Sm(\sum_i (e_i - f_i))$.

$f_i = 0$ with at least one $f_{j \neq i} > 0$, $j \in \mathcal{N}$.¹⁸

Given a ranking $p(\mathbf{f})$, a (subgame perfect) equilibrium in this contest game consists of two elements: a pair of sharing rules (α, β) specifying the prizes in the reduction tournament and a pair of efforts (e, f) determining output and the winning probabilities. Since we are implementing efficiency we are looking for a symmetric equilibrium in pure strategies.¹⁹

2.1 Timing and agreement participation mechanism

Since the players' expected payoffs are symmetric, we can think of a simple proposal game in which the design parameters $\langle \alpha, \beta, p(\mathbf{f}) \rangle$ are proposed by one player and the game is played if and only if all others simultaneously agree to the proposed parameters. The equilibrium of such a game is subgame perfect. In order to allow for more realistic, gradual negotiations, our design is slightly more involved: we propose a two-stage mechanism at the first stage of which an arbitrary player (called player 1) is chosen to propose the two balanced budget contracts $C = \langle \alpha, \beta, p(\mathbf{f}) \rangle$ and $C' = \langle \alpha', \beta', p(\mathbf{f})' \rangle$. The first contract C is invoked if all players agree to participate in the agreement. It implements efficient efforts and is subgame perfect. The second contract C' is invoked by the agreeing players if at least one player fails to participate and implements inefficient efforts which successfully deter non-agreement.²⁰

More precisely, at the first stage of the game, if all players accept C , then the contest specified by C is set up, players commit their output shares $(1 - \alpha)y(e_i)$ and the game proceeds to the next stage. If at least one player rejects C , the agreeing players form a residual agreement, implement C' and again proceed to the second stage. If less than two players agree to setting up the mechanism (C, C') , then the game ends and each player obtains their individual utility without agreement. At the second stage, conditional on the formation of an agreement, players choose their efforts simultaneously to maximise own expected utilities. The noisy ranking of reductive efforts specifies a winner, second, etc, final output realises, and the prize pool is redistributed to the winner, second, etc, according to the contract specified by C , or C' , respectively.

One of the main stumbling blocks for IEAs is the participants' commitment. Countries may sign the agreement but no supranational entity exists which punishes defection. Thus, countries can always choose inefficient efforts and free-ride once the agreement is signed by other countries. In

¹⁸ Since this contest success function is general, the reductive efforts determining the contest outcome can be easily normalised with respect to, for instance, the individual (perceived) emission consumption share s_i . As usual, this ranking technology can be interpreted as monitoring technology, i.e., the slope of the function can be determined, e.g., by the frequency of inspections or the design of surveillance equipment. From a design point of view, the underlying assumption is that higher monitoring precision comes at a higher cost; infinite precision is not attainable. The inclusion of some monitoring cost ω financed out of the prize pool $P = (\beta, 1 - \beta - \omega, \omega)$ is straightforward and does not qualitatively change any of our results.

¹⁹ Especially in the more complicated model variants discussed in the extensions, there may well be other (mixed) equilibria, perhaps of an asymmetric nature, which we disregard for the present analysis. The reason is that they can never implement the efficient allocation.

²⁰ Formally, this second problem is equivalent to allowing a signatory to exit the agreement (i.e., renege on his commitments) after the agreement is formed. As pointed out by Chander and Tulken (2006), this contract will typically not be renegotiation proof and commitment to C' is crucial. We discuss other enforcement measures in section 5.1 but reiterate that the focus of this paper is on achieving first-best outcomes, not the participation game. For a general analysis of the latter, we refer to Bos and Schweinzer (2012).

our setup, both free-riding inside the agreement and non-participation are effectively discouraged. Since a participant has to commit $1 - \alpha$ of GDP through signing the agreement, it is optimal for her to choose efficient efforts inside the agreement (by the design of the mechanism). Non-participation in the agreement can be discouraged by either the simple simultaneous agreement game described above, or the threat of agreement members to implement C' . It is easy to see that such a sufficiently strong punishment contract C' always exists: setting $C' = \langle \alpha' = 1, \cdot, \cdot \rangle$ replicates the pre-agreement scenario in which all players are worse off than with an agreement.²¹ This extreme form of punishment, however, will typically not be necessary. As illustrated in subsection 4.2, a second-best contract C' will generally be able to implement higher levels of abatement than those materialising absent an IEA.²²

3 Efficiency

Much of the economics behind our results can be understood from the simple symmetric two players case on which the main body of the paper rests. For this two-player setup, we label players as i, j with $i = 1, 2$ and $j = 3 - i$. We define the efficient levels of both productive and reductive efforts (e^*, f^*) as those maximising social welfare absent of incentive aspects

$$\begin{aligned} \max_{(e, f)} u(\mathbf{e}, \mathbf{f}) &= 2y(e) - m(2e - 2f) - 2c_e(e) - 2c_f(f) \\ &\Leftrightarrow \begin{cases} y'(e^*) = m'(2e^* - 2f^*) + c'_e(e^*), \\ m'(2e^* - 2f^*) = c'_f(f^*). \end{cases} \end{aligned} \quad (1)$$

(The expressions following the curly bracket are the resulting necessary first-order conditions for optimality.) In the absence of an incentive scheme, a player $i = 1, 2$ individually maximises

$$y(e_i) - s_i m(e_i + e_j - f_i - f_j) - c_e(e_i) - c_f(f_i) \Leftrightarrow \begin{cases} y'(e^*) = s_i m'(2e^* - 2f^*) + c'_e(e^*, f^*), \\ s_i m'(2e^* - 2f^*) = c'_f(e^*, f^*). \end{cases} \quad (2)$$

where s_i is player i 's local share of global emissions. Since at any particular point in time, a kilogram of GHG must be somewhere on (or over) the globe, $s_i + s_j = 1$ and the individual first-order conditions in (2) cannot both equal those in (1). In order to overcome this inefficiency in *both* dimensions, we introduce an endogenised rank-order emissions reduction reward scheme, i.e., a contest. We ask each participating nation to commit to contributing a share $(1 - \alpha)$ of their individual output $y(e_i)$ to the mechanism and therefore form a pool of prize money of size $P = (1 - \alpha)(y(e_i) + y(e_j))$. In a contest specifying player i 's winning probability as $p_i(\mathbf{f})$ based on both players' reductive efforts, we want to assign βP to the winner and $(1 - \beta)P$ to the player coming second. Notice that such a mechanism redistributes income. The individual problem under

²¹ For a detailed study of how punishments can be used to force agreement, see Chander and Tulkens (1995).

²² Designing C' just sufficiently bad to serve as a deterrent resembles the idea of γ -core stability in Chander (2007). An alternative way of deterring this kind of free-riding on the agreement is to grant most favoured 'green' trading terms only to participating nations. Both ideas are further explored in subsection 5.1.

our incentive mechanism therefore is

$$\max_{(e_i, f_i)} \underbrace{\alpha y(e_i)}_{\text{retained output}} + \underbrace{p_i^h(\mathbf{f})\beta^h P}_{\text{first prize}} + \underbrace{(1 - p_i^h(\mathbf{f}))(1 - \beta^h)P}_{\text{second prize}} - \underbrace{s_i m(e_i + e_j - f_i - f_j)}_{\text{damage from emissions}} - \underbrace{(c_e(e_i) + c_f(f_i))}_{\text{effort costs}}.$$

We define individual rationality as the requirement that the utility from participating in this mechanism for appropriately chosen $\langle \alpha, \beta, p(\mathbf{f}) \rangle$ exceeds *i*) the utility from non-formation of the agreement (*ii*) of free-riding on the others' reductive efforts *within* the agreement and *iii*) on free-riding on the others' reductive efforts *outside* the agreement. In the first case, no agreement exists at all while in the third case, an agreement outsider benefits from the reductive efforts of the agreement members. The second case concerns an agreement member's inefficient effort provision with committed output share.

3.1 Example of the efficient mechanism

We now use a simple, symmetric example with quadratic costs and square root production function to demonstrate the basic idea of our mechanism.²³ In this setup, a benevolent planner maximising the sum of social utility net of total cost (1) would want to maximise the objective

$$\max_{(e, f)} 2e^{1/2} - (2e - 2f)^2 - 2(e^2 + f^2) \Leftrightarrow \begin{cases} e^* \approx 0.2823, \\ f^* \approx 0.1882. \end{cases} \quad (3)$$

The corresponding individual problem (in the absence of an incentive mechanism) leads to inefficient provision of efforts

$$\max_{(e_i, f_i)} e_i^{1/2} - s_i(e_i + e_j - f_i - f_j)^2 - (e_i^2 + f_i^2) \Leftrightarrow \begin{cases} \hat{e}_i \approx 0.3029 > e^*, \\ \hat{f}_i \approx 0.1514 < f^*, \end{cases} \quad (4)$$

for symmetric damage shares $s_1 = s_2 = 1/2$. Notice that, with respect to the efficient efforts, the combined externality and free-riding inherent in the problem imply that players both produce too much and abate too little.

