First Draft, March 19, 2013
This version, September 3, 2013

The Contingent Expectations Hypothesis:

Conditional Rationality in Macroeconomics and Finance Theory[*]

Roman Frydman[**] and Michael D. Goldberg[***]

[*]The authors are grateful to the Institute for New Economic Thinking (INET) for supporting this research. The first draft of this paper was presented at the INET Annual Plenary conference in Hong Kong in April 2013.
[**]Department of Economics, New York University and INET Program on Imperfect Knowledge Economics (IKE)
http://www.econ.nyu.edu/user/frydmanr and http://ineteconomics.org/research-program/imperfect-knowledge-economics, respectively.
[***]Peter T. Paul College of Business and Economics, University of New Hampshire, http://pubpages.unh.edu/~michaelg/, and INET Program on Imperfect Knowledge Economics

1 Introduction

For macroeconomists, an individual is rational if she uses her understanding of how the economy works to forecast outcomes and make decisions that do not conflict with her objectives, such as profit-seeking. In order to build models that are compatible with rational decision-making, an economist must formalize participants' understanding of the process driving outcomes.

John Muth's striking insight was that "the relevant economic theory" could serve as the basis for representing how market participants understand the economy. Muth (1961, p. 316) formulated this insight with the rational expectations hypothesis (REH): participants' "expectations are essentially the same as the predictions of the relevant economic theory." In order to implement REH, Muth supposed that an economist's model adequately represents "the structure of the relevant system describing the economy," and when and how it might change.

Muth (1961, p.316) was well aware that the term "rational expectations" suggests some notion of rationality. Indeed, he explicitly recognized the possibility that the name itself would create confusion: "At the risk of confusing this *purely descriptive* hypothesis with a pronouncement as to what firms *ought to do*, we call such expectations 'rational"' (p. 316, emphasis added).

However, Robert Lucas pointed out that once an economist supposes his model provides "the relevant economic theory," he should impose internal consistency between his understanding of the economy and that of market participants. Otherwise, his model would presume that participants endlessly forego profit opportunities. Lucas regarded REH as the way to rid macroeconomics and finance models of such obvious irrationality.[1] By the 1980s, the vast majority of the economics profession embraced REH as the way to represent how profit-seeking market participants understand the economy and use this knowledge to forecast the future.[2]

[1] Lucas (1995, p. 255 and 2001, p.130) recounts how recognition of the key role of internal consistency altered the direction of macroeconomic research.

[2] Over the last four decades, REH has gained wide acceptance among macroeconomists and finance theorists spanning all major schools of thought: Chicago free-market adherents, New Keynesians, and asymmetric-information theorists. Even behavioral economists, who, in large part, emerged on the strength of evidence that REH models cannot explain how market participants actually behave, have continued to subscribe to the conventional view that REH does adequately represent rational forecasting. Consequently, they have interpreted the empirical failures of REH models as a symptom of participants' irrationality.

In the first part of this paper, we advance a novel *theoretical* argument that REH models are incompatible with rational decision-making in real-world markets. We depart sharply from earlier lines of theoretical critique, including our own.[3] We acknowledge that criticizing REH models for abstracting from many, arguably important, aspects of the forecasting process in real-world markets is "misleading" Sargent (1993, p. 21). Proponents of REH have rightly argued that the hypothesis is merely a bold abstraction.

Sargent (1993, p. 7) regarded REH models as abstract approximations of "the outcome of [an admittedly very complex market] process in which people have optimally chosen their perceptions." However, internal consistency ensures only that such approximations are abstractions of rational decision-making in markets in which an economist's model is "the relevant economic theory." Imposing internal consistency in a model that did not adequately account for the observed time-series regularities would imply that its representation of market participants' forecasting strategies was also inconsistent with those regularities. This, in turn, would presume that market participants endlessly forego obvious profit opportunities. As Lucas (2001, p.13) clearly understood, the widely supposed rationality of REH representations is *conditional* on the adequacy and domain of applicability of an economist's model.

Building on Karl Popper's fundamental insights, we point out that how REH models represent change in participants' understanding of the economy severely limits their domain of applicability. We argue that REH models are abstractions of rational decision-making, but only in "markets" in which participants can fully anticipate when and how their understanding of the process driving outcomes might change. Popper (1957, 1982) showed on purely logical grounds that characterizing individuals' understanding as never changing in unforeseen ways presumes that they make decisions in a "world" in which knowledge does not grow. As he put it, "If there is such a thing as growing human knowledge, we cannot anticipate today what we shall only

[3] In Frydman and Goldberg (2007,2011) we advanced a related argument on empirical grounds.In a nutshell, earlier critics of REH have argued that profit-seeking participants would not actually forecast according to an economist's model (Frydman and Phelps, 1983), owing to learning and coordination problems (Frydman, 1982, 1983; Evans and Honkophja, 2001, 2013; Guesnerie, 2005, 2013), insufficient capacity to calculate and psychological biases (Shleifer, 2000; Barberis and Thaler, 2003). For an overview and interpretation of the major arguments advanced in these studies, see Frydman and Phelps (2013) and references therein.

know tomorrow" (Popper, 1957, p. xii).[4]

Over the last four decades, however, a vast majority of economists and finance theorists have relied on REH models to represent participants' decision-making. We point out that, once we regard REH models as abstractions of decision-making in real-world markets, where knowledge does grow, their internal consistency does not ensure their compatibility with rational decision-making. Using a simple algebraic example, we show that REH models represent forecasting in real-world markets by participants who endlessly forego profit opportunities.

REH's inapplicability in representing rational decision-making in real-world markets leads us to propose the contingent expectations hypothesis (CEH) as an approach to representing rationality in these markets. Like REH, rationality of CEH representations is conditional on the adequacy of an economist's model. However, Popper's proposition implies that to serve as the basis for such representations, models should be open to unanticipated changes in how the model builder and market participants understand the economy. This "principle of model contingency" is one of CEH's two conceptual pillars.

In Frydman and Goldberg (2007), we proposed imperfect knowledge economics (IKE) as an approach to building models that are open to unanticipated changes in the model builder's and market participants' understanding of the economy and are thus, by design, consistent with the principle of model contingency. And yet, because these models are only partly open to the growth of knowledge, they have testable implications for time-series data.

However, we show in this paper that not all IKE models are compatible with conditional rationality. Remarkably, Muth's and Lucas's insight concerning the importance of a model's internal consistency is also relevant in ridding IKE models of the presumption that participants endlessly forego profit opportunities. For the purposes of CEH, we refer to this insight as the

[4]Popper's (1946, 1957, and 1983) insights on the growth of knowledge have had a profound impact on our understanding of the scientific process and our thinking about social change. Our reliance on these insights builds on George Soros's (1987) use of them in placing "fallibility" at the center of his framework: every understanding of markets is necessarily contingent, eventually becomes inadequate, and thus requires revision. Soros (2009, 2012) makes use of this to explain the global financial crisis that began in 2008 and the eurozone crisis. For further discussion of Soros's conceptual framework and how it differs from ours, see Frydman and Goldberg (2014).

"principle of qualitative model coherence." This principle – CEH's second conceptual pillar – plays a crucial role in restricting structural change in an IKE model, which formalizes the model builder's understanding of contingent change in the economy and is required for deriving time-series implications from his model.

We show how CEH provides a way to synthesize REH's focus on the importance of fundamental considerations in underpinning rational forecasting (for example, that inflation expectations depend on interest rates) with the two other major advances in macroeconomics over the last four decades. One is Phelps's et al (1970) research program of basing aggregate relationships on micro-foundations that accord an autonomous role to participants' expectations, and the other is behavioral economists' use of empirical observation in representing individuals' decision-making. These advances are usually thought to be incompatible with the assumption that market participants do not forego profit opportunities or make decisions that conflict with their objectives. By recognizing the importance of the growth of economists' and participants' knowledge, CEH-based analysis can incorporate these advances into macroeconomics and finance models, and yet maintain the assumption of conditional rationality.

2 The Choice for Macroeconomics

CEH builds on several main strands in macroeconomics over the last four decades, most notably the development of the rational expectation hypothesis (REH) and the research program pioneered by Phelps et al (1970), which accords expectations an autonomous role. In this section, we provide an overview of our approach and how it relates to these advances.

In proposing REH, Muth (1961, p. 315) suggested that it would address an important problem: in developing "a systematic theory of fluctuations in markets or in the economy..., it is often necessary to make sensible predictions about the way expectations would change when... the structure of the system is changed."[5] Muth's formulation of REH highlighted a crucial point: In order for an economist's model to serve as the basis for repre-

[5] This point is particularly important in policy analysis. In his seminal critique of Keynesian policy analysis, Lucas (1976) noted its assumption that participants' expectations would be unaffected by policy changes. He relied on REH to account for such changes, which has since become standard in policy analysis by central banks around the world.

senting how participants would change the way they form expectations, it should adequately account for regularities in time-series data and any structural change that such regularities might undergo. However, the internal consistency that REH imposes provides no guidance concerning which class of models has the potential to be empirically adequate and thus serve as "the relevant economic theory."

Empirical testing may reject a particular model's adequacy,[6] but it cannot rule out an entire class of models. Consequently, rejection of one REH model does not preclude the possibility that another, either existing or yet to be invented, might adequately account for regularities in time-series data. Indeed, macroeconomists continue to expend enormous talent and other resources in this search.

The problem is that implementing Muth's idea requires ascertaining empirical adequacy at the model-building stage, when this adequacy is inherently unknowable. This underscores the need for an *a priori* criterion by which to select the class of models that might yield "the relevant economic theory." We advance such a criterion here, thereby enabling us to ascertain which models have the potential to serve as the basis for representing how a profit-seeking individual understands the process driving outcomes in real-world markets, forecasts these outcomes, and makes decisions.

To this end, we demarcate two mutually exclusive classes of models. Models in both classes relate outcomes to a set of causal factors and characterize the processes that govern those factors. This "structure" formalizes an economist's understanding of the processes driving aggregate outcomes, including how market participants forecast. Model structures in the two classes may share specifications of participants' preferences and other components. But, as time passes, the process that underpins economic outcomes may change, implying that distinct structures may be required to represent how this process unfolds over time.

The two classes of models formalize sharply different conceptions of such change. One class leaves its models partly open to unanticipated structural change. Change in such a model's structure is constrained; but, conditional

[6] On this score, REH models' performance has been unimpressive; they have encountered widespread empirical difficulties in many markets. For example, in their magisterial treatise on REH-based international macroeconomics, Obstfeld and Rogoff (1996, p. 625) concluded that, "the undeniable difficulties that international economists encounter in empirically explaining nominal exchange-rate movements are an embarrassment, but one shared with virtually any other field that attempts to explain asset price data."

on a causal structure at any point in time, a partly open model does not specify in advance the exact structures that may be needed to represent the market process at any other point in time.

The other class represents change as determinate: conditional on their structure at any point in time, these models specify in advance all potential structures that might represent the process driving outcomes at any other point in time.[7]

Models that are partly open represent decision-making in real-world markets, in which participants' understanding does change at times and in ways that no one can fully foresee. Thus, they enable economists to recognize that market participants' understanding of the economy, as well as their own, will change in ways that they cannot fully anticipate in advance. However, if the economist's understanding is to serve as the basis for representing conditionally rational forecasting, he must suppose that, although contingent, change in the macroeconomy exhibits some regularity. In this sense, the model is only partly open to the growth of knowledge: there are certain qualitative aspects of an economist's understanding that do not change over time.

