Debt dynamics and fiscal policy in the euro area

Christin Kyrme Tuxen*

Department of Economics, University of Copenhagen

CHRISTIN.KYRME.TUXEN@ECON.KU.DK

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VERY PRELIMINARY DRAFT

Abstract

This paper studies aggregate euro-area fiscal policy using a cointegrated VAR (CVAR) approach in order to evaluate the effects of government actions on area-wide activity, interest rates and inflation. In the European welfare-state system the public sector constitutes an essential part of the economy, and with the introduction of the European Monetary Union (EMU) fiscal policy has become ever more important in dealing with asymmetric shocks. We test for Ricardian Equivalence in the euro area by mapping a small theoretical model into a set of cointegrating relations. We also address explicitly the large degree of time-series persistency in the fiscal variables and study the importance of cross-country differences. Our results suggest that while fiscal policy has had little effect on inflation and interest rates, it has had distorting effects on employment. These findings highlight the potential need for structural reforms in some euro-area countries.

JEL Classification: C32, E62, H62, H63

Keywords: Time-Series Models, Fiscal Policy, Deficit; Surplus, Debt; Debt Management

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1 Introduction

Fiscal policies can have significant effects on demand and price pressures via the level and composition of government revenue and expenditure as well as via public deficits and debt. As a result of the European welfare-state system the public sector constitutes an essential part of the euro-area economy. With the introduction of the European Monetary Union (EMU), which combines a centralised monetary policy with decentralised fiscal and structural policies, the latter have become ever more important in dealing with, in particular, asymmetric shocks.

The notion of 'unpleasant monetarist arithmetic' of Sargent and Wallace (1981) underlines the importance of monetary- and fiscal-policy interactions. Given failure of Ricardian equivalence (REq), government actions affect a range of economic variables such as aggregate demand, potential output, real interest rates, risk premia and prices. An inflation-targeting central bank should thus care about the fiscal stance. A range of theoretical papers address monetary-fiscal interdependency, see *inter alia* Leith and Wren-Lewis (2000), Woodford (2001), Beetsma and Jensen (2002) and Leith and Thadden (2006). Overall, the recommendation of this strand of literature is the use of non-discretionary rules and clear mandates in order to induce predictability and accountability and thereby anchor private-sector expectations and reduce uncertainty.

In a monetary union with a common central bank but no coordination of fiscal policy, fiscal incentives are distorted as the punishment by financial markets for high levels of debt in one country is shared by all member states. This creates a moral-hazard problem. The institutional framework for the EMU laid down by the Treaty and the Stability and Growth Pact requires member states to 'avoid excessive government deficits'. This is to ensure sustainability and to allow automatic stabilisers to work better, in turn achieving a more efficient resource allocation. These guidelines have however been continuously violated by some member states, raising the question whether the EMU is a regime of monetary or fiscal dominance.

Juselius (2002) speculates that economic integration in Europe initially produced a vicious circle where high levels of unemployment produced large public deficits, which in turned increased the demand for (unproductive) capital, thereby raising the level of long-term interest rate.

Although empirical evidence on REq in the euro area remains inconclusive, there is a consensus that fiscal polices have significant effects on demand. Failure of REq may be attributed to, for example, the discount rate of households being higher than that of financial markets, some households being subject by credit constraints, and/or taxes having a distortionary impact.

Henry, de Cos, and Momigliano (2004) study the short-term effects on prices in the euro area using a range of macroeconometric models. They take a disaggregated approach allowing to assess the impact of changes in individual government-budget items. Their results show while effects on prices usually takes over a year to materialise, indirect taxes and employers' social-security contributions have important effects on prices in the short run. Longer-term effects on inflation are significant in most cases, in particular following a public-consumption shock.

Cuaresma and Reitschuler (2007) finds evidence of REq in the EU-15 member states over the last four decades using a single-equation cointegration model. Notably their results suggest that the enforcement of the Maastricht criteria have implied that consumers have started to behave Ricardian after 1993 in some countries (France and Netherlands) whereas the opposite change in behaviour is also found (Austria and Ireland).

Nickel and Vansteenkiste (2008) investigate the Keynesian "twin-deficit hypothesis" vs. REq for a range of industrial countries using a dynamic panel-data threshold model. Whereas the Keynesian hypothesis should imply a positive relationship between fiscal deficits and current-account deficits, the Ricardian implies a negative one. They find evidence that the relationship between the two types of deficits is positive when the public debt-to-GDP ratio is below 80 per cent.

Afonso and Sousa (2009) consider the macroeconomic effects of fiscal policy using a Bayesian Structural VAR approach. They find significant "crowding-out" effects of government spending shocks whereas revenue shocks have lagged effects on GDP; neither type of shock seem to affect the price level. When debt dynamics are explicitly incorporated, long-term interest rates and GDP become more responsive and effects of fiscal policy become more persistent.

This paper studies area-wide and aggregate fiscal policy in the euro zone using a cointegrated VAR (CVAR) approach in order to evaluate its effects on activity and inflation. We also discuss some cross-country differences and their implications. Our results point to distorting effects of government policy on economic activity, in turn highlighting the potential need for structural reforms in the euro area. In addition, the model in this paper provides input for a broader model of euro-area inflation determination, see Tuxen

(2009).

The rest of the paper is organised as follows. Section 2 presents the economic framework used in identification of the statistical model which is introduced in Section 3. The data are presented in Section 4.1 and Section 4.2 derives the cointegrating implications of the economic model given the chosen data vector. Aggregate euro-area results are presented in Section 5 while Section 6 discusses differences across countries. Section 7 concludes.

2 Economic model

Our starting point is the 'five-equation approach' to macroeconomics of Kirsanova, Stehn, and Vines (2005). These authors note that most short-run macroeconomic analysis is essentially based on a system containing an IS curve, a Phillips curve and a Taylor-rule. This model includes a description of monetary policy but fiscal policy is taken as exogenous or left out altogether. Kirsanova et al. (2005) endogenise fiscal policy by adding to the three baseline relations a fiscal rule and an equation describing debt accumulation.

Reade (2007) and Tuxen(2006,2009) model interactions of monetary and fiscal policy for the US and the euro area, respectively, but here we abstract from monetary policy and focus solely on the effects of debt dynamics. As a result, we exclude the monetary-policy rule from the Kirsanova et al. (2005) set-up.

The theoretical model thus consists of the following four relations. First, a fiscal-policy rule,

$$b_t = -\varphi_3 d_{t-1} - \varphi_4 \left(U_t - \overline{U}^{NAIRU} \right) \quad (1)$$

with b_t the primary balance-to-GDP, d_t government debt-to-GDP and U_t is the unemployment rate. Here fiscal policy is assumed to react to a rise in government debt by increasing the primary balance in an attempt to ensure fiscal sustainability. Stabilisation of unemployment around its long-run (NAIRU) level is assumed to be the goal of fiscal policy such that policy becomes expansionary in response to a rise in unemployment. Stabilisation of inflation is left to the monetary authorities (excluded here). The associated government budget constraint is,

$$d_t = (1 + R_{t-1})d_{t-1} - b_t \quad (2)$$

with R_t the real long-term interest rate. In effect, (2) is simply an accounting identify. A

Phillips curve describes the evolution of prices,

$$\Delta p_t = \varphi_1 \Delta p_{t-1} - \varphi_2 U_{t-1} \tag{3}$$

where Δp_t is the rate of inflation. This represents the standard inverse relation between inflation and unemployment. Finally, an IS curve describes equilibrium in the market for public and private goods,

$$y_t^r = \varphi_5 y_{t-1}^r - \varphi_6 R_{t-1} + \varphi_7 b_t - \varphi_8 d_t \tag{4}$$

with y_t^r the level of real GDP (or alternatively the output gap). Output is assumed to be negatively affected by rising real rates. Fiscal policy may affect aggregate demand directly with multiplier φ_7 and indirectly with multiplier φ_8 as a rise in public debt will in part be treated as a rise in private-sector net wealth when Ricardian equivalence fails. This completes the description of the economic model that we use as guidance in identifying the long-run relations in the statistical model below.

