DISCUSSION PAPERS
Institute of Economics
University of Copenhagen

04-27

Social Fractionalization, Endogenous Property Rights, and Economic Development

Ines Lindner and Holger Strulik

Studiestræde 6, DK-1455 Copenhagen K., Denmark
Tel. +45 35 32 30 82 - Fax +45 35 32 30 00
http://www.econ.ku.dk

Social Fractionalization, Endogenous Property Rights, and Economic Development*

Ines Lindner† and Holger Strulik‡

November 2004

We investigate how social composition affects competitive and cooperative behavior in a linear growth model without a priori secure property rights. If a society is homogenous or highly fractionalized it is in the self-interest of people to cooperate. The first best allocation is enforced through trigger strategies, property rights turn out to be secure, and growth is independent from social fractionalization. If a society is polarized, i.e. if it consists of a small number of equally sized groups, property rights can turn out as unenforceable. If so, groups follow an exploitive strategy that leads to low investment and growth. In this case the rate of growth is continuously decreasing in the degree of fractionalization and possibly negative.

Keywords: Africa's Growth Tragedy, Property Rights, Social Fractionalization, Differential Games, Trigger Strategies.

JEL Classification: C73, D74, O40.

---

*We would like to thank Oded Galor, Nikolaus Siegfried, Stefanie Strulik, participants of research seminars at Hamburg, Jerusalem, and Zuerich Universities, and at the European Economic Association Annual Conference, Madrid 2004, for useful comments.

†Universite Catholique de Louvain, Center for Operations Research and Econometrics, 34 Voie du Romain Pays, B-1348 Louvain-la-Neuve, Belgium, email: lindner@core.ucl.ac.be.

‡Institute of Economics, University of Copenhagen, Studiestraede 6, 1455 Copenhagen K, Denmark, email: holger.strulik@econ.ku.dk.

1. Introduction

In this paper we offer a theory of how social composition of a population may determine its economic and social performance. The question why some of the world's countries perform so much worse than others has alternatively been posed as "Why not Africa?" (Freeman and Lindauer, 1999) because the majority of the worst performing countries can be found in tropical Sub Saharan Africa. Geographical location appears to be a self-evident explanatory variable for poor economic development. Yet, recently a couple of studies have provided evidence for a predominantly indirect effect of geography on development through the shaping of institutions during colonial time. At the same time the quality of institutions and in particular measures of the security of property rights have been identified as a fundamental explanation of the world income distribution. As one study concludes, "the quality of institutions trumps everything else" (Rodrik et al., 2002).[1]

If property rights are missing or insecure, an economy's wealth can be treated as a common pool resource and exploitive behavior can explain underinvestment (in physical capital or education) and slow, absent, or negative growth. Generally this result has been derived as a non-cooperative Nash equilibrium of the common pool problem that precludes cooperative behavior. Here, we refer to this solution as *exploitive equilibrium*. From standard game theory we know that the toughness of competition (here the degree of resource exploitation) is increasing in the number of players. In other words, if we imagine a society subdivided in competitive groups as the players of the game, the exploitive Nash equilibrium suggests that the damage caused by missing property rights is increasing in the degree of social fractionalization. Empirical work by Easterly and Levine (1997) on "Africa's Growth Tragedy" supports this view. They reveal a strong negative link between ethnolinguistic fractionalization and economic development according to which "ethnic diversity alone accounts for about 28 percent of the growth differential between the countries of Africa and East Asia". A follow-up study by Alesina et al. (2003) confirms these results.

An alternative measure of social diversity has been developed by Esteban and Ray (1994). They argue that social tension depends on social distance and relative group size and derive

[1]For geography see e.g. Gallup et al. (1999) and Bloom and Sachs (1998). For institutions see e.g. Mauro (1995), Keefer and Knack (1997), and Hall and Jones, (1999). For the impact of property rights on investment see e.g. Svensson (1998), and Johnson et al. (2002). For the indirect effect of location on growth via institutional choice see Acemoglu et al. (2001, 2002), Rodrik et al. (2002), and Easterly and Levine (2003). For surveys on Africa's growth problem see Freeman and Lindauer (1999) and Collier and Gunning (1999).

an index of social polarization. Assuming equal distance between groups the polarization index suggests that the potential for conflict is highest in a society of two groups of equal size and subsequently decreasing in the number of social groups. In other words, the incentive for civic cooperation first decreases and then increases with social diversity. This hypothesis is supported empirically by Keefer and Knack (2002) who find a non-linear relationship between ethnic homogeneity and the security of property rights implying that the risk of expropriation is highest for intermediate values of ethnic fractionalization. A similar result is found by Zak and Knack (2001) with respect to trust in economic transactions.[2]

Interestingly, Easterly and Levine (1997) cannot find such a non-linear correlation between growth and ethnic fractionalization. Alesina et al. (2003) perform growth regressions using both indices and conclude that results are substantially weaker using the polarization index. When both indices are used together, the fractionalization index typically remains significant while the polarization index is insignificant.

At first sight the above empirical studies may suggest an inconsistency in the argument that bad economic performance is explained by insecure property rights. To see this more clearly, consider a society subdivided in symmetric groups. The index of fractionalization (i.e. the probability to draw randomly from the population two people of different social affiliation) assumes its mean value of 0.5 when the society consists of two groups. Further rising group number increases the index of fractionalization and decreases the index of polarization. Because most of the conflict-ridden and slow growing countries show values of ethnic fractionalization well above 0.5, the above empirical studies suggest that *both*, economic growth *and* the risk of expropriation, decrease as the number of ethnic groups rises.