For example, it is reasonable in many contexts for a partly open model to characterize structural change as moderate for protracted stretches of time. During these periods, such models imply that co-movements between outcomes and causal variables are characterized by distinct qualitative relationships, for example, that they co-move positively. However, because they are partly open, these models also imply that, sooner or later, significant structural change will occur, and that it could bring different qualitative relationships or a different set of variables to the fore in the process driving outcomes.[8] Supposing that the model's qualitative implications adequately characterize the process driving outcomes, partly open models represent how participants understand this process and use this knowledge to forecast out-

[7] As we show in Frydman and Goldberg (2007), the class of determinate models includes standard REH models, in which the causal structure is constrained to be time-invariant. It also includes REH bubble and multiple-equilibrium models, as well as behavioral-finance models. In section 3, we provide a formal definition of a model's causal structure, as well as an example of a determinate model that is typical of macroeconomic and finance theory.

[8] The example of the CEH model in section 5 implies that the economy's inflation rate co-moves positively with the real interest rate over stretches of time. Moreover, changes in tax policy can cause a shift in the relationship between these variables from a positive to a negative co-movement, as well as alter the composition of the causal factors needed to understand the inflation process and market participants' forecasting of future inflation. The model leaves open the exact timing and nature of such change.

comes in ways that do not conflict with the hypothesized qualitative and contingent regularities in time-series data.

REH rules out the relevance of unanticipated structural changes in the process driving outcomes, and in participants' understanding of this process, by hypothesizing that the relevant regularities for modeling rational forecasting are determinate. Typically, these models characterize the process driving outcomes with an overarching probability distribution.[9] The moments of this distribution represent exactly how outcomes and causal variables move and co-move over stretches of time spanning decades. Supposing that the model's quantitative implications adequately characterize observed time-series, REH models represent the implications of how a participant understands the economy and forecasts outcomes to be "essentially the same" as the hypothesized determinate regularities.

Distinguishing between partly open and determinate models leads to an important conclusion: Every macroeconomic and finance model that aims to serve as the basis for representing rational forecasting should be seen as an abstraction of decision-making in only one of two "worlds."

We show in section 3 that, because they are determinate, REH models are abstractions of rational decision making in a world in which participants' knowledge of the economy does not grow. Models that are partly open represent decision-making in markets in which participants' knowledge does change in ways that no one can fully foresee.

The conclusion that the class of REH models represents conditional rationality in "markets" in which knowledge does not grow leaves open the question of whether these models could serve as abstract approximations of decision-making in real-world markets, in which knowledge grows. In Appendix A, we show that imposing REH to represent forecasting in these markets presumes that market participants forego obvious profit opportunities. Thus, REH models cannot serve as the basis for representing how profit-seeking participants understand the process driving outcomes in real-world markets, or, as a result, how these outcomes unfold over time.

Over the last four decades, however, REH has been regarded as the only sound way to represent rational forecasting in macroeconomic models. In-

[9] In Frydman and Goldberg (2007) and our subsequent work, we refer to models that fully prespecify change in probabilistic terms as "fully predetermined." In Frydman and Goldberg (2013a), we pointed out that all such models are determinate. Fully predetermined models specify in advance not only all potential structures, but also when and how changes between structures might occur.

deed, the vast majority of economists have relied on determinate models in their efforts to understand how the economy works. However, ever since economists embraced determinate models, they have faced a stark dilemma. If they wanted to accord expectations an autonomous role, à la Phelps et al (1970), they would be able to incorporate behavioral economists' findings concerning the relevance of psychological and social considerations in participants' forecasting. But, because according expectations such a role in a determinate model presumes that participants forego obvious profit opportunities, their models would conflict with the assumption of profit-seeking behavior.

Lucas (1972, 1973) and Sargent (1981) persuasively argued that models that presume such irrationality could not adequately account for longer-term regularities in time-series data. Eventually, participants would recognize that their forecasting strategies implied systematic forecast errors and revise them. But, as soon as they did, the economist's non-REH determinate model could not be "the relevant economic theory" of time-series regularities. This argument led the vast majority of economists to maintain profit seeking as a core assumption of macroeconomic analysis. Because these economists also continued to rely on determinate models, they were led to adopt REH and thereby jettison an autonomous role for participants' forecasting.

However, as evidence concerning the empirical failures of REH models accumulated, Lucas's and Sargent's arguments were set aside by so-called behavioral-finance theorists. In order to represent how market participants actually behave, these theorists reintroduced autonomous expectations into their models.[10] This was regarded as crucial to allowing the role of psychological considerations, such as confidence, to be included in representations of participants' forecasting and decision-making. Because behavioral-finance theorists have continued to rely on determinate models, they have, by design, rendered their models quantitatively incoherent: Conditional on the hypothesized adequacy of their quantitative implications for the time-series data, these models imply that market participants' ignore these regularities in making decisions, and thus forego obvious profit opportunities.

The reliance on determinate models by both REH and behavioral-finance theorists has overlooked the inherent limits that the growth of knowledge

[10] For an overview of behavioral-finance models, see Shleifer (2000).. Frydman and Phelps (2013) extensively discuss how the REH revolution derailed the development of the "microfoundations" approach pioneered by Phelps et al (1970).

poses for macroeconomic theory's ability to account for time-series data. Because participants in real-world markets revise their understanding of the economy at times and in ways that neither they nor economists can fully anticipate, quantitative predictions of time-series regularities that are implied by any determinate model will sooner or later become obsolete. Thus, analogously to Lucas's argument that non-REH determinate models are "the wrong theory," we argue that the search for quantitative predictions of longer-term regularities in real-world markets is futile.

Foreshadowing our CEH analysis of inflation in section 5, we present in section 4 an example of a determinate behavioral-finance model that accords participants' expectations a partly autonomous role. The example highlights how jettisoning determinate models has the potential to represent conditional rationality in macroeconomic models and accord market participants' expectations an autonomous role. The model's representation of forecasting incorporates the influences of both behavioral and fundamental factors, such as interest rates or income. We show that the model is qualitatively coherent: its qualitative implications concerning movements and co-movements of outcomes and causal factors are compatible with its representation of market participants' forecasting strategies. However, like many behavioral models, the model in our example is determinate. Thus, it is quantitatively incoherent: conditional on the hypothesized adequacy of its quantitative predictions, the model implies that market participants' systematically ignore these predictions and thus forego obvious profit opportunities.

Our example makes clear that the irrationality presumed by behavioral-finance models stems from their effort to represent time-series regularities with an overarching probability distribution.[11] It suggests that partly open models, which do not represent time-series regularities with such distributions and instead give rise only to qualitative and contingent predictions of market outcomes, can accord expectations an autonomous role without abandoning the assumption of conditional rationality. But, as Lucas and Sargent have argued, in order for any model to be able to account for longer-term time-series regularities, it should not presume that market participants' forego profit opportunities. CEH provides a way to build macroeconomic models

[11] There are no doubt many reasons for the economic profession's near-consensus that only models that generate quantitative predictions should be regarded as scientific. In his Nobel lecture, Hayek (1974) referred to the goal of explaining exactly the market process as the "pretence of exact knowledge." As he put it, "I confess that I prefer true but imperfect knowledge...to a pretence of exact knowledge that is likely to be false."

that avoid this presumption, while recognizing that in real-world markets knowledge about the economy grows.

CEH's first pillar – the principle of model contingency – selects models that are open to unanticipated changes in the economy, thereby ruling out determinate models. However, models that are consistent with this principle include those that do not impose any constraints on structural change. These fully open models do not have any implications for time-series data and thus, ipso facto, cannot represent economists' or market participants' understanding of how the process underpinning market outcomes changes over time. This uncontroversial point implies that when applying the principle of model contingency to select models that could represent conditionally rational forecasting, macroeconomists should exclude models that are fully open.

IKE provides a way to build formal models that are partly open to the growth of knowledge. This partial openness is required for any model to be an abstraction of conditionally rational forecasting in markets in which knowledge grows. In section 5, we provide a formal example of how IKE constrains structural change its models.

In sections 4 and 5, we also show how, like internal consistency in REH models, CEH's second pillar – the principle of qualitative model coherence – links an economist's representation of forecasting to the specifications of his models' other components, and how this connection restricts change in a model's structure over time. Qualitative model coherence plays a key role in opening macroeconomic models to unanticipated structural change, yet it does so in IKE models without abandoning either time-series implications or conditionally rational decision-making.

CEH provides a way to advance the research program proposed by Phelps et al. (1970), which was initially implemented within the context of determinate models. When the REH revolution made clear that these internally inconsistent models could not serve as a basis for representing rational decision-making, expectations' autonomous role was jettisoned from macroeconomic models. But it was the class of determinate models, not autonomous expectations, that should have been abandoned.

Our argument implies that behavioral findings should not be ignored in building macroeconomic models, as REH theorists typically do. Recognizing that individuals do not endlessly forego obvious profit opportunities does not mean that behavioral economists' emphasis on realism and the importance of psychological and social considerations is unimportant for understanding regularities in time-series data. Indeed, such considerations are crucial for understanding how *rational* individuals make decisions and how aggregate outcomes unfold over time. As Keynes clearly saw early on, recognizing the role of psychological and social considerations does not necessarily imply irrationality:

> We are merely reminding ourselves that our rational selves [are] choosing between alternatives as best as we are able, calculating where we can, but often falling back for our motive on whim or sentiment or [social convention]. (Keynes, 1936, p. 136)

In section 5, we show how CEH enables us to incorporate psychological considerations into IKE models in ways that are compatible with conditionally rational decision-making. Applying the principle of model coherence in a partly open model implies a qualitative and contingent regularity: there are stretches of time during which participants either maintain the ways in which they forecast outcomes or revise them gradually. This implication can be traced back to Keynes (1936) and has been documented by psychologists (Edwards, 1968). But, in the context of determinate behavioral-finance models, such behavior is interpreted as evidence of market participants' irrationality. By contrast, the results in section 5 show that, in IKE models, this finding by behavioral economists about how individuals actually revise their understanding of market processes is compatible with conditional rationality.

3 Conditional Rationality in a Determinate World

Prior to REH, economists often portrayed forecast revisions with error-correcting rules, such as adaptive expectations. Muth (1961, pp. 315-316) argued that such rules "do not assume enough rationality": market participants would relate their forecasts to their understanding of "the way the economy works...[and] when [and how] the structure of the system is changed."[12]

[12] For early use of adaptive expectations in macroeconomic modeling, see Cagan (1956) and Friedman (1957). Muth noted that his comment concerning insufficient rationality of adaptive expectations also applies to dynamic theories in which expectations do not explicitly appear. For example, he referred to the competitive equilibrium analysis of Arrow and Hurwicz (1958) and Arrow, Block, and Hurwicz (1959).

Muth's idea was that, because an economist's model provides "informed predictions of future events," it could serve as the basis for representing how "the way expectations [are formed] would change." But, in order to use his model to represent participants' understanding of how the economy unfolds over time, an economist must choose which of the two classes of models he considers relevant for representing change. Muth and later REH theorists chose determinate models.