3 Statistical model

The CVAR framework provides a convenient way of statistically separating the pulling (cointegration) forces from the pushing (common trends) forces in non-stationary data. Within a VAR set-up this leads us to consider a vector equilibrium correction model (VECM) formulated in terms of the first differences of the process. For a p-dimensional VAR($k = 2$) model, this takes the form of a VECM($k - 1 = 1$),

$$\Delta \mathbf{x}_t = \mathbf{\Pi} \mathbf{x}_{t-1} + \mathbf{\Gamma}_1 \Delta \mathbf{x}_{t-1} + \boldsymbol{\phi} \mathbf{D}_t + \boldsymbol{\varepsilon}_t, \qquad t = 1, 2, ..., T \tag{5}$$

where $\mathbf{x}_t$ is a $p \times 1$ vector of variables, $\mathbf{D}_t$ a vector of deterministic components with coefficient matrix $\boldsymbol{\phi}$, and $\boldsymbol{\varepsilon}_t$ is a $p \times 1$ vector of errors. We assume $\boldsymbol{\varepsilon}_t \sim iid\ N_p(\mathbf{0}, \mathbf{\Omega})$ with $\mathbf{\Omega}$ the variance-covariance matrix. The hypothesis of cointegration takes the form of a reduced-rank restriction: rank$(\mathbf{\Pi} = \boldsymbol{\alpha}\boldsymbol{\beta}') = r < p$ where both $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are $p \times r$ matrices with r the number of cointegrating relations. We also require $|\boldsymbol{\alpha}'_\perp \mathbf{\Gamma} \boldsymbol{\beta}_\perp| \neq 0$ with $\boldsymbol{\alpha}_\perp$ and $\boldsymbol{\beta}_\perp$ the orthogonal complements of $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ and $\mathbf{\Gamma} = \mathbf{I} - \mathbf{\Gamma}_1$. Estimation and test procedures for this model are given in Johansen (1988 , 1991, 1996).

While the VECM specification illustrates the equilibrium-correcting forces of the model, the moving average (MA) representation reveals how the common stochastic trends are driving the model. When $\mathbf{\Pi}$ has reduced rank, the levels of the process has the representation (see Engle and Granger (1987) and Johansen (1996)),

$$\mathbf{x}_t = \mathbf{C}\Sigma_{i=1}^{t}(\boldsymbol{\varepsilon}_i + \boldsymbol{\phi}\mathbf{D}_i) + \text{stationary components} \tag{6}$$

where the long-run impact matrix, $\mathbf{C} = \boldsymbol{\beta}_\perp(\boldsymbol{\alpha}'_\perp\mathbf{\Gamma}\boldsymbol{\beta}_\perp)^{-1}\boldsymbol{\alpha}'_\perp$, shows how a shock to one variable causes the rest of the variables in the system to react. The common trends are defined as $\boldsymbol{\alpha}'_\perp\Sigma_{i=0}^{t}\boldsymbol{\varepsilon}_i$ and $\widetilde{\beta}= \boldsymbol{\beta}_\perp(\boldsymbol{\alpha}'_\perp\mathbf{\Gamma}\boldsymbol{\beta}_\perp)^{-1}$ provides the loadings to these.

The CVAR allows a flexible modelling of the regularities in the data, i.e. in its unrestricted form the CVAR is just a reformulation of the covariances in the data. In order to conduct policy analysis we need to attach a structural economic interpretation of the dynamics. We thus use the economic model of Section (2) to guide the identification of the cointegrating relations.

4 Linking theory and data

We present the data set and derive a set of potential cointegrating relations based on the economic model in Section 2.

4.1 Data

We use quarterly data series from the ECB's Area-Wide Model (AWM) (see FHM) and consider the period 1982:4 to 2002:4. An overview of the data and sources is given in Table 9 in Appendix A. Figure 1 plots the key variables. The sample is set to start in the early 1980s as tests indicate that the transition to a more strict regime of the European Monetary System (EMS) and the demolition of capital restrictions are likely to constitute a structural break around that time.

In the empirical analysis we consider the data vector,

$$\mathbf{x}_t = (PB/Y, GD/Y, \Delta y, \Delta p, I_l, U)'_t \tag{7}$$

where PB/Y_t is the primary balance-to-GDP[1], GD/Y_t is the level of public (gross) debt-to-GDP[2], Δp_t is the (quarterly) inflation rate, Δy_t is (quarterly) growth in (nominal) GDP, $I_{l,t}$ is the ten-year bond yield, and U_t is the unemployment rate.

[1] The primary balance excludes interest payments on debt.

[2] The gross-debt variable from the AWM database may not be the ideal measure of public indebtedness; net wealth is likely to constitute a better measure but is not available. Alternatively one might use net financial liabilities as proxy; this turns out to cointegrate $(1, -1)$ with GD/Y. For comparability with the AWM we use gross debt.

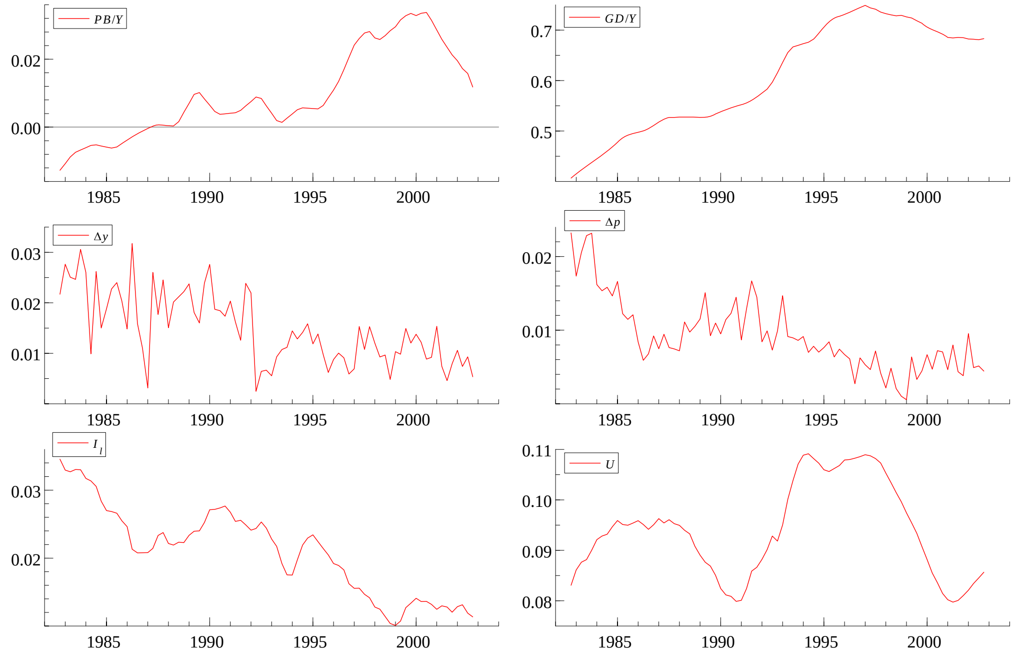

Figure 1: Euro-area data series, 1982:4-2002:4.

While a range of issues arise in aggregating national data of countries with flexible exchange rates in part of the sample (see Beyer, Doornik, and Hendry (2001) and Beyer and Juselius (2008)), the AWM data set is widely used for area-wide analysis of the euro zone and we abstract from aggregation issues here. National differences in government policies may complicate the interpretation of the area-wide stance of fiscal policy; we address this issue separately in Section 6.

Figure 1shows the levels of the data series in (7). Figure 2 shows the co-omvements of the primary deficit, i.e. $-PB_t$, and the change in the level of public debt, i.e. ΔGD_t. The difference between the two are the interest payments on existing debt. Despite falling bond yields the interest burden have weighed heavily on public deficits throughout the sample period as a result of high initial debt levels.

4.2 Statistical implications of the theoretical model

Given the data vector (7) we can now derive implications of the theoretical relations in terms of the CVAR. In all cases we add constant and error term to relations. For the fiscal rule (1) we obtain,

$$PB/Y_t - \beta_{11} GD/Y_t - \beta_{12} U_t - \beta_{10} + \nu_{1,t} \sim I(0) \tag{8}$$

where we expect $\beta_{11} > 0$ and $\beta_{12} < 0$.

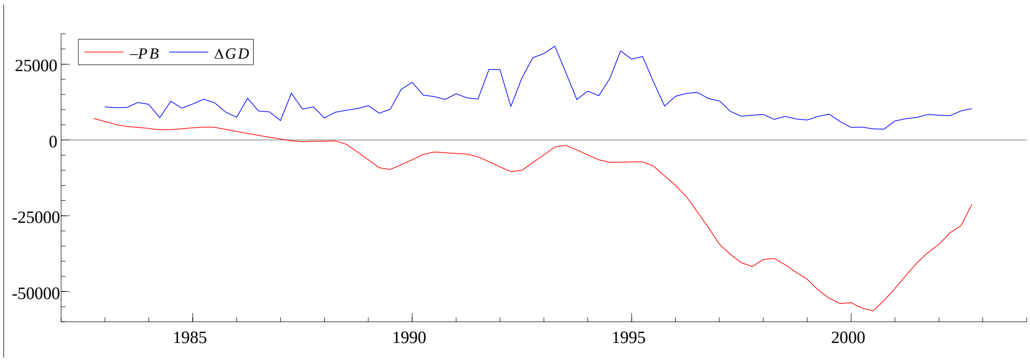

Figure 2: Euro-area primary *deficit* and change in the level of public debt, 1982:4-2002:4.

