Offshore Production and Business Cycle Dynamics with Heterogeneous Firms*

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Abstract

Cross-country variation in production costs encourages the relocation of production facilities to other countries, a process known as offshoring through vertical foreign direct investment. I examine the effect of offshoring on the international transmission of business cycles. Unlike the existing macroeconomic literature, I distinguish between fluctuations in the number of offshoring firms (the extensive margin) and in the value added per offshoring firm (the intensive margin) as separate transmission mechanisms. In the model, firms are heterogeneous in labor productivity. They face a sunk entry cost in the domestic market and an additional fixed cost to produce offshore. Offshoring increases with the difference between the domestic and foreign cost of effective labor and with firm-specific productivity. The key results are: (1) The model replicates the procyclical pattern of offshoring and the extensive and intensive margin dynamics that I document using data from Mexico's *maquiladora* sector; (2) Offshoring enhances the comovement of output between the countries involved; and (3) Offshoring reduces price dispersion across countries, because it dampens the real exchange rate appreciation that follows a productivity increase in the parent country (the Harrod-Balassa-Samuelson effect). The results are relevant for the study of macroeconomic interdependence between countries separated by persistent wage differences, such as the U.S. and Mexico or the original and new EU members.

JEL classification: F23, F41

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1 Introduction

Firms often follow strategies that involve the fragmentation of production chains and the establishment of foreign affiliates at locations with relatively lower labor costs, an activity known in the international trade literature as offshoring through vertical foreign direct investment (FDI) (Helpman, 1984).[1] Unlike production under horizontal FDI - which means that foreign affiliates attempt to gain market access by replicating the operations of their parent firms in the country where final consumption takes place - the type of vertically-integrated production that I model is primarily motivated by lower production costs, as foreign affiliates add value to the final goods sold in the multinationals' country of origin or in third countries.[2] The number of offshoring firms (which I refer to as the extensive margin of offshoring) and the real value added per offshoring firm (the intensive margin) fluctuate over the business cycle, and thus affect output, prices and wages in both the parent and the host countries.[3]

This paper contributes to the international macroeconomics literature by analyzing the extensive and intensive margins of offshoring as separate transmission mechanisms of business cycles between the parent and the host country. I model offshoring as an endogenous, firm-level decision that depends on the difference between the domestic and the foreign cost per unit of effective labor, the fixed cost of offshore production, and the trade cost of shipping output back to the parent country. Fluctuations in the number of offshoring firms are linked to domestic firm entry and to the resulting changes in the relative cost of effective labor. Thus, an aggregate productivity increase in the parent country encourages domestic firm entry and causes domestic wages to rise faster than productivity, as labor demand increases to cover entry requirements. In turn, the increase in the domestic cost of effective labor causes more firms to relocate production offshore. The gradual increase in the number of offshoring firms mirrors the gradual appreciation of the cost of effective labor generated by domestic firm entry.

I document a set of stylized facts that characterize the cyclicality of offshoring from U.S. manufacturing to Mexico's *maquiladora* sector.[4] Using the number of maquiladora establishments as an

[1] I attach different meanings to "offshoring" and "outsourcing." The former concerns the integration of multinational firms across borders through vertical or/and horizontal FDI. In contrast, the latter takes place when a firm purchases intermediate inputs or services from an unaffiliated supplier - either at home or abroad - rather than producing it in house. See Helpman (2006) for an extensive discussion of the related literature.

[2] Helpman, Melitz and Yeaple (2004) model horizontal FDI and exports as alternative internationalization strategies for multinational firms; Contessi (2006) analyzes this tradeoff in a dynamic framework.

[3] In this paper I maintain a one-to-one identification between an offshoring firm, a final good variety, and an offshore plant. Under this assumption, offshoring at the extensive margin can also be interpreted as the number of offshoring plants every period; offshoring at the intensive margin can be interpreted as the value added per offshoring plant.

[4] Mexico's *maquiladora* sector consists of manufacturing plants that import intermediate goods, process them, and export the resulting output (Gruben, 2001). The sector accounts for 20 percent of the Mexico's manufacturing value

empirical proxy for the extensive margin, I find that the value added offshore is procyclical with U.S. manufacturing output, and also that the extensive margin varies notably over the business cycle (Figure 6).[5] I also show that expansions in U.S. output precede increases in the number of maquiladora establishments by at least three quarters, a result that highlights the inter-temporal link between U.S. manufacturing and the extensive margin of offshoring. Bergin, Feenstra, and Hanson (2008) also analyze the extent to which fluctuations in the extensive margin of offshoring account for variations in Mexico's maquiladora employment. They show that more than one third of the adjustment in industry-level employment and nearly one half of the adjustment in maquiladora's total employment occur at the extensive margin (i.e. through variation in the number of establishments over time).[6]

Despite the empirical evidence, the theoretical macroeconomic literature addressing the business cycle implications of offshore production does not fully capture the inter-temporal dynamics of offshoring at the extensive and the intensive margins. For instance, Burstein, Kurz, and Tesar (2008) examine the role of production sharing in the transmission of business cycles in a two-country model in which the location of plants is fixed over time.[7] Bergin, Feenstra, and Hanson (2007) also focus on the importance of offshore production in amplifying the transmission of shocks across countries, but they do so with a model in which the intensive margin does not react, and the number of offshoring firms makes an abrupt shift rather than a gradual adjustment over time (as in the data) in response to aggregate shocks.

I address these deficiencies by incorporating the endogenous determination of the number of offshoring firms in a two-country (North and South), dynamic stochastic general equilibrium (DSGE) model with firm entry and firm heterogeneity, along the lines of Melitz (2003) and Ghironi and Melitz (2005). Firm entry is subject to a sunk cost reflecting the regulation associated with starting a business in the country of origin. Following entry, each firm can use either domestic or foreign inputs in the production of a different variety of final goods. The use of foreign inputs involves the establishment

added (INEGI), nearly 50 percent of the Mexico's exports, and approximately 25 percent of Mexico's employment (Bergin, Feenstra, Hanson, 2007, 2008).

[5] Although not entirely owned by U.S. multinationals, the maquiladora plants import most of their inputs (82 percent) and send most of their gross output (90 percent) from/to the U.S. (Hausman and Kaytko, 2003; Burstein, Kurtz and Tesar, 2008). For this reason, I use the number of maquiladora establishments as an empirical proxy for the extensive margin of offshoring.

[6] Recent empirical literature highlights the role of the extensive margin in international trade and the presence of fixed exporting costs: Baldwin and Harrigan (2007) show that the number of traded goods (the extensive margin) decreases with distance and increases with the size of the importing country. Besedes and Prusa (2006) find that the survival rate of exports for differentiated good varieties increases with the initial transaction size and also with the length of the relationship. Hummels and Klenow (2005) show that larger economies have larger exports, and that the extensive margin accounts for as much as 60 percent of this difference.

[7] In Burstein, Kurtz and Tesar (2008), the low substitutability between the domestic and foreign varieties entering an Armington composite enhances the cross-country co-movement of output.

of an offshore production plant and is subject to a fixed offshoring cost per-period. Also, offshoring involves the so-called iceberg trade costs that reflect transportation, insurance, and trade barriers, costs incurred in the shipping of final good varieties produced offshore back to the country of origin. Thus, when deciding on where to locate production (domestically vs. offshore), each firm balances the lower foreign costs of effective labor against the fixed and trade costs associated with offshore production.[8] Since firms are heterogeneous in productivity, the decision to produce offshore is firm-specific: Only the more productive firms can afford the fixed costs of offshoring, and their number varies over time. The model also implies that the relocation of production offshore takes place one-way: Since the cost of effective labor is relatively lower in the South, only Northern firms have the incentive to relocate production offshore. All Southern firms produce domestically.[9] The implications of the model are consistent with the empirical evidence provided by recent studies on the determinants of vertical production networks. For instance, Hanson, Mataloni, and Slaughter (2005) show that U.S. multinational firms import more intermediate inputs when their foreign affiliates benefit from lower wages and lower trade costs in the host economy. Kurz (2006) shows that U.S. firms choosing to offshore are *ex-ante* larger and more productive than their domestic counterparts, as their higher idiosyncratic productivity levels allow them to cover the fixed costs of offshoring.

The key results of the paper are as follows. First, the model generates a procyclical pattern of offshoring that is consistent with the stylized facts from Mexico's maquiladora industry. In particular, following an economic expansion in the parent country, the value added per offshoring firm spikes on impact. However, domestic firm entry leads to an appreciation of the terms of labor and generates a gradual increase in the number of offshoring firms over time.[10] Second, offshoring enhances the comovement of output relative to the benchmark model with exports developed in Ghironi and Melitz (2005). As firm entry places upward pressure on the domestic effective wage and causes more firms to relocate production offshore, higher demand for domestic labor (due to firm entry) and sequentially higher demand for offshore labor (due to the relocation of production) enhance the comovement of wages and aggregate incomes.[11] The result is consistent with the empirical regularity documented by

[8] I define the cost of effective labor as the ratio between the real wage and aggregate productivity (w_t/Z_t in the North and w_t^*/Z_t^* in the South).

[9] I derive an asymmetric steady state in which differences in the regulation of firm entry in the country of origin are translated in differences in real effective wages across countries. In the model, I set firm entry costs to be higher in the South; in turn, since the more regulated economy attracts a smaller number of firms, labor demand and the cost of effective labor are lower in the South.

[10] The terms of labor is defined as the ratio between the Southern and Northern real cost of effective labor expressed in units of the same consumption basket, $TOL_t = \frac{Q_t w_t^*/Z_t^*}{w_t/Z_t}$, as in Ghironi and Melitz (2005). An increase in the cost of effective labor in the North would cause the terms of labor to appreciate (i.e. TOL decreases).

[11] In contrast, in the traditional IRBC literature, a domestic increase in aggregate productivity leads to increased

Burstein, Kurz, and Tesar (2007) that countries with higher production-sharing trade links tend to exhibit higher correlations of manufacturing output. Third, offshoring narrows the price dispersion across countries, as it dampens the appreciation of the real exchange rate that follows an increase in aggregate productivity in the parent country (the Harrod-Balassa-Samuelson effect). This result is driven by several channels, including the upward pressure on the foreign wage, the decrease in the size of the domestic non-traded sector, the decline in import prices and in the availability of foreign varieties as offshoring crowds out the less productive foreign exporters. Fourth, offshoring enhances the procyclicality of investment and firm entry in the parent country relative to the benchmark model with exports only, as the lower-cost alternative of offshoring increases the profitability of domestic firms. In turn, the employment loss caused by offshoring is partially offset by the employment gain generated by greater domestic firm entry.

This paper is related to a growing body of macroeconomic literature that focuses on endogenous firm entry and adjustments along the extensive margin of exports (but not of offshoring). For example, Ghironi and Melitz (2005) study the export decision of firms in the presence of fixed exporting costs, in a framework with firm entry and firm heterogeneity. Alessandria and Choi (2007) analyze the extensive margin of exports in a model with sunk and continuation fixed costs that explains the "exporter hysteresis" behavior.[12] Corsetti, Martin, and Pesenti (2007) examine the terms-of-trade implications of productivity improvements affecting the entry of firms and the production sector, in a model in which the extensive margin of exports is endogenous. And Mejean (2006) emphasizes the implications of endogenous firm entry in the tradable sector for the real exchange rate dynamics and the Harrod-Balassa-Samuelson effect.

The study of macroeconomic implications of offshoring through vertical FDI is particularly relevant for pairs of countries and for economic areas that are separated by persistent differences in the cost of effective labor. For instance, offshoring through vertical FDI has been important for the U.S. multinational firms acting within the NAFTA region, and also within Central and South America: As much as 50 percent of the manufacturing sales of U.S. affiliates in Mexico and 26 percent in Latin America as a whole were directed towards their U.S. parent firms in 2005 (as opposed to only 3 percent for the U.S. affiliates in Europe and 5 percent in the Asia-Pacific region; BEA, 2007). A similar pattern exists between Western Europe and the new member countries of the European Union (Marin, 2006; Meyer, 2006).

production at home but not offshore, such as in Backus, Kehoe, Kydland (1992).

[12] "Exporter hysteresis" refers to the behavior of firms that continue to serve the foreign market even after a real exchange rate appreciation reduces their export competitiveness.

The rest of this paper is organized as follows: Section 2 introduces a DSGE model of offshoring that allows for fluctuations in offshoring at both the extensive and the intensive margins; Section 3 defines the average productivity levels of the firms producing domestically and offshore; Section 4 discusses the model calibration; Section 5 presents the results, including the macroeconomic dynamics in the presence of aggregate productivity shocks, as well as a comparison of the empirical moments of offshoring to Mexico and their model counterparts; Section 6 concludes with a summary and proposed extensions of the model.

2 Model of Offshoring with Heterogeneous Firms

2.1 Model Setup: Markets and Production Strategies

This section summarizes the two-stage model of firm entry and offshore production, illustrated in Figure 1. In the first stage, an unbounded pool of potential entrant firms face a trade-off between the sunk entry cost (reflecting the cost of starting a business in the firms' country of origin), the expected stream of future monopolistic profits, and the probability of exit very period, as in Ghironi and Melitz (2005, henceforth GM2005). Only after paying the sunk entry cost, each firm is assigned an idiosyncratic labor productivity factor that is drawn independently from a common distribution over a support interval, and that the firm keeps for the entire duration of its life.

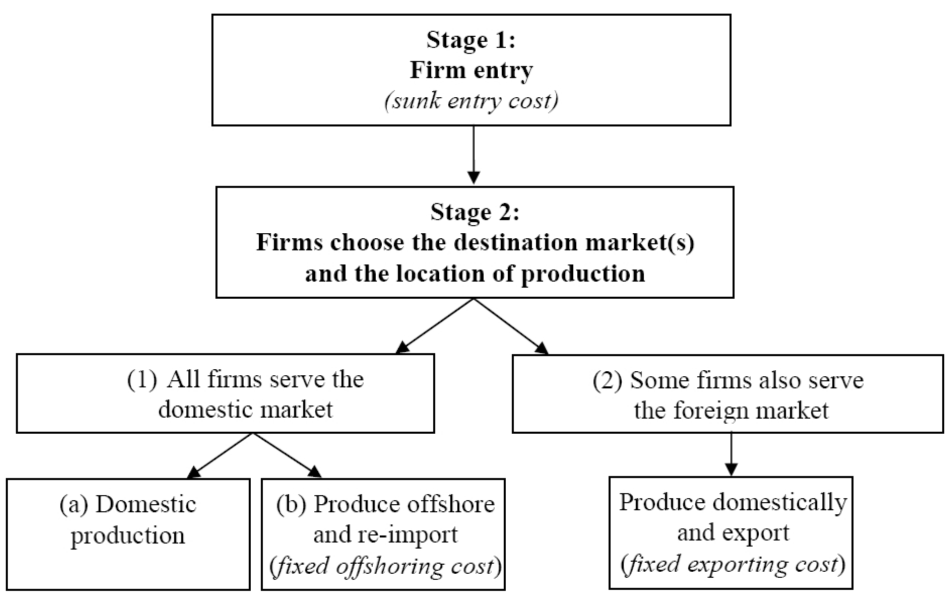

Figure 1. Destination markets and production strategies of firms

In the second stage, post-entry firms are monopolistically competitive and heterogeneous in labor productivity. Every period after entry, firms choose the destination market(s) that they serve as well as the location of production, as follows: (1) Firms serving their domestic market can use either domestic or foreign inputs in production. The use of foreign labor involves the establishment of offshore production plants (offshoring through vertical FDI). It offers the advantage of a lower production cost, but is subject to a per-period fixed offshoring cost, and to an iceberg trade cost that affects the final goods shipped back to the country of origin for final consumption. (2) Some of the firms can also serve the foreign market. To this end, they use domestic labor as the only input in the production of final goods which they export subject to a per-period fixed exporting cost, as in GM2005. Thus, I nest the model of GM2005 (exports only, no offshoring) as a special case in my model with offshoring.[13]

The following sections describe in detail the model with firm entry and offshore production, starting with the latter (sections 2.2) and ending with the former (section 2.5).

2.2 Firms Serving the Domestic Market: Domestic Production vs. Offshoring

This section describes the mechanisms of domestic and offshore production for the Northern firms. It does not concern the Southern firms, as offshoring takes place one-way, from the Northern economy to the low-wage South.

In the North, a continuum of monopolistically-competitive firms produce final goods for the domestic market. Firms are heterogeneous in productivity, with each firm producing a different variety of final goods.[14] Since each firm produces one variety, the firm-specific labor productivity z also serves as an index for the existing varieties of final goods. Every period, firms producing for the domestic market can choose one of two possible locations of production:

[13] I abstract from the possibility of offshoring through horizontal FDI: As an alternative to exports, firms serving the foreign market may produce abroad using the local labor of the country whose market they target, as in Contessi (2006), a strategy which I refer to as offshoring through horizontal FDI. The difference between offshore production under vertical and horizontal FDI is that, under vertical FDI, firms serving their domestic market shift part of the production chain offshore in order to take advantage of the relatively lower cost of effective labor. This reflects the relocation of downstream production activities offshore (e.g. manufacturing, assembly and packaging) by the vertically-integrated firms that continue to perform the upstream operations (e.g. research, marketing and sales) in the country of origin. In contrast to vertical FDI, production under horizontal FDI is motivated by improved access to the foreign market (i.e. obtained by avoiding the trade cost), and involves the simultaneous production of the same final good both at home (i.e. to be sold in the home market) and offshore (i.e. to be sold in the host market).

[14] The model implies that firms with productivity below the endogenous productivity cutoff $z_{V,t}$ (i.e. the less productive firms) produce their varieties of final goods domestically, whereas those above the cutoff produce offshore.

(1) Domestic production: The Northern firm with idiosyncratic labor productivity z employs labor l_t to obtain an amount of final goods:

$$y_{D,t}(z) = Z_t z l_t, \tag{1}$$

where Z_t is the aggregate productivity of labor in the North and z is the firm-specific labor productivity;

(2) Offshore production: Alternatively, the firm with idiosyncratic labor productivity z may choose to relocate production offshore using Southern labor l_t^* as the only input in production:

$$y_{V,t}(z) = Z_t^* z l_t^*. \tag{2}$$

Thus, I assume that while the offshoring firm becomes subject to the Southern aggregate labor productivity Z^*, it is able to carry its own idiosyncratic labor productivity z to the Southern economy.[15] Given the demand for final good varieties produced domestically, $y_{D,t}(z) = \rho_{D,t}(z)^{-\theta} C_t$, and also the demand for varieties produced offshore by the vertically-integrated firms, $y_{V,t}(z) = \rho_{V,t}(z)^{-\theta} C_t$,[16] the monopolistically-competitive firms solve their profit-maximization problem:

$$\max_{\{\rho_{D,t}(z)\}} d_D(z) = \rho_{D,t}(z) y_{D,t}(z) - \frac{w_t}{Z_t z} y_{D,t}(z), \tag{3}$$

$$\max_{\{\rho_{V,t}(z)\}} d_V(z) = \rho_{V,t}(z) y_{V,t}(z) - \tau \frac{w_t^* Q_t}{Z_t^* z} y_{V,t}(z) - f_V \frac{w_t^* Q_t}{Z_t^*}. \tag{4}$$

The cost of producing one unit of output either domestically or offshore varies not only with the cost of effective labor $\frac{w_t}{Z_t}$ and $\frac{w_t^* Q_t}{Z_t^*}$ across countries, but also with the level of idiosyncratic labor productivity

[15] Strategies (1) and (2) are extreme cases of a broader framework of offshoring, which would allow the offshoring firm with idiosyncratic labor productivity z to use a mix of Northern and Southern labor, l_t and l_t^*. Following the specification in Antras and Helpman (2004), I write output of every variety of final goods z as a Cobb-Douglas function of the domestic and foreign inputs:

$$y_{V,t}(z) = \left[\frac{Z_t z l_t}{\alpha}\right]^{\alpha} \left[\frac{Z_t^* z l_t^*}{1-\alpha}\right]^{1-\alpha}.$$

In this paper, I explore the economic implications of offshoring of two extreme scenarios under different calibrations of parameter α: At one extreme, setting $\alpha = 1$ shuts down offshoring under vertical FDI, as firms use exclusively domestic inputs in production. At the other extreme, setting $\alpha = 0$ implies that offshoring firms use exclusively foreign inputs in the production of final goods. The smaller α is, the more intensive production is in foreign inputs, i.e. the larger is the range of operations that the multinational firm locates offshore (e.g. manufacturing, assembly, packaging, customer service). For the two extreme cases, I use the l'Hôpital rule to obtain: $\underset{\alpha\to 0}{lim} \left(\frac{1}{\alpha}\right)^{\alpha} = \underset{\beta\to\infty}{lim} \beta^{1/\beta} = \underset{\beta\to\infty}{lim} e^{(1/\beta)\ln\beta} = e^{\underset{\beta\to\infty}{lim}\left(\frac{\ln\beta}{\beta}\right)} \overset{l\prime H\hat{o}pital}{=} e^{\underset{\beta\to\infty}{lim}(1/\beta)} = e^0 = 1.$

[16] I provide their derivation in the Appendix.

z across firms.[17] I define the real exchange rate $Q_t = \frac{P_t^* \varepsilon_t}{P_t}$ as the ratio between the price indexes in the South and North expressed in the same currency, where ε_t is the nominal exchange rate. In addition to the marginal cost, the Northern firms producing offshore incur a period-by-period fixed offshoring cost equal to f_V units of Southern effective labor, a cost that reflects the building and maintenance of the offshore production facility.[18] They also face an iceberg trade cost ($\tau > 1$) associated with the shipping of goods produced offshore back to the parent country: For every τ units produced offshore, only one unit reaches the Northern consumers, as the difference is lost due to costs associated with trade barriers, transportation, insurance, language barriers, and differences in the legal systems, as discussed in Anderson and Wincoop (2004).

