# Corporate Debt Maturity and Investment over the Business Cycle

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**Abstract**

I document that the share of long-term debt in total debt of US non-financial firms is pro-cyclical. Furthermore, the long-term debt share of small firms has a higher standard deviation and correlation with output than the long-term debt share of large firms. I construct a quantitative model in which firms optimally choose investment, leverage, debt maturity, dividends, and default. Firms face idiosyncratic and aggregate risk. When they choose their debt maturity, firms trade off default premia and roll-over costs. As a result, financially constrained firms endogenously prefer to issue short-term debt, because they face high default premia on long-term debt. Financially unconstrained firms issue long-term debt, because it has lower roll-over costs. The model, which is parameterized to match cross-sectional moments, can match stylized facts about the level and dynamics of the maturity structure of debt both in the aggregate and along the firm size distribution. Regarding the effects of outstanding debt on investment, it is not short-term debt, but long-term debt which leads to substantial under-investment due to a debt overhang effect.

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# 1 Introduction

The maturity structure of debt was a key factor in explaining the fall in investment during the 2007-2009 financial crisis. A high level of short-term debt at the brink of the crisis exposed firms to the risk of an unexpected decrease in credit availability, which in turn forced them to cut down investment to repay outstanding debt. Duchin, Ozbas, and Sensoy (2010) report that firms with high short-term debt restricted investment more during the financial crisis, while firms with high long-term debt did not significantly reduce investment. Almeida, Campello, Laranjeira, and Weisbrenner (2012) report that firms with a high fraction of debt maturing when the crisis hit decreased investment substantially more than firms without such a debt position.

Despite these observations, the macroeconomic literature has so far not considered which factors determine the maturity structure of corporate debt. Typically, macroeconomic models with financial frictions treat all debt as short-term. Yet for an average publicly traded U.S. firm between 1984 and 2012, only 36.8 percent of outstanding debt matured within the next year. In the aggregate, only 15.54 percent of corporate debt matured within the next year for the same time period. This discrepancy between the standard model assumption of short-term debt and the actual maturity structure observed in the data is economically important for how corporate debt affects corporate investment over the business cycle: Theories where firms rely exclusively on short-term debt emphasize liquidity constraints, such that firms reduce investment in a recession due to a high cost of refinancing. Theories where firms will mostly use long-term debt emphasize other frictions like the debt overhang problem first introduced by Myers (1977). According to this theory, firms reduce investment due to the failure of shareholders to internalize the benefits of investment to holders of outstanding debt.

In this paper, I study the determinants of the maturity structure of debt, both in the cross-section and over the business cycle. Specifically, I consider the cyclical dynamics of the maturity structure for firms of different size. I document that in the aggregate, the share of long-term debt in total debt is pro-cyclical. This correlation varies substantially by firm size: For the largest 10 percent of firms by assets, the share of long-term debt in total debt is *counter*-cyclical, while it is pro-cyclical for all other firms. In addition, large firms tend to use a larger share of long-term debt in general. Because the firm size distribution is very right-skewed, the behavior of large firms dominates the aggregate effect, such that it is easy to overlook the pro-cyclical debt maturity dynamics for the vast majority of small firms.

I construct a quantitative dynamic model that allows for a rich capital structure of firms: firms can issue short-term debt, long-term debt and equity. They accumulate productive capital, which is illiquid due to capital adjustment costs. Firms face both idiosyncratic and aggregate productivity risk. They also face aggregate consumption risk which affects asset prices in two ways: First, consumption risk creates a risk premium for assets whose returns co-move with consumption. Second, it creates a time-varying term structure of risk free interest rates: short-term interest rates are lower than long-term interest rates in a recession and higher in an expansion. Due to limited liability, firms can default on outstanding debt. Because default risk is endogenously pro-cyclical, bond prices reflect both expected losses from default and a risk premium.

The key ingredients of the model are endogenous default premia, debt and equity issuance costs and a tax benefit of debt. Since equity issuance costs in my parametrization are higher than debt issuance costs, they create an incentive for low productivity firms to issue debt to avoid having to issue equity. The tax deductibility of interest expenses creates an incentive to issue debt for high productivity firms.

The main trade-off between short-term debt and long-term is the following: On the one hand, financially unconstrained firms that want to maintain a high leverage ratio for the tax benefit want to keep the expected cost of rolling over debt low. They do so by issuing long-term debt, since long-term debt only has to be rolled over infrequently. On the other hand, financially constrained firms that issue debt because they lack internal funds want to keep the default premium low on newly issued debt. They do so by issuing short-term debt, since the default premium on short-term debt is endogenously lower than the default premium on long-term debt. There are two reasons for this lower default premium: First, long-term debt prices default risk over a longer time horizon and second, long-term debt creates an incentive misalignment between the firm owners and the long-term creditors that increases the probability of default. A pecking order theory arises: Very high productivity firms will issue long-term debt. Medium productivity firms will rely on internal funds. Low productivity firms will use short-term debt and very low productivity firms will issue equity. Since productivity is positively correlated with firm size, my model can match the stylized fact that small firms use a larger share of short-term debt.

The model can also match the stylized fact that the corporate debt maturity structure is pro-cyclical: If aggregate productivity decreases, the productivity distribution shifts to the left, such that firms will use more equity and short-term debt and less long-term debt. The model can also match the stylized fact that the debt maturity structure of small firms is more pro-cyclical than the debt maturity structure of large firms, because small firms tend to be in the region of the productivity distribution where firms issue short-term debt.

To understand the incentive misalignment better, consider the case of a firm that has some outstanding long-term debt and a positive default probability in the next period. This firm has to decide how much to invest today. For simplicity, assume that the investment is financed with internal funds. If the firm will default in some states of the world in the next period, an investment today constitutes an intertemporal transfer from the owners of the firm to the creditors, because the firm owners carry the entire cost of the investment today, while the benefit of the investment in default states in the next period accrues to the creditors. The firm has therefore an incentive to underinvest, if it does not care about the value of debt. For short-term debt, this incentive to underinvest does not arise, because the effect of the investment on the value of short-term debt is internalized by the firm through the effect on current bond issuance revenue. But for long-term debt, this effect is not internalized, because newly issued long-term debt constitutes only a fraction of all outstanding long-term debt. Knowing that the firm will not act in their interest ex post, long-term creditors will hence demand a higher default premium ex ante.

I calibrate the model to match several cross-sectional moments, among them the average share of long-term debt in the cross-section and the default rate. The calibrated model captures that firms endogenously use mostly long-term debt and that larger firms use a larger share of long-term debt. While I choose the model parameters to match cross-sectional moments, it can also match aggregate correlations, notably the pro-cyclicality of investment, the long-term debt share and long-term debt issuance and the counter-cyclicality of equity issuance, leverage and the default rate.

I show that issuance costs of debt and equity are important for match ing the level and dynamics of the debt maturity structure: Absent of equity issuance costs, there is no motive to issue short-term debt, because firms are never financially constrained. Firms will then exclusively use long-term debt, independent of their size. Absent of debt issuance costs firms find it optimal to avoid using long-term debt, also independent of their size. As a consequence, a model without debt and equity issuance costs cannot explain the size heterogeneity in the level and dynamics of the debt maturity structure. This results complements Crouzet (2015), who shows in a similar quantitative model without debt issuance costs that the optimality of short-term debt is a result of the incentive problem between the firm and the creditors. Finally, consumption risk helps to improve the model fit of maturity dynamics substantially, because it leads to a counter-cyclical slope term structure of interest rates over the business cycle.

I conclude by discussing the channels through which the corporate debt affects investment in the model. The model nests several channels discussed in the literature, namely the debt overhang channel and the credit constraints channel. These channels lead to under-investment relative to an unlevered firm. The debt overhang channel is the incentive to under-invest in the presence of long-term debt discussed above. The credit constraints channel arises because access to external funds is costly, such that firms will under-invest due to a higher cost of capital. I show that the debt overhang channel is quantitatively more important channel in my model, and it is primarily small firms which under-invest. Under-investment is quantitatively large: On average, the 25 percent smallest firms would choose an investment-capital ratio that is 37.5 percent higher if they had no debt. The debt overhang channel contributes 94 percent of this under-investment. Coming back to the motivation, it is exactly not short-term debt, but long-term debt, which causes under-investment in this model. Therefore, it is not a priori clear that a regulator should encourage firms to use more long-term debt, unless the different channels through which outstanding debt affects investment are well understood.

## 2 Review of the Literature

My paper is related to the literature on the cyclicality of the capital structure of firms. The closest paper to mine is Jungherr and Schott (2016). They focus on the maturity dynamics of aggregate liabilities from the financial accounts and find them to be *counter*-cyclical. There are two reasons why their data leads them to a different conclusion: First, their measure of short-term liabilities includes trade credit. This is important, since it is well known that during and after the financial crisis, there has been a collapse of trade and hence trade credit (Chor and Manova (2012)). This would show up as an increase in the maturity structure of debt in their data. In contrast, trade credit is a separate credit category in Compustat. Second, their measure of short-term debt includes all loans except mortgages and excludes all bonds. In contrast, loans and bonds in Compustat are classified according to their actual maturity. This is potentially an issue, because the fraction of loans in total debt financing is known to be pro-cyclical (de Fiore and Uhlig (2011)), which would show up as an increase in the maturity structure of debt during recessions in their data. In terms of the theoretical analysis, my focus is more on firm heterogeneity over the business cycle, whereas they focus on aggregate dynamics.

Various other characteristics of the business cycle dynamics of the capital structure have been studied in the literature, for example the choice of debt vs equity in Covas and Haan (2011) and Jermann and Quadrini (2012), of loans vs bonds in de Fiore and Uhlig (2011), Crouzet (2016) and Xiao (2017) or the choice between unsecured vs secured debt in Azariadis, Kaas, and Wen (2016).

My paper is also related to the literature on how financial risk amplifies business cycle fluctuations. Khan and Thomas (2013) develop a heterogeneous firm model in which firms issue secured short-term debt if they lack internal funds for investment. They do not consider default decisions. Furthermore, the debt issuance motive and hence leverage of firms in their model is negatively related to firm size.[1] By allowing firms to choose between different types of debt for different debt issuance motives, I can endogenously achieve a positive cross-sectional relation between firm size and leverage. Gilchrist, Sim, and Zakrajsek (2014) study uncertainty shocks in a heterogeneous agent model with financial frictions and defaultable, short-term debt. Gomes, Jermann, and Schmid (2016) develop a heterogeneous agent model with nominal long-term debt, in which inflation risk affects investment and default through a debt overhang channel. None of this papers consider a dynamic debt maturity choice.

There is also a literature in corporate finance which studies the maturity structure of corporate debt. He and Milbradt (2014) discuss the dynamics of debt maturity in a continuous time model. They solve their model in closed form and provide a theoretical discussion of the existence of various equilibria. My focus is different: I use a quantitative model to study the dynamics of debt maturity in a setting with rich cross-sectional heterogeneity of firms. Importantly, I discuss

[1] Specifically, this is true for their baseline firm setup. They improve their cross-sectional fit by adding a second type of firms to the model which instead of optimally choosing how much debt to issue use a simple rule that relates debt issuance to the square of the capital stock of the firm.

the role of aggregate uncertainty and investment for the maturity structure of debt. The focus on investment distinguishes my paper also from Chen, Xu, and Yang (2013), who investigate maturity choice in a He and Milbradt (2014)-type model with illiquid bond markets and endogenous default. They show that a liquidity-default spiral may lead firms to shorten their maturity structure during recessions, despite the existence of rollover risk. They do however not discuss the dynamics of maturity choice: conditional on the aggregate state variable, debt maturity is static in their model.

There is furthermore a large literature in corporate finance and asset pricing that studies the role of macroeconomic risk for the investment and financing decisions of firms in dynamic models, starting with Gomes (2001). Crouzet (2015) discusses the determinants of the debt maturity structure in a stylized model with frictionless investment, infinite equity issuance costs and no aggregate uncertainty. My paper builds on the model of Kuehn and Schmid (2014), who find than in a framework with long-term debt, investment options are crucial to account for the cross section of credit spreads. Hackbarth, Miao, and Morellec (2006) study leverage dynamics with long-term debt and aggregate uncertainty in a continuous-time framework and show that leverage is counter-cyclical. Bolton, Chen, and Wang (2013) study cash holdings in a single-factor model and find that firms issue external funds in times of low funding costs to build up precautionary cash buffers. The financing costs in their model are exogenous. In a similar framework, Eisfeldt and Muir (2014) use an estimated model to provide evidence of a separate financial factor for the build up of precautionary cash buffers through the issuance of external funds. Warusawitharana and Whited (2014) have a similar focus as Eisfeldt and Muir (2014), but provide a behavioral foundation for their financial factor in the form of equity misvaluation. Relative to these papers, I introduce a maturity choice and consider aggregate dynamics.

Finally, my paper is related to the literature which studies how the maturity structure of debt affects investment. Moyen (2007) discusses the role of different maturity structures for the quantitative importance of debt overhang. There are some differences between her framework and mine, most notably that in her model, firms hold either only short-term debt or long-term debt. Also, firms that hold long-term debt only issue a perpetual debt claim once at the beginning of their life, whereas firms in my model can dynamically adjust long-term leverage. Diamond and He (2014) also discuss the effect of different maturity structures on debt overhang. Investment in their model is however only a single expansion option.

I proceed as follows: In Section 3, I show the main facts about the maturity structure of debt of U.S. firms. In Section 4, I outline the model of the decision problem of an equity-value maximizing firm and the bond pricing equations. Section 5 illustrates the determinants of the maturity structure of debt. Section 6 discusses my calibration strategy. In Section 7, I present the numerical results for aggregate debt maturity dynamics. Section 9 concludes.

