# Labor Market and Regional Reallocation Effects of Housing Busts

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#### Abstract

The present paper studies the impact of a housing bust on regional labor reallocation and the labor market. I document a novel empirical fact, which suggests that, by increasing the fraction of households with negative housing equity, a housing bust hinders interregional mobility. I then study a multi-region economy with local labor and housing markets and worker reallocation. The model can account for the positive co-movement of relative house prices and unemployment with gross out-migration and negative co-movement with in-migration observed in the cross section of states. A housing bust creates debt overhang for some workers, which distorts their migration decisions and increases aggregate unemployment in the economy. This adverse effect is amplified when regional slumps are particularly deep as in the recent U.S. recession. In a calibrated version of the model, I find that the regional reallocation effect of the housing bust can account for between 0.2 and 0.5 percentage points of aggregate unemployment and 0.4 and 1.2 percentage points of unemployment in metropolitan areas experiencing deep local recessions in 2010. Finally, I study the labor market effects of two policies proposed for addressing the U.S. mortgage crisis.

JEL Codes: E24, E44, J61, J64, R23

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## 1 Introduction

The recent recession has been characterized by significant divergence in regional economic fortunes within the U.S. In particular, in 2010, unemployment dispersion across metropolitan statistical areas (MSAs) was 2.3% compared to around 1% in the early 2000s, including the 2001 recession. At the same time, internal migration in the U.S. fell in the aftermath of the recession, with inter-state migration at an all time low of 1.4% in 2009-2010 compared to 2% in the early 2000s. Parallel to the recession, the U.S. experienced a housing bust that has had a profound impact not only on the financial system but also on households themselves. Many were left owing more on their home mortgages than the value of the underlying houses, the so called "negative equity problem". For example, in 2009, 5 states had 17% or more of homeowners with negative housing equity, with Nevada being at the top with almost a third of homeowners in negative equity.

A common hypothesis for the decline in mobility, which popular media and commentators have extensively discussed, involves the distortion that negative equity may create in households migration decisions.<sup>1</sup> This possibility has raised questions about the implications of the housing bust for the performance of the labor market.<sup>2</sup> Despite this interest, however, the impact of a housing bust on the labor market through its effect on regional reallocation has remained largely unexplored.

This paper studies the labor market and regional reallocation effects of housing busts and addresses the likely quantitative relevance of the mobility hypothesis in the context of the recent recession. I start by documenting a novel empirical fact, which shows that the fraction of households with negative equity within a state is associated with decreases in state gross out-migration, while having no effect on gross in-migration. This observation complements previous micro-studies on the link between negative equity and household mobility and suggests that by increasing the fraction of households in negative equity, a housing bust may have an adverse effect on aggregate migration and regional labor reallocation.

I then study a multi-region economy with segmented labor markets and limited mobility. The economy consists of a continuum of islands (regions) with regional labor markets characterized by search and matching frictions, competitive housing markets and local recessions

<sup>&</sup>lt;sup>1</sup>There are many other important economic implications of negative equity beside the mobility effects that this paper focuses on. For example, households with negative equity invest less in maintaining their homes (Melzer (2010). Furthermore, foreclosures have substantial private costs but also result in lower prices for neighboring houses (Campbell, Giglio, and Pathak (2011)). Since, negative equity is related to increased risk of foreclosure (Bajari, Chu, and Park (2008) and Bhutta, Dokko, and Shan (2010)), the resulting deadweight costs would be substantial.

<sup>&</sup>lt;sup>2</sup>For example, as the Economist, notes: "Homeowners that are reluctant to default but unable to sell at a loss are left stuck where they are. This throws sand in the gears of America's famously fluid labour market. (Economist (2010))".

and booms. Workers reside across the regions of this economy and migrate out of a region for idiosyncratic reasons and due to regional labor and housing market conditions. Their migration is directed, i.e. they migrate to regions offering the most favorable labor and housing market conditions. Each region is endowed with a fixed supply of durable housing that can be used either by workers for housing services or for production by a local sector with a concave production function. Workers own housing by holding a mortgage, collateralized by the housing unit. However, a housing bust, which I model as an unexpected depreciation shock to some housing units, reduces the value of these units. The lack of contingency of mortgages on this shock creates debt overhang for the workers owning this housing, and the penalty incurred when defaulting distorts worker migration decisions. The higher the penalty, the lower the fraction of affected workers who out-migrate and the less the reallocation.

I characterize stationary equilibria of this economy under full immobility of workers with debt overhang and show that the model can generate the positive co-movements of relative unemployment and house prices with gross out-migration and the negative co-movement with in-migration observed in the cross section of states. Three features of the model are important for this result. First, durable housing and a downward sloping housing demand by the local sector lead to house price differences across regions with different populations. This, combined with idiosyncratic regional preferences by workers, which act as a migration barrier and lead to limited arbitrage of regional differences, creates a dependence of regional house prices on the history of labor market shocks and, consequently, a rich distribution of regional house prices. The resulting equilibrium house price heterogeneity drives the positive co-movement between out-migration and house prices. Limited spatial arbitrage also leads to the co-movement between out-migration and unemployment. Last, directed migration implies that regions with booming labor markets and lower populations and consequently, house prices, have larger population inflows leading to the negative co-movement between house prices and unemployment with in-migration.

In this framework, I show that a housing bust has a negative impact on aggregate unemployment and that this effect is amplified by a "regional shock", by which I mean an aggregate shock that causes deeper local recession, as in the recent recession. The intuition for these effects is straightforward: the migration distortion for workers with debt overhang hinders regional reallocation, leaving more workers in depressed regions compared to an economy without such a penalty. Regional reallocation, however, is more important whenever regional disparities are larger, which results in a positive interaction.

To examine the magnitude of these effects in the recent recession, I calibrate a version of the model economy to match pre- and post-recession facts about unemployment, unemployment dispersion, and migration. Similarly to Shimer (2005), but in the context of a model with regional labor markets, I find that if wages are set via Nash bargaining, then the calibrated model cannot account for the observed regional unemployment dispersion given the volatility of regional productivity shocks. This is no longer the case if regional wages are rigid as in Hall (2005). Using the calibrated model with wage rigidity, I find that the mobility distortions from "negative equity" can account for around 0.2 percentage points of the aggregate unemployment and 0.4 percentage points of the unemployment in metropolitan areas experiencing deeper local recessions in 2010. Furthermore, an upper bound on the regional reallocation distortions of the housing bust corresponds to an effect of 0.5 percentage points of aggregate unemployment and 1.2 percentage points of unemployment in depressed metropolitan areas. This corresponds to between 4 and 10% of the increase in aggregate unemployment from 2007 to 2010 and to between 7 and 20% of the increase in unemployment in depressed metropolitan areas.

The calibrated model can match some of the motivating cross-state empirical facts. It can also account for the observed relation between regional unemployment dispersion and shifts of the Beveridge curve. In particular, I show that holding the aggregate vacancy-tounemployment ratio (market tightness) constant, larger regional shocks in the model lead to both higher unemployment and larger unemployment dispersion. Given the constant aggregate vacancy-to-unemployment ratio, this implies that unemployment for a given level of vacancies is increased, which corresponds exactly to a shift of the Beveridge curve.

Lastly, I use the calibrated model to evaluate the labor market effects of two policies, proposed for solving the mortgage crisis. The first is a monthly mortgage payment reduction proposal, in the spirit of the "Home Affordable Modification Program", while the second is a mortgage principal reduction proposal (Pozner and Zingales (2009)). The former leads to a marginal increase in aggregate unemployment since it eliminates involuntary default, which effectively increases the default penalty that workers with debt overhang face, which slows down reallocation additionally. The latter policy proposal decreases unemployment since it eliminates the default penalty.

**Related literature.** This paper spans several strands of literature. On the empirical side my paper is related to microdata studies dealing with the effects of negative equity on household mobility (Henley (1998), Chan (2001), Ferreira, Gyourko, and Tracy (2010), Schulhofer-Wohl (2010), Ferreira, Gyourko, and Tracy (2011)). These studies have to deal with different issues arising from the general absence good quality data on both household balance sheets and migration decisions that spans a sufficiently large number of households with negative equity. Henley (1998) uses the first four waves of the British Household

Panel Survey and finds a strong adverse effect of negative equity on mobility but with a very small sample of households in negative equity. Similarly, Chan (2001) documents a negative effect using mortgage data from the U.S. with mortgage pre-payments serving as proxy for house moves. More recently, Ferreira, Gyourko, and Tracy (2010) examine data from the American Housing Survey, which tracks a sample of housing units in the U.S.. Identifying moves from housing ownership changes, these authors also find an adverse effect of negative equity. However, using a dataset that tracks housing units instead of households themselves creates potentially serious mis-measurement of household moves.<sup>3</sup> The empirical fact I document, though not at the micro level, complements these studies by side-stepping some of these issues as well as any compositional effects, which may make household level mobility distortions of negative equity irrelevant for aggregate migration.

The theoretical model is close in spirit to previous work on regional reallocation (Lucas and Prescott (1974) and more recently Alvarez and Shimer (2011), Carrillo-Tudela and Visschers (2011), Lkhagvasuren (2011), Coen-Pirani (2010), and Shimer (2007)). In these papers, dispersion in economic conditions across islands induces worker reallocation, which has implications for aggregate unemployment. However, these papers do not investigate the effects of the housing market on mobility and none of them addresses the co-movement between gross migration and local labor and housing market conditions.

The paper is also related to studies of the effect of homeownership on mobility and unemployment (Oswald (1996), Head and Lloyd-Ellis (2010)). Head and Lloyd-Ellis (2010) investigate the impact of home ownership on mobility and aggregate unemployment and build a theory that accounts for the reduced mobility of homeowners versus renters observed in micro-data and the negative correlation between regional unemployment and homeownership. In their models, housing markets affect the regional allocation of workers, although their model cannot account for the co-movement between gross migration rates and local labor and housing market conditions. Also, their modeling focus is more on the implications of the liquidity of the market for owner-occupied housing on migration decisions. As a result, their treatment of labor market frictions cannot allow for investigating the regional reallocation effects of a housing bust or its interaction with regional shock. Another paper that deals with the interactions between the housing market and the labor market is Nieuwerburgh and Weill (2010). The authors, however, address a very different issue, documenting the secular increase in dispersion in regional house prices and wages and building

<sup>&</sup>lt;sup>3</sup>In particular, Schulhofer-Wohl (2010) re-examines the data and observes that Ferreira, Gyourko, and Tracy (2010) drop observations for some uncertain housing tenure transitions, which if treated as moves, reverse their results. Nevertheless, Ferreira, Gyourko, and Tracy (2011) argue that such an approach significantly overstates actual moves leading to too many false positives once future observations are incorporated. Using an improved measure of moves that better utilizes the panel structure of the AHS they recover the results from Ferreira, Gyourko, and Tracy (2010).

a model that explains and quantitatively matches this increase.<sup>4</sup>

Examining the quantitative implications of a model with regional wage rigidity connects this paper to the growing literature on rigid wages in search and matching models of aggregate unemployment (Hall (2005), Shimer (2010), Gertler and Tigari (2009))

Recent work that also deals with the labor market implications of the housing bust includes Estevao and Tsounta (2011), Sterk (2010) and Karahan and Rhee (2011). Estevao and Tsounta (2011) look at the effects of house price declines on state structural unemployment and use those to infer the effect of housing market deterioration on aggregate unemployment. I use an alternative strategy to address this issue by quantitatively evaluating the aggregate unemployment implications of the housing bust through the lense of a structural model. Sterk (2010) builds a DSGE model with housing frictions and a reduced form migration decision by workers to show the dynamic impact of a housing bust on unemployment and the Beveridge curve. However, unlike my modeling framework, this reduced form migration approach cannot account for the observed shifts of the Beveridge curve due to regional mismatch. This would overstate the quantitative effects of decreased mobility from the recent housing bust given the parallel increase in regional unemployment dispersion. Finally, in related but independent work, Karahan and Rhee (2011) uncover an effect of house price declines on aggregate unemployment that is slightly higher in magnitude to the effects in my model.

The rest of the paper is organized as follows. Section 2 presents the motivating empirical facts. In Section 3, I present the basic model. In Section 4, I provide a characterization of stationary equilibria. In particular, I show how the model can account for the co-movement of state gross flows and labor and housing market conditions, and also show the housing bust effects. Finally, Section 5 contains the calibration results and counterfactual experiments for the housing bust, and Section 6 concludes. Additionally, the Appendix contains data description, proofs of results omitted from the main text, as well as details of the calibrated model and computational procedures used.

## 2 Empirical Facts

Do housing busts and the resulting negative equity problem for some households affect mobility? There are several economic mechanisms that can lead to the distortion of a household's decision to move due to negative equity. For example, such households may

<sup>&</sup>lt;sup>4</sup>Notowidigdo (2010) and Winkler (2010) are recent empirical contributions into the effect of the housing market on labor mobility. The first one deals with how house prices affect the mobility of workers with different skill levels, while the second examines the link between homeownership and mobility.

face pecuniary or non-pecuniary default penalties.<sup>5</sup> Another possible mechanism comes from the combination of low homeowner wealth, together with a downpayment requirement on new housing purchases, which would also affect mobility.<sup>6</sup>.

As discussed in the Introduction, conducting microstudies on the mobility distortion effect of negative equity has been hindered by data quality and availability. Furthermore, although ideal for identifying an effect at the household level, these microstudies may not be able to tell whether such an effect is important for inter-regional migration, which is the channel that is important for the labor market. For example, it may be that negative equity does not affect long-distance migration decisions even if it affects a household's decision to change houses. Alternatively, there may be compositional issues as negative neighborhood peer effects from households with negative equity may actually stimulate migration for other households thus increasing aggregate migration.

In this section, I take a complementary approach and examine state level aggregated data. I find suggestive evidence that a housing bust may have an adverse effect on interregional migration and regional labor reallocation. In particular, I look at the comovements of the fraction of households with negative housing equity on their primary residence with the gross in- and out-migration rate of households across states.

The data I use is an unbalanced panel of annual household state out- and in-migration rates and the estimated fraction of households with negative equity for 45 of the 50 states plus the District of Columbia, from 1993-2008.<sup>7</sup>. I obtain data on annual state gross migration rates from the IRS U.S. Population Migration Database. I construct state level estimates of the fraction of households in negative equity using household level information from the Interview Survey section of the Consumer Expenditure Survey (CE). I compute the housing equity for a household's primary residence, using information on the balance outstanding on all mortgages and equity lines of credit that the property collateralizes as well as the reported subjective property value, according to the equation:

$$E_{ist} = \frac{\tilde{V}_{ist} - D_{ist}}{\tilde{V}_{ist}} \tag{1}$$

Here,  $E_{ist}$  is the housing equity of household *i* living in state *s* in year *t*,  $D_{ist}$  is the total bal-

<sup>&</sup>lt;sup>5</sup>There is an extensive empirical literature dealing with homeowner default decisions that uncovers such default penalties, see for example recent work by Bhutta, Dokko, and Shan (2010), Guiso, Sapienza, and Zingales (2009) and Foote, Gerardi, and Willen (2008).

 $<sup>^{6}</sup>$ Such a mechanism is in the spirit of Stein (1995).

<sup>&</sup>lt;sup>7</sup>The states that are not included because of missing observations on fraction of households in negative equity for all years between 1993 and 2008 for them are Mississippi, Montana, New Mexico, North Dakota, South Dakota, and Wyoming. Appendix A contains information on how I construct all the relevant variables and controls that comprise the panel as well as details on the data sources. Here, I just provide a brief description.

ance outstanding on all mortgages and home equity lines of credit, and  $\tilde{V}_{ist}$  is the subjective property value. I then construct an estimate of the fraction of homeowners with negative equity in state s and year t by counting the number of sampled homeowners in state s and year t with  $E_{ist} < 0$  and dividing by the total number of sampled homeowners in that state and year. Multiplying by the homeownership rate for state s and year t, I get the estimated fraction of households with negative equity in state s,  $\hat{neg}_{st}$ .

I focus on the following panel regressions:

$$log(out_{st}) = \alpha_s^{out} + \zeta_t^{out} + \beta^{out} \ \widehat{neg}_{st} + \mathbf{x}'_{st} \ \gamma^{\mathbf{out}} + \epsilon_{st}^{out}$$
(2)

$$log(in_{st}) = \alpha_s^{in} + \zeta_t^{in} + \beta^{in} \ \widehat{neg}_{st} + \mathbf{x}'_{st} \ \gamma^{\mathbf{in}} + \epsilon_{st}^{in}$$
(3)

where  $out_{st}$  is the gross out-migration rate for state s and year t,  $in_{st}$  is the the gross inmigration rate,  $\widehat{neg}_{st}$  is the estimated fraction of households with negative equity in state s at time t,  $\alpha_s$  and  $\zeta_t$  are state and year fixed effects and  $\mathbf{x}_{st}$  is a vector of other covariates.

I control for state economic and housing market conditions by including the log of the state unemployment rate, and the log of state house prices relative to the national level. In addition, I include a measure of the relative wage as the log of the ratio of state average hourly wage in manufacturing to the national counterpart and the log of state income per capita relative to national income. I control for mortgage credit conditions proxied by the average debt-to-value ratio (computed from the CE) and the home ownership rate.

There are two potential issues with using the estimate  $\widehat{neg}_{st}$  rather than the true fraction  $neg_{st}$ . The first issue comes from potential misclassification problems of households, since I use subjective property values, and these are noisy estimates of the actual property price (Ferreira, Gyourko, and Tracy (2010)). To address this, I compare my estimates of the fraction of negative equity by state with estimates from First American CoreLogic for 2009. The CoreLogic estimates are based on much more precisely measured house price data and hence are prone to much lower misclassification problems but are available for a much shorter time period (2008-2010) (see CoreLogic (2009) for details). Comparing the two series reveals a very high cross-sectional correlation of  $\rho = 0.731$ . Hence, misclassification does not appear to be a problem for the state-level variation within a given year.<sup>8</sup> As an additional robustness check, I also run the panel regression using the estimated fraction of households with equity away from zero, i.e. the number of sampled homeowners in state s and year t with  $E_{ist} \leq c < 0$  divided by the total number of sampled homeowners in that state and

<sup>&</sup>lt;sup>8</sup>Figure 3 in Appendix A plots the two series and a linear regression line. This also highlights the high co-movement between the two series with any discrepancy likely due to measurement error. The only difference is that the CE estimates predict lower values for all states.

year.<sup>9</sup> The second issue is classical measurement error that comes from using an estimate rather than the true value. I address this issue below.

The estimation results for the panel regressions are shown in Table 1. The first and third columns present panel regression results with no additional controls, while the second and fourth column show results with year fixed effects and state level controls. The coefficients on households with negative equity for the regressions with out- and in-migration and no controls are negative and statistically significant at the 5% level. With the controls, the coefficient on households with negative equity is still negative and significant at the 10% level for the regression with state out-migration rate as the dependent variable. In contrast the coefficient in the regression with state in-migration as a dependent variable is insignificant at close to 0. Using other cutoffs produces similar results as Table 10 in Appendix A shows.

The asymmetric effect of  $\widehat{neg}$  on out- versus in- migration is very intriguing. One possible explanation, which I explore in the model in Section 3 and in the rest of the paper, is that negative equity affects household regional reallocation decisions, which ultimately leads to these co-movements. Studying a multi-region equilibrium model with worker reallocation will also allow me to see how the regional reallocation effects of a housing bust map into an effect on both local and aggregate labor markets, since it is hard to empirically establish a direct link between a housing bust and the labor market.

According to the coefficient estimates, a 10 percentage point increase in the fraction of households in negative equity decreases the out-migration rate by around 2.5%. However, as mentioned above, it is important to note that measurement error in the estimated negative equity fractions,  $\widehat{neg}_{st}$ , biases down the regression coefficient estimates. Having a notion of the size of attenuation bias is necessary for assessing the quantitative importance of this channel, as I do in Section 5. Fortunately, the equity data, used to construct  $\widehat{neg}_{st}$ , can provide information on that bias.

Since  $\widehat{neg}_{st}$  is an estimate of the true fraction,  $neg_{st}$ , it contains estimation error. This implies that the relationship between  $neg_{st}$  and  $\widehat{neg}_{st}$  can be represented by:

$$\widehat{neg}_{st} = neg_{st} + \nu_{st} \tag{4}$$

where  $\nu_{st}$  is a random variable with mean zero and variance  $\sigma_{\nu}^2$ , distributed i.i.d. over s and t and independent of  $neg_{st}$ ,  $\forall s, t$ . I can then estimate  $\sigma_{\nu}^2$  by using estimates of the sampling variance of  $\widehat{neg}_{st}$ .<sup>10</sup> Using the estimates for  $\sigma_{\nu}^2$  and  $Var(\widehat{neg}_{st})$ , and independence

<sup>&</sup>lt;sup>9</sup>In particular, Table 10 in Appendix A shows results for c = -0.1 and c = -0.2.

<sup>&</sup>lt;sup>10</sup>More specifically, I construct estimates of the sampling variance for each observation  $\widehat{neg}_{st}$ , using a bootstrap procedure on the equity data for each state and year, and then average over all observations to

Dep. Variable:	out-migratio	n rate, $100 \ln(out)$	in-migratior	1 rate, $100 \ln(in)$
households with negative equity, (%)	$-0.521^{**}$ (0.198)	-0.243* (0.143)	$-0.502^{**}$ (0.195)	0.0442 $(0.173)$
relative unemployment $100 \ln(\frac{u_s}{u})$		$0.168^{***}$ $(0.0359)$		$-0.237^{***}$ $(0.0388)$
relative house price $100 \ln(\frac{p_s}{p})$		$0.200^{***}$ $(0.0679)$		$-0.170^{***}$ $(0.0592)$
relative wage rate $100 \ln(\frac{w_s}{w})$		0.0140 (0.0829)		-0.0115 (0.165)
relative income $100 \ln(\frac{y_s}{y})$		-0.0895 $(0.302)$		$0.647^{**}$ $(0.246)$
ave. debt-to-value ratio (%)		0.0396 (0.0496)		0.0838 (0.0521)
home ownership rate (%)		0.424 (0.394)		-0.346 $(0.321)$
Year FE	No	Yes	No	$\mathbf{Yes}$
Ν	625	606	625	606

Table 1: State panel regressions

state and year divided by national unemployment rate for that year. Relative wage rate is the log of the average hourly average manufacturing wage for that state and year divided by the national average hourly manufacturing wage for that year. Relative income is the log of income per capita for that state and year divided by national income per capita for that year. Housing equity is defined as  $E = \frac{V-D}{V}$  where D is the total debt balance outstanding on all mortgages and home equity loans that a property collateralizes and V is the subjective property value. Households with negative Notes: Robust standard errors with clustering on state in parenthesis; \* = significant at 10%; \*\* = significant at 5%; \*\*\* = significant at 1%. Source: Own calculations from BLS, IRS, CE, FMHPI, and US Census Bureau. See Data Appendix for detailed description. Relative house price is constructed from the CMHPI index and median house value by state from the 2000 Census. Relative unemployment is the log of the unemployment rate for that housing equity is the fraction of the population who are homeowners with E < 0. Average debt-to-value ratio is defined as the average of  $\frac{D}{V}$ . Home ownership rate is the percent of households in the state that own a house. of  $\nu_{st}$  and  $neg_{st}$ , I compute  $Var(neg_{st})$ . I derive a lower bound on the effect of attenuation bias by constructing a reliability ratio,  $\lambda$ , for a univariate regression, which is given by  $\lambda = \frac{Var(neg_{st})}{Var(neg_{st})} = 0.43$ . This means that  $|\beta_{neg}| > \left|\frac{1}{\lambda}\widehat{\beta_{neg}}\right| = 0.565$ . This, however, is a lower bound on the effect of attenuation bias given the high  $R^2$  of 0.53 from regressing  $\widehat{neg}$  on the other controls in the panel regression and also given the panel nature of the data. Therefore, attentuation bias appears very important, and the true coefficient,  $\beta_{neg}$ , is substantially larger in magnitude than the estimate,  $\widehat{\beta_{neg}}$ . This in turn implies that the regional reallocation distortions of a housing bust may be large. I investigate this further in Section 5.