In this paper we offer an economic model that explains the observed empirical regularities by endogenizing property rights.[3] We consider a priori insecure property rights in a simple linear growth model and investigate whether a society is capable to develop social norms (through trigger strategies) in order to self-enforce the first-best solution of secure property rights. In their calculus whether to respect property rights or not groups have to take into account a short

[2]Both studies use ethnic homogeneity measured as the percentage of population belonging to a country's largest ethnic group. Garcia- Montalvo and Reynal-Querol (2002) present one of the rare studies employing directly an index of polarization. They report that ethnic polarization has a positive effect on civil wars which cannot be found for ethnic fractionalization. Theoretical reasoning shows that less unambiguous and possibly non-linear effects of group size on polarization are possible if social distance and asymmetric group size are taken into account, see Esteban and Ray (1994, 1999).

[3]Of course, an ad hoc solution of the apparent puzzle may be that the above reasoning neglects social distance and group asymmetries. Yet, the present paper maintains the symmetry assumption.

run gain from defection, a loss of growth during the defection period, and a further loss of growth during the punishment period when all groups relapse to exploitive strategies. The interplay of these effects explains a non-linear, inverted u-shaped correlation between social fractionalization and the incentive to defect, i.e. the insecurity of property rights. However, *if* property rights turn out to be unenforceable, economic growth decreases continuously as the number of social groups rises. Finally, economic growth is independent from social composition whenever a society is capable to enforce secure property rights.[4]

The present paper is related to two earlier investigations on social cooperation and economic growth by Benhabib and Radner (1992) and Benhabib and Rustichini (1996). These studies differ from the current one in their assumption about the strategy that groups announce to play in case of defection. There, it is assumed that groups play a fast consumption strategy according to which they extract resources at the maximum feasible rate. Here, we assume that groups play a Markovian Nash strategy after defection. We show that this interior solution constitutes a credible threat because it turns out as an equilibrium in a world without cooperation. The main difference between the papers, however, is the focus of investigation. While Benhabib and his co-authors explore the effects of initial conditions for societies consisting of two groups we investigate how social composition is related to conflict, cooperation, and economic development.

As a benchmark we use an otherwise identical economy where property rights are secure per definition. For that purpose the next section briefly reviews the Ak growth model and determines the first best intertemporal allocation. In Section 3 we discuss the Markovian Nash equilibrium that occurs when property rights are absent and unenforceable and groups act competitively. This exploitive solution has been investigated by Lane and Tornell (1996) but with a different focus (on the so called voracity effect). Here we concentrate on the economic effects of social fractionalization. The fourth section contains the main part of the paper. Groups are allowed to enforce the first best solution of secure property rights. Using a particular solution technique (by representing the value of consumption throughout the punishment phase as scrap value in a finite horizon problem of the potential defector) we are able to prove our propositions analytically. Some proofs, however, are quite long and are relegated to an Appendix for better readability.

[4]Fearon and Laitin (1996) emphasize the view that cooperation between (ethnic) groups is the rule rather than an exception. They provide empirical illustration and theoretical explanation within a social matching game that informal institutions develop which enable local-level interethnic cooperation. See also Axelrod (1984) for a theory on the evolution of cooperation. The complementing view that conflict rather than peace is the rule has been investigated in a series of papers based on Grossman (1991) and Hirshleifer (1995).

2. The Economy and its Performance under Secure Property Rights

Consider an economy populated by a continuum $[0, n]$ of people. The society is subdivided in $n \geq 2$ homogenous groups. We define a group as a set of people who cooperate with each other but not necessarily with members of other groups.[5] Each group $i = 1, 2, \ldots, n$ consists of a continuum $[0, 1]$ of agents with time preference rate $\rho > 0$ who maximize intertemporal utility of consumption c_i facing an iso-elastic utility function

$$\int_0^\infty \log(c_i)\mathrm{e}^{-\rho t}\mathrm{d}t \ . \tag{1}$$

Using capital $k \geq 0$ a single output is produced via a linear production technology. In order to elaborate the consequences of missing property rights we first consider an economy where property rights are secure by assumption. This is the familiar textbook model of growth of an economy populated by a continuum $[0, n]$ of identical utility maximizing consumers. Given a non-expropriable personal endowment of capital k_j, average productivity A, and depreciation rate δ, each consumer $j \in [0, n]$ faces an individual budget constraint $\dot{k}_j = Ak_j - \delta k_j - c_j$. Following, for example, Barro and Sala-i-Martin (1995, Ch. 4.1), economic development is described as follows.

If

$$A - \delta > \rho, \tag{2}$$

then an economy with secure property rights develops along a path of positive constant growth where every agent $j \in [0, n]$ consumes $c_j = \rho k_j$, implying that a group of size $[0, 1]$ consumes

$$c = \frac{\rho}{n} k \tag{3}$$

and the economy grows at rate

$$g_{FB} := \frac{\dot{c}}{c} = \frac{\dot{k}}{k} = A - \delta - \rho \ . \tag{4}$$

Given secure property rights, growth is independent from any subdivision of society in n groups.

[5]For theoretical analysis we must not determine *how* people are allocated to groups. For example, regional clusters, political opinions, language, ethnicity, or religion could be used as allocation device. Here, the attitude to cooperate can be thought of as the group defining "boundary" according to Barth (1969).

3. Unenforceable Property Rights: The Exploitive Equilibrium

In this section we consider the economic outcome when property rights are absent or unenforceable. Following the literature we describe this phenomenon as a common pool problem where agents of each group are free to invest in and appropriate from a common stock k which evolves according to

$$\dot{k} = Ak - \delta k - \sum_{i=1}^{n} c_i \ . \tag{5}$$

Given $k(0) = k_0 \geq 0$ groups maximize utility of their representative member (1) with respect to (5) using a Markovian Nash-strategy, $c_i(k)$. Since all groups share symmetric utility functions and the same state equation, we confine the analysis to Nash-equilibria in symmetric strategies $c_i = c$ for $i = 1, \ldots, n$.