Conditional on the empirical adequacy of the economist's determinate model, imposing REH does indeed rid it of the presumption that market participants forego obvious profit opportunities. Consequently, such models do imply longer-term regularities. But, by design, these regularities represent how outcomes and causal factors move over time in "markets" in which an economist's and participants' understanding of the economy either does not change or changes only in ways that can be fully anticipated in advance. We point out that, on theoretical grounds, REH models are not "the relevant economic theory" of outcomes in real-world markets, in which knowledge about the process driving these outcomes changes in unanticipated ways.[13] In this section and the appendix we illustrate formally how this conclusion is implied by Popper's proposition.

3.1 REH: Abstracting from the Growth of Knowledge

We make use of a stripped-down version of a model that is typical in contemporary macroeconomics and finance theory. The vast majority of such models are not only determinate, but also time-invariant. The following semi-reduced form represents an aggregate outcome, say, the market price:[14]

$$P_t = bX_t + c\hat{P}_{t|t+1} + \varepsilon_t^{\mathrm{M}} \quad \text{for all } t \tag{1}$$

[13] In our earlier work, we traced the epistemological shortcomings of determinate models to their presumption that unanticipated changes in how the economy works are unimportant for understanding outcomes. See Frydman and Goldberg (2013a, b) and references therein.

[14] Equation (1) has been used to model the money, currency, and equity markets, where X_t would include money and income in the first two cases and dividends in the third case. In all of these contexts, equation (1) can be derived from explicit microfoundations. For an excellent treatment of this issue, see Obstfeld and Rogoff (1996).

where $\hat{P}_{t|t+1}$ is the market's forecast formed at t of the price at $t+1$ and (b,c) is a vector of parameters. X_t is a set of causal factors, which are characterized with random walks:

$$X_t = \mu_{\mathrm{X}} + X_{t-1} + \varepsilon_t^{\mathrm{X}} \quad \text{for all } t \tag{2}$$

where μ_{X} are drifts and ε_t^x is vector of independently distributed disturbances ("the news"), which is characterized by an invariant probability distribution with the mean zero and some finite variance-covariance matrix. $\varepsilon_t^{\mathrm{M}}$ denotes the "error term" – the difference between the actual time-series observations of the market price and the economist's representation of the process driving this price. The model is considered empirically adequate if this error term is the mean-zero random variable that is distributed independently over time and independently of the model's causal variables.

Formal accounts of how individuals understand the economy relate market outcomes to a set of causal factors, which portray the information ("facts") that market participants consider relevant. A typical representation of the knowledge that underpins the market's forecast can be written as

$$\hat{P}_{t|t+1} = \beta Z_t \quad \text{for all } t \tag{3}$$

where β is a vector of parameters. and Z_t characterizes the union of information sets used by market participants, which, for simplicity, we also represent with random walks.

The functional forms of the explanatory part of the model, $bX_t + c\hat{P}_{t|t+1}$ and its representation of the market's forecast in (3), together with the processes driving the causal factors, such as in (2), constitute the model's causal structure. At any point in time, this structure formalizes an economist's understanding of the causal factors that he considers relevant, and the process by which outcomes – the price and the market's forecast – are related to those factors.

Supposing that an economist's model is "the relevant economic theory"– that it is empirically adequate– imposing internal consistency represents participants' understanding of the process driving outcomes at every t to be "essentially the same" as that of the economist. REH then sets the market's forecast to equal the mathematical expectation of the price process, P_t in (1), conditional on time-t information, I_t, $\hat{P}^{\mathrm{RE}}_{t|t+1} = E\left[P_t | I_t\right]$.

Imposing REH in (1) implies the following RE representations of the price process and the market's forecast:

$$P_t^{\mathrm{RE}} = \frac{bc}{(1-c)^2}\mu_{\mathrm{X}} + \frac{b}{1-c}X_t \text{...for all } t \tag{4}$$

$$\hat{P}_{t|t+1}^{\mathrm{RE}} = \frac{bc}{(1-c)^2}\mu_{\mathrm{X}} + \frac{b}{1-c}X_t \quad \text{for all } t \tag{5}$$

Like a vast majority of macroeconomic and finance models, the structure of the foregoing example is constrained not to change over time. By design, such time-invariant models represent participants' forecasting strategy in "markets" in which participants' understanding of the price process is static.

3.1.1 Allowing for Fully Anticipated Change in Structure

Some REH models recognize the possibility that, as time passes, different structures may be needed to represent change in how the causal factors, the process driving the price, and market participants' understanding of this process unfold over time. But, even if an REH model does explicitly allow for change, it represents this change as determinate: it fully specifies in advance when and how market participants might revise their understanding of the process driving outcomes.

By design, REH models specify participants' forecasting endogenously. Thus, in order to allow for structural change in his representation of the market's forecast, an REH theorist must move away from time-invariant specifications of the non-expectational components of his model. Such modifications have sometimes allowed for stochastic preferences (for example, Barberis et al. 2001). However, following Lucas (1976), revisions of participants' forecasting strategies are usually modeled as arising from changes in the stochastic process that characterizes government policymaking.

Hamilton (1988, 1994) developed a particularly influential class of models that represent change as determinate. This approach specifies a set of time-invariant REH models, typically two, to represent outcomes during different time periods and fully specifies all model structures that would be needed to represent adequately how the process driving market outcomes, and how participants' understanding of this process, might unfold over time. Hamilton's approach also specifies the timing of these changes with a probabilistic Markov switching rule. We now show that, like their time-invariant counterparts, such determinate models of change also abstract from the growth of knowledge in representing how individuals make decisions and how aggregate outcomes unfold over time.

We consider the case in which the vector of causal factors, X_t, consists of only one variable, say, money supply. We set the drift in μ_{X} of the process for X_t in (2) to undergo a one-time shift. The simplest way to represent this change as determinate is to suppose that it is known that at t and earlier, the drift is equal to μ_{X}^1, and that at $t+1$ the drift will shift permanently to μ_{X}^2 and remain at the new level thereafter. Our determinate restriction on change in this structure takes a particularly simple form:

$$\bar{A} = \mu_{\text{X}}^2 - \mu_{\text{X}}^1 \tag{6}$$

With this fully anticipated change in the process governing the policy variable at $t+1$, REH representations for the price process and the market's forecast take the following form:

$$P_{t+\tau}^{RE} = \frac{bc}{(1-c)^2}\left(\mu_{\text{X}}^1 + \bar{A} + D\,(c-1)\,\bar{A}\right) + \frac{b}{1-c}X_{t+\tau} \quad \text{for all } \tau \tag{7}$$

$$\hat{P}_{t+\tau|t+\tau+1}^{RE} = \frac{b}{(1-c)^2}\left(\mu_{\text{X}}^1 + \bar{A} + D\,(c-1)\,\bar{A}\right) + \frac{b}{1-c}X_{t+\tau} \quad \text{for all } \tau \tag{8}$$

where D is a dummy variable,

$$\begin{aligned} D &= 1 \text{ for } \tau = 0 \text{ and} \\ D &= 0 \text{ for all } \tau > 0 \end{aligned}$$

Specification in (8) represents the market's forecast in terms of a set of causal factors, X_t, in all time periods. However, although the model allows for change in the market participants' understanding, it specifies in advance that they will revise it at $t+1$. The model also fully specifies the structure in (8) for $\tau \geq 1$, which is assumed to represent adequately the post-change market's forecast.[15]

[15] The deterministic switching rule simplifies our presentation without altering any of the conclusions. In Frydman and Goldberg (2007, chapter 6) we show that Hamilton's model represents when and how participants might revise their forecasting strategies with a single overarching probability distribution. Thus, models with probabilistic switching rules assume away the possibility that participants may revise their forecasting strategies in ways that they have not fully anticipated in advance.

This specification thus represents decision-making in markets in which participants can fully anticipate today what they will know in the future about the process driving the price. As Popper (1957, 1982) showed,

> If there is such a thing as growing human knowledge, then no individual, such as an economist or a market participant, or group of individuals, such as market participants in the aggregate, can a anticipate today what they shall only know tomorrow. (Popper, 1957, xii)

By ruling out unanticipated change in how market participants might understand the economy, REH models represent decision-making in markets in which knowledge does not grow. This conclusion leaves open the question of whether these models are nonetheless relevant in modeling outcomes in which participants' knowledge might change in ways that they could not have anticipated in advance. After all, REH models are not literal descriptions, but bold abstractions.

According to Popper's proposition, in order to serve as the basis for representing the growth of participants' knowledge, a model would have to recognize that an economist's understanding of the economy – and that of market participants – might change at times and in ways that they cannot fully anticipate in advance. Any formal representation of decision-making in markets in which knowledge grows must therefore allow for an unanticipated change either in the structure of a semi-reduced form of an economist's model, or in the model's representation of the market's forecast. Thus, recognizing the growth of knowledge in an economist's model necessarily implies REH models' inadequacy in representing rational decision-making and longer-term regularities in time-series data. Simply put, even if we were to suppose that an REH model might have adequately accounted for aggregate outcomes and the market's forecast during some stretch of time, sooner or later the growth of knowledge would render the model inconsistent with the assumption that market participants are profit-seeking.

As Lucas has persuasively argued, conditional on the empirical adequacy of an economist's determinate model, REH rids the model of obvious irrationality on the part of market participants. Remarkably, once we consider models of markets in which knowledge grows, Lucas's reasoning leads us to the opposite conclusion: supposing that REH models are "the relevant economic theory" of these markets implies that these models represent decision-making by irrational individuals who forego obvious profit opportunities. In Appendix A, we provide a formal illustration of this argument.

4 The Contingent Expectations Hypothesis

REH models, we have shown, represent rational decision-making, but only in "markets" in which participants' knowledge of the process driving outcomes does not grow. In his seminal refutation of the belief that "lay[ing] bare the law of evolution of society in order to foretell its future" is within reach of social science, Popper (1957, 105-106, xi-xii) pointed out "the course of human history is strongly influenced by the growth of human knowledge." He went on to argue that "the truth of this premise must be admitted even by those [Marxist theorists] who see in our ideas, including our scientific ideas, merely the by-products of material developments of some kind or other." Indeed, it is self-evident that what REH determinate models exclude – the growth of knowledge – has been the central driver of human development, including economic outcomes.

Capitalist economies' hallmark is the establishment of powerful incentives that motivate individuals to search for new ways of understanding the world. The growth of knowledge engendered by profit-seeking may lead to relatively small modifications of existing production or marketing processes, or to large changes, such as the commercial development of major technological innovations (for example, personal computers, the Internet, or new sources of energy). These developments have the potential to alter significantly the way the economy works and thereby spur individuals to revise how they understand the process underlying aggregate outcomes.[16] Such revisions can, in turn, give rise to further change in the way the economy works by triggering changes in the social context within which individuals make decisions, such as policy developments following presidential or parliamentary elections or appointments of important officials, such as central bank governors. Change in the social context also includes institutional and political developments, such as the establishment of GATT and other Bretton Woods institutions, the introduction of the euro, or German reunification.

The two-way interdependence between the growth of knowledge and how

[16] We are thus drawing a distinction between knowledge of how the economy works, which underpins market participants' rational forecasting, and knowledge of how to produce goods and services, which is emphasized by the literature on long-run economic growth.

the economy works, along with Popper's proposition, implies that the process underpinning economic outcomes changes at times and in ways that no one can fully anticipate. These considerations, together with the arguments of the preceding section and Appendix A, underpin one of CEH's two pillars.