By simple accounting, for the government budget constraint, (2), we have,

$$GD_t = (1 + I_{l,t-1})GD_{t-1} - PB_t \tag{9}$$

$$\left(\frac{GD}{Y}\right)_t = (1 + I_{l,t-1})\left(\frac{GD}{Y}\right)_{t-1}\underbrace{\frac{Y_{t-1}}{Y_t}}_{\simeq(1-g_{t-1})} - \left(\frac{PB}{Y}\right)_t$$

$$\simeq (1 + I_{l,t-1} - g_{t-1})\left(\frac{GD}{Y}\right)_{t-1} - \left(\frac{PB}{Y}\right)_t \tag{10}$$

with I_l the long term interest rate and $g_{t-1} = \frac{Y_t - Y_{t-1}}{Y_{t-1}}$ GDP growth. Using $g_{t-1} \simeq \Delta y_t$ and a multivariate first order Taylor approx. e.g. around $\left(\frac{GD}{Y}\right)^* = 0.60$ and $I_l = 0.01 < \Delta y = 0.0125$ we obtain,

$$GD/Y_t - \beta_{21}PB/Y_t - \beta_{22}(I_l - \Delta y)_t - \beta_{20} + \nu_{2,t} \sim I(0) \tag{11}$$

where $\beta_{21} < 0$ and $\beta_{22} > 0$.

The Phillips curve, (3), becomes,

$$\Delta p_t - \beta_{31}U_t - \beta_{30} + \nu_{3,t} \sim I(0) \tag{12}$$

where we have assumed a constant NAIRU (implicitly included in β_{30}) and we look for $\beta_{31} < 0$.

We exclude the IS curve, (4), as our choice of data does not allow this to be identified. Even if we use $(\Delta y - \Delta p)_t$ or U_t as a proxy for activity, an IS curve cannot be separately identified in a statistical sense given the relations (8)-(12).

We can summarise the steady-state (cointegrating) relations suggested by the Kirsanova et al. (2005) model as,

$$\alpha\widetilde{\beta}'\widetilde{x}_{t-1} = \underbrace{\begin{pmatrix} \alpha_{11} & 0 & \alpha_{13} \\ \boldsymbol{\alpha}_{21} & 0 & \alpha_{23} \\ \alpha_{31} & 0 & \alpha_{33} \\ \alpha_{41} & 0 & \boldsymbol{\alpha}_{43} \\ \alpha_{51} & 0 & \alpha_{53} \\ \alpha_{61} & 0 & \boldsymbol{\alpha}_{63} \end{pmatrix}}_{\boldsymbol{\alpha}} \underbrace{\begin{pmatrix} \beta_{11} & 1 & 0 & 0 & 0 & \beta_{12} & \beta_{10} \\ 1 & \beta_{21} & -\beta_{22} & 0 & \beta_{22} & 0 & \beta_{20} \\ 0 & 0 & 0 & 1 & 0 & \beta_{31} & \beta_{30} \end{pmatrix}}_{\widetilde{\beta}'} \underbrace{\begin{pmatrix} GD/Y \\ PB/Y \\ \Delta y \\ \Delta p \\ I_l \\ U \\ 1 \end{pmatrix}_{t-1}}_{\widetilde{\mathbf{x}}_{t-1}} \tag{13}$$

The signs of the long-run (β-) coefficients supporting the theoretical interpretation were given above. The short-run (α-) coefficients are also of interest as these show the effects of temporary disequilibria on the adjustment dynamics.

Deviations from the fiscal rule suggest that policy is either looser or tighter than the rule dictates. If the first relation constitutes a fiscal rule the primary balance should be equilibrium-correcting and hence we expect $\alpha_{21} < 0$. If fiscal disequilibria have no effects on the real economy, i.e. of REq holds, unemployment should be unaffected by deviations from the rule and thus $\alpha_{61} = 0$. Effectively, REq would also imply that neither of the two nominal growth rates, Δy_t and Δp_t, should be affected by fiscal disequilibria. Failure to reject $\alpha_{51} < 0$ would provide some support for the Juselius (2002) hypothesis that high deficits have caused a rise in bond yields.

Deviations from the budget constraint simply represent approximation and/or measurement errors and hence are uninteresting from an economic point of view (but might be of interest in evaluating data quality).

In order to support the interpretation of the Phillips curve we would need either inflation or unemployment to equilibrium-correct to the third relation and thus expect α_{43} and/or α_{63}.

5 Empirical results

Due to signs of I(2) problems in the I(1) model we first test the nominal-to-real transformation (NRT) for a subset of the variables. After concluding that an I(1) set-up is nevertheless appropriate for our purpose we estimate a structural I(1) model. Appendix D offers some simple simulations of the effect of a large (but non-unit) root on the size and power of the LR-test for restrictions.

5.1 Testing the nominal-to-real transformation

The system based on 7 shows signs of an I(2) component. First, the spectral densities of GD/Y and PB/Y (not reported) indicate that the government variables need to be differenced twice to become stationary. Moreover, the dynamics of the Z- and R-form of $\widehat{\widetilde{\beta}_1'}\widetilde{x}_t$ differ markedly, and for any reasonable choice of rank a companion-form root of ≥ 0.93 remain in the model. Finally, the difference between the baseline and the Bartlett-corrected trace test statistics is rather large. Taken together this suggests that there is an I(2) component in the I(1) model. As shown by **?**) this may distort inference on the model.

Economic theory prescribes the existence of one single trend underlying the growth in all nominal variables. Assuming the central bank is in control of inflation the nominal anchor is thus set by monetary policy. In terms of econometrics, this implies that all nominal variables should share a common I(2) trend which can be removed by a simple transformation to I(1) space: the nominal trend in each series is eliminated by subtracting one of the other nominal series. The linear combination (1,-1) of each pair of variables thus ensures cointegration from I(2) to I(1) space and the I(2) component is assumed to cancel in this nominal-to-real transformation (NRT). The transformed system can thus be analysed in an I(1) model. In order to keep track of the nominal growth rate the first difference of either of the nominal variables must be included in the model and as long as the restrictions imposed via the NRT are valid (see **?**)), no information is lost in the transformation.

The nature of government variables complicates a test of the standard NRT: because the primary balance can turn negative (i.e. if there is a deficit) we cannot take logs and thus cannot conduct the simple (1,-1) test for long-run proportionality of $log(PB_t)$ with $y_t = log(Y_t)$. Moreover, from an economic point of view PB_t, a flow variable, is unlikely to be an I(2) variable; the I(2) component observed in PB/Y_t is thus likely to result from Y_t. In fact, the specification of the economic model led us to include two nominal growth rates, Δy_t and Δp_t, which should cointegrate if the NRT is valid.

To investigate whether the level of nominal debt, GDP and consumer prices share a common I(2) trend, we consider a subset of the variables in (7),

$$x_t^{nom} = (gd, y, p)_t' \tag{14}$$

where $gd_t = log(GD_t)$ and similarly for y_t and p_t. Based on (14) we estimate a VAR(2)