For our incentive agreement we assume in the present example that the probability of winning the reduction award is given by the (generalised) Tullock success function specifying a player's probability of winning as that player's effort over the total sum of efforts.²⁴ The prize pool which we collect for incentive purposes is $P = (1 - \alpha)(e_i^{1/2} + e_j^{1/2})$. Then, an individual's problem under the incentive scheme is

$$\max_{(e_i, f_i)} \alpha e_i^{1/2} + \frac{f_i^r}{f_i^r + f_j^r} \beta P + \frac{f_j^r}{f_i^r + f_j^r} (1 - \beta) P - s_i(e_i + e_j - f_i - f_j)^2 - (e_i^2 + f_i^2) \quad (5)$$

²³ We will return to this example setup throughout the paper to illustrate further ideas and extensions.

²⁴ Under a Tullock contest success function, the contestant with the highest effort does not necessarily win the prize. Hence the resulting ranking has occasionally been referred to as 'non-fully discriminatory,' 'non-deterministic,' 'noisy,' or 'fuzzy.' Our interpretation is that the ranking is inexact in the sense that the monitoring technology it is based on is not perfect. The Tullock (or Logit) form has been axiomatized by Skaperdas (1996) and follows naturally from micro-foundations à la Fu and Lu (2007) or Jia (2008).

for some exponent $r > 0$ specifying the precision with which the ranking selects the highest reduction effort nation among the set of competitors.²⁵ We interpret this exponent as the accuracy with which the agreement monitors the emissions reduction efforts of its members. Upon maximisation, this gives the set of simultaneous first-order conditions as

$$16e_i = 8f_i + \frac{1 + \alpha}{\sqrt{e_i}}, \quad 2e_i = 4f_i + \frac{\sqrt{e_i}r(\alpha - 1)(2\beta - 1)}{2f_i}. \quad (6)$$

We again invoke the fact that we assume that symmetric nations are identical (as we did before for the planner) and thus set $e = e_1 = e_2$, $f = f_1 = f_2$, with $s_i = 1/2$. We then simply force the resulting efforts in line with the efficient efforts by imposing $e = e^*$ and $f = f^*$ from (3) and solve (6) for the efficiency inducing design parameters $\langle \alpha, \beta, r \rangle$

$$\alpha^* = \frac{3}{5}, \quad \beta^* = \frac{1}{2} + \frac{1}{6r}. \quad (7)$$

As β^* depends on the precision of the monitoring technology r , the rewards scheme—and in particular the relative size of the prizes paid to the winner and loser given by β —can be designed as seen fit and compatible with the chosen monitoring technology.²⁶ The mechanism satisfies $\beta \in [\frac{1}{2}, 1]$ if $r \geq 1/3$, implying that the losing nation needs never pay more than the committed share $1 - \alpha$. Figure 1 shows that participating in the contest gives higher utility than staying out and free-riding on the other's effort. It confirms $(\alpha^*, \beta^*, e^*, f^*)$ as equilibrium in pure strategies with full participation (on an appropriately chosen plot-range outside of which utility is negative).

The economics behind this result is simple: An increase in productive efforts e_i causes individual output $y(e_i)$ —and, hence, the prize pool P —as well as global pollution $m(\sum_h e_h - \sum_h f_h)$ to rise. Of these, the player retains shares α and s_i , respectively. An increase in reductive efforts f_i enlarges the player's chance to win the prize share β in the reduction contest (while decreasing the competitors' chances) and simultaneously decreases global pollution. Trading off α against β allows us to fine-tune efforts to their efficient levels.

To get a feeling for the magnitudes implied by our mechanism we plug in the 2008 global GDP of \$61tr, or, among two identical players, \$30.5tr GDP per player. Thus, our proposed mechanism collects $(2/5)61 = \$24.4\text{tr}$ or \$12.2tr from each player. The following table lists the redistribution implied by the efficient mechanism. Depending on the precision of monitoring r , it pays out

²⁵ The particular monitoring technology is not very important as we generalise over the set of applicable success functions in section 5.4. What is important is that the success function incorporates enough randomness in its outcome. If the ranking is too precise (as is the case with the all-pay auction—which can be viewed as the $r = \infty$ limit-case of the Tullock function) then equilibria in pure strategies typically fail to exist. This would be problematic as our contest strives to implement the efficient pure effort choices.

²⁶ There are well-known existence issues with symmetric pure strategy equilibrium with $r > 2$ in rent-seeking contests (see, e.g., Schweinzer and Segev (2011)) but, as shown in proposition 2, these do not apply with the same severity to our problem where costs are convex and the prize pool is endogenous.

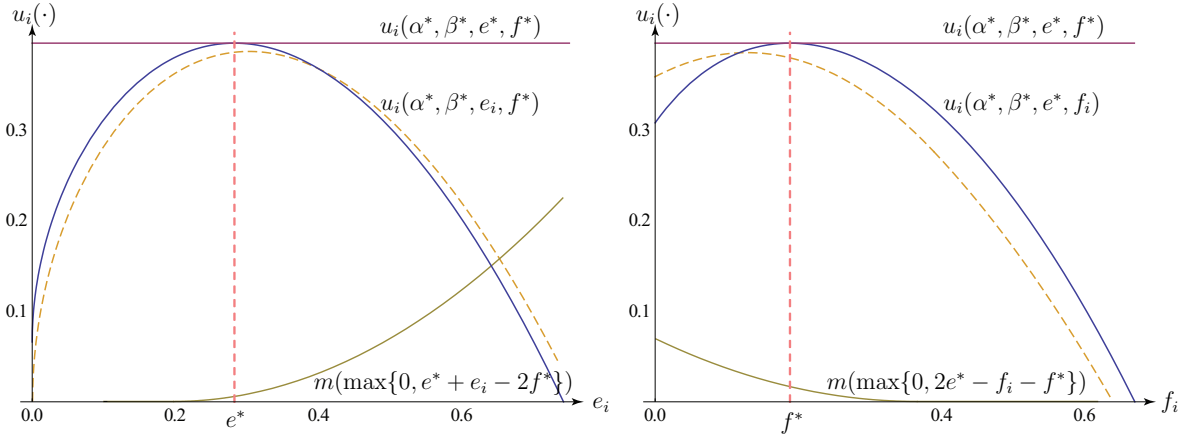


Figure 1: The top, horizontal line is the equilibrium utility from $(\alpha^*, \beta^*, e^*, f^*)$ implementing efficient efforts e^* and f^* . The curves below show the utility from unilaterally deviating in either effort dimension. Notice the positive utility from free-riding at zero efforts. The dashed curves give the (outside) utility from no agreement formation exhibiting both overproduction in e_i and underprovision of abatement f_i relative to the socially efficient levels.

r	β	$1 - \beta$	1 st	2 nd	transfer	%
1	$2/3$	$1/3$	\$16.3tr	\$8.1tr	$\pm\$4.1tr$	$\pm 13.3\%$
2	$7/12$	$5/12$	\$14.2tr	\$10.2tr	$\pm\$2.0tr$	$\pm 6.7\%$
.
5	$8/15$	$7/15$	\$13.0tr	\$11.4tr	$\pm\$0.8tr$	$\pm 2.7\%$
.
11	$17/33$	$16/33$	\$12.6tr	\$11.8tr	$\pm\$0.4tr$	$\pm 1.1\%$

where the rightmost column's percentages are taken from the total symmetric \$30.5tr GDP. Our sufficient existence condition derived in proposition 2 guarantees existence up to and including $r = 5$. Actual existence, however, is only lost for $r > 11$. At this monitoring level, a transfer of 1.1% of GDP is sufficient to implement both efficient abatement and efficient production.

4 Main results

4.1 Equilibrium characterisation and existence

We begin by characterising the design parameters which induce efficiency in our model. Recall that under the contest scheme, an individual participant $i = 1, 2$ chooses a pair of efforts (e_i, f_i) to

$$\max_{(e_i, f_i)} \alpha y(e_i) + p(\mathbf{f})\beta P + (1 - p(\mathbf{f}))(1 - \beta)P - s_i m(e_i + e_j - f_i - f_j) - c_e(e_i) - c_f(f_i) \quad (8)$$

where $p(\mathbf{f})$ is the probability of coming first in a ranking of reductive efforts f and the prize pool is $P = (1 - \alpha)(y(e_1) + y(e_2))$. We require that $y' > 0$, $y'' < 0$, $m' > 0$, $m'' > 0$, and $c'_{1,2} > 0$, $c''_{1,2} > 0$. We moreover assume that $m(\cdot)$ only depends on the difference of total productive minus reductive efforts. Taking derivatives with respect to both effort types, we obtain the simultaneous

pair of first-order conditions defining individually optimal efforts (e_i, f_i) as

$$\begin{aligned} c_e(e_i, f_i) + s_i m'(e_i + e_j - f_i - f_j) &= (1 - \beta + \alpha\beta + (1 - \alpha)(2\beta - 1)p(f_i, f_j))y'(e_i), \\ c_f(e_i, f_i) + (\alpha - 1)(2\beta - 1)(y(e_i) + y(e_j))p'(f_i, f_j) &= s_i m'(e_i + e_j - f_i - f_j). \end{aligned} \quad (9)$$