> **Principle of Model Contingency**
>
> In order to be relevant for representing how a rational individual understands and forecasts market outcomes in the pursuit of profits or other objectives, it should be partly open to unanticipated *structural* change**.**

This principle implies that to serve as the basis for representing rational forecasting, conditional on the adequacy on the theorist's understanding of the economy, an economic model must recognize that the process driving outcomes is contingent: it is subject to change at times and in ways that no one can fully foresee. But to do so adequately, a model must also imply regularities in time-series data and thus be only partly, rather than completely, open.

However, although IKE models are, by design, partly open, not all of them can serve as the basis for representing conditional rationality in macroeconomics and finance theory. The second pillar of CEH, which we call the principle of qualitative model coherence, selects the subclass of IKE models that can do so. This principle consists of two conditions that extend to IKE models the insights of Muth and Lucas for representing rational forecasting.

> **Principle of Qualitative Model Coherence**
>
> To serve as the basis for representing conditionally rational forecasting, a model should embody two conditions:
>
> **a. Cognitive Coherence**
>
> A model's representations of how individuals understand the economy should be compatible with the economist's own qualitative understanding of the economy as formalized by his model;
>
> **b. Predictive Coherence**
>
> A model's representations of how profit-seeking participants forecast market outcomes should imply predictions that are compatible with those that are implied by the model's reduced form.

Cognitive coherence builds on Muth's (1961, p. 316) insight into how to address pre-REH macroeconomic models' failure to "assume enough rationality": an economist can use his own understanding of the economy in representing that of market participants. Predictive coherence appeals to Lucas's (2001, p. 13) insight that models in which individuals are assumed to forego obvious profit opportunities are the "wrong theory." To avoid the presumption of such irrationality, a model cannot represent market participants' forecasting strategies in ways that conflict with its time-series implications.

4.1 Quantitative Cognitive Coherence of REH Models

The terms cognitive and predictive coherence do not appear in discussions of REH models. However, REH imposes a quantitative version of both conditions. Like any model builder, an REH theorist formalizes his understanding of the economy by his semi-reduced form, his specification of the process driving the causal variables, and restrictions on how this structure might change over time. He also hypothesizes that his understanding is adequate for representing the economy in all time periods. In terms of our REH example, we suppose that the theorist's understanding is formalized by equations (1) and 2).[17]

The understanding of change that is embodied in these equations supposes that the processes underlying both the market price and the causal variables can be represented as determinate, with time-invariant probability distributions. It also supposes that P_t and X_t both depend on a particular set of causal variables and that the effects of these variables (b and c for the price process) are set equal to some precise values. Moreover, a determinate model's understanding typically implies that these effects take on specific algebraic signs in all time periods. The X_t process is understood to involve constant drifts that take on precise values with explicit algebraic signs.

4.1.1 The Market's Epistemic Superiority

Cognitive coherence in the model entails using the knowledge that it formalizes to represent how market participants understand and forecast how the market price and causal variables unfold. Imposing this condition in a macroeconomic model can be seen as an implication of the following premise:

[17]This understanding would also include any microfoundations and aggregation rule that was used in obtaining the semi-reduced form.

Premise of the Market's Epistemic Superiority

If an economist's understanding of the process underlying economic outcomes is adequate, the market's understanding of this process encompasses that of the economist.

REH models embody a particularly restrictive form of this premise: They presume that an economist's understanding is "essentially the same ," in quantitative terms, as that of the market.

The notion that an economist should not presume that the market does not understand what he does about the economy has underpinned REH's appeal from the very beginning. For example, REH-based analysis of the consequences of alternative economic policies has been presented colloquially as simply a formalization of the idea that it is foolhardy to suppose, as the Keynesians of the 1960s did, that changes in government policy will have no effect on how the market understands the process driving outcomes. As Sargent (2008) put it,

The concept [of rational expectations] is motivated by the same thinking that led Abraham Lincoln to assert, 'You can fool some of the people all of the time, and all of the people some of the time, but you cannot fool all of the people all of the time'....Economists who believe in rational expectations base their belief on the standard economic assumption that people behave in ways that maximize their utility (their enjoyment of life) or profits.

We have argued, however, that REH supposes much more than Lincoln's adage: it presumes that "people's" understanding can be exactly specified in probabilistic terms. This presumption implies that "people [who] behave in ways that maximize ... profits " expect not to revise their understanding of the economy in ways that they could not foresee in advance. Moreover, the assumption of profit-seeking in determinate models implies that cognitive coherence in REH models supposes far more than that an economist or policymaker "cannot fool the people all the time." An REH model presumes that the economist is omniscient: he not only knows what the market does about how the economy works at any time, but can also predict in exact probabilistic terms how the market will understand the economy at any time in the future.

The observation that REH puts the market and economists on an equal footing is well understood by macroeconomists. But, what popular discussions of the meaning of REH, including as those appealing to Lincoln's adage, have overlooked is that REH models assume away the market's epistemic superiority relative to any individual or a group of individuals. As Hayek (1945, 1948) so well understood in his prescient argument that socialist planning is, in principle, impossible, markets play an essential role in modern economies because no one – whether an economist, a policymaker, or a planner – can have access to the "totality of knowledge," which is dispersed among market participants. In section 5, we show how IKE models embody the premise of the market's epistemic superiority.

4.1.2 Cognitive and Predictive Coherence in a Determinate Model

The procedure of solving REH models is well known. But we sketch the steps to highlight how the conditions of cognitive and predictive coherence are used in the familiar context of these models. We show that cognitive coherence in a determinate model amounts to hypothesizing that an economist knows what the market does about the process driving outcomes. We also show that cognitive coherence in a determinate model implies predictive coherence at every point in time.

The understanding of the price process that we have sketched implies that iterating equation (1) forward one period provides an adequate portrayal of the price process at $t+1$:

$$P_{t+1} = bX_{t+1} + c\hat{P}_{t+1|t+2} \tag{9}$$

It also implies the following conditional mathematical expectation of P_{t+1}:

$$E_t P_{t+1} = b\mu^x + cE_t\hat{P}_{t+1|t+2} \tag{10}$$

where we have used the understanding formalized by equation (2). Equation (10) represents the REH theorist's understanding of how to forecast next period's price. Applying the cognitive coherence condition, an economist uses this understanding to set $\hat{P}_{t|t+1} = E_t P_{t+1}$, thereby imposing predictive coherence in the model at $t+1$.

The expression $E_t\hat{P}_{t+1|t+2}$ implies that the theorist must represent at time t how the market will forecast the price at $t+1$ for $t+2$. Cognitive coherence leads him to impose predictive coherence at $t+1$ by setting $\hat{P}_{t+1|t+2} = E_{t+1}P_{t+2}$. The understanding that the price process can be characterized as determinate implies that the law of iterated expectations (LIE) can be applied, so that $E_t\hat{P}_{t+1|t+2} = E_t P_{t+2}$.

Having hypothesized that his understanding is adequate in all time periods, the REH theorist can derive how he and the market understands and forecasts the market price at all points in time beyond $t+1$ by iterating equation (1) progressively forward. At every step, he takes the conditional expectation, imposes cognitive and thus predictive coherence, and applies LIE. By doing so, he presumes that he knows exactly how he and the market will forecast market outcomes at every point in time, from t to eternity.

Repeated substitution at each iteration delivers the REH reduced form in equation (4) and the REH forecasting strategy in equation (5). The reduced form implies that if, for example, the economist were to expect ΔX_{t+1} to be positive on average and equal to μ^X, he would also expect the change in price to be positive on average and exactly related to μ^X, that is,

$$E\left[\Delta P_{t+1}\right] = \frac{b}{1-c}\mu_{\mathrm{X}} \tag{11}$$

The reduced form also implies that P_t and X_t co-move together, which we express with the following cross-moment:

$$E\left[\Delta P_{t+1}\Delta X_{t+1}\right] = \frac{b}{1-c}\left(\mu_{\mathrm{X}}^2 + \sigma_{\mathrm{X}}^2\right) \tag{12}$$

where σ_{X}^2 is a vector of second moments of the $\varepsilon_t^{\mathrm{X}}$ process.

Having imposed predictive coherence between price predictions at t and outcomes at $t+1$, the model implies a quantitative predictive coherence involving its (and the market's) average expected change in P_t and the average expected change in X_t:

$$E\left[\hat{P}_{t|t+1} - P\right] = \frac{b}{1-c}\mu_{\mathrm{X}} \tag{13}$$

The imposition of predictive coherence also implies that market participants expect the same co-movements between P_t and X_t as does the economist:

$$E\left(\hat{P}_{t|t+1} - P_t\right)\left(\widehat{X}_{t|t+1} - X_t\right) = \frac{b}{1-c}\left[\mu_{\mathrm{X}}^2 + \sigma_{\mathrm{X}}^2\right] \tag{14}$$

where $\widehat{X}_{t|t+1}$ represents the market's time-t conditional forecast of X_{t+1}.

4.2 Qualitative Cognitive and Predictive Coherence in a Behavioral Model

The understanding of the economy that is formalized by an IKE model is qualitative. Consequently, cognitive and predictive coherence in the model are defined only as qualitative conditions. In order to presage that analysis, we consider coherence in a behavioral model. Although such models are, by design, quantitatively incoherent both cognitively and predictively, they may nonetheless be qualitatively coherent. Like in a determinate model, qualitative cognitive coherence in both behavioral and IKE models can imply qualitative predictive coherence.

Although the understanding in our example implies that change is determinate, model builders typically take a stand on certain qualitative aspects of the macroeconomy that are formalized with qualitative constraints on the model's structure. For example, suppose that the understanding of the economy implies that there is one causal factor driving the market price, and that movements in this factor have a positive effect on P_t, that is, $b > 0$. Also suppose that $\mu^{\text{X}} > 0$. In this scenario, qualitative cognitive coherence implies the following representation of the market's forecast:

$$\hat{P}_{t|t+1} = P_t + \beta\hat{\mu}_{\text{X}} \tag{15}$$

where $\beta > 0$ and $\hat{\mu}^x > 0$ represents the market's time-t conditional forecast of ΔX_{t+1}. In a behavioral model, β and $\hat{\mu}^x$ are constrained to differ from the values that formalize the model builder's understanding of the economy. Such constraints are often used in behavioral-finance models to account for empirical observations that researchers interpret as implying systematic over- or under-reaction by market participants in how they forecast outcomes.[18]

Substituting the representation in (15) into (1) results in the following reduced-form model that formalizes an economist's understanding of the price process:

$$P_t = \frac{b}{1-c}X_t + \frac{c}{1-c}\beta\hat{\mu}_{\text{X}} + \varepsilon_t^{\text{M}} \tag{16}$$

This reduced form implies the following time-series regularities:

$$E\left[\Delta P_{t+1}\right] = \frac{b}{1-c}\mu_{\text{X}} \tag{17}$$

[18]For example, see Barberis et al. (1998) and Gourinchas and Tornell (2004).

$$E\left[\Delta P_{t+1}\Delta X_{t+1}\right] = \frac{b}{1+c}\left(\mu_{\text{X}}^2 + \sigma_{\text{X}}^2\right) > 0 \tag{18}$$

Using (15) the following moments are implied by the probability distribution formalizing market participants' understanding of the economy:

$$E\left[\hat{P}_{t|t+1} - P_t\right] = \beta\hat{\mu}_{\text{X}} \tag{19}$$

$$E\left(\hat{P}_{t|t+1} - P_t\right)\left(\widehat{X}_{t|t+1} - X_t\right) = \beta\hat{\mu}_{\text{X}}\mu_{\text{X}} \tag{20}$$

Thus, the model is predictively coherent in qualitative terms, but it is incoherent in quantitative terms.