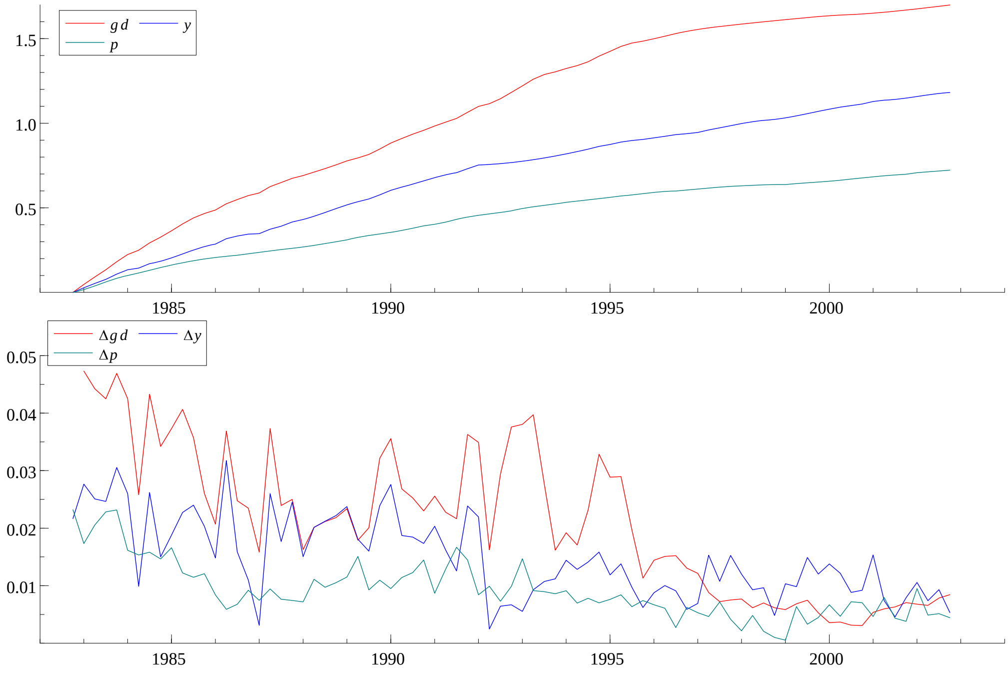

Figure 3: Euro-area variables, 1982:4-2002:4. Upper panel: levels of nominal debt, GDP and consumer prices. Lower panel: first differences of levels of nominal debt, GDP and consumer prices

with a trend restricted to the cointegration space. The I(2) rank test, see Table 10 in Appendix B, suggests one cointegrating relation and two I(2) trends. This choice of ranks leaves a root of 0.70 (alternatively with $r = 2, s_2 = 1$ we have a root of 0.86). Testing whether these two stochastic trends load proportionally into each of the three nominal variables leads to a test statistic of $\chi^2(1) = 11.06$ ($p = 0.00$) and thus rejection of the null of long-run price homogeneity. Table 11 in Appendix B reports the estimates of the I(2) model with the proportionality restrictions imposed. The $\alpha_{\perp 2}$-matrix suggests that the first I(2) trend is approximately made up of twice cumulated shocks to the debt-to-GDP ratio whereas the second one arise from shocks to prices.

[Conclude that I(1) analysis is appropriate nevertheless??]

5.2 I(1) results

Based on 7 we estimate an I(1) CVAR with a constant restricted to the cointegrating space. To take account of extraordinary events we include a set of dummy variables,

$$D_t = (DP_{86:2}, DP_{87:1}, DP_{97:2}, DP_{02:4}) \tag{15}$$

Lag	Test statistic	p-value
LM tests for no autocorrelation:		
1	$\chi^2(36) = 46.87$	0.11
2	$\chi^2(36) = 32.95$	0.61
3	$\chi^2(36) = 37.08$	0.42
4	$\chi^2(36) = 45.92$	0.12
Test for multivariate normality:		
	$\chi^2(12) = 18.18$	0.11
LM tests for no ARCH effects:		
1	$\chi^2(441) = 449.08$	0.38
2	$\chi^2(882) = 959.23$	0.04
3	$\chi^2(1323) = 1415.64$	0.04
4	$\chi^2(1764) = 1662.49$	0.96

Table 1: Misspecification tests

where $DP_{YY:Q}$ is a permanent impulse dummy which takes a value of one at time YY:Q and zero otherwise. Table 1 suggests that the model is well-specified: multivariate normality cannot be rejected and with two lags the autocorrelation tests do not indicate problems. At the five- (but not the one-) per cent level ARCH effects show up as significant but as shown by Dennis, Hansen, and Rahbek (2002) this is not critical for the distribution of the I(1) rank test.

Table 2 shows the trace test statistics. The baseline trace test point to a rank of two, three or four depending on the level of significance used. The Bartlett-corrected test suggest either $r = 0$ or $r = 1$. The difference between the baseline and corrected statistics is likely to result from the presence of an I(2) component as discussed above. We therefore need to consider other sources of information on the number of cointegrating relations present before concluding on the correct choice of rank. The modulus of the largest companion-form root minimised at 0.93 at $r = 3$, see Table 2. Graphs of the unrestricted cointegrating relations, the recursively calculated trace test statistic and significance of the α-coefficients also support a rank of three. The results reported below are thus based on $r = 3$ which is consistent with the theoretical predictions.

Table 3 reports a set of tests given $r = 3$. None of the variables appear to be long-run excludable, albeit the test hint that the constant may be excluded. This confirms that all six variables are needed in the modelling the long-run dynamics of the public sector. For all variables the hypothesis of stationarity (a unit vector in β including the constant) is rejected, although the conclusion is borderline for the unemployment rate at the one-per

$p-r$	r	Eigenvalue	Trace	Trace*	5% c.v.	p-value	p-value*	Largest root
6	0	0.67	191.16	108.09	103.68	0.00	0.02	0.95
5	1	0.42	103.78	59.06	76.81	0.00	0.52	0.95
4	2	0.28	61.32	33.93	53.94	0.01	0.77	0.96
3	3	0.19	34.97	22.24	35.07	0.05	0.58	0.93
2	4	0.17	17.99	9.24	20.16	0.10	0.72	0.97
1	5	0.04	3.22	1.87	9.14	0.55	0.80	0.97
0	6	-	-	-	-	-	-	0.94

Table 2: I(1) rank test and modulus of the largest companion-form root

	PB/Y	GD/Y	Δy	Δp	I_l	U	$const$
Tests for exclusion	17.71 [0.00]	21.59 [0.00]	65.11 [0.00]	23.54 [0.00]	22.52 [0.00]	20.78 [0.00]	5.59 [0.13]
Tests for stationarity	21.12 [0.00]	21.08 [0.00]	21.03 [0.00]	19.26 [0.00]	21.31 [0.00]	11.85 [0.01]	-
Tests for weak exogeneity	8.31 [0.04]	9.57 [0.02]	46.20 [0.00]	11.74 [0.01]	2.65 [0.45]	10.76 [0.01]	-
Tests for unit vector in α	8.46 [0.04]	17.71 [0.00]	0.42 [0.94]	4.11 [0.25]	14.84 [0.00]	5.83 [0.12]	-

Table 3: Tests for long-run exclusion, stationarity, weak exogeneity and unit vectors in α

cent level. The tests for weak exogeneity (a zero row in α) strongly suggests that the long-term interest rate is weakly exogenous to the system. Hence cumulated shocks to the long rate constitute a common stochastic trend. This is a first indication that the hypothesis that loose fiscal policy puts upward pressure on bond yields may be refuted. At the five-per cent level no other variables are found to be weakly exogenous but the test statistics for deficits and debt hint that fiscal policy is determined partly by outside factors. Finally, tests for unit vectors in α indicate that the nominal GDP growth, the inflation rate and the unemployment rate are purely adjusting within the model. The joint test of $(\Delta y, \Delta p, U)_t$ all having unit vectors in α is the mirror image of $(PB/Y, GD/Y, I_l)_t$ being weakly exogenous; this test results in a test statistic of $\chi^2(9) = 18.815$ ($p = 0.03$).

We impose the theoretical restrictions in (4.2) on the model but these are rejected ($\chi^2(4) = 30.85$; $p = 0.00$). However, if we include the long rate in the Phillips-curve relation, then the constant term, which presumably partly reflects the NAIRU level of unemployment, becomes excludable. We discuss the role of the long rate in the Phillips curve below. If we also allow for a non-homogenous relation between the long rate and nominal GDP growth in the budget constraint the restrictions cannot be rejected. If, in addition, we impose weak exogeneity on the long rate we are left with a test statistic of $\chi^2(7) = 6.30$ ($p = 0.51$). Table 12 through 15 in Appendix C report the matrices of both the VECM and MA representations.

Because of the large companion-form root p-values in tests for stationarity may be distorted and we need to study the graphs of the cointegrating relations to check for stationarity. Figure 4 shows that the fiscal-rule relation exhibits some fairly long swings around its mean but nevertheless look stationary. This persistency likely derives from the inherent persistency of the fiscal variables also visible from the $\mathbf{\Gamma}_1-$matrix: both the $\Delta PB/Y_t-$ and $\Delta GD/Y_t-$ equations has their own lagged value entering highly significantly and with a coefficient close to one (0.93 and 0.81, respectively). Recursive estimation does not indicate serious problems with non-constancy of the parameters; as an example, Table 5 shows the recursively calculated LR-test statistic which shows that for the R-form the test quickly becomes insignificant within the recursive estimation sample.