Theorem 3.1. *An economy without property rights has a Nash-equilibrium given by*

$$c_i = c = \rho k \tag{6}$$

for $i = 1, \ldots, n$.
The economy develops along a path of constant growth at rate

$$g_N := \frac{\dot{c}}{c} = \frac{\dot{k}}{k} = A - \delta - \rho n \ , \tag{7}$$

which is positive if

$$\frac{A - \delta}{n} > \rho \ . \tag{8}$$

The proof is delegated to the Appendix. The next two theorems compare with the outcome under secure property rights and show the effect of social fractionalization on economic performance in a non-cooperative equilibrium.

Theorem 3.2. *There exists a set of feasible parameter specifications for A, δ, ρ, n which enables an economy with given secure property rights to grow forever but not an otherwise identical economy without apriori given property rights. The possibility for stagnation and retrogression increases in the number of social groups.*

Proof. Conditions (2) and (8) have to hold for positive growth with secure property rights and without property rights, respectively. Condition (8) can always be violated by a sufficiently large n, whereas (2) is independent from n. $\square$

Condition (2) requires that the average net productivity (i.e. the net interest rate) is larger than the time preference rate for positive growth in a world with secure property rights. Without secure property rights, net productivity divided by the number of groups has to be larger than the time preference rate in order to generate growth promoting investment. Substraction of (7) from (4) provides the following result.

THEOREM 3.3. *If an economy is capable of long-run growth without property rights then growth in an otherwise identical economy with secure property rights is higher. The difference in growth rates is given by $(n-1)\rho$, which is increasing in the number of social groups.*

In a world with secure property rights everyone earns the fruits of his investment and the number of groups is a mere scale variable. Since we have assumed constant returns to scale in production, growth is independent from group number. Without secure property rights every investment in the capital stock bears the risk of expropriation, i.e. of not earning the fruits of an investment. Since $c_i'(k) > 0$ for all i, everyone knows that if he invests more (and raises k) his contenders will consume more in the future. This knowledge about possible expropriation drives down investment and growth. Because people cooperate with group affiliates and compete with members of other groups, the risk of being expropriated rises as people are increasingly surrounded by competing members of other groups. In conclusion, growth decreases with social fractionalization.[6]

[6]The results generalize with respect to iso-elastic utility functions, $c^{1-\theta}/(1-\theta)$, when $\theta > 1$. This requires that the intertemporal elasticity of substitution $(1/\theta)$ is not larger than one, a condition that finds empirical support, see e.g. Hall (1988) and Ogaki and Reinhart (1998). This assumption also prevents a voracity effect and a fast consumption strategy. For $\theta \geq 1$, utility goes to minus infinity as consumption goes to zero and groups will never exhaust an economy's resources in finite time. Nevertheless, condition (8) can be violated and an economy may retrogress at a constant rate and disappear eventually.

4. Social Norms and Self-enforceable Property Rights

Because under insecure property rights economic performance is always worse and possibly disastrous, it looks like as if it is in everybody's self-interest to obey the law. It is therefore interesting to investigate whether a society is capable to self-enforce secure property rights. Formally, this can be achieved through trigger strategies. Groups agree to follow the first best consumption strategy as determined for secure property rights in Section 2 and sustain their agreement by threatening to punish any defector. A credible threat is to relapse to the exploitive strategy as determined in Section 3 since we have already shown that it constitutes a non-cooperative Nash equilibrium.

For defection to be possible at all, there must be a delay between defection and punishment. If defectors were punished immediately at the time of defection without the possibility of a short-time gain, there would be no incentive to deviate from cooperation. Since $c = (\rho/n)k$ in first best equilibrium, the short-term gain of deviating from this consumption strategy is increasing in the number of groups. The long-term loss, however, is also increasing in the number of groups since we have shown that growth in a non-cooperative equilibrium depends negatively on the degree of fractionalization. The interplay between these three elements, delay of punishment, short term gain, and long-term loss, determines whether cooperation and self-enforcing laws are possible in a society or not.

The trigger strategy of each group is given by

$$\psi_i(k) = \begin{cases} (\rho/n)k & \text{if no player has defected at or before time } t-T; \\ \rho k & \text{if a defection has occured at or before time } t-T. \end{cases} \tag{9}$$

Hence, groups start cooperating and achieve the overall first best consumption strategy as long as nobody defects. Defection means a violation of the social norm by consuming more (and investing less) than allowed by secure property rights. In other words, a violation of property rights occurs. After a delay of T time units following the first defection all groups i switch to exploitive Markovian Nash strategies.

For the following discussion, we denote by $\Delta V_i(n, T, \tilde{t})$ the net value of defection, i.e. the net utility gain from defection at time $\tilde{t}$ for group i as compared to the cooperative solution. Group i has an incentive to defect at time $\tilde{t}$ if $\Delta V_i(n, T, \tilde{t}) > 0$. Since time enters the game explicity only via the exponential term in the utility functional, we restrict the analysis without loss of generality to $\tilde{t} = 0$. Further, since we consider only symmetric solutions the utility difference is $\Delta V_i = \Delta V$ for all groups i.

The solution is determined in the following way. We first calculate value of consumption in the punishment phase when all groups consume according to the non-cooperative strategy. This value can be expressed as a function of the capital stock at time T, the begin of punishment. It enters as a scrap value a finite horizon problem of optimal control that determines optimal consumption during the defection period. After optimal consumption of the defector has been found, total value during the phases of defection and punishment is calculated and compared to the first best solution. The following theorem (which is proven in the Appendix) shows that non-cooperation may indeed occur and characterizes its consequences on consumption and growth.