# 3 Stylized Facts about the Corporate Debt Maturity Structure

In this section, I describe the main facts that I want to explain with the model. All data are from Compustat. Long-term debt is defined as all debt with a residual maturity of at least one year. Debt includes notes, bonds, loans, credit lines and bankers acceptances. A detailed description of the data can be found in Section 6.1.

| | (1)<br>Mean,<br>LT Share | (2)<br>SD,<br>LT Share | (3)<br>FracAssets | (4)<br>FracDebt |
|---|---|---|---|---|
| 0% to 25% | 0.419 | 0.368 | 0.006 | 0.006 |
| 25% to 50% | 0.562 | 0.352 | 0.037 | 0.026 |
| 50% to 75% | 0.706 | 0.327 | 0.151 | 0.131 |
| 75% to 90% | 0.799 | 0.274 | 0.287 | 0.297 |
| 90% to 95% | 0.822 | 0.252 | 0.195 | 0.207 |
| 95% to 99% | 0.825 | 0.239 | 0.243 | 0.251 |
| 99% to 100% | 0.823 | 0.232 | 0.081 | 0.082 |
| 0% to 90% | 0.608 | 0.364 | 0.481 | 0.460 |
| All Firms | 0.632 | 0.359 | 1.000 | 1.000 |
| Aggregate | 0.845 | 0.012 | | |
| Observations | | | | 552987 |

Table 1: Summary statistics of the long-term debt share, by firm size.

First, consider the row ”Aggregate” of Table 1. The first column shows the share of long-term debt in total debt in the aggregate for the Compustat sample. On average, 84 percent of the outstanding debt of US firms has a residual maturity of at least one year. This is in stark contrast to the macroeconomic literature with financial frictions, which often assumes that the entire stock of corporate debt matures within the next quarter.

Second, consider the remaining rows of Column 1 of Table 1. These show the distribution of the long-term debt share conditional on the firm size distribution. A robust feature of the data is that larger firms tend to have a higher share of long-term debt. The average share of long-term debt of smallest quartile by firms size only holds on average 41.9 percent of debt as long-term debt, the largest one percent of firms holds on average 82.3 percent of debt debt as long-term debt. The second column shows that the standard deviation of the maturity structure within a given size quantile is decreasing in size, which is also a robust feature of the data. Not only do larger firms have a larger share of long-term debt, but their maturity structure is also less volatile.

In the last two columns, I show the fraction of assets and debt that firms within a given size quantile account for. These make it clear that while the distributions of debt and assets are very

| | (1) GDP(t-1) | (2) GDP(t) | (3) GDP(t+1) |
|---|---|---|---|
| 0% to 25% | 0.288** | 0.335*** | 0.283** |
| 25% to 50% | 0.265** | 0.408*** | 0.449*** |
| 50% to 75% | 0.621*** | 0.722*** | 0.760*** |
| 75% to 90% | 0.398*** | 0.547*** | 0.633*** |
| 90% to 95% | 0.135 | 0.227* | 0.286** |
| 95% to 99% | −0.324*** | −0.229* | −0.155 |
| 99% to 100% | −0.140 | −0.0633 | −0.006 32 |
| 0% to 90% | 0.467*** | 0.613*** | 0.690*** |
| All Firms | 0.227* | 0.396*** | 0.502*** |
| Observations | | | 116 |

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 2: Correlations of the detrended long-term debt share with detrended log real corporate sales, by firm size.

skewed, the smallest 90 percent account nonetheless for a nonnegligible fraction of 48 percent of assets and 46 percent of debt in the data.

The maturity structure of debt of nonfinancial US firms varies widely at the aggregate level. In Figure 1, I plot the share of long-term debt in total debt for all firms in the Compustat sample, computed using quarterly firm-level data from Compustat. It is evident that the share of long-term debt decreases during recessions and increases during expansions. For example, the long-term debt share during the financial crisis decreased from 88.75 percent in the first quarter of 2007 to 84.97 percent in the last quarter of 2008. In Table 2, I report the correlation coefficients between real corporate log sales and the share of long-term debt in total debt. Aggregate sales is my preferred measure of output, since it is the closest equivalent to output measured in my model and since it reflects cyclical fluctuations in the corporate sector better than real GDP. I present the correlations computed using real GDP or real corporate log profits in Appendix A. In the aggregate, the debt maturity structure is pro-cylical. This means that the fraction of due payments on outstanding debt increases exactly when internal funds are most valuable for firms, a seemingly puzzling observation.

The maturity structure also varies widely at the firm level. In Figure 2, I plot the time series for the long-term debt share of the smallest 50 percent, the firms in the 50-75, 75-90 and 90-100 percent size quantiles. I follow Covas and Haan (2011) in choosing these cutoffs. Figure 2 shows the long-term debt share of small firms is pro-cyclical, while the long-term debt share of large firms is counter-cyclical. This is confirmed by the correlations I report in Table 2: The long-term debt share of the firms up to the 90 percent size quantile is pro-cyclical, while for the largest 10 percent of firms, the long-term debt share is countercyclical. These differences are large: For the 50 percent to 75 percent quantile, the contemporaneous correlation between the long-term debt share and real output is 0.7222 and is significantly different from zero at the 0.1 percent level. For

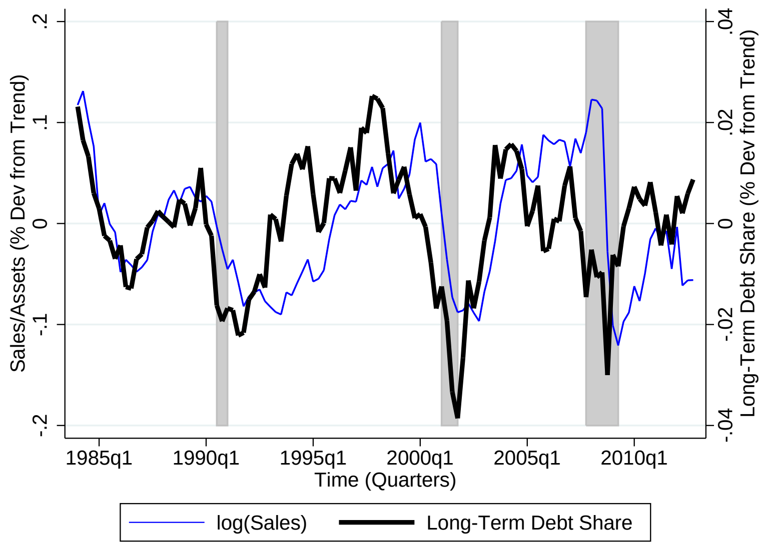


Figure 1: Figure 1 shows the cyclical component of the aggregate share of long-term debt in total debt for the bottom 90 % non-financial firms by size in the Compustat Quarterly database. The thin line is real aggregate sales. Both series are detrended with a linear-quadratic trend. The shaded areas indicate the NBER recession episodes.

the 95 to 99 percent size quantile, the contemporaneous correlation is -0.229 and is significantly different from zero at the 1 percent level.

To explain these patterns of the debt maturity structure is interesting for two reasons: First, they provide additional evidence about which financial frictions determine the capital structure decisions both at the micro-economic and macro-economic level. If firms predominantly rely on long-term debt, rollover costs are more important to the firms, if they use mostly short-term debt, default premia and default incentives are more important. Second, the maturity structure itself is a factor that determines through which channels outstanding debt affects the investment decisions of firms. Specifically, an important question that arises is whether rollover costs or debt overhang are more important for the investment behavior of financially constrained firms. Answering this question crucially depends on understanding the optimal maturity choices of firms, because firms take the effect of their current debt maturity choices on future investment decisions into account.

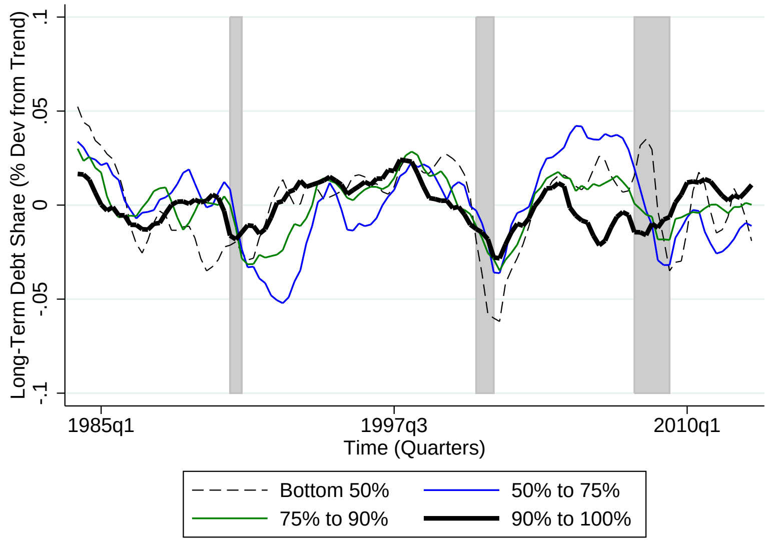


Figure 2: Figure 1 shows the cyclical component of the aggregate share of long-term debt for non-financial firms in the Compustat Quarterly database for different size bins. All series are detrended with a linear-quadratic trend. For better visibility, all series for this figure are smoothed using a moving average filter with two lags. The shaded areas indicate the NBER recession episodes.

# 4 Model

Consider the model of an infinitely-lived firm $i$ which makes both real and financial decisions in each period $t$. On the real side, the firm can invest in capital $K_{i,t+1}$, subject to capital adjustment costs. On the financial side, the firm can issue a short-term bond $B^S_{i,t+1}$, a long-term bond $B^L_{i,t+1}$ and pay out dividends $D_{i,t} > 0$ or issue equity $D_{i,t} < 0$. Debt and equity issuance are subject to non-convex adjustment costs. The firm can default on its outstanding liabilities. In this case, the firm is liquidated.

Bond prices take into account the future default probability of the firm as well as the expected loss in default. Bonds and equity charge a risk premium which reflects the co-movement between the exogenous consumption of the representative household and their endogenous cash flows.

In section 4.1, I present the firm problem. In section 4.2, I lay out how the bond prices are determined. I derive the stochastic discount factor from an exogenous consumption process and household preferences in section 4.3.

## 4.1 Firm Problem

### 4.1.1 Objective Function

The objective function of the firm is the present value of dividends. Future cash flows are discounted with a stochastic discount factor $\Lambda(C_t, C_s)$, where $C_t$ denotes aggregate consumption at time $t$.

The present value of dividends at time $t$ is given by

$$\mathbb{E}_t\left[\sum_{s=t}^{T}\Lambda(C_t, C_s)D_{i,s}\right]. \tag{4.1}$$

$T$ denotes the period in which it is optimal for the firm to default. In what follows, I will write the model in recursive form, using the notation that $X_t = X$ and $X_{t+1} = X'$ for any variable $X$.

### 4.1.2 Technology

The profit function of the firm is given by

$$\Pi(K_i, \tilde{A}_i, Z) = \tilde{A}_i Z K_i^{\alpha} - \psi, \tag{4.2}$$

where $\alpha < 1$ describes returns to scale at the firm level. $\tilde{A}_i$ is the idiosyncratic productivity of the firm, which evolves according to

$$\begin{aligned} \ln \tilde{A}_i' &= \rho \ln \tilde{A}_i + \sigma^A \varepsilon_i^A, \\ \varepsilon_i^A &\sim \text{i.i.d.} N(0,1). \end{aligned} \tag{4.3}$$

The idiosyncratic productivity shocks $\varepsilon_i^A$ are uncorrelated over time and across firms. $Z$ is the aggregate productivity, which also follows a first-order autoregressive process:

$$\begin{aligned} \ln Z' &= \rho \ln Z + \sigma^Z \varepsilon^Z, \\ \varepsilon^Z &\sim \text{i.i.d.} N(0,1). \end{aligned} \tag{4.4}$$

In addition, firms have to pay a fixed cost $\psi$. These costs arise only if the firm continues, independent of whether the firm produces or not. They can be interpreted as for example maintenance costs and administrative expenses. Such a fixed production cost is used, for example, in Gomes (2001).

Since idiosyncratic and aggregate productivity have the same persistence, they can be collapsed in the state variable $A_i = \tilde{A}_i Z$, which evolves according to

$$\ln A_i' = \rho \ln A_i + \sigma^A \varepsilon_i^A + \sigma^Z \varepsilon^Z. \tag{4.5}$$

The conditional density function of $A_i'$ is denoted by $f(A_i'|A_i)$.

### 4.1.3 Investment

Capital follows the standard law of motion:

$$K_i' = (1-\delta)K_i + I_i, \tag{4.6}$$

where $\delta$ is the depreciation rate and $I_i$ is investment. When installing new capital or selling old capital, the firm has to incur a quadratic capital adjustment cost with functional form

$$AC(K_i, K_i') = \frac{\theta}{2}\left(\frac{K_i'}{K_i} - 1 + \delta\right)^2 K_i. \tag{4.7}$$

With these capital adjustment costs, I capture in a simple way that capital is illiquid. This form of capital adjustment costs is common in the investment literature, see for example Hayashi (1982). It is widely used in the corporate finance literature, for example in Bolton, Chen, and Wang (2013) and Eisfeldt and Muir (2014). Furthermore, Bloom (2009) reports that at the firm level, quadratic capital adjustment costs yield a good description of firm level investment behavior, even if the capital adjustment costs are non-convex at the plant level.

### 4.1.4 Debt Financing

The firm can issue short-term debt $B_{S,i}$ and long-term debt, $B_{L,i}$. Short-term debt takes the form of a one-period contract. Long-term debt takes the form of a contract with stochastic maturity $\mu$. This formulation is a common way to model long-term debt without introducing too many state variables in the model. It is for example used in the corporate finance literature in Hackbarth, Miao, and Morellec (2006) and Kuehn and Schmid (2014), but also in the literature on sovereign debt, for example in Hatchondo and Martinez (2009). The level of long-term debt therefore evolves according to

$$B_{L,i}' = (1-\mu)B_{L,i} + J_i, \tag{4.8}$$

where $J_i$ denotes long-term debt issuance. Long-term debt cannot be repaid early: $J_i \geq 0$. Short-term debt and long-term debt pay a coupon $c$.

Debt is risky, because firms can default. Issuance occurs at state-contingent prices $Q_S$ and $Q_L$ for short-term debt and long-term debt respectively. I will explain how these bond prices are determined in equilibrium in section 4.2.