The profit-maximization problem under monopolistic competition implies the following equilibrium prices per unit of output produced domestically and offshore, respectively:[19]

$$\rho_{D,t}(z) = \frac{\theta}{\theta - 1} \frac{w_t}{Z_t z} \tag{5}$$

$$\rho_{V,t}(z) = \frac{\theta}{\theta - 1} \tau \frac{w_t^* Q_t}{Z_t^* z}. \tag{6}$$

The resulting profits from domestic and offshore production, both expressed in units of the Northern consumption basket C_t, are:[20]

$$d_{D,t}(z) = \frac{1}{\theta} \rho_{D,t}(z)^{1-\theta} C_t, \tag{7}$$

$$d_{V,t}(z) = \frac{1}{\theta} \rho_{V,t}(z)^{1-\theta} C_t - f_V \frac{w_t^* Q_t}{Z_t^*}. \tag{8}$$

To summarize the above, the profits associated with domestic and offshore production depend on the cost of effective labor in the North and South, the fixed offshoring cost, the iceberg trade cost, as well as the firm-specific labor productivity. Firms producing offshore benefit from the relatively lower cost of effective labor, but their profits decline with the per-period fixed offshoring cost, and also with the iceberg trade cost. Thus, when deciding upon the location of production every period, the firm with productivity z compares the profit $d_{D,t}(z)$ it would obtain from domestic production with the

[17] Given the domestic and offshore real wages, w_t and w_t^* respectively, the marginal cost of producing one unit of variety z domestically is $\frac{w_t}{Z_t z}$, and the marginal cost of producing it offshore is $\frac{w_t^* Q_t}{Z_t^* z}$. The real wage $w_t = W_t / P_t$ in North is expressed in units of the domestic consumption basket; the offshore real wage $w_t^* = W_t^* / P_t^*$ is expressed in units of the consumption basket in South.

[18] The mix of f_V units of Southern effective labor is equivalent to $f_V w_t^* / Z_t^*$ units of the Southern consumption basket.

[19] The derivations are provided in the Appendix.

[20] Using the demand $y_D(z) = \rho_D(z)^{-\theta} C$ and the price $\rho_D(z) = \frac{\theta}{\theta-1} \frac{w}{Zz}$ for domestic production, the corresponding profit is $d_D(z) = \frac{1}{\theta} \rho_D(z) y_D(z)$. The profit from offshore production is analogous.

profit $d_{V,t}(z)$ it would obtain from producing the same variety offshore.

The model implies that only the relatively more productive Northern firms find it profitable to locate production offshore every period. Despite the lower cost of effective labor in South relative to North, only firms with idiosyncratic productivity above a certain cutoff ($z > z_{V,t}$) obtain profits that are large enough to cover the fixed offshoring cost and the iceberg trade cost. This implication of the model is consistent with the empirical evidence provided by Kurz (2006) that firms choosing to produce offshore are *ex-ante* larger and more productive than their domestic counterparts, as the larger idiosyncratic productivity levels allow them to cover the fixed costs of offshoring.[21]

As a particular case, the firm with labor productivity equal to the cutoff $z_{V,t}$ is indifferent between producing domestically or offshore. After accounting for the fixed offshoring cost and the iceberg trade cost, the firm at the cutoff obtains equal profits from domestic and offshore production, a property which I use to solve for the endogenous productivity cutoff $z_{V,t}$ that governs the location decision of production:

$$z_{V,t} = \{z \mid d_{D,t}(z_{V,t}) = d_{V,t}(z_{V,t})\}. \tag{9}$$

Existence of the equilibrium productivity cutoff Next I show that the existence of the equilibrium productivity cutoff $z_{V,t}$ requires a cross-country asymmetry in the cost of effective labor, so that some of the Northern firms will always maintain an incentive to produce offshore.

I begin by re-writing the expressions for profits obtained from domestic and offshore production as $d_{D,t}(z) = B_t \left(\frac{w_t}{Z_t}\right)^{1-\theta} z^{\theta-1}$ and $d_{V,t}(z) = B_t \left(\tau \frac{w_t^* Q_t}{Z_t^*}\right)^{1-\theta} z^{\theta-1}$, respectively, where $B_t \equiv \frac{1}{\theta}\left(\frac{\theta}{1-\theta}\right)^{1-\theta} C_t$ is a measure of the market size in the North. In Figure 2, I plot the corresponding profits as functions of the idiosyncratic productivity parameter $z^{\theta-1}$ over the support interval $[z_{min}, \infty)$. The vertical intercepts represent the annualized value of the sunk entry cost for the case of domestic production ($-\Theta f_E \frac{w_t}{Z_t}$), and the annualized value of the sunk entry cost plus the period-by-period fixed cost for the case of offshore production ($-\Theta f_E \frac{w_t}{Z_t} - f_V \frac{w_t^* Q_t}{Z_t^*}$), where parameter $\Theta \equiv \frac{1-\beta(1-\delta)}{\beta(1-\delta)}$.

The existence of equilibrium productivity cutoff $z_{V,t}$ requires that the following condition holds every period: The profit function for offshoring must be steeper than that for domestic production, slope $\{d_{V,t}(z)\} >$ slope $\{d_{D,t}(z)\}$. When the condition is met, offshoring generates greater profits than domestic production for the Northern firms with idiosyncratic productivity z along the upper range

[21] A useful implication of firm heterogeneity along the lines of Melitz (2003) is that the more productive firms have larger output and revenue. Given two firms with idyodyncratic productivity $z_2 > z_1$, the ratios of output and profits are $\frac{y(z_2)}{y(z_1)} = \left(\frac{z_2}{z_1}\right)^{\theta} > 1$, $\frac{r(z_2)}{r(z_1)} = \left(\frac{z_2}{z_1}\right)^{\theta-1} > 1$. Empirical studies show that firms using imported inputs in production not only are more productive, but also are larger and employ more workers (Kurz, 2006; Yasar and Morrison Paul, 2006).

of the support interval. The slope inequality is equivalent to:

$$\tau \frac{w_t^* Q_t / Z_t^*}{w_t / Z_t} < 1, \tag{10}$$

which implies that the effective wage in the South must be sufficiently lower than that in the North, so that the difference covers the iceberg trade cost ($\tau > 1$) and thus provides an incentive for some of the Northern firms to produce offshore every period.[22]

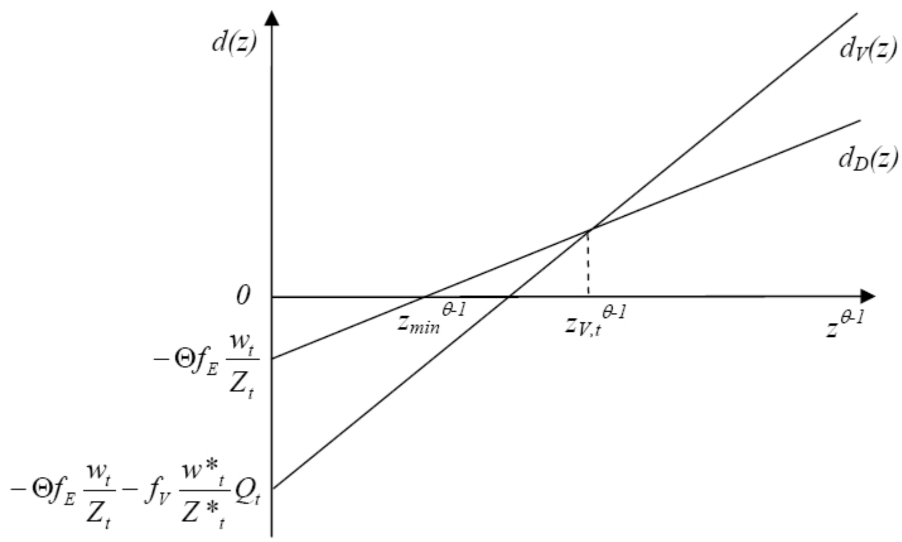

Figure 2. Existence of equilibrium productivity cutoff z_V, t.

2.3 Firms Serving the Foreign Market: Exports

In this section I describe the problem of the exporting firms originating in the North. The equations for the South are similar unless indicated otherwise. Variables for the Southern economy are marked with the (*) superscript.

Firms in each economy have the option to serve the foreign market through exports, as in GM2005. Thus, the Northern exporting firm with idiosyncratic productivity z uses an amount of domestic labor l_t in the production of final good varieties $y_{H,t}(z)$ exported the Southern market:[23]

[22] See Appendix 9. A second condion, necessary to avoid the corner solution when all firms would produce offshore, is that the productivity cutoff must be larger than the lower bound of the support interval: $z_{V,t} > z_{\min}$. This condition is equivalent to $d_{D,t}(z_{\min}) < \Theta f_E \frac{w_t}{Z_t} + f_V \frac{w_t^* Q_t}{Z_t^*}$,which shows that the firm with idiosyncratic productivity equal to the lower-bound level z_{min} would obtain zero profits from domestic production and negative profits from offshore production.

[23] I view exporting as a special case within a broader framework which allows firms to serve the foreign market using a

$$y_{H,t}(z) = Z_t z l_t. \tag{11}$$

The profit maximization problem of the Northern exporting firms generates the following price and profit functions:

$$\rho_{H,t}(z) = \frac{\theta}{\theta - 1}\tau^* \frac{w_t Q_t^{-1}}{Z_t z}, \tag{12}$$

$$d_{H,t}(z) = \frac{1}{\theta}\rho_{H,t}(z)^{1-\theta} C_t^* Q_t - f_H \frac{w_t}{Z_t}. \tag{13}$$

In particular, serving the foreign market involves a period-by-period fixed exporting cost equal to f_H units of Northern effective labor as well as the iceberg trade cost τ^*. The model implies that every period t, the Northern firms with idiosyncratic labor productivity above a certain cutoff ($z > z_{H,t}$) find it profitable to export to the Southern market at the same time with serving their domestic market (North). They obtain profits that are large enough to cover both the fixed cost and the iceberg trade cost of exporting.

In the presence of firm heterogeneity, the option to export is endogenous to the model. As in GM2005, the firm with the idiosyncratic labor productivity equal to the cutoff obtains zero profits from exporting. Thus, the time-variant productivity cutoff $z_{H,t}$ is obtained as:

$$z_{H,t} = \inf \left\{ z \mid d_{H,t}(z_{V,t}) > 0 \right\}. \tag{14}$$

mix of domestic and foreign inputs in production. Production is described by the following Cobb-Douglas specification:

$$y_{H,t}(z) = \left[\frac{Z_t z l_t}{\eta}\right]^{\eta} \left[\frac{Z_t^* z l_t^*}{1-\eta}\right]^{1-\eta},$$

where a larger η accounts for a smaller content of foreign inputs contributed by the offshore affiliates towards the production of final goods sold in the Southern market. Thus, In addition to nesting the model with endogenous exports as in GM2005 (which is focus of this paper, under the calibration $\eta = 1$), my model with offshoring can also nest the case in which firms serving the Southern market produce exclusively through their foreign affiliates as in Contessi (2006) (which I revisit by setting $\eta = 0$). Production in South through horizontal FDI allows firms to avoid the trade cost τ^* by using local inputs. Using the l'Hôpital rule, $\underset{\eta\to 0}{lim}\left(\frac{1}{\eta}\right)^{\eta} = \underset{\beta\to\infty}{lim}\,\beta^{1/\beta} = \underset{\beta\to\infty}{lim}\,e^{(1/\beta)\ln\beta} = e^{\underset{\beta\to\infty}{lim}\left(\frac{\ln\beta}{\beta}\right)} \overset{l\prime H\hat{o}pital}{=} e^{\underset{\beta\to\infty}{lim}(1/\beta)} = e^0 = 1$. The corresponding price and profit functions are $\rho_{H,t}(z) = \frac{\theta}{\theta-1}\left(\tau^* \frac{w_t Q_t^{-1}}{Z_t z}\right)^{\eta}\left(\frac{w_t^*}{Z_t^* z}\right)^{1-\eta}$ and $d_{H,t}(z) = \frac{1}{\theta}\rho_{H,t}(z)^{1-\theta} C_t^* Q_t - f_H\left(\frac{w_t}{Z_t}\right)^{\eta}\left(\frac{w_t^* Q_t}{Z_t^*}\right)^{1-\eta}$.

2.4 Households

Financial autarky Households in each country maximize the expected lifetime utility (as a function of consumption) and provide labor inelastically:

$$\max_{\{B_{t+1},\, x_{t+1}\}} \left[E_t \sum_{s=t}^{\infty} \beta^{s-t} \frac{C_s^{1-\gamma}}{1-\gamma} \right], \tag{15}$$

subject to the budget constraint:

$$(\widetilde{v}_t + \widetilde{d}_t) N_t x_t + (1 + r_t) B_t + w_t L \geqslant \widetilde{v}_t \left(N_t + N_{E,t} \right) x_{t+1} + B_{t+1} + C_t, \tag{16}$$

where $\beta \in (0, 1)$ is the subjective discount factor, C_t is the consumption basket, and $\gamma > 0$ is the inverse of the inter-temporal elasticity of substitution.

The Northern household starts every period t with mutual fund share holdings x_t (whose market value is $\widetilde{v}_t N_t$) and real bond holdings B_t. It receives dividend income $\widetilde{d}_t N_t$ on the mutual fund stocks (equal to the profit of the average firm times the number of producing firms) in proportion with its stock holdings x_t, interest $r_t B_t$ on bond holdings, and also labor income equal to the real wage w_t for the amount of labor $L = 1$ that it supplies inelastically. The Northern household purchases two types of assets every period. First, it purchases x_{t+1} shares in a mutual fund of Northern firms that includes: (i) N_t firms already producing at time t, either domestically or offshore, and (ii) $N_{E,t}$ new firms that enter the domestic market in period t. Each share is worth its market value $\widetilde{v}_t$, equal to the net present value of the expected stream of future profits of the average firm. Second, the household buys the risk-free bond B_{t+1} denominated in units of the Northern consumption basket. (Bond holdings play a role in the extended model with international trade in bonds, which I present in the Appendix.)[24]

In addition, households purchase the consumption basket C_t, which includes varieties of final goods produced by Northern firms ($\omega \in \Omega_t^{NN}$) either domestically or offshore; it also includes the imports of final good varieties produced by the Southern firms ($\omega \in \Omega_t^{SN}$):

$$C_t = \left[\underbrace{\int_{z_{\min}}^{z_{V,t}} y_{D,t}(\omega)^{\frac{\theta-1}{\theta}} d\omega}_{\text{Domestic production}} + \underbrace{\int_{z_{V,t}}^{\infty} y_{V,t}(\omega)^{\frac{\theta-1}{\theta}} d\omega}_{\text{Offshore production}} + \underbrace{\int_{z_{H,t}^*}^{\infty} y_{H,t}^*(\omega)^{\frac{\theta-1}{\theta}} d\omega}_{\Omega_t^{SN}} \right]^{\frac{\theta}{\theta-1}}, \tag{17}$$

[24] In the model with complete financial autarky (i.e. stocks in the mutual fund and bonds are not traded across countries), the equilibrium conditions for stock and bond holdings are $x_t = x_{t+1} = 1$ and $B_t = B_{t+1} = 0$.

where $\theta > 1$ is the symmetric elasticity of substitution across final good varieties. I use the home consumption basket C_t as the numeraire good; I define the real price of variety z in units of the Northern consumption basket as $\rho_t(z) = p_t(z)/P_t$. Thus, the the consumption-based price index in the North is:[25]

$$1 = \left[\int \rho_t(\omega)^{1-\theta} d\omega\right]^{\frac{1}{1-\theta}}, \ \omega \in \Omega_t^{NN} \cup \Omega_t^{SN}. \tag{18}$$

The first-order conditions generate the Euler equations for bonds and stocks:

$$C_t^{-\gamma} = \beta\left(1 + r_{t+1}\right) E_t\left[C_{t+1}^{-\gamma}\right], \tag{19}$$

$$\widetilde{v}_t = \beta(1-\delta) E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\left(\widetilde{d}_{t+1} + \widetilde{v}_{t+1}\right)\right]. \tag{20}$$

2.5 Firm Entry and Exit

Following GM2005, firm entry takes place every period. It requires a sunk entry cost (reflecting the cost of starting a business in the firms' country of origin) equal to f_E units of Northern effective labor.[26] Potential entrants become aware of their idiosyncratic labor productivity z only after entering the market. After paying the sunk entry cost, each firm is randomly assigned an idiosyncratic labor productivity z which is drawn independently from a common distribution $G(z)$ with support over the interval $[z_{min}, \infty)$, and which the firm keeps for the entire duration of its life.

The potential entrant firms are forward looking and correctly anticipate their expected post-entry value $\widetilde{v}_t$, which is given by the expected stream of future profits $\widetilde{d}_t$ and by the exogenous probability δ with which they receive an exit-inducing shock every period. The forward iteration of the Euler equation for stocks (20) generates the expression for the expected post-entry value of potential entrants:

$$\widetilde{v}_t = E_t\left\{\sum_{s=t+1}^{\infty} [\beta(1-\delta)]^{s-t}\left(\frac{C_s}{C_t}\right)^{-\gamma} \widetilde{d}_s\right\}. \tag{21}$$

In equilibrium, firm entry takes place until the value of the average firm $\widetilde{v}_t$ equals the sunk entry cost $f_E \frac{w_t}{Z_t}$, both expressed in units of the Northern consumption basket:

$$\widetilde{v}_t = f_E \frac{w_t}{Z_t}. \tag{22}$$

[25] If $p_t(z)$ denotes the price of each variety z, the price index of the home consumption basket is $P_t = \left[\int p_t(\omega)^{1-\theta} d\omega\right]^{\frac{1}{1-\theta}}$ for $\omega \in \Omega_t^{NN} \cup \Omega_t^{SN}$. The demand for each variety of final goods z is $y_t(z) = \left(p_t(z)/P_t\right)^{-\theta} C_t$.

[26] Or $f_E w_t / Z_t$ units of the consumption basket in the North.

The $N_{E,t}$ firms entering at time t do not produce until period $t+1$. Irrespective of their idiosyncratic productivity, all firms - including the new entrants - are subject to a random exit shock that occurs with probability δ at the end of every period after production has taken place. Thus, the law of motion for the number of producing firms is:

$$N_{t+1} = (1-\delta)(N_t + N_{E,t}), \tag{23}$$

where $N_t = N_{t,D} + N_{t,V}$ consists of firms producing either domestically or offshore every period.

3 Solving the Model with Firm Heterogeneity

A necessary step in solving the model with firm heterogeneity is to derive analytical solutions for the average productivity, prices and profits of the representative Northern firms that produce domestically and offshore every period. This section also provides the expressions for aggregate accounting and the balance of international payments that close the model with offshoring.

3.1 Average Firm Productivity Levels

Firms serving the domestic market I define two average labor productivity levels: (1) the average productivity $\widetilde{z}_{D,t}$ of the Northern firms producing domestically, and (2) the average productivity $\widetilde{z}_{V,t}$ of the Northern firms producing offshore. I illustrate them in Figure 3, in which I plot the density of the firm-specific labor productivity levels z over the support interval $[z_{min}, \infty)$. Every period t, there are $N_{D,t}$ of the relatively less productive Northern firms ($z < z_{V,t}$) that choose to produce domestically; their average productivity is $\widetilde{z}_{D,t}$. The remaining $N_{V,t}$ of the relatively more productive Northern firms ($z > z_{V,t}$) choose to produce offshore;[27] their average productivity is $\widetilde{z}_{V,t}$.[28]

Since the firm-specific labor productivities z are random draws from a common distribution $G(z)$ with density $g(z)$, I write the average idiosyncratic productivities of the Northern firms producing domestically and offshore as:

$$\widetilde{z}_{D,t} = \left[\frac{1}{G(z_{V,t})}\int_{z_{\min}}^{z_{V,t}} z^{\theta-1}g(z)dz\right]^{\frac{1}{\theta-1}} \quad \text{and} \quad \widetilde{z}_{V,t} = \left[\frac{1}{1-G(z_{V,t})}\int_{z_{V,t}}^{\infty} z^{\theta-1}g(z)dz\right]^{\frac{1}{\theta-1}}. \tag{24}$$

[27] The total number of Northern firms is $N_t = N_{V,t} + N_{D,t}$.

[28] The difference between $\widetilde{z}_{V,t}$ and $z_{V,t}$ is that the former is the average productivity of offshoring firms, whereas the latter is the cutoff productivity above which firms produce offshore.