Another salient set of results is the positive co-movement of relative unemployment and house prices with out-migration and negative co-movement with in-migration.<sup>11</sup> More specifically, a ten percent increase in state relative unemployment is associated with an increase in out-migration of around 1.7% and a decrease in in-migration of 2.4%. Overall, these estimates point to unemployment being an important driver of regional reallocation and, conversely, migration being an important adjustment mechanism in response to local recessions (Blanchard and Katz (1992)). Note, however, that both out- and in-migration co-move with changes in unemployment, which is an important observation that restricts the set of models of regional reallocation that can account for them as opposed to accounting for net migration responses only. For example, as I discuss in Section 3, models with undirected mobility, though accounting for net migration cannot generate the observed inand out-migration pattens.

The co-movement of relative house prices with out- and in-migration shows that even after controlling for labor market conditions, house price changes are associated with variation in migration. One possible explanation for this observation is through perfect spatial arbitrage and large variations in amenities or construction costs at annual frequency as in the classical spatial equilibrium framework (Roback (1982)). In Section 3, I show an alternative explanation, which relies on limited regional mobility and a durable housing stock.

Therefore, one can summarize the results of this section as follows:

- 1. A higher fraction of households with negative equity within a state correlates with decreases in migration out of that state but is not correlated with migration into the state;
- 2. Higher unemployment in a state relative to the national level correlates with increased migration out of the state and decreased migration into the state controlling for relative

obtain  $\hat{\sigma}_{\nu}^2$ .

<sup>&</sup>lt;sup>11</sup>Saks and Wozniak (2007) and Jackman and Savouri (1992) have obtained similar observations as side results but do not provide any implications they may have.

house price;

3. Higher relative house prices correlate with increased out-migration and decreased inmigration controlling for relative unemployment.

I turn next to a model of a multiple-region economy that can match these facts. I then use the model to analyze the labor market and regional reallocation effect of a housing bust and the likely magnitudes of these effects in the recent recession.

# **3** Basic Model of Regional Reallocation

In this section I propose a model of regional reallocation, which provides insights into how mobility distortions as a result of a housing bust affect regional reallocation and, ultimately, the labor market. I consider a discrete time economy with an infinite number of periods t = 0, 1, 2, ... The economy consists of a measure 2 of islands or regions. The economy is populated by a measure L of infinitely lived workers, distributed across regions of the economy. Workers are risk neutral, derive utility from consumption as well as from housing (see Section 3.3 below), and can supply 1 unit of labor. The initial measure of workers in each region j is given by  $l_{-1}^j \leq L$ , with  $\int l_{-1}^j dj = L$ . The end-of-period or post-migration measure of workers in a region j at time t is given by  $l_t^j \in [0, \overline{L}], \overline{L} \geq L$ .<sup>12</sup>

## 3.1 Regional labor markets, job creation, and destruction

In each region there is a representative firm that can open job vacancies at a per-period cost of k and recruit workers. For the basic model I consider in this section, I assume that jobs remain productive for one period only. The reason for this is analytical tractability, since under it there will be no agent heterogeneity in terms of the idiosyncratic employment state at the time of the migration decision. This leads to only one relevant endogenous state variable that affects migration decisions, which makes equilibrium characterization possible. However, in Section 5, where I look at the quantitative effect of the housing bust in the recent recession, I consider the more general set-up with stochastic job destruction.

Each job in region j has the capacity for the production of  $A_t^j$  units of the consumption good at time t if it can hire a worker. Regional productivity  $A_t^j$  can have two possible realizations  $\overline{A}$  or  $\underline{A}$  ( $\overline{A} \ge \underline{A}$ ) and follows a Markov chain with persistence  $\rho \ge \frac{1}{2}$ . Furthermore at any time t one half of regions have  $A_t^j = \overline{A}$  and the other half have  $A_t^j = \underline{A}$ . I refer

<sup>&</sup>lt;sup>12</sup>The upper bound of  $\overline{L}$  is a technical restriction necessary for showing the analytical results. It can be thought of as a limit on the space available within a region.

to the former as (relatively) "booming" regions and to the latter as (relatively) "depressed" regions.<sup>13</sup>

The labor market of each region is characterized by a search and matching friction as in the standard Diamond-Mortensen-Pissarides framework (Pissarides (2000)). In particular, in a region j at time t, after migration, a measure  $l_t^j$  of workers and measure  $v_t^j$  of vacancies try to match with each other. Matching is described by a standard regional reduced-form constant returns to scale (CRS) matching function  $m^j (l_t^j, v_t^j)$  giving the total number of regional matches per period. I assume that matching functions are identical across regions, i.e.  $m^j (l_t^j, v_t^j) = m (l_t^j, v_t^j) = l_t^j \cdot m (1, \frac{v_t^j}{l_t^j})$ . Letting  $\theta_t^j = \frac{v_t^j}{l_t^j}$  be the regional labor market tightness and defining  $\mu(\theta) = m(1, \theta)$ , we have that  $m (l_t^j, v_t^j) = l_t^j \mu (\theta_t^j)$ . This translates into a job finding probability for a worker in a given period of  $\mu (\theta_t^j)$  and a job filling probability for a vacancy of  $\frac{\mu(\theta_t^j)}{\theta_t^j}$ . Workers that remain unmatched in a given period are considered unemployed and receive a period payoff of e. The total measure of unemployed in region jis then given by  $(1 - \mu(\theta_t^j)) l_t^j$  and the unemployment rate is simply  $U_t^j = (1 - \mu(\theta_t^j))$ .<sup>14</sup>

#### 3.2 Job creation decisions and wage determination

I allow for wages to be determined either by Nash bargaining or to be rigid as in Hall (2005). The particular wage determination rule does not affect the results of this section, so I focus on a model with the more standard assumption of Nash bargaining. However, as I discuss in Section 5, the process of wage determination does matter for the calibrated model.

Given one period job length, the vacancy posting decision of the representative firm is straightforward. In particular, the firm opens vacant jobs until the cost of opening a vacancy equals the expected payoff from posting a vacancy, or:

$$k = \frac{\mu(\theta_t^j)}{\theta_t^j} \left( A_t^j - w_t^j \right) \tag{5}$$

where  $w_t^j$  is the wage rate in region j at time t. Letting workers' bargaining power be  $\eta \in [0, 1)$ , the wage rate is:

$$w_t^j = e + \eta \left( A_t^j - e \right) \tag{6}$$

where e is the outside option. Hence, equations (5) and (6) pin down the labor market tightness in a region as a function of  $A_t^j$ , so that one can write  $\theta(A)$  for a given region productivity A. Note that labor market tightness is a function only of regional productivity

<sup>&</sup>lt;sup>13</sup>The local productivity shocks can be considered a proxy for any shocks that cause local recessions and result in regional variations in unemployment and unemployment differences across regions.

<sup>&</sup>lt;sup>14</sup>This definition of unemployment comes from the one period job length assumption.

and not of the labor force in a region. This is because of the representative firm's production technology has constant returns.

#### 3.3 Regional housing markets, home financing and debt overhang

In every region j, there is a fixed supply L of undepreciated housing units trading in a competitive market at price  $p_t^j$ . Workers derive utility  $\gamma > 0$  at the end of each period, in which they own a single unit of housing. I normalize the period utility from renting to  $0.^{15}$  Apart from the demand by workers, there is also residual demand for housing from a sector of local firms that use housing (and housing only) for production, with concave production function g(h), where g(0) = 0,  $\lim_{h\to 0} g'(h) = \gamma$  and  $\lim_{h\to L} g'(h) = a \leq \gamma$  and  $g''(h) \leq 0$ . This pins down regional house prices and potentially creates house price differences across regions.<sup>16</sup>

Workers cannot borrow against their future income and have no access to a savings technology. Instead they buy housing via an infinite period financial contract (mortgage), which a housing unit collateralizes. Through that contract, they borrow from a competitive sector of financial intermediaries that face an exogenous interest rate of  $r = \frac{1}{\beta} - 1$ . The financial contract specifies a sequence of repayments,  $\left\{\tilde{d}_s^j\right\}_{s=t}^{\infty}$ , and an associated sequence of debt balance levels,  $\left\{b_s^j\right\}_{s=t}^{\infty}$ , which are linked via a promise-keeping constraint:

$$b_{s}^{j} = \tilde{d}_{s}^{j} + \frac{1}{1+r} E_{s} \left[ b_{s+1}^{j} \right], \ s \ge t$$
(7)

Furthermore, the mortgage is fully collateralized by the value of the housing unit, i.e.

$$b_s^j \le p_s^j, \ s \ge t \tag{8}$$

The promise keeping constraint is the recursive form of  $b_s^j = E_s \left[ \sum_{t=s}^{\infty} \tilde{d}_s^j \right]$  and a no-ponzi game condition, which is automatically satisfied in this case given full collateralization. I abstract away from the possibility of repayment risk for mortgages issued in equilibrium by restricting attention to economies, in which  $\tilde{d}_t^j \leq e, \forall t, j$ , i.e. I assume that even unemployed workers can cover mortgage repayments in every period.<sup>17</sup> Finally, a homeowner is free to terminate the mortgage contract at the beginning of every period, in which case the housing

<sup>15</sup> This is without loss of generality, since what matters for the home-ownership decision is the difference in utilities.

<sup>&</sup>lt;sup>16</sup>The simplistic housing market assumptions permit me to derive analytical results. One could include a housing construction sector and natural depreciation of the housing stock but that would not change the model's main implications.

 $<sup>^{17}\</sup>text{Assuming that}\ e \geq \gamma$  is sufficient for this.

unit is sold and the financial intermediary is repaid. Full collateralization and no repayment risk implies that default is not expected to occur on contracts issued in equilibrium.

Hence, this financial contract corresponds to a frictionless financing arrangement, in which repayments are free to vary over time and there is no default. At the same time, such an arrangement is sufficient to incorporate a relevance of regional house prices for worker's out- and in-migration decisions. Given frictionless financing and equal discount rates for the worker and the financial intermediary, it follows that workers will be indifferent over any contract  $\left\{ \left\{ \tilde{d}_s^j \right\}_{s=t}^{\infty}, \left\{ b_s^j \right\}_{s=t}^{\infty} \right\}$  that satisfies the promise keeping constraints (7), full collateralization constraints (8) and the no repayment risk constraints  $\tilde{d}_t^j \leq e, \forall t, j$ . Therefore, without loss of generality, I can restrict attention to a financial contract with debt balance levels  $b_t^j = p_t^j, \forall t, j$ . Selecting this contract significantly simplifies characterization of the worker's problem and hence equilibrium characterization as it makes the debt balance level redundant as a state variable when defining a value function for the worker. Additionally, I assume that this restriction also applies to workers who start at t = 0 as counterparties to a financial contract.

I model a housing bust as an unexpected housing depreciation shock at the beginning of t = 0 for a measure  $l_{0,-1}^{j}$  of homeowners in region j. By depreciated housing, I mean that new potential owners of the housing unit do not derive utility from it and local firms cannot use it in production so its price is 0. This assumption captures in a simple way the large heterogeneity of houses across local housing markets and the heterogeneity in homeowner balance sheets existing in reality (and therefore the differential impact of house price declines on homeowners).

I assume that the depreciation shock is unforseen, i.e. it is a state of the world, which financial contracts are not contingent on. Therefore workers with depreciated housing are bound by the terms of the frictionless financial contract, so  $b_0^j > 0$  for them as well, and they have debt overhang. However, these workers are free to default on that contract at any time but have to incur a default penalty of  $\zeta > 0$ . Workers with a depreciated housing unit still derive ownership utility  $\gamma$  from it until they default.<sup>18</sup>

Finally, note that given the concave production function, firms in the local sector may be making non-zero profits in equilibrium. Similarly, financial intermediaries may be making non-zero profits since they start the economy as counterparties to financial contracts already in place. In order to account for these I assume that there is a small measure of immobile risk neutral agents living on each island who own the local firms and intermediaries and consume the any profits from these.

<sup>&</sup>lt;sup>18</sup>Upon worker default the depreciated housing unit becomes useless to all agents in the economy.

#### 3.4 Regional migration

Workers have an idiosyncratic region preference  $\epsilon$  for regions they currently reside in. At the beginning of each period a worker draws a new  $\epsilon$  from a continuous distribution Fwith density function f with  $E[\epsilon] = 0$  and support over [-B, B] for some B > 0. After observing his match quality, a worker decides whether to move to a different region. Moving is instantaneous and entails a fixed cost of c. Upon moving, the worker terminates the mortgage debt contract, in which case the housing unit is sold if it is not depreciated and lenders are repaid. Otherwise workers default and incur the cost  $\zeta$ . Assuming that a worker with debt overhang has to terminate the mortgage when moving may at first seem problematic. After all, in reality, households with negative equity are free to move to a different region, while at the same time keeping the house and not defaulting. There are two issues with this argument. First of all, such behavior is not costless as a household still has to keep mortgage payments and pay for their new residence. Such costs would also affect a household's migration decision in the same way that forcing a household to incur a default penalty upon moving would. Secondly, such behavior by households does not seem relevant in the data.<sup>19</sup>

Finally, I assume that migration is directed, i.e. a worker observes regional characteristics and migrates to the region that gives him the highest expected value. After moving, the worker draws an  $\epsilon \sim F$  for the new region.

## 3.5 Timing

The timing within a period is as follows:

- 1. Agents observe the realization of regional productivity A;
- 2. Workers draw region specific payoffs and make migration and mortgage termination decisions;
- 3. Housing market opens;

<sup>&</sup>lt;sup>19</sup>The Consumer Expenditure Survey data, which I used in Section 2, contains information on whether a sampled household still owns their previous home, given their current housing status (renters or homeowners). It turns out that only 0.5% of households still own their previous home and have a mortgage on it for the period 2008-2010 compared to 0.3% for 1993-2007. This, however, also includes households in the process of selling or foreclosing on their previous home. Unfortunately, CE data from 2007-2010 does not include information on property values apart from the huseholds's primary residence, so one cannot examine what fraction of households actually still own their previous home and have negative equity on it. For 1993-2006, this fraction is effectively 0. Therefore, the actual fraction of households who keep their old home because of negative equity but move into a differnt one during the housing bust period of 2008-2010 is likely to be much smaller than 0.5%.

- 4. Firms make job creation decisions for new jobs;
- 5. Matching of workers and jobs;
- 6. Production occurs and wages are paid;

## 4 Equilibrium

I will be focusing on symmetric recursive equilibria, in which each region j is fully characterized by a vector of state variables  $\tilde{X}_t^j = (A_t^j, l_{0,t-1}^j, l_{1,t-1}^j, \overline{\nu}_t, \underline{\nu}_t)$ . This contains the current period productivity  $A_t^j$ , as well as the beginning-of-period measure of workers with and without debt overhang,  $l_{0,t-1}^j$  and  $l_{1,t-1}^j$ , respectively. Lastly,  $\overline{\nu}_t$  and  $\underline{\nu}_t$  denote the beginning-of-period distributions of workers in booming and depressed regions, respectively. Also, I define  $X_t^j = (A_t^j, l_{1,t-1}^j, \overline{\nu}_t, \underline{\nu}_t)$ . These variables are relevant for the worker's problem and regional house price determination, while  $l_{0,t-1}^j$  is only relevant for determining regional populations.<sup>20</sup>

A symmetric recursive equilibrium will then be defined by laws of motion for the endogenous variables,  $l_{0,t}^j = l'_0\left(\tilde{X}_t^j\right)$  and  $l'_1\left(X_t^j\right)$ , and  $\overline{\nu}_{t+1} = \overline{\Xi}\left(\overline{\nu}_t, \underline{\nu}_t\right)$ ,  $\underline{\nu}_{t+1} = \underline{\Xi}\left(\overline{\nu}_t, \underline{\nu}_t\right)$ , and by functions,  $\theta\left(A_t^j\right)$ ,  $w\left(A_t^j\right)$ ,  $p\left(X_t^j\right)$  giving regional market tightness and wages and regional house price as a function of the regional state such that (i) workers migration decisions are optimal given the laws of motions for A and the endogenous state variables, (ii) laws of motions for the endogenous state variables are consistent with workers migration decisions and with population constancy in the economy.

#### 4.1 Regional house prices

I first characterize regional house prices. The demand for non-depreciated housing of a representative firm from the local sector in region j solves:

$$\max_{h_t^j} \left\{ g\left(h_t^j\right) + \beta E_t\left[p\left(X_{t+1}^j\right)\right] h_t^j - p\left(X_t^j\right) h_t^j \right\} \right\}$$

which immediately implies that:

$$h_t^j > 0 \quad if \quad p\left(X_t^j\right) < \gamma + \beta E_t \left[p\left(X_{t+1}^j\right)\right] h_t^j = 0 \quad if \quad p\left(X_t^j\right) \ge \gamma + \beta E_t \left[p\left(X_{t+1}^j\right)\right]$$

$$(9)$$

 $<sup>^{20}</sup>$ This block recursive structure is standard for models with search frictions (Carrillo-Tudela and Visschers (2011)).

since  $g'(0) = \gamma$ . I can also derive the housing demand by workers. In Appendix B, I show that (i) workers with no debt overhang demand housing iff  $\tilde{d}(X_t^j) \leq \gamma$ , and (ii) workers with debt overhang do not demand non-depreciated housing. The first result, immediately implies that the equilibrium house price satisfies  $p(X_t^j) \leq \gamma + \beta E_t [p(X_{t+1}^j)]$  and therefore, in equilibrium, all workers with no debt overhang buy housing. Defining

$$d(X_{t}^{j}) = \begin{cases} g'(L - l_{1}'(X_{t}^{j})) & l_{1}'(X_{t}^{j}) < L \\ \gamma & l_{1}'(X_{t}^{j}) = L \end{cases}$$
(10)

it follows that

$$p_t^j = p\left(X_t^j\right) = d\left(X_t^j\right) + \beta E_t\left[p\left(X_{t+1}^j\right)\right]$$
(11)

This, together with a transversality condition on  $p_t^j$ ,  $\lim_{T\to\infty} \beta^T E_t \left[ p_{t+T}^j \right] = 0 \ \forall t, j$ , implies that

$$p_t^j = E_t \left[ \sum_{s=0}^{\infty} \beta^s d\left( X_{t+s}^j \right) \right]$$
(12)

where  $\tilde{d}_t^j = d\left(X_t^j\right)$ .

#### 4.2 Migration decisions

I next turn to characterizing the worker's migration decision. Given (10) and (11) it follows that all workers with no debt overhang buy housing in equilibrium. Furthermore, since there is no involuntary default (due to no repayment risk) all default is strategic and arises whenever a homeowner with debt overhang chooses to migrate (see Appendix B). This is because in the model the utility difference between having debt overhang and not is lower than the default penalty as the only benefit from defaulting without moving is the forgone cost of default one period later. Given these observations let  $V^h(X)$  be a worker's end-ofperiod value given the regional state X and the idiosyncratic housing state (h = 1 for no debt overhang and h = 0 for debt overhang). Then defining

$$\bar{V} = \max_{\tilde{x}} \left\{ V^1(\tilde{x}) \right\} \tag{13}$$

as the migration value, we have that:

$$V^{h}(X) = \gamma - d(X) + e + \mu(\theta(A))(w(A) - e) + \beta E_{X}\left[W^{h}\left(X'\right)\right]$$
(14)

where

$$W^{h}(X) = \max_{\bar{\epsilon}} \left\{ F(\bar{\epsilon}) \bar{V} + (1 - F(\bar{\epsilon})) V^{h}(X) - F(\bar{\epsilon}) (c + (1 - h)\zeta) + \int_{\bar{\epsilon}} \epsilon dF \right\}$$
(15)

is the beginning-of-period value function for the worker. The interpretation of these value functions is straightforward. For  $V^h$ , the first part captures the per-period utility from employment/unemployment as well as utility from owning housing, net of mortgage repayment. The second is the expected value in the next period, which takes into account the migration option of the worker. In particular for a given region preference  $\epsilon$ , a worker compares the value of staying in the region to the value of moving to a region that offers the highest expected utility, net of the migration cost and potential default penalties. Given the structure of the problem, migration will follow a cutoff rule for  $\epsilon$ , which I denote by  $\overline{\epsilon}$ , and which is given by:

$$\overline{\epsilon}(X,h) = \overline{V} - V^{h}(X) - c - (1-h)\zeta$$
(16)

Then for  $\epsilon < \overline{\epsilon}(X, h)$  the worker migrates and for  $\epsilon > \overline{\epsilon}(X, h)$  the worker stays, which means that the fraction of workers migrating from a region with state X is:

$$q(X,h) = Pr(\epsilon \le \overline{\epsilon}(X,h)) = F(\overline{\epsilon}(X,h))$$
(17)

which is also the ex ante probability of worker migration prior to realization of  $\epsilon$ . Note that it immediately follows that  $q(X, 0) \leq q(X, 1)$ , i.e. a worker with debt overhang will migrate out of a region less often than a worker with no debt overhang.

