Problem Set 4

Solve before the classes 4–6/3.

Exercise 1

The book introduces quasilinear preferences in its Definition 3.B.7, page 45. If the preferences are continuous and quasilinear with respect to commodity 1, it can be proved that the preferences can be represented by a utility function written as $u(x) = x_1 + \phi(x_2, \ldots, x_L)$, where $\phi : \mathbb{R}_+^{L-1} \to \mathbb{R}$ is a continuous function. We will not prove this result here.

a) Conversely, assume that a consumer, Kirsten, has consumption set $\mathbb{R} \times \mathbb{R}_+^{L-1}$ and preferences represented by a utility function $u(x) = x_1 + \phi(x_2, \ldots, x_L)$. Prove that Kirsten's preferences are quasilinear with respect to commodity 1.

b) Next, we wish to argue that Kirsten's demand for commodities $2, \ldots, L$ is independent of w. Assume thus that Kirsten takes as given a price vector $p \gg 0$ normalized with $p_1 = 1$. Assume that Kirsten finds the bundle x optimal when her wealth is w. Show, that the bundle $x' = (x_1 + \mathrm{w}' - \mathrm{w}, x_2, \ldots, x_L)$ solves Kirsten's utility maximization problem at wealth w'. Give this interpretation: a wealth change only affects commodity 1.

c) Argue that Kirsten's indirect utility function can be written as $v(p, \mathrm{w}) = \mathrm{w} + \eta(p_2, \ldots, p_L)$ where $\eta : \mathbb{R}_+^{L-1} \to \mathbb{R}$ (we still confine attention to price vectors with $p_1 = 1$).

d) Assume further, that Kirsten's preferences are monotone and strictly convex, and that the function ϕ is differentiable. Continue to consider only price vectors with $p_1 = 1$. Argue that the first order conditions to Kirsten's utility maximization problem can be summarized as $\forall \ell = 2, \ldots, L$:

$$\frac{\partial \phi(x_2, \ldots, x_L)}{\partial x_\ell} = p_\ell \text{ and } x_\ell > 0 \qquad \text{or} \qquad \frac{\partial \phi(x_2, \ldots, x_L)}{\partial x_\ell} \leq p_\ell \text{ and } x_\ell = 0.$$

Exercise 2

Problems 3.E.3 and 3.E.4 of the book.

Exercise 3

Karl's consumption set is $\mathbb{R}_+^L$. His Cobb-Douglas preferences are representes by the utility function $u(x_1, \ldots, x_L) = \prod_{\ell=1}^L x_\ell^{a_\ell}$ with every $a_\ell > 0$, and $\sum_{\ell=1}^L a_\ell = 1$. In Exercise 4 of Problem Set 3 you found Karl's demand for good ℓ as $x_\ell(p, \mathrm{w}) = a_\ell \mathrm{w}/p_\ell$.

a) Write down Karl's indirect utility function $v(p, \mathrm{w})$.

b) Apply duality $u = v(p, e(p, u))$ to find Karl's expenditure function $e(p, u)$.

c) Find Karl's Hicksian demand function $h(p, u)$ from $e(p, u)$.

d) Show that h satisfies $\partial h_\ell(p, u)/\partial p_k = \partial h_k(p, u)/\partial p_\ell$. It this a general property?