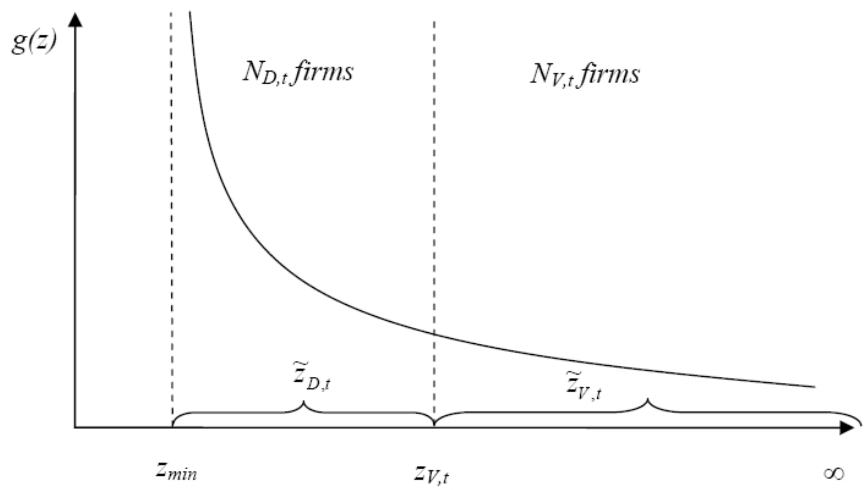

Figure 3. Average labor productivities for firms serving the domestic market through domestic $(\widetilde{z}_D, t)$ and offshore $(\widetilde{z}_V, t)$ production

Pareto-distributed firm productivity Following Melitz (2003) and GM2005, I assume that the firm-specific labor productivity draws z are Pareto-distributed, with p.d.f. $g(z) = kz_{min}/z^{k+1}$ and c.d.f. $G(z) = 1 - (z_{min}/z)^k$ over the support interval $[z_{min}, \infty)$. Using this assumption, I derive analytical solutions for the average productivities of the two representative Northern firms producing domestically and offshore as functions of the time-variant productivity cutoff $z_{V,t}$:[29]

$$\widetilde{z}_{D,t} = \nu z_{\min} z_{V,t} \left[\frac{z_{V,t}^{k-(\theta-1)} - z_{\min}^{k-(\theta-1)}}{z_{V,t}^{k} - z_{\min}^{k}} \right]^{\frac{1}{\theta-1}} \quad \text{and} \quad \widetilde{z}_{V,t} = \nu z_{V,t}, \tag{25}$$

where the productivity cutoff[30] is $z_{V,t} = z_{min}(N_t/N_{V,t})^{(1/k)}$, and the parameters are $\nu \equiv \left[\frac{k}{k-(\theta-1)}\right]^{\frac{1}{\theta-1}}$ and $k > \theta - 1$.[31] Since offshoring takes place one-way, from North to South, the Southern firms serving their domestic market (exclusively through domestic production) have a constant average productivity $\widetilde{z}_D^* = \nu z_{\min}^*$.

[29] I provide their derivation in the Appendix.

[30] The shares of Northern firms producing domestically and offshore, respectively, are $N_{D,t}/N_t = G(z_{V,t})$ and $N_{V,t}/N_t = 1 - G(z_{V,t})$, where the total number of Northern firms in every period is $N_t = N_{D,t} + N_{V,t}$. I use the functional form for the Pareto c.d.f. in order to derive the productivity cutoff as $z_{V,t} = z_{min}(N_t/N_{V,t})^{(1/k)}$.

[31] Parameter k reflects the dispersion of the productivity draws: A relatively larger k implies a smaller dispersion and a higher concentration of productivities z towards the lower productivity bound z_{min}. Also, the condition $k > \theta - 1$ ensures that the variance of firm size is finite, given the average productivities of the firms producing domestically and offshore.

Exporting firms Under the assumption of Pareto-distributed productivity draws, GM2005 define the average productivity levels of the exporting firms originating in each economy:

$$\widetilde{z}_{H,t} = \nu z_{\min} \left(\frac{N_t}{N_{H,t}} \right)^{1/k} \quad \text{and} \quad \widetilde{z}^*_{H,t} = \nu z^*_{\min} \left(\frac{N^*_{D,t}}{N^*_{H,t}} \right)^{1/k} . \tag{26}$$

3.2 Average Prices and Profits

After deriving the average productivities, I re-write the model in terms of *three representative Northern firms*: one producing domestically, another producing offshore (each serving the Northern market), and a third firm exporting to the foreign market. Since the Southern firms do not produce offshore due to the wage asymmetry across countries, there are only *two representative Southern firms*, one producing domestically for the local market, and the other exporting to the North.

I use the average firm productivities defined above to write the prices and profits associated with each representative firm, as shown in Table 1:

Table 1. Average prices and profits

Production strategy	Destination market	Prices	Profits
Domestic	Domestic	$\widetilde{\rho}_{D,t} = \frac{\theta}{\theta-1} \frac{w_t}{Z_t \widetilde{z}_{D,t}}$	$\widetilde{d}_{D,t} = \frac{1}{\theta} \left(\widetilde{\rho}_{D,t}\right)^{1-\theta} C_t$
		$\widetilde{\rho}^*_{D,t} = \frac{\theta}{\theta-1} \frac{w^*_t}{Z^*_t \widetilde{z}_{D,t^*}}$	$\widetilde{d}^*_{D,t} = \frac{1}{\theta} \left(\widetilde{\rho}^*_{D,t}\right)^{1-\theta} C^*_t$
Offshore	Domestic	$\widetilde{\rho}_{V,t} = \frac{\theta}{\theta-1} \tau \frac{w^*_t Q_t}{Z^*_t \widetilde{z}_{V,t}}$	$\widetilde{d}_{V,t} = \frac{1}{\theta} \left(\widetilde{\rho}_{V,t}\right)^{1-\theta} C_t - f_V \frac{w^*_t Q_t}{Z^*_t}$
Domestic	Export	$\widetilde{\rho}_{H,t} = \frac{\theta}{\theta-1} \tau^* \frac{w_t Q_t^{-1}}{Z_t \widetilde{z}_{H,t}}$	$\widetilde{d}_{H,t} = \frac{1}{\theta} \left(\widetilde{\rho}_{H,t}\right)^{1-\theta} C^*_t Q_t - f_H \frac{w_t}{Z_t}$
		$\widetilde{\rho}^*_{H,t} = \frac{\theta}{\theta-1} \tau \frac{w^*_t Q_t}{Z^*_t \widetilde{z}^*_{H,t}}$	$\widetilde{d}^*_{H,t} = \frac{1}{\theta} \left(\widetilde{\rho}^*_{H,t}\right)^{1-\theta} C_t Q_t^{-1} - f^*_H \frac{w^*_t}{Z^*_t}$

Endogenous productivity cutoff for offshoring Using the Pareto assumption and the property that the firm at the productivity cutoff $z_{V,t}$ is indifferent about the location of production, I write the link between the profits of the two representative firms as:[32]

$$\widetilde{d}_{V,t} = \frac{k}{k-(\theta-1)} \left(\frac{z_{V,t}}{\widetilde{z}_{D,t}} \right)^{\theta-1} \widetilde{d}_{D,t} + \frac{\theta-1}{k-(\theta-1)} f_V \left(\frac{w_t}{Z_t} \right)^{\alpha} \left(\frac{w^*_t Q_t}{Z^*_t} \right)^{1-\alpha} . \tag{27}$$

The productivity cutoff for exports is also endogenous, as in GM2005. Using the property that the firm at the productivity cutoff $z_{H,t}$ obtains zero profits from exporting, the average profits from

[32] See the Appendix for the derivation.

exports are:

$$\widetilde{d}_{H,t} = \frac{\theta - 1}{k - (\theta - 1)} f_H \frac{w_t}{Z_t}, \text{ and } \widetilde{d}^*_{H,t} = \frac{\theta - 1}{k - (\theta - 1)} f^*_H \frac{w^*_t}{Z^*_t}. \tag{28}$$

Price indexes The consumption price index in the Northern economy is an expression of the average prices of goods produced domestically and offshore by the Northern firms, as well as of the average price of goods imported from the South:

$$1 = N_{D,t} \left(\widetilde{\rho}_{D,t}\right)^{1-\theta} + N_{V,t} \left(\widetilde{\rho}_{V,t}\right)^{1-\theta} + N^*_{H,t} \left(\widetilde{\rho}^*_{H,t}\right)^{1-\theta}. \tag{29}$$

Due to the relative wage asymmetry, there is no representative Southern firm producing offshore. The consumption price index in the South includes only the average price of goods produced domestically by Southern firms and that of goods imported from the North:

$$1 = N^*_{D,t} \left(\widetilde{\rho}^*_{D,t}\right)^{1-\theta} + N_{H,t} \left(\widetilde{\rho}_{H,t}\right)^{1-\theta}. \tag{30}$$

Total profits The total profits of the Northern firms include the average profits from domestic production, from offshore production, and from exporting:

$$N_t \widetilde{d}_t = N_{D,t} \widetilde{d}_{D,t} + N_{V,t} \widetilde{d}_{V,t} + N_{H,t} \widetilde{d}_{H,t}. \tag{31}$$

In contrast to the Northern economy, the total profits of the Southern firms combine the profits from domestic production and from exports only:

$$N^*_{D,t} \widetilde{d}_t = N^*_{D,t} \widetilde{d}_{D,t} + N^*_{H,t} \widetilde{d}^*_{H,t}. \tag{32}$$

3.3 Aggregate Accounting and the Balance of International Payments

I use value added as a measure of the aggregate incomes of the two economies. The measure avoids the double-counting of offshore production by the Northern firms, which is captured by the value added of the Southern economy through the wage bill. Thus, I define value added as the sum of the wage bill and the amount of stock dividends that households in each country obtain every period:

$$Y_t = w_t + N_t \widetilde{d}_t \quad \text{and} \quad Y^*_t = w^*_t + N^*_{D,t} \widetilde{d}^*_t. \tag{33}$$

Under financial autarky in the markets for both bonds and stocks (i.e. $B_{t+1} = B_t = 0$ and

$x_{t+1} = x_t = 1$ in equilibrium), aggregate accounting implies that households spend their income from labor and stock holdings on consumption and investment in new firms:

$$C_t + N_{E,t}\widetilde{v}_t = Y_t \quad \text{and} \quad C_t^* + N_{E,t}^*\widetilde{v}_t^* = Y_t^*. \tag{34}$$

Finally, the real exchange rate Q_t is determined by the balanced current account condition for the Northern economy, which reflects the corresponding balance for the South:

$$CA_t^{Autarky} = \underbrace{N_{H,t}\left(\widetilde{\rho}_{H,t}\right)^{1-\theta} C_t^* Q_t}_{\text{Exports}} + \underbrace{N_{V,t}\widetilde{d}_{V,t}}_{\text{Repatriated profits}} - \underbrace{N_{V,t}\left(\widetilde{\rho}_{V,t}\right)^{1-\theta} C_t}_{\text{Offshore value added}} - \underbrace{N_{H,t}^*\left(\widetilde{\rho}_{H,t}^*\right)^{1-\theta} C_t}_{\text{Imports from Southern firms}}$$

Under financial autarky, the balanced current account condition ($CA_t^{Autarky} = 0$) implies that the sum of (a) exports by the Northern firms to the South and (b) repatriated profits of offshore affiliates must equal the sum of (c) the value of imports from offshore affiliates and (d) imports of final good varieties produced by the Southern firms.

3.4 Model Summary

As shown in Appendix A.1, the baseline model with financial autarky for the Northern economy can be summarized by 16 equations in 16 endogenous variables: N_t, $N_{D,t}$, $N_{V,t}$, $N_{H,t}$, $N_{E,t}$, $\widetilde{d}_t$, $\widetilde{d}_{D,t}$, $\widetilde{d}_{V,t}$, $\widetilde{d}_{H,t}$, $\widetilde{z}_{D,t}$, $\widetilde{z}_{V,t}$, $\widetilde{z}_{H,t}$, $\widetilde{v}_t$, r_t, w_t and C_t. As the Southern firms do not offshore to the high-wage North, the Southern economy is described by only 11 equations in 11 endogenous variables: There are no Southern counterparts for N_t, $N_{V,t}$, $\widetilde{d}_{V,t}$, $\widetilde{z}_{D,t}$ and $\widetilde{z}_{V,t}$. In particular, the average labor productivity of the representative Southern firm producing for the domestic market ($\widetilde{z}_D^*$) is constant over time. Variables $N_{D,t}$, r_t, N_t^* and r_t^* are predetermined.

4 Calibration

I use a standard quarterly calibration by setting the subjective rate of time discount $\beta = 0.99$ to match an average annualized interest rate of 4 percent. The coefficient of relative risk aversion is $\gamma = 2$. Following GM2005, I set the intra-temporal elasticity of substitution at $\theta = 3.8$, a value which corresponds to the U.S. plant and macro trade data. Although the resulting markup of 35.71 percent over the marginal cost might appear too large compared to the standard macroeconomic literature, its magnitude must be considered in the context of the sunk entry cost that places a wedge between the firms' marginal and average cost. I also calibrate the probability of firm exit $\delta = 0.025$ to match the annual 10 percent job destruction in the U.S.

As summarized in Table 2, I calibrate the fixed costs of offshoring (f_V) and exporting (f_H and f_H^*) as well as the Pareto distribution parameter (k) so that the model matches the importance of offshoring and trade for the Mexican economy, as illustrated by four empirical moments: (1) Maquiladora's value added represents approximately 20 percent of Mexico's manufacturing GDP (INEGI, 2008), as compared to 25 percent in the model; (2) Maquiladora's exports represent approximately half of Mexico's total exports (Bergin, Feenstra, and Hanson, 2008), as compared to 60 percent in the model; (3) Employment in the maquiladora sector accounts for approximately 25 percent of Mexico's total manufacturing employment (Bergin, Feenstra, and Hanson, 2008), as compared to 22 percent in the model; (4) Total imports represent the equivalent of 33 percent of Mexico's GDP (INEGI, 2008), as compared to 32 percent in the model. To this end, I set $f_V = 0.0057$ (the fixed cost of offshoring for Northern firms), $f_H = 0.032$ and $f_H^* = 0.018$ (the fixed costs of exporting for Northern and Southern firms, respectively), as well as $k = 4.2$ (the Pareto distribution coefficient).[33] Without loss of generality, I set the lower bound of the support interval for firm-specific productivities in the North and South at $z_{min} = z_{min}^* = 1$.

In order to derive an asymmetric steady state in which the cost of effective labor is lower in the South than in North, I set the sunk entry cost (i.e. which reflects the cost of starting a business) to be larger in the South than in North ($f_E^* = 4f_E$, while setting $f_E = 1$ without loss of generality).[34] The calibration reflects the considerable variation in the cost of starting a business across countries: the monetary cost is 3.3 times higher in Mexico than in the U.S. or Canada; it is 6.2 times higher in Hungary than in the U.K. (World Bank, 2007; see Appendix 5). The asymmetric sunk entry costs, along with the trade iceberg cost ($\tau = 1.3$) and the values for f_V, f_H and f_H^* reported above, generate a steady state value for the terms of labor that is less than one ($TOL = 0.76$). Thus, the steady state cost of effective labor in the South, defined as the real wage divided by the aggregate productivity

[33] In the model with exports only, I set $f_H = 0.0260$ and $f_H^* = 0.0226$ so that the fraction of Northern exporting firms (10 percent) and that of Southern exporting firms (63 percent) match the corresponding steady state values in the model with offshoring.

[34] The asymmetric sunk entry cost is one method that generates different effective wages across countries. The same result would be achieved with at least two other modeling devices: (1) Introduce a cross-country asymmetry in the size of firms (rather than in their number) through the price elasticity of demand. With identical sunk entry costs and equal average labor productivity levels in the two economies, firms in the economy with the lower price elasticity of demand charge higher markups and produce relatively lower ouput; in turn, the lower labor demand generates lower wages. (2) Another way to generate different firm sizes across countries, similar to the one I use in this paper, would be to allow for multi-product firms and sunk costs of creating new product varieties. While keeping the sunk firm entry costs equal across countries, there will be fewer varieties per firm and lower demand for labor in the economy with the higher sunk cost of creating a new variety.

level, is 76 percent of the corresponding value in the North. The calibration provides an incentive for the Northern firms to produce offshore in steady state.

Table 2. Baseline model with offshoring: calibration

$f_E = 1$	Sunk firm entry cost, North	$k = 4.2$	Pareto distribution coefficient
$f_E^* = 4f_E$	Sunk firm entry cost, South	$\beta = 0.99$	Standard quarterly calibration
$f_V = 0.0057$	Fixed cost of offshoring	$\gamma = 2$	CRRA coefficient
$f_H = 0.0320$	Fixed cost of exporting, North	$\delta = 0.025$	Probability of firm exit
$f_H^* = 0.0180$	Fixed cost of exporting, South	$\theta = 3.8$	Intra-temporal elasticity of substitution

The resulting steady-state fraction of the Northern firms that use inputs imported from their offshore affiliates (N_V/N) is 1.4 percent; the ratio of exporting firms in total (N_H/N) is 10.1 percent.[35] Since I model offshoring in an asymmetric two-country framework that abstracts from exchanges between U.S. firms and the rest of the world (other than Mexico), the steady state values reported above are less than their empirical counterparts that include all U.S. manufacturing firms. In the data, approximately 14 percent of the U.S. firms (other than domestic wholesalers) used imports from both Mexico and the rest of the world in 1997 (Bernard, Jensen, Redding and Schott, 2007),[36] out of which intra-firm imports (as opposed to arm's length transactions) represented half of the total (Bardhan and Jafee, 2004); approximately 21 percent of the U.S. manufacturing plants were exporters in 1992 (Bernard, Eaton, Jensen and Kortum, 2003).

The calibration also implies that the steady-state share of Northern expenditure on the varieties produced by Northern firms domestically (66.0 percent) - firms which are relatively less productive than the average - is less than their fraction in the total number of varieties available to Northern consumers (89.2 percent). In contrast, since the offshore varieties are produced by the relatively more productive Northern firms, their market share (21.2 percent) is more than their fraction in the total number of varieties available in North (1.2 percent).[37]

[35] In the Southern economy, the ratio of exporting firms in the total is 63 percent.

[36] The value would understate the fraction of *plants* that use imported inputs if the importing *firms* tend to operate multiple plants manufacturing multiple product varieties.

[37] The market share of the Southern varieties in the Southern market is 61.66 percent. Their fraction in the total number of varieties available in South (62.77) exceeds their market share.

5 Results

5.1 Offshoring Dynamics

Under financial autarky, I log-linearize the model with offshore production around the steady state and compute the impulse responses to a transitory 1 percent increase in aggregate productivity in the North. I assume that productivity is described by the univariate process $\log Z_{t+1} = \rho \log Z_t + u_t$, with the persistence parameter $\rho = 0.9$.

Offshoring at the intensive margin Figure 4 shows the impulse responses of the model with offshoring (solid line), and contrasts them with the impulse responses of the benchmark model with endogenous exports and no offshoring as in GM2005 (dotted line). For each variable, the horizontal axis illustrates quarters after the initial shock, and the vertical axis shows the percent deviations from the original steady state in each quarter.

On impact, the 1 percent increase in aggregate labor productivity in the North generates an equal increase in the real wage w_t. The increased demand for the final good varieties already produced offshore (offshoring at the intensive margin) causes the real wage in the South (w_t^*) and the terms of labor $\left(TOL_t = \frac{Q_t w_t^*/Z_t^*}{w_t/Z_t}\right)$ to spike upward on impact: Since the increase in aggregate labor productivity in the North is not replicated in the South, on impact there is excess demand for the Southern units of effective labor. Therefore, the number of Northern firms that produce offshore ($N_{V,t}$) declines on impact due to: (1) the increase in the cost of producing offshore, and (2) the increase in the fixed cost of relocation, both of which are sensitive to the cost of effective labor in the South.

Offshoring at the extensive margin Over the business cycle, as aggregate labor productivity in the North persists above the initial steady state, the larger market size encourages firm entry, which causes the number of Northern firms (N_t) to increase (Figure 4). The larger number of incumbent firms causes an increase in labor demand in the North, and thus leads to an appreciation of the cost of effective labor in the North relative to that in the South in the medium run (as shown by the decline of TOL_t below the initial steady state). Following the appreciation of the terms of labor, more of the Northern firms have an incentive to relocate production to the South (offshoring at the extensive margin). Hence, following the initial drop, the number of Northern firms that relocate production to the South ($N_{V,t}$) rises above the original steady state 4 quarters after the shock.

The initial upward spike in the Southern real wage, caused by the increased demand for final good varieties that were already produced offshore when the shock occurred (offshoring at the intensive margin), is followed by a hump-shaped increase in the Southern wage which occurs as more Northern firms relocate production offshore over the business cycle (offshoring at the extensive margin). Thus, the adjustment along the extensive margin of offshoring places additional upward pressure on the Southern wage, and thus causes the terms of labor to appreciate by less (TOL to decrease by less) in the medium run relative to the benchmark model with exports only. As a result, the higher labor demand in the North, caused by firm entry, and subsequently the higher labor demand in the South, caused by offshoring, enhance the cross-country comovements of wages and aggregate incomes relative to the benchmark model with exports only.