Assuming that a symmetric equilibrium $e = e_i = e_j$, $f = f_i = f_j$, $s_i = 1/2$ exists, this simplifies to

$$\begin{aligned} 2c_e(e, f) + m'(2e - 2f) &= (\alpha + 1)y'(e), \\ 2c_f(e, f) - m'(2e - 2f) &= 4(1 - \alpha)(2\beta - 1)p'(f, f)y(e). \end{aligned} \quad (10)$$

Equating these efforts to the efficient efforts e^* , f^* resulting from the solution to the social planner's problem in (1), we obtain

$$4p'(\mathbf{f}^*)(2\beta - 1) = \frac{y'(e^*)}{y(e^*)} \Leftrightarrow \begin{cases} c_e(e^*, f^*) = \alpha y'(e^*), \\ c_f(e^*, f^*) = 4(1 - \alpha)(2\beta - 1)p'(\mathbf{f}^*)y(e^*) \end{cases} \quad (11)$$

where $\mathbf{f}^* = (f^*, f^*)$. We know from (1) that there exists an $\alpha \in [0, 1]$ to satisfy the first equation. Substituting this α into the second equation determines $\beta \in [1/2, 1]$ for a suitably chosen ranking $p(\cdot)$. Without further restrictions on the design parameters—and in particular the slope of the ranking technology $p(\cdot)$ in equilibrium—the set of necessary conditions in (11) can always be satisfied. Taking equilibrium existence as given (until we verify it in proposition 2), the following proposition establishes the precise criteria on the parameters for both productive and reductive efficiency to obtain simultaneously for any number of players $n \geq 2$. In all following results we employ the simple prize structure $\beta = \left(\beta^1, \frac{1-\beta^1}{n-1}, \dots, \frac{1-\beta^1}{n-1}\right)$ assigning a single winning prize and another prize to all losers. This is not necessary but simplifies the exposition considerably.

Proposition 1. *For appropriately chosen $\langle \alpha, \beta, p(\mathbf{f}) \rangle$ and $P = (1 - \alpha) \sum_j y(e_j)$, player $i \in \mathcal{N}$ chooses efficient productive as well as reductive efforts (e^*, f^*) in*

$$\max_{(e_i, f_i)} \alpha y(e_i) + \sum_h (\beta^h p^h(\mathbf{f})P) - s_i m\left(\sum_h (e_h - f_h)\right) - c_e(e_i) - c_f(f_i). \quad (12)$$

The proof can be found in the appendix and follows the same intuition as described in the example. It is straightforward to show that proposing $C = \langle \alpha, \beta, p(\mathbf{f}) \rangle$ at the first stage of the game maximises player 1's expected utility, given that players' efforts are functions of the proposed mechanism $(e(\alpha, \beta, p), f(\alpha, \beta, p))$. This is unsurprising as payoffs are symmetric and what maximises welfare must also maximise the proposer's utility.²⁷

A consequence of this first result is that full efficiency in the symmetric n -player model can be obtained with just two different prizes: one for the winner and another for everyone else. As one only needs to check for a winning 'abatement-champion,' such a scheme is easy to monitor. Since the general objective (12) is not necessarily well-behaved without further assumptions on $p(\cdot)$, we proceed to show that equilibria exist for the subclass of problems governed by the Tullock success

²⁷ The design of the proposal stage is less straightforward in the asymmetric case.

function $p_i(f) = f_i^r / (f_1^r + \dots + f_n^r)$. Depending on the exponent r , this function may be first convex and then concave. So, again, the underlying optimisation problem is non-concave.

Proposition 2. *The provision of efficient efforts (e^*, f^*) is globally optimal provided that reduction costs are sufficiently convex. In particular, the sufficient threshold (40) ensures the existence of a symmetric pure strategy equilibrium for contests governed by the Tullock success function.*

The proof of this proposition establishes a sufficient condition for quasi-concavity of the players' objective. Therefore, as long as this condition is respected, an equilibrium which implements the efficient efforts characterised in proposition 1 is certain to exist. If the effort cost of abatement are insufficiently convex or, equivalently, the equilibrium monitoring precision desired from the ranking technology $p(\mathbf{f})$ is too high, then pure strategy equilibrium fails to exist. In that case, giving up the simple 'flat-looser' prize structure $\beta = \left(\beta^1, \frac{1-\beta^1}{n-1}, \dots, \frac{1-\beta^1}{n-1}\right)$ in favour of a structure which awards multiple first prizes $\beta' = (\beta^1 = \beta^2 \geq \dots \geq \beta^n)$ eases the problem at the expense of the implemented abatement efforts.²⁸

4.2 Participation

Equilibrium existence implies that free-riding on the reductive effort is not attractive once a nation is committed to the agreement.²⁹ As the number of participants in the mechanism n goes up, the utility from free-riding on the agreement increases as the disutility from pollution $m(\sum_h (e_h - f_h))$ approaches the efficient level. Hence, the only leverage left in the efficient contract C is the contest on the pre-committed output share of $(1 - \alpha)$ —which is generally not sufficient to deter free-riding once an agreement is in place. The alternative contract C' is, however, capable of eradicating all gains from free-riding on the agreement by—in its most extreme form—replicating non-agreement pollution levels.

Proposition 3. *Participation in the mechanism specifying the pair of contracts $C = \langle \alpha^*, \beta^*, p^*(\mathbf{f}) \rangle$ determined through (12) and $C' = \langle \alpha' = 1, \beta' = 1/2, \cdot \rangle$ is individually rational in the sense that the utility from free-riding efforts e^s, f^s on C' cannot exceed the utility obtained when agreeing to C*

$$y(e_i^s) - s_i m(e_i^s + (n-1)e'(\alpha', \beta', \cdot) - f_i^s - (n-1)f'(\alpha', \beta', \cdot)) - c_e(e_i^s) - c_f(f_i^s) \leq u_i(e^*, f^*). \quad (\text{IR})$$

This result is intuitive as the efficient allocation maximises welfare and the agreement will therefore always be formed. As participation in the agreement is individually rational, free-riding is fully deterred. Off the equilibrium path, the second-best contract C' —which is implemented if at least one player fails to participate—will generally still allow substantial emissions reductions. The following example shows that the agreement's raison d'être needs not necessarily be surrendered to holdup attempts.

²⁸ It is easy to see why this is the case: an equilibrium where every player gets the same prize must exist (with zero abatement efforts). By continuity, equilibria (with small reductive efforts) will exist under a prize structure which gives the same prize to every player except the one coming last.

²⁹ Once a nation has committed her share of output $(1 - \alpha)y(e_i)$ to the agreement, the only possibility for free-riding is on her reductive efforts—which we show in proposition 1 to be suboptimal.

Our argument in proposition 3 uses the maximal threat $C' = \langle \alpha' = 1, \beta', p(\mathbf{f})' \rangle$ to show that free-riding can always be discouraged. This extreme case, however, renders the agreement wholly ineffective if a punishment becomes necessary. The purpose of the following example is to show that such severe measures are not generally needed. Typically, the punishment of a deserter leaves enough freedom to increase abatement levels over those realising under no agreement. As outlined in the previous sections, contract C implements efficient efforts. Consider now a deviation by some player which triggers $C' = \langle \alpha', \beta', p(\mathbf{f})' \rangle$. Denote the equilibrium agreement utility attained by adhering to C' by $u'(\cdot)$ and the corresponding equilibrium efforts by $e'(\alpha', \beta', p')$, $f'(\alpha', \beta', p')$. By inflicting sufficient damage through $m(\cdot)$, we need to ascertain that free-riding utility $u_i^s(e_i^s, f_i^s)$ —with the agreement members adhering to C' —is smaller than what participation in C gives, that is

$$u_i^s(e_i^s, f_i^s | C') = y(e_i^s) - s_i m(e_i^s + (n-1)e'(\cdot) - f_i^s - (n-1)f'(\cdot)) - c_e(e_i^s) - c_f(f_i^s) \leq u_i(e^*, f^* | C).$$