We can summarise the separation of the long- and short-run dynamics as follows,

$$\begin{pmatrix} \Delta PB/Y \\ \Delta GD/Y \\ \Delta^2 y \\ \Delta^2 p \\ \Delta I_l \\ \Delta U \end{pmatrix}_t = \begin{pmatrix} \underset{[-3.63]}{-0.07} & * & * \\ * & * & \underset{[-1.88]}{-0.29} \\ \underset{[1.65]}{0.17} & \underset{[-8.54]}{-0.02} & \underset{[3.53]}{0.91} \\ * & * & \underset{[-4.58]}{-0.69} \\ 0 & 0 & 0 \\ \underset{[-4.63]}{-0.10} & * & * \end{pmatrix} \begin{pmatrix} \widetilde{\beta}_1'\widetilde{x}_{t-1} \\ \widetilde{\beta}_2'\widetilde{x}_{t-1} \\ \widetilde{\beta}_3'\widetilde{x}_{t-1} \end{pmatrix}_{t-1} + ... + \widehat{\varepsilon}_t \tag{16}$$

The first cointegrating relation is a fiscal-policy rule,

$$\widetilde{\beta}_1'\widetilde{x}_t = PB/Y_t \underset{[-13.18]}{-0.11} GD/Y_t + \underset{[10.76]}{0.65} U_t \tag{17}$$

where the fiscal stance is loosened in response to a rise in the unemployment rate and tightened and/or a fall in the debt-to-GDP ration. The primary balance equilibrium-corrects significantly to (5.2) supporting its interpretation as a fiscal rule. We find evidence of REq failure as unemployment reacts to fiscal disequilibria. However, expansionary fiscal policy has a tendency to be followed by rise in unemployment, i.e. $\alpha_{51} < 0$), contrary to the standard presumption. The $C-$matrix shows that in the long run there is a positive (negative) but insignificant effect of shocks to the level of debt-to-GDP on unemployment (nominal GDP growth).

The adverse effect of fiscal policy may be attributed to crowding-out and higher risk premia, but these explanations are no longer obvious given that the long rate seemed largely determined by global market conditions. Rather, it is possible that the demand-supporting effects of a rise in the government deficit are neutralised if the public becomes concerned about the sustainability of policy and the implications for future tax burdens and growth, i.e. if Ricardian equivalence holds, see ECB (2004). However, the fact that fiscal stimulus has had an outright negative effect on the economy is an indication that public expenditure has not been put to its most productive use in this period.[3] This could reflect the rigid institutional structures dominating many European labour markets with high minimum wages, inflexible hiring/firing regulations and high unemployment benefits disencouraging job search for some groups of workers.[4] A disaggregated model of the composition of public finances would be required to study more carefully the supply-side effects of public policy, see *inter alia* Henry, de Cos, and Momigliano (2004).

The second cointegrating relation is the government budget constraint,

$$\widetilde{\beta}_2'\widetilde{x}_t = GD/Y_t + \underset{[3.78]}{15.51}PB/Y_t \underset{[-6.42]}{-58.69}\, I_{l,t} + \underset{[12.59]}{90.15}\Delta y_t \underset{[-5.50]}{-1.04} \tag{18}$$

where deficits and a higher growth-adjusted interest rate - albeit homogeneity is rejected - add to the level of debt (mechanically). Deviations from this identity does not represent disequilibria errors but merely approximation and measurement errors. The fact that nominal GDP growth significantly adjusts towards (5.2) is a hint that *gross* debt is not the ideal measure of public indebtedness. Government-sector *net* wealth is likely to constitute a better measure but this is not available.

Finally, the third cointegrating relation is a variant of the Phillips curve,

$$\widetilde{\beta}_3'\widetilde{x}_t = \Delta p_t \underset{[-16.07]}{-0.74}\, I_{l,t} + \underset{[5.95]}{0.06}U_t \tag{19}$$

where we have normalised on inflation. Alternatively, normalising on unemployment we have,

$$\widetilde{\beta}_{3,alt}'\widetilde{x}_t = U_t + \underset{[8.98]}{4.49}\Delta p_t \underset{[-12.77]}{-12.49}\, \underbrace{(I_l - \Delta p)_t}_{\text{real rate}} \tag{20}$$

which sees the standard inverse relationship between inflation and unemployment augmented with the real long rate of interest. This relation resembles that proposed by Phelps (1994) who argues that a rise in the rental cost of capital depresses employment. Phelps (1994) emphasised the real rate as the core transmission mechanism between the labour market and the wider economy because changes in the cost of capital induces movements

[3]As the primary deficit excludes the costs of servicing debt this 'unproductive use' can not solely be attributed to high levels of outstanding debt-to-GDP.

[4]The Lisbon Strategy is one attempt to promote structural reforms and thereby growth to cope with the disincentives created by inflexible institutional arrangements in, for example, the labour market.

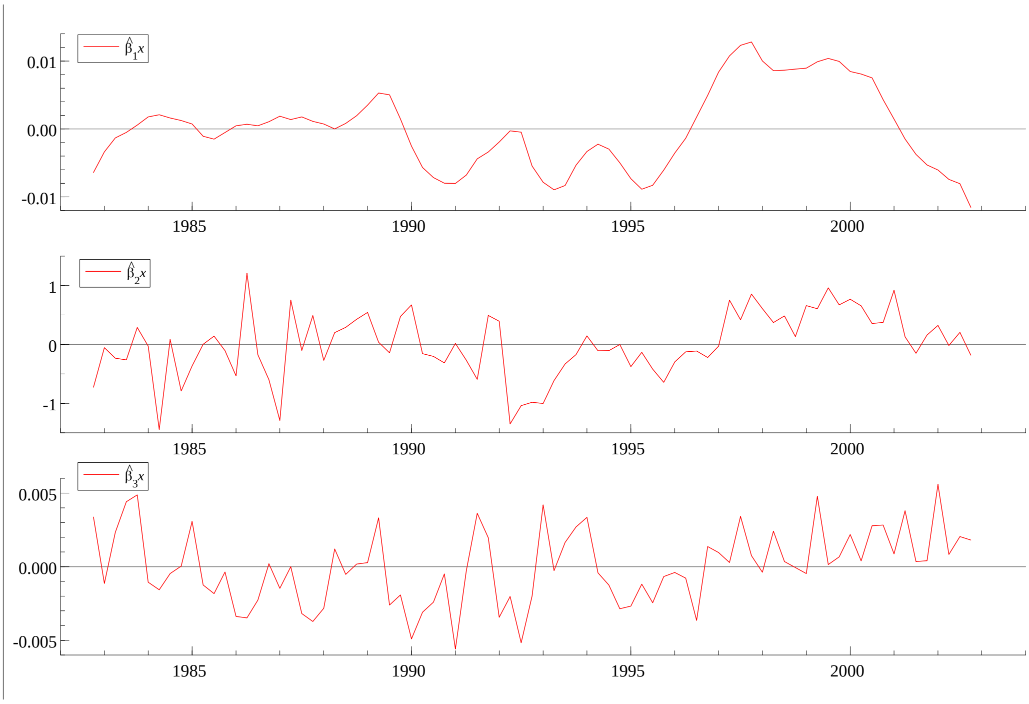

Figure 4: Deviations from the identified cointegrating relations, 1982:1-2002:4.

in the natural rate of unemployment (NAIRU). Thus (5.2) points to the existence of a non-constant NAIRU which moves with the real interest rate. Only when this is time-varying NAIRU level is taken into account does the standard trade-off between Δp_t and U_t show up.

Figure 4 shows that the fiscal rule ends in a negative disequilibrium end 2002. This suggests relatively loose fiscal policy relative to fundamentals at the end of our sample. Indeed, the fiscal stance was allowed to loosen extraordinarily in a range of countries in response to the 2000-01 economic downturn. The Phelps-Phillips curve points to relatively high quarterly inflation at the end of 2002 relative to long-run fundamentals.

[Discuss MA representation and common trends??]

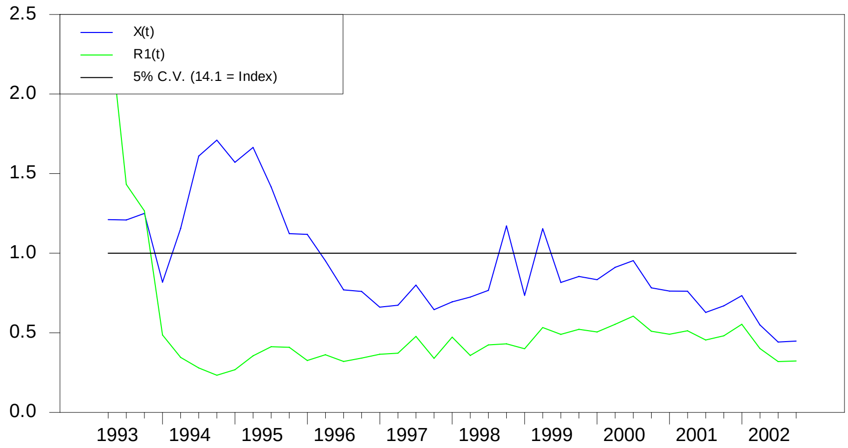

Figure 5: Recursively calculated LR-test statistic, 1993:3-2002:4.