#### 4.3 Laws of motion for endogenous state variables

Given the worker migration decisions above, it follows that the end-of-period measure of workers with no debt overhang in a given region j,  $l_{1,t}^{j}$ , is:

$$l_{1,t}^{j} = l_{1}^{\prime} \left( X_{t}^{j} \right) = \left( 1 - q \left( X_{t}^{j}, 1 \right) \right) l_{1,t-1}^{j} + \Psi \left( X_{t}^{j} \right)$$
(18)

where  $\Psi(X_t^j)$  is a function that gives the measure of workers migrating into the region at time t. In particular,

$$\Psi\left(X_{t}^{j}\right) \geq 0 \quad X_{t}^{j} \in \arg\max_{x}\left\{V^{1}(x)\right\}$$
$$\Psi\left(X_{t}^{j}\right) = 0 \qquad o.w.$$

i.e. due to directed migration, some regions do not experience worker inflows. The exact form of  $\Psi(X)$  for  $X \in \arg \max_x \{V^1(x)\}$  is determined in equilibrium. Similarly, the end-

of-period measure of workers with depreciated housing in a given region,  $l_{0,t}^{j}$  is given by:

$$l_{0,t}^{j} = l_{0}^{\prime} \left( \tilde{X}_{t}^{j} \right) = \left( 1 - q \left( X_{t}^{j}, 0 \right) \right) l_{0,t-1}^{j}$$
(19)

Therefore, if  $q(X,0) \neq 0$ ,  $\forall X$ ,  $\lim_{t\to\infty} l_{0,t}^j = 0$ , i.e. unless such workers are completely hindered from migrating, so that q(X,0) = 0 for some X, the measure of workers with depreciated housing goes to 0 over time.

#### 4.4 Stationary Equilibrium

I now turn to the first main result of this section, showing how the model can account for Facts 2 and 3 from Section 2. I will show this in a symmetric stationary equilibrium of this economy, in which each region has a measure  $l_0$  of workers with debt overhang. Given that I will be looking at a stationary equilibrium, it follows that the distributions  $\overline{\nu}$  and  $\underline{\nu}$  are time invariant, and so the state vector X will contain  $A_t^j$  and  $l_{1,t-1}^j$  only. Also, for notational convenience I use l to denote the beginning-of-period measure of workers with no debt overhang in a region in place of  $l_1$ .

I define a stationary equilibrium for economies that satisfy the following assumption:<sup>21</sup>

This assumption means that I consider economies, in which there is gross out-migration out of any region, while workers with debt overhang are completely immobile. The first part of the assumption is technical, while the second part allows me to have workers with debt overhang in a stationary equilibrium and hence to examine conceptually, the effects of a housing bust on the labor market. Therefore, for the rest of this Section, I refer to workers with debt overhang as immobile workers and to workers with no debt overhang as mobile workers. Also, I focus on equilibria in which both  $V^1(A, l)$  and  $l'_1(A, l)$  are continuous.<sup>22</sup>

I first characterize the regional dynamics in this economy and the link between migration and regional characteristics resulting from the equilibrium behavior of workers. A set of technical results in Appendix B, Lemmas 11, 12, and 13, which characterize the equilibrium properties of  $V^1(A, l_1)$  and  $l'_1(A, l_1)$  allow for this. Here, I only summarize their implications. First of all, Lemma 11 implies that workers with no debt overhang weakly prefer regions

 $<sup>^{21}</sup>$ The definition of a symmetric stationary recursive equilibrium is given in Appendix B and follows the broader definition from Section 4.

<sup>&</sup>lt;sup>22</sup>Note that I do not show existence of such equilibria. However, Lemma 10 in Appendix B shows that having  $V^1(A, l_1)$  and  $l'_1(A, l_1)$  continuous is mutually consistent and hence possible in equilibrium.

with lower populations. Secondly, the law of motion  $l'_1(A, l)$  is increasing in l, so that regions with a high beginning-of-period population of mobile workers can never end up with lower end-of-period populations compared to regions with low beginning-of-period population.

Lemma 12, in turn, shows that there are stable populations of mobile workers depending on regional labor productivity A,  $\underline{l}^*$  for regions with productivity  $\underline{A}$ , and  $\overline{l}^*$  for regions with productivity  $\overline{A}$ . Regions with productivity  $\overline{A}$  and with  $l < \overline{l}^*$  will be experiencing inflows that increase their population of mobile workers to  $\overline{l}^*$  or slow declines in population towards  $\overline{l}^*$ , if  $l > \overline{l}^*$  and similarly for regions with productivity  $\underline{A}$ . Therefore, Lemma 12 describes how the in-migration function  $\Psi(A, l)$  looks like in equilibrium.

Lastly, Lemma 13 shows that in equilibrium, regions with high productivity weakly dominate regions with low productivity in mobile workers migration decisions for any given population l. This implies that, depending on parameter values, there can be two types of equilibria. The first type is a "partial compensation" equilibrium with,  $\underline{l}^* = 0$  and  $V^1(\underline{A}, l) < V^1(\overline{A}, l), \forall l$ , while the second is a "full compensation" equilibrium with  $\underline{l}^* \geq 0$ and  $V^1(\underline{A}, \underline{l}^*) = V^1(\overline{A}, \overline{l}^*)$ , In the first type of equilibrium, house price differences cannot compensate for labor market differences for any population of mobile workers, whereas in the second house price differences do compensate fully for labor market differences as long as populations in low productivity regions fall sufficiently.

Therefore, we can make the following observations about the dynamic evoluation of regions in a stationary equilibrium of this economy:

**Lemma 1.** The following hold in a stationary equilibrium of this economy

- 1. Regional populations of mobile workers lie in the set  $[\underline{l}^*, \overline{l}^*]$ ;
- 2. Transitioning from depressed to booming, a region's population of mobile workers increases to  $\bar{l}^*$ ;
- A depressed region's population of mobile workers moves down towards <u>l</u>\* experiencing a decreasing out-migration rate as the population of mobile workers declines towards <u>l</u>\*;
- 4. Depressed regions experience no in-migration apart from regions with  $l \in [\underline{l}^*, \underline{l}]$ , where <u>l</u> is given by equation (29) in Appendix B;

5. The stationary distributions  $\underline{\nu}^*$  and  $\overline{\nu}^*$  are discrete.

Proof. See Appendix B.

Figure 1: Regional population dynamics



Figure 1 summarizes the implications of Lemma 1 for regional *net* migration in "full compensation" equilibria.<sup>23</sup> Regions with low productivity slowly lose population, whereas a region that experiences a positive productivity shock moves up to  $\bar{l}^*$ .

Therefore, idiosyncratic regional preferences and the mobility cost c lead to limited spatial arbitrage. Depressed regions lose population more slowly than in an economy with frictionless mobility. Limited spatial arbitrage, in turn, creates a non-degenerate distribution of regions over populations. This, combined with durable housing and a downward sloping demand by the local sector create a dependence of regional house prices on the history of labor market shocks. Figure 2, which graphs the simulated paths for regional productivity, unemployment rate and house prices for a region, clearly shows this history dependence.

Not surprisingly, given the one-period-job-length assumption, the regional unemployment rate simply jumps with regional productivity. The behavior of house prices, however, shows how a region hit by a negative productivity shock experiences an initial large drop in house prices and subsequent smaller declines. A spatial equilibrium model with perfect spatial arbitrage in the spirit of Roback (1982) would imply a jump in house prices in response to regional productivity only, similarly to the response of unemployment. In contrast, as

<sup>&</sup>lt;sup>23</sup>The regional evolution for " partial compensation" equilibria is similar to that of "full compensation" equilibria but with regions hit by a sequence of negative productivity shocks never actually reaching  $\underline{l}^* = 0$  but coming arbitrarily close.



#### Figure 2: Simulated regional dynamics

the Figure illustrates, limited spatial arbitrage creates very different house price dynamics with house prices movements occuring without labor market shocks. Lastly, Figure 2 also shows how population inflows into regions create house price jumps.<sup>24</sup>

The dependence of regional house prices on the history of labor market shocks also implies a rich distribution of regions over house prices. This variation, in turn, leads to the observed co-movements between regional house prices and regional out- and in-migration from Section 2 as I now show. First of all, I use the characterization results above to clarify how regional inflows and outflows are related to regional state variables.

**Proposition 2.** Let out(A, l) and in(A, l) be the out-migration and in-migration rates for a region described by (A, l). Then:

1.  $out(\overline{A}, l) \leq out(\underline{A}, l), \ l \in \left[\underline{l}^*, \overline{l}^*\right] \text{ and } in(\overline{A}, l) \geq in(\underline{A}, l), \ l \in \left[\underline{l}^*, \overline{l}^*\right] \text{ with the inequalities strict for some } l.$ 2.  $out(A, l) \text{ is increasing in } l \text{ and } in(A, l) \text{ is decreasing in } l \text{ for } l \in \left[\underline{l}^*, \overline{l}^*\right].$ 

Proof. See Appendix B

<sup>&</sup>lt;sup>24</sup>The large in-migration into booming regions with low population due to directed migration and the jump in population that this entails is, of course, unrealistic. However, one can smooth out these jumps by introducing a realistic convex cost of transforming housing into residential units from units used in production, for example.

The above result is intuitive: higher regional productivity, implies lower out-migration from that region and higher in-migration for a given population. On the other hand, for given labor productivity, a region with a smaller population experiences lower out-migration and higher in-migration. This effect is due to the difference in house prices across such regions, with workers migrating out less from regions with lower house prices and migrating more into such regions.

However, this result does not make it clear how house prices co-move with regional flows. In order to make this connection, I first characterize the behavior of house prices across regions. I will keep track of two prices, the regional house price prior to workers' migration decisions,  $\tilde{p}(A, l)$ , and the house price after migration takes place, p(A, l), i.e.  $\tilde{p}$ is the regional house price in the beginning of a period, while p is the regional house price in the end of a period. We have the following result.

**Proposition 3.** p(A, l) is increasing in A and l for  $l \in \left[\underline{l}^*, \overline{l}^*\right]$ , and  $\tilde{p}(A, l)$  is increasing in l for  $l \in \left[\underline{l}^*, \overline{l}^*\right]$ .

Proof. See Appendix B

The proposition establishes a tight link between beginning-of-period house prices  $\tilde{p}$  and the beginning-of-period measure of mobile workers in a region, l. Therefore, defining  $U(A, l) = (1 - \mu(\theta(A)))$  as the unemployment rate in region (A, l), we immediately have the following result.

**Proposition 4.** Consider a cross-sectional sample of J regions from the model economy. Let  $out^j = out(A^j, l^j)$  and  $in^j = in(A^j, l^j)$  be the out-migration and in-migration rates for region  $j \in \{1, 2, ..., J\}$ . Also, let  $U^j = U(A^j, l^j)$  be the unemployment rate and  $\tilde{p}^j = \tilde{p}(A^j, l^j)$  be the beginning-of-period house price for a region  $j \in \{1, 2, ..., J\}$ . Then:

- 1. for a given  $\tilde{p}^{j}$ , out<sup>j</sup> is increasing in  $U^{j}$  and  $in^{j}$  is decreasing in  $U^{j}$ ;
- 2. for a given  $U^j$ ,  $out^j$  is increasing in  $\tilde{p}^j$  and  $in^j$  is decreasing in  $\tilde{p}^j$ .

Proof. First, Proposition 3 implies that  $\tilde{p}^j = \tilde{p}(A^j, l^j)$  is increasing in  $l^j$  for  $l^j \in [\underline{l}^*, \overline{l}^*]$ . Secondly, note that  $U^j = U(A^j, l^j)$  is decreasing in A. Given these two observations, fact 1 follows from fact 1 in Proposition 2 and fact 2 follows from fact 2 in that Proposition as well.

Proposition 4 establishes that the model can account for the co-movements between relative unemployment and house prices and regional migration documented in Section 2. It implies that if one simulates data for many regions from the model and runs the panel regression from Section 2, one would obtain coefficient estimates with the same signs as in the data. The intuition for the result is that the unemployment rate  $U^j$  captures the variation in regional productivity  $A^j$ , while the beginning-of-period house price  $\tilde{p}^j$  captures the variation in  $l^j$ , the measure of mobile workers. In that sense, it is directly linked to the results from Proposition 2, however it casts it into co-movements based on observable variables, like  $U^j$  and  $\tilde{p}^j$  rather than the state variables A and l.

It is important to discuss, which components of the model drive these results. First of all, the equilibrium house price heterogeneity arising from regional histories, drives the positive co-movement between out-migration and house prices holding current labor market conditions fixed. Limited spatial arbitrage also leads to the co-movement between out-migration and unemployment controlling for house prices. Directed migration, on the other hand, implies that regions with booming labor markets and lower populations and consequently, house prices, have larger population inflows leading to the negative co-movement between house prices and unemployment with in-migration. Therefore, models with undirected migration would have troubles accounting for the in-migration co-movements. Additionally, models with frictionless regional mobility would need high frequency variation in the value of amenities or construction costs, that is independent from variation in labor market conditions to simultaneously account for the two facts.

#### 4.5 Housing bust and regional reallocation

I now turn to the second main result of this section, related to the labor market and regional reallocation effects of a housing bust. I focus on a stationary equilibrium analyzed in Section 4.4, switching off house price differences (g(.) is linear), and on how the measure of workers,  $l_0$  with debt overhang, affects the aggregate labor market. I show that increasing the measure of these immobile workers increases aggregate unemployment and that small regional shock amplifies that effect.

First of all, I define aggregate unemployment  $U_{agg}$  as a function of  $l_0$ . It immediately follows that:

$$U_{agg}(l_0) = \left(1 - \mu\left(\theta\left(\overline{A}\right)\right)\right) \left(\overline{l}^*(l_0) + l_0\right) + \left(1 - \mu\left(\theta\left(\underline{A}\right)\right)\right) \left(\int_{\underline{l}^*(l_0)}^{\overline{l}^*(l_0)} l_1'(\underline{A}, l; l_0) d\underline{\nu} + l_0\right)$$

$$(20)$$

or, alternatively, using equations (18) and (30):

$$U_{agg}(l_0) = (1 - \mu(\theta(\underline{A}))) L - (\mu(\theta(\overline{A})) - \mu(\theta(\underline{A}))) \underbrace{(\overline{l}^*(l_0) + l_0)}_{\overline{A} \text{ regions}}$$
(21)

In equation (21), the bracketed expression  $\bar{l}^*(l_0) + l_0$  is the end-of-period measure of workers

in booming regions with  $\bar{l}^*$  ( $l_0$ ) denoting the stationary equilibrium level of  $\bar{l}^*$  given  $l_0$ . The equation very clearly shows the importance of regional reallocation for aggregate unemployment. More workers in booming regions decreases unemployment as they face a higher job finding probability compared to workers in low productivity regions. Therefore, any interference with the movement of workers from low productivity to high productivity regions would increase aggregate unemployment. One can show that this is exactly what increases in  $l_0$  do.

**Lemma 5.** The end-of-period population of workers in booming regions,  $\bar{l}^* + l_0$ , is decreasing in  $l_0$ .

Proof. See Appendix B.

This result is intuitive, considering that the total population in the economy is constant. Reducing the fraction of mobile workers leads to lower population dispersion over regions, which implies that the population of booming regions declines. Then we immediately have that more immobile workers have a negative impact on unemployment but also that larger local recessions amplify that effect.

**Proposition 6.** Aggregate unemployment,  $U_{agg}(l_0)$ , is increasing in the measure of immobile workers,  $l_0$ . Furthermore,  $u(l_0) = -\frac{\partial - U_{agg}(l_0)}{\partial \underline{A}} \Big|_{\underline{A} = \overline{A}}$  is also increasing in  $l_0$ .

Proof.  $U_{agg}(l_0)$  increasing in  $l_0$  follows from immediately from inspection of (21) given that  $\bar{l}^* + l_0$  is decreasing in  $l_0$  and  $U_{agg}(l_0)$  is decreasing in  $\bar{l}^* + l_0$ . The second part is also straightforward since  $-\frac{\partial - U_{agg}(l_0)}{\partial \underline{A}} \Big|_{\underline{A} = \overline{A}} = \frac{d\mu}{d\underline{A}} \left( L - \bar{l}^*(l_0) - l_0 \right)$  and  $\bar{l}^* + l_0$  decreasing in  $l_0$ .  $\Box$ 

The reason for this result is straightforward: worker immobility hinders regional reallocation, which results in higher aggregate unemployment. On the other hand, regional reallocation is more important whenever regional disparities are larger, i.e. when regional recessions are deeper, which gives the amplification effect. These two results point to a potentially important quantitative effect of the housing bust on the labor market in the recent recession given the simultaneous large divergence in regional economic conditions. I turn now to addressing this question.

# 5 Quantitative Model

In this section I study the quantitative implications of my model of a housing bust. The purpose of this is to show some additional features of the model and give a sense of the potential magnitudes of the effects discussed in Section 4.5 in the context of the recent

recession. However, the basic model with one-period job length, which I examined in Section 3, is not appropriate for establishing quantitative effects. Furthermore, it is silent on the effects of regional reallocation distortions for regional unemployment. Therefore, in this section I calibrate a version of the model that is richer in terms of labor market dynamics.

#### 5.1 Model set-up

I first briefly describe how the calibrated model differs from the basic model. Appendix C contains a more detailed description of that model. Rather than one period long jobs, the calibrated model has stochastic job destruction, which is the standard assumption of search and matching models of the labor market. In particular, at the end of each period, after production takes place, with probability  $s \in (0, 1)$  a job becomes unproductive and is destroyed. This assumption is important for generating realistic employment flows and *ex ante* employment heterogeneity among workers. I also assume that there's no on-the-job search and that only unemployed workers match to vacant jobs.<sup>25</sup>

With *ex ante* employment heterogeneity among workers it also becomes necessary to specify migration decisions for both the employed and unemployed. I assume that only the unemployed suffer idiosyncratic region preference shocks, and migrate, while workers that are employed at the beginning of a period remain in the same region. This assumption is similar to the no on-the-job-search assumption of the standard search and matching model. However, it still allows for all important combinations of joint migration and employment flows observed in the data to be represented.<sup>26</sup>

I calibrate a model with no regional house price differences (g''(.) = 0), which implies that migration decisions are based on regional labor market conditions only. The reason for this is computational tractability, since some of the quantitative exercises below deal with simulating a transition to a steady state rather than a stationary equilibrium. Numerical simulations of the basic model, however, show that the effects of house price differences on reallocation are likely to be small.

Lastly, wages in the calibrated model are rigid in the sense of Hall (2005). As was first pointed out by Shimer (2005), the standard search model with Nash bargaining leads to a large response of wages to changes in labor productivity, unless the value of unemployment is very high (Hagedorn and Manovskii (2008)). The large sensitivity of the bargained wage implies that changes in labor productivity are mostly absorbed by changes in the

 $<sup>^{25}</sup>$ Also, I count as unemployed the workers who are not employed at the beginning of a period, not the workers who are unmatched at the end of a period, as in the basic model.

<sup>&</sup>lt;sup>26</sup>Also, CPS migration data shows that currently unemployed workers are much more likely to have migrated in the previous year than currently employed workers.

wage resulting in small effects on the job finding probability and from there on unemployment. As a result, the standard search model has problems accounting for the volatility of unemployment, given the observed volatility in labor productivity.

An analogous problem arises in the environment with regional labor markets that I consider. In particular, Lkhagvasuren (2011) shows that for the period 1974-2004 regional labor productivity volatility is  $\sigma_A = 1.2\%$ . Using this as the dispersion in productivities between booming and depressed regions and calibrating my model with wage determination via Nash bargaining produces regional unemployment dispersion  $\hat{\sigma}^u < 0.1\%$  for the baseline calibration versus  $\hat{\sigma}^u = 1\%$  in the data. Matching that regional unemployment dispersion entails setting regional productivity dispersion to around 6%, that is 5 times higher than the observed one. Trying to match even higher unemployment dispersion, such as the one from the recent recession, for example, requires an even higher number.

The modification of the standard search model that Hall (2005) proposes is to have a rigid wage, arising, for example, from a social norm, which does not vary with the aggregate business cycle, thus breaking the strong link between productivity and the wage. Furthermore, the wage lies in the bargaining set of a worker-job pair for every value of productivity over the cycle and hence does not violate individual rationality<sup>27</sup>. This is the approach I adopt as well.<sup>28</sup> A rigid wage also significantly simplifies the computation of equilibrium since the firm's problem does not depend on the whole distribution of workers over debt overhang and no debt overhang, as would be the case with a Nash bargained wage.

#### 5.2 Baseline calibration

I calibrate the model to monthly frequency. For the calibration, I consider a region in the model to correspond to a metropolitan statistical area (MSA) in the data. A metropolitan statistical area according to the US Office of Management and Budget definition is a city of at least 50,000 inhabitants and its adjacent areas that have a high degree of economic integration in terms of commuting time (OMB (2009)). Therefore, according to this definition workers within an MSA do not need to move in order to search and be matched to a vacant job in the MSA, which matches well the specification of regions in the model.<sup>29</sup>

The baseline calibration is for the low regional dispersion period of the early 2000s, when, as noted in the Introduction, MSA unemployment dispersion was around 1%. The model

 $<sup>^{27}</sup>$ There is a large subsequent literature dealing with rigid wages in search models (see Gertler and Tigari (2009) and Shimer (2010)).

 $<sup>^{28}</sup>$ See Appendix C for the exact conditions.

<sup>&</sup>lt;sup>29</sup>Note that MSAs may span one or several counties and may be contained in one or several states. However, data limitations preclude me from having the empirical facts from Section 2 at the MSA rather than the state level.

contains a set of fixed parameters and functional forms, as well as a set of parameters that I calibrate jointly based on matching data moments to corresponding moments simulated from the model.

I set the discount factor  $\beta$  to 0.995, which gives an annual discount rate of around 6%. For the flow benefit from unemployment I set e to 0.65, which lies between the values proposed by Shimer (2005) and Hagedorn and Manovskii (2008). The exogenous job destruction probability s is set to be 0.034, which is consistent with the rate used in Shimer (2005) and Hall (2005). For the regional productivity process, I assume  $\overline{A} = A + d$  and  $\underline{A} = A - d$ , where A is average regional productivity and d parametrizes regional productivity dispersion. Given the linear production technology, I normalize the average productivity A of a workerjob pair to 1. I set the persistence of regional productivity to  $\rho = 0.98$ . This is consistent with the high persistence of state unemployment in the data of around 0.99.<sup>30</sup>

Turning to the matching technology, I use a Cobb-Douglas matching function,  $m(\tilde{u}, v) = \kappa \tilde{u}^{1-\alpha}v^{\alpha}$ , which implies that  $\mu(\theta) = \kappa \theta^{\alpha}$  for  $\theta = \frac{v}{\tilde{u}}$ . I estimate  $\alpha$  and  $\kappa$  from JOLTS using monthly data from December, 2000 to December, 2007. Note that I need an estimate for a matching function at a low level of aggregation but have only aggregate data. This would be worrying, when regional dispersion is high since mismatch would affect the estimated aggregate matching function, which will no longer correspond to the matching function at a lower level of aggregation (Barnichon and Figura (2011). Therefore, I estimate the aggregate matching function for a time period when of low regional dispersion to ensure that estimate would be close to a matching function estimate at a lower level of aggregation.<sup>31</sup> The estimates I obtain for the matching function are  $\alpha = 0.605$  and  $\kappa = 1$ . The value of  $\alpha$  obtained lies in the middle of the set of estimates reported by Petrongolo and Pissarides (2001).