THEOREM 4.1. *The net value of defection is*

$$\begin{aligned}\Delta V(n,T) &= \{\rho \log[n] - (g_{FB} - g_D) - (g_D - g_N)\exp[-\rho T]\}/\rho^2 \\ &= \frac{1}{\rho}\left[\log[n] - \left(1 - \frac{1}{n}\right) - \left(n - 2 + \frac{1}{n}\right)\exp[-\rho T]\right] \ .\end{aligned} \tag{10}$$

The set of parameters $P = \{(n,T) \mid \Delta V(n,T) > 0\}$ *for which defection occurs is nonempty.*

In the phase of defection, the defecting group optimally consumes

$$c = \rho k, \tag{11}$$

which yields the growth rate

$$g_D = A - \delta - \frac{2n-1}{n}\rho. \tag{12}$$

It is interesting to note that during the defection period the deviating group consumes according to the non-cooperative strategy ρk, which would also be the first best strategy if it were the only group existing in the economy. In this phase the remaining $(n-1)$ groups still cooperate by consuming $(\rho/n)k$, which explains the overall growth rate (12).

The next theorem specifies the set of parameters $P = \{(n,T) \mid \Delta V(n,T) > 0\}$ for which defection occurs. It is proven in the Appendix.

THEOREM 4.2. *Defection occurs iff $T > \log(2)/\rho$. For fixed $T > \log(2)/\rho$ the curve $\Delta V(n,T)$ starts at $\Delta V(1,T) = 0$ and increases in n reaching a maximum at $n = \exp[\rho T] - 1$. For n sufficiently large $\Delta V(n,T)$ is negative.*

Generally, we observe cooperation independently from the degree of social fractionalization. Only if the delay of punishment and the time preference rate are sufficiently large such that $\exp[\rho T] > 2$, we observe conflict in polarized societies consisting of a few large groups. The gain of defection is inverted U-shaped in the number of groups and corresponds with the empirical finding of a U-shaped correlation between ethnic homogeneity and security of property rights (see Introduction).

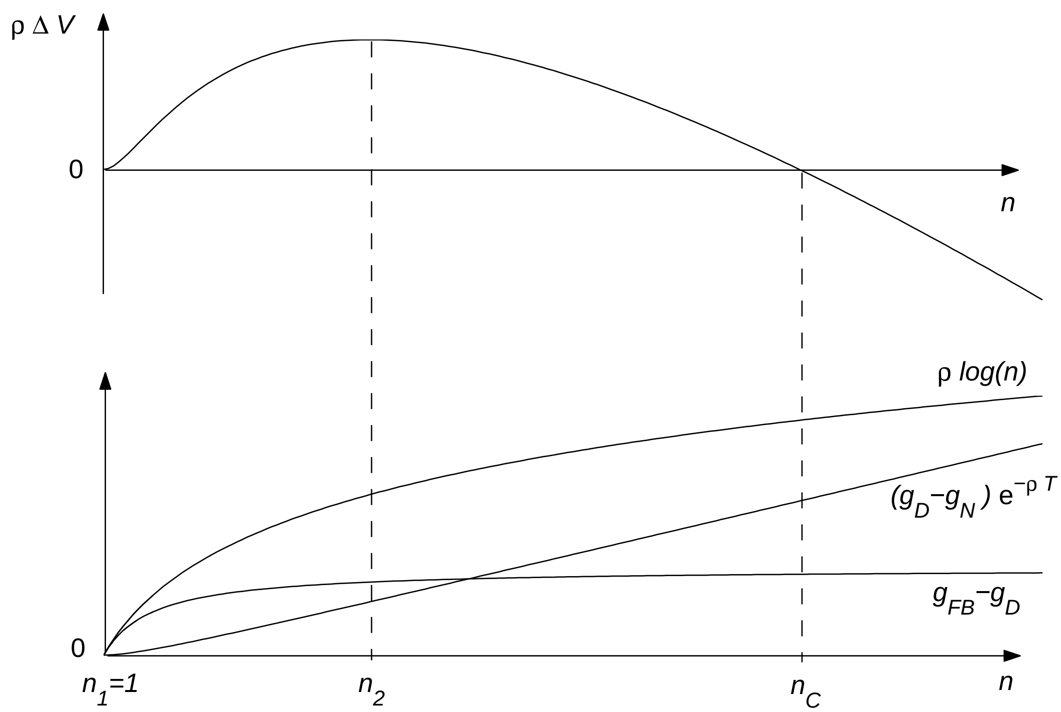
The nonlinear behavior of ΔV can best be explained using equation (10) and its graphical representation in Figure 1. The first term in braces, $\rho \log[n]$, results from the utility gain during the defection phase. Members of the defecting group consume ρk violating the "right" to consume $(\rho/n)k$ according to the social norm (9). This positive effect is increasing in n. When there are more competing groups in a society the appropriable share rises. This effect resembles the so called *business stealing effect* in the R&D literature, now stealing taken literally.

The second term represents the loss of growth that would result if growth during the defection phase applied at all times. Growth in the first best economy (g_{FB}) is independent from social fractionalization. Growth during the defection phase (g_D) depends negatively on n: the larger the degree of social fractionalization the larger is the loss of growth promoting investment when one defecting group consumes ρk instead of $(\rho/n)k$. Hence, loss of growth, $(g_{FB} - g_D) = (1 - 1/n)\rho$, depends positively on n. If this would be the only negative consequence of defection we would always observe non-cooperative behavior because $\log[n] > 1 - 1/n$ for all $n > 1$.