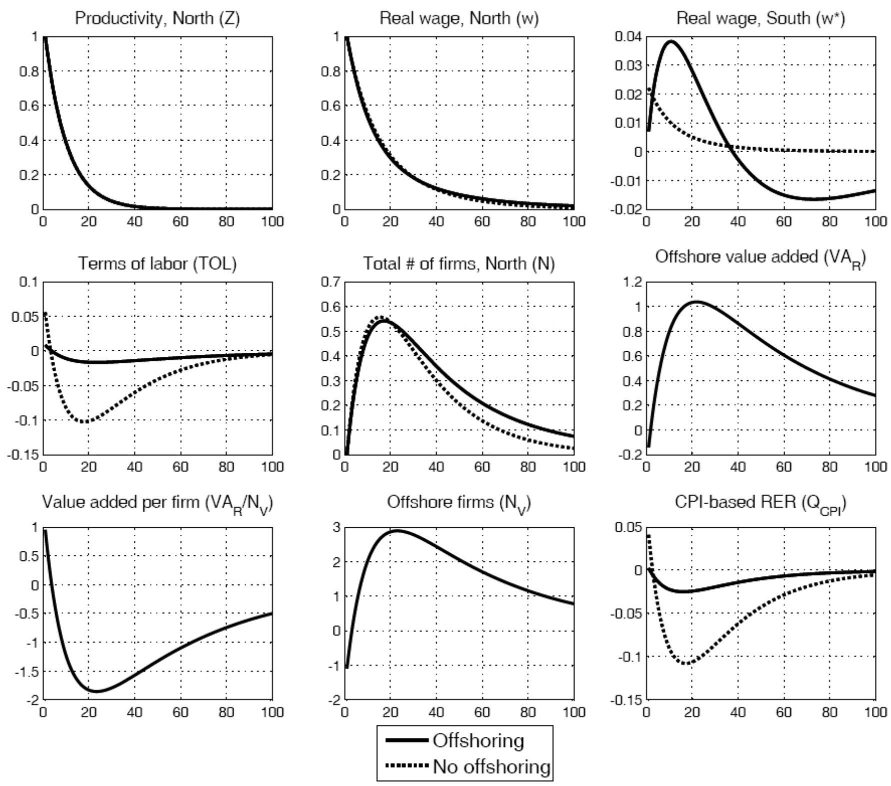

Figure 4. Endogenous offshoring (continuos line) vs. exports only (dotted line), impulse responses to a transitory 1 percent increase in aggregate productivity in the North

5.2 Real Exchange Rate Dynamics

Average prices and product variety In this section I analyze the model's predictions for the dynamics of relative prices in response to aggregate shocks. Due to the existence of endogenous product variety in the model, I use the consumer price index (CPI)-based real exchange rate $\widetilde{Q_t} = \varepsilon_t \widetilde{P_t^*}/\widetilde{P_t}$ as the theoretical counterpart to the empirical real exchange rate, since the average prices $\widetilde{P}_t$ and $\widetilde{P_t^*}$

best represent the corresponding empirical CPI levels, as discussed in Broda and Weinstein (2003). To this end, I break down the welfare-based price indexes P_t and P_t^* into (a) components reflecting average prices ($\widetilde{P_t}$ and $\widetilde{P_t^*}$) and (b) components reflecting product variety:[38]

$$P_t = \left(N_{D,t} + N_{V,t} + N^*_{H,t}\right)^{\frac{1}{1-\theta}} \widetilde{P_t} \text{ and } P_t^* = \left(N^*_{D,t} + N_{H,t}\right)^{\frac{1}{1-\theta}} \widetilde{P_t^*} \tag{35}$$

Thus, I write the CPI-based real exchange rate as:[39]

$$\widetilde{Q_t}^{1-\theta} = \left(\frac{N_{D,t} + N_{V,t} + N^*_{H,t}}{N^*_{D,t} + N_{H,t}}\right) \frac{N^*_{D,t}\left(\frac{TOL_t}{\widetilde{z}^*_{D,t}}\right)^{1-\theta} + N_{H,t}\left(\frac{\tau^*_t}{\widetilde{z}_{H,t}}\right)^{1-\theta}}{N_{D,t}\left(\frac{1}{\widetilde{z}_{D,t}}\right)^{1-\theta} + N_{V,t}\left(\frac{\tau_t TOL_t}{\widetilde{z}_{V,t}}\right)^{1-\theta} + N^*_{H,t}\left[\frac{\tau_t TOL_t}{\widetilde{z}^*_{H,t}}\right]^{1-\theta}}, \tag{36}$$

where the terms of labor $TOL_t = \frac{Q_t w^*_t / Z^*_t}{w_t / Z_t}$ measure the cost of effective labor in the South relative to the North; the iceberg trade costs τ_t and τ^*_t (which I allow to vary over time) affect the Northern and Southern imports, respectively. The expression nests the model with endogenous exports of GM2005; I shut down offshoring and revisit the GM2005 case when $N_{V,t} = 0$.

Analytical results While allowing for offshore production and nesting the benchmark model with exports only of GM2005, I log-linearize (36) to obtain:[40]

$$\widehat{\widetilde{Q}}_t = \left[s_D - s_V + s^*_D - 1\right]\widehat{TOL}_t + \tag{C1}$$

$$+ \left(s_D - s_V\right)\widehat{\widetilde{z}}_{D,t} + s_V \widehat{\widetilde{z}}_{V,t} - (1-\alpha) s_V \widehat{\tau}_t + \tag{C2}$$

$$+ \left(1 - s_D\right)\left(\widehat{\widetilde{z}}^*_{H,t} - \widehat{\tau}_t\right) - \left(1 - s^*_D\right)\left(\widehat{\widetilde{z}}_{H,t} - \widehat{\tau}^*_t\right) + \tag{C3}$$

$$+ \frac{1}{\theta - 1}\left(s_V - \frac{N_V}{N_D + N_V + N^*_H}\right)\left(\widehat{N}_{V,t} - \widehat{N}^*_{H,t}\right) + \tag{C4}$$

$$+ \frac{1}{\theta - 1}\left[\left(\frac{N^*_D}{N^*_D + N_H} - s^*_D\right)\left(\widehat{N}^*_{D,t} - \widehat{N}_{H,t}\right) - \left(\frac{N_D}{N_D + N_V + N^*_H} - (s_D - s_V)\right)\left(\widehat{N}_{D,t} - \widehat{N}^*_{H,t}\right)\right], \tag{C5}$$

[38] Variable $N_{D,t}$ represents the number of final good varieties produced by Northern firms and sold domestically, $N_{V,t}$ represents varieties produced by Northern firms offshore and sold in the Northern market, and $N^*_{H,t}$ reflects varieties produced by Southern firms and exported to the Northern market. It follows that $\widetilde{Q_t}^{1-\theta} = \left(\frac{N_{D,t}+N_{V,t}+N^*_{H,t}}{N^*_{D,t}+N_{H,t}}\right) Q_t^{1-\theta}$.

[39] The CPI-based real exchange rate $\widetilde{Q_t}$ deviates from the welfare-based real exchange rate $Q_t = \varepsilon_t P^*_t / P_t$ due to cross-country differences in product variety. As discussed in GM2005, an appreciation of the CPI-based real exchange rate $\widetilde{Q_t}$ (i.e. an increase in the CPI in North relative to that in South) may be offset by the increase in product variety in North ($N_{D,t} + N_{V,t} + N^*_{H,t}$) relative to South ($N^*_{D,t} + N_{H,t}$), so that the welfare-based real exchange rate Q_t depreciates (i.e. despite the increase in average prices, consumers derive higher utility in the market with the larger product variety).

[40] See the Appendix for the derivation.

where parameter s_D is the steady-state share of spending in the North on goods produced by Northern firms *both* domestically and offshore; s_V is the steady-state share of spending in the North *only* on goods produced by Northern firms offshore (I revisit GM2005 when $s_V = 0$); s_D^* is the steady-state share of spending in the South on goods produced by Southern firms domestically. The calibration of the model ensures that: (a) $(s_D - s_V) + s_D^* > 1$, as the domestically-produced varieties represent more than 50 percent of the consumption spending in each country; (b) $\left(\frac{N_D}{N_D+N_V+N_H^*} - (s_D - s_V)\right) > 0$ and $\left(\frac{N_D^*}{N_D^*+N_H} - s_D^*\right) > 0$, i.e. the market shares of varieties produced domestically by *the less productive* firms are *smaller* than their fraction in the total number of varieties; and (c) finally, $\left(s_V - \frac{N_V}{N_D+N_V+N_H^*}\right) > 0$, i.e. the market share of varieties produced offshore by *the more productive* Northern firms is *larger* than their fraction in the total number of varieties available in the North. Thus, the model implies that the more productive firms are larger and have larger market shares than their less productive counterparts, which is in line with the empirical evidence in Kurz (2006).

The log-linearized form of (36) outlines five channels (labeled C1-C5 in the log-linearized expression above) through which the CPI-based real exchange rate is affected by: (1) changes in the price of non-tradable goods induced by fluctuations in the terms of labor $(\widehat{TOL}_t)$; (2) changes in the price of offshored goods reflecting fluctuations in the average productivity of offshoring firms $\left(\widehat{\tilde{z}}_{V,t}\right)$ and in the magnitude of trade costs $(\widehat{\tau}_t)$; (3) changes in the relative import prices triggered by fluctuations in the average productivity of Northern exporters $(\widehat{\tilde{z}}_{H,t})$ relative to that of their Southern counterparts $(\widehat{\tilde{z}}^*_{H,t})$; (4) changes in the relative availability of varieties produced by Northern offshoring firms $(\widehat{N}_{V,t})$ relative to that of Southern exported varieties $(\widehat{N}^*_{H,t})$; and (5) changes in the relative availability of domestic varieties $(\widehat{N}_{D,t})$ relative to that of Southern exported varieties $(\widehat{N}^*_{H,t})$.

Impulse responses I find that, relative to the benchmark model with endogenous exports, offshoring dampens the appreciation of the real exchange rate following an aggregate productivity improvement in the North. Specifically, the effect occurs through channels C1 (price of non-traded goods), C3 (relative import prices) and C4 (availability of offshored vs. Southern imported varieties). The impulse responses for the variables of interest are outlined in Figure 5; their impact on the real exchange rate is described next.

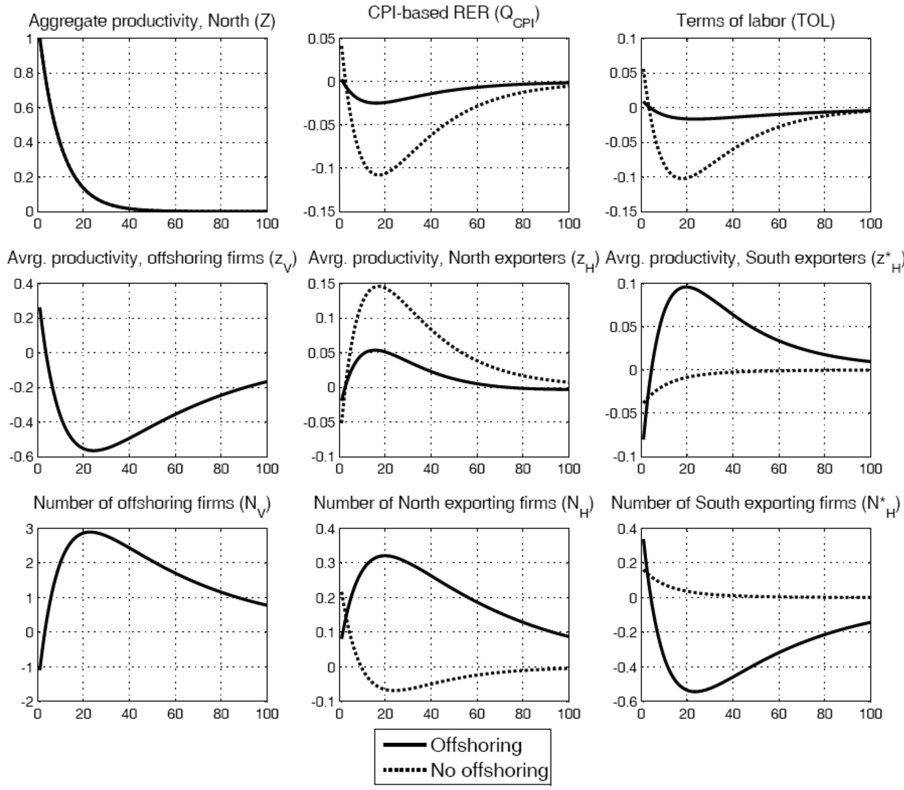

Figure 5. Endogenous offshoring (continuos line) vs. exports only (dotted line),

impulse responses to a transitory 1 percent increase in aggregate productivity in the North

(C1) Changes in the price of non-traded goods. In the benchmark model with endogenous exports and no offshoring, a productivity increase in the North encourages firm entry and leads to the appreciation of the terms of labor in the medium run (i.e. TOL_t decreases). In turn, this causes the average price of non-traded goods in the North to increase relative to that in the South, and thus leads to the appreciation of the real exchange rate (i.e. $\widetilde{Q_t}$ decreases).

In my model, offshoring dampens the appreciation of the real exchange rate taking effect through this channel in two ways: (a) Offshoring reduces the share of non-traded goods in total spending $(s_D - s_V)$ as $s_V > 0$; (b) Offshoring also dampens the appreciation of the terms of labor relative to the benchmark model with exports only (i.e. TOL_t decreases by less), as the relocation of production offshore transfers upward pressure from the domestic wage onto the foreign one.

(C2) Changes in the price of offshored goods. On impact, due to the initial spike in the Southern wage (caused by offshoring at the intensive margin), the number of offshoring firms declines and their average productivity rises. In the medium run, however, due to the appreciation of the cost of effective labor in the North relative to the South, offshoring becomes a more profitable option. Thus, the average productivity $\widetilde{z}_{V,t}$ of offshoring firms declines and their average price increases. Offshoring contributes to the appreciation of the real exchange rate through this channel.

Exogenous policy changes can also affect the price of goods produced offshore. For instance, tariff cuts for the varieties of final goods produced offshore (i.e. a policy measure reflected by a decrease in τ_t) would dampen the appreciation of the CPI-based real exchange rate.

(C3) Changes in relative import prices. In the benchmark model with exports only, the appreciation of the terms of labor reduces the export profitability of the Northern firms relative to that of their Southern counterparts. In turn, the average productivity of the surviving Northern exporters ($\widetilde{z}_{H,t}$) increases relative to that of the Southern exporters $\left(\widetilde{z}^*_{H,t}\right)$, and the average price of the Southern imports to decline relative to that of the Northern imports. This causes the real exchange rate to appreciate.

Offshoring reverses this effect. The upwards pressure on the Southern wage causes the export profitability of the Southern firms to decline, and thus the productivity of the surviving Southern exporters to increase by more than that of their Northern counterparts. In contrast to the benchmark model with exports only, offshoring causes the average price of the Northern imports to decline relative to the average price of the Southern imports, a result which dampens the appreciation of the real exchange rate through import prices.

(C4) Expenditure switching from imports towards offshored goods. Following an increase in aggregate productivity, offshoring puts upward pressure on the Southern wage and reduces the competitiveness of the Southern exports. Thus, Northern consumers switch their expenditure away from the increasingly less competitive Southern varieties ($N^*_{H,t}$ decreases) and towards the relatively cheaper varieties produced by Northern firms offshore ($N_{V,t}$ increases). The result dampens the appreciation of the real exchange rate in the medium run. It is consistent with the empirical evidence that FDI inflows in Mexico were associated with a decline in Mexico's low-wage competitiveness during the late 1990s (Gallagher and Zarsky, 2007).

(C5) Expenditure switching from imports towards domestic goods. Firm entry in the North generates an increase in the number of domestic varieties ($N_{D,t}$) relative to foreign imported varieties ($N^*_{H,t}$) available to Northern consumers. In turn, consumers switch their expenditures from imports towards the final good varieties produced domestically by the relatively less productive firms, and which are available at relatively higher average prices. As in the model with exports only, this channel works towards the appreciation of the real exchange rate.

5.3 Empirical Moments: The Cyclicality of Offshoring

In this section I show that offshore production to Mexico's maquiladora sector is pro-cyclical with fluctuations in the U.S. manufacturing output. The result is robust across several indicators of offshoring: real value added, hours worked, and the number of establishments in the maquiladora sector. The finding invites the construction of the model in which offshore production is procyclical with domestic output, and in which the extensive margin of offshoring (proxied empirically by the number of establishments) plays a special role in the cross-country transmission of business cycle fluctuations.

The absence of local consumption and the dominant share of the U.S. as the destination market make Mexico's maquiladora sector an appropriate empirical setup to study the cyclicality of offshoring through vertical FDI. By definition, plants operating under Mexico's maquiladora program import inputs, process them, and ship the resulting goods back to the country of origin (Gruben, 2001). Although not all plants in Mexico's maquiladora sector are owned by U.S. firms, most of the maquiladora's imported inputs (82 percent in 2001) originate in the U.S. (Hausman and Haytko, 2003), and most of the maquiladora's value added (roughly 90 percent) is exported to the U.S. (Burstein, Kurz, Tesar, 2007).

In panels A-C of Figure 6 (left) I plot the three maquiladora indicators (real value added, total hours worked, and the number of establishments) against the industrial production index for U.S. manufacturing.[41] I apply the Baxter-King bandpass-filter to the quarterly data in natural logs for the interval 1990:1-2006:4 in order to eliminate fluctuations with periodicity lower than 18 months and greater than eight years. The visual inspection of the filtered data suggests that the U.S. economic expansion throughout the 1990s, as well as the recession in 2001, were associated with similar developments in the maquiladora sector. Also, the unconditional correlations summarized in panels D-F of Figure 6 (right) suggest that offshoring to Mexico is procyclical with fluctuations in U.S. manufacturing. In particular, the correlations of offshoring with lags and leads of the U.S. manufacturing index suggest that U.S. output is contemporaneously correlated with the number of hours worked in the maquiladora sector, whereas it tends to lead the number of maquiladora establishments (offshoring at

[41] I use quarterly data for the interval between 1990:1 and 2006:4. The data for U.S. manufacturing (i.e. seasonally adjusted real industrial production and the nominal hourly wage in manufacturing) is provided by the Board of Governors of the Federal Reserve System and by the U.S. Bureau of Labor Statistics. Data for Mexico's maquiladora sector (real value added, hours worked and the number of plants), at monthly frequency and without seasonal adjustment, is provided by the Instituto Nacional de Estadística y Geografía (INEGI), Mexico. Thus, I take the quarterly averages of the Mexican data and perform the seasonal adjustment using the X-12-ARIMA method of the U.S. Census Bureau.

the extensive margin) by at least three quarters.[42]

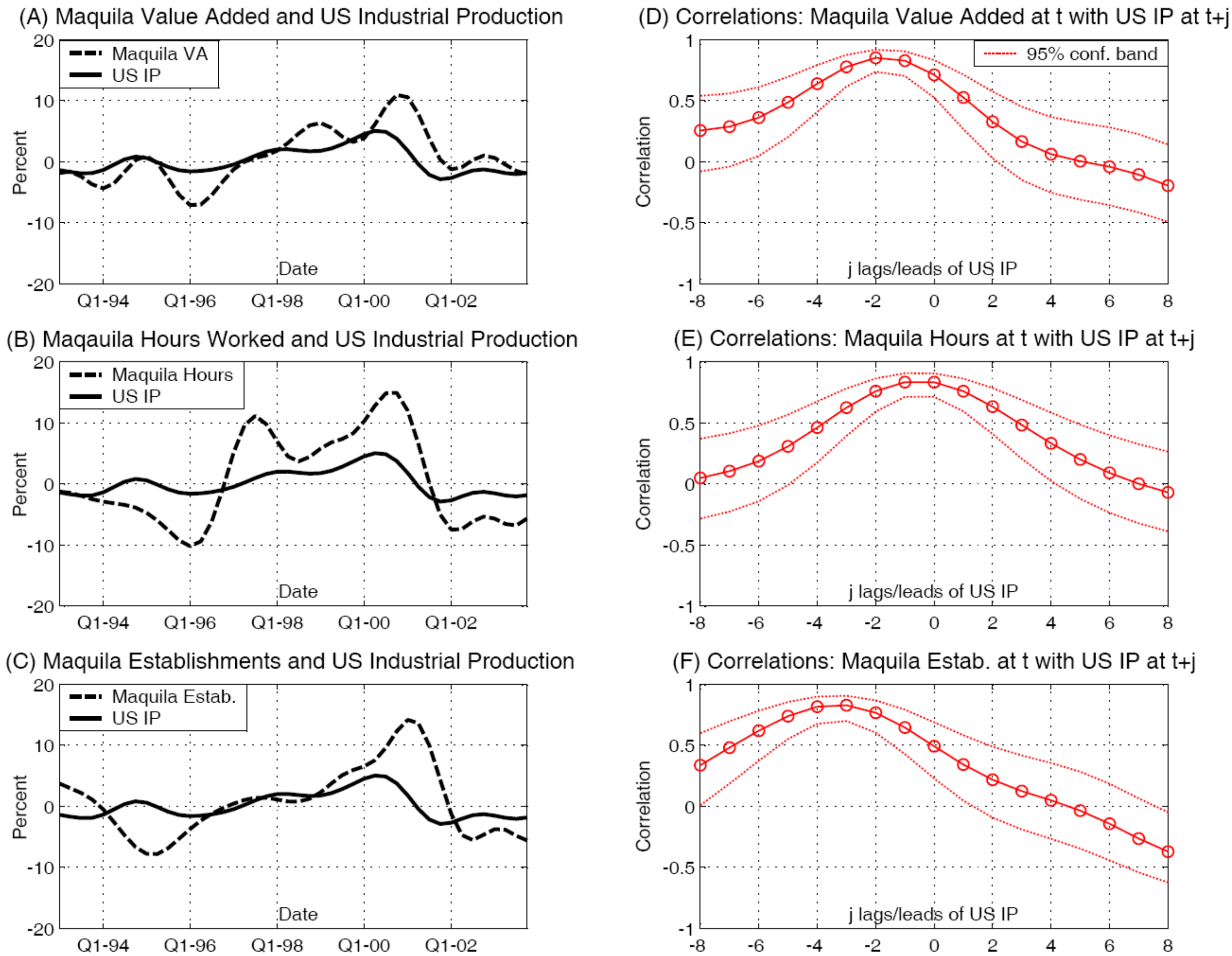

Figure 6. Mexico's maquiladora and U.S. manufacturing industrial production

5.4 Theoretical Moments: The Cyclicality of Offshoring

Theoretical measures of offshoring In this section I explore the ability of the model to replicate the cyclicality of offshoring to Mexico's maquiladora industry relative to fluctuations in U.S. manufacturing. In particular, I distinguish between the extensive margin of offshoring (the number of offshoring firms) and the intensive margin (the value added per offshoring firm). Using the usual demand and price functions under monopolistic competition for the varieties produced offshore, the

[42] Although the interval of three quarters may appear too short for the creation of new offshore plants, this finding must be considered in light of the fact that a considerable fraction of the non-U.S. owned maquiladora plants represent arm's length contractors that have the flexibility to enter into and exit from outsourcing relationships with U.S. firms over the business cycle.

total value added contributed by offshore affiliates is:[43]

$$\mathrm{VA}_t = N_{V,t} \left[\frac{\theta}{\theta - 1} \frac{w_t}{Z_t} \frac{\tau_t TOL_t}{\widetilde{z}_{V,t}} \right]^{1-\theta} C_t, \text{ with } \theta > 1. \tag{37}$$

I measure the *extensive margin* as the number of offshoring firms every period, $N_{V,t}$, and the *intensive margin* of offshoring as the real value added per offshoring firm, $\mathrm{VA}_{R,t}/N_{V,t}$.