Since (3) implies that efforts e' , f' are monotonic in α' , β' , payoff $u_i^s(\cdot | C')$ is continuous in α' and β' . Hence there exists an $\alpha' \in (\alpha, 1]$ which ensures the above inequality for suitable β' and $p'(\mathbf{f})$. Consider the case of $n+1$ players in the example setup of section 3.1. Then, full participation efficient efforts are given by

$$e^* = \frac{n+2}{2 \times 2^{1/3} ((2+n)(3+2n)^2)^{1/3}}, \quad f^* = \frac{n+1}{2 \times 2^{1/3} ((2+n)(3+2n)^2)^{1/3}} \quad (13)$$

which are implemented by

$$\alpha^* = \frac{4\sqrt{e^*}(2e^* - f^*)(n+1) - 1}{n}, \quad \beta^* = \frac{1}{n+1} + \frac{2(e^* - 2f^*)f^*}{\sqrt{e^*}r(\alpha - 1)} \quad (14)$$

for the Tullock success function parameterised by r . This determines $u_i(e^*, f^* | C)$. For the deviation utility $u_i^s(e_i^s, f_i^s | C')$, the deviation efforts e^d , f^d are determined by the first-order conditions

$$2e^s = \frac{1}{2\sqrt{e^s}} + \frac{2(f^s + n(f'(\cdot) - e'(\cdot)) - e^s)}{n+1}, \quad f^s = \frac{e^s + n(e'(\cdot) - f'(\cdot))}{n+2} \quad (15)$$

where $e'(\cdot)$, $f'(\cdot)$ are the agreement equilibrium efforts in the agreement under C' . These functions $e'(\cdot)$, $f'(\cdot)$, and therefore the damage they inflict on the deviator through $m(e^s + ne' - f^s - nf')$, are determined by

$$\alpha' = \frac{n(4\sqrt{e'}(e' + e^s - f^s + 2e'n - f'n) - 1) - 1}{n^2 - 1},$$

$$\beta' = \frac{(e'^2(4+8n) - 2f'(n-1)(f' + f^s + 2f'n - e^s) + 2e'(2e^s - 2f^s + f'(n-3)n) - \sqrt{e'}(n+1))}{(4e'^2n(1+2n) - 4e'n(-e^s + f^s + f'n) - \sqrt{e'}n(1+n))}.$$

In our example setup, it turns out that participation is individually rational for any number of players.

The details for the simplest case of three players (two in the agreement, one outside) are

$$e' = 0.302269, f' = 0.195276, e^s = 0.314075, f^s = 0.132016, (e^* = 0.273276, f^* = 0.204957)$$

for $C' = \langle \alpha' \approx 0.910124, \beta' = 1, r' = 1 \rangle$. If there is no agreement, efforts are $e^d = 0.30286, f^d = 0.15142$, so the 29% increased abatement efforts achieved by the punishment contract are substantial.

In conclusion of our discussion of the participation problem we would like to discuss a result from a companion paper, Bos and Schweinzer (2012), which is exclusively concerned with the formation of a redistributive agreement. Our contest-based mechanism is redistributive because a member state pays into the common pool and gets a (winner's or loser's) share back. In Bos and Schweinzer (2012), national income or GDP $y(e_i)$ is taken to be stochastic. In particular, the form $y(e_i) = \tilde{y}(e_i) + \varepsilon$ is assumed, where $\varepsilon \sim F(\mu = 0, \sigma^2)$ and $\tilde{y}(e_i)$ has all (concavity) properties we assume in this paper for $y(e_i)$. In various symmetric and asymmetric settings, Bos and Schweinzer (2012) show that the income distribution of a state which prefers to stay outside the agreement is a mean-preserving spread of the income distribution of a member of the contest-based, redistributive IEA. Therefore, agreement members can expect a 'smoother' income stream than free-riders. This property implies that there is a degree of income-shock-aversion under which a free-rider finds it profitable to join the agreement.

As it seems excessive to introduce the full stochastic framework of Bos and Schweinzer (2012) into this model, we just refer the reader to their example of subsection 3.4. It maintains the general form of the present model and is fully applicable to our argument. Therefore, sufficiently income-shock-averse nations will join our redistributive agreement. Several more focussed (albeit perhaps less elegant) ways of ensuring participation are discussed in subsection 5.1.

4.3 Asymmetries

Finally, we show that our efficiency result is not an artifact of our symmetry assumptions. The main argument is presented for any number of players $n \geq 2$ and identity dependent shares (α_i, β_i) . An illustrative example follows and further discussions of the asymmetric case can be found in subsection 5.2.

Let $i \in \mathcal{N}$ and $n \geq 2$. We illustrate that, for appropriately chosen $\langle \alpha_i, \beta_i; p(\mathbf{f}) \rangle_i^n$, prize pool $P = \sum_{j=1}^n (1 - \alpha_j) y_j(e_j)$, and prize structure $(\beta_i, \frac{1-\beta_i}{n-1}, \dots, \frac{1-\beta_i}{n-1})$, efficient solutions exist to player i 's asymmetric problem

$$\max_{(e_i, f_i)} \alpha_i y_i(e_i) + p_i^1(\mathbf{f}) \beta_i P + \sum_{i \neq j} p_j^1(\mathbf{f}) \left(\frac{1 - \beta_j}{n - 1} \right) P - s_i m \left(\sum_{i=1}^n e_i - f_i \right) - c_i(e_i, f_i). \quad (16)$$

Analogous to (1), let player i 's asymmetric efficient efforts be given by

$$y'_i(e_i^*) = m'(A) + c'_{e_i}(e_i^*) \text{ and } m'(A) = c'_{f_i}(f_i^*) \quad (17)$$

where $A = \sum_{j=1}^n (e_j - f_j)$. Let the payments' shares α_i and winning shares be identity-dependent, i.e., a winning player i gets share β_i and a winning j gets share β_j of the total prize pool $P = \sum_{j=1}^n (1 - \alpha_j)y_j(e_j)$. Thus, taking all $e_j^*, f_j^*, j \neq i$, as given, player i maximises (16). Taking derivatives with respect to e_i, f_i and inserting (17), determines player i 's best response through³⁰

$$\alpha_i = \frac{y'_i(e_i)(1 - B) - (1 - s_i)m'(A)}{y'_i(e_i)(1 - B)}, \text{ where } B = \beta_i p_i^1(\mathbf{f}) + \frac{\sum_{j \neq i} (1 - \beta_j) p_j^1(\mathbf{f})}{(n - 1)},$$

$$\beta_i = \frac{(n - 1) ((1 - s_i)m'(A)) - \sum_{j \neq i} (1 - \beta_j) p_{j(f_i)}^1(\mathbf{f}) P}{p_{i(f_i)}^1(\mathbf{f})(n - 1)P} \quad (18)$$

and $p_{i(f_i)}^1(\mathbf{f})$ denotes the $\frac{\partial}{\partial f_i} p_i^1(\mathbf{f})$. (18) corresponds to (34) and elicits asymmetric efficient efforts (e_i^*, f_i^*) . Without putting any restrictions on the numbers α_i and β_i , a (numerical) solution to the (18) can always be found. Since the same is true for the best responses of player i 's opponents, we confirm that a solution to the complete system $(\alpha_1, \dots, \alpha_n; \beta_1, \dots, \beta_n)$ exists. We cannot, however, make any general statements about feasibility and non-negativity of prizes in the asymmetric case.

In the introduction, we stress that diluted objectives among development nations may be problematic for an IEA. In order to address this problem, our asymmetric model extension views developing countries as exhibiting a smaller cost of emissions reduction than their fully developed counterparts. In our model, this increases a developing country's chance of winning the reduction contest. Similarly, in equilibrium, developed countries feature higher productive efforts and thus contribute a bigger share of the tournament prize pool than less productive countries.³¹

As an illustration of this asymmetric model we extend the example from subsection 3.1 to three players which we parameterise player $i = 1, 2, 3$'s cost with respect to productive and reductive efforts by the pair of scalars (γ_i, δ_i) .³² Following the general strategy outlined above, we use identity (class) dependent shares (α_i, β_i) and prize structures $(\beta_i, \frac{1-\beta_i}{n-1}, \dots, \frac{1-\beta_i}{n-1})$. The planner's objective is then to

$$\max_{(e_1, f_1, e_2, f_2, e_3, f_3)} \sum_i e_i^{1/2} - \left(\sum_i (e_i - f_i) \right)^2 - \sum_i \gamma_i e_i^2 - \sum_i \delta_i f_i^2. \quad (19)$$

For total prize pool $P = \sum_i (1 - \alpha_i) e_i^{1/2}$, individuals $i = 1, 2, 3$ choose (e_i, f_i) to maximise

$$\alpha_1 e_1^{1/2} + \frac{f_1^r}{\sum_h f_h^r} \beta_1 P + \left(1 - \frac{f_1^r}{\sum_h f_h^r} \right) \left(\frac{f_2^r}{f_2^r + f_3^r} \frac{1 - \beta_2}{2} + \frac{f_3^r}{f_2^r + f_3^r} \frac{1 - \beta_3}{2} \right) P - s_1 (\sum_h (e_h - f_h))^2 - \gamma_1 e_1^2 - \delta_1 f_1^2,$$

$$\alpha_2 e_2^{1/2} + \frac{f_2^r}{\sum_h f_h^r} \beta_2 P + \left(1 - \frac{f_2^r}{\sum_h f_h^r} \right) \left(\frac{f_1^r}{f_1^r + f_3^r} \frac{1 - \beta_1}{2} + \frac{f_3^r}{f_1^r + f_3^r} \frac{1 - \beta_3}{2} \right) P - s_2 (\sum_h (e_h - f_h))^2 - \gamma_2 e_2^2 - \delta_2 f_2^2,$$

$$\alpha_3 e_3^{1/2} + \frac{f_3^r}{\sum_h f_h^r} \beta_3 P + \left(1 - \frac{f_3^r}{\sum_h f_h^r} \right) \left(\frac{f_1^r}{f_1^r + f_2^r} \frac{1 - \beta_1}{2} + \frac{f_2^r}{f_1^r + f_2^r} \frac{1 - \beta_2}{2} \right) P - s_3 (\sum_h (e_h - f_h))^2 - \gamma_3 e_3^2 - \delta_3 f_3^2,$$

³⁰ The expressions (18) can be simplified further but then get excessively lengthy.