The overall loss of growth, however, is larger because after detection at time T all social groups switch to the exploitive Nash-consumption strategy ρk and the economy grows at rate $g_N < g_D$. The last term in (10) represents this effect. The (undiscounted) loss of growth, $(g_D - g_N) = (n + 1/n - 2)\rho$, depends positively on the number of social groups. Note that while the first two effects, $\log(n)$ and $(g_D - g_{FB})$, are concave in n, the loss of growth during punishment is slightly convex in n. Hence, the punishment effect is always dominating for sufficiently large n. This reflects the negative impact of social fractionalization: the larger the number of competing groups the lower an economy's growth rate and the larger the loss of value in the punishment phase. The overall consequences of this effect are the heavier the earlier defection entails punishment. If punishment follows sufficiently rapidly, defection would not occur. In other words, the earlier defection is punished the higher the degree of social polarization needed for non-cooperative behavior to occur.

Taken together, the incentive to violate property rights first increases and then decreases with increasing social diversity. For small n the short run gain from defection (the business stealing effect) dominates, reaching a maximum at n_2 in Figure 1. Marginal utility, however, is diminishing. For large n (i.e. small power of a single group) the utiltiy gain is overcompensated by the negative effect of fractionalization on growth caused by a whole society relapsing to non-cooperative behavior. The incentive to violate property rights is inverted u-shaped. At the same time, economic growth decreases continuously in social fractionalization [according to (7)] whenever property rights turn out to be insecure and is independent from social composition [according to (4)] whenever secure property can be enforced.

FIGURE 1. The Gain of Defection and its Components



5. Final Remarks

This paper has offered a theory of how social composition affects property rights and, through this channel, economic development. We have explained why social fractionalization can be non-linearly correlated with the risk of expropriation (with a highest incentive to violate property rights at intermediate values) and at the same time have a continuously negative impact on economic growth. Nevertheless, the model supports also the "institutions trumps everything view" in that social fractionalization is only harmful for growth if secure property rights cannot be enforced.

The use of a simple growth model has allowed us to derive our results analytically. Simplicity entails, of course, also some limitations. For deviation from cooperative behavior to occur, logarithmic utility requires high values for time preference and delay of punishment. For example, a society of two groups is not capable to enforce property rights if $\rho = 0.1$ and $T \geq 10$ (or $\rho = 0.05$ and $T \geq 20$). Numerically one can verify that the delay of punishment necessary for non-cooperative behavior can be reduced substantially using a general iso-elastic utility function. Yet, the findings of Section 4 cannot longer be proven analytically. On the other hand, the results highlights the importance of a sufficiently long time interval between defection and punishment. This feature has an interesting interpretation in light of Temple and Johnsson's (1998) reconsideration of the Adelman-Morris index of social capability. After decomposing the index, one single variable turns out to be robust in various growth regressions (as proxy for civic trust). This is the component based on newspaper circulation and the number of radios per head. Easterly and Levine (1997) provide a similar result. They calculate that Africa's telephone system accounts for a loss of one percent per year in growth. Assuming that a high provision of mass communication reduces the time needed to detect and publish defection, our model predicts insecure property rights and slow growth in particular for those countries where sufficiently polarized societies suffer from low social capability through low initial endowment of means of mass communication.

We have shown that a model-society divided in symmetric groups is sufficient to derive our results. Social structure of existing societies is, of course, far more complex in various aspects. Not only are groups of different size and power. Sociologists and political scientists also emphasize that people classify themselves and others along multiple dimensions of group membership. Heterogeneity of social affiliation, cross cutting ties, and conflicting group loyalties promote lower levels of intergroup conflict and decrease group cohesion (see, for example, Coser, 1956, Le Vine and Campbell, 1972). Finally, importance of a particular group membership may itself vary over time as, for example, class affiliation together with income distribution and ethnic fractionalization together with the evolution of nations (Anderson, 1983). In this paper we have proposed a theory of how given social fractionalization affects economic growth through the enforcement of property rights. In the long run, however, both property rights rights *and* social diversity are endogenous. A theory that considers an endogenous evolution of both variables simultaneously constitutes a promising field for future research.

Appendix

The proof of Theorem 3.1 uses the following Lemma.

LEMMA 5.1. *If a continuously differentiable function $V(k)$ can be found that satisfies*

$$\rho V(k) = \log[V'(k)] + V'(k)\left[Ak - \delta k - nV'(k)^{-1}\right] \tag{A.1}$$

subject to the boundary condition

$$\lim_{t\to\infty} V(k(t))\exp\left[-\rho t\right] = 0, \tag{A.2}$$

where $k(t)$ is the nonnegative solution to $\dot{k} = Ak - \delta k - c_1 - ... - c_n$ with $k(0) = k_0$ and

$$c(k) = V'(k)^{-1}, \tag{A.3}$$

then it generates a symmetric Markovian Nash-equilibrium with the strategy of each player defined by (A.3).

Proof. Using the Hamiltonian functions defined by

$$H_i(k, c_1, ..., c_n, \lambda_i, t) := \log(c_i)\exp\left[-\rho t\right] + \lambda_i\left[Ak - \delta k - c_1 - ... - c_n\right] \tag{A.4}$$

the Hamiltonian-Jacobi-Bellman equations can be written as

$$\begin{aligned} -\frac{\partial S_i(k,t)}{\partial t} &= \max_{c_i} H_i(k, c_1(k,t), ..., c_i, ..., c_n(k,t), \frac{\partial S_i(k,t)}{\partial k}, t), \\ c_i(k,t) &= \arg\max_{c_i} H_i(k, c_1(k,t), ..., c_i, ..., c_n(k,t), \frac{\partial S_i(k,t)}{\partial k}, t) \end{aligned} \tag{A.5}$$

with boundary conditions

$$\lim_{t\to\infty} S_i(k(t), t) = 0, \tag{A.6}$$

for $i = 1, ..., n$, where $k(t) \geq 0$ solves $\dot{k} = Ak - \delta k - c_1 - ... - c_n$ with $k(0) = k_0$.