This incoherence arises because the economist accords market participants' forecasting an autonomous role in a determinate model. Behavioral-finance models thus represent the decision-making of market participants who endlessly forego profit opportunities. Lucas (1972, 1973) and Sargent (1981) pointed out that such non-REH models could not adequately account for longer-term regularities in time-series data. Eventually, participants would recognize that their forecasting strategies implied systematic forecast errors and revise them. But, as soon as they did, the economist's non-REH model would no longer be "the relevant economic theory" of time-series regularities. This argument led the vast majority of economists to jettison an autonomous role in modeling participants' forecasting by embracing REH.

This methodological choice, however, overlooks the inherent limits that the growth of knowledge poses for macroeconomic theory's ability to account for regularities in observed time-series data. Because participants in real-world markets revise their understanding of the economy at times and in ways that neither they nor economists can fully anticipate, quantitative predictions of time-series regularities that are implied by any determinate model will sooner or later become obsolete, rendering the model predictively incoherent.

We have argued that the search for quantitative predictions of longer-term regularities in the economy is futile. However, there may qualitative regularities in time-series data that macroeconomic analysis can explain. In the next section, we jettison the practice of relying on determinate models to explore and understand these regularities. We show how the use of CEH enables us to accord market participants' forecasting a partly autonomous role, without presuming that they forego profit opportunities. We also show how CEH's principle of qualitative model coherence constrains IKE models sufficiently to imply longer-term qualitative regularities in time-series data.

5 CEH Representations of Conditional Rational Forecasting: An Inflation Example

In order to illustrate how CEH represents conditionally rational forecasting strategies, we consider modeling an economy's inflation rate. Our example is based on a New Keynesian model that has been used widely in the literature, which in a stripped-down version is reducible to the basic equations that we have used in previous sections. This version involves a single causal variable, the interest rate, which simplifies considerably the application of qualitative model coherence. We show how cognitive coherence restricts change in representing individuals' forecasting strategies and, in doing so, leads in our example to predictive coherence. We leave analysis of a multivariate CEH model for future research.

Our characterization of the interest-rate process and formalizations of contingent change in the macroeconomy and market participants' forecasting strategies provide just one example of how a macroeconomic model can be partly opened to the growth of knowledge. To be sure, there are other ways to characterize the causal variables and formalize contingent change. But, however one represents these features, the approach by which cognitive and predictive coherence is imposed in an IKE model would be the same. We strip our presentation of much of the detail, which we provide in Appendix B. This enables us to focus on the main features of our approach to representing conditional rationality in macroeconomic models.

5.1 Imperfect Knowledge of the Inflation Process

We characterize an economist's understanding of the process underlying the inflation rate at a point in time along the lines of Calvo (1983) and Rotemberg (1985):

$$P_t = b_{1t} Y_t + b_{2t} V_t + c\hat{P}_{t|t+1} + \varepsilon_t^{\mathrm{M}} \tag{21}$$

where P_t now denotes an economy's inflation rate between $t-1$ and t, Y_t is national output, V_t represents firms' marginal cost considerations other than those that are captured by Y_t, $0 < c < 1$, the parameters b_{1t} and b_{2t} are also assumed to be positive at every moment, but are allowed to vary over time, and $\varepsilon_t^{\mathrm{M}}$ is a white noise error that represents the influence of all other factors not explicitly accounted for in the model.[19]

We assume that V_t is related to firms' need to obtain credit in order to finance their wage bills. Labor costs, and thus V_t, depend on the cost of credit, which we assume is determined by the real interest rate, R_t. A rise in the real rate, therefore, has an inflationary impact through aggregate supply by increasing the marginal cost of labor.[20]

To model output, we assume a simple IS curve that relates Y_t to the real interest rate. With this assumption, we can express the inflation process as follows:

$$P_t = b_t R_t + c\hat{P}_{t|t+1} + \varepsilon_t^{\mathrm{M}} \tag{22}$$

The algebraic sign of b_t would be positive if the inflationary supply-side effect of a change in R_t were greater than the deflationary demand-side effect and negative if the opposite were true.

As for the interest rate, we suppose for simplicity that it follows a time-varying $AR(1)$ process:

$$R_t = a_t R_{t-1} + \varepsilon_t^{\mathrm{R}} \tag{23}$$

where a non-zero mean is omitted for ease of exposition, $\varepsilon_t^{\mathrm{R}}$ is a white noise error, and a_t is assumed to be positive. Alternative specifications of R_t could also be considered.[21]

Equations (21)-(23) formalize qualitative knowledge of the inflation and interest rate processes, for example, that inflation depends positively on firms' inflation expectations and either positively or negatively on the interest rate, and that R_t is persistent. This knowledge also implies that over time, movements in the inflation rate depend on changes in the inflation process itself, as represented by changes in b_t and a_t. Consequently, in order

[19] Setting c to a constant eases our exposition of CEH. In general, however, we would expect the inflation rate's sensitivity to a change in firms' expectations to vary over time as well.

[20] See Van Wijnbegen (1983, 1985) and Neumeyer and Perri (2005) for models in which credit financing of working capital plays a key role in business cycle fluctuations.

[21] For example, in Frydman and Goldberg (2013c,d) we develop a CEH model of exchange rate fluctuations whose structure is similar to the model presented here, but we specify interest rates and other causal variables as random walk processes with a small and persistent time-varying drift. Such processes imply persistence in first differences, whereas the AR(1) specification in (23) does not. Johansen et al. (2010), Juselius (2013) and others report strong support for this more persistent specification for nominal and real interest rates and other key macroeconomic variables.

for the model to generate time series implications, we must restrict how these structural parameters might change over time. Only then can the model serve as a basis for representing conditionally rational forecasting.

5.1.1 Representing Contingent Change in the Macroeconomy

The New Keynesian specification in (22) implies that any factor that significantly influences the supply side or demand side of the economy can cause a shift in the inflation process. Consider tax policy. Firms' interest payments are deductible as expenses under current law, implying that the real cost of working capital depends on firms' marginal tax rates. A lower marginal tax rate would lower the size of these deductions, thereby raising the cost of working capital. Shifts in tax policy, therefore, would influence the relative strength of the supply-side and demand-side effects of interest-rate changes on inflation. In fact, if the change in tax policy was large enough, it could be associated with a reversal, say, from positive to negative, in how inflation and the interest rate co-moved over time.[22]

To be sure, there are many other developments that can influence how the interest rate affects the inflation rate. These include changes in other supply-side factors (such as shifts in a country's trade openness) that affect firms' ability to pass changes in marginal costs on to customers, as well as changes in demand-side factors (such as shifts in consumers' debt burdens) that affect their willingness to borrow and spend at any interest rate.[23] However, these and other factors tend to remain largely unchanged or change very little for protracted stretches of time. During these periods, we would expect moderate or no change in the inflation process from one point in time to the next.

Similar reasoning applies to potential changes in the interest-rate process. Monetary policy plays a central role, but there are many other financial and economic factors that can influence how interest rates unfold over time. These

[22]For example, the Tax Reform Act of 1986 eliminated the investment tax credit, dramatically changed depreciation allowances, and lowered the top federal statutory tax rate for corporate income tax rate from 46% to 34% percent (which was increased in 1993 to its current rate of 35%). Cohen et al. (1999) find that this tax-policy change of tax policy significantly increased firms' user cost of capital.

[23]In explaining the lackluster response of private spending to a fall in interest rates in 1990s Japan and the U.S. and Europe after 2008, Koo (2008) and others emphasize high debt burdens and underwater balance sheets, owing to downswings in asset prices. As with relatively large shifts in tax policy, large shifts in debt burdens can cause a reversal in terms of which interest-rate effects – on the supply side or the demand side – are dominant.

factors include shifts in macroprudential policy, such as changes in banks' capital requirements, and institutional changes like German reunification and the creation of the European Monetary Union. As with the inflation process, these factors tend to remain unchanged or change very little for protracted stretches of time.

However, major shifts in policy, institutions, and other factors do eventually occur. We would thus expect that periods of moderate or no change in the inflation and interest rate processes would be punctuated by moments at which they shifted in relatively large ways. Major shifts in the macroeconomy depend on economic, political, and financial developments that for the most part can be anticipated only dimly, if at all. Consequently, no one can fully anticipate when time intervals of moderate or no change in the macroeconomy might begin or end.

Qualitative and Contingent Restrictions on Structural Change We formalize this qualitative understanding of contingent change in the inflation and interest-rate processes in a way that has time-series implications. Equations (21)-(23) imply that over time, movements in the inflation rate stemming from what we call the "direct" (or non-expectational) component depend on changes in the inflation process and movements in the interest rate:

$$\Delta P_t^D = \Delta b_t R_t + b_{t-1}\Delta R_t + \Delta\varepsilon_t^{\mathrm{M}} \tag{24}$$

where P_t^D represents the direct channel through which the interest rate influences the inflation rate and Δb_t characterizes change in the P_t^D process. Similarly, interest-rate movements depend on changes in the R_t process and lagged movements of R_t:

$$\Delta R_t = \Delta a_t R_{t-1} + a_{t-1}\Delta R_{t-1} + \Delta\varepsilon_t^{\mathrm{R}} \tag{25}$$

Given these representations, the qualitative understanding of change that we sketched above implies that there are protracted stretches of time in which change in the macroeconomy is sufficiently moderate so that the algebraic signs of b_t and a_t can be assumed to remain unchanged. However, this assumption alone is insufficient to generate time series implications from the model. Even if b_t was assumed to be positive or negative over a stretch of time, without further restriction on structural change, the model would be consistent with co-movements of P_t^D and R_t that are either positive, negative, or display no regularity at all.

In formalizing our understanding of contingent change, therefore, we specify moderate structural change over a time interval with the following qualitative and contingent restriction:

$$|\Delta b_t R_t| < |b_{t-1} \Delta R_t| \text{ for } t \text{ between } T_{i+1} - T_i \tag{26}$$

where interval i begins at $t = T_i$ and ends at T_{i+1}, $|\cdot|$ denotes an absolute value, and we refer to $b_{t-1}\Delta R_t$ as the linear impact of interest rate movements, and to $\Delta b_t R_t$ as the impact of structural change. We assume that this qualitative and contingent condition is satisfied in ways that do not alter qualitatively the relative balance between the supply- and demand-side effects of interest movements (that is, the sign of b_t) during an interval of moderate change. With this constraint, the representation in (24) implies that P_t^D tends to move either together (if $b_t > 0$) or inversely (if $b_t < 0$) with R_t at each point in time during a period of moderate change.

Similar reasoning leads to an analogous constraint on structural change in the interest rate process:

$$|\Delta a_t R_{t-1}| < |a_{t-1} \Delta R_{t-1}| \text{ for } t \text{ between } T_{j+1} - T_j \tag{27}$$

which implies that during a time interval of moderate change, interest-rate movements tend to be followed in the next period by movements in the same direction.

We note that because the constraints in (26) and (27) are qualitative, they are consistent with myriad possible changes in b_t and a_t over an interval, including no change at all. These constraints recognize that the timing and impact of any moderate shifts cannot be fully foreseen. The model is thus contingent and partly open to unanticipated change even during periods of moderate change.

The constraints in (26) and (27) themselves are also contingent; the model does not specify exactly when intervals of moderate change begin or end. However, our understanding of contingent change in the macroeconomy implies that these periods tend to be quite protracted and occur more frequently than the much shorter intervals during which the macroeconomy cannot be characterized by any qualitative regularity.