The determinants of offshoring Using the expression for total value added in (37), and holding fixed the cost of effective labor in the North, the following regularities become apparent: (a) Offshoring decreases with the terms of labor ($\partial \mathrm{VA}_t/\partial TOL_t < 0$), because the higher is the cost of effective labor in the South relative to North, the lower is the incentive to produce offshore; (b) Offshoring decreases with the trade cost affecting the shipping goods produced by offshore affiliates back to the country of origin ($\partial \mathrm{VA}_t/\partial \tau_t < 0$), cost which reflects the magnitude of tariffs, trade barriers, transportation and insurance costs; (c) The value added offshore increases with the average productivity of the offshoring firms ($\partial \mathrm{VA}_t/\partial \widetilde{z}_{V,t} > 0$); and (d) Offshoring depends on the number of firms that produce offshore every period ($\partial \mathrm{VA}_t/\partial N_{V,t} > 0$), a number that is inversely related to the fixed cost of offshoring (f_V). The predictions of the model are in line with the empirical evidence provided by Hanson, Mataloni and Slaugther (2005) that demand for imported inputs by U.S. multinational firms increases when their offshore affiliates benefit from relatively lower low-skilled foreign wages and face lower trade costs.

The productivity process I introduce elastic labor supply in the baseline model of offshoring under financial autarky, using the standard calibration described above. I also assume that aggregate productivity in the North and in the South follow a bivariate autoregressive process:

$$\begin{bmatrix} \log Z_t \\ \log Z_t^* \end{bmatrix} = \begin{bmatrix} \rho_Z & \rho_{ZZ^*} \\ \rho_{Z^*Z} & \rho_{Z^*} \end{bmatrix} \begin{bmatrix} \log Z_{t-1} \\ \log Z_{t-1}^* \end{bmatrix} + \begin{bmatrix} \xi_t \\ \xi_t^* \end{bmatrix}, \tag{38}$$

and that the productivity shocks are the only source of international business cycles in the model. Following Backus, Kehoe, and Kydland (1992), I set the persistence parameters to $\rho_Z = \rho_{Z^*} = 0.906$, and the spillover parameters to $\rho_{ZZ^*} = \rho_{Z^*Z} = 0.088$; the variance of the shocks is 0.00852 and the covariance is $0.18728 * 10^{-4}$, values which correspond to a correlation of innovations of 0.258.

[43] To compute moments, I deflate the value added offshore by the average CPI in the North economy in order to eliminate the variety effect, i.e. $(VA)_{R,t} = P_t\,(VA_t)\,/\widetilde{P}_t$, where $P_t = N_t^{\frac{1}{1-\theta}}\widetilde{P}_t$.

Empirical vs. theoretical correlations Table 3 (panel A) provides the empirical moments of offshoring from U.S. manufacturing to Mexico's maquiladora sector. It includes the empirical correlations of the maquiladora indicators (the number of establishments and the total value added per establishment[44]) with lags and leads of the industrial production index for U.S. manufacturing. In addition, Figure 7 contrasts the empirical moments with their theoretical counterparts generated by the model.

The model is successful in generating the procyclical variation in the total value added offshore relative to fluctuations in U.S. manufacturing output (Table 3). The contemporaneous correlation between the offshore value added and manufacturing output in the North (0.78) is remarkably close to the corresponding empirical correlation between the value added in Mexico's maquiladora and U.S. manufacturing output (0.71).

Table 3. Correlations of the maquiladora variables at t with GDP in North (y_R) at $t+j$

j	−8	−6	−4	−2	**0**	+2	+4	+6	+8
A. Empirical correlations with lags and leads of U.S. manufacturing IP									
Value added in maquiladora	0.26	0.36	0.64	0.85	**0.71**	0.33	0.06	−0.04	−0.20
Number of establishments	0.34	0.62	0.82	0.77	**0.49**	0.22	0.05	−0.14	−0.37
Value added per establishment	−0.23	−0.55	−0.51	-0.16	**0.08**	0.06	0.00	0.16	0.30
B. Model with elastic labor supply under financial autarky									
Value added (VA_R)	−0.04	0.09	0.28	0.51	**0.78**	0.86	0.92	0.93	0.93
Number of firms (N_V)	0.28	0.41	0.45	0.31	**−0.19**	-0.16	−0.11	−0.07	−0.03
Value added per firm ($\frac{VA_R}{N_V}$)	−0.29	−0.40	−0.43	−0.26	**0.27**	0.25	0.20	0.16	0.12

Turning towards the extensive margin (Figure 7, panel A), the data shows a strong and positive correlation between the number of maquiladora establishments and *past* U.S. manufacturing output, a result which suggests that U.S. economic expansions tend to lead the number of establishments by at least three quarters. The model is successful qualitatively in capturing this pattern: In the model, the correlation between the number of offshoring firms and *past* output in the North is positive (it peaks for Northern output lagged by four quarters). The result is caused by the fact that, following a productivity improvement in the North, offshoring at the extensive margin increases gradually over

[44] I use the number of establishments in the maquiladora sector as an empirical proxy for the extensive margin of offshoring, and the real value added per establishment as an empirical proxy for the intensive margin.

time, a pattern which mirrors the gradual appreciation of the Northern wage over time caused by domestic firm entry.

The model generates one implication that is at odds with the data, namely that the contemporaneous correlation between the number of offshoring firms and output in the North is negative (rather than positive as in the data). On impact, the greater demand for Southern varieties causes the Southern wage to spike upwards, thus reducing the number of offshoring firms and generating a negative contemporaneous correlation between the number of offshoring firms and Northern output. The introduction of a sunk offshoring cost in the model (to replace the fixed, per-period offshoring cost) would address this issue.

Regarding the intensive margin (Figure 7, panel B), the empirical correlation between value added per maquiladora establishment and the past manufacturing output is negative and statistically significant. The model is successful in replicating this pattern as well: In the model, the correlation between the intensive margin and past output in North is negative. The value added per offshoring firm declines several quarters after an increase in aggregate labor productivity in North: Following the appreciation of the terms of labor, the amount of value added per offshoring firm declines because the number of firms producing offshore increases faster than the total value added.

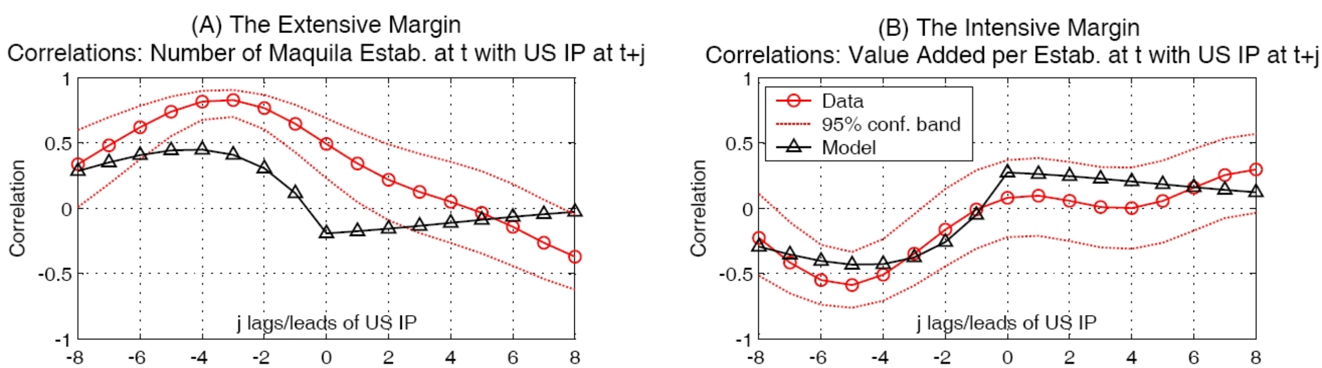

Figure 7. Empirical vs. theoretical moments, financial autarky with elastic labor supply

5.5 Theoretical Moments: Cross-Country Comovement of Output and Consumption

In this section I examine the cross-country correlations of national income and consumption generated by the model with offshoring relative to those from the model with exports only. I also conduct sensitivity analysis for a wide range of possible values of the key model parameters. Under the baseline framework of offshoring with financial autarky, I assume that productivity follows the bivariate autoregressive process in (38), and that aggregate productivity shocks are the only sources of business cycle fluctuations.

The productivity process I devote particular attention to the matrix of persistence and spillover coefficients in the bivariate productivity process, using parameter values that are in line with those used in the international real business cycle literature. In particular, I focus on three cases: (1) Low persistence ($\rho_Z = \rho_{Z^*} = 0.906$) and positive spillover parameters ($\rho_{ZZ^*} = \rho_{Z^*Z} = 0.088$) as in Backus, Kehoe, and Kydland (1992, 1994); (2) Near-unit persistence ($\rho_Z = \rho_{Z^*} = 0.999$) and zero spillover parameters as in Baxter and Farr (2005), with the variance of shocks 0.00852 and covariance $0.18728 * 10^{-4}$ (correlation 0.26) from Backus, Kehoe, and Kydland (1992); (3) Asymmetric persistence ($\rho_Z = 0.996$ and $\rho_{Z^*} = 0.951$) with the shock being more volatile for Mexico than for the U.S. (i.e. variances 0.0139570^{-2} vs. 0.0050939^{-2}) and the covariance $0.1898 * 10^{-4}$ (correlation 0.27), as estimated in Mandelman and Zlate (2008) using total factor productivity (TFP) data for the U.S. and Mexico.

Table 4 shows the cross-country correlations of output $Corr(Y_R, Y_R^*)$ and consumption $Corr(C_R, C_R^*)$, both for the model with offshoring and for the benchmark model with exports only, for each of the three productivity specifications described above.[45] The results show that offshoring enhances the cross-country comovement of both output and consumption relative to the benchmark model with exports only. In particular, under the specification with near-unit persistence, the model with offshoring under financial autarky reverses the ranking of correlations (the cross-country correlation of output exceeds that of consumption), thus addressing the output-consumption correlation puzzle observed in the international real business cycle literature, as discussed in Backus, Kehoe, and Kydland (1992, 1994).

[45] In order to compute the cross-country correlations of national income and consumption, I deflate the corresponding variables by the average price indices in each country. For instance, I deflate the national income in North as $Y_{R,t} = P_t Y_t / \widetilde{P}_t$, where $P_t = \left(N_{D,t} + N_{V,t} + N_{H,t}^*\right)^{\frac{1}{\theta-1}} \widetilde{P}_t$, since the empirical price deflators are best represented by the average price index $\widetilde{P}_t$ rather than the welfare-based price index P_t, as discussed in Ghironi and Melitz (2005).

Table 4. Output and consumption comovement, financial autarky

Calibration:	(1) Low persistence, $\rho_Z = \rho_{Z^*} = 0.906$		(2) High persistence, $\rho_Z = \rho_{Z^*} = 0.999$		(3) Asymmetric persistence, $\rho_Z = 0.996$, $\rho_{Z^*} = 0.951$	
Model:	**Offshoring**	No offshoring	**Offshoring**	No offshoring	**Offshoring**	No offshoring
$Corr(Y_R, Y_R^*)$	**0.41**	0.39	**0.33**	0.25	**0.35**	0.27
$Corr(C_R, C_R^*)$	**0.96**	0.92	**0.32**	0.28	**0.40**	0.28

Sensitivity to ρ, θ, τ I also check the sensitivity of the cross-country correlations of output and consumption to variations in the following parameters: (a) the persistence of the bivariate autoregressive productivity process ρ_Z (with zero spillovers); (b) the elasticity of substitution between the Northern and Southern final good varieties θ; and (c) the iceberg trade cost τ.

The results in Figure 8 show that the model with offshoring under financial autarky generates larger cross-country correlations for both output and consumption relative to the benchmark model with exports, a result which holds for a wide range of values for the persistence parameter $\phi_Z \in [0.9, 1)$, the elasticity of substitution $\theta \in [2.5, 4.1]$ and the iceberg trade cost $\tau \in [1.20, 1.33]$. In particular, the cross-country correlation of output declines with the iceberg trade cost. Following a positive shock in the North, a larger trade cost dampens the firms' incentive to relocate production offshore, which leads to a lower co-movement of output. The result is in line with the stylized facts documented in Burstein, Kurz, and Tesar (2007), namely that countries involved in production sharing more intensely with each other tend to display higher correlations of manufacturing output.

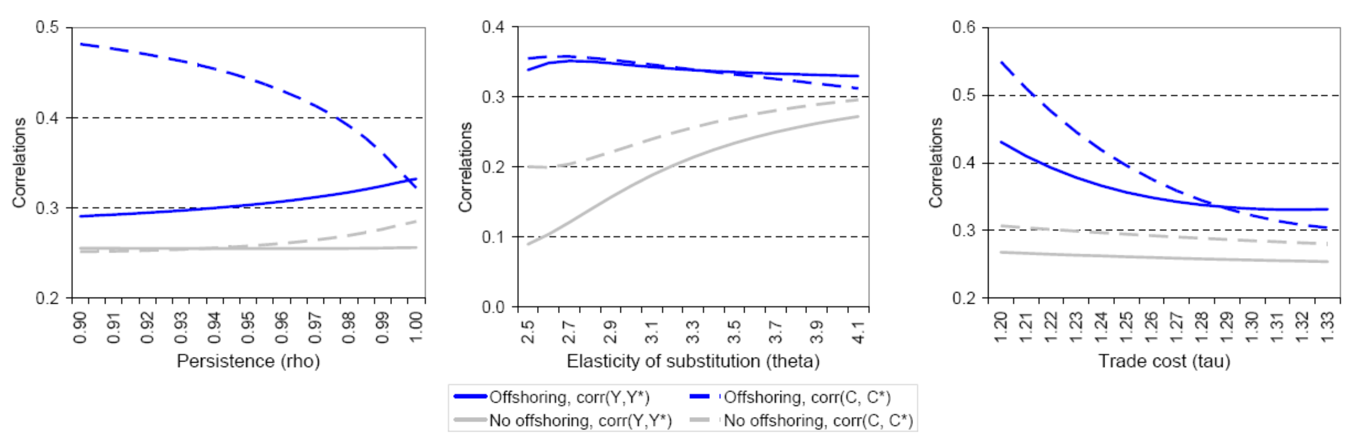

Figure 8. Offshoring under financial autarky: comovement sensitivity to ρ_Z, θ and τ.

5.6 Theoretical Moments: Offshoring and the Macroeconomy

Table 5 shows the theoretical moments generated by the models with offshoring (panel B) and with exports only as in GM2005 (panel C), under the baseline framework with inelastic labor supply augmented with *international trade in bonds*. (The framework with international trade in bonds is described in Appendix 2.) The empirical moments reported in the table are those computed in Mandelman and Zlate (2008) based on data for the U.S. and Mexico. I assume that productivity follows the bivariate productivity process in (38), with the persistence parameter $\rho_Z = \rho_{Z^*} = 0.906$, positive spillover parameters $\rho_{ZZ^*} = \rho_{Z^*Z} = 0.088$, the variance of shocks 0.00852 and the covariance $0.18728 * 10^{-4}$, as in Backus, Kehoe, and Kydland (1992, 1994).

The results are largely similar to the ones in GM2005, with a couple of exceptions on which I focus here. First, investment in the North becomes more pro-cyclical with the Northern output in the presence of offshoring. The correlation between output and investment in the North (where the latter is measured as both total investment in firm entry and the number of new entrant firms) is larger in the model with offshoring (0.89 for each) than in the model with exports only (0.86 and 0.87, respectively). The result is due to the fact that, when offshoring is available as a low-cost alternative to domestic production, the expected profitability of potential entrants in the Northern economy increases, and so do investment and firm entry. In turn, the employment loss caused by offshoring is partially offset by the employment gain generated by stronger firm entry in the presence of offshoring. (The result is in line with the employment dynamics discussed in Appendix 3).

Second, the trade balance for the North becomes more counter-cyclical in the model with offshoring than in the benchmark model with exports only: The correlation between the relative trade balance and output in the North is more negative with offshoring (−0.12) than with exports only (−0.10); the correlation between the offshore value added and the trade balance is negative (−0.23). The result is due to the fact that the imports of offshored varieties contribute to the expanding trade deficit that follows a productivity increase in the North; also, the stronger domestic firm entry in the presence of offshoring increases lending from the South to North.

Nonetheless, when allowing for international trade in bonds, there is no notable increase in the comovement of output in the model with offshoring relative to the model with exports only. The transfer of resources through lending towards the more productive Northern economy (and the resulting decline in Southern output) offsets the enhanced comovement generated by offshoring at the extensive margin.

Table 5. Offshoring and the macroeconomy: empirical vs. theoretical moments

	Absolute std. dev.		Relative std. dev.		Correlations with output in:		Other correlations	
(a) Empirical moments	U.S.	Mex	U.S.	Mex	U.S.	Mex		
Output	1.24	2.32	1.00	1.00	–	–	y_R, y_R^*	0.16
Consumption	0.93	2.84	0.75	1.23	0.83	0.92	C_R, C_R^*	-0.04
Investment	4.18	9.26	3.36	4.00	0.90	0.90	I, I^*	0.21
Trade balance $\left(\frac{TB}{GDP}\right)$	0.33	1.47	0.26	1.47	-0.42	-0.72	$\frac{C_R}{C_R^*}, Q_{CPI}$	-0.47
	North	South	North	South	North	South		
(b) Offshoring, financial integration								
Output (y_R and y_R^*)	0.95	0.92	1.00	1.00	–	–	y_R, y_R^*	0.40
Consumption (C_R and C_R^*)	0.64	0.60	0.67	0.65	0.91	0.87	C_R, C_R^*	0.97
Investment ($\widetilde{v}_R^E$ and $\widetilde{v}_R^{E*}$)	3.23	4.33	3.40	4.71	0.89	0.82	$\widetilde{v}_R^E, \widetilde{v}_R^{E*}$	-0.56
Firm entry (N_E and N_E^*)	3.26	4.40	3.43	4.78	0.89	0.83	$\frac{y_R - C_R}{y_R}, \frac{\widetilde{v}_R^E}{y_R}$	0.97
Trade balance $\left(\frac{TB_R}{GDP_R}\right)$	0.10	0.11	0.11	0.12	-0.12	0.20	$\frac{C_R}{C_R^*}, Q_{CPI}$	0.14
CPI-based RER (Q_{CPI})	0.06		0.06				$\frac{C}{C_R}, Q$	0.74
							TB_R, VA_R	-0.23
(c) No offshoring (GM2005), financial integration								
Output (y_R and y_R^*)	0.95	0.92	1.00	1.00	–	–	y_R, y_R^*	0.40
Consumption (C_R and C_R^*)	0.65	0.61	0.68	0.66	0.91	0.89	C_R, C_R^*	0.96
Investment ($\widetilde{v}_R^E$ and $\widetilde{v}_R^{E*}$)	3.64	3.83	3.83	4.16	0.86	0.84	$\widetilde{v}_R^E, \widetilde{v}_R^{E*}$	-0.55
Firm entry (N_E and N_E^*)	3.66	3.93	3.85	4.27	0.87	0.85	$\frac{y_R - C_R}{y_R}, \frac{\widetilde{v}_R^E}{y_R}$	0.97
Trade balance $\left(\frac{TB_R}{GDP_R}\right)$	0.11	0.11	0.12	0.12	-0.10	-0.04	$\frac{C_R}{C_R^*}, Q_{CPI}$	-0.02
CPI-based RER (Q_{CPI})	0.04		0.04				$\frac{C}{C_R}, Q$	0.74

6 Conclusion

In this paper I study the way in which the relocation of production to other countries alters the cross-country transmission of business cycles. In particular, I focus on the fluctuations in the extensive and intensive margins of offshoring (the number of firms and the value added per firm, respectively), and analyze their impact on output, wages and relative prices in the parent and the host country. Thus, my study adds to the theoretical literature of macroeconomics and international trade which abstracts from the dynamics of offshoring along the extensive and the intensive margins. In the model, offshore production is determined endogenously in the presence of domestic firm entry and heterogeneity in labor productivity across firms. Offshoring depends on the difference between the domestic and foreign cost of effective labor, on the firm-specific labor productivity, as well as on the fixed and iceberg trade costs.