If there are continuously differentiable functions $S_1(k,t), ..., S_n(k,t)$ which satisfy (A.5) and (A.6), then they generate a Markovian Nash-equilibrium by maximizing the Hamiltonians (A.5) (see for example Theorem 4.1 in Dockner et al. (2000).

By setting $S_i(k,t) = V_i(k)\exp\left[-\rho t\right]$ solving equations (A.5) and (A.6) simplifies to solving the following system of ordinary differential equations:

$$\begin{aligned} \rho V_i(k) &= \max_{c_i} H_i(k, c_1(k), ..., c_i, ..., c_n(k), V_i'(k), 0), \\ c_i(k) &= \arg\max_{c_i} H_i(k, c_1(k), ..., c_i, ..., c_n(k), V_i'(k), 0), \end{aligned} \tag{A.7}$$

with boundary conditions

$$\lim_{t\to\infty} V_i(k(t))\exp\left[-\rho t\right] = 0. \tag{A.8}$$

Maximization of the Hamiltonians provides

$$1/c_i = V_i'(k), \tag{A.9}$$

and (A.7) can be rewritten as

$$\rho V_i(k) = H_i(k, V_i'(k)^{-1}, ..., V_i'(k)^{-1}, V_i'(k), 0). \tag{A.10}$$

For symmetric solutions (A.8) and (5) simplify to

$$\rho V(k) = H(k, V'(k)^{-1}, ..., V'(k)^{-1}, V'(k), 0),$$

and with (A.4) Lemma 1 follows. □

Proof of Theorem 3.1. Let $V(k)$ be defined as

$$V(k) := \int_{k_0}^{k} u'(c(y))dy + V(k_0), \tag{A.11}$$

with $V(k_0)$ given by

$$V(k_0) = \frac{1}{\rho}\{u(c_0) + u'(c_0)[Ak_0 - \delta k_0 - nc_0]\}$$

and $u(y) = \log(y)$. Differentiating (A.1) with respect to k and substituting $u'(c) = V'(k)$ and $u''(c)c'(k) = V''(k)$ provides

$$c'(k) = \frac{[A - \delta - \rho]\,c(k)}{Ak - \delta k - nc(k) + (n-1)c(k)},$$

solving by method of undetermined coefficients yields (6). From (A.11) follows

$$V(k) = 1/\rho\,[\log(k) - \log(k_0)] + V(k_0)$$

and insertion of $V(k)$ into (A.1) using $V'(k_0) = u'(k_0)$ yields the initial value $V(k_0)$.

For (A.2) follows

$$\begin{aligned}
\lim_{t\to\infty} V(k(t))\exp[-\rho t] &= \lim_{t\to\infty}\frac{1}{\rho}[\log(k(t)) - \log(k_0)]\exp[-\rho t] \\
&= \frac{1}{\rho}\lim_{t\to\infty}\log(k(t))\exp[-\rho t] \\
&= \frac{1}{\rho}\lim_{t\to\infty}[\log(k_0) + (A - \delta - \rho n)t]\exp[-\rho t] = 0\ .
\end{aligned}$$

□

Proof of Theorem 4.1. The proof is given in three steps. Step 1 derives (11), Step 2 determines (12) and ΔV, and Step 3 proves the non-emptiness of P.

Step 1: The value in the punishment phase is given by

$$\begin{aligned}
V_P &= \int_T^\infty \log[\rho k(T)\exp[g_N(t-T)]]\exp[-\rho t]\,dt \\
&= \exp[-\rho T]\int_T^\infty \{\log[\rho k(T)] + g_N(t-T)\}\exp[-\rho(t-T)]\,dt \\
&= \exp[-\rho T]\,S(k(T)),
\end{aligned}$$

with

$$S(k(T)) = \int_T^\infty \{\log[\rho k(T)] + g_N(t-T)\}\exp[-\rho(t-T)]\,dt.$$

For $S(k(T))$ follows

$$\begin{aligned}
S(k(T)) &= \log[\rho k(T)]\int_T^\infty \exp[-\rho(t-T)]\,dt + g_N\int_T^\infty (t-T)\exp[-\rho(t-T)]\,dt \\
&= \frac{1}{\rho}\log[\rho k(T)] + \frac{g_N}{\rho^2},
\end{aligned}$$

and hence

$$V_P = \exp[-\rho T]\left(\frac{1}{\rho}\log[\rho k(T)] + \frac{g_N}{\rho^2}\right). \tag{A.12}$$

If a player considers defection at time $\tilde{t} = 0$, he has to solve the optimal control problem

$$\max_c \int_0^T \log c \exp[-\rho t]\, dt + V_P$$

subject to

$$\dot{k} = \left(A - \delta - \frac{n-1}{n}\rho\right) k - c, \qquad k(0) = k_0.$$

The Hamiltonian follows as

$$H(k, c, t) = \log[c] + \lambda \left[\left(A - \delta - \frac{n-1}{n}\rho\right) k - c\right]$$

and the first order conditions follow as

$$\text{(A.13)} \qquad \begin{aligned} \lambda &= \frac{1}{c}, \\ \frac{\dot{\lambda}}{\lambda} &= -\left(A - \delta - \frac{2n-1}{n}\rho\right), \\ \lambda(T) &= S'(k(T)) = \frac{1}{\rho} k(T), \end{aligned}$$

Consider the strategy $c(k) = \rho k$. This implies $\dot{k}/k = g_D$ and hence $\lambda = 1/(\rho k)$ fulfills (A.13).