5.2 Qualitative Model Coherence and Conditional Rationality

We now use the principle of qualitative model coherence to represent rational forecasting, conditional on the adequacy of the qualitative understanding of the macroeconomy as formalized by equations (21)-(27).

5.2.1 Cognitive Coherence in an IKE Model

Imposing cognitive coherence, as in the simple behavioral model of section 4.2, entails using the qualitative knowledge that underpins the model's direct component in equations (22) and (24) and the interest rate process in equations (23) and (25) to represent the market's forecasting strategy:

$$\hat{R}_{t|t+1} = R_t + \alpha_t \Delta R_t + \Delta\hat{\alpha}_{t|t+1} R_t \tag{28}$$

$$\hat{P}_{t|t+1} = P_t + \beta_t \left(\hat{R}_{t|t+1} - R_t\right) + \Delta\hat{\beta}_{t|t+1}\hat{R}_{t|t+1} \tag{29}$$

which implies

$$\hat{P}_{t|t+1} = P_t + \gamma_t \Delta R_t - \rho_t R_t \tag{30}$$

where $\gamma_t = \alpha_t \delta_t$, $\delta_t = \left(\beta_t + \Delta\hat{\beta}_{t|t+1}\right)$, $\rho_t = \left(\delta_t \Delta\hat{\alpha}_{t|t+1} + \Delta\hat{\beta}_{t|t+1}\right)$, and $\Delta\hat{\beta}_{t|t+1}$ and $\Delta\hat{\alpha}_{t|t+1}$ represent the market's prediction of how its forecasting strategy might change from t to $t+1$.[24] Internal coherence implies that $sgn\left(\alpha_t\right) = sgn\left(a_t\right) > 0$ and $sgn\left(\beta_t\right) = sgn\left(b_t\right)$. But, these qualitative restrictions alone are insufficient for the representations in equations (28)-(30) to yield time-series implications. To do so, the model must constrain, *ex ante*, how the structure of these representations might change over time.

In general, participants decide to revise their forecasting strategies for various reasons, including the performance of their current strategies and news about a range macroeconomic, political, and other developments that

[24] In general, cognitive coherence in the model is consistent with representations of forecasting that explicitly include variables other than the interest rate. The inclusion of such variables in a CEH representation of forecasting would be guided by theoretical and empirical considerations. See Frydman et al. (2013) for such a CEH model of asset prices. There is much evidence in asset markets that participants rely on a broad range of fundamental, psychological, and technical considerations in forecasting outcomes. See Mangee (2013) and Sullivan (2013), which construct novel datasets based on scoring daily market-wrap stories from *Bloomberg News* and the *Wall Street Journal*, respectively.

leads them to rethink how the causal variables influence inflation. Their decision to revise their strategies also depends on psychological factors, such as their confidence in their current strategies and their intuition for the possibility that structural change in the underlying economy has occurred or may occur over the forecast horizon. And, of course, there is a great diversity in how market participants might revise their forecasting strategies.

The qualitative and contingent nature of these considerations implies the futility of relying on a determinate rule to constrain revisions of forecasting strategies in the model. As we show in Appendix A, determinate accounts are inadequate, because market participants eventually revise their strategies in ways that neither they nor economists can fully foresee. IKE models explore the possibility that such revisions may nonetheless exhibit regularities that can be formalized *ex ante* with qualitative and contingent constraints on structural change.

Internal coherence provides theoretical justification for such regularities. It implies that market participants tend to revise their forecasting strategies in moderate ways for protracted stretches of time. Consequently, the model shows how, contrary to the view implied by behavioral-finance models, a tendency toward moderate revisions is compatible with conditional rationality.

In Appendix B, we show how moderate revisions can be formalized in the context of our model. Two sets of qualitative and contingent constraints on revisions are needed to generate time series implications. One set restricts the model's characterization of the market's prediction concerning change in its forecasting strategy over the coming period, that is, $\Delta\hat{\alpha}_{t|t+1}$ and $\Delta\hat{\beta}_{t|t+1}$. The other set constrains the characterization of how market participants' forecasting strategies might change between adjacent points in time, that is, $\Delta\alpha_t$ and $\Delta\beta_t$. In both cases, we use the same formalization of moderate change as before, that linear impacts dominate.

The resulting CEH model does not specify ex ante when periods of moderate change in the macroeconomy or moderate revisions of forecasting strategies begin or end. However, in order to derive time-series regularities from the model, we must assume that whenever an economist's model characterizes change in the macroeconomy as moderate, it makes sense for the model to represent forecast revisions as being moderate as well. The premise of the market's epistemic superiority implies that the market understands before the economist that an interval of moderate change has begun or ended. One can assume that some time is needed before the market understands that periods of moderate change have begun or ended. But, to simplify our exposition, we assume that the market recognizes this timing immediately.

Qualitative and Contingent Predictions We show in the appendix that given the set of constraints on $\Delta\hat{\alpha}_{t|t+1}$ and $\Delta\hat{\beta}_{t|t+1}$ in equations 44-46, respectively, the representations of forecasting in equations (28)-(30) imply that interest-rate movements lead the market to forecast further interest-rate movements in the same direction during an interval of moderate change in the macroeconomy:[25]

$$sgn\left(\hat{R}_{t|t+1} - R_t\right) = sgn\left(\alpha_t \Delta R_t\right) \tag{31}$$

for t between $T_{i+1} - T_i$. These constraints imply that movements of R_t also lead the market to forecast a change in the inflation rate either in the same or opposite direction depending on the sign of β_t:[26]

$$sgn\left(\hat{P}_{t|t+1} - P_t\right) = sgn\left(\beta_t \Delta R_t\right) \tag{32}$$

for t between $T_{i+1} - T_i$.

The qualitative and contingent implications in (31) and (32), in turn, imply that during periods of moderate change, the market forecasts that P_{t+1} and R_{t+1} will co-move qualitatively, either positively or negatively. We represent this qualitative and contingent empirical regularity with an analogue to determinate models' cross-moment measure that we call the "qualitative co-movement measure" (QCM):

$$QCM_i\left[\left(\hat{P}_{t|t+1} - P_t\right)\left(\hat{R}_{t|t+1} - R_t\right)\right] = \frac{\sum_{t=T_i}^{T_{i+1}}\left(\hat{P}_{t|t+1} - P_t\right)\left(\hat{R}_{t|t+1} - R_t\right)}{\left(T_{i+1} - T_i\right)} \begin{matrix} > \\ < \end{matrix} 0 \tag{33}$$

for t between $T_{i+1} - T_i$ and the algebraic sign of this QCM is determined by the sign of β_t and thus of b_t. As for the interest rate, the market forecasts a positive co-movement between R_{t+1} and R_t during periods of moderate change:

[25] This follows because the constraints imply that $\left|\Delta\hat{\alpha}_{t|t+1}\right| < |\alpha_t|$.

[26] This follows because the constraints imply that $\left|\Delta\hat{\beta}_{t|t+1}\right| < |\beta_t|$, so that $sgn\left(\delta_t\right) = sgn\left(\beta_t\right)$, and $|\gamma_t| < |\rho_t|$.

$$QCM_i\left[\left(\hat{R}_{t|t+1}-R_t\right)\Delta R_t\right]=\frac{\sum_{t=T_i}^{T_{i+1}}\left(\hat{R}_{t|t+1}-R_t\right)\Delta R_t}{(T_{i+1}-T_i)}>0 \tag{34}$$

for t between $T_{i+1}-T_i$.

5.2.2 Predictive Coherence in an IKE Model

The model's time series implications for the inflation rate are obtained by plugging the representation of the market's forecast of P_{t+1} into the semi-reduced form in (22), yielding the following reduced-form specification:

$$P_t=\lambda_t^1 R_t+\lambda_t^2\Delta R_t+\varepsilon_t^{\mathrm{M}} \tag{35}$$

where $\lambda_t^1=\frac{b_t+c\rho_t}{1-c}$ and $\lambda_t^2=\frac{c\gamma_t}{1-c}$. The qualitative and contingent constraints on revisions entailed by cognitive coherence, which are in equations (44)-(46), imply that the algebraic signs of λ_t^1 and λ_t^2 are determined by the sign of b_t. Cognitive coherence also implies constrains on how α_t, λ_t^1, and λ_t^2 might change during time intervals in which change in the macroeconomy is characterized as moderate. (These constraints are shown in equations (49)-(50) and (56)-(57)

We show in Appendix B that given these constraints, the model is coherent in terms of its predictions concerning the interest rate, because

$$sgn\left(R_{t+1}-R_t\right)=sgn\left(a_t\Delta R_t\right)=sgn\left(\hat{R}_{t|t+1}-R_t\right) \tag{36}$$

$$sgn\left\{QCM\left[\left(R_{t+1}-R_t\right)\Delta R_t\right]\right\}=sgn\left\{QCM_i\left[\left(\hat{R}_{t|t+1}-R_t\right)\Delta R_t\right]\right\}>0 \tag{37}$$

for t between $T_{i+1}-T_i$.

As for the model's implications for time series regularities in the inflation rate, the reduced form in (35) implies:[27]

$$\Delta P_{t+1}=\lambda_t^1\Delta R_{t+1}+\lambda_t^2\Delta^2 R_{t+1}+\Delta\varepsilon_{t+1}^{\mathrm{M}} \tag{38}$$

[27] We have omitted terms involving $\Delta\lambda_t^1$, $\Delta\lambda_t^2$, and Δa_{t+1} because the model implies that during intervals of moderate change, the effects of structural change are smaller than the linear effects appearing in (38). See the appendix.

Consequently, the model's reduced-form qualitative and contingent predictions for how the inflation rate unfolds are given by

$$sgn\left(P_{t+1}-P_t\right)=sgn\left(\lambda_t^1 \Delta R_{t+1}\right)=sgn\left(\hat{P}_{t|t+1}-P_t\right) \tag{39}$$

$$\begin{aligned} sgn\left\{QCM\left[\left(P_{t+1}-P_t\right)\Delta R_{t+1}\right]\right\} &= sgn\left(b_t\right) \\ &= sgn\left\{QCM_i\left[\left(\hat{P}_{t|t+1}-P_t\right)\left(\hat{R}_{t|t+1}-R_t\right)\right]\right\} \end{aligned} \tag{40}$$

for t between $T_{i+1}-T_i$.[28] Thus, the the model is predictively coherent: qualitative time-series implications of the economist's reduced- form model in (35) match those of his representation of the market participants' forecasting in (29).

[28]In deriving these predictions, we have used our assumption that α_t is sufficiently close to unity, thereby implying that the second component in (38) can be ignored. Emprical researchers rountinely report estimates of α_t of 0.99 in monthly data.

References

[1] Barberis, Nicholas., Shleifer, Andrei., and Robert W. Vishny (1998), "A Model of Investor Sentiment," *Journal of Financial Economics,* 49, 307-343.

[2] Barberis, Nicolas C., Huang, Ming. and Tano Santos (2001), "Prospect Theory and Asset Prices," *Quarterly Journal of Economic*s, 1-53.

[3] Barberis, Nicholas C. and Richard H. Thaler (2003), "A Survey of Behavioral Finance," in Constantinides, George, Harris, Milton and Rene Stulz (eds.), *Handbook of the Economics of Finance*, Amsterdam: North-Holland, 1050-1121.