³¹ Thus, in our asymmetric model, the larger part of emissions reductions are implemented where they are the cheapest, i.e., in emerging economies.

³² The two players asymmetric case in the example setup is rather special since $\beta_1 + (1 - \beta_2) = 1$ implies $\beta_1 = \beta_2$. Hence, in order to lend itself to a solution, the two players case requires success function slopes of precisely

$$\frac{\partial}{\partial f_i} p_i(\mathbf{f}) = \frac{(1 - s_i) c'_f(f_i)}{(\beta_1 + \beta_2 - 1)P}.$$

Cases with higher numbers of players than two do not imply equal prize shares and do not induce complications.

The result for $(\gamma_1 = 1, \delta_1 = 1, \gamma_2 = 2/3, \delta_2 = 3/4, \gamma_3 = 1/3, \delta_3 = 1/2)$ and $s_i = 1/3$ leads to the efficiency inducing asymmetric shares of³³

$$\alpha_1 = 0.618533, \beta_1 = 0.511359, \alpha_2 = 0.552969, \beta_2 = 0.549507, \alpha_3 = 0.312778, \beta_3 = 0.711638, \\ e_1^* = 0.273795, f_1^* = 0.203984, e_2^* = 0.338534, f_2^* = 0.271979, e_3^* = 0.475588, f_3^* = 0.407969.$$

Although we demonstrate most of our results in a simplified symmetric setup, the previous proposition shows that this can be done without loss of generality. The following section extends our basic model in several directions and also shows that our results are robust to the choice of contest success function. Together with the above results, we hope that this convinces the reader that the proposed contest mechanism indeed provides incentives for efficient provision in the targeted setup.

5 Extensions and robustness

5.1 Exclusive trade agreements and enforcing standards

The purpose of this subsection is to show that a simple way to deter free-riding of individual nations is to grant most favoured ‘green’ trading terms only to participating nations. Similarly, environmental certification conditional on treaty commitment can be a powerful complementary tool to enforce participation in the IEA. The expansion of equilibrium abatement efforts from their no agreement level f^0 to the level within the agreement f^* creates ‘green’ products; the higher abatement efforts, the greener is productive output. The idea is to label free-riding countries’ products and thereby creating (political) incentives to respect commitments to the IEA. Consequently, firms’ lobbying against environmental regulation may result in that country’s desertion followed by the IEA labelling its goods. We thus propose a negative label which signals a product lacking the ‘green’ environmental standards enforced by the IEA.³⁴

This idea can be formalised in our setup by decreasing the value of a deserting country’s output. Loosely speaking, we require that—once certified as environmentally unfriendly—a consumer’s willingness to pay for labelled products decreases³⁵. Therefore the revenue generated from the production of labelled products sinks as these products suffer a decrease in price of $x(\mathbf{f}) \in [0, 1]$. This fraction corresponds to the deserter’s deviation from agreed abatement levels. Denoting the outside

³³ In this first example, we use countries of same ‘size’ s_i . This is done to concentrate on the effect of cost asymmetries. The example of subsection 5.2 focuses on the effect of size asymmetries.

³⁴ There are many green labelling examples: UK supermarket chain Tesco has recently introduced a promotional campaign on carbon labels. US Walmart and French Casino have similar ambitions. Examples of negative labelling campaigns are the mandatory GMO labelling implemented in Europe, and “we’re Greenpeace, and we want a fresh green Apple” targeted at US computer maker Apple. Grankvist, Dahstrand, and Biel (2004) argue that negative labelling may have a higher consumption impact than positive labelling. Engel (2004) underlines the necessity to inform consumers, especially when a firm is found cheating on its environmental claims.

³⁵ Models on certification and standard settings have been studied intensely; see for instance Lerner and Tirole (2006) or Harbaugh, Maxwell, and Roussillon (2011)

equilibrium efforts of a single deserter by (e^d, f^d) , desertion utility is

$$u_i^d(e_d, f_d) = \underbrace{\left(\frac{f^* - f_i^0}{f^*} \right)}_{=x \in [0,1]} y(e_i^d) - s_i m(e_i^d + (n-1)e^* - f_i^d - (n-1)f^*) - c_e(e_i^d) - c_f(f_i^d) \quad (20)$$

for $i \in \mathcal{N}$ and $n > 2$ (since there must be at least two players left in the agreement after i deserts). A sufficiently large fraction x will successfully deter free-riding on reductive efforts and can be seen as alternative to the blunt global threat represented by contract C' .³⁶

A further step in this direction is the formation of an exclusive trade agreement. If a deserter can be excluded from the fraction of trade corresponding to the necessary abatement investments within the agreement, then individual desertion can be again discouraged. As above, green production—generated by the expanded abatement effort—is traded among countries and produces wealth. Our model does not take into account the international trading aspects of production and thus there is no direct way of measuring the involved consequences to individual wealth.³⁷ A simple (ad-hoc) way of nevertheless capturing the idea of enforcement through an exclusive trading agreement is to restrict trade on (and therefore capitalising on) the fraction of productive output corresponding to the reductive investments within the treaty to agreement members. Then player i 's desertion utility in (20) can be reduced by using

$$x(\mathbf{f}) = \left(1 - \frac{f^* - f_i^0}{e^*} \right) \quad (21)$$

where reductive effort f_i^0 is the equilibrium abatement level without agreement. As indicated above, to some extent this choice is arbitrary.³⁸ The intuition for (21) is that an agreement defector j can free-ride on the reductive efforts of agreement members (through a cleaner environment) but is punished by restricted access to the agreement market consisting of the tradables $\sum_{h \neq j} y(e_h)$.

Returning to the simple quadratic cost, square-root production example of example section 3.1, this implies that it is individually rational to participate in the agreement if

$$u_i(e^*, f^*) \geq x(\mathbf{f}) e_i^{\frac{1}{2}} - \frac{1}{n} (e_i + (n-1)e^* - f_i - (n-1)f^*)^2 - e_i^2 - f_i^2. \quad (22)$$

Solving the deserter's maximisation problem (on the rhs) for the exclusive trade agreement under (21) leads to the first-order conditions

$$2f^* + \frac{e^* - f^* + f_i^0}{2e^* \sqrt{e_i^d}} = \frac{2(e_i^d + f^* - f_i^d + e^*(n-1) + e_i^d n)}{n}, \quad f_i^d = \frac{e_i^d + (e^* - f^*)(n-1)}{n+1} \quad (23)$$

where (e^*, f^*) are the equilibrium effort levels provided inside the agreement. Plotting the utilities from the desertion efforts (e_i^d, f_i^d) solving above first-order conditions results in a graph similar to

³⁶ To some extent, the particular choice of $x(\mathbf{f})$ is arbitrary. Any function of abatement efforts implementing sufficient deterrence (such as the function used for the exclusive trade example below) could be used instead.

³⁷ Modelling both these aspects formally is possible and certainly provides grounds for future research.

³⁸ If monitoring of the deserter's abatement efforts is good, then the actual level f_i^d could be used. (In the present example setup, this leads to a rather unintuitive corner solution.) The idea, however, seems important for dealing with competing abatement agreements: As long as they are effective, there is no reason to punish them.

figure 1 for the case of $n = 3$ showing that deviations are not profitable. Thus, the exclusive trade agreement ensures participation. (The analysis is nearly identical for the labelling setup discussed above and therefore not replicated.) As the severity of punishment $(1 - x(\mathbf{f}))$ in (21) is given by

$$\frac{f^* - f_i^0}{e^*} = \frac{6 - 3^{1/3}(-2 - 4n)^{2/3}(-1 - n)^{1/3}}{6(1 + n)} \quad (24)$$

which is increasing in n , the punishment gets more severe for larger n and participation is easier to obtain in the general case. (The limit as $n \rightarrow \infty$ equals $2^{1/3}/3^{2/3} \approx .606$.)

5.2 Asymmetric pollution shares

In this subsection, we extend the example setup of section 3.1 with unequal relative damage shares $s_i \in (0, 1)$, $i = 1, 2$. Since shares sum to 1, both efficient effort types are still given by (3). Player i 's problem is unchanged and imposing efficiency (3) we obtain the shares

$$\alpha_i^* = \frac{1}{5}(1 + 4s_i), \quad \beta^* = \frac{1}{2} + \frac{1}{6r}. \quad (25)$$

Notice that only α_i^* turns out to depend on the player's identity (class), the efficiency-inducing prize structure β is identical to the symmetric case. As to be expected, the share $(1 - \alpha_i^*)$ of output which has to be committed to the contest gets arbitrarily small when the public bad problem disappears as s_i approaches 1. On the other extreme, a player who does not suffer from the effects of global warming at all must be asked to commit close to $4/5$ of her output to the contest in order to induce efficient efforts on her behalf. A numerical example taking relative damage shares of $s_1 = 1/4$, $s_2 = 3/4$ requires $\alpha_1 = 0.4$ and $\alpha_2 = 0.8$ in order to implement efficiency.