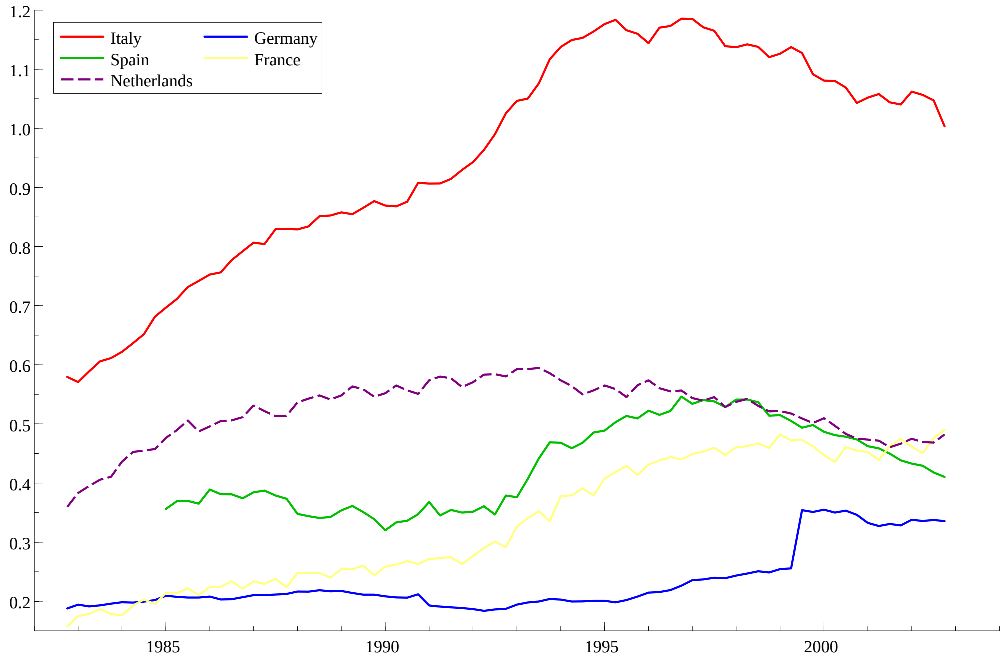

Figure 6: Public debt-to-GDP in different euro-area countries, 1982:1-2002:4.

6 Comparison of euro-area results with individual countries

National differences in government policies may distort results based on an area-wide consideration of fiscal policy. First of all, the markets for public goods in the euro-area countries are disintegrated to a much larger extent than is the case for monetary policy. Due to a large degree of integration in financial markets monetary-policy actions affect investor decisions abroad and thus led to exchange-rate movements. Public services are inherently non-traded goods and the international transmission of domestic fiscal policy actions is much less direct.

Moreover, major differences exist between countries. Figure 6 and 7 show the level of public debt-to-GDP and the budget balance-to-GDP, respectively, in a range of euro-area countries. For example, highly indebted Italy contrasts with the more prudent Netherlands. A study of cross-country differences within the euro area provides a validity check of the aggregate analysis presented above.

Data availability:

- Germany: re-unification implies jump in 1991:1

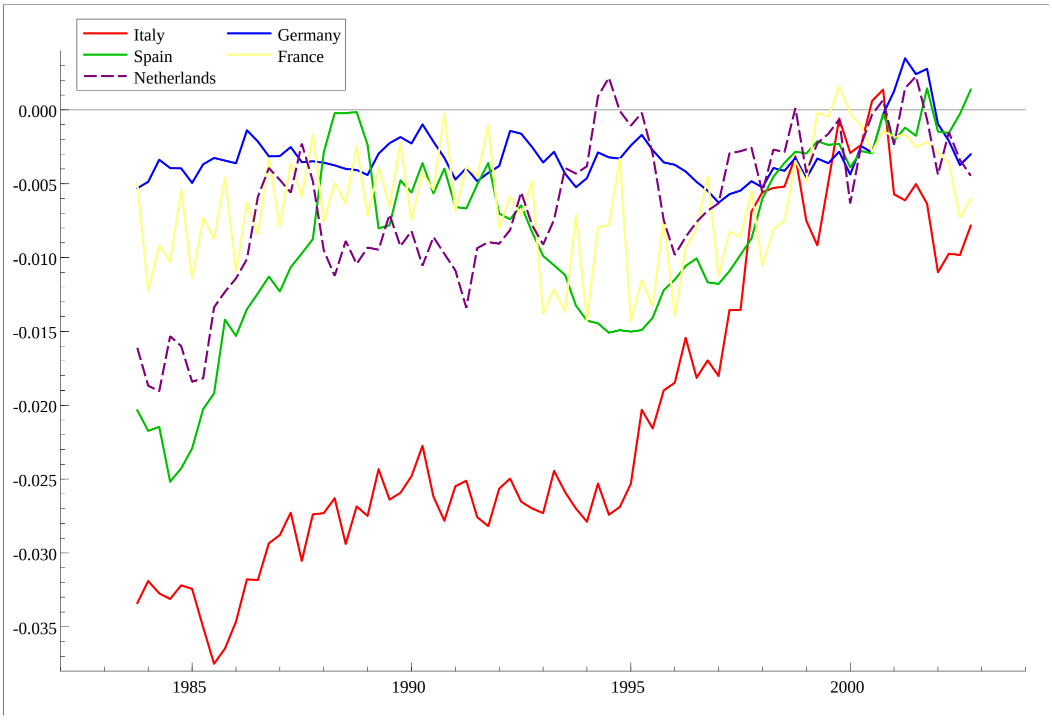

Figure 7: Budget balance-to-GDP in different euro-area countries, (4-quarter moving averages), 1983:4-2002:4.

- Spain: data on fiscal variables only available from 1985:1 and onwards
- Italy: yearly data from 1999 and onwards (but quarterly data from IFS if spliced 1998/99)

We propose a range of methods for comparing individual country models with each other and with the aggregate model:

1. Choice of rank and interpretation of cointegrating relations
2. Tests on the adjustment structure
3. Simple correlation of 'similar' ECM terms
4. Principal components analysis of 'similar' ECM terms
5. Tests of national information set vs. area-wide (using automatic model selection)

Results on comparing models are reported in Table 4 through 8. Overall, the results can be summarised as follows:

1. National CVAR models: $r = 3$ in both cases but $PB/Y \sim$ I(0); otherwise similar cointegrating relations

2. Correlation of 'similar' ECM terms high with euro area; lower between France and the Netherlands

3. First principal component of 'similar' ECM terms explains more than 50 per cent of the variation

4. Simultaneous equations model (SEM) (short run structure of CVAR): national country variables do not add much to the explanation of euro area variables (debt and deficit ratios for Italy, Germany and Spain included as well; source: IMF)

Our preliminary conclusion is that the Netherlands is 'quite similar' to euro area, France 'somewhat different' while Finland is 'in-between'.

[Other methods/techniques for comparing countries: panel cointegration analysis??]

	Euro area	Netherlands	France
Weight in AWM	–	0.06	0.20
Trace correlation	0.70	0.71	0.66
Modulus of largest root			
$r = 2$	0.96	0.98	0.85
$r = 3$	0.97	0.98	0.85
$r = 4$	0.96	0.94	0.93
Choice of rank, r	3	3	3 (or 2)
Tests on α			
Zero row in α	$(PB/Y, I_l)$	$(PB/Y, I_l)$	(GD/Y)
Unit column in α	$(U, \Delta y)$	$(\Delta p, U)$	$(\Delta p, I_l)$

Table 4: Key characteristics of each model and tests on α

	Euro area	Netherlands	France
$ECM_1 = \widehat{\widetilde{\beta}'_1}\widetilde{x}_t$	fiscal policy rule	fiscal policy rule	fiscal policy rule
$ECM_2 = \widehat{\widetilde{\beta}'_2}\widetilde{x}_t$	Phelps-PC	Phelps-PC	Fisher parity
$ECM_3 = \widehat{\widetilde{\beta}'_3}\widetilde{x}_t$	budget constraint	budget-real growth	budget-real growth
Weakly exogenous	I_l	I_l	GD/Y_n
Test of restrictions	$\chi^2(3) = \underset{[0.41]}{2.91}$ (slightly different!)	$\chi^2(6) = \underset{[0.62]}{4.39}$	$\chi^2(5) = \underset{[0.57]}{3.86}$