I assume that regional preferences are drawn from a truncated normal distribution with zero mean and variance  $\sigma_{\epsilon}^2$ , where the domain is given by [-B, B]. The value of B has a small effect on the results as long as it is larger than the mobility cost c, and so I set  $B = 4\sigma_{\epsilon}$ .

I calibrate the vacancy posting cost k, volatility of preference shock,  $\sigma_{\epsilon}$ , and regional productivity dispersion d jointly by matching the following data moments using the corresponding simulated moments from the model:

1. unemployment rate of u = 5%. I obtain this as the average unemployment rate for the period 2000-2007;

 $<sup>^{30}</sup>$ I also set the total population of workers in the economy to L = 2, for convenience.

<sup>&</sup>lt;sup>31</sup>JOLTS contains information on total hires per month, which, when divided by the total stock of unemployed, gives the job finding probability  $\mu(\theta)$ . The value of  $\theta$  is similarly obtained as the total vacancies divided by the stock of unemployed.

- 2. annual migration rate of q = 5%. I obtain this from the CPS using aggregate data on mobility rates for people in the labor force, which corresponds to the workers in my model. Since the CPS does not track migration rate at the MSA level, I look at the average of inter-state migration rate for the period 2000-2007 and average of inter-county migration rate for the same period and take a value that lies between these two;
- 3. unemployment dispersion of  $\hat{\sigma}_u = 1\%$ . I obtain this by computing

$$\hat{\sigma}_t^u = \sqrt{\sum_{i=1}^n \frac{l_{i,t}}{l_t} (u_{i,t} - u_t)^2}$$

where  $l_{i,t}$  is labor force in MSA *i* at time *t*,  $l_t$  is total labor force at time *t*,  $u_{i,t}$  is the unemployment rate in MSA *i* at time *t* and  $u_t$  is national unemployment rate and taking the average over the period 2000-2007.

Note that each of the above moments roughly corresponds to the particular parameter that it identifies, k,  $\sigma_{\epsilon}$ , and d, respectively. I simulate a steady state equilibrium of this economy and compute the moments as in the data.<sup>32</sup> Additionally, I set the mobility cost to c = 12w, the annual wage in my model. This estimate lies in between the mobility cost estimates found in Bayer and Juessen (2008) and Kennan and Walker (2011), who report a mobility cost of around 6 and 36 monthly wages, respectively based on estimations of structural models. Lastly, the regional wage rate is set at the wage rate from a standard search and matching model with symmetric Nash bargaining and no regional productivity dispersion but otherwise parametrized as my model. This is similar to the approach from Hall (2005). I summarize this baseline calibration procedure in Table 2.

Before examining the effects of a housing bust and calibrating the parameters necessary for that, I first show how the model can account for the co-movement between regional unemployment dispersion and shifts in the Beveridge curve observed in the data. Showing this comovement is independent of any housing bust effects and hence can be done without a housing depreciation shock.

#### 5.3 Regional shocks and Beveridge curve shifts

In her study on the shifts of the U.S. Beveridge curve, Abraham (1987) conjectures that dispersion in regional economic conditions are associated with shifts in the curve. I confirm

 $<sup>^{32}</sup>$ Appendix C contains information on the algorithm I use to simulate this economy. For the parameters values I obtain, the simulated moments match the data moments almost exactly.

parameter	description	value
$\beta$	discount factor	0.995
e	flow unemployment payoff	0.65
s	job destruction probability	0.034
A	average regional productivity	1
ρ	regional productivity persistence	0.98
$\mu( heta)$	matching function	$ heta^{0.605}$
$\sigma_\epsilon$	volatility of preference shock	8.467
$F(\epsilon)$	distribution of preference shocks	$N(0, \sigma_{\epsilon}^{2})$ with symmetric truncation at $B = 4\sigma_{\epsilon}$
k	vacancy posting cost	0.633
с	mobility cost	11.776
d	baseline regional productivity dispersions	0.0056
w	wage rate	0.9813

## Table 2: Baseline Parameters

Table 3: Beveridge curve regressions with vacancy rate as dependent variable (a) and unemployment rate as dependent variable (b)

Dep Variable		vacan	cv rate	
Dop. (anabio.		v	59 1000	ln(v)
$\frac{1}{(u)}$	$-0.447^{***}$ (0.0510)	$-0.498^{***}$ (0.0380)	$-0.965^{***}$ (0.170)	
MSA Unemployment Dispersion	( )	$0.533^{**}$ (0.235)	$0.528^{**}$ (0.189)	$0.0941 \\ (0.0623)$
$u^2$			$0.0371^{**}$ (0.0132)	
ln(u)				$-0.933^{***}$ (0.0664)
$R^2$	0.970	0.977	0.988	0.983

(a)

(b)

Dep. Variable:	unemployment rate				
		u		ln(u)	
vacancy rate	-2.036***	-1.841***	-3.495**		
(v)	(0.165)	(0.0944)	(1.201)		
MSA Unemployment		$1.287^{*}$	0.954	$0.135^{*}$	
Dispersion		(0.669)	(0.643)	(0.0680)	
$v^2$			0.250		
			(0.173)		
ln(v)				-1.001***	
				(0.0763)	
$R^2$	0.970	0.982	0.984	0.983	

Notes: Own calculations from BLS and Conference Board. Annual data from 1990-2010. Newey-West robust standard errors in parenthesis; \* = significant at 10%; \*\* = significant at 5%; \*\*\* = significant at 1%. All regressions include a cubic time trend. v is national vacancy rate and u is national unemployment rate. Dispersion measure for variable x is defined as  $\hat{\sigma}_t^x = \sqrt{\sum_{i=1}^n \frac{e_{i,t}}{e_t}(x_{i,t}-x_t)^2}$  where  $e_{i,t}$  is labor force (employment) in MSA (sector) i at time t,  $e_t$  is national labor force (employment) at time t,  $x_{i,t}$  is the realization in MSA/sector i at time t and  $x_t$  is weighted average of MSA (sector) realizations weighted by labor force (employment) weights.

this by first showing that there is a positive co-movement between regional unemployment dispersion and shifts in the Beveridge curve and then showing how my model can account for it.

I look at annual data for the U.S. and construct a synthetic vacancy rate using the Conference Board HWI and JOLTS (see Appendix A for details), as well as annual employment weighted state and MSA unemployment dispersion series. I estimate a standard Beveridge curve regression, adding a cubic time trend to control for secular shifts in the curve as well as for trends in the vacancy or unemployment rates, and also include my unemployment dispersion measure to the regression. The main regressions I run are of the form:

$$v_t = \alpha_0 + \alpha_1 \, u_t + \alpha_2 \, \hat{\sigma}_t^u + \sum_{i=1}^3 \alpha_{2+i} \, t^i + \epsilon_t \tag{22}$$

and

$$u_{t} = \beta_{0} + \beta_{1} v_{t} + \beta_{2} \hat{\sigma}_{t}^{u} + \sum_{i=1}^{3} \beta_{2+i} t^{i} + \epsilon_{t}$$
(23)

where  $v_t$  is the aggregate vacancy rate in year t,  $u_t$  is the national unemployment rate and  $\hat{\sigma}_t^u$  is regional unemployment dispersion. I also augment (22) and (23) with the square of unemployment rate and vacancy rate, respectively and also run a regression with the vacancy and unemployment rates in logs to control for non-linearities.

Table 3 shows the regression results for MSA unemployment dispersion.<sup>33</sup> Not surprisingly there is a very significant negative relation between vacancies and unemployment. More interestingly, there is also a positive relation between MSA unemployment dispersion and vacancies, controlling for unemployment (or vice versa in the regressions where unemployment is the dependent variable).

This relation implies a positive co-movement between state/MSA unemployment dispersion and shifts in the Beveridge curve. The recent recession and its aftermath are examples of this positive co-movement as both state and MSA unemployment dispersion have increased significantly, and at the same time the Beveridge appears to have shifted out.<sup>34</sup>

I now show how regional shocks in my model can generate this co-movement. I calibrate regional productivities <u>A</u> and  $\overline{A}$  to match regional unemployment dispersions of  $\hat{\sigma}^u = 2\%$ and  $\hat{\sigma}^u = 3\%$ , while keeping aggregate market tightness,  $\theta_{agg}$ , at the level of the baseline calibration. Keeping  $\theta_{agg}$  constant controls for an aggregate shock that would move vacancies

<sup>&</sup>lt;sup>33</sup>Table 11 contains the results for state unemployment dispersion.

<sup>&</sup>lt;sup>34</sup>The possibility of an outward shift in the curve has renewed interest among academics and policy makers about the importance of mismatch for the labor market. (Katz (2010) and Kocherlakota (2010)). Empirical work on the Beveridge curve and JOLTS data by Barnichon and Figura (2010) and Barnichon, Elsby, Hobijn, and Sahin (2010) provides additional evidence for the magnitude of the recent shift.

$\overline{\overline{A}} - \underline{A}$	$U_{agg}$	$\hat{\sigma}^u$
0.0112	5%	1%
0.0231	5.22%	2%
0.0318	5.56%	3%

Table 4: Regional shocks and Beveridge Curve shifts

Notes: Model simulation results. See text for details.  $\overline{A} - \underline{A}$  is the productivity difference between booming and depressed regions,  $U_{agg}$  is the aggregate unemployment rate and  $\hat{\sigma}^u$  is unemployment rate dispersion.

and unemployment in the opposite direction. Any changes in unemployment, keeping  $\theta_{agg}$  constant, will then be associated with changes in vacancies in the same direction, that is shifting of the vacancy-unemployment locus. Table 4 contains the results for aggregate unemployment  $(U_{agg})$  and unemployment dispersion  $(\hat{\sigma}^u)$ . Increasing the dispersion in regional productivity, while keeping  $\theta_{agg}$  constant, increases unemployment dispersion and aggregate unemployment, which is exactly the observed co-movement in the data. Therefore, the model generate shifts in the Beveridge curve accompanied by increases in unemployment dispersion. The quantitative effect it generates is smaller than in the data, so the model can explain only part of that co-movement.

#### 5.4 Housing bust effects

I now look at the quantitative magnitudes of the labor market effects of a housing bust in my model by performing two exercises. In the first one, I show the effect through migration distortions due to negative equity and default, while the second one provides an upper bound on any regional reallocation distortions that the housing bust may have. In each exercise, there are two parameters to calibrate, the default penalty,  $\zeta$ , and the fraction of the regional population that has debt overhang, which I denote by  $\lambda$ . Given  $\zeta$  and  $\lambda$ , I simulate the effect of the recent recession by considering a housing depreciation shock and a simultaneous permanent shock to average productivity,  $A_{2010}$ , and productivity dispersion,  $d_{2010}$ .<sup>35</sup> I look at a transition path for the model economy, simulating 24 months of data,

<sup>&</sup>lt;sup>35</sup>Note that the recent housing bust may have had a direct effect on the severity of the aggregate and local recessions by affecting financial intermediaries balance sheets or even through its effect on household's consumption decisions (Iacoviello (2005), Mian and Sufi (2011), Midrigan and Philippon (2011)). However, for this paper, I want to focus on one particular channel - the indirect impact of a housing bust on the labor market through regional reallocation. Modeling the impact of the recession through these reduced form productivity shocks allows me to do that by switching off these other channels.

Parameter:	$A_{2010}$	$d_{2010}$	ζ	λ
Quantitative Exercise 1	0.9931	0.007	17.66	0.13
Quantitative Exercise 2	0.9934	0.006	6.94	1

Table 5: Parameter values for Housing Bust Exercises

and use the last 12 months to set  $A_{2010}$  and  $d_{2010}$  by matching the average unemployment rate and MSA unemployment dispersion for 2010 of u = 9.5% and  $\hat{\sigma}^u = 2.3\%$ .<sup>36</sup> Table 5 below summarizes the parameter values used in the two exercises.

#### 5.4.1 Effects through "negative equity"

I calibrate the default penalty,  $\zeta$ , by using a microestimate of the cost of default implied by default decisions of households with negative equity. I use information from Bhutta, Dokko, and Shan (2010), who use a sample of non-prime borrowers from the states affected most deeply by the recent housing bust to estimate the negative equity threshold that is associated with voluntary default. Such a threshold can be taken as the combination of pecuniary and non-pecuniary costs that are associated with default, since if default were costless, a household would default as soon as its housing equity goes below zero (Deng, Quigley, and Order (2000)).<sup>37</sup> These authors find that if both voluntary and involunatry default are treated as observationally equivalent, as is the case in my model (since I do not allow for involuntary default), then the median household in negative equity defaults at around -30% of the value of their house (negative equity of -30%). Given a median house value of \$177,000 and average monthly wage of around \$3000 for 2009, it follows that this default cost translates into approximately 18 months of wage income. Therefore, I set a value of  $\zeta = 18w \approx 17.7$  in my model.

I first show how the model can account for the correlation between the fraction of households in negative equity and out-migration, documented in Section 2. Using the baseline calibration for the early 2000s, I look at a small housing depreciation shock with the fraction

<sup>&</sup>lt;sup>36</sup>Since the total period length for the simulation is only 24 months, the assumption of a permanent shock is fine as long as shocks are fairly persistent at monthly frequency in the data. While, I cannot estimate the persistence of a regional shock, state unemployment dispersion data points to a regime switching process for regional shocks given the high unemployment dispersion in the pre-Great Moderation period and the low dispersion thereafter up to the recent recession. A test for multiple structural breaks (Bai and Perron (1998)) confirms this.

<sup>&</sup>lt;sup>37</sup>Their estimates appears to be a lower bound compared to other studies of the implied cost of default (Foote, Gerardi, and Willen (2008), Guiso, Sapienza, and Zingales (2009)).

	Da	ta	Data (EIV correction)	Mo	del
Dep. Variable:	$100 \ln(out)$	$100 \ln(in)$	$100 \ln(out)$	$100 \ln(out)$	$100 \ln(in)$
households/workers with negative equity (%)	-0.243 (0.143)	0.0442 (0.173)	-0.565	-0.844 (0.247)	-7.708 (6.934)
relative unemployment $100 \ln(\frac{u_s}{u})$	$0.168 \\ (0.0359)$	-0.237 (0.0388)		0.821 (0.0258)	-4.579 (0.5866)

Table 6: Model and data comparisons

Notes: Standard errors in parenthesis. EIV correction refers to using a reliability ratio of 0.43 to correct for errors-in-variables in the coefficient on "households with negative equity". See text for details.

of workers in debt overhang uniformly distributed across regions with a mean of 0.03 (the average fraction of homeowners with negative equity during the late 90s and early 2000s according to the CE data) and support of [0,0.06] (the dispersion across states for that period). I then simulate 10 years of data for 50 regions from the transition path of the model. Using this simulated data from the model, I create a regional panel, which I use to run a panel regression similar to that in Section 2. Table 6 compares the regression result for the simulated data to the regression from Section 2.

First of all, looking at the coefficient estimates on households/workers with negative equity, the model generates the negative effect observed in the data for the out-migration regression. The coefficient in the model is higher than in the data. However, this is not unexpected, given the substantial measurement error in the fraction of households with negative equity and the resulting large attenuation bias, as discussed in Section 2. In particular, the third column of Table 6 shows the corrected coefficient,  $\beta_{neg}$ , using the reliability ratio of 0.43 derived in that Section. That reliability ratio is a conservative lower bound on the effect of measurement error, so the discrepancy between the model coefficient and the estimated coefficient from the data is indeed small. Secondly, the estimated coefficient on negative equity for the in-mingration regression is not significant. Therefore, the calibrated model can account for the asymmetric effect of fraction of negative equity on out- and inmigration.

Comparing the coefficients on unemployment, we see that they have the same sign but the coefficients from the model are larger. Note, however, that it is hard to compare the magnitudes of the coefficients, as I am using state level data of household migration, whereas the model simulates MSA level data for individuals that are part of the labor force. Both the use of state level and household migration data will lead to weaker co-movements between unemployment and migration rates. However, the model does produce a much larger effect
	$U_{agg}$	$U_{\underline{A}}$	$U_{\overline{A}}$	$\hat{\sigma}^u$	migration rate
2010 (housing bust)	9.50%	12.12%	7.54%	2.27%	8.45%
2010 (no housing bust)	9.33%	11.71%	7.60%	2.03%	9.67%

Table 7: Results for Quantitative Exercise 1 ( $\zeta = 17.7, \lambda = 0.13$ )

Notes: Model simulation results. See text for details  $U_{agg}$  is the aggregate unemployment rate and  $U_{\underline{A}}$  and  $U_{\overline{A}}$  are average unemployment rates for depressed and booming regions respectively.  $\hat{\sigma}^u$  is unemployment rate dispersion and the "migration rate" column gives the annual inter-MSA migration rate.

of unemployment on in-migration than on out-migration, while in the data that difference is much smaller. This is due to the large variation in the in-migration rate in the model as regions can vary from having no in-migration to having very large in-migration flows. This leads to the large point estimate for the effect of fraction of workers with debt overhang on in-migration as well. Nevertheless, this discrepancy between the model and data affects only how migrating workers are distributed across booming regions, and so it is not important for the labor market effect of the housing bust implied by the model, since what is relevant for that effect is how a housing bust impacts aggregate migration from depressed to booming regions.<sup>38</sup>

Returning to the quantitative exercises, I set the fraction of workers with debt overhang to  $\lambda = 0.13$ , which corresponds to the fraction of households in negative equity in the 5 worst affected states in 2009, based on the CE data. I use this particular number to account for a salient feature of the recent recession, that states with more severe labor market shocks in 2010 were also states, where a higher fraction of households had negative equity. For example, the two states with the largest unemployment problems during 2010 - Nevada and California (with 2010 unemployment rates of 14.3% and 12.3% respectively) - have also experienced some of the most severe negative equity problems. Using the national average would significantly understate the scope of the reallocation distortion for workers. At the same time, having booming regions with a high fraction of less mobile workers affects only the migration rate and not unemployment, as workers in booming regions migrate only to other booming regions in the model.

Table 7 contains the result for the counterfactual experiment (setting  $\lambda = 0$ , i.e. no housing depreciation shock). Through the "negative equity" channel the housing bust can account for around 0.2 percentage points on aggregate unemployment and 0.4 percentage

<sup>&</sup>lt;sup>38</sup>Improving the model fit along this dimension will require a channel for migration into depressed regions as well as for lower inflow rates, for example due to convex costs of transforming housing units for production into residential units.

	$U_{agg}$	$U_{\underline{A}}$	$U_{\overline{A}}$	$\hat{\sigma}^u$	migration rate
2010 (housing bust)	9.50%	12.04%	7.44%	2.29%	3.5%
2010 (no housing bust)	9%	10.79%	7.70%	1.53%	9.2%

Table 8: Results for Quantitative Exercise 2 ( $\zeta = 7.9, \lambda = 1$ )

Notes: Model simulation results. See text for details.  $U_{agg}$  is the aggregate unemployment rate and  $U_{\underline{A}}$  and  $U_{\overline{A}}$  are average unemployment rates for depressed and booming regions respectively.  $\hat{\sigma}^u$  is unemployment rate dispersion and the "migration rate" column gives the annual inter-MSA migration rate.

points of unemployment in depressed metropolitan areas. Without the housing bust unemployment in booming metropolitan areas is slightly higher since such regions experience a higher inflow of unemployed workers. Also, unemployment dispersion falls by almost 0.25 percentage points. Note, however, that the migration rate is almost 8.5%, which is still substantially higher than the post recession migration rate observed. In reality, the housing bust may have an adverse effect on reallocation not only because of negative equity and default but also because of a housing wealth shock by increasing the fraction of households who cannot afford to make a downpayment on a new house and also by increasing downpayment requirements because of credit market freezes. Additionally, the calibrated fraction of workers in debt overhang,  $\lambda$ , may be understating the true fraction of affected households since as discussed in Section 2 the Consumer Expenditure Survey data may be misclassifying households. Nevertheless, I can use the post-recession migration rate to show an upper bound on any regional reallocation effects the housing bust may have according to my model and the implications for unemployment.

### 5.4.2 An upper bound

In this exercise, I use the observed post-recession migration rate to provide an upper bound on the regional reallocation and labor market effects of the housing bust according to the calibrated model. I compute the 2009 MSA migration rate as the average of inter-county and inter-state migration from the CPS and obtain a number of 3.5%. I then set the fraction of affected workers,  $\lambda$ , and default penalty,  $\zeta$ , to match this migration rate. Since there are two parameters and one target, there are multiple combinations of  $\lambda$  and  $\zeta$  that can achieve this. Therefore, I fix  $\lambda = 1$  and set  $\zeta$ , i.e. I look at the increase in mobility cost for all workers that would explain the observed post-recession migration rate.

The results for this experiment are shown in Table 8. The effect on aggregate unemployment is 0.5%, while that on unemployment in depressed metropolitan areas is around 1.2%. Since there are other possible mechanisms operating that decrease mobility, these results are an upper bound on the labor reallocation effect of the housing bust.

### 5.5 Discussion

To summarize the results from the two quantitative exercises, the effect on aggregate unemployment is between 0.2 and 0.5 percentage points, while the effect on unemployment in depressed metropolitan areas is between 0.4 and 1.2 percentage points. This corresponds to between 4 and 10% of the increase in aggregate unemployment from 2007 to 2010 and to between 7 and 20% of the increase in unemployment in depressed metropolitan areas.

It is interesting to examine what the importance of the amplification effect of a regional shock is. To see this, I consider the baseline (pre-recession) calibration, and include a housing bust as the counterfactual experiment, using the parametrizations from the two exercises above. I find that for the first exercise aggregate unemployment increases by only 0.03 percentage points due to the housing bust, while for the second exercise the increase is 0.14 percentage points. Furthermore, unemployment in depressed metropolitan areas increases only by 0.05 and 0.28 percentage points, respectively. These effects are substantially smaller than the counterfactual effects from the recent recession. Therefore, hindering regional reallocation during a period when regional disparities are large as in the aftermath of the recession is particularly important. Had the housing bust somehow occurred without an increase in regional dispersion, its reallocation distortion effect on the labor market would have been substantially smaller. Consequently, diminishing of the current regional disparities would have the added effect of decreasing the impact of the housing bust on the labor market.

I also consider the robustness of the effect to the use of a different migration rate for the baseline calibration. Due to unavailability of inter-MSA migration rate data, I used an average of inter-county and inter-state migration, which implied a 5% migration rate. Here, I consider what the effects would be given a migration rate of 4% and 6%. I repeat the baseline calibration to match each of these rates and then perform the first housing bust exercise for each case. For the first case of a migration rate of 4%, the effect of the housing bust decreases slightly to 0.13 percentage points for aggregate unemployment and 0.34 percentage points for unemployment in depressed metropolitan areas. For the second case of a migration of 6%, the effect increases to 0.19 percentage points and 0.48 percentage points, respectively. Therefore, the magnitude of the effects is fairly robust to a calibration with a different migration rate.<sup>39</sup>

 $<sup>^{39}\</sup>text{Checking}$  for robustness with respect to  $\rho$  also produces small variations in the effects.