In the more general case of $n > 2$ players with damage shares parameterised by $s_i = \frac{2i}{n+n^2}$, $i = 1, 2, \dots, n$, with $\sum_{i=1}^n s_i = 1$, efficient efforts are given by

$$4e(1 + n) = \frac{1}{\sqrt{e}} + 4fn, \quad en = f(1 + n) \Leftrightarrow \begin{cases} e^* = \frac{1 + n}{2 \times 2^{1/3} ((1 + n)(1 + 2n)^2)^{1/3}}, \\ f^* = \frac{n}{2 \times 2^{1/3} ((1 + n)(1 + 2n)^2)^{1/3}}. \end{cases} \quad (26)$$

The n -player individual asymmetric problem in the example setup under the two-part price structure $(\beta, \frac{1-\beta}{n-1}, \dots, \frac{1-\beta}{n-1})$ employed previously is

$$\max_{(e_i, f_i)} \alpha_i e_i^{1/2} + \frac{f_i^r}{f_i^r + (n-1)(f^*)^r} \beta P + \left(1 - \frac{f_i^r}{f_i^r + (n-1)(f^*)^r}\right) \left(\frac{1-\beta}{n-1}\right) P - \frac{s_i(e_i + (n-1)e^* - f_i - (n-1)f^*)^2 - (e_i^2 + f_i^2)}{2} \quad (27)$$

which is solved by the intimidating but straightforward efficient parameters

$$\alpha_i^* = \frac{\sqrt{\frac{1+n}{((1+n)(1+2n)^2)^{1/3}}}(1+n+ns_i)n - ((1+n)(1+2n)^2)^{1/3}}{(n-1)((1+n)(1+2n)^2)^{1/3}}, \quad (28)$$

$$\beta^* = \frac{(n-1)n(1+2n)^2 \left(\frac{1+n}{((1+n)(1+2n)^2)^{1/3}} \right)^{3/2} + n(n^2-1)(1+n+ns_i) + 2(1+n)^2 r(2+n(3+s_i))}{2n(1+n)^2 r(2+n(3+s_i))}$$

where, again, β is not identity dependent. A numerical example for $n = 187$, $r = 3$ gives for ‘type’ $s_i = 1/n$ (i.e., $i = 94$) an efficiency-inducing $\alpha_i^* = 0.50133$ and redistribution vector of ($\beta^* = 0.17024$, $\frac{1-\beta^*}{n-1} = 0.00446$, \dots , $\frac{1-\beta^*}{n-1} = 0.00446$) which compares to the flat $1/n = 0.00534$. Under the contest, type $s_i = 1/n$ gives up roughly 50% of her output but gets back 41.6% even if losing the contest. She gets almost 16 times her output if she wins. Notice that if equilibrium existence allows, then the two can be further equalised by employing a more precise ranking and thereby increasing r .

A maybe more realistic alternative (or addition) to the framework we examine here is to assign a vector of individual weights $\psi = (\psi_1, \dots, \psi_n)$, with each $\psi_i > 0$, to the heterogenous contests’ reductive efforts turning the basic success function into

$$p_i(f, \psi) = \frac{\psi_i f_i^r}{\sum_j \psi_j f_j^r}, \quad \text{where } \frac{\psi_i}{\delta_i^r} = \frac{\psi_j}{\delta_j^r} \quad \forall j \neq i \quad (29)$$

and thus ‘levelling the playing field.’ This idea of creating a more symmetric contest through the appropriate choice of ψ was studied recently, among others, by Franke, Kanzow, Leininger, and Schwartz (2011), and Franke (2012). Following the assignment of ψ , one can proceed with the analysis of section 4. Bos and Schweinzer (2012) develop a similar asymmetric example where individual abatement efforts are normalised by the player’s (endogenously chosen) GDP before entering the success function.

5.3 Comparative statics

In this subsection we compare the wealth distribution without an international environmental agreement (IEA) to the distribution under the proposed contest scheme. The three players example of the previous subsection suggests that the most productive countries (with the lowest γ, δ in (19)) will exert higher efforts of both types in equilibrium—as would be the case without an IEA. As is the case for taxes achieving emissions reduction, effectiveness is ensured at the smallest possible cost.

The output share α and the prize pool share β are mainly determined by the relative efficiency of the different countries’ reductive efforts. The country with the lowest reductive effort cost (lowest δ) is willing to contribute the most to the prize pool—smallest α —accepting unequal prize pool shares—high β —in order to achieve efficient efforts in the output and reductive dimensions. Indeed, improved reductive efforts increase the probability of winning the contest, thus augmenting the country’s willingness to participate in the contest and also the desired prize pool share.

The relative efficiency of productive efforts influences the proposed scheme in a continuous

fashion. Low output costs induce higher prize pool contributions—lower α —as the country is getting increasingly more productive. However, low efficiency of reductive effort partially offsets this, as the probability of winning is the driving force. Hence, even if efficiency in both dimensions seems to work in the same direction, the relative efficiency of reductive efforts is decisive.

Regarding the impact on the players' utility, the relative efficiency of productive effort is decisive for the impact on a country's welfare. Countries with low efficiency of productive effort are likely to benefit from the implementation of the proposed scheme. The most productive nations, however, are unlikely to benefit from the contest scheme even if they are efficient in the reductive dimension.

The remainder of this subsection tentatively applies the proposed mechanism to the debate on international cooperation on climate change. We consider three different groups of countries suggestively labelled EU, US and developing poor (DP). We postulate that EU and US are more efficient in productive output than DP. For the reductive efforts, we assume that US and DP have low costs but EU has not. In particular, we assume³⁹

$$\delta_{EU} = 1, \delta_{US} = \delta_{DP} = 0.5, \text{ and } \gamma_{EU} = \gamma_{US} = 0.5, \gamma_{DP} = 1.$$

This seems to coincide with reality as rich countries are arguably more efficient in producing output than poor states. Metaphorically speaking, US and DP have lots of low hanging fruit to take advantage of since they are quite inefficient in their energy use, whereas EU is relatively energy efficient and subsequent improvements are costly.

Our results suggest that the proposed scheme will improve the welfare distribution as the range of expected utility narrows. Under the proposed mechanism, we obtain expected utilities of

$$EU_{EU}^* = 0.44, EU_{US}^* = 0.46, EU_{DP}^* = 0.45$$

whereas without an IEA, the corresponding wealth distribution is $U_{EU} = 0.46, U_{US} = 0.46, U_{DP} = 0.37$. We thus find that global welfare increases by 7% thanks to the introduction of the IEA. EU, however, could loose in comparison with the status quo and therefore needs to be compensated. Other considerations, such as her historical responsibility, may also be underlined to motivate her participation in the contest.

Finally, the question whether and in how far the efficient asymmetric abatement levels can be used directly as inputs into the contest success function or not is, of course, politically charged. We therefore restrict ourselves to pointing out that any standard normalisation of input efforts is feasible. For instance, it is perfectly possible to normalise inputs such that each country which exerts its efficient abatement level has the same chance of winning the first prize $1/n$. A strength of the contest mechanism is its ability to adopt different abatement-cost distribution rules. Our concept of reductive effort can encompass population size, GDP, areal expansion or incorporate fairness considerations.

³⁹ The numbers we use are inspired by Ellerman and Decaux (1998). The debate on marginal abatement cost of greenhouse gases is not settled, however. For a recent discussion see Morris, Paltsev, and Reilly (2008) and the references therein.