The key results of the paper are as follows. First, the model is successful in replicating the pro-cyclical pattern of offshoring as well as the dynamics of offshoring along the extensive and intensive margins, which I document using data from Mexico's maquiladora sector. Following an aggregate productivity increase in the country of origin, the amount of value added per offshoring firm (the intensive margin) spikes upward on impact, and decreases afterwards. In the medium run, however, domestic firm entry causes the domestic wage to increase faster than aggregate productivity, which in turn determines some of the more productive firms to relocate production offshore. The gradual response of the number of offshoring firms (the extensive margin) mirrors the steady appreciation of the cost of effective labor that follows domestic firm entry. Thus, the model is consistent with the empirical regularity that expansions in U.S. manufacturing output precede increases in the number of offshore plants in Mexico's maquiladora sector.

Second, offshoring enhances the cross-country comovement of manufacturing output relative to the model with endogenous exports. As firm entry in the parent country leads to the appreciation of the terms of labor, the higher demand for domestic labor (due to firm entry) and sequentially higher demand for labor offshore (due to the relocation of production) enhance the co-movement of wages and aggregate incomes. The result is consistent with the stylized fact outlined in Burstein, Kurz, and Tesar (2008), that countries with stronger production sharing trade links tend to display a closer comovement of manufacturing output.

Third, offshoring reduces the price level gap between the countries involved, because it dampens the appreciation of the real exchange rate that follows an aggregate productivity improvement in the parent country (the Harrod-Balassa-Samuelson effect) through several channels. In particular, offshoring transfers some of the upward pressure from the domestic wage (caused by domestic firm entry) onto the foreign wage (through the relocation of production), and thus dampens the appreciation of the terms of labor; offshoring also leads to a decrease in the average import prices and crowds out the increasingly less competitive foreign exports.

I recognize the possibility for several interesting extensions to this paper. First, the model with endogenous offshoring allows for an in-depth analysis of the impact that the relocation of production makes on employment and wages, both in the parent and in the host countries. The employment dynamics discussed in Appendix 3 show that, as offshoring enhances firm entry in the parent country, the domestic job loss caused by offshoring is partially offset by the creation of new jobs associated with new product varieties. Second, the model allows for the study of welfare implications of offshoring and trade liberalization, as discussed in Appendix 4. Third, in an empirical extension of the paper, I study the dynamic response of the extensive and intensive margins of offshore production in Mexico's maquiladora sector to long-run labor productivity shocks in U.S. manufacturing that I identify as permanent, country-specific technology shocks as in Gali (1999). Fourth and finally, one extension with rich policy implications involves the study of dynamic interactions between offshore production and labor migration within an integrated framework in which labor mobility is driven by fluctuations in relative wages, as in Mandelman and Zlate (2008).

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A Appendix

A.1 Offshoring under Financial Autarky

Table A.1. Model Summary, Financial Autarky

Euler equation, bonds	$C_t^{-\gamma} = \beta\left(1 + r_{t+1}\right) E_t\left[C_{t+1}^{-\gamma}\right]$ $C_t^{*-\gamma} = \beta\left(1 + r_{t+1}^*\right) E_t\left[C_{t+1}^{*-\gamma}\right]$
Euler equation, stocks	$\widetilde{v}_t = \beta(1-\delta)E_t\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}(\widetilde{d}_{t+1} + \widetilde{v}_{t+1})$ $\widetilde{v}_t^* = \beta^*(1-\delta^*)E_t\left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\gamma}(\widetilde{d}_{t+1}^* + \widetilde{v}_{t+1}^*)$
Free entry	$\widetilde{v}_t = \frac{f_E w_t}{Z_t}$ $\widetilde{v}_t^* = \frac{f_E^* w_t^*}{Z_t^*}$
Rule of motion, # firms	$N_{t+1} = (1-\delta)(N_t + N_{E,t})$ $N_{D,t+1}^* = (1-\delta)(N_{D,t}^* + N_{E,t}^*)$
Aggregate accounting	$C_t + N_{E,t}\widetilde{v}_t = w_t L + N_t\widetilde{d}_t$ $C_t^* + N_{E,t}^*\widetilde{v}_t^* = w_t^* L^* + N_{D,t}^*\widetilde{d}_t^*$
Consumption price index	$1 = N_{D,t}\left(\widetilde{\rho}_{D,t}\right)^{1-\theta} + N_{V,t}\left(\widetilde{\rho}_{V,t}\right)^{1-\theta} + N_{H,t}^*\left(\widetilde{\rho}_{H,t}^*\right)^{1-\theta}$ $1 = N_{D,t}^*\left(\widetilde{\rho}_{D,t}^*\right)^{1-\theta} + N_{H,t}\left(\widetilde{\rho}_{H,t}\right)^{1-\theta}$
Total profits	$N_t\widetilde{d}_t = N_{D,t}\widetilde{d}_{D,t} + N_{V,t}\widetilde{d}_{V,t} + N_{H,t}\widetilde{d}_{H,t}$ $N_{D,t}^*\widetilde{d}_t^* = N_{D,t}^*\widetilde{d}_{D,t}^* + N_{H,t}^*\widetilde{d}_{H,t}^*$
Number of firms (Home)	$N_t = N_{D,t} + N_{V,t}$
VFDI profits link (Home) HFDI profits link	$\widetilde{d}_{V,t} = \frac{k}{k-(\theta-1)}\left(\frac{z_{V,t}}{\widetilde{z}_{D,t}}\right)^{\theta-1}\widetilde{d}_{D,t} + \frac{\theta-1}{k-(\theta-1)} f_V \frac{w_t^* Q_t}{Z_t^*}$ $\widetilde{d}_{H,t} = \frac{\theta-1}{k-(\theta-1)} f_H \frac{w_t}{Z_t}$ $\widetilde{d}_{H,t}^* = \frac{\theta-1}{k-(\theta-1)} f_H^* \frac{w_t^*}{Z_t^*}$
Dom. productivity (Home) VFDI productivity (Home) HFDI productivity	$\widetilde{z}_{D,t} = \nu z_{\min} z_{V,t}\left[\frac{z_{V,t}^{k-(\theta-1)} - z_{\min}^{k-(\theta-1)}}{z_{V,t}^k - z_{\min}^k}\right]^{\frac{1}{\theta-1}}$ $\widetilde{z}_{V,t} = \nu z_{\min}\left(\frac{N_t}{N_{V,t}}\right)^{1/k}$ $\widetilde{z}_{H,t} = \nu z_{\min}\left(\frac{N_t}{N_{H,t}}\right)^{1/k}$ $\widetilde{z}_{H,t}^* = \nu z_{\min}^*\left(\frac{N_{D,t}^*}{N_{H,t}^*}\right)^{1/k}$
Balanced trade	$N_{H,t}\left(\widetilde{\rho}_{H,t}\right)^{1-\theta} C_t^* Q_t + N_{V,t}\widetilde{d}_{V,t} = N_{V,t}\left(\widetilde{\rho}_{V,t}\right)^{1-\theta} C_t + N_{H,t}^*\left(\widetilde{\rho}_{H,t}^*\right)^{1-\theta} C_t$

The baseline model with financial autarky for the Northern economy is summarized by 16 equations in 16 endogenous variables: N_t, $N_{D,t}$, $N_{V,t}$, $N_{H,t}$, $N_{E,t}$, $\widetilde{d}_t$, $\widetilde{d}_{D,t}$, $\widetilde{d}_{V,t}$, $\widetilde{d}_{H,t}$, $\widetilde{z}_{D,t}$, $\widetilde{z}_{V,t}$, $\widetilde{z}_{H,t}$, $\widetilde{v}_t$, r_t, w_t and C_t. As the Southern firms do not offshore to the high-wage North, the Southern economy is described by only 11 equations in 11 endogenous variables: There are no Southern counterparts for N_t, $N_{V,t}$, $\widetilde{d}_{V,t}$, $\widetilde{z}_{D,t}$ and $\widetilde{z}_{V,t}$. In particular, the average labor productivity of the representative domestic Southern firm ($\widetilde{z}^*_D$) is constant over time. Variables $N_{D,t}$, r_t, N^*_t and r^*_t are predetermined.

A.2 Offshoring with Financial Integration

I allow for trade in international bonds in an extended version of the model with endogenous offshoring. Following GM2005, I assume that: (1) International asset markets are incomplete, as households in each country issue risk-free bonds denominated in their own currency. (2) Nominal returns are indexed to inflation in each economy, so that each type of bonds provides a real return denominated in units of that country's consumption basket. (3) In order to avoid the indeterminacy of steady-state net foreign assets and non-stationarity, I introduce quadratic costs of adjustment for bond holdings, a tool which allows to pin down the steady state and also to ensure stationarity in the presence of temporary shocks.

The infinitely-lived representative household in the North maximizes the inter-temporal utility subject to the constraint:

$$\begin{aligned}&(\widetilde{d}_t+\widetilde{v}_t)N_t x_t + w_t L + (1+r_t)\,B_{h,t} + (1+r^*_t)\,Q_t B_{f,t} + T_t \\ &\geqslant C_t + \widetilde{v}_t\,(N_t+N_{E,t})\,x_{t+1} + B_{h,t+1} + \frac{\pi}{2}\,(B_{h,t+1})^2 + Q_t B_{f,t+1} + \frac{\pi}{2} Q_t\,(B_{f,t+1})^2\,,\end{aligned} \tag{39}$$

where r_t and r^*_t are the rates of return of the North and South-specific bonds; $(1+r_t)B_{h,t}$ and $(1+r^*_t)Q_tB_{f,t}$ denote the principal and interest income from holdings of each type of bonds; $\frac{\pi}{2}\,(B_{h,t+1})^2$ and $\frac{\pi}{2}Q_t\,(B_{f,t+1})^2$ are the cost of adjusting bond holdings, respectively; T_t is the fee rebate. Setting $\pi=0.0025$, I add the two Euler equations for bonds to the baseline model:

$$1+\pi B_{h,t+1} = \beta(1+r_{t+1})E_t\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}, \tag{40}$$

$$1+\pi B_{f,t+1} = \beta(1+r^*_{t+1})E_t\frac{Q_{t+1}}{Q_t}\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}. \tag{41}$$

The budget constraint of the Southern household is similar, and the corresponding Euler equations for bonds are:

$$1+\pi B^{*}_{h,t+1}=\beta^{*}(1+r_{t+1})E_t\frac{Q_t}{Q_{t+1}}\left(\frac{C^{*}_{t+1}}{C^{*}_t}\right)^{-\gamma}, \tag{42}$$

$$1+\pi B^{*}_{f,t+1}=\beta^{*}(1+r^{*}_{t+1})E_t\left(\frac{C^{*}_{t+1}}{C^{*}_t}\right)^{-\gamma}. \tag{43}$$

The market clearing conditions for bonds are:

$$B_{h,t+1}+B^{*}_{h,t+1}=0, \tag{44}$$

$$B_{f,t+1}+B^{*}_{f,t+1}=0. \tag{45}$$

Thus, financial integration through trade in bonds adds 4 new variables ($B_{h,t}, B_{f,t}, B^{*}_{h,t}, B^{*}_{f,t}$) and 6 new equations (40, 41, 42, 43, 44 and 45) while removing the original two Euler equations from the baseline model with financial autarky. Trade in bonds also involves changes in the aggregate accounting equations and in the balanced current account condition. I re-write the expressions for aggregate accounting in the North and South as:

$$C_t+N_{E,t}\widetilde{v}_t+B_{h,t+1}+Q_tB_{f,t+1}=w_tL+N_t\widetilde{d}_t+(1+r_t)\,B_{h,t}+(1+r^{*}_t)\,Q_tB_{f,t}, \tag{46}$$

$$C^{*}_t+N^{*}_{E,t}\widetilde{v}^{*}_t+Q^{-1}_tB^{*}_{h,t+1}+B^{*}_{f,t+1}=w^{*}_tL^{*}+N^{*}_{D,t}\widetilde{d}^{*}_t+(1+r_t)\,Q^{-1}_tB^{*}_{h,t}+(1+r^{*}_t)\,B^{*}_{f,t}. \tag{47}$$

I also replace the balanced current account condition from the model with financial autarky with the expression for the balance of international payments:

$$TB_t+\underbrace{N_{V,t}\widetilde{d}_{V,t}}_{\text{Repatriated profits}}+\underbrace{r_tB_{h,t}+r^{*}_tQ_tB_{f,t}}_{\text{Income from bonds}}=\underbrace{(B_{h,t+1}-B_{h,t})-Q_t\,(B_{f,t+1}-B_{f,t})}_{\text{Change in bond holdings}} \tag{48}$$

which shows that the current account balance (the trade balance plus repatriated profits of foreign affiliates plus investment income) must equal the negative of the financial account balance (the change in bond holdings).

A.3 Employment Dynamics

Theoretical measures of sectoral employment In this section I study the effect of offshoring on employment in both the North and the South. To this end, I focus on the offshoring sector in the Southern economy in addition to the three employment sectors in each economy (entry, domestic and exporting) described in GM2005.[46] The representative Northern offshoring firm hires Southern labor both for covering the fixed cost of offshoring (f_V/Z_t^* units of Southern labor every period) and for production ($\widetilde{l}_{V,t}^* = \widetilde{d}_{V,t}\frac{\theta-1}{w_t^* Q_t} + f_V\frac{\theta-1}{Z_t^*}$). Thus, I write the total employment in the offshoring sector as:

$$L_{V,t}^* = N_{V,t}\left(\widetilde{l}_{V,t}^* + f_V/Z_t^*\right). \tag{49}$$

The log-linearized expressions for total employment in each economy are:

$$\widehat{L}_t = \frac{L_E}{L}\widehat{L}_{E,t} + \frac{L_D}{L}\widehat{L}_{D,t} + \frac{L_H}{L}\widehat{L}_{H,t}, \tag{50}$$

$$\widehat{L}_t^* = \frac{L_V^*}{L^*}\widehat{L}_{V,t}^* + \frac{L_E^*}{L^*}\widehat{L}_{E,t}^* + \frac{L_D^*}{L^*}\widehat{L}_{D,t}^* + \frac{L_H^*}{L^*}\widehat{L}_{H,t}^*, \tag{51}$$

where the calibration implies that the steady state shares of employment in the North are 22, 53 and 25 percent for the entry, domestic and exporting sectors. In the South, they are 15, 48 and 15 percent respectively, plus the remaining 22 percent in the offshoring sector.

Impulse responses for a productivity increase in the North Figure A.1 illustrates the employment dynamics in the offshoring model in response to a positive productivity shock in the North, when productivity follows the autoregressive univariate process $\log Z_{t+1} = \rho \log Z_t + u_t$ with persistence parameter $\rho = 0.9$. In order to analyze the employment dynamics, I add elastic labor supply to the framework with offshoring under financial autarky.[47]

[46] In North, labor is hired for production by the representative *firm serving the domestic market* ($\widetilde{l}_{D,t} = \frac{\theta-1}{w_t}\widetilde{d}_{D,t}$ units of labor), as well as by the representative *exporting firm* that hires labor both for production ($\widetilde{l}_{H,t} = \widetilde{d}_{H,t}\frac{\theta-1}{w_t} + f_H\frac{\theta-1}{Z_t}$) and for covering the fixed cost of exporting every period ($\frac{f_H}{Z_t}$). Thus, the total amount of labor hired by each sector in North is $L_{D,t} = N_{D,t}\widetilde{l}_{D,t}$ and $L_{H,t} = N_{H,t}\left(\widetilde{l}_{H,t} + \frac{f_H}{Z_t}\right)$, respectively. In addition, the *new entrants* in each economy hire labor to cover the sunk entry costs ($\frac{f_E}{Z_t}$ units of labor per new entrant in North); the total amount of labor hired by the entry sector in North is $N_{E,t}\frac{f_H}{Z_t}$ every period.

[47] The representative household aiming to maximize the expected inter-temporal utility $\max_{\{B_t,\, x_t, L_t\}}\left[E_t\sum_{s=t}^{\infty}\beta^{s-t}\left(\ln C_s - \chi\frac{L_s^{1+1/\psi}}{1+1/\psi}\right)\right]$ consumes and supplies L_t working hours elastically in a competitive labor market subject to the budget constraint $B_{t+1} + \widetilde{v}_t\left(N_t + N_{E,t}\right)x_{t+1} + C_t = (1+r_t)B_t + (\widetilde{d}_t + \widetilde{v}_t)N_t x_t + w_t L_t$,where $\chi > 0$ is the weight of disutility from labor in the period utility function, and $\psi \geq 0$ is the Frisch elasticity of labor supply to wages and the inter-temporal elasticity of substitution in labor supply. Following King, Plosser and Rebello (1988) and the discussions in Campbell (1994) and Bilbie et al. (2006), I use log utility for consumption (which is equivalent to setting $\gamma = 1$ in the baseline model) in order to obtain constant steady state labor supply in a model in which utility is additively separable over consumption and hours. I incorporate the usual first order conditions with respect to hours worked into the model, $\chi\left(L_t\right)^{\frac{1}{\psi}} = w_t C_t^{-1}$ and $\chi^*\left(L_t\right)^{\frac{1}{\psi^*}} = w_t^* C_t^{*-1}$.Using the baseline model calibration, I set the weight parameter $\chi = 0.9188$ and $\chi^* = 0.9458$, so that the steady-state level of hours worked is equal to unit, $\overline{L} = \left\{\frac{1}{\chi}\frac{\overline{w}}{\overline{C}}\right\}^{\psi} = 1$. The wage elasticity of labor supply in North and South is $\psi = 3$.

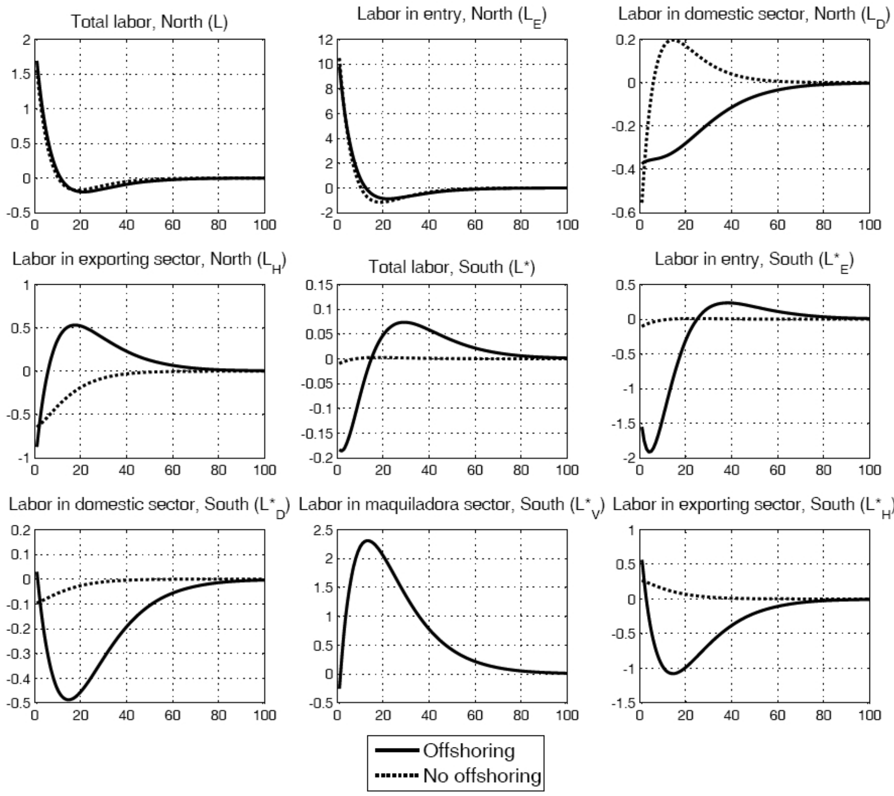

Figure A.1. Employment dynamics,

impulse responses to a transitory 1 percent productivity shock in North

In the North, on impact, employment rises in the entry sector and declines in the domestic and exporting sectors. Thus, the reallocation of labor across sectors supports the creation of new product varieties following the productivity improvement in the Northern economy, both in the model with offshoring and in the model with exports only.

Important differences in employment dynamics across the two models become visible in the medium run. As the option to produce offshore improves the average profitability of prospective entrants, firm entry is more persistent and employment in the entry sector declines by less in the medium run in the model with offshoring than in the model with exports only. Also in the medium run, offshoring stimulates employment in the Northern exporting sector, as the creation of new product varieties and the dampened appreciation of the terms of labor enhance the competitiveness of the Northern exports relative to the model with exports only. However, offshoring reduces employment in the Northern domestic sector, partly due to the relocation of production to the South, and partly due to the within-country reallocation of employment towards the entry and the exporting sectors in the North. Overall, the employment loss in the North caused by offshoring is (partly) offset by the employment gains generated by enhanced product creation and export competitiveness in the North.

In the Southern economy, the increase in employment in the offshoring sector offsets the loss in the domestic and exporting sectors, as well as the loss in the entry sector in the short run. The result is in line with the empirical evidence that given the crowding out of domestic investment, most of the new jobs in Mexico's manufacturing (96 percent) during 1994-2002 were in the maquiladora sector (Gallagher and Zarsky, 2007).

A.4 Welfare Analysis: Trade Costs and Offshore Production

In this section I analyze the welfare effect of a sudden and permanent decrease in the iceberg trade cost affecting the imports of offshored product varieties from $\tau_0 = 1.3$ to $\tau_1 = 1.2$ (a deterministic shock), in addition to the stochastic transitory shocks to aggregate productivity. To this end, I log-linearize the model using a second order approximation around the steady state, and assume that productivity follows the bivariate autoregressive process described in expression (38), with the persistence, spillover and variance-covariance matrix of shocks from Backus, Kehoe, and Kydland (1992, 1994).