Table 5: Comparison of interpretation of identified cointegrating relations

	$ECM_{1,EA}$	$ECM_{1,NE}$	$ECM_{1,FR}$
$ECM_{1,NE}$	0.65	-	-
$ECM_{1,FR}$	0.09	0.09	-
	$ECM_{2,EA}$	$ECM_{2,NE}$	$ECM_{2,FR}$
$ECM_{2,NE}$	0.72	-	-
$ECM_{2,FR}$	0.45	0.30	-
	$ECM_{3,EA}$	$ECM_{3,NE}$	$ECM_{3,FR}$
$ECM_{3,NE}$	0.16	-	-
$ECM_{3,FR}$	-0.49	0.13	-

Table 6: Simple correlation of comparable ECM terms. EA=Euro area; NE=Netherlands; FR=France

	Euro area	Netherlands	France	Finland	Explanatory power
ECM_1					
PC_1	-0.62	-0.58	0.11	-0.52	49%
PC_2	0.25	0.29	0.80	-0.45	32%
ECM_2					
PC_1	-0.58	-0.49	-0.55	-0.34	53%
PC_2	0.32	0.53	-0.38	-0.69	34%
ECM_3					
PC_1	0.42	0.62	-0.17	0.64	44%
PC_2	0.56	-0.33	-0.72	-0.23	36%

Table 7: Principal components analysis of comparable ECM terms. Eigenvectors and explanatory power for first two principal components, PC_1 and PC_2. Note: Finland included

	Netherlands	France	Finland
$\Delta PB/Y$	✓	✓	✓
$\Delta GD/Y$	✓	÷	✓
$\Delta^2 p$	÷	✓	÷
ΔI_l	✓	÷	✓
ΔU	÷	÷	÷
$\Delta^2 y$	✓	÷	÷

Table 8: PcGets (liberal strategy) single equation results: national information set vs. information set including area wide ECM terms. Results of encompassing tests (or LR-tests if models nested): V-sign means AWM information set is sufficient; minus means national data yields extra information

7 Conclusions

We have studied aggregate euro-area fiscal policy using a CVAR approach in order to evaluate its effects on area-wide activity, interest rates and inflation. High persistency of fiscal variables complicates the CVAR analysis but we were able to replicate crucial theoretical relations empirically. Notably a Phillips-curve trade-off was identified once a non-constant NAIRU level, which varied with the cost of capital, was taken into account.

Our results suggest that government policy has not significantly affected interest rates and inflation. In fact, bond yields were found to be determined by factors outside our model. In contrast, public policy has had distorting effects on employment, i.e. we reject the REq hypothesis for the euro area. This could reflect that public spending diverts resources from their most productive use. The restrictions on public finances laid down in the Stability and Growth Pact thus seem reasonable. Our findings thus highlight the potential need for structural reforms in some euro-area countries as in fact proposed by the Lisbon agenda.

Preliminary results suggest that national dynamics are not crucial for analysing the aggregate stance of fiscal policy in the euro area.

With the introduction of the EMU, which combines a centralised monetary policy with decentralised fiscal and structural policies, the latter have become ever more important in dealing with asymmetric shocks. More recently, as monetary policy has found it difficult to steer money-market rates as a result of the credit crisis, the need for additional stimulus from the government sector has become apparent. In the current situation, our results point to the importance of designing fiscal policy with a view to improve, for example, the functioning of the labour market.

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A Data series

Variable	Notation	Source
Consumer price index (CPI)	p	AWM: PCD
CPI inflation	Δp	-
Public debt-to-GDP	GD/Y	AWM: GDN_YEN
Primary deficit-to-GDP	PD/Y	AWM: (-) GPN_YEN
Nominal output (GDP)	y	AWM: YEN
Real GDP	$y_r \equiv y - p$	-
Nominal GDP growth	Δy	-
Long-term interest rate	I_l	AWM: LTN

Table 9: Variables and data sources. AWM data is the 4th. update of the database. Lowercase letters denote (natural) logarithmic transformations.

B I(2) results

$p-r$	r	$s_2 = 3$	$s_2 = 2$	$s_2 = 1$	$s_2 = 0$
3	0	130.49 [0.00]	82.28 [0.00]	69.36 [0.00]	62.45 [0.00]
2	1		44.50 [0.12]	33.55 [0.07]	30.74 [0.01]
1	2			13.81 [0.31]	10.72 [0.10]

Table 10: I(2) rank test

β'	gd	y	p	$trend$
β'_1	1.00	-5.27	4.27	0.04

$\tilde{\boldsymbol{\beta}}'_2$	gd	y	p
$\tilde{\beta}'_{21}$	0.36	0.55	0.60
$\tilde{\beta}'_{22}$	-0.74	0.16	0.37

$\boldsymbol{\alpha}$	α
$\Delta^2 gd$	-0.01 [-8.89]
$\Delta^2 y$	-0.01 [-7.29]
$\Delta^2 p$	-0.00 [-0.66]

$\boldsymbol{\alpha}_{\perp 2}$	$\alpha_{\perp 21}$	$\alpha_{\perp 22}$
$\sum\sum\varepsilon_{gd}$	-0.81 [-12.75]	-0.04 [-0.65]
$\sum\sum\varepsilon_{y}$	1.00 [NA]	0.00 [NA]
$\sum\sum\varepsilon_{p}$	0.00 [NA]	1.00 [NA]

$\boldsymbol{\zeta}$	ζ_1
$\Delta^2 gd$	0.11 [6.86]
$\Delta^2 y$	0.11 [7.30]
$\Delta^2 p$	-0.02 [-1.92]

$\boldsymbol{\delta}'$	gd	y	p	$trend$
δ'_1	46.61	14.59	6.75	38.10

Table 11: I(2) estimates with long-run proportionality imposed

C I(1) results

$\boldsymbol{\beta}'$	PB/Y	GD/Y	Δy	Δp	I_l	U	$const$
β_1'	1.00 [NA]	−0.11 [−13.18]	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.65 [10.76]	0.00 [NA]
β_2'	15.51 [3.78]	1.00 [NA]	90.15 [12.59]	0.00 [NA]	−58.69 [−6.42]	0.00 [NA]	−1.04 [−5.50]
β_3'	0.00 [NA]	0.00 [NA]	0.00 [NA]	1.00 [NA]	−0.74 [−16.07]	0.06 [5.95]	0.00 [NA]

$\boldsymbol{\alpha}$	α_1	α_2	α_3
$\Delta PB/Y$	−0.07 [−3.63]	0.00 [1.42]	−0.00 [−0.03]
$\Delta GD/Y$	0.09 [1.59]	−0.00 [−1.91]	−0.29 [−1.88]
$\Delta^2 y$	0.17 [1.65]	−0.02 [−8.54]	0.91 [3.53]
$\Delta^2 p$	0.02 [0.29]	−0.00 [−0.54]	−0.69 [−4.58]
ΔI_l	0.00 [0.00]	0.00 [0.00]	0.00 [0.00]
ΔU	−0.10 [−4.63]	0.00 [1.42]	0.07 [1.13]

Table 12: Cointegrating relations and adjustment structure

$\boldsymbol{\Gamma}_1$	$\Delta PB/Y_{-1}$	$\Delta GD/Y_{-1}$	$\Delta^2 y_{-1}$	$\Delta^2 p_{-1}$	$\Delta I_{l,-1}$	ΔU_{-1}
$\Delta PB/Y$	0.93 [15.30]	0.02 [0.71]	−0.01 [−0.52]	0.00 [0.09]	−0.28 [−3.15]	−0.08 [−0.88]
$\Delta GD/Y$	0.05 [0.26]	0.81 [11.95]	0.08 [1.41]	0.12 [1.00]	0.47 [1.82]	0.13 [0.50]
$\Delta^2 y$	−0.28 [−0.94]	−0.55 [−4.74]	0.21 [2.15]	−0.30 [−1.40]	0.93 [2.13]	−0.93 [−2.03]
$\Delta^2 p$	−0.13 [−0.74]	−0.18 [−2.67]	0.01 [0.14]	−0.00 [−0.02]	−0.36 [−1.39]	0.13 [0.50]
ΔI_l	0.10 [1.61]	−0.03 [−1.32]	0.00 [0.09]	0.04 [0.98]	0.45 [4.78]	0.02 [0.17]
ΔU	−0.05 [−0.69]	0.07 [2.88]	−0.03 [−1.34]	0.04 [0.86]	−0.30 [−3.05]	0.43 [4.24]