5.4 The choice of ranking technology

Consider a n -player extension of the problem of subsection 3.1 with prize structure $(\beta, \frac{1-\beta}{n-1}, \dots, \frac{1-\beta}{n-1})$. The present example shows that efficiency can be obtained in proposition 2 for a ‘difference-form’ success function. The specific properties of the generalised Tullock success function which we use in the remainder of the paper are therefore not crucial to our results. Difference-form success functions have been widely used in the literature, for instance by Che and Gale (2000), but suffer from the lack of a generally accepted, simple extension to more than two players. We define player i ’s probability of winning as

$$p_i(\Delta) = \frac{\exp^{\Delta_i^r}}{\sum_{j=1}^n \exp^{\Delta_j^r}}, \text{ where } \Delta = (\Delta_1, \dots, \Delta_n), \Delta_i = f_i - \frac{\sum_{j \neq i} f_j}{n-1}, \text{ and } r > 0. \quad (30)$$

Setting $P = (1 - \alpha)(e_i^y + (n - 1)e_j^y)$, $y \in (0, 1)$, $m, b > 1$ and all $j \neq i$ equal, player i ’s individual problem is to

$$\max_{(e_i, f_i)} \alpha e_i^y + p_i(\Delta)\beta P + (1 - p_i(\Delta))\frac{1 - \beta}{n - 1}P - s_i(e_i + (n - 1)e_j - f_i - (n - 1)f_j)^m - (e_i^b + f_i^b)$$

which, in symmetric equilibrium $e = e_i = e_j$, $f = f_i = f_j$ gives for any $p_i(\Delta)$

$$\alpha = \frac{e^{-y} ((e - f)(be^b n - e^y y) + em((e - f)n)^m s_i)}{(e - f)(n - 1)y},$$

$$\beta = \frac{e^{-y} (-b(e - f)f^b(n - 1)n + f(m(n - 1)((e - f)n)^m s_i + e^y(e - f)n^2(\alpha - 1)p'_i(0)))}{(e - f)fn^3(\alpha - 1)p'(0)}$$

where $\Delta = 0$ is the equilibrium vector of deviations. Plugging in the efficient efforts from (3), employing (30), and returning to the example setup from section 3.1: $n = 2$, $y = 1/2$, $b = m = 2$, and $s_i = 1/2$, this results in a very similar efficient mechanism as under the Tullock success function

$$\alpha^* = \frac{3}{5}, \beta^* = \frac{r + (5/6)^{2/3}}{2r} \quad (31)$$

where $\beta^* \in (.5, 1]$ is ensured for $r \geq (5/6)^{2/3} \approx 0.89$. A picture nearly identical to figure 1 confirms, for instance, $(\alpha^*, \beta^*, r = 2)$ as equilibrium. The precise form of ranking technology employed is thus immaterial to our results.

6 Concluding remarks

We show that a simple, redistributive contest organised among nations can implement both efficient productive and reductive efforts. Our model provides a benchmark for the cost of achieving efficiency in terms of model-GDP. Many desirable generalisations of the model must be left for future work: Which share of global (per capita) GDP would have to be redistributed—in reality—to the country with the highest emissions reduction in order to implement our results? Is the resulting wealth

redistribution one we would like to see? Can the mechanism's design parameters be effectively negotiated? These questions are to a large extent empirical and all have significant policy implications. At any rate we do not feel qualified to answer these questions now. What we do provide, however, are firm results showing that a mechanism along the lines we indicate can *in principle* correct nations' combined incentives to emit too much while abating too little.

Appendix

Proof of proposition 1. Efficient efforts are extending (1) as the pair (e^*, f^*) solving

$$y'(e) = m'(ne - nf) + c_e(e, f), \quad m'(ne - nf) = c_f(e, f). \quad (32)$$

Let $P = (1 - \alpha) \sum_{h=1}^n y(e_h)$. Since we are only interested in deviations from symmetric equilibrium, we set $e_j = e_{-i}$. Rewriting (12) for the 2-prize structure $(\beta^1, \frac{1-\beta^1}{n-1}, \dots, \frac{1-\beta^1}{n-1})$ results in

$$\alpha y(e_i) + \beta^1 p_i^1(\mathbf{f})P + \sum_{h=2}^n \frac{1 - \beta^1}{n - 1} p_i^h(\mathbf{f})P - s_i m(e_i + (n - 1)e_j - f_i - (n - 1)f_j) - c_e(e_i) - c_f(f_i)$$

which simplifies to

$$\alpha y(e_i) + \beta^1 p_i^1(\mathbf{f})P + \frac{1 - \beta^1}{n - 1} (1 - p_i^1(\mathbf{f}))P - s_i m(e_i + (n - 1)e_j - f_i - (n - 1)f_j) - c_e(e_i) - c_f(f_i).$$

The symmetric $e = e_i = e_j$, $f = f_i = f_j$, first-order conditions for this problem are

$$\begin{aligned} c'_e(e) + s_i m'(ne - nf) &= \frac{1 - \beta^1 + \alpha(n + \beta^1 - 2) + (1 - \alpha)(n\beta^1 - 1)}{n - 1} p(f) y'(e), \\ c'_f(f) &= s_i m'((e - f)n) + \frac{n(1 - \alpha)(n\beta^1 - 1)}{n - 1} p'(f) y(e). \end{aligned} \quad (33)$$

Plugging in (32) and imposing $s_i = 1/n$, one obtains

$$\alpha^* = 1 - \frac{y'(e^*) - c'_e(e^*)}{y'(e^*)} \quad \text{and} \quad \beta^* = \frac{1}{n} + \frac{(n - 1)^2 y'(e^*)}{n^3 y(e^*) p'(\mathbf{f}^*)} \quad (34)$$

which can always be achieved by picking a suitably steep ranking technology $p(\mathbf{f}^*)$. \square

Proof of proposition 2. Since under our assumptions (12) is fully separable we can split the problem into two independent problems along the respective effort dimensions. Setting $P = (1 - \alpha) \sum_{h=1}^n y(e_h)$, the two separate problems are

$$\begin{aligned} \alpha y(e_i) + \beta^1 p_i^1(\mathbf{f}^*)P + \frac{1 - \beta^1}{n - 1} (1 - p_i^1(\mathbf{f}^*))P - s_i m(e_i + (n - 1)e^* - n f^*) - c_e(e_i) - c_f(f^*), \\ \alpha y(e_i^*) + \beta^1 p_i^1(\mathbf{f})P + \frac{1 - \beta^1}{n - 1} (1 - p_i^1(\mathbf{f}))P - s_i m(ne^* - f_i - (n - 1)f^*) - c_e(e^*) - c_f(f_i). \end{aligned} \quad (35)$$

1) We show that exerting productive effort $e_i = e^*$ gives a global maximum. As players are symmetric and we are looking for a profitable deviation from the efficient level we set $\mathbf{f}^* = (f_1 = f^*, \dots, f_n = f^*)$ implying that the probability of winning is $p_i^1(\mathbf{f}^*) = 1/n$. Thus the problem simplifies to

$$\alpha y(e_i) + \frac{1}{n}P - s_i m(e_i + (n-1)e^* - (n)f^*) - c_e(e_i) - c_f(f^*) \quad (36)$$

giving the first-order condition for productive effort e_i as⁴⁰

$$\underbrace{y'(e_i) \left(\alpha + \frac{1}{n}(1-\alpha) \right)}_{\searrow} = \underbrace{s_i m'(\max\{0, e_i + (n-1)e_j^* - (n)f^*\})}_{\nearrow} + \underbrace{c'_{e_i}(e_i)}_{\nearrow}.$$

Notice that output is strictly increasing in e_i and is strictly concave. Thus $y''(e_i) < 0$ and $y'(e_i)$ is decreasing. Both cost functions are increasing and convex, therefore $s_i m''(\cdot) + c''(e_i) > 0$ and the rhs is increasing. As $y'(0) > s_i m'(\max\{0, (n-1)e^* - nf^*\}) + c'(0)$,⁴¹ this confirms single crossing of rhs and lhs and ensures the existence of an equilibrium.

2) We now demonstrate global optimality of $f_i = f^*$. Assuming efficient productive effort provision, the first-order condition for reductive effort is

$$\underbrace{ny(e^*)(1-\alpha)(\beta n-1)p'(f_i, f^*)}_{=B} = \underbrace{c'_{f_i}(f_i)}_{=C \nearrow} - \underbrace{s_i m'(\max\{0, ne^* - (n-1)f^* - f_i\})}_{=A \searrow}. \quad (37)$$

Notice that the rhs is strictly increasing as we know that, with respect to f_i , $s_i m''(\cdot) \leq 0$ and thus that A is decreasing and the cost function is convex. Without further assumptions on the monitoring technology $p(\cdot)$ we cannot sign the slope of B . Notice, however, that increasing the slope of the (convex) cost function $c'_{f_i}(f_i)$ sufficiently guarantees single crossing and thus a unique maximum whatever the precise specification of $p(\cdot)$.

3) We now show that (37) identifies a global maximum for the Tullock success function.⁴² Again, $s_i m(\max\{0, ne^* - (n-1)f^* - f_i\}) > 0$ for $f_i = 0$ while $p'(f_i, f^*) = 0$ and thus the lhs of (37) is zero at $f_i = 0$ while the rhs is negative. Single crossing is immediate for the case of $r \in (0, 1]$ as B is (weakly) decreasing. In the general case of

$$p_i(\mathbf{f}) = \frac{f_i^r}{\sum_{j=1}^n f_j^r}, \quad r > 1, \quad (38)$$

the function B has a single critical point and is decreasing when $f_i \geq f^* \left(\frac{(n-1)(r-1)}{r+1} \right)^{1/r}$.

To get single crossing if the two curves are increasing we need to ensure either strict concavity or convexity for the lhs and strict convexity for the rhs and prove that if $f_i = 0$, lhs is larger than the rhs. As we have not specified anything about our functions regarding the third derivative we illustrate this point using the specific $c'_{f_i}(f_i) = bf^{b-1}$ and $s_i m(\max\{0, ne^* - (n-1)f^* - f_i\}) =$

⁴⁰ It is routine to verify that both first-order conditions identify a maximum.

⁴¹ Since output is concave and the sum of cost functions is convex in e_i , the above inequality holds.

⁴² A nearly identical argument can be made for any other ratio-based success function. In that more general case, however, we cannot derive an explicit existence threshold.