Figure A.2 plots the transition paths to the new steady state for key variables of the model. The lower trade cost associated with offshoring increases profitability and hence stimulates firm entry in the North. In turn, the total number of Northern firms, the real wage, output and consumption converge to relatively higher steady state levels, an outcome which is welfare-enhancing for the Northern economy. The total value added offshore and the number of offshoring firms also converge to higher steady state levels. In the Southern economy, the real wage, consumption and output decrease to lower steady state values due to the crowding out of domestic entry by offshoring.

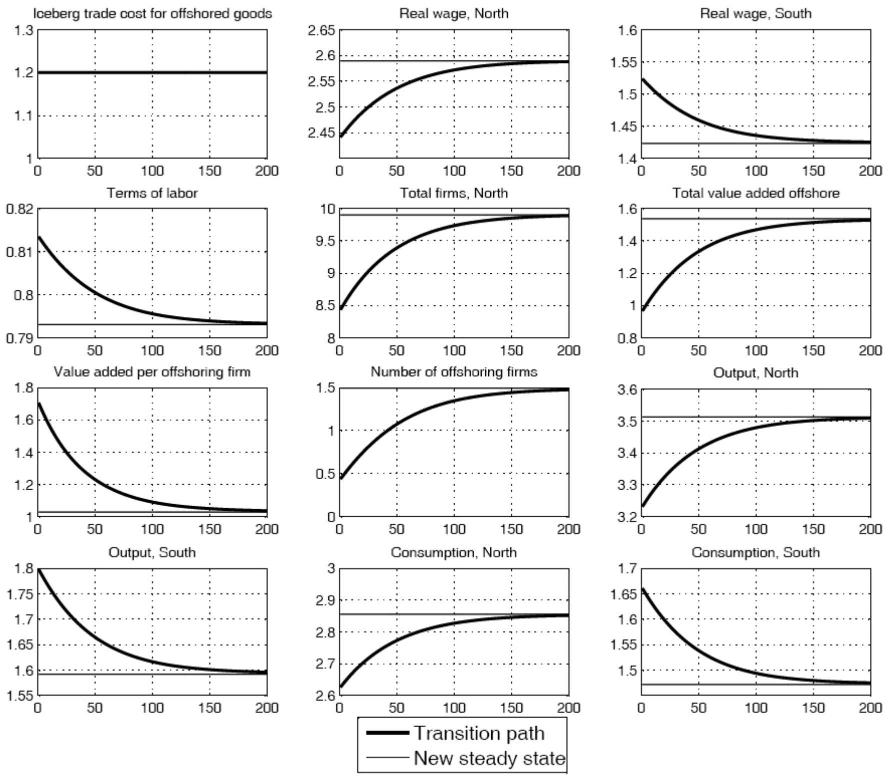

Figure A.2. Transition paths to new steady states, following a permanent decrease in the iceberg trade cost of offshoring (from $\tau_0 = 1.3$ to $\tau_1 = 1.2$)

In order to compute the consumption-equivalent gain for the North, I compare the level of welfare that the Northern household obtains in the initial steady state (V_0) with the level of welfare it holds as of period t' when the decrease in the trade cost of offshoring takes place ($V_{t'}$):

$$V_0 = \frac{1}{1-\beta} U\left(\overline{C}_{\tau_0=1.3}\right) \text{ and } V_{t'} = E_{t'} \sum_{v=t'}^{\infty} \beta^v U\left(\overline{C}_v\right), \tag{52}$$

where the welfare level of period t' takes into account the discounted stream of utilities that the Northern household achieves at all future periods during the transition path to the new steady state. Then I define the constants $\overline{C_0}$ and $\overline{C_1}$ to denote the permanent streams of consumption necessary to generate the welfare values V_0 and $V_{t'}$:

$$V_0 = \frac{1}{1-\beta} \frac{\overline{C_0}^{1-\gamma}}{1-\gamma} \text{ and } V_{t'} = \frac{1}{1-\beta} \frac{\overline{C_1}^{1-\gamma}}{1-\gamma}, \tag{53}$$

and compute the consumption-equivalent welfare gain ($\lambda > 0$) or loss ($\lambda < 0$) that corresponds to the permanent decrease of the iceberg trade cost for offshored goods:

$$\lambda = \left(\frac{\overline{C_1}}{\overline{C_0}} - 1 \right) \times 100. \tag{54}$$

Figure A.3 (continuous line) plots the consumption-equivalent welfare gain measured as a percentage increase in steady-state consumption (on the vertical axis) associated with the permanent decrease in the trade cost for offshored goods; I allow the elasticity of substitution between domestic and offshored varieties (on the horizontal axis) to vary over $\theta \in [3.1, 3.9]$. The results show that the Northern economy obtains a welfare gain that exceeds the equivalent of 5 percent of initial consumption for the entire range of elasticity values. Moreover, the gain increases with the degree of complementarity between the domestic and offshored varieties.

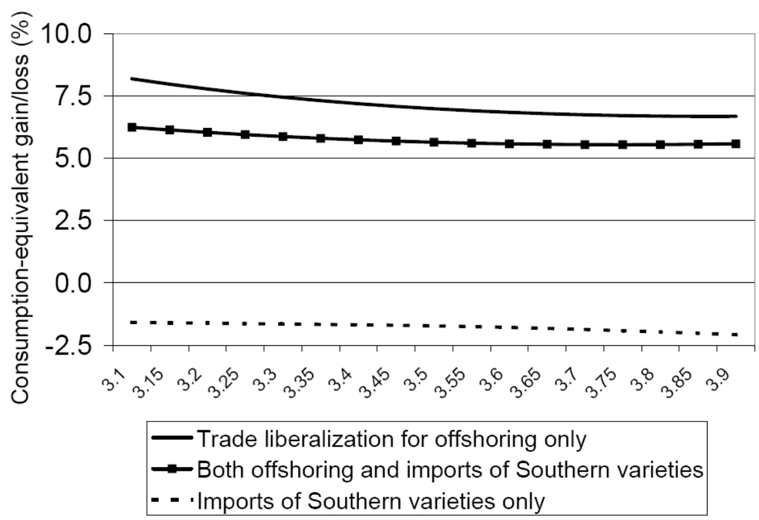

Figure A.3. Consumption-equivalent welfare gain/loss, following a permanent decrease in the iceberg trade cost of offshoring (from $\tau_0 = 1.3$ to $\tau_1 = 1.2$)

A.5 Asymmetric Firm Entry Costs

The World Bank's *Doing Business* report outlines the large variation in the regulation of starting a business across countries at different levels of economic development (Table A.2). For instance, the monetary cost is 3.3 times higher in Mexico than in the U.S. or Canada; it is 6.2 times higher in

Hungary than in the U.K.

Table A.2. Firm entry costs, selected economies

Economy	Procedures (number)	Duration (days)	Monetary Cost (USD)	**Relative Cost (U.S.=1.0)**
U.S.	6	6	314.79	**1.0**
Canada	2	3	325.53	**1.0**
Mexico	8	27	1,046.71	**3.3**
Germany	9	18	2,087.34	**6.6**
U.K.	6	13	321.44	**1.0**
France	5	7	402.05	**1.3**
Poland	10	31	1,736.28	**5.5**
Czech Republic	10	17	1,344.08	**4.3**
Hungary	6	16	1,938.15	**6.2**

(Source: World Bank, *Doing Business* Report, 2007)

A.6 Solution for the Asymmetric Steady State ($TOL < 1$)

In this section I provide the steady state solution for the offshoring model with cross-country differences in the cost of effective labor in steady state ($TOL < 1$). To this end, I use an integrated framework that nests both the baseline model with offshoring ($\alpha = 0, \eta = 1$) and the benchmark model with exports ($\alpha = 1, \eta = 1$) as in Ghironi and Melitz (2005), as described in the text.

I provide a numerical solution for the unique steady state using a non-linear system of 12 equations (listed below) in 12 unknowns. The unknowns are the steady state values of: z_V (the offshoring cutoff productivity), z_H (the exporting cutoff productivity in North), TOL, $\frac{C}{C^*Q}$, Q, $\frac{\widetilde{d}_D}{w}$, $\frac{\widetilde{d}_V}{w}$, $\frac{\widetilde{d}_H}{w}$, z_H^* (the exporting cutoff productivity in South), $\widetilde{\rho}_H$, $\widetilde{\rho}_H^*$, and $\frac{N}{N_D^*}$. Subsequently, the numerical solution for the initial 12 variables allows for an analytical solution for the steady state values of the remaining variables of the model. A technical appendix providing their complete derivation is available upon request.

I use the following pricing and profit formulas (in which $Z = Z^* = 1$) in order to derive the steady state solution:

Table A.3. Prices and Profits

	Prices	Profits
Domestic prod., North	$\widetilde{\rho}_D = \frac{\theta}{\theta-1}\frac{w_t}{\widetilde{z}_D}$	$\widetilde{d}_{D,t} = \frac{1}{\theta}\left(\widetilde{\rho}_{D,t}\right)^{1-\theta} C_t$
Domestic prod., South	$\widetilde{\rho}^*_D = \frac{\theta}{\theta-1}\frac{w^*_t}{\widetilde{z}^*_D}$	$\widetilde{d}^*_{D,t} = \frac{1}{\theta}\left(\widetilde{\rho}^*_{D,t}\right)^{1-\theta} C^*_t$
Offshore prod. (VFDI)	$\widetilde{\rho}_V = \frac{\theta}{\theta-1}\frac{w}{\widetilde{z}_V}\left(\tau TOL\right)^{1-\alpha}$	$\widetilde{d}_{V,t} = \frac{1}{\theta}\left(\widetilde{\rho}_{V,t}\right)^{1-\theta} C_t - f_V w TOL^{1-\alpha}$
Exports/HFDI, North	$\widetilde{\rho}_H = \frac{\theta}{\theta-1}\frac{\tau^{*\eta} w Q^{-1}}{\widetilde{z}_H} TOL^{1-\eta}$	$\widetilde{d}_{H,t} = \frac{1}{\theta}\left(\widetilde{\rho}_{H,t}\right)^{1-\theta} C^*_t Q_t - f_H w TOL^{1-\eta}$
Exports/HFDI, South	$\widetilde{\rho}^*_H = \frac{\theta}{\theta-1}\frac{\tau^{\eta^*} w^* Q}{\widetilde{z}^*_H}\left(\frac{1}{TOL}\right)^{1-\eta^*}$	$\widetilde{d}^*_{H,t} = \frac{1}{\theta}\left(\widetilde{\rho}^*_{H,t}\right)^{1-\theta} C_t Q_t^{-1} - f^*_H w^* \left(\frac{w^*_t}{Z^*_t}\right)\left(\frac{1}{TOL}\right)^{1-\eta^*}$

In addition, using that $v = \frac{\beta(1-\delta)}{1-\beta(1-\delta)} d$, $N_E = \frac{\delta}{1-\delta} N$, and $v = f_e w$ in the expression for total profits in the Northern economy, the first equation in the system is:

$$\frac{1-\beta(1-\delta)}{\beta(1-\delta)} f_E = \frac{N_D}{N}\frac{\widetilde{d}_D}{w} + \frac{N_V}{N}\frac{\widetilde{d}_V}{w} + \frac{N_H}{N}\frac{\widetilde{d}_H}{w}, \tag{55}$$

where $\frac{N_H}{N} = \left(\frac{1}{z_H}\right)^k$, $\frac{N_D}{N} = 1 - \left(\frac{1}{z_H}\right)^k$, $\frac{N_V}{N} = \left(\frac{1}{z_V}\right)^k$.

Next, the profit formulas for the Northern economy imply:

$$\frac{\widetilde{d}_D}{w} = \frac{k}{k-(\theta-1)} f_H TOL^{\theta(1-\eta)} \frac{C}{C^* Q} Q^{1-\theta} \tau^{*(\theta-1)\eta} \left(\frac{z_V}{z_H}\right)^{\theta-1} \frac{z_V^{k-(\theta-1)} - 1}{z_V^k - 1}, \tag{56}$$

$$\frac{\widetilde{d}_V}{w} = \frac{k}{k-(\theta-1)} f_H TOL^{1-\alpha+\theta(\alpha-\eta)} \frac{C}{C^* Q} Q^{1-\theta} \left(\frac{z_V}{z_H}\right)^{\theta-1} \frac{\tau^{*(\theta-1)\eta}}{\tau^{(1-\alpha)(\theta-1)}} - f_V TOL^{1-\alpha}, \tag{57}$$

$$\frac{\widetilde{d}_H}{w} = \frac{(\theta-1)}{k-(\theta-1)} f_H w TOL^{1-\eta}, \tag{58}$$

$$\frac{\widetilde{d}_V}{w} = \frac{z_V^k - 1}{z_V^{k-(\theta-1)} - 1}\frac{\widetilde{d}_D}{w} + \frac{(\theta-1)}{k-(\theta-1)} f_V w TOL^{1-\alpha}. \tag{59}$$

The expression for total profits in the Southern economy implies:

$$\frac{1-\beta^*(1-\delta^*)}{\beta^*(1-\delta^*)} f^*_E = \frac{k}{k-(\theta-1)} f^*_H TOL^{\theta(\eta^*-1)} \tau^{\eta^*(\theta-1)} Q^{\theta-1} z_H^{*1-\theta} \frac{C^* Q}{C} + \\ + \frac{\theta-1}{k-(\theta-1)} \left(\frac{1}{z^*_H}\right)^k f^*_H TOL^{\eta^*-1}, \tag{60}$$

Next, the consumption ratio adjusted for the real exchange rate is:

$$\frac{C}{C^{*}Q}=\frac{f_{H}^{*}}{f_{H}}TOL^{\theta(\eta+\eta^{*}-1)}Q^{\theta-1}\left(\frac{z_{H}\tau^{\eta^{*}}}{z_{H}^{*}\tau^{*\eta}}\right)^{\theta-1}. \tag{61}$$

Using the balanced current account condition, I obtain:

$$\begin{aligned}&(1-\alpha)\,z_{V}^{-k}TOL^{-\alpha}\left[\frac{(\theta-1)\,k}{k-(\theta-1)}f_{H}^{*}\left(\frac{z_{V}}{z_{H}^{*}}\right)^{\theta-1}\frac{TOL^{\theta(\alpha+\eta^{*}-1)}}{\tau^{(\theta-1)(1-\alpha-\eta^{*})}}+f_{V}\right] \qquad (62)\\&=\Lambda\frac{f_{H}}{z_{H}^{k}}\left(\frac{N}{N_{D}^{*}}\right)^{-1}TOL^{-\eta}-\Lambda^{*}\frac{f_{H}^{*}}{z_{H}^{*k}}\left(\frac{N}{N_{D}^{*}}\right)^{-1}TOL^{\eta^{*}-1},\end{aligned}$$

where $\Lambda=\left(\eta+\frac{(1-\eta)}{\theta}\right)\frac{k\theta}{k-(\theta-1)}-(1-\eta)$ and $\Lambda^{*}=\left(\eta^{*}+\frac{(1-\eta^{*})}{\theta}\right)\frac{k\theta}{k-(\theta-1)}-(1-\eta^{*})$.

The expression for the real exchange rate in steady state is:

$$Q^{1-\theta}=\frac{TOL^{1-\theta}+\left(\tau^{*\eta}TOL^{1-\eta}\right)^{1-\theta}z_{H}^{\theta-1-k}\frac{N}{N_{D}^{*}}}{\left(1-z_{V}^{-k}\right)z_{V}^{\theta-1}\frac{z_{V}^{k-(\theta-1)}-1}{z_{V}^{k}-1}\frac{N}{N_{D}^{*}}+z_{V}^{\theta-1-k}\frac{N}{N_{D}^{*}}\left(\tau TOL\right)^{(1-\alpha)(1-\theta)}+z_{H}^{*\theta-1-k}\frac{N}{N_{D}^{*}}\left(\tau TOL\right)^{\eta^{*}(1-\theta)}} \tag{63}$$

The remaining equations are:

$$\frac{\theta k}{k-(\theta-1)}f_{H}\frac{\widetilde{\rho}_{H}^{\theta-1}}{TOL^{\eta}}=1+\frac{1-\beta^{*}}{\beta^{*}\left(1-\delta^{*}\right)}f_{E}^{*}\frac{\widetilde{\rho}_{H}^{\theta-1}}{\Xi_{t}}, \tag{64}$$

$$\frac{\theta k}{k-(\theta-1)}f_{H}^{*}TOL^{\eta^{*}}\widetilde{\rho}_{H}^{*\theta-1}=1+\frac{1-\beta}{\beta\left(1-\delta\right)}f_{E}\frac{\widetilde{\rho}_{H}^{*\theta-1}}{\Omega_{t}}, \tag{65}$$

$$\frac{N}{N_{D}^{*}}\left(\frac{\widetilde{\rho}_{H}}{\widetilde{\rho}_{H}^{*}}\right)^{\theta-1}=\frac{\Xi_{t}}{\Omega_{t}}, \tag{66}$$

where:

$$\Xi_{t}=\left[\frac{1}{z_{H}}\left(\frac{\tau^{*}}{TOL}\right)^{\eta}\right]^{\theta-1}+z_{H}^{-k}\frac{N}{N_{D}^{*}}, \tag{67}$$

$$\Omega_{t}=\left(1-z_{V}^{-k}\right)\left(\frac{z_{V}}{z_{H}^{*}}\right)^{\theta-1}\frac{z_{V}^{k-(\theta-1)}-1}{z_{V}^{k}-1}\left(\tau TOL\right)^{\eta^{*}(\theta-1)}+z_{V}^{-k}\left[\frac{z_{V}}{z_{H}^{*}}\left(\tau TOL\right)^{\eta^{*}+\alpha-1}\right]^{\theta-1}+z_{H}^{*-k}\left(\frac{N}{N_{D}^{*}}\right)^{-1}. \tag{68}$$

A.7 Demand Functions and the Welfare-Based Price Index

The Northern representative household minimizes the total expenditure associated with the consumption basket C_t, which includes final good varieties produced by the Northern firms domestically ($y_{D,t}$), offshore ($y_{V,t}$), as well as final good varieties produced by Southern firms ($y^*_{H,t}$):

$$\min_{\left\{y_{D,t}(z),\, y_{V,t}(z), y^*_{H,t}(z)\right\}} P_t C_t = \underbrace{\int_{z_{\min}}^{z_{V,t}} p_{D,t}(z) y_{D,t}(z) dz}_{\text{Produced domestically}} + \underbrace{\int_{z_{V,t}}^{\infty} p_{V,t}(z) y_{V,t}(z) dz}_{\text{Produced offshore}} + \underbrace{\int_{z^*_{H,t}}^{\infty} p^*_{H,t}(z) y^*_{H,t}(z) dz}_{\text{Produced by Southern firms}} , \quad (69)$$

subject to $C_t = \left[\int_{z_{\min}}^{z_{V,t}} y_{D,t}(z)^{\frac{\theta-1}{\theta}} dz + \int_{z_{V,t}}^{\infty} y_{V,t}(z)^{\frac{\theta-1}{\theta}} dz + \int_{z^*_{H,t}}^{\infty} y^*_{H,t}(z)^{\frac{\theta-1}{\theta}} dz\right]^{\frac{\theta}{\theta-1}}$. The first-order conditions with respect to $y_{D,t}(z)$, $y_{V,t}(z)$ and $y^*_{H,t}(z)$ imply:

$$p_{D,t}(z) = \lambda_t C_t^{\frac{1}{\theta}} y_{D,t}(z)^{-\frac{1}{\theta}}, \; p_{V,t}(z) = \lambda_t C_t^{\frac{1}{\theta}} y_{V,t}(z)^{-\frac{1}{\theta}} \text{ and } p^*_{H,t}(z) = \lambda_t C_t^{\frac{1}{\theta}} y^*_{H,t}(z)^{-\frac{1}{\theta}}, \quad (70)$$

which I use to re-write the total expenditure amount:

$$P_t C_t = \int_{z_{\min}}^{z_{V,t}} p_{D,t}(z) y_{D,t}(z) dz + \int_{z_{V,t}}^{\infty} p_{V,t}(z) y_{V,t}(z) dz + \int_{z^*_{H,t}}^{\infty} p_{H,t}(z) y^*_{H,t}(z) dz = \quad (71)$$

$$= \lambda_t C_t^{\frac{1}{\theta}} \underbrace{\left[\int_{z_{\min}}^{z_{V,t}} y_{D,t}(z)^{\frac{\theta-1}{\theta}} dz + \int_{z_{V,t}}^{\infty} y_{V,t}(z)^{\frac{\theta-1}{\theta}} dz + \int_{z^*_{H,t}}^{\infty} y^*_{H,t}(z)^{\frac{\theta-1}{\theta}} dz\right]}_{C_t^{\frac{\theta-1}{\theta}}} = \lambda_t C_t. \quad (72)$$

Next I insert the resulting identity $\lambda_t = P_t$ and the demand functions $y_{D,t}(z) = \left(p_{D,t}(z)/P_t\right)^{-\theta} C_t$, $y_{V,t}(z) = \left(p_{V,t}(z)/P_t\right)^{-\theta} C_t$, $y^*_{H,t}(z) = \left(p^*_{H,t}(z)/P_t\right)^{-\theta} C_t$ into the expression for total expenditure, $P_t C_t = \int_{z_{\min}}^{z_{V,t}} p_{D,t}(z) y_{D,t}(z) dz + \int_{z_{V,t}}^{\infty} p_{V,t}(z) y_{V,t}(z) dz + \int_{z^*_{H,t}}^{\infty} p^*_{H,t}(z) y^*_{H,t}(z) dz$, in order to derive the price index:

$$P_t = \left[\int_{z_{\min}}^{z_{V,t}} p_{D,t}(z)^{1-\theta} dz + \int_{z_{V,t}}^{\infty} p_{V,t}(z)^{1-\theta} dz + \int_{z^*_{H,t}}^{\infty} p_{H,t}(z)^{1-\theta} dz\right]^{\frac{1}{1-\theta}}. \quad (73)$$