$V_U^h(A)$ $A = \underline{A}  A = \overline{A}$		$V^h_E(A)$	$A = \underline{A}$	$A = \overline{A}$	
debt overhang, $h = 0$ 189.83191.1no debt overhang, $h = 1$ 195.5195.9		$191.18 \\ 195.93$	debt overhang, $h = 0$ no debt overhang, $h = 1$	$190.97 \\ 195.89$	$191.53 \\ 196.06$
(a)			(b)		

Table 9: Value functions for unemployed (a) and employed (b) workers

Beyond the consequences for unemployment, examined up to now, it is important to examine the welfare implications of these effects. First of all, I define total welfare in the economy. Let  $\mathcal{W}_t(\zeta, \lambda)$  be total welfare at the beginning of period t as a function of the default penalty  $\zeta$  and fraction of affected workers  $\lambda$ . Then

$$\mathcal{W}_t\left(\zeta,\lambda\right) = \sum_{A,h} \left[ U\left(A,h\right) V_U^h\left(A\right) + \left(L\left(A,h\right) - U\left(A,h\right)\right) V_E^h\left(A\right) \right]$$
(24)

where U(A, h) is the beginning-of-period measure of unemployed workers with housing state  $h \in \{0, 1\}$  (0 denotes workers with debt overhang and 1 denotes workers with no debt overhang) in regions with productivity A, L(A, h) is the beginning-of-period population of workers with housing state  $h \in \{0, 1\}$ , and  $V_U^h(A)$  and  $V_E^h(A)$  are the value functions of unemployed and employed workers, respectively. I also suppress the dependence of these objects on  $\zeta$  and  $\lambda$  for notational convenience. To understand the welfare effects of the regional reallocation distortion of the housing bust, similarly to the quantitative exercises above, I look at the proportional change in welfare,  $\frac{W_t(\zeta, 0) - W_t(\zeta, \lambda)}{W_t(\zeta, \lambda)}$ , from the counterfactual case of removing the housing depreciation shock relative to welfare under a housing depreciation shock and average that over the 24 months, for which I simulate the model economy.

First of all, Table 9 shows the value functions calibration used in the first quantitative exercise ( $\zeta = 17.66$  and  $\lambda = 0.13$ ) for unemployed and employed workers with and without debt overhang that reside in regions with productivity  $\overline{A}$  or  $\underline{A}$ . One can make two important observations from these values. First of all, experiencing debt overhang substantially decreases a worker's utility irrespective of their current employment state or the region they reside in. The reason for this is that in the model workers experience idiosyncratic regional preference shocks that may induce them to migrate independently from local labor market conditions. These have a large effect on a worker's utility as he has to either incur a large negative preference shock or the penalty for default. The second important observation, is that unemployed workers with debt overhang who reside in depressed regions have a larger discount in their lifetime expected utility relative to being employed, compared to unem-

ployed workers without debt overhang. The reason for this is that the former are exposed to a much longer unemployment spell compared to the latter as the latter can more easily movel to regions where finding a job is easier. This shows an important interaction of debt overhang and labor market outcomes at the individual level. Debt overhang leads to longer unemployment spells as unemployed workers effectively face a lower job finding probability.

Turning to the effects on welfare, I get that for the first quantitative exercise the improvement in total welfare is on the order of 0.3%, while for the second exercise it is around 1.5%. These numbers should, of course, be taken with caution given some of the assumptions in the model, most importantly, that agents in the model are risk neutral. Additionally, a thorough welfare analysis of the housing bust requires incorporating additional channels through which the housing bust affected the real economy in the recession.

Finally, I use the calibrated model to address the labor market effects of two policies proposed for dealing with the mortgage crisis. I focus on the first housing bust exercise since it has a clear channel of action of the housing bust on regional reallocation through the default penalty. The first policy I consider is the "Home Affordable Modification Program", which has been in place since the beginning of 2009. The main objective of the program is to reduce the monthly mortgage payments of borrowers who face imminent risk of default to levels, commensurate with borrower monthly income.<sup>40</sup> Therefore, the program effectively removes involuntary default by homeowners leaving only voluntary or strategic default. However, Bhutta, Dokko, and Shan (2010) find that accounting for involuntary default, the default threshold, associated with purely strategic default for the median household in negative equity is around 60% of home value. This corresponds to a doubling of the default cost that is implied when involuntary and voluntary default are not distinguished. It follows that under this policy the effective default penalty in the model would also double to  $\zeta = 35.32$  from the level in Section 5.4.1. The result for the labor market is a marginal increase in aggregate unemployment and unemployment in depressed metropolitan areas of 0.01 and 0.03 percentage points, respectively.

The second policy is a proposal for broader principal reduction through a modification of personal bankruptcy law. Pozner and Zingales (2009) discuss such a policy proposal, which is exactly targeted to home owners in negative equity and calls for principal reduction to the current value of a property. Thus the proposal forces a renegotiation between borrower and lender and removes any debt overhang problem the borrower may have and any adverse consequences stemming from it. In the context of my model, this is equivalent to a removal of the default penalty on workers in debt overhang. The labor market effect of such a policy is equivalent to the effect of the counterfactual experiment in Section 5.4.1. This

<sup>&</sup>lt;sup>40</sup>See https://www.hmpadmin.com/portal/programs/hamp.jsp for details.

leads to a decrease in aggregate unemployment of 0.17 percentage points and a decrease of unemployment in depressed metropolitan areas of 0.4 percentage points. Therefore, from the perspective of the labor market, the second policy has a more beneficial effect compared to the first.

## 6 Conclusion

This paper addresses how a housing bust may affect the labor market indirectly through its impact on regional reallocation. I document how the fraction of households with negative equity in a state correlates with state out-migration and in-migration as well as comovements between gross migration rates and state labor and housing market conditions. I then study an multi-region model with regional labor and housing markets, which accounts for the co-movements between unemployment, house prices and gross migration due to the assumptions of directed but limited regional mobility. That model allows me to study the regional reallocation effect of a housing bust and the consequent labor market implications. A housing bust increases aggregate unemployment by hindering regional reallocation, while a regional shock amplifies that effect. Finally, I quantitatively evaluate how much of the recent rise in unemployment can be attributed to the housing bust.

There are several venues for future research arising from this work. First of all, the combination of large regional dispersion in economic conditions, together with a housing market related mobility slow-down is not a peculiarity of the recent recession only. The recession of the early 80s was characterized by an increase in regional unemployment dispersion as well, and the high interest rates of that period discouraged many households from taking on new mortgages, thus affecting their mobility. That recession was also characterized by a very high level of unemployment in the U.S. Hence, the conditions observed in the recent recession were present in that period as well, although in a slightly different form.

Second, the model economy that I studied here assumes no occupational or skill heterogeneity or equivalently, no occupational mobility costs for workers. Introducing occupational heterogeneity and limited occupational mobility for workers would lead to another channel of mismatch apart from regional mismatch and would have interesting interaction with regional reallocation decisions. Such interaction may also affect the reallocation effects of a housing bust.

Lastly, the paper shows that limited mobility has implications for regional population and house price dynamics, which are not present in the benchmark framework of frictionless mobility. Studying these implications in greater detail may lead to important insights for models with regional heterogeneity.

# Appendix A - Data Appendix

### Data for panel regression

The data for the state panel regression results reported in Section 2 come from a number of sources:

- 1. Data on household state in- and out-migration rates I obtain from the IRS U.S. Population Migration Database, which is available up to 2008. The data is based on the year-to-year address changes of individual income tax returns. From that raw data, for each state the IRS computes the total number of tax returns, which approximates the number of households, that have migrated into and out of that state. From this data I compute state in- and out- migration rates for a given year as the ratio of the number of movers into or out of the state to sum of the number of movers (into and out of the state, respectively) and non-movers. Note that the period covered does not correspond exactly to the calendar year as it covers moves from April of a given year to April of the next year. However, I treat it as corresponding to the calendar year. The advantage of using the IRS data for tracking state migration patterns as opposed to, for example the Current Population Survey micro-data, is that it is continuously available from up to 2008, whereas CPS has a gap in 1995. Furthermore, the CPS is a survey with a limited sample of individuals so computing state inflow and outflow rates would introduce significant measurement error, which would be problematic for having precise estimates. On the other hand, the IRS data is not completely representative since it excludes the very poor and elderly. However, that should not create problems since the very poor are not homeowners and the elderly do not generally migrate for employment reasons. Also, the IRS data and looks at mobility of all households rather than mobility of individuals that are part of the labor force.
- 2. Data on fraction of homeowners in negative equity by state I obtain from the Interview Survey of the Consumer Expenditure Survey (CE). The Interview Survey of the CE is a quarterly survey of consumption patterns and expenditures of American consumers. The survey collects data on household characteristics, income, and major expenditures. The sample design is a rotary panel survey with data on each household available for 4 quarters. The survey includes questions on ownership of real estate, including subjective property valuation as well as principal balance outstanding on all mortgages and home equity credit lines that a specific property collateralizes. The household characteristics include information on homeownership as well as state of residence, although the state identifier is suppressed for several states. Data on house-

holds mortgage balance outstanding, which is necessary for constructing estimates of a household's housing equity, becomes available from 1988. However, household state identifiers are only available from 1993. Using this information for each unique homeowner in a given year I construct their housing equity E for their primary residence as  $E = \frac{V-D}{V}$ , where D is the total balance outstanding on all mortgages and home equity lines of credit that a property collateralizes and V is the value of the property. Note that I remove homeowners with E < -2, as those are homeowners that report either very low home values or do not report home values. Together with the state identifier for that homeowner, I can then construct an estimate for the fraction of homeowners in negative equity for each state for the given year. As discussed in Section 2, for robustness to household level misclassification, I also create estimates of negative equity by using a cutoff c, i.e. I count households with  $E \leq c < 0$ . Table 10 contains these results. I also use this data to construct estimates of average mortgage debt ratio  $\frac{D}{V}$ by state.

- 3. Data on homeownership rates by state I obtain from the US Census Bureau's Housing Vacancies and Homeownership data.
- 4. Data on relative house prices I obtain by using the Freddie Mac House Price Index (FMHPI), formerly known as the Conventional Mortgage House Price Index (CMHPI) as well as the 2000 U.S. Census data on single family median home values by states and at the national level. In particular, the single family median house price in state s at time t,  $p_{s,t} = p_{s,2000} \cdot \frac{FMHPI_{s,t}}{FMHPI_{s,2000}}$  and similarly for national house prices,  $p_t$ . Relative house price in state s and time t is them  $\frac{p_{s,t}}{p_t}$ .
- 5. Data on relative unemployment rate is constructed using data from the BLS LAUS database.
- Data on relative wage rates is constructed using the BLS CES database. I use the average hourly manufacturing wage as it is the only wage series that spans my sample period 1993-2007.
- 7. Data on relative income is constructed using annual data on state level income from the BEA and population data from the US Census.



Figure 3: Negative equity fractions from First American CoreLogic vs. CE Data

Notes: State level estimates from 2009 of fraction of mortgage holders in negative equity from Consumer Expenditure Survey versus First American CoreLogic. CE data is from own calculations. CoreLogic data is from CoreLogic (2009). Regression coefficient is 1.13 and intercept is 13.4. Correlation coefficient is  $\rho = 0.731$ .

#### Data for the Beveridge curve regressions

I use the Conference Board Help Wanted Index and data from the BLS including the national unemployment rate (derived from the CPS), state and MSA unemployment rates and employment and labor force levels (derived from the Local Area Unemployment Statistics database), and the national vacancy rate (derived from JOLTS). Additionally I take the state unemployment dispersion data for the period 1960-1975 from Abraham (1987).

The Conference Board Help Wanted Index is the source for data on job vacancies prior to the introduction of JOLTS in December 2000. It tracks the number of help wanted advertisings in the newspapers of 51 cities, which are then aggregated into 9 Census Divison and 1 National Index. There are several well known problems with using the HWI as a proxy for vacancies particularly for looking at shifts in the Beveridge curve. Most recently, there has been a downward secular trend in the HWI, which has been attributed to the more extensive use of online job advertising. To remedy this I construct a synthetic vacancy rate by regressing monthly vacancy data from JOLTS for the period December 2000 to December 2003 on data from the HWI and a constant, this is a period where the two series track each other well. I then use these coefficient estimates to construct a synthetic vacancy rate for the entire sample period, taking annual averages to get annual data for the period 1960 to 2000. For years 2001 to 2010 I use the JOLTS vacancy data. This approach, however, may be unsatisfactory since it does not take care of trends in the HWI in earlier years unrelated to the labor market Abraham (1987)), which get transferred directly to the synthetic vacancy index. To address this, I include a cubic trend in all regressions, which should account for these additional secular trends.

For the national unemployment rate data, I take annual averages of the monthly seasonally adjusted series. Similarly to the HWI there may be a secular trend in the unemployment rate because of compositional changes with the aging of the baby boomer generation (Shimer (1999)). Again a cubic trend in the Beveridge curve regression would account for such a trend as well.

Following Abraham (1987) and Lilien (1982), I construct employment-weighted state unemployment rate dispersion according to the formula:  $\hat{\sigma}_t^x = \sqrt{\sum_{i=1}^n \frac{e_{i,t}}{e_t} (x_{i,t} - x_t)^2}$ , where  $e_{i,t}$ is employment in state *i* at time *t*,  $e_t$  is national employment at time *t*,  $x_{i,t}$  is the realization in state *i* at time *t* and  $x_t$  is weighted average of state realizations weighted by employment weights. For the unemployment dispersion I use annual averages of monthly seasonally adjusted state unemployment rates as well as annual averages of seasonally adjusted monthly state employment levels. Finally, I construct MSA unemployment dispersion according to:  $\hat{\sigma}_t^x = \sqrt{\sum_{i=1}^n \frac{e_{i,t}}{e_t} (x_{i,t} - x_t)^2}$  where  $e_{i,t}$  is labor force in MSA *i* at time *t*,  $e_t$  is national labor force at time *t*,  $x_{i,t}$  is the realization in MSA *i* at time *t* and  $x_t$  is weighted average of MSA realizations weighted by labor force weights. Table 11: Beveridge curve regressions with vacancy rate as dependent variable (a) and unemployment rate as dependent variable (b)

Dep. Variable:		vacano		
		v		ln(v)
unemployment rate	-0.203***	-0.458***	-0.455	
(u)	(0.0327)	(0.101)	(0.311)	
State Unemployment		1.146**	1.146**	$0.246^{*}$
Dispersion		(0.458)	(0.459)	(0.129)
$u^2$			-0.000255	
u			(0.0217)	
ln(u)				-0.742***
				(0.172)
0				
$R^2$	0.801	0.851	0.851	0.881

(a)

ſ	b)
ſ	0)

Dep. Variable:				
		u		ln(u)
V	-2.294***	-1.044***	-4.081***	
	(0.415)	(0.224)	(1.286)	
State Unemployment		3.177***	2.803***	0.475***
Dispersion		(0.237)	(0.252)	(0.0421)
$v^2$			0.411**	
			(0.191)	
ln(v)				-0.662***
				(0.117)
$R^2$	0.801	0.851	0.851	0.881

Notes: Own calculations from BLS, Conference Board and Abraham (1987). Annual data from 1960-2010. Newey-West robust standard errors in parenthesis; \* = significant at 10%; \*\* = significant at 5%; \*\*\* = significant at 1%. All regressions include a cubic time trend. v is national vacancy rate and u is national unemployment rate. Dispersion measure for unemployment is defined as  $\hat{\sigma}_t^u = \sqrt{\sum_{i=1}^n \frac{e_{i,t}}{e_t}(u_{i,t} - u_t)^2}$  where  $e_{i,t}$  is employment in state *i* at time *t*,  $e_t$  is national employment at time *t*,  $u_{i,t}$  is unemployment rate state *i* at time *t* and  $u_t$  is the national unemployment rate.

.1, (b) $c = -0.2$
c = -0
a
cutoffs: (
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regressions
Panel
10:
Table

 $(\mathbf{q})$ 

(a)

in-migration rate $100 \ln(in)$	-0.0404 (0.168)	$-0.237^{***}$ $(0.0389)$	$-0.169^{***}$ (0.0588)	-0.00940 (0.164)	$0.645^{**}$ $(0.246)$	$0.0984^{**}$ (0.0488)	-0.337 $(0.321)$	606
out-migration rate $100 \ln(out)$	$-0.377^{**}$ $(0.175)$	$0.166^{***}$ $(0.0358)$	$0.204^{***}$ $(0.0683)$	0.0175 (0.0828)	-0.0970 $(0.303)$	0.0448 (0.0477)	0.427 (0.396)	606
Dep. Variable:	households with equity $< -0.2$ , (%)	relative unemployment $100 \ln(\frac{u_s}{u})$	relative house price $100 \ln(\frac{p_s}{p})$	relative wage rate $100 \ln(\frac{w_s}{w})$	relative income $100 \ln(\frac{y_s}{y})$	ave. debt-to-value ratio (%)	home ownership rate (%)	Ν
in-migration rate $100 \ln(in)$	-0.0413 (0.185)	$-0.237^{***}$ $(0.0388)$	$-0.169^{***}$ $(0.0590)$	-0.00955 $(0.165)$	$0.646^{**}$ $(0.246)$	0.0995* $(0.0526)$	-0.335 $(0.323)$	606
out-migration rate $100 \ln(out)$	$-0.301^{*}$ (0.160)	$0.167^{***}$ (0.0359)	$0.203^{***}$ $(0.0686)$	0.0143 (0.0840)	-0.0909 $(0.303)$	0.0404 (0.0494)	0.434 ( $0.392$ )	606
Dep. Variable:	households with equity $< -0.1$ , (%)	relative unemployment $100 \ln(\frac{u_s}{u})$	relative house price $100 \ln(\frac{p_s}{p})$	relative wage rate $100 \ln(\frac{w_s}{w})$	relative income $100 \ln(\frac{y_s}{y})$	ave. debt-to-value ratio (%)	home ownership rate (%)	Ν

Notes: Robust standard errors with clustering on state in parenthesis; \* = significant at 10%; \*\* = significant at 5%; \*\*\* = significant at 1%. Source: Own calculations from BLS, IRS, CE, FMHPI, and US Census Bureau. See Data Appendix for detailed description.

## Appendix B - Proofs and auxiliary results

This part of the appendix contains proofs of all results not contained in the body of the paper.

#### Worker value functions, homeownership and default:

Let me first define the worker's value function generally. Let  $V^{1}(X)$  be the end-of-period value function of a worker who owns a non-depreciated unit of housing and  $W^{1}(X)$  be his beginning-of-period value function. Similarly, let  $V^{0}(X)$  be the end-of-period value function of a worker who owns a depreciated unit of housing and  $W^{0}(X)$  be his beginning-of-period value function. Also, let  $\hat{V}(X)$  be the end-of-period value function of a worker who does not own housing at the end of a period and  $\hat{W}(X)$  be his beginning-of period value function.

Then, we have that:

$$V^{1}(X) = \gamma + e - \tilde{d}(X) + \mu(\theta(A))(w(A) - e) + \beta E_{X} \left[ W^{1}(X') \right]$$

with

$$W^{1}(X) = \max_{\overline{\epsilon}} \left\{ F\left(\overline{\epsilon}\right) \overline{V} + \left(1 - F\left(\overline{\epsilon}\right)\right) \max\left\{V^{1}\left(X\right), \ \widehat{V}\left(X\right)\right\} - F\left(\overline{\epsilon}\right)c + \int_{\overline{\epsilon}} \epsilon dF \right\}$$

and

$$\bar{V} = \max_{\tilde{x}} \left\{ \max \left\{ V^1(\tilde{x}), \ \hat{V}(\tilde{x}) \right\} \right\}$$

Also,

$$V^{0}(X) = \gamma + e - \tilde{d}(X) + \mu(\theta(A))(w(A) - e) + \beta E_{X}\left[W^{0}\left(X'\right)\right]$$

with

$$W^{0}(X) = \max_{\overline{\epsilon}} \left\{ F(\overline{\epsilon}) \,\overline{V} - F(\overline{\epsilon}) \,(c+\zeta) + \int_{\overline{\epsilon}} \epsilon dF + (1 - F(\overline{\epsilon})) \max\left\{ V^{0}(X) \,, \, \max\left\{ V^{1}(X) \,, \, \widehat{V}(X) \right\} - \zeta \right\} \right\}$$

Finally,

$$\hat{V}(X) = e + \mu\left(\theta\left(A\right)\right)\left(w(A) - e\right) + \beta E_X\left[\hat{W}\left(X'\right)\right]$$
(25)

and

$$\hat{W}(X) = \max_{\bar{\epsilon}} \left\{ F(\bar{\epsilon}) \, \bar{V} + (1 - F(\bar{\epsilon})) \max \left\{ V^1(X), \, \hat{V}(X) \right\} - F(\bar{\epsilon}) \, c + \int_{\bar{\epsilon}} \epsilon dF \right\}$$

or  $\hat{W}(X) = W^{1}(X)$ , i.e. house purchases are completely reversible. Inspection of the value functions immediately implies that

**Lemma 7.** A worker with no debt overhang prefers to buy housing iff  $\tilde{d}(X) \leq \gamma$ .

Secondly, I show that workers with debt overhang do not demand non-depreciated housing and default only when migrating. This follows from the following result:

Lemma 8.  $V^0(X) \ge V^1(X) - \beta \cdot \zeta, \ \forall X.$ 

*Proof.* First I show that  $W^1(X) - W^0(X) \leq \zeta$ ,  $\forall X$ . We have that:

$$W^{1}(X) = F\left(\overline{\epsilon}\left(X,1\right)\right)\left(\overline{V}-c\right) + \left(1-F\left(\overline{\epsilon}\left(X,1\right)\right)\right)\max\left\{V^{1}\left(X\right),\ \widehat{V}\left(X\right)\right\} + \int_{\overline{\epsilon}\left(X,1\right)}\epsilon dF$$

and

$$W^{0}(X) = F\left(\overline{\epsilon}\left(X,0\right)\right)\left(\overline{V} - c - \zeta\right) + \left(1 - F\left(\overline{\epsilon}\left(X,0\right)\right)\right) \max\left\{V^{0}\left(X\right), \max\left\{V^{1}\left(X\right), \ \widehat{V}\left(X\right)\right\} - \zeta\right\} + \int_{\overline{\epsilon}\left(X,0\right)} \epsilon dF$$

Therefore,

$$\begin{split} W^{0}(X) &= F\left(\overline{\epsilon}\left(X,0\right)\right)\left(\overline{V}-c-\zeta\right) + \int_{\overline{\epsilon}(X,0)} \epsilon dF + \\ &+ \left(1-F\left(\overline{\epsilon}\left(X,0\right)\right)\right) \max\left\{V^{1}\left(X\right), \ \max\left\{V^{1}\left(X\right), \ \widehat{V}\left(X\right)\right\} - \zeta\right\} \\ &\geq F\left(\overline{\epsilon}\left(X,1\right)\right)\left(\overline{V}-c-\zeta\right) + \int_{\overline{\epsilon}(X,1)} \epsilon dF + \\ &+ \left(1-F\left(\overline{\epsilon}\left(X,1\right)\right)\right) \max\left\{V^{1}\left(X\right), \ \max\left\{V^{1}\left(X\right), \ \widehat{V}\left(X\right)\right\} - \zeta\right\} + \\ &\geq F\left(\overline{\epsilon}\left(X,1\right)\right)\left(\overline{V}-c-\zeta\right) + \int_{\overline{\epsilon}(X,1)} \epsilon dF + \\ &+ \left(1-F\left(\overline{\epsilon}\left(X,1\right)\right)\right)\left(\max\left\{V^{1}\left(X\right), \ \widehat{V}\left(X\right)\right\} - \zeta\right) = \tilde{W} \end{split}$$

The first inequality comes from not using the optimal cutoff  $\overline{\epsilon}(X,0)$  but rather  $\overline{\epsilon}(X,1)$  and the second inequality comes from disregarding the max operator. Hence,  $W^1(X) - W^0(X) \leq W^1(X) - \tilde{W}$ . However, note that  $W^1(X) - \tilde{W} = \zeta$  and so  $W^1(X) - W^0(X) \leq \zeta$ . Observing that  $V^1(X) - V^0(X) = \beta E_X \left[ W^1(X') - W^0(X') \right]$ , we have our result.  $\Box$ 

Lemma 8 implies that if  $V^{1}(X) \geq \widehat{V}(X)$ , then  $\max \left\{ V^{0}(X), \max \left\{ V^{1}(X), \widehat{V}(X) \right\} - \zeta \right\} = V^{0}(X)$ , i.e. a worker with debt overhang default only when migrating and therefore does not

demand non-depreciated housing from the region he currently resides in. If  $V^1(X) < \hat{V}(X)$  it may be that a worker may default without migrating but he still does not demand non-depreciated housing. Therefore, we have that if  $\tilde{d}(X) < \gamma$ , a worker with deb overhang does not demand non-depreciated housing and default only when migrating.