[4] Arrow, Kenneth J., and Leonid Hurwicz (1958), "On the stability of competitive equilibrium," *Econometrica,* 26, 522-552.

[5] Arrow, Kenneth J., Henry D. Block, and Leonid Hurwicz (1959), "On the stability of competitive equilibrium II," *Econometrica*, 27: 82-109.

[6] Cagan Phillip (1956), "The monetary dynamics of inflation," in: Milton Friedman, (ed.), *Studies in the quantity theory of money,* University of Chicago Press, 25-117.

[7] Calvo, Guillermo A. (1983), "Staggered Prices in a Utility-Maximizing Framework," *Journal of Monetary Economics*, 12, 383–398.

[8] Cohen, Darrel, Keven Hassett, and R. Glenn Hubbard (1999), "Inflation and the User Cost of Capital: Does Inflation Still Matter," in Martin Feldstein (ed.), *The Costs and Benefits of Price Stability*, Chicago: University of Chicago Press, 199-234.

[9] Edwards, Ward (1968), "Conservatism in Human Information Processing," in B. Kleinmuth (ed.), *Formal Representation of Human Judgement*, New York: John Wiley and Sons.

[10] Evans, George W. and Seppo Honkapohja (2001), *Learning and Expectations in Macroeconomics,* Princeton, NJ: Princeton University Press.

[11] Evans, George W. and Seppo Honkapohja (2013), "Learning as a Rational Foundation of Macroeconomics and Finance," in Frydman, Roman

and Edmund S. Phelps (eds.), *Rethinking Expectations: The Way Forward for Macroeconomics*, NJ: Princeton University Press, 130-165.

[12] Friedman, Milton (1957), *Theory of the Consumption function*, Princeton University Press.

[13] Frydman, Roman (1982), "Towards an Understanding of Market Processes: Individual Expectations, Learning and Convergence To Rational Expectations Equilibrium," *American Economic Review*, 72, 652-668.

[14] Frydman, Roman (1983), "Individual Rationality, Decentralization and the Rational Expectations Hypothesis," in Frydman, Roman and Edmund S. Phelps (eds.), *Individual Forecasting and Aggregate Outcomes: "Rational Expectations" Examined*, New York: Cambridge University Press, 97-122.

[15] Frydman, Roman and Michael D. Goldberg (2007), *Imperfect Knowledge Economics: Exchange Rates and Risk*, Princeton, NJ: Princeton University Press.

[16] Frydman, Roman and Michael D. Goldberg (2011), *Beyond Mechanical Markets: Asset Price Swings, Risk,and the Role of the State*, Princeton, NJ: Princeton University Press.

[17] Frydman, Roman and Michael D. Goldberg (2013a), "The Imperfect Knowledge Imperative in Modern Macroeconomics and Finance Theory," in Frydman, Roman and Edmund S. Phelps (eds.), *Rethinking Expectations: The Way Forward for Macroeconomics*, NJ: Princeton University Press, 130-165.

[18] Frydman, Roman and Michael D. Goldberg (2013b), "Change and Expectations in Macroeconomic Models:Recognizing the Limits to Knowability," *Journal of Economic Methodology,* 118-138.

[19] Frydman, Roman and Michael D. Goldberg (2013c), "Opening Models of Asset Prices and Risk to Non-Routine Change," in Frydman, Roman and Edmund S. Phelps (eds.), *Rethinking Expectations: The Way Forward for Macroeconomics*, NJ: Princeton University Press, 130-165.

[20] Frydman, Roman and Michael D. Goldberg (2013d), "An IKE Models of Asset Price Swings: Conditional Rationality, Fundamentals, and Psychology," in preparation.

[21] Frydman, Roman and Michael D. Goldberg (2014), "Fallibility in Formal Macroeconomics and Finance Theory," forthcoming in the *Journal of Economic Methodology.*

[22] Frydman, Roman, Michael D. Goldberg, and Peter Sullivan (2013), "Conditional Rationality and the Meese and Rogoff Exchange-Rate-Disconnect Puzzle: Learning vs. Contingent Knowledge," in preparation.

[23] Frydman, Roman and Edmund .S. Phelps (1983), "Introduction" in Frydman, Roman and Edmund S. Phelps (eds.), *Individual Forecasting and Aggregate Outcomes: "Rational Expectations" Examined*, New York: Cambridge University Press, 1-30.

[24] Frydman, Roman and Edmund S. Phelps (2013), "Which Way Forward for Macroeconomics and Policy Analysis?," in Frydman, Roman and Edmund S. Phelps (eds.), *Rethinking Expectations: The Way Forward for Macroeconomics*, Princeton University Press, 1-46.

[25] Gourinchas, Pierre-Oliver and Aaron Tornell (2004), "Exchange Rate Puzzles and Distorted Beliefs," *Journal of International Economics*, 64, 303-333.

[26] Guesnerie, Roger (2005), *Assessing Rational Expectations: "Eductive" Stability in Economics,* Cambridge, MA: MIT Press.

[27] Guesnerie, Roger (2013), "Expectational Coordination Failures and Market Volatility," in Frydman, Roman and Edmund S. Phelps (eds.), *Rethinking Expectations: The Way Forward for Macroeconomics*, Princeton University Press, 49-67.

[28] Hamilton, J.D. (1988), "Rational-Expectations Econometric Analysis of Changes in Regime: An Investigation of the Term Structure of Interest Rates," *Journal of Economics Dynamics and Control,* 12, pp. 385-423.

[29] Hamilton, James D. (1994), *Time Series Analysis*, Princeton, NJ: Princeton University Press.

[30] Hayek, Friedrich A. (1945), "The Use of Knowledge in Society," *American Economic Review*, 35, 519-30.

[31] Hayek, Friedrich A. (1948), *Individualism and Economic Order*, Chicago: The University of Chicago Press.

[32] Hayek, Friedrich.A. (1974), "The Pretence of Knowledge," *Nobel Lecture*, in *New Studies in Philosophy, Politics, Economics and History of Ideas*, Chicago: The University of Chicago Press, 1978.

[33] Johansen, Søren, Katarina Juselius, Roman Frydman, and Michael D. Goldberg (2010), "Testing Hypotheses in an I(2) Model With Piecewise Linear Trends. An Analysis of the Persistent Long Swings in the Dmk/$ Rate," *Journal of Econometrics*, 158, 117–29.

[34] Juselius, Katarina (2013),

[35] Keynes, John Maynard. (1936), *The General Theory of Employment, Interest and Money*, Harcourt, Brace and World.

[36] Koo, Richard C. (2008), *The Holy Grail of Macroeconomics: Lessons From Japan's Great Recession,* John Wiley and Sons (Asia): Singapore.

[37] Lucas, Robert E. Jr. (1972), "Econometric Testing of the Natural Rate Hypothesis," in Otto Ecstein (ed.), *Econometrics of Price Determination*, Washington, DC: Board of Governors of the Federal Reserve System.

[38] Lucas, Robert E. Jr. (1973), "Some International Evidence on Output-Inflation Trade-offs," *American Economic Review*, 63, 326-334.

[39] Lucas, Robert E. Jr. (1976), "Econometric Policy Evaluation: A Critique," in Brunner Karl and Allan H. Meltzer, *The Phillips Curve and Labor Markets*, Carnegie-Rochester Conference Series on Public Policy, Amsterdam: North-Holland Publishing Company, 19-46.

[40] Lucas, Robert E. Jr. (1995), "The Monetary Neutrality," *The Nobel Lecture*, Stockholm: The Nobel Foundation: http://www.nobelprize.org/nobel_prizes/economics/laureates/1995/lucas-lecture.pdf.

[41] Lucas, Robert E. Jr. (2001), “Professional Memoir,” mimeo. http://home.uchicago.edu.

[42] Mangee, Nicholas (2013), *The Puzzle of Long Swings in Equity Markets*, PhD Dissertation, University of New Hampshire, June.

[43] Muth, John F. (1961), “Rational Expectations and the Theory of Price Movements,” *Econometrica*, 29, 315-335.

[44] Neumeyer, Pablo A. and Fabrizio Perri (2005), “Business Cycles in Emerging Economies: The Role of Interest Rates, *Journal of Monetary Economics*, 52, 383–398.

[45] Obstfeld, Maurice and Kenneth Rogoff (1996), *Foundations of International Macroeconomics*, Cambridge, MA: MIT Press.

[46] Phelps, Edmund S. et al. (1970), *Microeconomic Foundations of Employment and Inflation,* New York: Norton.

[47] Popper, Karl R. (1946), *The Open Society and Its Enemies*, Princeton, NJ: Princeton University Press, 1962.

[48] Popper, Karl R. (1957), *The Poverty of Historicism*, London and New York: Routledge.

[49] Popper, Karl R. (1982), *The Open Universe: An Argument for Indeterminism*, London and New York: Routledge.

[50] Rotemberg, Julio J. (1985), “Aggregate Consequences of Fixed Costs of Price Adjustment,” *American Economic Review*, 73, 433–436.

[51] Sargent, Thomas J. (1993), *Bounded Rationality in Macroeconomics*, Oxford, UK: Oxford University Press.

[52] Sargent, Thomas J. (1981),. “Interpreting Economic Time Series,” *Journal of Political Economy*, 89, 213-248.

[53] Sargent, Thomas J. (2008), “Rational Expectations," in *the Concise Encyclopedia of Economics,* (Second Edition), Liberty Fund, http://www.econlib.org/library/Enc/RationalExpectations.html.

[54] Shleifer, Andrei (2000), *Inefficient Markets*, Oxford: Oxford University Press.

[55] Soros, George (1987), *The Alchemy of Finance*, New York: Wiley.

[56] Soros, George (2009), "Financial Markets," *Lecture 2 of the Central European University Lectures,* October: http://www.ft.com/cms/s/2/dbc0e0c6-bfe9-11de-aed2-00144feab49a.html.

[57] Soros, George (2012), "Remarks at the Festival of Economics," Trento, Italy, June: http://www.georgesoros.com/interviews-speeches/entry/remarks_at_the_festival_of_economics_trento_italy/.

[58] Sullivan, Peter (2013), *What Drives Exchange Rates?: Reexamining the Exchange-Rate-Disconnect Puzzle*, PhD Dissertation, University of New Hampshire, forthcoming.

[59] S. Van Wijnbergen (1983), "Interest rate management in LDC's," *Journal Monetary Economics*, 12, 433-452.

[60] S. Van Wijnbergen (1985), "Macro-economic effects of changes in bank interest rates: Simulation results for South Korea," *Journal of Development Economics*, 18, 541-554.