There is a linear issuance cost $\xi$ for debt. Debt issuance costs are equal for short-term debt issuance and long-term debt issuance. The functional form for debt issuance costs is given by

$$DIC(M_i, B_i, M_i', B_i') = \xi\left(|(1-M_i')B_i'| + |M_i'B_i' - (1-\mu)M_iB_i|\right). \tag{4.9}$$

These issuance costs can be interpreted as flotation fees for new bond issues or bank fees for new loans. Such costs can arise in addition to the endogenous default premium. Typically, the literature considers either a combination of fixed and linear debt issuance costs or one of the two. An example for the former is Kuehn and Schmid (2014). A model which uses a purely linear issuance cost is Titman and Tsyplakov (2007).

### 4.1.5 Corporate Income Tax

There is a proportional corporate income tax $\tau$. Consistent with the U.S. tax code, taxable income is calculated as income less operating costs, depreciation and interest expense. This implies there is a tax benefit for investment as well as debt issuance. As a consequence, from the perspective of the shareholder, debt issuance is cheaper than equity issuance, because a fraction of interest expense is paid by the government. There is therefore an incentive for the firm to increase leverage up to the point where issuance costs and the default premium on newly issued debt cancel out the tax benefit of debt. This trade-off between the tax shield and the default premium is an important determinant of leverage, see for example Kraus and Litzenberger (1973) or Fischer, Heinkel, and Zechner (1989). In this model, it is the reason why large, financially unconstrained firms issue debt.

### 4.1.6 Dividends and Equity Financing

Dividends are given residually by the budget constraint of the firm,

$$\begin{aligned} D_i = (1-\tau) & \underbrace{\left[\Pi(K_i, A_i) - \delta K_i - \psi - (c^S(1-M_i) + c^L M_i)B_i\right]}_{\text{Taxable Income}} \\ & - \underbrace{((1-M_i) + \mu M_i)B_i}_{\text{Principal Repayment}} + \underbrace{K_i - K_i'}_{\text{Gross Investment}} - AC(K_i, K_i') + \underbrace{Q_S(1-M_i')B_i'}_{\text{Revenue from ST Debt Issuance}} \\ & + \underbrace{Q_L(M_i'B_i' - (1-\mu)M_iB_i)}_{\text{Revenue from LT Debt Issuance}} - \underbrace{DIC(M_i, B_i, M_i', B_i')}_{\text{Debt Issuance Cost}}, \end{aligned} \tag{4.10}$$

where $\tau$ is the corporate tax rate, $\delta$ is the deprecation rate. $\psi$ is a fixed cost. $c^S$ and $c^L$ are the coupons on short-term debt and long-term debt, respectively. $\mu$ is the repayment rate on long-term debt $AC(K_i, K_i')$ is a quadratic capital adjustment cost. $Q_S$ and $Q_L$ are the endogenous, state-dependent bond prices for short-term debt and long-term debt, respectively. $DIC(M_i, B_i, M_i', B_i')$ is a debt issuance cost.

Dividends $D_i$ can be negative. In this case, the firm issues seasoned equity and has to pay an equity issuance cost. These costs are meant to capture monetary costs, such as underwriting fees, but also non-monetary costs like managerial effort and signaling costs conveyed through the issues. The equity issuance cost consists of a fixed component $\phi_0$ and a linear component $\phi_1$, such that average cost of issuing equity is decreasing in the size of the issue. The functional form is

$$EIC(D_i) = (\phi_0 + \phi_1 |D_i|)\, \mathbb{1}_{(D_i<0)}. \tag{4.11}$$

This form of equity issuance costs is for example used in Gomes (2001), Cooper and Ejarque (2003) and Hennessy and Whited (2007).

### 4.1.7 Firm Problem and Default

For convenience, I use the total amount of outstanding debt, $B_i = B_{S,i} + B_{L,i}$ and the fraction of long-term debt, $M_i = B_{L,i}/B_i$, as state variables. I collect the endogenous state variables of firm $i$ in the tuple $\mathcal{S}_i = (K_i, B_i, M_i)$. $K_i$ is the capital stock of the firm, $B_i$ is the total amount of outstanding debt, and $M_i$ is the fraction of outstanding debt that was issued in the form of long-term debt.

If the firm decides not to default, its problem is then to maximize the present value of dividends by choosing the capital stock $K_i'$, debt $B_i'$, the fraction of long-term debt $M_i'$, and dividends $D_i$. The value function of a continuing firm $i$ can be summarized as

$$\begin{aligned} V^C(\mathcal{S}_i, A_i, C) = \max_{K_i', B_i', M_i', D_i} & \Big\{ D_i - EIC(D_i) \\ & + \mathbb{E}\left[ \Lambda(C, C') \int_{-\infty}^{\infty} V(\mathcal{S}_i', A_i', C') f(A_i'|A_i) dA_i' | C \right] \Big\}, \end{aligned} \tag{4.12}$$

subject to the budget constraint 4.10 and the constraints $K_i' \geq 0$, $B_i' \geq (1-\mu) M_i B_i$ and $(1-\mu) M_i B_i / B_i' \leq M_i' \leq 1$. The last two constraints arise due to the assumption that long-term debt cannot be repurchased.

The value of equity in default is 0. The total value of equity is then

$$V(\mathcal{S}_i, A_i, C) = \max \left\{ V^C(\mathcal{S}_i, A_i, C), 0 \right\}. \tag{4.13}$$

Default occurs if the firm does not repay its debt, either for strategic reasons or because the firm cannot raise sufficient funds to repay outstanding liabilities. Since equity owners can simply walk away if the value of owning the firm becomes negative, the value of the firm in default to shareholders is 0.

## 4.2 Bond Markets

### 4.2.1 Payouts to Creditors

The bond markets are competitive. Bonds are discounted with the same discount factor as equity. Both bonds pay a fixed coupon. Coupons are calculated according to

$$c_S = c_L = \frac{1}{\beta} - 1. \tag{4.14}$$

That is, coupons are chosen such that the values of risk-free bond prices in the absence of aggregate risk are both equal to 1.

If the firm does not default, the payment to the short-term creditors is $1 + c^S$. The payment to the long-term creditors is $\mu + c^L$. The outstanding fraction $(1 - \mu)$ of the long-term bond is valued by bond-holders at the end-of period bond price $Q'_L$, such that the value of owning one unit of owning one unit of a long-term bond that is not in default is given by $\mu + c^L + (1 - \mu)Q'_L$.

If the firm decides to default on the outstanding debt, the firm is liquidated after production has taken place. There is a cross-default clause: a default on short-term debt triggers a default on long-term debt and vice versa. In addition, there is a pari passu clause: bond holders have equal claims on the liquidation value of the firm, independent of the maturity of their bond. The liquidation value consists of the profits plus the depreciated capital stock. Consistent with the U.S. tax code, it is not possible to deduct interest expense from taxable income in default.

A complication of the model with quadratic capital adjustment costs is that capital is illiquid. I interpret the capital adjustment cost as a primitive of the model that has also to be paid if the firm is liquidated. Therefore, it is not optimal to uninstall the entire capital stock of the firm, because beyond some optimal disinvestment $I^* < 0$, the adjustment cost from disinvesting an additional unit of the capital stock outweigh the gain. This optimal level of disinvestment is the solution to

$$\max_I = -I - \frac{\theta}{2}\left(\frac{I}{K}\right)^2 K.$$

The solution is given by $I^* = -\frac{K}{\theta}$. With this disinvestment, the adjustment cost is given by $\frac{K}{2\theta}$, such that creditors receive $I^* - \frac{K}{2\theta}$ from the liquidation of the capital stock in the current period. The recovery value per unit of the bond is therefore

$$R(\mathcal{S}_i, A_i) = \chi \max\left(\frac{(1-\tau)(\Pi(K_i, A_i) - \psi - \delta K_i) - I^* - \frac{K}{2\theta}}{B_i}, 0\right). \tag{4.15}$$

Note that the model has an endogenous liquidation loss on capital due to the capital adjustment cost. Hennessy and Whited (2007) and Kuehn and Schmid (2014) assume an exogenous liquidation loss.

### 4.2.2 Bond Pricing Equations

It is useful to define a default threshold set for idiosyncratic productivity. The default threshold set is implicitly defined by

$$a^*(\mathcal{S}_i, C) = \left\{A \in \mathcal{A} : V^C(\mathcal{S}_i, A, C) = 0\right\}. \tag{4.16}$$

Suppose that the value function is strictly increasing in idiosyncratic productivity. Then, for each $(\mathcal{S}_i, C)$, the default threshold is unique and equation 4.16 defines a function for the default threshold productivity: For $A \leq a^*(\mathcal{S}_i, C)$, the firm will default, for $A > a^*(\mathcal{S}_i, C)$, the firm will continue.

The bond price functions are then

$$\begin{aligned} q_S(\mathcal{S}_i', A_i, C) = \mathbb{E}\Bigg[\Lambda(C, C')\Bigg(&\int_{a^*(\mathcal{S}_i', C')}^{\infty} (1 + c_S) f(A_i'|A_i) dA_i' + \\ &\int_{-\infty}^{a^*(\mathcal{S}_i', C')} R(\mathcal{S}_i', A_i') f(A_i'|A_i) dA_i'\Bigg) |C\Bigg], \end{aligned} \tag{4.17}$$

$$\begin{aligned} q_L(\mathcal{S}_i', A_i, C) = \mathbb{E}\Bigg[\Lambda(C, C')\Bigg(&\int_{a^*(\mathcal{S}_i', C')}^{\infty} (\mu + c_L + (1-\mu) Q_L') f(A_i'|A_i) dA_i' \\ &+ \int_{-\infty}^{a^*(\mathcal{S}_i', C')} R(\mathcal{S}_i', A_i') f(A_i'|A_i) dA_i'\Bigg) |C\Bigg], \end{aligned} \tag{4.18}$$

$$Q_L' = q_L(\mathcal{S}_i'', A_i', C').$$

That is, bond prices reflect the future default probabilities and the value of the firm in default. Future cash flows are discounted at the stochastic discount factor. Notably, while the short-term bond price only reflects the next period default probability, the long-term bond price captures the entire future path of default probabilities through its dependence on $Q_L'$.

## 4.3 Stochastic Discount Factor

Equity and debt payouts are discounted with the stochastic discount factor

$$\Lambda(C, C') = \beta \left(\frac{C'}{C}\right)^{-\sigma}. \tag{4.19}$$

This discount factor is derived from a household whose consumption process moves with the aggregate productivity process: Household preferences are time-separable with discount factor $\beta$. The period felicity function has a constant relative risk aversion $\sigma$. The utility function therefore takes the recursive form

$$U(C) = \frac{C^{1-\sigma} - 1}{1-\sigma} + \beta \mathbb{E}_{C'}\left[U(C')|C\right] \tag{4.20}$$

The consumption process is driven by aggregate productivity and a process $\tilde{C}$ that is uncorrelated to productivity. $\tilde{C}$ represents cyclical movements in aggregate consumption that are unrelated to productivity, for example due to other economic shocks. It also follows a first order autoregressive process:

$$\ln \tilde{C}' = \rho \ln \tilde{C} + \sigma^C \varepsilon^C. \tag{4.21}$$

Aggregate consumption depends on productivity $Z$ with weight $\lambda_1$ and on $\tilde{C}$ with weight $\lambda_2$:

$$\ln C = \lambda_0 + \lambda_1 \ln Z + \lambda_2 \ln \tilde{C}. \tag{4.22}$$

Combining equations 4.19 and 4.22 yields the stochastic discount factor. This discount factor leads to a risk premium in the model, which has been established to be an important component of bond yields.[2] While the risk premium in the model is exogenous, default premia do reflect the endogenous default decisions of firms in the model, and are therefore endogenously determined as well. I will come back to the bond pricing equation in section 4.2.

Aggregate consumption is exogenous, which by itself is not important for the model results. What matters is that as a consequence, the term structure of risk free interest rates is exogenous. Note however that I allow for correlation between aggregate productivity and aggregate consumption. The exogeneity of the term structure of risk-free interest rates is plausible, since I only model the markets for risky nonfinancial corporate debt and equity. The markets for government debt or household debt, for example, are outside the model. According to the Financial Accounts of the US, corporate business debt constituted only 17.9 percent of all outstanding debt in 2015. The market value of equity ofnonfinancial domestic corporations constituted about 27.2 percent of the net wealth of the US for the same year. In addition, the assumption of an exogenous aggregate consumption or an exogenous stochastic discount factor is common in the asset pricing and corporate finance literature. It is for example used in Campbell and Cochrane (1999).

## 4.4 Equilibrium

The recursive competitive equilibrium for this economy is given by a set of policy functions $h : (\mathcal{S}_i, A_i, C) \to \mathbb{R}^3$ for capital, debt, and the share of long-term debt; a default policy characterized by $a^* : (\mathcal{S}_i, C, ) \to \mathbb{R}$, value functions $V_C : (\mathcal{S}_i, A_i, C) \to \mathbb{R}$ and $V : (\mathcal{S}_i, A_i, C) \to \mathbb{R}$; and bond price functions $q_S : (\mathcal{S}_i', A_i, C) \to \mathbb{R}$ and $q_L : (\mathcal{S}_i', A_i, c) \to \mathbb{R}$, such that for every firm $i$:

- for any $(\mathcal{S}_i, A_i, C) \in \mathbb{S} \times \mathbb{A} \times \mathbb{C}$, given $q_S$ and $q_L$, $h(\mathcal{S}_i, A_i, C)$, maximizes the continuation problem in equation 4.12, with the solution to the firm problem given by $V_C(\mathcal{S}_i, A_i, C)$.

[2]See for example Chen (2010) or Bhamra, Kuehn, and Strebulaev (2010)

- for any $(\mathcal{S}_i, A_i, C) \in \mathbb{S} \times \mathbb{A} \times \mathbb{C}$, given $q_S$ and $q_L$, the firm chooses $a^*$ such that $V^C(\mathcal{S}_i, A_i, C) \geq 0 \iff A_i \geq a^*(\mathcal{S}_i, C)$ and

$$V(\mathcal{S}_i, A_i, C) = \begin{cases} V^C(\mathcal{S}_i, A_i, C) & \text{if } A_i > a^*(\mathcal{S}_i, C) \\ 0 & \text{if } A_i \leq a^*(\mathcal{S}_i, C) \end{cases}$$

- for any $(\mathcal{S}'_i, A_i, C) \in \mathbb{S} \times \mathbb{A} \times \mathbb{C}$, given $h$, $V_C$, $V_D$ and $V$, $q_S(\mathcal{S}'_i, A_i, C)$ and $q_L(\mathcal{S}'_i, A_i, C)$ are the solutions to the bond pricing equations 4.17 and 4.18.