Step 2: For the growth rates in the defection phase follows

$$\frac{\dot{k}}{k} = \left(A - \delta - \frac{n-1}{n}\rho - \rho\right) = A - \delta - \frac{2n-1}{n}\rho$$

and with (A.13) follows

$$g_D := A - \delta - \frac{2n-1}{n}\rho = \frac{\dot{c}}{c} = \frac{\dot{k}}{k}.$$

This provides $k(T) = k_0 \exp[g_D T]$.

The value in the defection phase is given by

$$\text{(A.14)} \qquad \begin{aligned} V_D &= \int_0^T \log[\rho k_0 \exp[g_D t]] \exp[-\rho t]\, dt \\ &= \log[\rho k_0] \int_0^T \exp[-\rho t]\, dt + g_D \int_0^T t \exp[-\rho t]\, dt \\ &= \frac{1}{\rho} \log[\rho k_0] (1 - \exp[-\rho T]) + \frac{g_D}{\rho}\left(\frac{1}{\rho} - \exp[-\rho T](T + \frac{1}{\rho})\right). \end{aligned}$$

From (A.12) follows with $k(T) = k_0 \exp[g_D T]$

$$\text{(A.15)} \qquad V_P = \frac{\exp[-\rho T]}{\rho}\left(\log[\rho k_0] + g_D T + \frac{g_N}{\rho}\right)$$

The total value of defection follows with (A.14) and (A.15) after some simplification as

$$\text{(A.16)} \qquad \begin{aligned} V_{D+P} &= V_D + V_P \\ &= \frac{1}{\rho^2}\left\{\rho \log[\rho k_0] + g_D + (g_N - g_D)\exp[-\rho T]\right\}. \end{aligned}$$

The value of cooperation and hence the value of the first best solution follows with (4) as

$$\text{(A.17)} \qquad \begin{aligned} V_{FB} &= \int_0^\infty \log\left[\frac{\rho}{n} k_0 \exp[g_{FB} t]\right] \exp[-\rho t]\, dt \\ &= \log\left[\frac{\rho}{n} k_0\right] \int_0^\infty \exp[-\rho t]\, dt + g_{FB} \int_0^\infty t \exp[-\rho t]\, dt \end{aligned}$$

$$= \frac{1}{\rho}\log\left[\frac{\rho}{n}k_0\right] + \frac{g_{FB}}{\rho^2}.$$

Set $\Delta V := V_{D+P} - V_{FB}$. From (A.16) and (A.17) follows

$$\Delta V(n,T) = \frac{1}{\rho^2}\left[(g_D - g_{FB}) - (g_D - g_N)\exp\left[-\rho T\right] + \rho\log\left[n\right]\right] \tag{A.18}$$

$$= \frac{1}{\rho}\left[-1 + \frac{1}{n} + \left(2 - n - \frac{1}{n}\right)\exp\left[-\rho T\right] + \log\left[n\right]\right]. \tag{A.19}$$

Step 3: A player has an incentive to defect if $\Delta V := V_{D+P} - V_{FB} > 0$. If n is fixed it follows for ΔV

$$\lim_{T\to 0}\Delta V = \frac{1}{\rho}\left[1 - n + \log\left[n\right]\right] < 0 \qquad \text{for } n > 1.$$

Further

$$\lim_{T\to\infty}\Delta V = \frac{1}{\rho}\left[-1 + \frac{1}{n} + \log\left[n\right]\right] > 0 \qquad \text{for } n > 1.$$

This proves that the set of parameters $P = \{(n,T) \mid \Delta V(n,T) > 0\}$ is nonempty. □

Proof of Theorem 4.2.

Set $x := \exp[-\rho T]$ then from (A.19) follows

$$\Delta V(n,x) = \frac{1}{\rho}\left[-1 + \frac{1}{n} + (2 - n - \frac{1}{n})x + \log[n]\right], \tag{A.20}$$

where $n \in \mathbb{N}$ and $x \in [0,1]$.
It is easy to see that $\Delta V(1,x) = 0$.

For the partial derivative of ΔV with respect to n follows

$$\frac{\partial}{\partial n}\Delta V(n,x) = \frac{1}{\rho}\left[-\frac{1}{n^2} + (-1 + \frac{1}{n^2})x + \frac{1}{n}\right]$$

which provides the extrema $n_1 = 1$ and $n_2 = 1/x - 1$.
For the second partial derivative follows

$$\frac{\partial^2}{(\partial n)^2}\Delta V(n,x) = \frac{1}{\rho}\left[2\frac{1}{n^3} - 2\frac{1}{n^3}x - \frac{1}{n^2}\right]$$

and hence

$$\frac{\partial^2}{(\partial n)^2}\Delta V(n,x)|_{n=n_1} = \frac{1}{\rho}\left[1 - 2x\right]$$

$$\frac{\partial^2}{(\partial n)^2}\Delta V(n,x)|_{n=n_2} = \frac{1}{\rho}\frac{x^2(2x-1)}{(x-1)^2}. \tag{A.21}$$

Case differentiation:

(1) The inequality $n_1 > n_2$ is equivalent to $x > 1/2$ in which case (A.21) provides that n_1 is a Maximum and n_2 a Minimum. This yields $\Delta V(n,x) \leq 0$ for $n \geq 1$ and hence there is no incentive for defection.

(2) $n_1 = n_2$ is equivalent to $x = 1/2$ and also $\Delta V(n,x) \leq 0$.