Throughout the model I use the consumption basket as the numeraire good in each economy. Thus, the real prices of final good varieties expressed in units of the Northern consumption basket are:

$$\rho_{D,t}(z) \equiv \frac{p_{D,t}(z)}{P_t}, \ \rho_{V,t}(z) \equiv \frac{p_{V,t}(z)}{P_t} \text{ and } \rho^*_{H,t}(z) \equiv \frac{p^*_{H,t}(z)}{P_t}, \tag{74}$$

and the demand functions for final good varieties become:

$$y_{D,t}(z) = \rho_{D,t}(z)^{-\theta}C_t, \ y_{V,t}(z) = \rho_{V,t}(z)^{-\theta}C_t, \text{ and } y^*_{H,t}(z) = \rho^*_{H,t}(z)^{-\theta}C_t. \tag{75}$$

A.8 Profit Maximization with Domestic and Offshore Production

Northern firms producing domestically Firms set optimal prices by solving the profit maximization problem:

$$\max_{\{\rho_{D,t}(z)\}} \rho_{D,t}(z)y_{D,t}(z) - \frac{w_t}{Z_t z}y_{D,t}(z). \tag{76}$$

Using the demand function $y_{D,t}(z) = \rho_{D,t}(z)^{-\theta}C_t$, the price is equal to the marginal cost multiplied by the markup:

$$y_{D,t}(z) + \rho_{D,t}(z)\frac{\partial y_{D,t}(z)}{\partial \rho_{D,t}(z)} - \frac{w_t}{Z_t z}\frac{\partial y_{D,t}(z)}{\partial \rho_{D,t}(z)} = 0 \Rightarrow \rho_{D,t}(z) = \frac{\theta}{\theta-1}\frac{w_t}{Z_t z}. \tag{77}$$

Northern firms producing offshore The firm with idiosyncratic labor productivity z that produces final goods using a mix of domestic and offshore inputs solves the following profit maximizing problem:[48]

$$\max_{\{\rho_{V,t}\}} \rho_{V,t}(z)y_{V,t}(z) - \left(\frac{w_t}{Z_t z}\right)^{\alpha}\left(\tau\frac{w^*_t Q_t}{Z^*_t z}\right)^{1-\alpha} y_{D,t}(z) - f_V\left(\frac{w_t}{Z_t}\right)^{\alpha}\left(\frac{w^*_t Q_t}{Z^*_t}\right)^{1-\alpha}. \tag{78}$$

Using the demand function $y_{V,t}(z) = \rho_{V,t}(z)^{-\theta}C_t$, the resulting price formula is:

$$\rho_{V,t}(z) = \frac{\theta}{\theta-1}\left(\frac{w_t}{Z_t z}\right)^{\alpha}\left(\tau\frac{w^*_t Q_t}{Z^*_t z}\right)^{1-\alpha}. \tag{79}$$

[48] The cost minimization problem in the broader framework of offshoring, $\min_{\{l_t, l^*_t\}} w_t l_t + \tau w^*_t Q_t l^*_t$ so that $y_{V,t}(z) = \left[\frac{Z_t z l_t}{\alpha}\right]^{\alpha}\left[\frac{Z^*_t z l^*_t}{1-\alpha}\right]^{1-\alpha}$, leads to the following expression for the marginal cost: $MC_t = \left(\frac{w_t}{Z_t z}\right)^{\alpha}\left(\frac{\tau w^*_t Q_t}{Z^*_t z}\right)^{1-\alpha}$.

Firms serving the foreign market The pricing formulas for the firms from either country serving the foreign market through either exports or horizontal FDI can be obtained in a similar way:

$$\rho_{H,t}(z) = \frac{\theta}{\theta-1}\left(\tau^* \frac{w_t Q_t^{-1}}{Z_t z}\right)^{\eta}\left(\frac{w_t^*}{Z_t^* z}\right)^{1-\eta} \text{ and } \rho_{H,t}^*(z) = \frac{\theta}{\theta-1}\left(\tau \frac{w_t^* Q_t}{Z_t^* z}\right)^{\eta^*}\left(\frac{w_t}{Z_t z}\right)^{1-\eta^*}. \tag{80}$$

A.9 Existence of Equilibrium for the Offshoring Productivity Cutoff ($z_{V,t}$)

As discussed in the text, two conditions must hold every period in order to ensure the existence of equilibrium productivity cutoff $z_{V,t}$: (1) $d_{V,t}(z)$ is steeper than $d_{D,t}(z)$, and (2) $z_{\min} < z_{V,t}$. The first condition implies that the effective wage in South must be low enough relative to the effective wage in North ($TOL_t < 1$) in order to offset the iceberg trade cost ($\tau > 1$):

$$\tau \frac{w_t^* Q_t}{Z_t^*} < \frac{w_t}{Z_t z} \Longleftrightarrow \tau TOL_t < 1. \tag{81}$$

The second condition, $z_{\min} < z_{V,t}$, requires that:

$$\begin{aligned} Slope(d_{V,t}(z)) &< \frac{\Theta f_E \frac{w_t}{Z_t} + f_V \frac{w_t^* Q_t}{Z_t^*}}{z_{\min}^{\theta-1}}, \\ z_{\min}^{\theta-1}\frac{1}{\theta}\left(\frac{\theta}{\theta-1}\frac{w_t}{Z_t}\right)^{1-\theta} C_t &< \Theta f_E \frac{w_t}{Z_t} + f_V \frac{w_t^* Q_t}{Z_t^*}, \\ \frac{1}{\theta}\left(\frac{\theta}{\theta-1}\frac{w_t}{Z_t z_{\min}}\right)^{1-\theta} C_t &< \Theta f_E \frac{w_t}{Z_t} + f_V \frac{w_t^* Q_t}{Z_t^*}, \\ d_{D,t}(z_{\min}) &< \Theta f_E \frac{w_t}{Z_t} + f_V \frac{w_t^* Q_t}{Z_t^*}, \end{aligned} \tag{82}$$

where $\Theta = \frac{1-\beta(1-\delta)}{\beta(1-\delta)}$. The last inequality shows that the profit obtained from domestic production by the firm with the minimum productivity z_{min} must be smaller than the sum of the per-period value of the sunk entry cost and the fixed cost of offshoring. In other words, the firm that obtains zero profit from domestic production would make negative profits if it engages in offshore production, i.e. the firm with productivity z_{min} does not produce in either country.

A.10 Average Firm-Specific Productivities under the Pareto Distribution

Northern firms producing offshore

$$\widetilde{z}_{V,t} = \left[\frac{1}{1 - G(z_{V,t})} \int_{z_{V,t}}^{\infty} z^{\theta-1} g(z) dz \right]^{\frac{1}{\theta-1}} = \left[\left(\frac{z_{V,t}}{z_{\min}} \right)^k \int_{z_{V,t}}^{\infty} z^{\theta-1} \frac{k z_{\min}^k}{z^{k+1}} dz \right]^{\frac{1}{\theta-1}} =$$

$$= \left[\left(\frac{z_{V,t}}{z_{\min}} \right)^k \frac{k z_{\min}^k}{k - (\theta-1)} z_{V,t}^{\theta-1-k} \right]^{\frac{1}{\theta-1}} = \nu z_{V,t}, \text{ where } \nu \equiv \left[\frac{k}{k - (\theta-1)} \right]^{\frac{1}{\theta-1}}. \tag{83}$$

Northern firms producing domestically

$$\widetilde{z}_{D,t} = \left[\frac{1}{G(z_{V,t})} \int_{z_{\min}}^{z_{V,t}} z^{\theta-1} g(z) dz \right]^{\frac{1}{\theta-1}} = \left[\frac{z_{V,t}^k}{z_{V,t}^k - z_{\min}^k} \int_{z_{\min}}^{z_{V,t}} z^{\theta-1} \frac{k z_{\min}^k}{z^{k+1}} dz \right]^{\frac{1}{\theta-1}} =$$

$$= \left[\frac{z_{V,t}^k}{z_{V,t}^k - z_{\min}^k} \frac{k z_{\min}^k}{(\theta - k - 1)} \left(z_{V,t}^{\theta-1-k} - z_{\min}^{\theta-1-k} \right) \right]^{\frac{1}{\theta-1}} =$$

$$= \nu \left[\frac{(z_{\min} z_{V,t})^k}{z_{\min}^k - z_{V,t}^k} \left(\frac{1}{z_{V,t}^{k-(\theta-1)}} - \frac{1}{z_{\min}^{k-(\theta-1)}} \right) \right]^{\frac{1}{\theta-1}} = \nu z_{\min} z_{V,t} \left[\frac{z_{V,t}^{k-(\theta-1)} - z_{\min}^{k-(\theta-1)}}{z_{V,t}^k - z_{\min}^k} \right]^{\frac{1}{\theta-1}}. \tag{84}$$

A.11 Average Profits: Domestic and Offshore Production

The average profit of the Northern firms producing domestically is:

$$\widetilde{d}_{D,t} = d_{D,t}(\widetilde{z}_{D,t}) = \frac{1}{\theta} \left[\frac{\theta}{\theta-1} \frac{w_t}{Z_t \widetilde{z}_{D,t}} \right]^{1-\theta} C_t = \frac{1}{\theta} \left[\frac{\theta}{\theta-1} \frac{w_t}{Z_t} \right]^{1-\theta} C_t \widetilde{z}_{D,t}^{\theta-1} =$$

$$= \frac{1}{\theta} \left[\frac{\theta}{\theta-1} \frac{w_t}{Z_t} \right]^{1-\theta} C_t \left(\nu z_{\min} z_{V,t} \right)^{\theta-1} \left[\frac{z_{V,t}^{k-(\theta-1)} - z_{\min}^{k-(\theta-1)}}{z_{V,t}^k - z_{\min}^k} \right] =$$

$$= \underbrace{\frac{1}{\theta} \left[\frac{\theta}{\theta-1} \frac{w_t}{Z_t z_{V,t}} \right]^{1-\theta} C_t}_{d_{D,t}(z_{V,t})} \left(\nu z_{\min} \right)^{\theta-1} \left[\frac{z_{V,t}^{k-(\theta-1)} - z_{\min}^{k-(\theta-1)}}{z_{V,t}^k - z_{\min}^k} \right] =$$

$$= d_{D,t}(z_{V,t}) \left(\nu z_{\min} \right)^{\theta-1} \left[\frac{z_{V,t}^{k-(\theta-1)} - z_{\min}^{k-(\theta-1)}}{z_{V,t}^k - z_{\min}^k} \right].$$

Similarly, the average profit of the Northern firm producing offshore through vertical FDI is:

$$\begin{aligned}
\widetilde{d}_{V,t} &= d_{V,t}(\widetilde{z}_{V,t}) = \frac{1}{\theta}\left[\tau\frac{\theta}{\theta-1}\frac{w_t^* Q_t}{Z_t^* \widetilde{z}_{V,t}}\right]^{1-\theta} C_t - f_V \left(\frac{w_t}{Z_t}\right)^{\alpha}\left(\frac{w_t^* Q_t}{Z_t^*}\right)^{1-\alpha} = \\
&= \frac{1}{\theta}\left[\tau\frac{\theta}{\theta-1}\frac{w_t^* Q_t}{Z_t^*}\right]^{1-\theta} C_t \widetilde{z}_{V,t}^{\theta-1} - f_V \left(\frac{w_t}{Z_t}\right)^{\alpha}\left(\frac{w_t^* Q_t}{Z_t^*}\right)^{1-\alpha} = \\
&= \underbrace{\left\{\frac{1}{\theta}\left[\tau\frac{\theta}{\theta-1}\frac{w_t^* Q_t}{Z_t^* z_{V,t}}\right]^{1-\theta} C_t - f_V \left(\frac{w_t}{Z_t}\right)^{\alpha}\left(\frac{w_t^* Q_t}{Z_t^*}\right)^{1-\alpha}\right\}}_{d_{V,t}(z_{V,t})}\nu^{\theta-1} + \left(\nu^{\theta-1}-1\right) f_V \left(\frac{w_t}{Z_t}\right)^{\alpha}\left(\frac{w_t^* Q_t}{Z_t^*}\right)^{1-\alpha} = \\
&= d_{V,t}(z_{V,t})\nu^{\theta-1} + \frac{\theta-1}{k-(\theta-1)} f_V \left(\frac{w_t}{Z_t}\right)^{\alpha}\left(\frac{w_t^* Q_t}{Z_t^*}\right)^{1-\alpha}.
\end{aligned}$$

The Northern firm with productivity equal to the cutoff $z_{V,t}$ is indifferent between locating production domestically or offshore. Hence, the identity of profits at the productivity cutoff, $d_{D,t}(z_{V,t}) = d_{V,t}(z_{V,t})$, and the two expressions above allow to write the link between the profits to the two representative Northern firms:

$$\begin{aligned}
\widetilde{d}_{V,t} &= \underbrace{\left(\frac{1}{\nu z_{\min}}\right)^{\theta-1}\left[\frac{z_{V,t}^{k-(\theta-1)} - z_{\min}^{k-(\theta-1)}}{z_{V,t}^{k} - z_{\min}^{k}}\right]^{-1}\widetilde{d}_{D,t}}_{=d_{D,t}(z_{V,t})}\nu^{\theta-1} + \frac{\theta-1}{k-(\theta-1)} f_V \left(\frac{w_t}{Z_t}\right)^{\alpha}\left(\frac{w_t^* Q_t}{Z_t^*}\right)^{1-\alpha} = \\
&= z_{\min}^{1-\theta}\left[\frac{z_{V,t}^{k-(\theta-1)} - z_{\min}^{k-(\theta-1)}}{z_{V,t}^{k} - z_{\min}^{k}}\right]^{-1}\widetilde{d}_{D,t} + \frac{\theta-1}{k-(\theta-1)} f_V \left(\frac{w_t}{Z_t}\right)^{\alpha}\left(\frac{w_t^* Q_t}{Z_t^*}\right)^{1-\alpha} = \\
&= \frac{k}{k-(\theta-1)}\left(\frac{z_{V,t}}{\widetilde{z}_{D,t}}\right)^{\theta-1}\widetilde{d}_{D,t} + \frac{\theta-1}{k-(\theta-1)} f_V \left(\frac{w_t}{Z_t}\right)^{\alpha}\left(\frac{w_t^* Q_t}{Z_t^*}\right)^{1-\alpha}. \qquad (85)
\end{aligned}$$

A.12 The Real Exchange Rate

Using the definition $\widetilde{Q}_t^{1-\theta} = \left(\frac{N_{D,t}+N_{V,t}+N^*_{H,t}}{N^*_{D,t}+N_{H,t}}\right) Q_t^{1-\theta}$ and the notation $\widetilde{N}_t \equiv N_{D,t} + N_{V,t} + N^*_{H,t}$, $\widetilde{N}^*_t \equiv N^*_{D,t} + N_{H,t}$; I re-write the CPI-based real exchange rate as:

$$\widetilde{Q}_t^{1-\theta} = \frac{\widetilde{N}_t}{\widetilde{N}^*_t} \frac{N^*_{D,t}\left(\widetilde{\rho}^*_{D,t} P^*_t \varepsilon_t\right)^{1-\theta} + N_{H,t}\left(\widetilde{\rho}_{H,t} P^*_t \varepsilon_t\right)^{1-\theta}}{N_{D,t}\left(\widetilde{\rho}_{D,t} P_t\right)^{1-\theta} + N_{V,t}\left(\widetilde{\rho}_{V,t} P_t\right)^{1-\theta} + N^*_{H,t}\left[\widetilde{\rho}^*_{H,t} P_t\right]^{1-\theta}} =$$
$$= \frac{\widetilde{N}_t}{\widetilde{N}^*_t} \frac{N^*_{D,t}\left(\frac{w^*_t P^*_t \varepsilon_t}{Z^*_t \widetilde{z}^*_{D,t}}\right)^{1-\theta} + N_{H,t}\left[\left(\frac{\tau^*_t}{TOL_t}\right)^{\eta} \frac{w^*_t P^*_t \varepsilon_t}{Z^*_t \widetilde{z}_{H,t}}\right]^{1-\theta}}{N_{D,t}\left(\frac{w_t P_t}{Z_t \widetilde{z}_{D,t}}\right)^{1-\theta} + N_{V,t}\left[\frac{(\tau_t TOL_t)^{1-\alpha} w_t P_t}{Z_t \widetilde{z}_{V,t}}\right]^{1-\theta} + N^*_{H,t}\left[\frac{(\tau_t TOL_t)^{\eta^*} w_t P_t}{Z_t \widetilde{z}^*_{H,t}}\right]^{1-\theta}} =$$
$$= \frac{\frac{N^*_{D,t}}{\widetilde{N}^*_t}\left[\frac{TOL_t}{\widetilde{z}^*_{D,t}}\right]^{1-\theta} + \frac{N_{H,t}}{\widetilde{N}^*_t}\left[\frac{(\tau^*_t)^{\eta} TOL_t^{1-\eta}}{\widetilde{z}_{H,t}}\right]^{1-\theta}}{\frac{N_{D,t}}{\widetilde{N}_t}\left(\frac{1}{\widetilde{z}_{D,t}}\right)^{1-\theta} + \frac{N_{V,t}}{\widetilde{N}_t}\left[\frac{(\tau_t TOL_t)^{1-\alpha}}{\widetilde{z}_{V,t}}\right]^{1-\theta} + \frac{N^*_{H,t}}{\widetilde{N}_t}\left[\frac{(\tau_t TOL_t)^{\eta^*}}{\widetilde{z}^*_{H,t}}\right]^{1-\theta}}. \tag{86}$$

In what follows I use the notation $s_D \equiv N_D\left(\frac{w}{Z\widetilde{z}_D}\right)^{1-\theta} + N_V\left[\frac{w}{Z\widetilde{z}_V}(\tau TOL)^{1-\alpha}\right]^{1-\theta}$ to denote the steady-state share of spending in North on goods produced by Northern firms both domestically and offshore. Expression $s_V \equiv N_V\left[\frac{w}{Z\widetilde{z}_V}(\tau TOL)^{1-\alpha}\right]^{1-\theta}$ denotes the steady-state share of spending in North on goods produced by Northern firms offshore only (i.e. $s_V < s_D$). Expression $s^*_D \equiv N^*_D\left(\frac{w^* Q}{Z^*\widetilde{z}^*_D}\right)^{1-\theta}$ denotes the steady-state share of spending in South on goods produced by Southern firms domestically. I also take into account that the average productivity of Southern firms producing domestically $\widetilde{z}^*_{D,t}$ is a constant. Using all of the above, I log-linearize the CPI-based real exchange rate:

$$\begin{aligned}(1-\theta)\widehat{\widetilde{Q}}_t = {} & s^*_D\left[\widehat{N}^*_{D,t} - \widehat{\widetilde{N}}^*_t + (1-\theta)\widehat{TOL}_t\right] + \\ & + (1-s^*_D)\left[\widehat{N}_{H,t} - \widehat{\widetilde{N}}^*_t + (1-\theta)\left(\eta\widehat{\tau}^*_t + (1-\eta)\widehat{TOL}_t - \widehat{\widetilde{z}}_{H,t}\right)\right] - \\ & - (s_D - s_V)\left[\widehat{N}_{D,t} - \widehat{\widetilde{N}}_t - (1-\theta)\widehat{\widetilde{z}}_{D,t}\right] - \\ & - s_V\left[\widehat{N}_{V,t} - \widehat{\widetilde{N}}_t + (1-\theta)\left((1-\alpha)(\widehat{\tau}_t + \widehat{TOL}_t) - \widehat{\widetilde{z}}_{V,t}\right)\right] - \\ & - (1-s_D)\left[\widehat{N}^*_{H,t} - \widehat{\widetilde{N}}_t + (1-\theta)\left(\eta^*(\widehat{\tau}_t + \widehat{TOL}_t) - \widehat{\widetilde{z}}^*_{H,t}\right)\right].\end{aligned}$$

Setting $\eta = \eta^* = 1$ to nest the Ghironi and Melitz (2005) model with endogenous exports (i.e. firms in each economy serve the foreign markets through exports), the log-linearized expression for the CPI-based real exchange rate becomes:

$$
\begin{aligned}
\widehat{\widehat{Q}}_t = & \left[s_D - (1-\alpha)s_V + s_D^* - 1\right]\widehat{TOL}_t + \\
& + (s_D - s_V)\,\widehat{\widehat{z}}_{D,t} + s_V\,\widehat{\widehat{z}}_{V,t} - (1-\alpha)s_V\widehat{\tau}_t + \\
& + (1-s_D)\left(\widehat{\widehat{z}}^*_{H,t} - \widehat{\tau}_t\right) - (1-s_D^*)\left(\widehat{\widehat{z}}_{H,t} - \widehat{\tau}^*_t\right) + \\
& + \frac{1}{\theta-1}\left(s_V - \frac{N_V}{N}\right)\left(\widehat{N}_{V,t} - \widehat{N}^*_{H,t}\right) + \\
& + \frac{1}{\theta-1}\left[\left(\frac{N_D^*}{N^*} - s_D^*\right)\left(\widehat{N}^*_{D,t} - \widehat{N}_{H,t}\right) - \left(\frac{N_D}{N} - (s_D - s_V)\right)\left(\widehat{N}_{D,t} - \widehat{N}^*_{H,t}\right)\right].
\end{aligned}
\tag{87}
$$