The first equilibrium condition states that the firm makes optimal investment and debt issuance decisions, taking the bond prices as given, the second states that the firm makes an optimal default decision, taking the bond prices as given, and the third condition states that bond price schedules incorporate the true default probability of the firm, taking firm policies as given.

# 5 The Determinants of Debt Maturity

In this section, I will outline the determinants of the maturity structure of a single firm. First, I will explain the different channels that determine the maturity structure of the firm. Then, I will discuss how aggregate consumption shocks affect the optimal maturity choice.

I will denote $Q_L = q_L(\mathcal{S}'_i, A_i, C)$, $Q_S = q_S(\mathcal{S}'_i, A_i, C)$, $V = V(\mathcal{S}_i, A_i, C)$ and $V^C = V^C(\mathcal{S}_i, A_i, C)$ to economize on notation.

Throughout this section, I assume that the value function $V$ is once differentiable in $K$, $B$, $M$ and $A$ and the bond price functions $Q_S$ and $Q_L$ are differentiable in $K'$, $B'$, $M'$ and $A$. I further assume that the short-term and long-term bond prices are weakly increasing in $A$, i.e. $\frac{\partial Q_S}{\partial A} \geq 0$ and $\frac{\partial Q_L}{\partial A} \geq 0$. I do not make these assumptions when I solve the model numerically later on.

## 5.1 The General Case

The optimal maturity choice is given by the first order condition with respect to $M'$ in the continuation problem of the firm presented in equation 4.12. I denote as $\lambda_D$ the contemporaneous shadow cost of internal funds. $\lambda^D$ is positive if the firm has to issue costly equity to avoid bankruptcy.[3] Further, I denote as $\lambda_{M,0}$ and $\lambda_{M,1}$ the multipliers for the constraints $M' \geq (1 - \mu MB)/B'$ and $M' \leq 1$, respectively.[4]

[3]The non-convex issuance costs for debt and equity require the introduction of this additional multiplier. With respect to equity issuance, the firm problem can be split into three sub-problems: In problem (1) the firm pays dividends. Then, $\lambda_D = 0$. In problem (2), the firm does not pay dividends, but also does not issue equity. Then, $D_i = 0$ becomes one of the equilibrium conditions of the model and $\lambda_D$ is found residually from the first-order condition for investment. Finally, in problem (3), the firm issues equity. In that case, $\lambda_D = \phi_1$. The value function $V^C$ is the envelope of these three subproblems.

[4]The lower bound on $M'$ arises from the assumption of no debt repurchases.

The first order condition for $M'$ is

$$\begin{aligned}\frac{\partial V^C}{\partial M'} = \Bigg[&(Q_L - Q_S)B' + \\ &\frac{\partial Q_S}{\partial M'}(1-M')B' + \frac{\partial Q_L}{\partial M'}\left(M'B' - (1-\mu)MB\right)\Bigg](1+\lambda_D) + \\ &\mathbb{E}\left[\Lambda(C,C')\frac{\partial V'}{\partial M'}|A,C\right] = \lambda_{M,1} - \lambda_{M,0}. \end{aligned} \tag{5.1}$$

The envelope condition yields

$$\begin{aligned}\frac{\partial V}{\partial M} &= (\mu + (1-\mu)Q_L - 1 + (1-\mu)\xi)B(1+\lambda_D) \\ &= (1-\mu)(Q_L - 1 + \xi)B(1+\lambda_D). \end{aligned} \tag{5.2}$$

if the firm is in a no default state. Combining 5.1 and 5.2, we get

$$\begin{aligned}\frac{\partial V^C}{\partial M'} = \Bigg[&(Q_L - Q_S)B' + \\ &\frac{\partial Q_S}{\partial M'}(1-M')B' + \frac{\partial Q_L}{\partial M'}\left(M'B' - (1-\mu)MB\right)\Bigg](1+\lambda_D) + \\ &\mathbb{E}\left[\Lambda(C,C')(1-\mu)(Q_L' - 1 + \xi)B'(1+\lambda_D')|A,C\right] = \lambda_{M,1} - \lambda_{M,0}. \end{aligned} \tag{5.3}$$

The interpretation of this first order condition is that the benefit and the cost of marginally increasing the share of long-term debt must be equal. This decision concerns only the *maturity structure* of debt in the next period, but not the *leverage*. The total amount of debt issuance and hence the leverage choice is given by the first-order condition for total debt, $B'$.

The choice of $M'$ is a portfolio choice how to allocate borrowing among different types of debt for a given amount of total debt $B'$. The cost of issuing marginally more long-term debt is here the opportunity cost of issuing marginally less short-term debt.[5]

## 5.2 Optimal Maturity Choice without Consumption Risk

First, I will focus on the main trade-off between short-term debt and long-term debt in a setup without consumption risk. That is, $\lambda_1 = \lambda_2 = 0$. In this case, the discount factor $\Lambda(1,1)$ equals $\beta$ and the risk-free bond prices for short-term debt and long-term debt are both equal to 1.

The first order condition contains many different terms, so I will consider three different types of firms: In the first case, I discuss the case of a firm which never defaults. This is the case of a low leverage, high productivity firm. For such a firm, neither short-term debt nor long-term debt are risky. In the second case, the firm may default only after the next period. The firm hence has

[5] Of course, $B'$ and $M'$ are in the end jointly determined.

no short-run default risk, but some long-run default risk. Then, short-term debt is risk-free while long-term debt is risky. In the third case, the firm also has some short-run default risk, such that both short-term debt and long-term debt are risky.

In this way, I can introduce the channels that affect maturity choice one by one. I will first focus on the main trade-off between rollover costs and default risk and then add other channels.

### 5.2.1 Case I: No Default Risk

In the case of no default risk and no consumption risk, bond prices do not include a default premium: $Q_S = Q_L = 1$. In addition, bond prices are insensitive to changes in the maturity structure of the firm: $\partial Q_S / \partial M' = \partial Q_L / \partial M' = 0$. The first order condition reduces to

$$\frac{\partial V^C}{\partial M'} = \beta(1-\mu)\xi B' \mathbb{E}\left[(1+\lambda'_D)|\mathcal{Y}\right] = \lambda_{M,1} > 0.$$

This optimality condition states that the benefit of increasing the long-term debt share is that the firm has to pay less rollover costs, $\xi$, in the next period if it uses relatively more long-term debt. Consequently, a firm that can issue debt without risk wants to set the long-term debt share as high as possible: $M' = 1$, which implies a positive multiplier $\lambda_{M,1} > 0$.

### 5.2.2 Case II: Risk-Free Short-Term Debt, Risky Long-Term Debt

If default cannot occur in the next period, but may occur after the next period, the short-term bond price does not include a default premium, while the long-term bond price does: $Q_S = 1, Q_L < 1$. The short-term bond price remains insensitive to the maturity structure of the firm: $\partial Q_S / \partial M' = 0$. The first-order condition for the long-term debt share becomes[6]

$$\begin{aligned}\frac{\partial V^C}{\partial M'} = &\left[\underbrace{(Q_L - 1)\, B'}_{\text{Change, Marginal Revenue}} + \underbrace{\frac{\partial Q_L}{\partial M'}\left(M'B' - (1-\mu)MB\right)}_{\text{Change, Intramarginal LT Revenue}}\right](1+\lambda_D) + \\ &\beta\mathbb{E}\left[(1-\mu)(\underbrace{1-Q'_L}_{\text{Change, Endogenous Rollover Cost}} + \underbrace{\xi}_{\text{Change, Exogenous Rollover Cost}})B'(1+\lambda'_D)|A_i\right] \\ &= 0.\end{aligned}$$

There are four important terms: The first two terms describe the change in the revenue from issuing new bonds if the firm decides to issue long-term bonds instead of short-term bonds. These are the costs of issuing a higher share of long-term debt. The last two terms describe the change in future rollover costs if the firm issues marginally more long-term debt. These are the benefits of

[6] From this section onward, I focus on the case of an interior solution, so $\lambda_{M,1} = \lambda_{M,0} = 0$.

issuing a higher share of long-term debt. Relative Case I in Section 5.2.1, the first three terms are new, whereas the exogenous rollover cost also arises in a situation with risk-free short-term *and* long-term debt.

The first term is the *change in the marginal revenue*: If the firm issues marginally more debt as risky long-term debt instead of risk-free short-term debt, it has to incur a default premium, captured by the term $(Q_L - 1)$. If this default premium is high, the firm prefers to issue short-term debt by setting a low $M'$. The default premium arises entirely from default risk after the next period. It is instructive to rewrite the bond price for long-term debt as a function of the bond price for short-term debt:

$$\begin{aligned} Q_L &= Q_S + (1-\mu)\beta \underbrace{\mathbb{E}\left[(Q'_L - 1)\,|A_i, C\right]}_{\leq 0} \\ &= 1 + (1-\mu)\beta \underbrace{\mathbb{E}\left[(Q'_L - 1)\,|A_i\right]}_{\leq 0} \leq 1. \end{aligned}$$

The bond price for long-term debt differs from the bond price for short-term debt, because it incorporates future default risk, which is embedded in the future long-term bond price $Q'_L$.

The second term captures how a marginally larger long-term debt share affects the *intramarginal revenue from long-term debt issuance*. This effect arises, since a higher share of long-term debt today adversely affects firm policies in the future: The derivative $\frac{\partial Q_L}{\partial M'}$ is given by

$$\frac{\partial Q_L}{\partial M'} = \beta(1-\mu)\mathbb{E}\left[\frac{\partial Q'_L}{\partial K''}\frac{\partial K''}{\partial M'} + \frac{\partial Q'_L}{\partial B''}\frac{\partial B''}{\partial M'} + \frac{\partial Q'_L}{\partial M''}\frac{\partial M''}{\partial M'}|A_i\right].$$

Since the envelope theorem does not apply to the bond price, the effects of current choices on future choices enter the current bond price.[7] It is not possible to find analytic expressions for $\frac{\partial K'}{\partial M}$, $\frac{\partial B'}{\partial M}$ and $\frac{\partial M'}{\partial M}$. In the numerical solution to my model, the policy function for the next period capital stock $K''$ is decreasing in $M'$, while the policy function for the next period level of debt $B''$ is increasing in $M'$. This is because the firm acts only in the interest of the shareholder and does therefore not internalize the effect of its decisions in default states. Since the benefits of investment and the costs of debt issuance arise in the future, the larger the share of firm value that accrues to long-term debt, the lower will be investment and the higher debt issuance. As a consequence, a higher share of long-term debt will in general increase default risk after the next period, which drives down the price of long-term debt today. In this case, $\frac{\partial Q_L}{\partial M'} < 0$.

The third term is the *endogenous rollover cost*. If the firm issues short-term debt, it has to repay the entire amount at the face value in the next period. If the firm instead issues long-term debt, it can roll over a fraction $(1-\mu)$ at the market value. The market value of long-term debt

[7] The reason for why the envelope theorem does not apply to the bond prices is that firms do maximize over the market value of equity, but not over the market value of debt.

is below the face value because of future default risk. Therefore, being able to roll over long-term debt at the market value leads to lower rollover costs for the firm.

In this case, the firm trades off roll-over costs of short-term debt against the long-term default premium and the negative incentive effect of long-term debt issuance. This is the main trade-off I consider and therefore deserves to be discussed in more detail. Consider the case of a firm with low productivity and a low capital stock. This firm issues debt due to a high value of internal funds, that is, since $\lambda_D$ is high. It will have a low probability of long-term survival, and hence face a high long-term default premium on long-term debt. This is captured by the term $Q_L - 1 < 0$. Furthermore, by issuing long-term debt, such a firm would decrease the incentive for future investment, since a part of that investment would essentially be an intertemporal transfer of current shareholder funds to future bondholder funds. This is captured by the term $\frac{\partial Q_L}{\partial M'} < 0$. If these effects outweigh the rollover costs, such a liquidity-constrained firm will choose a low long-term debt share.

Now consider a firm with a high capital stock and a high productivity. The motive for such a firm to issue debt is not a high value of $\lambda_D$, but the tax benefit of debt. Such a firm has a low long-term default probability, and hence $Q_L - 1$ and $\frac{\partial Q_L}{\partial M'}$ will be small. Therefore, such a firm will mostly be concerned about the rollover costs of debt and will issue long-term debt.

So in this model, firms endogenously use different types of debt for different motives: Liquidity constrained firms use short-term debt, while firms which care mostly about the tax benefit of debt use long-term debt. These two motives will later on give rise to the cyclical dynamics of debt maturity: Intuitively, the fraction of firms which issue short-term debt due to liquidity constraints increases in a recession, while the fraction of firms which issue long-term debt due to the tax benefit decreases. The motive to issue debt due to a liquidity shortfall is counter-cyclical, while the tax benefit of debt net of the default premium is pro-cyclical. As a consequence, the aggregate long-term debt share in the model is pro-cyclical.