(3) $n_1 < n_2$ is equivalent to $x < 1/2$. For ΔV follows

$$\Delta V(n_2,x) = -2 + 4x + \log[1/x - 1] > 0 \text{ for } x < 1/2.$$

Hence we can observe defection for a range of values of $n \geq 1$ for $x < 1/2$, where the latter inequality is equivalent to $T > \log 2/\rho$.

For fixed $x > 0$ it follows from (A.20) that

$$\lim_{n \to \infty} \Delta V(n, x) < 0,$$

which implies that for fixed $T > \log 2/\rho$ the range of values of n for which $\Delta V > 0$ is bounded. $\square$

References

Acemoglu, D., S. Johnson, and J.A. Robinson, 2001, The Colonial Origins of Comparative Development: An Empirical Investigation, *American Economic Review*, 91(5), 1369-1401.

Acemoglu, D., S. Johnson, and J.A. Robinson, 2002, Reversal of Fortune: Geography and Institutions in the Making of the Modern Word Income Distribution, *Quarterly Journal of Economics* 117(4), 1231-1294.

Alesina, A., A. Devleeschauwer, W. Easterly, S. Kurlat, and R. Wacziarg, 2003, Fractionalization, *Journal of Economic Growth* 8, 55-194.

Anderson, B., 1983, *Imagined Communities, Reflections on the Origin and Spread of Nationalism*, Verso Editions, London.

Axelrod, R., 1984, *The Evolution of Cooperation*, Basic Books, New York.

Barro, R.J. and X. Sala-i-Martin, 1995, *Economic Growth*, McGraw-Hill, New York.

Barth, F., 1999, Introduction, in: *Ethnic Groups and Boundaries*, ed. by F. Barth, Little, Brown, and Company, Boston, MA.

Benhabib, J. and R. Radner, 1992, The Joint Exploitation of a Productive Asset: A Game Theoretic Approach, *Journal of Economic Theory* 2, 155-190.

Benhabib, J. and A. Rustichini, 1996, Social Conflict and Growth, *Journal of Economic Growth* 1, 125-142.

Bloom, D. E. and J.D. Sachs, 1998, Geography, Demography, and Economic Growth in Africa, *Brookings Papers on Economic Activity*, 1998:2, 207-295.

Collier, P. and J.W. Gunning, 1999, Explaining African Economic Performance, *Journal of Economic Literature* 37, 64-111.

Coser, L.A., 1956, *The Functions of Social Conflict*, The Free Press, Glencoe, IL.

Dockner, E.J., S. Jorgenson, N.V. Long, and G. Sorger, 2000, *Differential Games in Economics and Management Science*, Cambridge University Press, Cambridge, U.K.

Easterly, W. and R. Levine, 1997, Africa's Growth Tragedy: Policies and Ethnic Divisions, *Quarterly Journal of Economics* 111, 1203-1250.

Easterly, W. and R. Levine, 2003, Tropics, Germs, and Crops: How Endowments Influence Economic Development, *Journal of Monetary Economics* 50(1), 3-39.

Esteban, J.-M. and D. Ray, 1994, On the Measurement of Polarization, *Econometrica* 62, 819-851.

Esteban, J.-M. and D. Ray, 1999, Conflict and Distribution, *Journal of Economic Theory* 87, 379-415.

Fearon, J.D. and D.D. Laitin, 1996, Explaining Interethnic Cooperation, *American Political Science Review* 90, 715-735.

Freeman, R. B. and D. L. Lindauer, 1999, Why Not Africa ?, NBER Working Paper 6942.

Gallup, J.L., A.D. Mellinger, and J.D. Sachs, 1998, Geography and Economic Development, NBER Working Paper 6849.

Garcia-Montalvo, J. and M. Reynal-Querol, 2002, Why Ethnic Fractionalizaton? Polarization, Ethnic Conflict and Growth, Working Paper, Universitat Pompeo Fabra.

Grossman, H., 1991, A General Equilibrium Model of Insurrections, *American Economic Review* 81, 912-921.

Hall, R.E., 1988, Intertemporal Substitution in Consumption, *Journal of Political Economiy* 96, 330-357.

Hirshleifer, J., 1995, Anarchy and its Breakdown, *Journal of Political Economy* 103, 26-52.

Johnson, S., J. McMillan, and C. Woodruff, 2002, Property Rights and Finance, *American Economic Review* 92(5), 1335-1356

Keefer, P. and S. Knack, 1997, Why Don't Poor Countries Catch Up? A Cross-National Test of Institutional Explanation, *Economic Inquiry* 35, 590-602.

Keefer, P. and S. Knack, 2002, Polarization, Politics, and Property Rights: Links between Inequality and Growth, *Public Choice* 111, 127-54.

Lane, P.R. and A. Tornell, 1996, Power, Growth, and the Voracity Effect, *Journal of Economic Growth* 1, 213-241.

LeVine, R.A. and D.T. Campbell, 1972, *Ethnocentrism: Theories of conflict, ethnic attitudes and group behavior*, Wiley, New York.

Ogaki, M. and C. M. Reinhart, 1998, Measuring Intertemporal Substitution: The Role of Durable Goods, *Journal of Political Economy* 106, 1078-98.

Rodrik, D., A. Subramanian, and F. Trebbi, 2002, Institutions Rule: The Primacy of Institutions over Geography and Integration in Economic Development, NBER Working Paper 9305.

Svensson, J., 1998, Investment, Property Rights and Political Instability: Theory and Evidence, *European Economic Review* 42, 1317-1342.

Temple, J. and P.A. Johnson, 1998, Social Capability and Economic Growth, *Quarterly Journal of Economics* 113, 965-990.

Zak, P.J. and S. Knack, 2001, Trust and Growth, *Economic Journal* 111, 295-321.