### Law of motion for $\overline{\nu}_t$ and $\underline{\nu}_t$ :

The laws of motion for the distributions  $\overline{\nu}_t$  and  $\underline{\nu}_t$ ,  $\overline{\nu}_{t+1} = \overline{\Xi}(\overline{\nu}_t, \underline{\nu}_t)$ ,  $\underline{\nu}_{t+1} = \underline{\Xi}(\overline{\nu}_t, \underline{\nu}_t)$  are given by,

$$\overline{\nu}_{t+1}(l_0, l_1) = \overline{\Xi} \left( \overline{\nu}_t \left( (l_0, l_1) \right), \underline{\nu}_t \left( (l_0, l_1) \right) \right) = \\ = \int \rho \cdot I \left\{ l_0 = l'_0 \left( \overline{A}, l_{0,-1}, l_{1,-1}, \overline{\nu}_t, \underline{\nu}_t \right), l_1 = l'_1 \left( \overline{A}, l_{0,-1}, l_{1,-1}, \overline{\nu}_t, \underline{\nu}_t \right) \right\} d\overline{\nu}_t \left( l_{0,-1}, l_{1,-1} \right) + \\ + \int (1-\rho) \cdot I \left\{ l_0 = l'_0 \left( \underline{A}, l_{0,-1}, l_{1,-1}, \overline{\nu}_t, \underline{\nu}_t \right), l_1 = l'_1 \left( \underline{A}, l_{0,-1}, l_{1,-1}, \overline{\nu}_t, \underline{\nu}_t \right) \right\} d\underline{\nu}_t \left( l_{0,-1}, l_{1,-1} \right)$$
(26)

and similarly,

$$\underline{\nu}_{t+1} (l_0, l_1) = \underline{\Xi} \left( \overline{\nu}_t \left( (l_0, l_1) \right), \underline{\nu}_t \left( (l_0, l_1) \right) \right) = \\ = \int \rho \cdot I \left\{ l_0 = l'_0 \left( \underline{A}, l_{0,-1}, l_{1,-1}, \overline{\nu}_t, \underline{\nu}_t \right), l_1 = l'_1 \left( \underline{A}, l_{0,-1}, l_{1,-1}, \overline{\nu}_t, \underline{\nu}_t \right) \right\} d\underline{\nu}_t \left( l_{0,-1}, l_{1,-1} \right) + \\ + \int (1-\rho) \cdot \left\{ l_0 = l'_0 \left( \overline{A}, l_{0,-1}, l_{1,-1}, \overline{\nu}_t, \underline{\nu}_t \right), l_1 = l'_1 \left( \overline{A}, l_{0,-1}, l_{1,-1}, \overline{\nu}_t, \underline{\nu}_t \right) \right\} d\overline{\nu}_t \left( l_{0,-1}, l_{1,-1} \right)$$
(27)

#### Definition of stationary recursive equilibrium:

**Definition 9.** A symmetric stationary recursive equilibrium for the economy described above consists of market tightness  $\theta(A)$ , wages w(A), worker value functions  $V^{\chi}(X)$ , migration value  $\overline{V}$ , migration thresholds  $\overline{\epsilon}(X,\chi)$ , regional house prices p(X), laws of motion,  $\Gamma$ , for X, and distributions  $(\overline{\nu}^*, \underline{\nu}^*)$  such that:

- 1.  $\theta(A)$  satisfies (5) given w(A);
- 2. w(A) satisfies (6);
- 3.  $V^{\chi}(X)$  satisfies (14) given  $\theta(A)$ ,  $\overline{V}$ , and  $\Gamma$ ;
- 4.  $\overline{V}$  satisfies (13) given  $V^{\chi}(X)$ ;

- 5.  $\overline{\epsilon}(X,\chi)$  satisfies (16) given  $\overline{V}$  and  $V^{\chi}(X)$ ;
- 6. p(X) satisfies (11) given  $\Gamma$ .
- 7.  $\Gamma$  and satisfies 18 and law of motions for A given  $\overline{\epsilon}(X, \chi)$ ;
- 8.  $(\overline{\nu}^*, \underline{\nu}^*)$  are a fixed point of (26) and (27).
- 9.  $\int l_1'(\overline{A}, l_1) d\overline{\nu}^* + \int l_1'(\underline{A}, l_1) d\underline{\nu}^* + 2l_0 = L$  (population constancy).

### Stationary equilibrium characterization:

**Lemma 10.** Suppose that  $l'_{1}(A, l)$  is a continuous function of l for  $A \in \{\underline{A}, \overline{A}\}$ . Then  $V^{1}(A, l)$  is a bounded continuous function of l for  $A \in \{\underline{A}, \overline{A}\}$ . Conversely, suppose that  $V^{1}(A, l)$  is a continuous function of l for  $A \in \{\underline{A}, \overline{A}\}$ . Then  $l'_{1}(A, l)$  is a continuous function of l for  $A \in \{\underline{A}, \overline{A}\}$ . Then  $l'_{1}(A, l)$  is a continuous function of l for  $A \in \{\underline{A}, \overline{A}\}$ .

*Proof.* Let us define the operator

$$T\left[v(A,l)\right] = \max_{\left\{\bar{\epsilon}(A',l')\right\}_{A'\in\left\{\underline{A},\overline{A}\right\}}} \left\{\gamma - d\left(A,l\right) + e + \mu\left(\left(\theta(A)\right)\left(w\left(A\right) - e\right) + \beta E_{A}\left[F\left(\bar{\epsilon}\left(A',l'\right)\right)\bar{V} + \left(1 - F\left(\bar{\epsilon}\left(A',l'\right)\right)\right)v\left(A',l'\right) - F\left(\bar{\epsilon}\left(A',l'\right)\right)c + \int_{\bar{\epsilon}(A',l')} d\epsilon\right]\right\}$$

with  $l' = l'_1(A, l)$  and a given  $\overline{V}$ . Note that d(A, l) = g'(L - l') is a continuous function of l'by assumptions on g'() and  $l'_1(A, l)$  is also a continuous function of l. Both are also clearly bounded. Hence T maps bounded continuous functions into bounded continuous functions. Furthermore, T satisfies Blackwell's sufficient conditions for a contraction. Hence, by the Contraction Mapping Theorem (Stokey and Lucas (1989)), T has a unique fixed point in the space of bounded continuous functions. Hence,  $V^1(A, l)$ , is a bounded continuous function.

Suppose now that  $V^1(A, l)$  is continuous in l. Given this it follows immediately that d(A, l) must be continuous in l and since d = g'(L - l') is continuous in l', it follows that  $l'_1$  must be continuous in l.

As mentioned in Section 4.4, I focus on equilibria, in which both  $V^1(A, l)$  and  $l'_1(A, l)$ are continuous. Given continuity of  $V^1(A, l_1)$  and the compactness of the domain of X it follows immediately that the set  $\arg \max_x \{V(x)\}$  is nonempty. I then have the following result.

**Lemma 11.**  $V^1(A, l)$  is a non-increasing function of l, and  $l'_1(A, l)$  is a non-decreasing function of l.

*Proof.* The basic idea behind showing this result is showing that, if  $V^1(A, l)$  is a nonincreasing function of l, then  $l'_1(A, l)$  is a non-decreasing function of l and vice versa, and then showing that it is impossible for  $l'_1$  to be strictly decreasing in l if  $V^1(A, l)$  is strictly increasing in l, i.e. the only property that is mutually consistent and hence possible in equilibrium is for  $V^1(A, l)$  to be a non-increasing function of l and  $l'_1$  to be a non-decreasing function of l. I proceed to show this in three steps.

**Step 1:** I show that if  $l'_1(A, l)$  is a non-decreasing function of l then  $V^1(A, l), A \in \{\underline{A}, \overline{A}\}$  is a non-increasing function of l. Define again the operator

$$T[v(A, l)] = \gamma - d(A, l) + e + \mu((\theta(A))(w(A) - e) + \beta E_A \left[\omega(A', l')\right]$$

where

$$\omega(A,l) = \max_{\overline{\epsilon}} \left\{ F(\overline{\epsilon}) \, \overline{V} + (1 - F(\overline{\epsilon})) \, v(A,l) - F(\overline{\epsilon}) \, c + \int_{\overline{\epsilon}} d\epsilon \right\}$$

We have that l' is non-decreasing in l and hence -d(A, l) is non-decreasing in l. Furthermore, for  $l_1 < l_2$ :

$$\omega \left(A, l_{1}\right) - \omega \left(A, l_{2}\right) =$$

$$= \left(1 - F\left(\overline{\epsilon}\left(A, l_{1}\right)\right)\right) v\left(A, l_{1}\right) + \int_{\overline{\epsilon}\left(A, l_{1}\right)} d\epsilon + F\left(\overline{\epsilon}\left(A, l_{1}\right)\right)\left(\overline{V} - c\right) -$$

$$- \left(1 - F\left(\overline{\epsilon}\left(A, l_{2}\right)\right)\right) v\left(A, l_{2}\right) + \int_{\overline{\epsilon}\left(A, l_{2}\right)} d\epsilon + F\left(\overline{\epsilon}\left(A, l_{2}\right)\right)\left(\overline{V} - c\right) \geq$$

$$\geq \left(1 - F\left(\overline{\epsilon}\left(A, l_{2}\right)\right)\right) v\left(A, l_{1}\right) + \int_{\overline{\epsilon}\left(A, l_{2}\right)} d\epsilon + F\left(\overline{\epsilon}\left(A, l_{2}\right)\right)\left(\overline{V} - c\right) -$$

$$- \left(1 - F\left(\overline{\epsilon}\left(A, l_{2}\right)\right)\right) v\left(A, l_{2}\right) + \int_{\overline{\epsilon}\left(A, l_{2}\right)} d\epsilon + F\left(\overline{\epsilon}\left(A, l_{2}\right)\right)\left(\overline{V} - c\right) +$$

$$\geq \left(1 - F\left(\overline{\epsilon}\left(A, l_{2}\right)\right)\right) \left(v\left(A, l_{1}\right) - v\left(A, l_{2}\right)\right)$$

where the inequality comes from the fact that a different cutoff from the optimal,  $\bar{\epsilon}(A, l_1)$ , is used. Hence, if v(A, l) is non-increasing in l then so is  $\omega(A, l)$ . Therefore,  $\omega(A', l')$  is non-increasing in l. Then it follows that T[V(A, l)] is non-increasing in l. Hence, T maps non-increasing functions to non-increasing functions. Therefore, its fixed point,  $V^1(A, l)$ , is non-increasing in l.

Step 2: I show that if  $V^1(A, l)$ ,  $A \in \{\underline{A}, \overline{A}\}$  is a non-increasing function of l then  $l'_1(A, l)$  is a non-decreasing function of l. Suppose that V(A, l) is non-increasing in l but  $\exists l_1, l_2$  with  $l_1 < l_2$  s.t.  $l'_1 = l'_1(A, l_1) > l'_1(A, l_2) = l'_2$ . Then

$$0 \le V^{1}(A, l_{1}) - V^{1}(A, l_{2}) = -d(A, l_{1}) + d(A, l_{2}) + \beta E_{A} \left[ W^{1} \left( A', l_{1}' \right) - W^{1} \left( A', l_{2}' \right) \right] < 0$$

since  $-d(A, l_1) + d(A, l_2) < 0$  given that  $l'_1 > l'_2$  and  $E_A \left[ W^1 \left( A', l'_1 \right) - W^1 \left( A', l'_2 \right) \right] < 0$  since  $V^1(A, l)$  is non-increasing in l and  $l'_1 > l'_2$ . Hence, we have a contradiction, which is due to the assumption that  $\exists l_1, l_2$  with  $l_1 < l_2$  s.t.  $l'_1 = l'_1(A, l_1) > l'_1(A, l_2) = l'_2$ .

Step 3: I show that if  $V^1(A, l)$  is strictly increasing in l then  $l'_1$  is increasing in l. Let  $l_1 < l_2$ . Then  $V(A, l_1) < V(A, l_2)$ ,  $A \in \{\underline{A}, \overline{A}\}$  and so  $\overline{\epsilon}(A, l_1, 1) > \overline{\epsilon}(A, l_2, 1)$ . Hence,  $l'_1(A, l_1) = (1 - F(\overline{\epsilon}(A, l_1, 1))) l_1 + \Psi(A, l_1)$  and  $l'_1(A, l_2) = (1 - F(\overline{\epsilon}(A, l_2, 1))) l_2 + \Psi(A, l_2)$ . Now, observe that  $\Psi(A, l_1) \leq \Psi(A, l_2)$  and  $(1 - F(\overline{\epsilon}(A, l_1, 1))) l_1 < (1 - F(\overline{\epsilon}(A, l_2, 1))) l_2$ , which implies that  $l'_1(A, l_1) < l'_1(A, l_2)$ .

Steps 1 and 2 show that it is mutually consistent for  $V^1$  and  $l'_1$  to be non-increasing and non-decreasing in l, respectively. However, Step 3 shows that it is not mutually consistent for  $V^1$  and  $l'_1$  to be strictly increasing and strictly decreasing in l, respectively. Hence, in any equilibrium  $V^1$  and  $l'_1$  are non-increasing and non-decreasing, respectively.

Next, I define

$$\bar{l} = \begin{cases} \sup_{l} \left\{ l : (\overline{A}, l) \in \arg\max_{\tilde{l}} \left\{ V^{1} \left( A, \tilde{l} \right) \right\} \right\} &, \left\{ l : (\overline{A}, l) \in \arg\max_{\tilde{l}} \left\{ V^{1} \left( A, \tilde{l} \right) \right\} \right\} \neq \emptyset \\ 0 &, o.w. \end{cases}$$

$$(28)$$

for  $\overline{l}$  and similarly

$$\underline{l} = \begin{cases} \sup_{l} \left\{ l : (\underline{A}, l) \in \arg\max_{\tilde{l}} \left\{ V^{1}\left(A, \tilde{l}\right) \right\} \right\} &, \left\{ l : (\underline{A}, l) \in \arg\max_{\tilde{l}} \left\{ V^{1}\left(A, \tilde{l}\right) \right\} \right\} \neq \emptyset \\ 0 &, o.w. \end{cases}$$

$$(29)$$

for  $\underline{l}^{41}$  Therefore, regions with populations of mobile workers above  $\overline{l}$  and  $\underline{l}$  cannot attract workers and therefore only lose population, i.e.  $l'_1(A, l) < l$ , for  $l > \overline{l}$ . It follows that the functions  $l'_1(\overline{A}, l)$  and  $l'_1(\underline{A}, l)$  have a fixed point, which is unique whenever there are potential house price differences across regions, that is when g(.) is strictly concave.

**Lemma 12.** There exist  $\overline{l}^* \leq \overline{l}$  and  $\underline{l}^* \leq \underline{l}$  such that  $l'_1(\overline{A}, \overline{l}^*) = \overline{l}^*$  and  $l'_1(\underline{A}, \underline{l}^*) = \underline{l}^*$ . They are unique if g(.) is strictly concave and, furthermore,  $l'_1(\overline{A}, l) = \overline{l}^*$  for  $l \in [0, \overline{l}]$  and  $l'_1(\underline{A}, l) = \underline{l}^*$  for  $l \in [0, \underline{l}]$ .

*Proof.* I show this for  $A = \overline{A}$  since the other case is analogous. Note that if

$$\left\{l:\left(\overline{A},l\right)\in\arg\max_{\tilde{l}}\left\{V^{1}\left(A,\tilde{l}\right)\right\}\right\}\neq\emptyset$$

<sup>&</sup>lt;sup>41</sup>These bounds always exist, given that  $l_1 \leq L$ .

then  $\overline{l} \in \left\{ l : (\overline{A}, l) \in \arg \max_{\widetilde{l}} \left\{ V^1 \left( A, \widetilde{l} \right) \right\} \right\}$  since  $V^1 \left( \overline{A}, l \right)$  is continuous in l. Hence,  $\left\{ l : (\overline{A}, l) \in \arg \max_{\widetilde{l}} \left\{ V^1 \left( A, \widetilde{l} \right) \right\} \right\}$ 

is a compact set. Now l' is continuous. Hence, by Brower's fixed point theorem there exists a  $\overline{l}^*$  such that  $l'_1(\overline{A}, \overline{l}^*) = \overline{l}^*$  and  $\overline{l}^* \leq \overline{l}$ . If  $\left\{ l : (\overline{A}, l) \in \arg \max_{\overline{l}} \left\{ V^1(A, \overline{l}) \right\} \right\} = \emptyset$  then  $\overline{l} = 0$  and clearly  $l'_1(\overline{A}, 0) = 0$ . Hence, can define  $\overline{l}^* = 0$ . Noting that for  $l > \overline{l}$ ,  $l'_1(A, l) < l$ , it follows that no fixed point of  $l'_1(\overline{A}, l)$  can be greater than  $\overline{l}$ .

It is straightforward to show that for  $a < \gamma \ l'_1(\overline{A}, l)$  is constant for  $l \in [0, \overline{l}]$  by using Lemma 11 and proceeding by contradiction. In particular, suppose that  $l'_1(\overline{A}, l_1) > l'_1(\overline{A}, l_2)$ for some  $l_1, l_2$ , such that  $\overline{l} \ge l_1 > l_2$ . However, this implies that  $V^1(\overline{A}, l_1) < V^1(\overline{A}, l_2)$  and so  $l_1 > \overline{l}$ . Now, to show uniqueness, I first show that if  $(A, l) \in \arg \max \{V^1(A, l)\}$  then  $(A, l'_1(A, l)) \in \arg \max \{V^1(A, l)\}$  as well. I can show this by contradiction. Suppose  $A = \underline{A}$ and  $(\underline{A}, l) \in \arg \max \{V^1(A, l)\}$  but  $(\underline{A}, l'_1(\underline{A}, l)) \notin \arg \max \{V^1(A, l)\}$ . Hence by Lemma 11 it follows that  $l'_1(\underline{A}, l) > l$ . However, this also implies that  $l'_1(\underline{A}, l'_1(\underline{A}, l)) \ge l'_1(\underline{A}, l)$ . But if  $(\underline{A}, l'_1(\underline{A}, l)) \notin \arg \max \{V^1(A, l)\}$  then by Equation 18 and since  $\Psi(\underline{A}, l'_1(\underline{A}, l)) = 0$  it must be that  $l'_1(\underline{A}, l'_1(\underline{A}, l)) < l'_1(\underline{A}, l)$ , which is a contradiction. Uniqueness follows immediately from this result and the constancy of  $l'_1(\underline{A}, l)$ .

Lemma 12 shows that regions will either be experiencing inflows that equalize the population of mobile workers or slow declines in population if they are too big relative to regional productivity (i.e.  $l > \overline{l}$  for booming regions or  $l > \underline{l}$  for depressed regions). Lemma 12 also describes what  $\Psi(A, l)$  looks like in equilibrium. In particular, it follows that

$$\Psi(\overline{A}, l) = \begin{cases} \overline{l}^* - (1 - F(-c))l & , \ l < \overline{l} \\ 0 & , \ o.w. \end{cases}$$
(30)

with  $\bar{l}^* = (1 - F(-c)) \bar{l}$  and similarly for  $\Psi(\underline{A}, l)$ .<sup>42</sup> Lastly, I compare mobile workers' value function for regions with the same populations but different productivities, A:

Lemma 13.  $V^1(\overline{A}, l) \geq V^1(\underline{A}, l)$ ,  $\forall l$ 

*Proof.* Suppose that  $V^1(\underline{A}, l) > V^1(\overline{A}, l)$  some l. Hence,  $(\overline{A}, l) \notin \arg \max \{V^1(A, l)\}$  and hence by equation Equation 18 and since  $\Psi(\underline{A}, l'_1(\underline{A}, l)) = 0, l'_1(\overline{A}, l) < l$  and  $l'_1(\overline{A}, l) < l$ 

<sup>&</sup>lt;sup>42</sup>Note that this need not be the case for a linear g(.), i.e. when  $a = \gamma$ , since in that case there are no house price differences across regions, so workers are indifferent between migrating to regions with any l. However, for continuity with respect to a, I will also look at the equilibrium, where  $\Psi$  takes this form for the case of a linear g(.).