### 5.2.3 Case III: Risky Short-Term Debt and Long-Term Debt

If short-term debt is also risky, the first order condition for $M'$ is

$$\frac{\partial V^C}{\partial M'} = \Bigg[ \underbrace{(Q_L - Q_S)B'}_{\text{Change, Marginal Revenue}} + \underbrace{\frac{\partial Q_S}{\partial M'}(1-M')B'}_{\text{Change, Intramarginal ST Revenue}} + \underbrace{\frac{\partial Q_L}{\partial M'}\left(M'B' - (1-\mu)MB\right)}_{\text{Change, Intramarginal LT Revenue}} \Bigg] (1+\lambda_D) + \beta \mathbb{E}\left[(1-\mu)(1-Q_L'+\xi)B'(1+\lambda_D')|A_i\right] = 0.$$

In this case, there are two new terms relative to the case in which only long-term debt is risky: First, the short-term bond price and the long-term bond price also incorporate a premium for the possibility of default in the next period. Similar to the case with only long-term default risk, the long-term bond price can be written as

$$Q_L = Q_S + (1-\mu)\beta\mathbb{E}\left[(Q'_L - 1)\,\mathbb{1}_{(A'>a^*)}|A_i\right].$$

Hence, $Q_L - Q_S = (1-\mu)\beta\mathbb{E}\left[(Q'_L - 1)\,\mathbb{1}_{(A'>a^*)}|A_i\right]$, which is almost exactly the same term as in case II. The difference is that long-term default risk is only relevant when the firm does not default in the next period. The premium for default after the next period is the only change in the marginal revenue that arises when the firm increases the long-term debt share.

Second, if short-term debt is risky, the short-term bond price is also sensitive to the maturity structure of the firm. What matters for the short-term bond price is how a change in the maturity structure of debt affects the probability of default of the firm in the next period: The derivatives of the short-term and long-term bond prices in the case of risky short-term debt and long-term debt are given by:

$$\frac{\partial Q_S}{\partial M'} = \underbrace{[1 + c - R(K,B,a^*,1)]}_{\text{Change in Repayment}} \qquad \underbrace{\frac{\frac{\partial V'}{\partial M'}}{\frac{\partial V'}{\partial A'}} f(a^*|A)}_{\text{Change in Next Period Default Probability}} \qquad > 0,$$

$$\frac{\partial Q_L}{\partial M'} = \underbrace{[\mu + c + (1-\mu)Q'_L - R(K,B,a^*,1)]}_{\text{Change in Repayment}} \qquad \underbrace{\frac{\frac{\partial V'}{\partial M'}}{\frac{\partial V'}{\partial A'}} f(a^*|A)}_{\text{Change in Next Period Default Probability}}$$
$$+ \beta(1-\mu)\underbrace{\int_{a^*}^{\infty}\left(\frac{\partial Q'_L}{\partial K''}\frac{\partial K''}{\partial M'} + \frac{\partial Q'_L}{\partial B''}\frac{\partial B''}{\partial M'} + \frac{\partial Q'_L}{\partial M''}\frac{\partial M''}{\partial M'}\right) f(A'|A)dA}_{\text{Change in Long-Term Repayment and Default Probability}}.$$

Interestingly, increasing the long-term debt share can have opposite effects on the bond prices: A higher long-term debt share increases the price of short-term debt, because it reduces the probability of default in the next period. However, a higher long-term share also reduces investment in the next period and increases debt issuance in the next period, such that default risk after the next period may actually increase. There are therefore two effects of a higher long-term debt share on the long-term bond price: The next period default probability decreases, but the default probability after the next period increases. In the quantitative version below, the latter effect dominates.

In Figure 3, I depict the bond price as a function of the long-term share for two different levels of debt. All other variables are chosen to be the same. The parameters are from my baseline calibration. In the left panel, the level of debt chosen is high and as a consequence, the next period default probability is high. The long-term bond price and the short-term bond price both increase for the most part in response to an increase in the long-term debt share. This is because

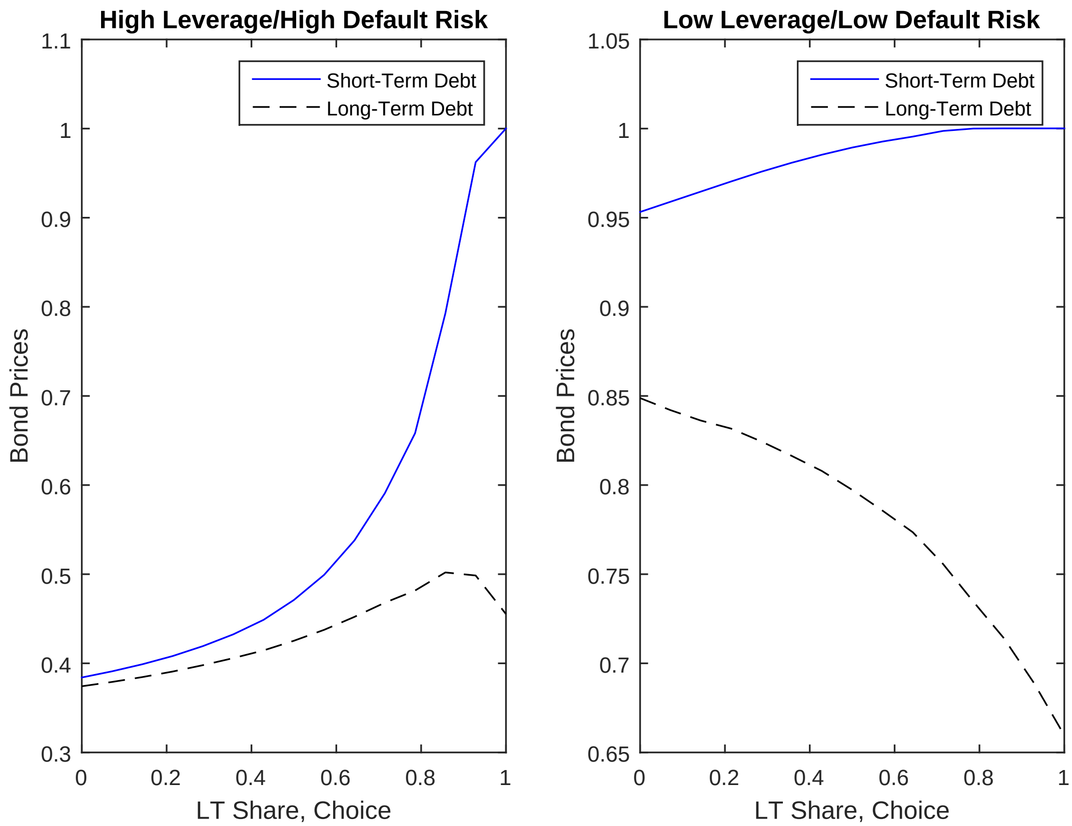


Figure 3: Bond prices as a function of the long-term debt share choice $M'$. The left panel depicts a situation with high debt and high next period default risk, the right panel depicts a situation with low debt and low next period default risk. Capital, productivity and the aggregate state are the same in the left and the right panel.

such an increase lowers the next period default probability, which is here the dominant effect.

In the right panel, the level of debt is low and therefore the next period default probability is low. The long-term bond price decreases mostly if the long-term debt share increases. This is because a higher long-term debt share in this case leads to a higher long-term probability of default. In contrast to that, the short-term bond price increases, as in the left panel, monotonically with a higher long-term debt share.

In summary, if short-term debt and long-term debt are risky, the trade-off is fundamentally the same as in the case of risk-free short-term debt and risky long-term debt. Relative to the case with risk-free short-term debt, there is an additional benefit of issuing long-term debt, since a higher long-term debt share reduces the next period default probability. However, in the quantitative model below, firms will still prefer to issue short-term debt if they are liquidity constrained.

## 5.3 The Effect of Consumption Risk on the Optimal Maturity Structure

In the presence of consumption risk, the difference in bond prices can be decomposed into two terms:

$$Q_L - Q_S = \underbrace{Q_L^{RF} - Q_S^{RF}}_{\text{Risk-Free Term Spread}} + \underbrace{(Q_L - Q_L^{RF}) - (Q_S - Q_S^{RF})}_{\text{Long-Term Default Premium}}.$$

The new first term is the difference in risk-free bond prices. The second term is the long-term default premium. Bonds yield a fixed stream of income which is $1+c$ for short-term bonds and $\mu+c$ per period for long-term bonds. In the presence of aggregate risk, a marginal unit of consumption more in a recession is more valuable than a marginal unit of consumption in an expansion. Hence, conditional on being in a recession, the fact that shocks are mean reverting imply that risk-free bond prices in a recession are lower than in an expansion. Further, the positive autocorrelation of the shocks implies that the risk-free long-term bond price in the recession is lower than the risk-free short-term bond price.

With this decomposition, the first order condition for the long-term debt share can be rewritten as

$$\begin{aligned}\frac{\partial V^C}{\partial M'} = \Bigg[ & \left((Q_L^{RF} - Q_S^{RF}) + (Q_L - Q_L^{RF} - Q_S + Q_S^{RF})\right) B' + \\ & \frac{\partial Q_S}{\partial M'}(1-M')B' + \frac{\partial Q_L}{\partial M'}\left(M'B' - (1-\mu)MB\right)\Bigg](1+\lambda_D) + \\ & \mathbb{E}\left[\Lambda(C,C')\frac{\partial V'}{\partial M'}|A_i\right] = 0.\end{aligned}$$

Consumption risk is important for two reasons. First, the risk-free short-term bond price is higher than the risk-free long-term bond price in a recession and lower in expansions. In other words, the term structure of risk-free bond yields is downward sloping in expansions and upward sloping in recessions. Without the shadow cost of internal funds, $\lambda_D$, this would not matter, since firms discount future cash-flows at the same discount factor as creditors. However, a firm that places a high value of internal funds today versus tomorrow, i.e. with $\lambda_D > \lambda_D'$, will prefer short-term debt relative to long-term debt more in a recession.

Second, as outlined in Chen (2010) and Bhamra, Kuehn, and Strebulaev (2010), in the presence of aggregate risk, there is an additional risk-premium on the default premium if the default probability is higher in recessions. Then, cash-flows from the firms to creditors are low exactly when creditors value cash flows highly. This negative covariance between the default probability and the marginal utility of consumption increases the default risk premium on debt.

In summary, consumption risk introduces a new channel for the determination of the maturity structure through the time variation in the term structure of risk-free rates and emphasizes the importance of the default channel relative to the rollover channel. The fact that the short-term bond price is higher than the long-term bond price in recessions makes short-term debt even more attractive for liquidity constraints in recessions. This channel should amplify the counter-cyclicality of short-term debt issuance. In addition, the higher and more cyclical default premia reduce the net tax benefit of long-term debt issuance, particularly in recessions. This effect should amplify the pro-cyclicality of long-term debt issuance.

# 6 Mapping the Model to the Data

In this section, I will describe the data and the selection of parameters. I divide the set of parameters into three subsets: I take the first set from the literature. I estimate the second set of parameters directly from aggregate data. The third set of parameters is chosen to match cross-sectional data moments in simulations of the model. I solve the model using value function iteration. The interested reader will find a detailed description of the solution algorithm in Appendix C.

## 6.1 Data

### 6.1.1 Cross-Sectional Data

Firm data are from the merged CRSP Compustat Quarterly North America database. The observation unit is a firm-quarter. I use data from the first quarter of 1984 to the last quarter of 2012. I exclude regulated firms (SIC code 4900-4999), financial firms (SIC code 6000-6999) and non-profit firms (SIC code 9000-9999) from my sample. Furthermore, I exclude those observations which do not report total assets and those which report either negative assets or a negative net capital stock.

I calculate all flow variables from the cash flow statements of firms. Investment is capital expenditure minus sales of property, plant and equipment. Short-term debt issuance is defined as change in current debt. Long-term debt issuance is long-term debt issuance minus long-term debt reduction. Equity issuance is sale of common and preferred stock minus purchase of common and preferred stock minus dividends. All flows are normalized by lagged total assets. I calculate the share of long-term debt to total debt as long-term debt divided by long-term debt plus current debt. I follow Whited (1992) to calculate market leverage: It is defined as the book value of short-term debt plus the market value of long-term debt divided by the sum of the market value of debt and equity. To calculate the market value of long-term debt, I use the method by Bernanke et al. (1988). The market value of common stock is defined as the share price times the number of shares. The market value of preferred stock is defined as the current dividend for preferred stock divided by the current federal funds rate.

| Name | Value | Role | Reference |
|---|---|---|---|
| Parameters from the Literature | | | |
| $\beta$ | $1.04^{-1/4}$ | Discount Factor | 4% Annual Risk Free Rate |
| $\sigma$ | 2 | Risk Aversion | |
| $\mu$ | 0.05 | Long-Term Debt Repayment Rate | 5 Year Maturity |
| $\tau$ | 0.14 | Corporate Income Tax Rate | Graham (2000) |
| $\chi$ | 0.8 | Recovery Rate | Kuehn and Schmid (2014) |
| $\theta$ | 4 | Capital Adjustment Cost | Warusawitharana and Whited (2014) |
| $\alpha$ | 0.35 | Prod Function Curvature | Moyen (2007) |
| $\rho_A$ | 0.95 | Persistence, Idiosyncratic Productivity | Katagiri (2014) |
| $\sigma_A$ | 0.1 | Volatility, Idiosyncratic Productivity | Katagiri (2014) |
| Calibrated Parameters | | | |
| $\xi$ | 0.0025 | Linear Debt Issuance Cost | Average Long-Term Debt Share |
| $\phi_0$ | 0.1 | Fixed Equity Issuance Cost | Size, Equity Issuance |
| $\phi_1$ | 0.04 | Linear Equity Issuance Cost | Frequency, Equity Issuance |
| $\psi$ | 1.8 | Fixed Production Cost | Annual Default Rate |
| Estimated Parameters | | | |
| $\sigma^Z$ | 0.0071 | Volatility, Aggregate Productivity | |
| $\sigma^C$ | 0.0058 | Volatility, Consumption | |
| $\lambda^Z$ | 0.113 | Productivity Coefficient in Consumption | |
| $\lambda^C$ | 1 | Consumption Coefficient | |

Table 3: Parameter Choices. This table shows all model parameters, grouped into three categories: The first category shows parameters chosen from the literature, the second category shows parameters from the literature, the third category shows parameters taken from a production function estimation using a dynamic panel data estimator.

The data for the default rate are taken from Ou et al. (2011). I use the default rate for the largest sample, namely all firms from 1920 to 2011, which corresponds to 1.1 percent.

### 6.1.2 Aggregate Data

I use the productivity time series from Fernald (2014) to compute productivity shocks and real personal consumption expenditure from the Bureau of Economic Analysis (BEA) to compute the stochastic process for consumption. For real GDP, I either use real GDP from the BEA or aggregate sales from Compustat. All series are detrended with a quadratic trend, as are the other time series I aggregate from Compustat data.