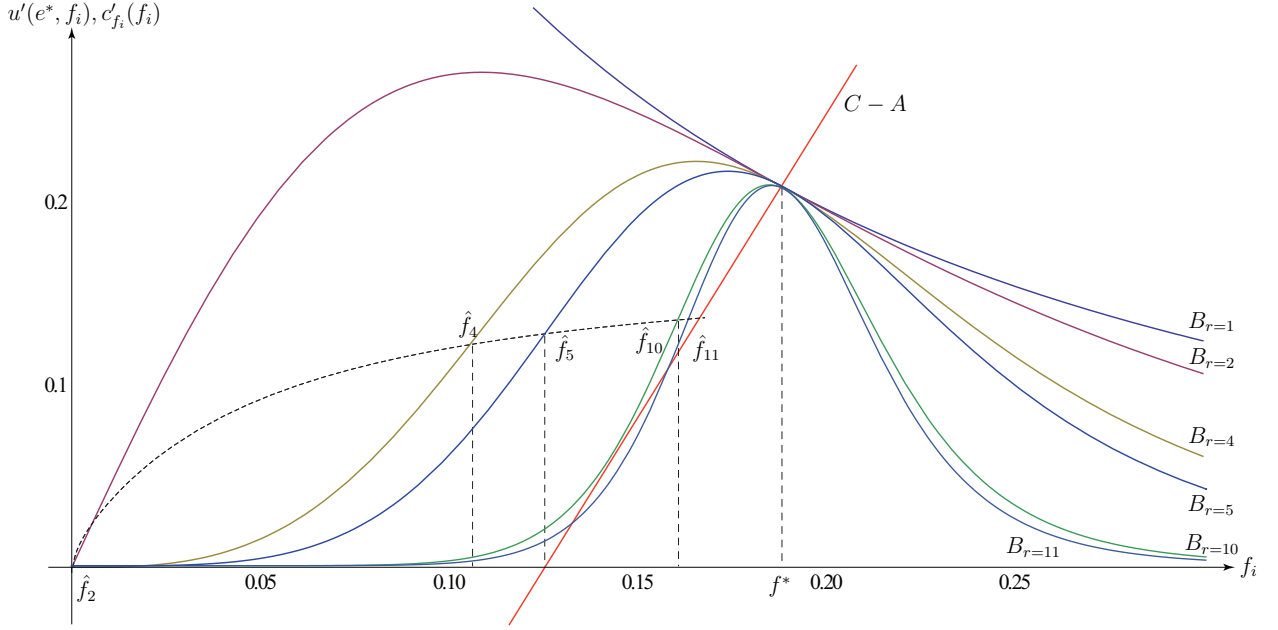


Figure 2: Single crossing in equation (37) ensures a unique global maximum at $f_i = f^*$ for the example setup of section 3.1. The dotted line gives the location of inflection points \hat{f} for different r .

$s_i(\max\{0, ne^* - (n-1)f^* - f_i\})^b$. We also set $s_i = \frac{1}{n}$. We find that both curves have an inflection point, thus we need to find a condition to ensure single crossing.

We first show that the rhs starts out negative and eventually becomes positive as for $f_i = 0$ we have $C - A = -s_i m'(\max\{0, ne^* - (n-1)f^*\}) < 0$. Therefore, as long as the lhs is positive and the rhs negative, the two curves cannot cross. We find that $C - A < 0$ for $f_i < f^* \frac{2}{n^{\frac{1}{b-1}} + 1}$ because

$$C - A = f_i^{b-1}b - \frac{\overbrace{(ne^* - (n-1)f^* - f_i)}^{=2f^*}}{n} = 0 \Leftrightarrow (2f^* - f_i)^{b-1} = n f_i^{b-1}. \quad (39)$$

Moreover, for the rhs, the inflection point occurs when the curve is negative, and it is first concave and then convex. Thus we can conclude that when the curve is above zero, it is strictly convex. We find that $(C - A)'' < 0$ for $f_i < f^* \frac{2}{n^{\frac{1}{b-3}} + 1}$ and $f_i < f^* \frac{2}{n^{\frac{1}{b-3}} + 1} < f^* \frac{2}{n^{\frac{1}{b-1}} + 1}$ because⁴³

$$\begin{aligned} (C - A)'' &= f_i^{b-3}(b-2)(b-1)b - \frac{(2f^* - f_i)^{b-3}(b-2)(b-1)b}{n} = 0 \Leftrightarrow (2f^* - f_i)^{b-3} = n f_i^{b-3}, \\ &\Leftrightarrow f^* \frac{2}{n^{\frac{1}{b-1}} + 1} - f^* \frac{2}{n^{\frac{1}{b-3}} + 1} = 2 \frac{f^* \left(n^{\frac{1}{b-3}} - n^{\frac{1}{b-1}} \right)}{\left(n^{\frac{1}{b-1}} + 1 \right) \left(n^{\frac{1}{b-3}} + 1 \right)} \geq 0. \end{aligned}$$

We conclude that the rhs is strictly increasing and convex when it is positive.

For the lhs, there are two inflection points: one in the increasing part and the other in the decreasing part. In the increasing part we find a condition which implies that the inflection occurs

⁴³ This is true for any $b \geq 3$.

if the rhs is negative.⁴⁴ A sufficient condition for single-peakedness is therefore that

$$\frac{2^r (f^*)^r}{\left(n^{\frac{1}{b-1}} + 1\right)^r} \geq \underbrace{\frac{(n-1) \left(2(f^*)^r (r^2 - 1) - \sqrt{3} \sqrt{(f^*)^{2r} r^2 (r^2 - 1)}\right)}{2 + 3r + r^2}}_{=: \hat{f}}. \quad (40)$$

Thus if the rhs of (37) is positive, it is also strictly convex. If (40) is respected, the lhs is strictly concave or convex. Notice also that at the inflection point, the rhs is positive and the lhs is negative and therefore the lhs is larger than the rhs. The geometric intuition of (40) is shown in figure 2 for the setup of the example section 3.1. The figure shows a family of curves B for $r \in \{1, 2, 4, 10, 11\}$ with inflection points labelled $\hat{f}_2, \hat{f}_4, \hat{f}_{10}$, and \hat{f}_{11} , respectively. Condition (40) is fulfilled as long as the red cost curve $C - A$ is negative at the respective inflection point. This is true for $r = 2$ and $r = 4$ soon after which (40) starts failing. Uniqueness, however, is actually only lost for $r > 10$. \square

Proof of proposition 3. Player i 's equilibrium participation utility for $P = (1 - \alpha)ny(e^*)$ under the efficiency-inducing contract $C = \langle \alpha^*, \beta^*, p^*(\mathbf{f}) \rangle$ defined in (12) with prizes $(\beta, \frac{1-\beta}{n-1}, \dots, \frac{1-\beta}{n-1})$ is

$$\begin{aligned} u_i(e^*, f^*) &= \alpha^* y(e^*) + \frac{1}{n} \beta^* P + (n-1) \frac{1-\beta^*}{n} P - s_i m(ne^* - nf^*) - c_e(e^*) - c_f(f^*) \\ &= \alpha y(e^*) + \frac{1}{n} (1 - \alpha^*) ny(e^*) - s_i m(ne^* - nf^*) - c_e(e^*) - c_f(f^*) \\ &= y(e^*) - s_i m(ne^* - nf^*) - c_e(e^*) - c_f(f^*). \end{aligned} \quad (41)$$

Now consider the contract $C' = \langle \alpha', \beta', p' \rangle$. Since second-stage efforts $e(\alpha', \beta', p'), f(\alpha', \beta', p')$ are continuous in α' , it is sufficient to consider the extreme case of $\alpha' = 1$ which implements no contest at all. A deserter's utility when subjected to C' can therefore be driven down to⁴⁵

$$u_i^s(e_i^s, f_i^s) = y(e_i^s) - s_i m(e_i^s + (n-1)e^a - f_i^s - (n-1)f^a) - c_e(e_i^s) - c_f(f_i^s) \quad (42)$$

where e^a and f^a are the equilibrium inside agreement efforts prescribed by C' . For $\alpha' = 1$, these equal the equilibrium efforts without agreement e_i^s, f_i^s . Hence (41) and (42) are identical but implement different efforts. Since both $e^s > e^*$ and $f^s < f^*$, it is therefore individually rational to join the agreement implementing C if the alternative is C' because the cost differential

$$s_i(m(ne^s - nf^s) - m(ne^* - nf^*)) \quad (43)$$

is increasing in n and convex. As the efficient allocation is welfare maximising, this outweighs any productivity gains from free-riding $y(e^s) - y(e^*) + c_e(e^*) + c_f(f^*) - c_e(e^s) - c_f(f^s)$. Thus, every player finds it individually rational to join the reductive contest if threatened by the alternative C' . \square

⁴⁴ The inflection point in the decreasing part does not matter. As long as one curve is increasing and the other is decreasing they can only cross once.

⁴⁵ Notice that the latter formulation requires $n > 2$ as the contest can only produce incentives if at least two players participate in the contest.

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