Appendix A

The Growth of Knowledge and the Irrationality of Rational Expectations

To frame the argument, we suppose that the time-invariant REH model in (4) and (5) is relevant for representing the price process and the market forecast during some time period, $[T_1, T]$ .We now suppose that at time T, there is an unanticipated change in the economy that renders the semi-reduced form in (1) empirically inadequate, in the sense that the error between the observed market price and an economist's representation of it, which is equal to $\left(bX_t + c\hat{P}^{\text{RE}}_{t|t+1}\right)$, ceases to have a mean zero mean and be distributed independently over time and independently of X_t. We also suppose that the following semi-reduced form adequately characterizes the process driving the price during the interval $[T+1, T_1)]$ after the structural change:[29]

$$P_t = a_1 V_t + a_2 \hat{P}_{t|t+1} + \varepsilon^{\text{M}}_t \quad \text{for all } t \in [T+1, T_1] \tag{41}$$

where, (a_1, a_2) are parameters, and V_t is the set of causal factors , which, analogously to (2) is characterized as a random walk with the drift, μ_{V}, and the disturbance term ε^{V}.[30]

Now, suppose that an economist continues to represent the market's forecast with $\hat{P}^{\text{RE}}_{t|t+1}$ which is precisely what REH theory would lead him to do. Substituting (**??**) into (41) implies that the following reduced form adequately approximates the price process during $[T+1, T_1]$:

$$P_t = a_0 + a_1 V_t + a_3 X_t + \varepsilon^{\text{M}}_t \quad \text{for all } t \in [T+1, T_1] \tag{42}$$

[29] In general, there may be a stretch of time after T during which so much change is going onoccurring in the economy that no semi-reduced form would adequately approximate the price process in terms of some set of causal factors. Our conclusion that representing the market's forecast after T with (**??**) would assumespresume that market participants ignore systematic forecast errors would also hold during such an interval. For simplicity, we do not consider such intervals here.

[30] For example, in Frydman and Goldberg (2007, chapter 15), we showed that there are intermittent structural breaks in the relationship between major currency rates and a set of causal factors drawn from a large class of the monetary models for the exchange rate. Moreover, according to standard statistical criteria, for about a year after the break, there appears to be no relationship between the currency movements and the pre-break set of causal factors. Subsequently, a statistically adequate relationship emerges for a while, though it involves a different set of causal factors than the pre-change one.

where $a_0 = a_2 \frac{b}{(1-c)^2} \mu_{\mathrm{X}}$ and $a_3 = a_2 \frac{b}{1-c}$. Iterating (42) forward implies that the REH's presumption that (5) adequately represents the market's forecasts during the interval would imply the following representation of participants' forecast errors:

$$FE_{t+1}^{\mathrm{RE}} = \hat{P}_{t|t+1}^{\mathrm{RE}} - P_{t+1} = a_0^{\mathrm{FE}} + a_1^{\mathrm{FE}} V_t + a_3^{\mathrm{FE}} X_t + \varepsilon_{t|t+1}^{FE} \quad \text{for all } t \in [T+1, T_1] \tag{43}$$

where $\varepsilon_{t|t+1}^{FE} = \varepsilon_{t+1}^{\mathrm{M}} + a_1 \varepsilon_{t+1}^{\mathrm{V}} + a_3 \varepsilon_{t+1}^{\mathrm{X}}$ is the mean-zero random error that is distributed independently over time and independently of the model's causal variables. However, because the parameters $(a_0^{\mathrm{FE}}, a_1^{\mathrm{FE}}, a_3^{\mathrm{FE}})$ are not equal to zero, the RE forecast errors, in (43), are correlated over time and depend systematically on V_t and X_t.

In the context of the REH model considered here, the variables in X_t represent the information set that underpins the market forecast. Thus, were the REH representation to continue to represent adequately participants' forecasting after T, individuals would have to ignore the obvious under-performance of their forecasting strategies: these strategies' forecast errors are systematically correlated over time and with the information on which market participants supposedly based their forecasts. Assuming that a time-invariant REH representation in (5) adequately approximates market participants' forecasting in all time periods implies that sooner or later they forego obvious profit opportunities.

Appendix B

Qualitative and Contingent Restrictions and Predictive Coherence

This appendix provides the two sets of constraints that cognitive coherence implies for characterizing how market participants might revise their forecasting strategies (given in equations (28)-(30) over time. It then shows how these conditions, together with those that characterize change in the macroeconomy, lead to predictive coherence.

5.3 Qualitative and Contingent Restrictions on Revisions

One set of qualitative and contingent conditions constrains the model's representation of market participants' predictions of how they might revise their forecasting strategies in the next period, given by $\Delta\hat{\alpha}_{t|t+1}$ and $\Delta\hat{\beta}_{t|t+1}$, during intervals of moderate change in the macroeconomy:

$$\left|\Delta\hat{\alpha}_{t|t+1}R_t\right| < |\alpha_t \Delta R_t| \tag{44}$$

$$\left|\Delta\hat{\beta}_{t|t+1}\hat{R}_{t|t+1}\right| < \left|\beta_t\left(\hat{R}_{t|t+1} - R_t\right)\right| \tag{45}$$

$$|\rho_t R_t| < |\gamma_t \Delta R_t| \tag{46}$$

for t between $T_{i+1} - T_i$.[31] The one-period movement in the interest rate, which typically entails a change of ten basis points or less, is almost invariably much smaller in size than the level of R_t. As such, the constraints in equations (44)-(46) largely hold in ways in which $\left|\Delta\hat{\beta}_{t|t+1}\right| < |\beta_t|$, $\left|\Delta\hat{\alpha}_{t|t+1}\right| < |\alpha_t|$, and $|\rho_t| < |\gamma_t|$, where we recall that $\gamma_t = \alpha_t\delta_t$, $\delta_t = \left(\beta_t + \Delta\hat{\beta}_{t|t+1}\right)$, $\rho_t = \left(\delta_t\Delta\hat{\alpha}_{t|t+1} + \Delta\hat{\beta}_{t|t+1}\right)$. Internal coherence then implies that the algebraic signs of β_t and γ_t are largely the same during any period of moderate change in the macroeconomy. This qualitative understanding leads directly to the result in section 4.1.2 that the model's representations of forecasting in equations (28)-(30) imply the qualitative and contingent predictions in equations (31)-(34).

[31] The constraint in (**??**) does not necessarily follow from the other two. But, we interpret the second component in (31) as representing the linear impact on inflation.

The second set of qualitative and contingent conditions constrains the model's representations of how market participants' forecasting strategies might change between adjacent points in time, that is, $\Delta\alpha_t$ and $\Delta\gamma_t$ (and thus $\Delta\beta_t$), during periods of moderate change in the macroeconomy. The representations in equations (28) and (30) and the constraints in equations (44)-(46) imply the following characterizations of how the market's forecasts of the interest rate and inflation rate change over time during an interval of moderate change in the macroeconomy:

$$\Delta\hat{R}_{t|t+1} = \Delta R_t + \alpha_{t-1}\Delta^2 R_t + \Delta\alpha_t \Delta R_t \tag{47}$$

$$\Delta\hat{P}_{t|t+1} = \Delta P_t + \gamma_{t-1}\Delta^2 R_t + \Delta\gamma_t \Delta R_t \tag{48}$$

for t between $T_{i+1} - T_i$ and $\Delta^2 R_t = \Delta R_t - \Delta R_{t-1}$ and we have assumed that changes in $\Delta\hat{\alpha}_{t|t+1}$ and ρ_t during time intervals of moderate change, which are constrained to involve moderate changes themselves, are of second-order importance and can be ignored.

Internal coherence implies assuming that any structural shifts have a smaller impact than the linear effects of interest-rate movements:

$$|\Delta\alpha_t \Delta R_t| < \left|\alpha_{t-1}\Delta^2 R_t\right| \tag{49}$$

$$|\Delta\gamma_t \Delta R_t| < \left|\gamma_{t-1}\Delta^2 R_t\right| \tag{50}$$

for t between $T_{i+1} - T_i$. We assume that periods of moderate change in the economy are characterized by a pronounced tendency for $\Delta^2 R_t$ to be much smaller in size than ΔR_t. With this assumption, the constraints in equations (49)-(50) imply that movements of R_t tend to lead the market to alter its forecasts of R_t and P_t in the same direction.[32]

5.3.1 Predictive Coherence in an IKE Model

We recall that the unfolding of the interest rate is characterized by:

$$\Delta R_t = \Delta a_t R_{t-1} + a_{t-1}\Delta R_{t-1} + \Delta\varepsilon_t^{\mathrm{R}} \tag{51}$$

and that change in this process is represented with the following qualitative and contingent constraint:

$$|\Delta a_t R_{t-1}| < |a_{t-1}\Delta R_{t-1}| \text{ for } t \text{ between } T_{j+1} - T_j \tag{52}$$

[32] The implication concerning $\Delta\hat{P}_{t|t+1}$ follows from substituting the reduced-form solution for ΔP_t in the next section into (48).

It is immediately clear that the model is coherent in terms of its predictions concerning the interest rate and those implied by its representation of forecasting in (28):

$$sgn\,(R_{t+1} - R_t) = sgn\,(a_t \Delta R_t) = sgn\left(\hat{R}_{t|t+1} - R_t\right) \tag{53}$$

$$sgn\,\{QCM\,[(R_{t+1} - R_t)\,\Delta R_t]\} = sgn\left\{QCM\left[\left(\hat{R}_{t|t+1} - R_t\right)\Delta R_t\right]\right\} > 0 \tag{54}$$

for t between $T_{i+1} - T_i$.

The model is also coherent in terms of its reduced-from predictions of the inflation rate and those implied by its representation of forecasting in (30). The reduced-form representation of ΔP_{t+1} is given as follows:

$$\Delta P_{t+1} = \left[\lambda_t^1 \Delta R_{t+1} + \Delta\lambda_{t+1}^1 R_{t+1}\right] + \left[\lambda_t^2 \Delta^2 R_{t+1} + \Delta\lambda_{t+1}^2 \Delta R_{t+1}\right] + \Delta\varepsilon_{t+1}^{\mathrm{M}} \tag{55}$$

where we recall that $\lambda_t^1 = \frac{b_t + c\rho_t}{1-c}$ and $\lambda_t^2 = \frac{c\gamma_t}{1-c}$ and that the algebraic signs of λ_t^1 and λ_t^2 are determined by the sign of b_t. No one can know whether the direct linear effect of R_{t+1} on inflation, as represented by b_{t+1}, will be larger or smaller in size than the impact of the market's predictions concerning how to forecast revisions, which is represented by ρ_{t+1}. However, ex ante, cognitive coherence implies that the linear impact dominates, so that in representing the macroeconomy and forecasting, we can assume $sgn\left(\lambda_{t+1}^1\right) = sgn\,(b_{t+1})$.

We point out that the constraint on changes in b_t in (26), together with the assumption that $\Delta\rho_t$ is of second-order importance, implies the following qualitative and contingent constraint on changes in λ_t^1:

$$\left|\Delta\lambda_{t+1}^1 \Delta R_{t+1}\right| < \left|\lambda_t^1 \Delta R_t\right| \tag{56}$$

Similarly, the constraint in (50) implies the following qualitative and contingent constraint on changes in λ_t^2:

$$\left|\Delta\lambda_{t+1}^2 \Delta R_{t+1}\right| < \left|\lambda_t^2 \Delta^2 R_{t+1}\right| \tag{57}$$

These constraints, together with the assumption that $\Delta^2 R_t$ tends to be much smaller in size than ΔR_t, imply that the model's reduced form qualitative and contingent predictions concerning the inflation rate are compatible with the predictions implied by its representation of forecasting:

$$sgn\,(P_{t+1} - P_t) = sgn\left(\lambda_t^1 \Delta R_{t+1}\right) = sgn\left(\hat{P}_{t|t+1} - P_t\right) \tag{58}$$

$$sgn\left\{QCM\left[\left(P_{t+1}-P_{t}\right)\Delta R_{t+1}\right]\right\}=sgn\left(\lambda_{t}^{1}\right)=sgn\left\{QCM\left[\left(P_{t+1}-P_{t}\right)\Delta R_{t+1}\right]\right\} \tag{59}$$

for t between $T_{i+1}-T_{i}$.