## 6.2 Parameter Choices

### 6.2.1 Parameters from the Literature

For the preferences of the representative household, I use a time preference rate, $\frac{1}{\beta}-1$, of 4 percent per year and a risk aversion coefficient $\sigma$ of 2. These values are standard in the macroeconomic

| | (1) ln(Z) | (2) ln(C) |
|---|---|---|
| L.ln(Z) | 0.9662*** | -0.0263 |
| | (34.14) | (-0.32) |
| ln(Z) | | 0.1128 |
| | | (1.44) |
| L.ln(C) | | 0.9172*** |
| | | (29.37) |
| Observations | 115 | 115 |

$t$ statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 4: Estimation of aggregate processes.

literature. In the baseline calibration, I set the maturity of long-term debt to 5 years. This implies a quarterly repayment rate $\mu$ of 5 percent. Following Graham (2000), I set the corporate income tax rate $\tau$ to 14 percent. This is substantially lower than the true marginal US corporate income tax rate, but corresponds to about the actual average tax rate of firms in the US. I set the recovery rate in default to 0.8 as in Kuehn and Schmid (2014). I set the autocorrelation of the productivity shock to 0.95 and the standard deviation of idiosyncratic productivity to 0.1, which is similar to Katagiri (2014). I use $\alpha = 0.35$ for the production function curvature. I set the capital adjustment cost $\theta$ to 4. Warusawitharana and Whited (2014) estimate an adjustment cost between 4 and 6 for large firms in Compustat. Bloom (2009) estimates an adjustment cost between 4 and 10 in his purely quadratic adjustment cost specifications.

### 6.2.2 Estimated Parameters

There are three parameters that govern aggregate uncertainty in the model: The volatility $\sigma^Z$ of the aggregate productivity shock, the volatility $\sigma^C$ of the residual consumption shock and the coefficient of productivity in consumption, $\lambda^Z$. The persistence of the aggregate shocks is equal to the persistence of the idiosyncratic shocks to keep the state space tractable. I estimate the followings regression on detrended productivity and consumption data:

$$\ln Z_t = \rho^Z \ln Z_{t-1} + \eta_t^Z$$
$$\ln C_t = \lambda^Z \rho^Z \ln Z_{t-1} + \lambda^Z (\ln Z_t - \rho^Z \ln Z_{t-1}) + \rho^C \ln C_{t-1} + \eta_t^C$$

The results are in Table 4. The standard deviation of $\eta_t^Z$ is 0.0071, the standard deviation of $\eta_t^C$ 0.0058. Hence, I set $\lambda^Z = 0.113$, $\sigma^Z = 0.0071$, and $\sigma^C = 0.0058$. The regression further shows that setting $\rho^Z = \rho^C = 0.95$ is well within the range of plausible parameters.

### 6.2.3 Parameters Set to Match Cross-Sectional Moments

I choose the debt issuance cost parameter $\xi$, the equity issuance cost parameters $\phi_0$ and $\phi_1$ and the fixed production cost $\psi$ to match a set of cross-sectional moments. I choose to match the average share of long-term debt, the average size and frequency of equity issuance, the cumulative one year default rate and the cross-sectional standard deviation of the investment capital ratio.

The average share of long-term debt is informative about the linear debt issuance cost $\xi$: the higher is $\xi$, the less attractive is short-term debt relative to long-term debt for the purpose of the tax benefit, and the higher is the share of long-term debt. The size and frequency of equity issuance are informative about the equity issuance costs $\phi_0$ and $\phi_1$: A higher fixed cost $\phi_0$ leads to a larger size of equity issuance conditional on issuance and to a lower frequency of equity issuance. A higher linear cost $\phi_1$ leads to a smaller size of equity issuance conditonal on issuing, and to a lower frequency of equity issuance. The default rate helps to identify the fixed cost $\psi$: A higher value for $\psi$ implies a higher default rate.

Table 3 shows the parameters resulting from the moment matching exercise. The fixed cost $\psi$ corresponds to 10.54% of the steady state capital stock of the model. Kuehn and Schmid (2014) use a linear production cost that corresponds to 4% of the lagged capital stock.

| | Model | Data |
|---|---|---|
| Long-Term Debt Share, Mean | 76.894 | 63.200 |
| Equity Issuance, Size | 7.549 | 12.200 |
| Equity Issuance, Frequency | 24.068 | 10.100 |
| Def Rate | 0.776 | 1.146 |

Table 5: Model fit.

Table 5 reports targeted moments from a numerical simulation. The model matches the average long-term debt share well. The default rate in the model is close to the default rate in the data, with the model default rate at about 1 percent and the data default rate at about 1.1 percent. The model can also match the size and frequency of equity issuance. Overall, while the match between the model and the data is by no means perfect, it delivers plausible numbers for all targeted moments.

## 6.3 Simulation Procedure

To compute the firm-level moments from the numerical solution, I simulate a panel of 5000 firms for 5200 quarters. I use a burn-in period of 200 quarters. Defaulted firms are replaced with new firms which draw a new productivity shock from the unconditional productivity distribution. Changing these values does not affect the results. These firms start out with zero debt and a very small capital stock. I exclude defaulted firms and firms for the first two quarters after entry when I calculate the statistics for the model.

In the data, short-term debt is defined as debt with a maturity of less than 1 year. This definition includes long-term debt with a residual maturity of less than 1 year. In the model, debt with a maturity of less than 1 year is given by

$$(1 - M_{i,t})B_{i,t} + (1 - (1 - \mu)^4)M_{i,t}B_{i,t}. \tag{6.1}$$

Therefore, the share of long-term debt in total debt at the firm level is given by

$$\frac{B_{i,t} - (1 - M_{i,t})B_{i,t} - (1 - (1 - \mu)^4)M_{i,t}B_{i,t}}{B_{i,t}} = (1 - \mu)^4 M_{i,t}. \tag{6.2}$$

Market leverage at the firm level is calculated as the market value of debt divided by the market value of debt plus the ex dividend value of equity:

$$\frac{Q_{S,i,t}B_{S,i,t+1} + Q_{L,i,t}B_{L,i,t+1}}{V_{i,t} - D_{i,t} - EIC(D_{i,t}) + Q_{S,i,t}B_{S,i,t+1} + Q_{L,i,t}B_{L,i,t+1}}. \tag{6.3}$$

Book leverage is given by

$$\frac{B_{S,i,t+1} + B_{L,i,t+1}}{K_{i,t+1}}. \tag{6.4}$$

Finally, the Market-to-Book ratio is

$$\frac{V_{i,t} - D_{i,t} - EIC(D_{i,t}) + Q_{S,i,t}B_{S,i,t+1} + Q_{L,i,t}B_{L,i,t+1}}{K_{i,t+1}}. \tag{6.5}$$

# 7 Results

## 7.1 Aggregate Results

In this section, I report the implications of the model for the dynamics of the aggregate maturity structure. The aim of this section is to show that the model, which is parameterized to match firm-level moments, can also match aggregate dynamics. First, I will report aggregate means and correlations. Second, I consider how well the model fits the cross-sectional distributions of leverage and the debt maturity structure in the data and third how well it replicates the dynamics of the debt maturity structure across the size distribution of firms. Explaining the heterogeneity in the level and dynamics of the maturity structure for firms of different size classes is a key contribution of the paper relative to the existing literature.

### 7.1.1 Aggregate First Moments

In Table 6, I report aggregate summary statistics. I report moments for the aggregate long-term debt share, which is the key variable the model should explain, as well as book and market leverage,

| | Data | Model |
|---|---|---|
| Long-Term Debt Share, Mean | 0.845 | 0.763 |
| Long-Term Debt Share, StDev | 0.012 | 0.014 |
| Book Leverage, Mean | 0.433 | 0.525 |
| Book Leverage, StDev | 0.033 | 0.032 |
| Market Leverage, Mean | 0.228 | 0.199 |
| Market Leverage, StDev | 0.040 | 0.011 |
| Investment/Capital, Mean | 0.032 | 0.037 |
| Investment/Capital, StDev | 0.005 | 0.003 |

Table 6: Aggregate summary statistics

which together with the long-term debt share fully characterize the capital structure of the firms in the model. Since investment is a key reason why firms want to issue debt in this model, I also report moments for aggregate investment.

The mean and standard deviation of the aggregate long-term debt share are well matched by the model. As in the data, the aggregate fraction of debt maturing within the next year is relatively low. The standard deviation of the long-term debt share in the model is similar to the data. Overall, despite the fact that that the model only has two contracts with different debt maturity, compared to the limitless contracting options in the data, it explains the aggregate maturity structure in the data well.

Aggregate book leverage in the model is much higher than in the data, but the volatility is similar. Book leverage is of separate interest, because it is much easier to measure in the data than market leverage, because it does not depend on prices and hence expectations. It is however market leverage which is relevant for corporate decisions, since market leverage reflect expectations about the fraction of future cash flows accruing to the creditors of the firm. The model can also match the aggregate market leverage, but the volatility of market leverage is too low. This is because market leverage is calculated using stock prices in the data, and the model does not generate sufficient stock price volatility.

The model can also match the mean and standard deviation of the aggregate investment capital ratio well. The mean of the investment capital ratio is by construction equal to the deprecation rate, which corresponds surprisingly well to the average aggregate investment capital ratio. The volatility of investment in the data is higher than in the model. Arguably, the model misses many important drivers of aggregate investment like uncertainty shocks (Bloom (2009)) or investment-specific technology shocks (Fisher (2006)).

### 7.1.2 Aggregate Correlations

In Table 7, I report the correlations of the aggregate time series of the model with my preferred measure of output, aggregate sales. This is the better measure of corporate cash flows than real

| | Data | Model |
|---|---|---|
| Long-Term Debt Share | 0.386 | 0.324 |
| Book Leverage | −0.099 | −0.400 |
| Market Leverage | −0.277 | −0.551 |
| Short-Term Debt Issuance/Capital | 0.179 | −0.295 |
| Long Term Debt Issuance/Capital | 0.510 | 0.222 |
| Debt Issuance/Capital | 0.383 | −0.256 |
| Equity Issuance/Capital | −0.390 | −0.510 |
| Investment/Capital | 0.390 | 0.670 |

Table 7: Aggregate correlations with log(output).

GDP and is also strongly pro-cyclical and hence a good proxy variable for the business cycle. Aggregate output in the model is computed as

$$Y_t = \sum_{i=1}^{N} A_{i,t} Z_t K_{i,t}^{\alpha}.$$

The model matches the signs of the correlations of the long-term debt share, book leverage and market leverage with output. In terms of flows, it matches the signs of investment, long-term debt issuance, equity issuance and the default rate with output.

Importantly, the long-term debt share in the model is pro-cyclical. The model matches the correlation in the data quantitatively well.

Both book leverage and market leverage in the model are counter-cyclical. Market leverage has a more negative correlation with output than book leverage, because the market value of equity is more volatile than the market value of debt. This is also true in the data, although the model overstates the cyclicality of leverage in the data. A negative correlation between market leverage and output implies that the fraction of cash flows accruing to creditors increases, which increases debt overhang problems in recessions and leads to a counter-cyclical default rate.

Short-term debt issuance is counter-cyclical in the baseline model, in contrast to the data. The main motive for short-term debt issuance are financial constraints in the form of a shadow cost of internal funds $\lambda_D$. As financial constraints are more severe in recessions, short-term debt issuance is higher during recessions. My measure of short-term debt issuance in the model is also not perfect, since I only observe the change in current debt, which also includes maturing long-term debt. Since I do not observe long-term debt maturing and repurchased separately, this problem cannot be easily remedies.

In line with the data, long-term debt issuance is pro-cyclical, since long-term debt is issued by unconstrained firms due to the tax benefit of debt. In a recession, default premia increase, while the tax benefit is constant. As a consequence, firms will issue less long-term debt.

Total debt issuance is counter-cyclical in the baseline model, also in contrast with the data.

The reason for this is that counter-cyclical short-term debt issuance is too high relative to pro-cyclical long-term debt issuance. A model in which the maturity of long-term debt $\mu$ is also a choice variable could potentially better match both the dynamics of the long-term debt share and debt issuance.

Equity issuance is counter-cyclical in the baseline model as well as in the data. Financially constrained firms who cannot or do not want to issue debt will issue equity instead. This result is in line with the results in Jermann and Quadrini (2012), who also report counter-cyclical equity issuance. I interpret equity issuance strictly as liquidity injections due to a lack of internal funds, and measure it in the data accordingly. Other motives for equity issuance outlined in Fama and French (2005) exist, but those are not captured by this model. This is of relevance, since Covas and Haan (2011) document that other measures, which include for example stock compensation for employees or equity swaps during mergers are actually pro-cyclical.

Finally, the investment-capital ratio in the model is pro-cyclical, as it is in the data. The relatively weak pro-cyclicality is surprising. This is due to the fact that I scale investment by the last period capital ratio, which is also pro-cyclical.

Overall, while the model predicts a too high counter-cyclicality of short-term debt issuance and hence total debt issuance, it replicates aggregate dynamics in the data well. In particular, it matches the main fact that the aggregate maturity structure of debt is pro-cyclical.

## 7.2 Untargeted Cross-Sectional Moments

| | Long-Term Debt Share | | Market Leverage | |
|---|---|---|---|---|
| | Data | Model | Data | Model |
| Mean | 63.210 | 76.896 | 22.862 | 24.472 |
| StDev | 35.914 | 8.186 | 26.292 | 16.830 |
| Correlation with Size | 0.414 | 0.635 | 0.068 | −0.647 |
| Correlation with Market To Book | −0.190 | 0.059 | −0.236 | −0.295 |
| Correlation with Book Leverage | −0.030 | −0.508 | 0.413 | 0.903 |

Table 8: Cross-sectional summary statistics.

In Columns 1 and 2 of Table 8, I show moments of the cross-sectional distribution of the long-term debt share of the model. I report the mean, standard deviation and the median, as well as correlations with firm size and the market to book ratio, which are important determinants of leverage in the corporate finance literature, as well as the correlation with book leverage. Size is an important proxy variable for financial constraints, while the market to book ratio of the firm measures its growth opportunities and is highly correlated with its productivity. Note that I only target the mean for the long-term debt share. All numbers report percentages.