Table 13: Short-run dynamics

$\boldsymbol{\alpha}'_\perp$	PB/Y	GD/Y	Δy	Δp	I_l	U
$\alpha'_{\perp,1}$	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	1.00 [NA]	0.00 [NA]
$\alpha'_{\perp,2}$	0.98 [0.95]	1.00 [NA]	−0.08 [−0.87]	−0.52 [−2.19]	0.00 [NA]	0.00 [NA]
$\alpha'_{\perp,3}$	−1.43 [−2.35]	0.00 [NA]	−0.01 [−0.25]	0.08 [0.57]	0.00 [NA]	1.00 [NA]

$\tilde{\boldsymbol{\beta}}_\perp$	$\tilde{\beta}_{\perp,1}$	$\tilde{\beta}_{\perp,2}$	$\tilde{\beta}_{\perp,3}$
PB/Y	0.99 [0.76]	0.62 [0.96]	−1.53 [−2.24]
GD/Y	22.64 [1.35]	13.25 [1.59]	−2.85 [−0.32]
Δy	0.08 [0.09]	−0.69 [−1.64]	0.24 [0.55]
Δp	0.42 [0.56]	−0.58 [−1.54]	−0.17 [−0.42]
I_l	0.77 [0.84]	−0.67 [−1.49]	−0.08 [−0.16]
U	2.44 [1.08]	1.37 [1.22]	1.86 [1.57]

Table 14: I(1) common stochastic trends and loadings

$\mathbf{C}$	$\Sigma\varepsilon_{PB/Y}$	$\Sigma\varepsilon_{GD/Y}$	$\Sigma\varepsilon_{\Delta y}$	$\Sigma\varepsilon_{\Delta p}$	$\Sigma\varepsilon_{I_l}$	$\Sigma\varepsilon_U$
PB/Y	2.79 [1.95]	0.62 [0.96]	−0.03 [−0.30]	−0.44 [−1.09]	0.99 [0.76]	−1.53 [−2.24]
GD/Y	17.11 [0.93]	13.25 [1.59]	−1.02 [−0.82]	−7.15 [−1.36]	22.64 [1.35]	−2.85 [−0.32]
Δy	−1.03 [−1.10]	−0.69 [−1.64]	0.05 [0.82]	0.38 [1.43]	0.08 [0.09]	0.24 [0.55]
Δp	−0.33 [−0.40]	−0.58 [−1.54]	0.05 [0.85]	0.29 [1.21]	0.42 [0.56]	−0.17 [−0.42]
I_l	−0.55 [−0.55]	−0.67 [−1.49]	0.05 [0.80]	0.35 [1.21]	0.77 [0.84]	−0.08 [−0.16]
U	−1.31 [−0.53]	1.37 [1.22]	−0.13 [−0.80]	−0.57 [−0.80]	2.44 [1.08]	1.86 [1.57]

Table 15: Long-run impact of shocks

D Simulation of LR-test for restrictions on $\boldsymbol{\beta}$

In order to understand the effect on inference of a near-unit root in the I(1) CVAR we simulate the distribution of the likelihood ratio test for restrictions on $\boldsymbol{\beta}$. We set up a Monte-Carlo experiment based on a simple CVAR with/without a large remaining root for a true/false null hypothesis and compare results on rejection frequencies to investigate size/power.

We find that the presence of a large root remaining after setting the rank of Π does indeed matter for the distribution of the LR test statistic as the test becomes over-sized and has low power.

D.1 Monte-Carlo set-up

Consider the cointegrated VAR (CVAR) model,

$$\Delta x_t = \underbrace{\alpha\beta'}_{=\Pi} x_{t-1} + \varepsilon_t \tag{21}$$

with $\varepsilon_t \sim N_p(\mu, \Omega_p)$ and $T = 100$, $p = 3$, $k = 1$ and $r = 2$. We have thus made the following simplifications in relation to the model specification of Section (5): $\phi = 0$ and $\Gamma_1 = 0$ (i.e. $k = 1$).

Example of a well-behaved case,

$$\beta = \begin{pmatrix} 1 & 1 \\ -0.8 & 0 \\ 0 & -1 \end{pmatrix} \quad \alpha = \begin{pmatrix} -0.1 & -0.3 \\ 0.2 & -0.1 \\ -0.2 & 0.4 \end{pmatrix} \quad \Pi = \begin{pmatrix} -0.4 & 0.08 & 0.3 \\ 0.1 & -0.16 & 0.1 \\ 0.2 & 0.16 & -0.4 \end{pmatrix} \tag{22}$$

$$eig(I_p + \Pi) = (1.00; 0.68; 0.36)$$

Example of a large-root case,

$$\beta = \begin{pmatrix} 1 & 1 \\ -0.8 & 0 \\ 0 & -1 \end{pmatrix} \quad \alpha = \begin{pmatrix} -0.08 & -0.03 \\ 0.04 & -0.01 \\ -0.03 & 0.05 \end{pmatrix} \quad \Pi = \begin{pmatrix} -0.11 & 0.064 & 0.03 \\ 0.03 & -0.032 & 0.01 \\ 0.02 & 0.024 & -0.05 \end{pmatrix} \tag{23}$$

$$eig(I_p + \Pi) = (1.00; 0.94; 0.87)$$

D.2 Test for over-identifying restrictions

Conditional on the choice of true rank we estimate the CVAR with restrictions.[5] Restrictions are specified on the form: $H_0 : \beta = H\phi$ with H the restriction matrix and ϕ the

[5] Thanks a lot to Jimmy Reade for supplying the Ox code for the switching algorithm.

	True null (size)	False null (power)
Well-behaved case	0.0568	0.9999
Large-root case	0.1128	0.5790

Table 16: Rejection frequencies ($M = 5000$)

free parameters.

An example of a *true* null hypothesis with one over-identifying restriction,

$$H_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \quad H_2 = \begin{pmatrix} & 1 \\ 0 & -1 \end{pmatrix} \tag{24}$$

where H_1 is the restriction matrix for the first cointegrating relation and similarly for H_2. This test can be used to calculate size of test (type I error); the critical value is $\chi^2_{0.95}(1) = 3.84$.

A example of a *false* null hypothesis (i.e. first two variables are stationary) with two over-identifying restrictions,

$$H_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad H_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \tag{25}$$

This can be used to calculate power of test (one minus type II error); the critical value is $\chi^2_{0.95}(2) = 5.99$.

D.3 Some first simulation results

We conduct simulations using $M = 5000$ replications and a five-per cent significance level yield the results presented in Table 16. Figure 8 through 11 show the distribution of the test statistics in the four combinations of model specification and hypothesis.

Outstanding issues:

- Choice of DGP: how to generate the appropriate set of eigenvalues from choice of α and β?
- Choice of H_1 and H_2: which hypotheses are interesting to consider?
- Would a simple bootstrap be reasonable when DGP close to I(2) border?

[References: has anybody done this before??]

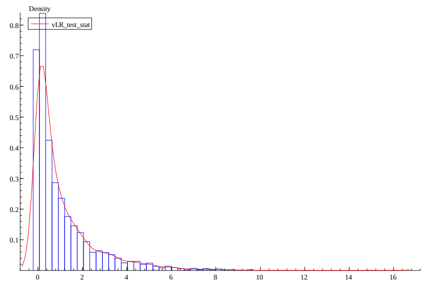

Figure 8: Well-behaved case. True null.

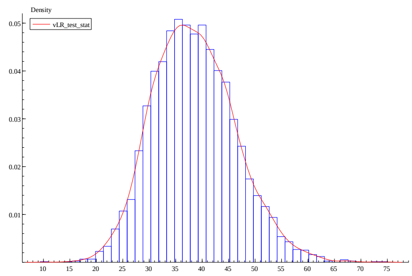

Figure 9: Well-behaved case. False null.

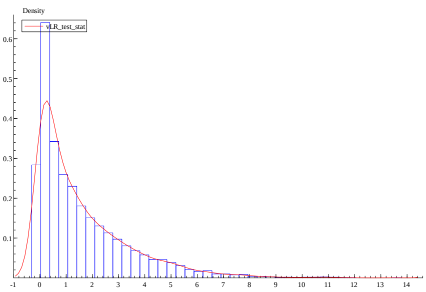

Figure 10: Large-root case. True null.

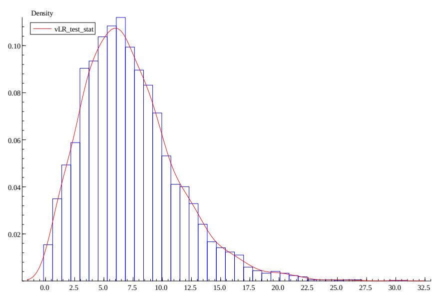

Figure 11: Large-root case. False null.