 $l'_1(\underline{A}, l)$ . Then,

$$V^{1}(\overline{A}, l) - V^{1}(\underline{A}, l) = \gamma - d(\overline{A}, l) - \gamma + d(\underline{A}, l) + \mu(\theta(\overline{A}))(w(\overline{A}) - e) - \mu(\theta(\underline{A}))(w(\underline{A}) - e) + \beta E[W^{1}(A', l') | (\overline{A}, l)] - \beta E[W^{1}(A', l') | (\underline{A}, l)]$$

However, given that  $l'_1(\overline{A}, l) < l'_1(\underline{A}, l)$ , it follows that  $W^1\left(A, l'_1(\overline{A}, l)\right) > W^1\left(A, l'_1(\underline{A}, l)\right)$  for  $A \in \{\underline{A}, \overline{A}\}$  from Lemma 11. Now, if  $W^1\left(\overline{A}, l'_1(\overline{A}, l)\right) \ge W^1\left(\underline{A}, l'_1(\overline{A}, l)\right)$  or  $W^1\left(\overline{A}, l'_1(\underline{A}, l)\right) \ge W^1\left(\underline{A}, l'_1(\underline{A}, l)\right)$ , then  $\beta E\left[W^1\left(A', l'\right) \mid (\overline{A}, l)\right] - \beta E\left[W^1\left(A', l'\right) \mid (\underline{A}, l)\right] > 0$  and we have a contradiction. Hence,  $W^1\left(\overline{A}, l'(\overline{A}, l)\right) < W^1\left(\underline{A}, l'(\overline{A}, l)\right)$  and  $W^1\left(\overline{A}, l'_1(\underline{A}, l)\right) < W^1\left(\underline{A}, l'_1(\underline{A}, l)\right)$ . If  $W^1\left(\overline{A}, l'_1(\overline{A}, l)\right) \ge W^1\left(\underline{A}, l'_1(\underline{A}, l)\right)$ , then, again we will arrive at a contradiction. Hence,  $W^1\left(\overline{A}, l'_1(\underline{A}, l)\right)$ , which implies that  $V^1\left(\overline{A}, l'_1(\overline{A}, l)\right) < V^1\left(\underline{A}, l'_1(\underline{A}, l)\right)$ .

Now, define,  $\bar{l}^0 = l$ ,  $\bar{l}^1 = l'_1(\bar{A}, \bar{l}^0)$ ,  $\bar{l}^2 = l'(\bar{A}, \bar{l}^1)$ , etc. and similarly  $\underline{l}^0 = l$ ,  $\underline{l}^1 = l'(\bar{A}, \underline{l}^0)$ ,  $\underline{l}^2 = l'(\bar{A}, \underline{l}^1)$ , etc. Hence, by induction we have that  $V\left(\bar{A}, \bar{l}^i\right) < V\left(\underline{A}, \underline{l}^i\right)$  for i = 1, 2, ...Now, clearly each sequence is either decreasing and bounded below by  $\bar{l}$  and  $\underline{l}$ , respectively, or increasing and bounded above by  $\bar{l}$  and  $\underline{l}$ , respectively. Hence, by continuity of  $V^1(A, l)$  and from Lemma 12 it follows that the sequences would converge to unique fixed points of  $l'_1(\bar{A}, l)$ and  $l'_1(\bar{A}, l)$ ,  $\bar{l}^*$  and  $\underline{l}^*$ , respectively and by continuity of  $V^1$  then  $V^1\left(\bar{A}, \bar{l}^*\right) < V^1(\underline{A}, \underline{l}^*)$ , which is only possible if the set  $\left\{l: (\bar{A}, l) \in \arg\max_{\bar{l}}\left\{V^1\left(A, \tilde{l}\right)\right\}\right\}$  is empty, in which case  $\bar{l} = 0$  and so  $V^1(\bar{A}, 0) < V^1(\underline{A}, \underline{l})$ . However, note that

$$V^{1}(\overline{A}, 0) = \gamma - 0 + e + (\mu(\theta(\overline{A})))(w(\overline{A}) - e) + \beta E[W^{1}(A', 0)|\overline{A}] \geq 2$$

$$\geq \gamma + e + (\mu(\theta(\overline{A})))(w(\overline{A}) - e) + \beta \cdot (V^{1}(\overline{A}, 0) - F(c) \cdot c + \int_{c} \epsilon dF) = 2$$

$$\gamma + e + (\mu(\theta(\overline{A})))(w(\overline{A}) - e) + \beta (\gamma + e + (\mu(\theta(\overline{A})))(w(\overline{A}) - e) + 2$$

$$+\beta E[W^{1}(A', 0)|\overline{A}] - F(c) \cdot c + \int_{c} \epsilon dF) \geq ... \geq 2$$

$$\geq \sum_{j=0}^{\infty} \beta^{j} (\gamma + e + (\mu(\theta(\overline{A})))(w(\overline{A}) - e)) + \sum_{j=1}^{\infty} \beta^{j} (-F(c) \cdot c + \int_{c} \epsilon dF)$$

Similarly, one can show that

$$V^{1}\left(\underline{A},\underline{l}\right) = \gamma - d\left(\underline{A},\underline{l}\right) + e + \left(\mu\left(\theta\left(\underline{A}\right)\right)\right)\left(w\left(\underline{A}\right) - e\right) + \beta E\left[W^{1}\left(A',\underline{l}\right)|\underline{A}\right] \leq \frac{1}{2}$$

$$\leq \gamma - d\left(\underline{A}, \underline{l}\right) + e + \left(\mu\left(\theta\left(\underline{A}\right)\right)\right) \left(w\left(\underline{A}\right) - e\right) + \beta \cdot \left(V^{1}\left(\underline{A}, \underline{l}\right) - F\left(c\right) \cdot c + \int_{c} \epsilon dF\right) = \\ \gamma - d\left(\underline{A}, \underline{l}\right) + e + \left(\mu\left(\theta\left(\underline{A}\right)\right)\right) \left(w\left(\underline{A}\right) - e\right) + \beta \left(\gamma - d\left(\underline{A}, \underline{l}\right) + e + \left(\mu\left(\theta\left(\underline{A}\right)\right)\right)\right) \left(w\left(\underline{A}\right) - e\right) + \\ + \beta E \left[W^{1}\left(A', \underline{l}\right) |\underline{A}\right] - F\left(c\right) \cdot c + \int_{c} \epsilon dF\right) \leq \dots \leq \\ \leq \sum_{j=0}^{\infty} \beta^{j} \left(\gamma - d\left(\underline{A}, \underline{l}\right) + e + \left(\mu\left(\theta\left(\underline{A}\right)\right)\right)\left(w\left(\underline{A}\right) - e\right)\right) + \sum_{j=1}^{\infty} \beta^{j} \left(-F\left(c\right) \cdot c + \int_{c} \epsilon dF\right)$$

Hence,  $V^{1}(\underline{A}, \underline{l}) > V^{1}(\overline{A}, 0)$  implies that  $\sum_{j=0}^{\infty} \beta^{j} (\gamma - d(\underline{A}, \underline{l}) + e + (\mu(\theta(\underline{A})))(w(\underline{A}) - e)) > \sum_{j=0}^{\infty} \beta^{j} (\gamma + e + (\mu(\theta(\overline{A})))(w(\overline{A}) - e))$ , which is a contradiction.

### Proof of Lemma 1

Results 1 through 4 follow directly from the implication of Lemmas 11, 12, and 13 for the law of motion for  $l'_1$ . To show result 5, Lemma 12 implies that regions with productivity  $\overline{A}$  move to a population of mobile workers of  $\overline{l}^*$ . Furthermore, Lemma 13 implies that regions with  $\overline{l}^*$  that experience a sequence of negative productivity shocks always move deterministically through a set  $\mathcal{L}$  of population levels, which is countably infinite in the case equilibria with  $\underline{l}^* = 0$  or finite in the case of equilibria with  $\underline{l}^* > 0$  since out-migration rates are always bounded below by F(-c) > 0 (the natural out-migration rate from regions  $X \in \arg \max_{\tilde{X}} \left\{ V^1(\tilde{X}) \right\}$ ). Since regions with populations  $l \notin \mathcal{L}$  eventually move to  $\overline{l}^*$  with probability 1 but conditional on being in  $\overline{l}^*$  they never move to population states outside  $\mathcal{L}$ , it follows that all states  $l \notin \mathcal{L}$  are transient, i.e. the ergodic set of the process governing regional evolutions is given by  $\mathcal{L}$ . Therefore, the stationary distribution of populations across regions  $\underline{\nu}^*$  and  $\overline{\nu}^*$  are given by the set  $\mathcal{L}$ .

### **Proof of Proposition 2**

We have that  $out(A, l) = \frac{q((A,l),1)\cdot l}{l+l_0}$  and similarly  $in(A, l) = \frac{\Psi(A,l)}{l+l_0}$ . To show Result 1, first of all, Lemma 13 implies that  $\overline{V} - V^1(\overline{A}, l) \leq \overline{V} - V^1(\underline{A}, l)$ ,  $\forall l$  and hence,  $\overline{\epsilon}(\overline{A}, l, 1) \leq \overline{\epsilon}(\underline{A}, l, 1)$ , which immediately implies that  $out(\overline{A}, l) \leq out(\underline{A}, l)$ . This inequality is of course strict for  $l = \epsilon [l, \overline{l}^*]$ . Similarly, from (30) it follows that  $in(\overline{A}, l) \geq in(\underline{A}, l)$ . To show Result 2, Lemma 11 implies that out(A, l) is increasing in l and strictly so for  $A = \underline{A}$ , which immediately implies that out(A, l) is increasing in l. Turning to in(A, l), note that  $\Psi(A, l)$  is decreasing in l and hence so will  $\frac{\Psi(A,l)}{l+l_0}$ .

### **Proof of Proposition 3**

We have that  $p(A, l) = d(A, l) + \beta \cdot E_A[p(A', l'_1(A, l))]$ . First of all note that  $d(A, l) = g'(L - l'_1(A, l))$  is continuous in l by the continuity of  $l'_1(A, l)$  and g'(.). Furthermore, it is bounded and increasing in (A, l) since  $l'_1(A, l)$  is increasing in (A, l) by Lemma 11 and equation (30) and g'(.) is decreasing. Now, clearly the operator  $T[f(A, l)] = d(A, l) + \beta \cdot E_A[f(A', l'_1(A, l))]$  maps bounded continuous functions into bounded continuous functions and, furthermore, satisfies Blackwell's sufficient conditions for a contraction. Hence, by the Contraction Mapping Theorem, T has a unique fixed point in the space of bounded continuous functions. Therefore, since d(A, l) and  $l'_1(A, l)$  are increasing in (A, l) it follows that T maps increasing functions into increasing functions and therefore its unique fixed point is increasing.

To show the second part of the proposition, first of all, note that  $\tilde{p}(A, \bar{l}^*) = p\left(\overline{A}, \bar{l}^*\right)$  and  $\tilde{p}(A, \underline{l}^*) = p\left(\underline{A}, \underline{l}^*\right)$ . Furthermore,  $\tilde{p}(A, l) = p\left(\underline{A}, l_1^{\prime-1}(\underline{A}, l)\right)$  for  $l \in (\underline{l}^*, \overline{l}^*)$  given equation (30) and the inverse  $l_1^{\prime-1}$  is well-defined. Furthermore, given that  $l_1'(A, l)$  is increasing in l it follows that  $l_1'^{-1}(\underline{A}, l)$  is increasing in l as well. Hence, by the properties of p it follows that  $p\left(\underline{A}, l_1'^{-1}(\underline{A}, l)\right)$  is increasing in l for  $l \in (\underline{l}^*, \overline{l}^*)$  and hence so is  $\tilde{p}(A, l)$ . Now, noting that  $p\left(\overline{A}, \overline{l}^*\right) \ge p(A, l) \ l < \overline{l}^*$ , it follows that  $\tilde{p}(A, l)$  is increasing in l for  $l \in [\underline{l}^*, \overline{l}^*]$ .

### Proof of Lemma 5

Consider the population constancy condition

$$\int l_1' \left(\overline{A}, l_1\right) d\overline{\nu}^* + \int l_1' \left(\underline{A}, l_1\right) d\underline{\nu}^* + 2l_0 = L$$

It follows from Lemma 12 that  $l'_1(\overline{A}, l_1) = \overline{l}^*$  for every  $l_1$  in the support of  $\overline{\nu}^*$ . Hence,  $\int l'_1(\overline{A}, l_1) d\overline{\nu}^* = \overline{l}^*$  and so we have that:

$$L - 2l_0 - \bar{l}^* = \int_0^{\bar{l}^*} l_1'(\underline{A}, l_1) \, d\underline{\nu}^*(l_1)$$

where the integral on the RHS is a sum over members in the support of  $\underline{\nu}^*$ ,  $\mathcal{L}$ , which is given recursively by  $\{l_1^n\}_{n=0}^{\infty}$  with  $l_1^0 = \overline{l}^*$  and  $l_1^n = (1 - F(\overline{\epsilon}(\underline{A}, l_1^{n-1}))) l_1^{n-1}$ , for  $n \ge 1$ , i.e.

we have:

$$G(l_0, \bar{l}^*) = L - 2l_0 - \bar{l}^* - \sum_{n=0}^{\infty} l'_1(\underline{A}, l_1^n) \cdot \underline{\nu}^*(l_1^n) = 0$$
(31)

where  $\underline{\nu}^*(l_1^n)$  is the fraction of regions with population  $l_1^n$ . First of all, note that changes in  $l_0$  have no direct effect on the law of motion  $l_1'(\underline{A}, l_1)$  or on the stationary distribution  $\underline{\nu}^*$  since  $l_0$  does not enter directly into workers value functions and hence does not enter into  $\overline{\epsilon}(\underline{A}, l_1)$  and neither does it enter the equation for the stationary distribution  $\underline{\nu}^*$ . Hence,  $\frac{\partial G}{\partial l_0} = -2$ .

Now, turning to the effect of  $\overline{l}^*$ , first of all note that  $\overline{l}^*$  only affects the ergodic set of  $\underline{\nu}^*$  but does not affect the equations for the distribution (26) and (27), i.e. while the set  $\{l_1^n\}_{n=0}^{\infty}$  may change, the distribution over that set does not change with  $\overline{l}^*$ . It is actually straightforward to explicitly solve for the distributions  $\underline{\nu}^*$  and  $\overline{\nu}^*$  from (26) and (27) for an equilibrium of type 1. In particular, we have that

$$\overline{\nu}^* \left( l_1^0 \right) = \rho \cdot \sum_{n=0}^{\infty} \overline{\nu}^* \left( l_1^n \right)$$
$$\underline{\nu}^* \left( l_1^0 \right) = (1-\rho) \cdot \sum_{n=0}^{\infty} \overline{\nu}^* \left( l_1^n \right)$$
$$\overline{\nu}^* \left( l_1^n \right) = (1-\rho) \underline{\nu}^* \left( l_1^{n-1} \right)$$
$$\underline{\nu}^* \left( l_1^n \right) = \rho \cdot \underline{\nu}^* \left( l_1^{n-1} \right)$$

Noting that  $\sum_{n=0}^{\infty} \overline{\nu}^* (l_1^n) = 1$ , we get that  $\underline{\nu}^*$  follows a geometric distribution with parameter  $1 - \rho$ , i.e.  $\underline{\nu}^* (l_1^n) = \rho^n (1 - \rho)$ .

Turning to the effect of  $\bar{l}^*$  on  $l_1'(\underline{A}, l_1^n)$ , first of all observe that for  $a = \gamma$ ,  $\bar{\epsilon}(\underline{A}, l_1) = \bar{\epsilon}(\underline{A})$ , since there are no house price differences across regions. Therefore,  $\sum_{n=0}^{\infty} l_1'(\underline{A}, l_1^n) \cdot \underline{\nu}^*(l_1^n) = \sum_{n=0}^{\infty} (1 - F(\bar{\epsilon}(\underline{A})))^{n+1} \bar{l}^* \rho^n (1-\rho) = \frac{(1-F(\bar{\epsilon}(\underline{A})))(1-\rho)}{1-(1-F(\bar{\epsilon}(\underline{A})))\rho} \bar{l}^*$ . Hence,  $\frac{\partial G}{\partial \bar{l}^*} = -1 - \frac{(1-F(\bar{\epsilon}(\underline{A})))(1-\rho)}{1-(1-F(\bar{\epsilon}(\underline{A})))\rho}$  and so by the implicit function theorem,  $\frac{d\bar{l}^*}{dl_0} = -\frac{2}{1+\frac{(1-F(\bar{\epsilon}(\underline{A})))(1-\rho)}{1-(1-F(\bar{\epsilon}(\underline{A})))\rho}}$ . Note that  $\frac{(1-F(\bar{\epsilon}(\underline{A})))(1-\rho)}{1-(1-F(\bar{\epsilon}(\underline{A})))\rho} < 1$  and so  $\frac{d\bar{l}^*}{dl_0} < -1$ , which immediately implies that  $\bar{l}^* + l_0$  is decreasing in  $l_0$ .

## Appendix C - Computational

### Model for calibration

In this section I provide a description of the differences between the model used for calibration in Section 5 and the basic model from Section 3. I only include model assumptions that differ across the two models.

### Regional labor markets, job creation, and destruction

As before, I consider a discrete time economy with infinite number of periods t = 0, 1, 2, ...The economy consists of a measure M = 2 of islands or regions. The economy is populated by a measure L of infinitely lived workers, residing in different regions who are risk neutral and derive utility from consumption as well as from housing services. Workers can supply 1 unit of labor.

Regional productivity follows a two state Markov chain as before. In each region there is a representative firm that can open job vacancies at a per period cost of k and recruit workers for production in the same period. At the end of each period, after production takes place, with probability s a job becomes unproductive and is destroyed. The labor market of each region is characterized by a search and matching friction. After migration decisions have been made, a measure  $\tilde{u}_t^j$  of unemployed workers and a measure  $v_t^j$  of vacancies try to match with each other. Matching is described by a CRS matching function  $m^j (\tilde{u}_t^j, v_t^j)$ giving the total number of regional matches per period. Defining  $\theta_t^j = \frac{v_t^j}{\tilde{u}_t^j}$  as the regional market tightness and  $\mu(\theta) = m(1, \theta)$ , we have that the job finding probability for a worker is  $\mu (\theta_t^j)$  and a job filling probability for a vacancy is  $\frac{\mu(\theta_t^j)}{\theta_t^j}$ . Workers that remain unmatched in a given period receive a period payoff of e.

### Job creation decisions

Let J(A) be the value from posting a vacant job in a region with productivity j. Then

$$J(A) = -k + \frac{\mu(\theta(A))}{\theta(A)}K(A) + \left(1 - \frac{\mu(\theta(A))}{\theta(A)}\right)\beta(1-s)E_t[J(A)]$$
(32)

Similarly, let K(A) be the value from a matched job. Then

 $K(A) = A - w + \beta(1 - s) \cdot E_t[K(A)]$  (33)

where w is the wage rate paid. The firm is owned by the workers in the economy and discounts payoffs at the discount rate of workers  $\beta$ . The firm opens vacant jobs until the

cost of opening a vacancy equals the expected payoff from a filled vacancy, or:

$$J(A) = 0 = -k + \frac{\mu(\theta(A))}{\theta(A)}K(A)$$

or

$$K(A) = k \frac{\theta(A)}{\mu(\theta(A))}$$

Then, substituting in for K(A), we get that:

$$\frac{\theta(\overline{A})}{\mu(\theta(\overline{A}))} = \frac{\overline{A} - w}{k} + \beta(1 - s) \left(\rho \frac{\theta(\overline{A})}{\mu(\theta(\overline{A}))} + (1 - \rho) \frac{\theta(\underline{A})}{\mu(\theta(\underline{A}))}\right)$$
(34)

and

$$\frac{\theta(\underline{A})}{\mu(\theta(\underline{A}))} = \frac{\underline{A} - w}{k} + \beta(1 - s) \left( \rho \frac{\theta(\underline{A})}{\mu(\theta(\underline{A}))} + (1 - \rho) \frac{\theta(\overline{A})}{\mu(\theta(\overline{A}))} \right)$$
(35)

### Regional housing market and depreciation shock

The set-up for regional housing markets, home financing and depreciation shock are identical as before.

#### Worker migration

Unemployed workers have an idiosyncratic region preference  $\epsilon$  for the current region that they reside in. At the beginning of each period an unemployed worker gets a new draw of  $\epsilon$  from a continuous distribution F with density function f with  $E[\epsilon] = 0$  and support over [-B, B] for some B > 0. Upon observing his match quality, a worker decides whether to move to a different region. Moving is instantaneous and entails a fixed cost of c. Upon moving, the worker terminates the mortgage debt contract, in which case the housing unit is sold if it is not depreciated and lenders are repaid or the worker defaults and incurs the penalty  $\zeta$ . Upon moving, the worker draws a new  $\epsilon \sim F$  for the new region. Migration is directed, a worker migrates to the region that gives him the highest expected value.

### **Regional state variables**

In contrast to the one-period job model, in this model each region j will be fully characterized by its current period productivity  $A_t^j$  plus the beginning-of-period measure of workers with and without debt overhang,  $l_{j,t}^h$ ,  $h \in \{0, 1\}$ , the beginning-of-period measure of unemployed workers with and without debt overhang,  $u_{j,t}^h$ ,  $h \in \{0, 1\}$  and the beginning-of-period distributions of workers over employment and housing states,  $\overline{\nu}_t$  and  $\underline{\nu}_t$ . However, since there are no house prices differences across regions only the region's current productivity,  $A_{j,t}$ , will be relevant for worker migration decisions. Also, I let  $\tilde{u}_{j,t}^h$  to be the post-migration measure of unemployed workers with housing state  $h \in \{0, 1\}$  in region j, while  $\tilde{l}_{j,t}^h$  is the end-of-period labor force.

### Worker value functions

Similarly to the one-period job model all workers (weakly) prefer to be homeowners in equilibrium. I define  $V_U^h(A)$  to be an unemployed worker's value function, given regional state A, and housing state h. and similarly an employed worker's value function is given by  $V_E^h(A)$ . Then, letting

$$\bar{V} = \max_{A} \left\{ V_U^1(A) \right\} \tag{36}$$

be migration value, we have that:

$$V_{U}^{h}(A) = \mu(\theta(A)) V_{E}^{h}(A) + (1 - \mu(\theta(A))) \left(e - d + \gamma + \beta E_{A} \left[W^{h}(A')\right]\right)$$
(37)

and

$$V_{E}^{h}(A) = w - d + \gamma + \beta \cdot E_{A}\left[(1 - s) V_{E}^{h}(A') + s \cdot \left[W^{h}(A')\right]\right]$$
(38)

where

$$W^{h}(A) = \max_{\bar{\epsilon}(A,\chi)} \left\{ F\left(\bar{\epsilon}\left(A,h\right)\right)\bar{V} + \left(1 - F\left(\bar{\epsilon}\left(A,h\right)\right)\right)V^{h}(A) - F\left(\bar{\epsilon}\left(A,h\right)\right)\left(c + \left(1 - h\right)\zeta\right) + \int_{\bar{\epsilon}(A,h)} \epsilon dF \right) \right\}$$
(39)

The function  $W^{h}(A)$  takes this form since as I now show, only unemployed workers who migrate default (this is in the spirit of Lemma 8 above).