The model can accurately capture that the mean for the long-term share is lower than the median. The standard deviation of the long-term debt share is too low relative to the data. One reason is that in the data, there is a non-negligible fraction of firms which use exclusively short-term debt. This cannot be rationalized in this model.

The model can also correctly account for the positive correlation between the share of long-term debt and firm size, which is crucial to match aggregate moments. The correlation with the market to book ratio is low, as in the data. However, the correlation with the market to book ratio is too weak in the model. The correlation with book leverage is too high. One issue is that the motive to use leverage in the model is too weak for large firms, such that small firms will simultaneously have a high book leverage and a short debt maturity structure, which leads to this strong negative correlation.

In Columns 3 and 4 of Table 8, I show the distribution of market leverage. The model can match the average market leverage as well as the standard deviation of the market leverage well. However, the model predicts a strong negative correlation between market leverage and firm size, while this correlation is weakly pro-cyclical in the data. The reason is that due to the fixed production cost, it is mostly small firms which issue debt because they are liquidity constrained. Firms in which growth options constitute a large fraction of the firm value use less leverage, as shown by the negative correlation between the market-to-book ratio and market leverage. As expected, the correlation between book leverage and market leverage is positive both in the model and the data.

## 7.3 The Cross-Section over the Cycle

| | Data | Model |
|---|---|---|
| 0% to 25% | 0.333 | 0.541 |
| 25% to 50% | 0.473 | 0.204 |
| 50% to 75% | 0.658 | 0.123 |
| 75% to 90% | 0.519 | 0.227 |
| 90% to 95% | 0.269 | 0.100 |
| 95% to 99% | −0.214 | 0.053 |
| 99% to 100% | −0.086 | 0.027 |

Table 9: Correlations between the long-term debt share and log(output) across the firm size distribution.

To investigate how firms adjust their issuance in response to macroeconomic shocks, I group the firms from the simulated panel into 7 size quantiles and compute the correlation of the long-term debt share with output within each of these size quantiles.

As in the data, the cyclicality of the long-term debt shares varies substantially by firm size. Both in the model and in the data, it is the medium sized firms for which the long-term debt share is the most sensitive to cyclical fluctuations. Small firms in the model are constrained and use predominantly short-term debt even during expansions, while medium sized firms are unconstrained during expansions, but constrained during recessions. The larger the firms, the less likely it is that they are financially constrained during a recession, and the lower is their need to issue short-term debt.

These results support the hypothesis that the main driver of short-term debt issuance and hence a short maturity structure are liquidity constraints. In addition, the model also lends support to the theory that the fraction of liquidity constrained firms increases during a recession.

Note also that for the largest firms, the maturity structure is basically acylical both in the model and in the data. The puzzling firms, from the perspective of the model, are the firms in the 95 to 99 percent size quantile for which the maturity structure of debt is *counter*-cyclical. Jungherr and Schott (2016) construct a model in which the share of long-term debt is counter-cylical due to debt dilution: As firms in their model issue more debt, they shorten the maturity structure of debt because newly issued debt constitutes a very large share of newly issued debt, which aligns incentives between firm owners and creditors. Creditors prefer short-term debt because long-term debt creates debt dilution issues. The distinction between their model and my model is that my model has a clear pecking order in which short-term and long-term debt are issued for different reasons, whereas in their model, short-term debt and long-term debt are to some extent substitutes.

One possible way to reconcile these results is if fixed costs constitutes a large share of debt issuance costs, such that it is beneficial for large firms to also issue short-term debt due to the tax benefit. However, such a model, large firms would choose a short debt maturity structure, which runs counter to the observation that the debt maturity structure is strictly increasing in firm size in the data.

## 7.4 Robustness

In this section, I want to further illustrate the importance of issuance costs for the main results. In addition, I investigate the role of consumption risk in the model.

Table 10 shows aggregate moments for alternative model specifications. In the first alternative model in column (3), I eliminate debt issuance costs. Specifically, I set $\xi$ to zero. In this case, the exogenous component of the debt rollover costs in the first order condition 5.3 is zero. Hence, this model substantially reduces rollover costs. In the absence of debt issuance costs, firms prefer to mostly issue short-term debt to attain the tax benefit of debt. Short-term debt avoids the problem of the incentive misalignment between shareholders and creditors with respect to future investment and debt issuance decisions. As a consequence, the decision short-term debt can be issued at lower default risk premia and hence at lower costs. The model without issuance costs is at odds with the data, because it predicts a very low long-term debt share.

The second alternative model in column (4) has no equity issuance costs. I set $\phi_0$ and $\phi_1$ equal

| | (1) Data | (2) Baseline | (3) No DIC | (4) No EIC | (5) $\sigma = 0$ |
|---|---|---|---|---|---|
| Long-Term Debt Share, Mean | 0.845 | 0.763 | 0.620 | 0.681 | 0.561 |
| Long-Term Debt Share, StDev | 0.012 | 0.014 | 0.058 | 0.050 | 0.009 |
| Book Leverage, Mean | 0.433 | 0.525 | 0.547 | 1.169 | 0.515 |
| Book Leverage, StDev | 0.033 | 0.032 | 0.024 | 0.191 | 0.010 |
| Market Leverage, Mean | 0.228 | 0.199 | 0.221 | 0.438 | 0.213 |
| Market Leverage, StDev | 0.040 | 0.011 | 0.013 | 0.068 | 0.004 |
| Investment/Capital, Mean | 0.032 | 0.037 | 0.037 | 0.037 | 0.037 |
| Investment/Capital, StDev | 0.005 | 0.003 | 0.003 | 0.004 | 0.001 |

Table 10: Aggregate moments, alternative models. "No DIC" refers to the model without debt issuance costs, "No EIC" to the model without equity issuance costs and $\sigma = 0$ to the model with a risk neutral representative household.

to zero. Without equity issuance costs, firms can issue equity at all time at the price of one. This shuts down the variation in the shadow cost of internal funds $\lambda^D$ over time. As a consequence, the motive to issue short-term debt shoud decrease in this model. Firms also issue a substantially higher amount of debt than in the baseline model: Leverage is about twice as high as in the baseline model.

For the third alternative model version in column (5), I consider the case of a risk-neutral representative household. I set the risk aversion parameter $\sigma$ in this model to zero. Without risk-aversion, the household's consumption risk is irrelevant for firm decisions. In particular, there is no term structure of risk-free interest rates and default is priced at the risk-neutral default premium. In terms of aggregate first moments, eliminating the risk aversion of the representative household substantially reduces the long-term debt share and the volatility of all aggregate variables.

| | (1) Data | (2) Baseline | (3) No DIC | (4) No EIC | (5) $\sigma = 0$ |
|---|---|---|---|---|---|
| Long-Term Debt Share | 0.386 | 0.324 | 0.346 | 0.475 | 0.463 |
| Book Leverage | -0.099 | -0.400 | -0.531 | -0.578 | -0.129 |
| Market Leverage | -0.277 | -0.551 | -0.635 | -0.598 | -0.615 |
| Short-Term Debt Issuance/Capital | 0.179 | -0.295 | -0.308 | -0.441 | -0.145 |
| Long Term Debt Issuance/Capital | 0.510 | 0.222 | 0.231 | -0.123 | 0.708 |
| Debt Issuance/Capital | 0.383 | -0.256 | -0.297 | -0.436 | -0.039 |
| Equity Issuance/Capital | -0.390 | -0.510 | -0.403 | -0.248 | -0.747 |
| Investment/Capital | 0.390 | 0.670 | 0.648 | 0.566 | 0.875 |

Table 11: Aggregate correlations, alternative models. "No DIC" refers to the model without debt issuance costs, "No EIC" to the model without equity issuance costs and $\sigma = 0$ to the model with a risk neutral representative household.

Table 11 shows aggregate correlations for alternative model specifications. The model without debt issuance cost overall matches the signs of the correlations in the data well. However, in contrast to the data the correlation of short-term debt issuance with output and total debt issuance with output are negative. In addition, even with a small issuance cost, the correlation of leverage with output is much lower. In addition, the correlation of equity issuance with output is lower. The small debt issuance cost however does not substantially affect the correlation between output and investment.

The model without equity issuance costs predicts a much lower correlation between total debt issuance and output. In particular, even the correlation between long-term debt issuance and output becomes negative. A potential explanation is that debt dilution is much more prominent in this model, since leverage is almost twice as high. The incentive to dilute debt is stronger in recessions. The correlation between equity issuance and output is much more negative in this model. Finally, market leverage is strongly counter-cyclical and the long-term debt share in this model is very pro-cyclical. This shows that variation in the shadow cost of internal funds $\lambda^D$ is an important driver of aggregate debt maturity dynamics in this model.

The model with a risk-neutral representative household generates in general much stronger correlations, since the aggregate productivity shock is now the only shock which drives aggregate dynamics. The counter-cyclicality of equity issuance increases dramatically, while the counter-cyclicality of short-term debt issuance decreases, the pro-cyclicality of long-term debt issuance increases. This shows that the term structure implied by the stochastic discount factor of the representative investor matters both for the optimal long-term leverage and the choice between equity and short-term debt when the firm is liquidity constrained.

# 8 Debt Maturity, Leverage and Investment

## 8.1 Debt Overhang or Credit Constraints?

I now discuss how debt overhang affects investment and debt issuance decisions in the model, and how the maturity structure of debt affects the severity of debt overhang. In the model high leverage can lead to under-investment relative to an otherwise identical unlevered firm. I define under-investment $I^-$ as the difference between the investment decision of an unlevered and a levered firm which are otherwise identical. Let $I^*(K, B, M, A, C)$ denote the optimal investment policy of a firm. Then, under-investment can be computed as

$$I^-(K, B, M, A, C) = I^*(K, 0, M, A, C) - I^*(K, B, M, A, C). \tag{8.1}$$

Under-investment can arise for two reasons: First, a high level of debt maturing in the current period reduces the amount of internal funds available to the firm, such that the firm has to access some costly external funding. Due to the higher cost of capital compared to the case with internal

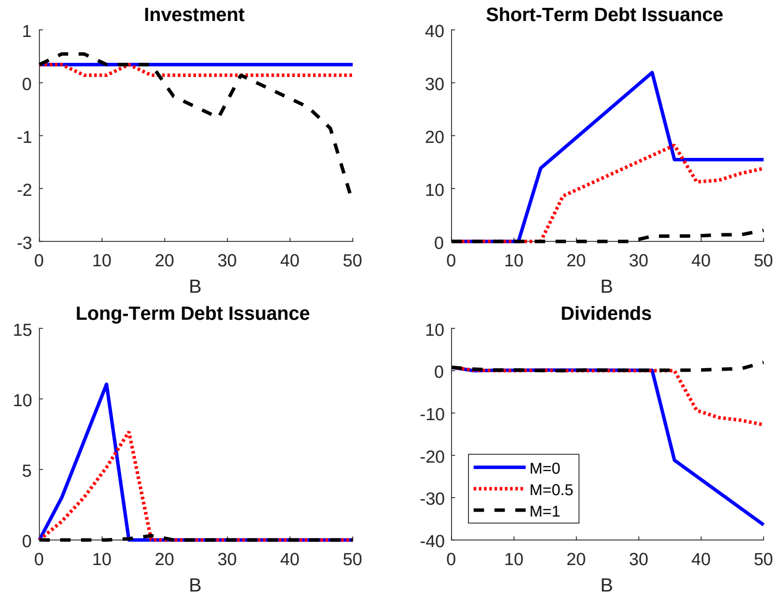


Figure 4: Policy functions for capital and debt as a function of debt state variable for three different values of the long-term debt share state variable. The capital stock and idiosyncratic productivity are held constant at the mean level., Aggregate consumption is set at the lower level.

funding, the firm will choose a lower level of investment. Second, if the firm has a high level of long-term debt outstanding, it will under-invest due to the debt overhang effect.

In Figure 4, I plot the policy functions as a function of outstanding debt for three different values of the maturity of outstanding debt. The blue line is for a firm with only short-term debt outstanding, the red line for a firm with an intermediate maturity structure and the black line for a firm with only long-term debt outstanding. The other state variables are held constant at the mean level.

For firm with only short-term debt, a higher level of debt reduces investment due to a higher cost of capital if the firm has to issue external funds. Since the firm in the figure is always constrained, the level of investment is insensitive to further increases in the level of debt. For a higher maturity structure of debt, the reduction in investment is more dramatic. The reason for this is that a higher capital stock and a lower leverage are costly to the shareholder today. However, a large part of their future benefit accrues to the creditors instead of the shareholders, either through a lower default risk or through a higher recovery value in default. As a consequence, investment and lower leverage levels if the default rate is high constitute costly transfers from shareholders to creditors. With only short-term debt, this transfer is priced into the bond price of newly issued debt. Since with short-term debt, all future outstanding debt is issued in the current period, the value of this transfer is fully internalized by the shareholder. With long-term debt, however, the effect on the value of outstanding debt carried over from the last period is not internalized by the shareholder today, and as such the firm will use more leverage and a lower capital stock than an unlevered, otherwise identical firm.

## 8.2 Measuring Under-Investment

| | Actual Investment Rate | Under-investment, Total | Underinvestment, Due to Lack of Internal Funds | Underinvestment, Due to Debt Overhang |
|---|---|---|---|---|
| 0% to 25% | 0.047 | 0.018 | 0.001 | 0.017 |
| 25% to 50% | 0.037 | 0.006 | -0.002 | 0.007 |
| 50% to 75% | 0.037 | 0.007 | -0.000 | 0.007 |
| 75% to 90% | 0.038 | 0.001 | 0.000 | 0.000 |
| 90% to 95% | 0.038 | -0.001 | 0.000 | -0.001 |
| 95% to 99% | 0.038 | -0.000 | 0.000 | -0.001 |
| 99% to 100% | 0.039 | -0.000 | 0.000 | -0.001 |
| All Firms | 0.040 | 0.008 | -0.000 | 0.008 |

Table 12: Underinvestment, total and decomposition.