**Lemma 14.** For  $A \in \{\underline{A}, \overline{A}\}$ ,  $V_E^0(A) > V_E^1(A) - \zeta$  and  $V_U^0(A) > V_U^1(A) - \zeta$ .

*Proof.* First of all note that  $W^1(A) - W^0(A) \leq \zeta$ . The proof of this is identical to the proof of Lemma 8. We have that:

$$V_E^1(A_t) = w - d + \gamma + \beta \cdot \left[ (1 - s) E_t \left[ V_E^1(A_{t+1}) \right] + s E_t \left[ W^1(A_{t+1}) \right] \right]$$

or

$$V_E^1(A_t) = \frac{w + \gamma - d}{1 - \beta(1 - s)} + s\beta E_t \left[\sum_{j=0}^{\infty} \beta^j (1 - s)^j W^1(A_{t+j+1})\right]$$

Hence,

$$V_E^1(A_t) - V_E^0(A_t) \le s\beta E_t \left[ \sum_{j=0}^{\infty} \beta^j (1-s)^j \left( W^1(A_{t+j+1}) - W^0(A_{t+j+1}) \right) \right]$$

Therefore, we have that

$$V_{E}^{1}(A_{t}) - V_{E}^{1}(A_{t}) \leq s\beta \sum_{j=0}^{\infty} \beta^{j} (1-s)^{j} \gamma$$

or

$$V_{E}^{1}(A_{t}) - V_{E}^{0}(A_{t}) \le s\beta \frac{\gamma}{1 - \beta(1 - s)}$$

Note that  $\frac{s\beta}{1-\beta(1-s)} < 1$  and so  $V_E^1(A_t) - V_E^0(A_t) < \gamma$  or  $V_E^0(A_t) > V_E^1(A_t) - \gamma$ . Now, consider  $V_U^0(A_t)$  and  $V_U^1(A_t)$ . We have:

$$V_{U}^{h}(A_{t}) = \mu(\theta(A_{t})) V_{E}^{h}(A_{t}) + (1 - \mu(\theta(A_{t}))) \left(e - d + \gamma + \beta \cdot E_{t} \left[W^{h}(A_{t+1})\right]\right)$$

Hence,

$$V_{U}^{1}(A_{t}) - V_{U}^{0}(A_{t}) \leq \mu(\theta(A_{t})) \left( V_{E}^{1}(A_{t}) - V_{E}^{0}(A_{t}) \right) + \left( 1 - \mu(\theta(A_{t})) \right) \beta E_{t} \left[ W^{1}(A_{t+1}) - W^{0}(A_{t+1}) \right]$$

and so:

$$V_U^1(A_t) - V_U^0(A_t) \le \mu\left(\theta(A_t)\right)\gamma + \left(1 - \mu\left(\theta(A_t)\right)\right)\beta \cdot \gamma$$

This implies that,  $V_U^1(A_t) - V_U^1(A_t) < \gamma$  or  $V_U^0(A_t) > V_U^1(A_t) - \gamma$ .

Lemma 14 implies that a worker defaults only when unemployed and migrating out of the region.

### Wage determination

Rather than through Nash bargaining, the wage rate in each region is pinned down at w. However, w lies within the bargaining set of a worker-job match. In particular, at bargaining stage, in region j, the outside option of a worker is  $\tilde{V}_U^h(A) = e + \beta E_A [W^h(A')]$ . Hence, the minimum wage a worker would accept,  $\underline{w}^h(A)$ , leaves him indifferent between employment and unemployment, i.e. it solves:

$$V_E^h(A, \underline{w}^h) = \tilde{V}_U^h(A) \tag{40}$$

or

$$\underline{w}^{h}(A) = e + \beta E_{t} \left[ W^{h} \left( A_{t+1} \right) \right] - \beta \left[ (1-s) E_{t} \left[ V_{E}^{h} \left( A_{t+1} \right) \right] + s E_{t} \left[ W^{h} \left( A_{t+1} \right) \right] \right]$$
(41)

which implies that

$$\underline{w}^{h}(A) = e + (1-s)\beta \left[ E_{t} \left[ W^{h} \left( A_{t+1} \right) \right] - E_{t} \left[ V_{E}^{h} \left( A_{t+1} \right) \right] \right]$$

$$\tag{42}$$

Similarly, the maximum wage a firm would accept,  $\overline{w}(A)$ , leaves it indifferent between employing the worker and keeping the job vacant, i.e. it solves:

$$K(A,\overline{w}) = 0 \tag{43}$$

or

$$\overline{w}^{j}(A) = A + \beta(1-s) \cdot E_t \left[ K(A_{t+1}) \right]$$
(44)

Using (34) and (35) we get that:

$$\overline{w}(\overline{A}) = \overline{A} + k\beta(1-s) \left(\rho \frac{\theta(\overline{A})}{\mu(\theta(\overline{A}))} + (1-\rho)\frac{\theta(\underline{A})}{\mu(\theta(\underline{A}))}\right)$$
(45)

$$\overline{w}(\underline{A}) = \underline{A} + k\beta(1-s) \left( \rho \frac{\theta(\underline{A})}{\mu(\theta(\underline{A}))} + (1-\rho) \frac{\theta(\overline{A})}{\mu(\theta(\overline{A}))} \right)$$
(46)

Therefore,

$$w \in \left[\max\left\{\underline{w}^{0}(\overline{A}), \underline{w}^{1}(\overline{A})\right\}, \ \overline{w}(\overline{A})\right]$$
(47)

Note that with wages determined via Nash bargaining there will be two wages depending on a worker's housing state h, with the wage rate being a weighted average of the two boundaries.

### Laws of motion for regional unemployment and labor force

We have the following laws of motion:

$$\tilde{u}_t^0(A, u_t^0) = \left(1 - q^0(A)\right) u_t^0 \tag{48}$$

$$\tilde{l}_t^0(A, l_t^0, u_t^0) = l_t^0 - q^0(A)u_t^0$$
(49)

$$\tilde{u}_t^1(A, u_t^1, l_t^1) = \begin{cases} (1 - q^1(\underline{A})) \, u_t^1 & \text{if } A = \underline{A} \\ (1 - q^1(\overline{A})) \, u_t^1 + \Phi_t \, (u_t^1, l_t^1) & \text{if } A = \overline{A} \end{cases}$$
(50)

$$\tilde{l}_t^1\left(A, u_t^1, l_t^1\right) = \begin{cases} l_t^1 - q^1(\underline{A})u_t^1 & \text{if } A = \underline{A}\\ l_t^1 - q^1(\overline{A})u_t^1 + \Phi_t\left(u_t^1, l_t^1\right) & \text{if } A = \overline{A} \end{cases}$$
(51)

I describe  $\Phi_t(u^1, l^1)$  below. Using these laws of motion one can derive laws of motion for the distributions  $\underline{\nu}_t$  and  $\overline{\nu}_t$ . However, I am more interested in deriving laws of motions for the first moments of these distributions, i.e. for the total (beginning-of-period) unemployment and total labor force in booming and depressed regions conditional on housing state h. Denote these by  $\overline{U}_t^h$  and  $\overline{L}_t^h$  for booming regions and  $\underline{U}_t^h$  and  $\underline{L}_t^h$  for depressed regions, where  $h \in \{0, 1\}$ . Then,

$$\begin{split} \overline{U}_{t+1}^{h} &= \rho \left[ \int_{\left(u^{h}, l^{h}\right)} \left(1 - \mu\left(\theta(\overline{A})\right)\right) (1 - s) \tilde{u}_{t}^{h}\left(\overline{A}, u^{h}, l^{h}\right) + s \tilde{l}_{t}^{h}\left(\overline{A}, u^{h}, l^{h}\right) d\overline{\nu}_{t} \right] \\ &+ (1 - \rho) \left[ \int_{\left(u^{h}, l^{h}\right)} \left(1 - \mu\left(\theta(\underline{A})\right)\right) (1 - s) \tilde{u}_{t}^{h}\left(\underline{A}, u^{h}, l^{h}\right) + s \tilde{l}_{t}^{h}\left(\underline{A}, u^{h}, l^{h}\right) d\underline{\nu}_{t} \right] \\ \overline{L}_{t+1}^{h} &= \rho \int_{\left(u^{h}, l^{h}\right)} \tilde{l}_{t}^{h}\left(\overline{A}, u^{h}, l^{h}\right) d\overline{\nu}_{t} + (1 - \rho) \int_{\left(u^{h}, l^{h}\right)} \tilde{l}_{t}^{h}\left(\underline{A}, u^{h}, l^{h}\right) d\underline{\nu}_{t} \end{split}$$

and similarly for  $\underline{U}_{t+1}^h$  and  $\underline{L}_{t+1}^h$ . Additionally, we have a population constancy condition:

$$\sum_{h} \overline{L}^{h} + \underline{L}^{h} = L$$

Given that agents are indifferent between migrating to any booming regions, there will be some indeterminacy in the in-migration function  $\Phi_t(u^1, l^1)$ . Similarly to the one-period job model I will focus on a migration function where all booming regions have the same end-of-period population of workers with no debt overhang in a given time period, which I denote by  $\bar{l}_t^1$ . Therefore,

$$\Phi_t\left(u^1, l^1\right) = \overline{l}_t^1 - l^1 + q^1(\overline{A}) \cdot u^1$$

and, solving for the above laws of motion, we get that:

$$\begin{split} \overline{U}_{t+1}^{1} &= \rho \left[ \int_{\left(u^{h}, l^{h}\right)} \left(1 - \mu \left(\theta(\overline{A})\right)\right) \left(1 - s\right) \left(u^{1} + \overline{l}_{t}^{1} - l^{1}\right) + s\overline{l}_{t}^{1} d\overline{\nu}_{t} \right] \\ &+ \left(1 - \rho\right) \left[ \int_{\left(u^{h}, l^{h}\right)} \left(1 - \mu \left(\theta(\underline{A})\right)\right) \left(1 - s\right) \left(1 - q^{1} \left(\underline{A}\right)\right) u^{1} + sl^{1} - q^{1} \left(\underline{A}\right) u^{1} d\underline{\nu}_{t} \right] \end{split}$$

or

$$\begin{split} \overline{U}_{t+1}^{1} &= \rho \left[ \left( 1 - \mu \left( \theta(\overline{A}) \right) \right) (1-s) \left( \overline{U}_{t}^{1} + \overline{l}_{t}^{1} - \overline{L}_{t}^{1} \right) + s\overline{l}_{t}^{1} \right] \\ &+ \left( 1 - \rho \right) \left[ \left( 1 - \mu \left( \theta(\underline{A}) \right) \right) (1-s) \left( 1 - q^{1} \left( \underline{A} \right) \right) \underline{U}_{t}^{1} + s \left( \underline{L}_{t}^{1} - q^{1} \left( \underline{A} \right) \underline{U}_{t}^{1} \right) \right] \end{split}$$

Similarly,

$$\begin{split} \underline{U}_{t+1}^{1} &= \rho \left[ \left( 1 - \mu \left( \boldsymbol{\theta}(\overline{A}) \right) \right) \left( 1 - s \right) \left( 1 - q^{1} \left( \underline{A} \right) \right) \underline{U}_{t}^{1} + s \left( \underline{L}_{t}^{1} - q^{1} \left( \underline{A} \right) \underline{U}_{t}^{1} \right) \right] \\ &+ \left( 1 - \rho \right) \left[ \left( 1 - \mu \left( \boldsymbol{\theta}(\underline{A}) \right) \right) \left( 1 - s \right) \left( \overline{U}_{t}^{1} + \overline{l}_{t}^{1} - \overline{L}_{t}^{1} \right) + s \overline{l}_{t}^{1} \right] \end{split}$$

with an equivalent expression for  $\overline{U}_{t+1}^0$  and  $\underline{U}_{t+1}^0.$  Furthermore,

$$\begin{split} \overline{L}_{t+1}^{1} &= \rho \overline{l}_{t}^{1} + (1-\rho) \left( \underline{L}_{t}^{1} - q^{1} \left( \underline{A} \right) \underline{U}_{t}^{1} \right) \\ \underline{L}_{t+1}^{1} &= \rho \left( \underline{L}_{t}^{1} - q^{1} \left( \underline{A} \right) \underline{U}_{t}^{1} \right) + (1-\rho) \overline{l}_{t}^{1} \\ \overline{L}_{t+1}^{0} &= \rho \left( \overline{L}_{t}^{0} - q^{0} \left( \overline{A} \right) \overline{U}_{t}^{0} \right) + (1-\rho) \left( \underline{L}_{t}^{0} - q^{0} \left( \underline{A} \right) \underline{U}_{t}^{0} \right) \\ \underline{L}_{t+1}^{0} &= \rho \left( \underline{L}_{t}^{0} - q^{0} \left( \underline{A} \right) \underline{U}_{t}^{0} \right) + (1-\rho) \left( \overline{L}_{t}^{0} - q^{0} \left( \overline{A} \right) \overline{U}_{t}^{0} \right) \end{split}$$

Summing up  $\overline{L}_{t+1}^1$ ,  $\underline{L}_{t+1}^1$ ,  $\underline{L}_{t+1}^0$  and  $\overline{L}_{t+1}^0$  and using the population constancy condition, we have that:

$$\overline{l}_{t}^{1} = \overline{L}_{t}^{1} + q^{1} (\underline{A}) \underline{U}_{t}^{1} + q^{0} (\underline{A}) \underline{U}_{t}^{0} + q^{0} (\overline{A}) \overline{U}_{t}^{0}$$

Hence, finally we get:

$$\begin{split} \overline{U}_{t+1}^{1} &= \rho \left[ \left( 1 - \mu \left( \theta(\overline{A}) \right) \right) \left( 1 - s \right) \left( \overline{U}_{t}^{1} + q^{1} \left( \underline{A} \right) \underline{U}_{t}^{1} + q^{0} \left( \underline{A} \right) \underline{U}_{t}^{0} + q^{0} \left( \overline{A} \right) \overline{U}_{t}^{0} \right) + s \left( \overline{L}_{t}^{1} + q^{1} \left( \underline{A} \right) \underline{U}_{t}^{1} + q^{0} \left( \underline{A} \right) \underline{U}_{t}^{0} + q^{0} \left( \overline{A} \right) \overline{U}_{t}^{0} \right) \right] \\ &+ \left( 1 - \rho \right) \left[ \left( 1 - \mu \left( \theta(\underline{A}) \right) \right) \left( 1 - s \right) \left( 1 - q^{1} \left( \underline{A} \right) \right) \underline{U}_{t}^{1} + s \left( \underline{L}_{t}^{1} - q^{1} \left( \underline{A} \right) \underline{U}_{t}^{1} \right) \right] \right] \end{split}$$

$$\begin{split} \underline{U}_{t+1}^{1} &= \rho \left[ \left( 1 - \mu \left( \theta(\overline{A}) \right) \right) \left( 1 - s \right) \left( 1 - q^{1} \left( \underline{A} \right) \right) \underline{U}_{t}^{1} + s \left( \underline{L}_{t}^{1} - q^{1} \left( \underline{A} \right) \underline{U}_{t}^{1} \right) \right] \\ &+ \left( 1 - \rho \right) \left[ \left( 1 - \mu \left( \theta(\underline{A}) \right) \right) \left( 1 - s \right) \left( \overline{U}_{t}^{1} + q^{1} \left( \underline{A} \right) \underline{U}_{t}^{1} + q^{0} \left( \underline{A} \right) \underline{U}_{t}^{0} + q^{0} \left( \overline{A} \right) \overline{U}_{t}^{0} \right) + s \left( \overline{L}_{t}^{1} + q^{1} \left( \underline{A} \right) \underline{U}_{t}^{1} + q^{0} \left( \underline{A} \right) \overline{U}_{t}^{0} \right) \right] \\ &= \overline{U}_{t+1}^{0} = \rho \left[ \left( 1 - \mu \left( \theta(\overline{A}) \right) \right) \left( 1 - s \right) \left( 1 - q^{0} \left( \overline{A} \right) \right) \overline{U}_{t}^{0} + s \left( \overline{L}_{t}^{0} - q^{0} \left( \overline{A} \right) \overline{U}_{t}^{0} \right) \right] \\ &+ \left( 1 - \rho \right) \left[ \left( 1 - \mu \left( \theta(\underline{A}) \right) \right) \left( 1 - s \right) \left( 1 - q^{0} \left( \underline{A} \right) \right) \underline{U}_{t}^{0} + s \left( \underline{L}_{t}^{0} - q^{0} \left( \underline{A} \right) \underline{U}_{t}^{0} \right) \right] \\ &= \frac{U_{t+1}^{0}}{U_{t+1}^{0}} = \rho \left[ \left( 1 - \mu \left( \theta(\underline{A}) \right) \right) \left( 1 - s \right) \left( 1 - q^{0} \left( \underline{A} \right) \right) \underline{U}_{t}^{0} + s \left( \underline{L}_{t}^{0} - q^{0} \left( \underline{A} \right) \underline{U}_{t}^{0} \right) \right] \\ &+ \left( 1 - \rho \right) \left[ \left( 1 - \mu \left( \theta(\overline{A}) \right) \right) \left( 1 - s \right) \left( 1 - q^{0} \left( \underline{A} \right) \right) \overline{U}_{t}^{0} + s \left( \overline{L}_{t}^{0} - q^{0} \left( \underline{A} \right) \underline{U}_{t}^{0} \right) \right] \end{aligned}$$

and for the labor force measures:

$$\begin{split} \overline{L}_{t+1}^{1} &= \rho \left( \overline{L}_{t}^{1} + q^{1} \left( \underline{A} \right) \underline{U}_{t}^{1} + q^{0} \left( \underline{A} \right) \underline{U}_{t}^{0} + q^{0} \left( \overline{A} \right) \overline{U}_{t}^{0} \right) + (1 - \rho) \left( \underline{L}_{t}^{1} - q^{1} \left( \underline{A} \right) \underline{U}_{t}^{1} \right) \\ \underline{L}_{t+1}^{1} &= \rho \left( \underline{L}_{t}^{1} - q^{1} \left( \underline{A} \right) \underline{U}_{t}^{1} \right) + (1 - \rho) \left( \overline{L}_{t}^{1} + q^{1} \left( \underline{A} \right) \underline{U}_{t}^{1} + q^{0} \left( \underline{A} \right) \underline{U}_{t}^{0} + q^{0} \left( \overline{A} \right) \overline{U}_{t}^{0} \right) \\ \overline{L}_{t+1}^{0} &= \rho \left( \overline{L}_{t}^{0} - q^{0} \left( \overline{A} \right) \overline{U}_{t}^{0} \right) + (1 - \rho) \left( \underline{L}_{t}^{0} - q^{0} \left( \underline{A} \right) \underline{U}_{t}^{0} \right) \\ \underline{L}_{t+1}^{0} &= \rho \left( \underline{L}_{t}^{0} - q^{0} \left( \underline{A} \right) \underline{U}_{t}^{0} \right) + (1 - \rho) \left( \overline{L}_{t}^{0} - q^{0} \left( \overline{A} \right) \overline{U}_{t}^{0} \right) \end{split}$$

## **Computational Algorithms**

Here I describes the algorithms used for simulating the stationary equilibrium for the oneperiod job model as well as the equilibrium of the calibrated model.

### Simulating the one-period job model

Computing the stationary equilibrium of the one-period job model requires a nested fixed point approach. The inner fixed point requires computing value function  $V^1$  and law of motion  $l^1$  for a given value of  $\underline{l}^*$  and  $\overline{l}^*$ . In the outer loop, I vary  $\underline{l}^*$  and  $\overline{l}^*$  to satisfy  $\underline{V}^1 \leq \overline{V}^1$ and the population constancy condition, respectively. I use the following algorithm:

- 1. I construct a grid  $\mathcal{G}$  for l;
- 2. Guess a value of  $\bar{l}_0^*$  on the grid and set  $\underline{l}_0^* = 0$ :
- 3. Pick initial value functions  $\overline{V}_0^1$  and  $\underline{V}_0^1$  on  $\mathcal{G}$ . Iterate the following steps until convergence of  $V^1$ :
  - Given  $\overline{V}_i^1$  and  $\underline{V}_i^1$  derive the implied law of motion for  $l_i^1$ ;
  - Given law of motion for  $l_i^1$  and solve  $\overline{V}_i^1$  and  $\underline{V}_i^1$  for  $\overline{V}_{i+1}^1$  and  $\underline{V}_{i+1}^1$ . Note that I use linear interpolation for values of  $V_n^1$  that are not on the grid, when solving for  $V_{i+1}^1$ .
- 4. Check whether  $\underline{V}^1 \overline{V}^1 \leq \epsilon_V > 0$  for all  $l \in \mathcal{G}$ . If not, increase  $\underline{l}_0^*$  and go back to Step 3;
- 5. Given  $\overline{V}^1$  and  $\underline{V}^1$ , compute  $l^1$  and use it to simulate *B* regions for *T* periods starting from random initial conditions (for productivity state and labor force levels);
- 6. Using the empirical distribution at time T, check population constancy condition. If population constancy condition is sufficiently close to being valid terminate, otherwise increase or decrease  $\bar{l}^*$  so that the population constancy condition holds and go back to Step 1.

### Simulating the model for calibration

- Given the block-recursive nature of the equilibrium in this case (due to the assumption of no house price differences across regions), I first solve for the worker value functions (which depend on the productivity of the region and the housing and employment state of the worker) as well as migration cutoffs. I also solve for regional market tightness;
- Using the solved value functions and market tightness I check whether the wage rate lies in the worker-job bargaining set; If not, then equilibrium does not exist for this wage rate;

- If the wage rate lies in the bargaining set, I proceed to solve for the average measure of unemployed in booming and depressed regions,  $\overline{U}^h$ ,  $\underline{U}^h$ ,  $h \in \{0, 1\}$  as well as the total labor force in booming and depressed regions  $\overline{L}^h$  and  $\underline{L}^h$ . Based on these I can solve for aggregate unemployment rate, aggregate market tightness and migration rate.
- To determine unemployment dispersion for the stationary equilibrium, I simulate the stationary distribution of unemployment and labor force over regions. I simulate B = 2000 regions for T = 1000 periods starting from random initial conditions (productivity state, labor force and unemployment level) using the outflow probabilities from workers problems and job finding probabilities for booming and depressed regions. I then take the resulting distribution at time T and compute unemployment dispersion and average unemployment rate in booming and depressed regions from it.
- Simulating the non-stationary equilibrium is identical apart from using the dispersion in unemployment across booming and depressed regions as the measure of unemployment dispersion in this case because of computational issues in simulating the non-stationary distribution over regions.

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