To investigate whether under-investment is quantitatively important in the model and whether this under-investment arises due to costly external funding or debt overhang, I conduct the following decomposition: Consider a firm with states $(K, B, M, A, C)$ and another firm that has the same amount of debt coming due in the current period as the original firm, but no further debt. So this firm is described by the states $(K, (\mu M + (1-M))B, 0, A, C)$. The latter firm has the same net worth as the original firm, but it does not have a debt overhang, since it has no outstanding long-term debt. Then, the part of under-investment that is due to costly external funds is given by

$$I^{-,CF}(K, B, M, A, C) = I^*(K, 0, M, A, C) - I^*(K, (\mu M + (1-M))B, 0, A, C). \tag{8.2}$$

The part of under-investment due to debt overhang is then given by

$$I^{-,DO}(K, B, M, A, C) = I^-(K, B, M, A, C) - I^{-,CF}(K, B, M, A, C). \tag{8.3}$$

I report the average under-investment and the decomposition for different firm size classes in Table 12. I normalize under-investment by the capital stock of the respective firm. Overall, the average firm under-invests about 0.8 percent relative to the capital stock compared to a firm without any debt. Most of that under-investment is driven by the debt overhang effect. In addition, smaller firms under-invest more: the smallest 25 percent of firms by size under-invest about 1.8 percent relative to the capital stock, which corresponds to an investment rate which is 38 percent too low, while the largest 25 percent of firms basically do not under-invest. Most of the under-investment is driven by the debt overhang effect, while the lack of internal funds plays a negligible role.

In summary, in contrast to many macroeconomic models with only short-term debt, debt overhang is the most important driver of under-investment in this model. This often overlooked channel can have a quantitatively large impact on investment behavior, in particular for small firms.

# 9 Conclusion

I study the determinants of aggregate corporate debt maturity dynamics in a quantitative model with rich cross-sectional heterogeneity. In the model, firms prefer long-term debt, because issuance costs imply that the tax advantage of long-term debt is much bigger than the tax advantage of short-term debt. Maturity dynamics are driven by liquidity constrained firms issuing short-term debt to cover liquidity shortfalls.

The model can explain match levels and dynamics of the debt maturity structure both in the cross-section and in the aggregate, and is consistent with other established facts about the dynamics of corporate financing and investment decisions.

The question of regulation arises naturally, given the agency problem outlined in the model. Can and should regulatory authorities develop rules such that the preferences of bondholders are better reflected in the decisions of firms? The results in this paper suggest that such rules can lead to higher investment rates.

Some interesting extensions of the model are left for future research. For example, I abstract from cash holdings and credit lines, which are additional sources of funds firms can use to reduce the incidence of liquidity shortfalls. There are no labor market frictions in the model, which might be another important reason to issue short-term debt through a working capital requirement as in Jermann and Quadrini (2012).

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# A Other Measures of Corporate Cash Flows

In this section, I describe how using other measures of corporate cash flows affects the main empirical facts.

| | (1) GDP(t-1) | (2) GDP(t) | (3) GDP(t+1) |
|---|---|---|---|
| 0% to 25% | 0.176 | 0.202* | 0.195* |
| 25% to 50% | −0.0398 | 0.0518 | 0.101 |
| 50% to 75% | 0.569*** | 0.651*** | 0.707*** |
| 75% to 90% | 0.127 | 0.233* | 0.311*** |
| 90% to 95% | −0.256** | −0.205* | −0.157 |
| 95% to 99% | −0.488*** | −0.444*** | −0.373*** |
| 99% to 100% | −0.302** | −0.234* | −0.170 |
| 0% to 90% | 0.158 | 0.256** | 0.328*** |
| All Firms | −0.114 | 0.0001 | 0.101 |
| Observations | 116 | 116 | 116 |

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 13: Correlations of the Detrended Long-Term Debt Share with Detrended Logged real GDP, by Firm Size.

In Table 13, I report the correlation between the long-term debt share and real GDP. The upside of using real GDP is that it is conventionally used in the business cycle literature to compute the cyclicality of other variables. The downside is that it is only imperfectly related to corporate cash flows. This is problematic from a theoretical and empirical perspective: The mapping to the model counterpart, namely real aggregate corporate cash flow, is imperfect. For example, real GDP might include shocks to other sectors of the economy which are uncorrelated with cash flow in the corporate sector. For example, a sharp decrease in added value from the financial sector or the public sector would not be contemporaneously reflected in corporate cash flow.

Indeed, using real GDP instead of real aggregate sales changes the correlations substantially: The correlation between the long-term debt share of all firms and real GDP is zero, and the correlation between the long-term debt share and the bottom 90 percent of firms by size is only one third as high compared to the correlation when using sales.

In Table 14, I report the correlations of the long-term debt share with aggregate log real corporate profits. Using profits instead of sales is a better measure of corporate cash flows if there is significant cyclicality in the operating costs of firms, such that fluctuations in revenues do not present a full picture of the funds firms have available. However, the correlations are broadly similar to those in Table 2. The biggest difference is that with profits as measure of corporate cash flows, the maturity structure of the largest five percent is practically acyclical.

| | (1) GDP(t-1) | (2) GDP(t) | (3) GDP(t+1) |
|---|---|---|---|
| 0% to 25% | 0.248** | 0.274** | 0.137 |
| 25% to 50% | 0.311*** | 0.415*** | 0.344*** |
| 50% to 75% | 0.589*** | 0.716*** | 0.705*** |
| 75% to 90% | 0.405*** | 0.548*** | 0.559*** |
| 90% to 95% | 0.231* | 0.300** | 0.297** |
| 95% to 99% | −0.162 | −0.0648 | 0.0110 |
| 99% to 100% | −0.176 | −0.0592 | 0.0191 |
| 0% to 90% | 0.530*** | 0.666*** | 0.659*** |
| All Firms | 0.322*** | 0.494*** | 0.539*** |
| Observations | 116 | 116 | 116 |

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 14: Correlations of the Detrended Long-Term Debt Share with Detrended Log Real Corporate Profits, by Firm Size.

# B Derivation of the Derivatives in the Text

As in section 5, I assume that the bond prices are differentiable once in $K'$, $B'$, $M'$ and $A$ and value function is differentiable once in $K$, $B$, $M$ and $A$. Further, I assume that the short-term and the long-term bond price are non-decreasing in $A$.

## B.1 Value Function

The first order condition for the long-term debt share is given by

$$\frac{\partial V}{\partial M'} = \Bigg[(Q_L - Q_S)B' + \frac{\partial Q_S}{\partial M'}(1-M')B' + \frac{\partial Q_L}{\partial M'}\left(M'B' - (1-\mu)MB\right)\Bigg](1+\lambda_D) + \mathbb{E}\left[\Lambda(C,C')\frac{\partial V'}{\partial M'}|\mathcal{Y}\right] = \lambda_{M,1} - \lambda_{M,0}$$

The envelope condition yields

$$\begin{aligned}\frac{\partial V}{\partial M} &= (\mu + (1-\mu)Q_L - 1 + (1-\mu)\xi)B(1+\lambda_D)\\ &= (1-\mu)(Q_L - 1 + \xi)B(1+\lambda_D)\end{aligned}$$

if the firm is in a no default state and

$$\frac{\partial V}{\partial M} = 0$$

if the firm is in a default state.

The derivative of the value function with respect to $A$ is given by

$$\frac{\partial V}{\partial A_i} = (1-\tau)K^\alpha + \frac{\partial Q_S}{\partial A}(1-M')B' + \frac{\partial Q_L}{\partial A}(M'B' - (1-\mu)MB) + \\ \mathbb{E}\left[\Lambda(C, C') \int_{-\infty}^{\infty} \frac{\partial V'}{\partial A_i'} \frac{\partial A_i'}{\partial A_i} f(A_i'|A_i) dA_i' | C\right]$$

Using $\frac{\partial A_i'}{\partial A_i} = \rho \frac{A_i'}{A_i}$ yields

$$\frac{\partial V}{\partial A_i} = (1-\tau)K^\alpha + \frac{\partial Q_S}{\partial A}(1-M')B' + \frac{\partial Q_L}{\partial A}(M'B' - (1-\mu)MB) + \\ \mathbb{E}\left[\Lambda(C, C') \int_{-\infty}^{\infty} \frac{\partial V'}{\partial A_i'} \rho \frac{A_i'}{A_i} f(A_i'|A_i) dA_i' | C\right]$$

in non-default states and

$$\frac{\partial V}{\partial A_i} = 0$$

in default states.

Expanding the recursion, one can see that this value function derivative depends on the entire future path of the derivatives of the production function and the bond prices with respect to how the sequence of productivity changes with a current change in the productivity. The production function derivative is positive. As shown below, the short-term bond price derivative is nonnegative. It not possible to analytically determine the sign of these bond prices. If these derivatives are nonnegative, which I assume and which is the case in simulations, the derivative of the value function with respect to idiosyncratic productivity is positive, i.e. $\frac{\partial V}{\partial A_i} > 0$, if the firm is in a non-default state.

## B.2 Short-Term Bond Price

The sign of the short-term bond price derivative depends on how the default cutoff varies with the share of long-term debt. The default cutoff function $a^*(K_i, B_i, M_i, C)$, which exists if the value function derivative with respect to idiosyncratic is positive, i.e. $\frac{\partial V}{\partial A_i} > 0$, is implicitly defined by the equation

$$V(K_i, B_i, M_i, a^*(K_i, B_i, M_i, C), C) = 0$$

Using the implicit function theorem, the derivative for $a^*$ is given by:

$$\frac{\partial a^*}{\partial M_i} = -\frac{\frac{\partial V}{\partial M_i}}{\frac{\partial V}{\partial A_i}}$$

Since $\frac{\partial V}{\partial M_i} > 0$ and $\frac{\partial V}{\partial A_i} > 0$, $\frac{\partial a^*}{\partial M_i} < 0$, i.e. the default threshold in the next period is decreasing in $M$.

With this information and using Leibniz rule, the short-term bond price derivative with respect to the long-term debt share can be computed as

$$\frac{\partial Q_S}{\partial M_i'} = \mathbb{E}\left[\Lambda(C,C')\left(R(K_i',B_i',a^*,C') - (1+c)\right) f(a^*|A)\frac{\partial a^*}{\partial M_i'}|C\right]$$

Since $1+c \geq R(K,B,a^*,C)$, i.e. the creditor cannot recover more than his claim per unit of the bond in default, and $\frac{\partial a^*}{\partial M'} \leq 0$, i.e. the default cutoff for the next period is lower if the long-term share in the next period is higher, this derivative is positive.

## B.3 Long-Term Bond Price

For the long-term bond price, the derivative with respect to the long-term debt share is given by

$$\begin{aligned}\frac{\partial Q_L}{\partial M'} = \mathbb{E}\Bigg[\Lambda(C,C')\Bigg[&\left[R(K_i',B_i',a^*,C') - (\mu + c + (1-\mu)Q_L')\right] f(a^*|A)\frac{\partial a^*}{\partial M_i'} \\ &+(1-\mu)\int_{a^*}^{\infty}\left(\frac{\partial Q_L'}{\partial K''}\frac{\partial K''}{\partial M'} + \frac{\partial Q_L'}{\partial B''}\frac{\partial B''}{\partial M'} + \frac{\partial Q_L'}{\partial M''}\frac{\partial M''}{\partial M'}\right) f(A'|A)dA\Bigg]|C\Bigg]\end{aligned}$$

It is not possible to determine the sign of this derivative, for two reasons: First, it is not necessarily the case that $(\mu + c + (1-\mu)Q_L') > R(K_i',B_i',a^*,C')$, since the continuation bond price $Q_L' = Q_L((K_i',B_i',M_i',a^*,C'))$, which represents a part of the claim of the creditor to the firm, might be low.

Second, the future bond price also changes with future firm decisions, which depend on the policy for the long-term debt share today. Since the policy functions are unknown, it is not possible to determine these derivatives.

In general, the long-term bond price can therefore decrease in the long-term debt share. This can be the case in two situations: First, if defaulting would actually lead to a higher payoff for creditors. Second, if a higher long-term share increases default risk after the next period through adversely affecting the firm policies in the next period.

# C Numerical Algorithm

My solution algorithm is a value function iteration algorithm based on Hatchondo et al. (2015). It works as follows.

1. Start with a guess for the expected value function and bond prices. The equilibrium for the infinite horizon model might not be unique. I therefore follow Hatchondo and Martinez (2009) and approximate the infinite horizon value functions by finite horizon value functions for the first period. Therefore, the initial guesses are the terminal value function and the terminal bond prices.

2. Compute the policy functions and value function. I approximate the value function between grid points using linear interpolation. For capital and debt, I use grids with 15 points, respectively. For the share of long-term debt, I use 5 grid points. For the idiosyncratic productivity shock, I use 9 and for the aggregate productivity shock 5 grid points. I use a choice grid with 250 points for capital, 250 points for debt and 100 points for the long-term debt share.

3. Update the expected value function and bond prices. For the calculation of expectations, I approximate the expected value function $\mathbb{E}\left[V(\mathcal{S}_i', A_i, C)\right]$, $q_S(\mathcal{S}_i', A_i, C)$, and $q_L(\mathcal{S}_i', A_i, C)$ using linear interpolation on $(\mathcal{S}_i', A_i)$. I treat $A_i$ as continuous, using Gauss-Legendre quadrature to calculate the expectation over $A_i'$. I compute these expectations on a Grid with 30 points for $K_i'$, 30 for $B_i'$ and 5 for $M_i'$ and 9 points for $A_i$. I use 25 quadrature points to compute the expectations approximating the integrals piecewise in the default and no default regions, using the exact default cutoffs for $A_i$. I use three Gauss-Hermite quadrature nodes for $\epsilon^Z$ and $\varepsilon^C$. By approximating the expectations, I only have to calculate expectations only once outside the maximization step instead of many times within the maximization step.

4. Repeat until the updating errors in the expected value function and bond prices are smaller than 1e-4.

The long-term bond price function requires some smoothing to converge. I use a smoothing weight of 0.001 on the bond price and no smoothing on the value function, once the bond price error is sufficiently small.