Problem Set 3

Solve before the classes 25–27/2.

Exercise 1

a) Problem 3.B.2 of the book. Hint: given $x \gg y$, place a very small ball around y, and find in this ball some z with $x \succsim z$ and $z \succ y$.

b) Assume now that $\succsim$ is monotone and continuous. Show that $\succsim$ is weakly monotone (as defined in problem 3.B.2).

Exercise 2

Irina's utility function over consumption bundles in $\mathbb{R}^2_+$ is of the Leontief type $u(x_1, x_2) = \min\{ax_1, bx_2\}$ where $a, b > 0$. Draw some of Irina's indifference curves and solve her utility maximization problem given $(p, \mathrm{w}) \gg 0$.

Exercise 3

Wassily has somewhat unusual preferences over $\mathbb{R}^2_+$, described on the basis of two Leontief utility functions. His utility function is

$$u(x_1, x_2) = \max\left\{\min\{2x_1, x_2\}, \min\{x_1, 2x_2\}\right\}$$

Thus, Wassily first evaluates the two Leontief expressions $\min\{2x_1, x_2\}$ og $\min\{x_1, 2x_2\}$, and then finds his utility as the larger of the two expressions.

a) Show that $x_1 > x_2$ implies $u(x_1, x_2) = \min\{x_1, 2x_2\}$.

b) Show that $x_1 < x_2$ implies $u(x_1, x_2) = \min\{2x_1, x_2\}$.

c) Show that $x_1 = x_2$ implies $u(x_1, x_2) = \min\{x_1, 2x_2\} = \min\{2x_1, x_2\}$.

d) Draw some of Wassily's indifference curves, using results a)–c).

e) How monotone are Wassily's preferences? Locally nonsatiated, monotone, or strongly monotone? Are these preferences convex?

f) Solve Wassily's utility maximization problem given $(p, \mathrm{w}) \gg 0$.

Exercise 4

Karl's consumption set is $\mathbb{R}^L_+$. He has preferences represented by a Cobb-Douglas utility function $u(x_1, \ldots, x_L) = \prod_{\ell=1}^L x_\ell^{\alpha_\ell}$ with every $\alpha_\ell > 0$. Recall that the symbol $\prod_{\ell=1}^L$ denotes multiplication, i.e. $u(x_1, \ldots, x_L)$ is the product $x_1^{\alpha_1} \cdots x_L^{\alpha_L}$.

a) Show that Karl's preferences can also be represented by another Cobb-Douglas utility function, $u(x_1, \ldots, x_L) = \prod_{\ell=1}^L x_\ell^{a_\ell}$, with every $a_\ell > 0$, now also satisfying that $\sum_{\ell=1}^L a_\ell = 1$.

b) Karl faces given prices $p \gg 0$ and wealth $\mathrm{w} > 0$, and desires to maximize his utility. Argue that Karl chooses a bundle on the budget hyperplane ($p \cdot x = \mathrm{w}$) which is not on the boundary of the consumption set, and write down the first order conditions characterizing solutions to Karl's problem.

c) Using the first order conditions, show that $p_\ell x_\ell = (a_\ell / a_1) p_1 x_1$ for every ℓ.

d) Find the unique solution to Karl's utility maximization problem, $x(p, \mathrm{w})$. Verify that Karl spends a fixed share of his wealth on each commodity.

e) Check explicitly that your solution from d) satisfies homogeneity of degree zero and Walras' law. Argue also, that it is continuous and differentiable in $(p, \mathrm{w}) \gg 0$.

Excercise 5 (as time permits)

This exercise treats homothetic preferences, as introduced by the book in Definition 3.B.6, page 45. The four questions below can be answered independently.

a) Assume that the rational preference relation $\succsim$ is continuous, monotone and homothetic, and that the consumption set is $\mathbb{R}^L_+$. Show that the utility function constructed in the proof of Proposition 3.C.1 will be homogeneous of degree one. This means that $u(\alpha x) = \alpha u(x)$ for every $\alpha > 0$.

b) Conversely, assume that the consumer Lars has his monotone preferences over $\mathbb{R}^L_+$ represented by a utility function which is homogeneous of degree one. Prove that Lars' preferences are homothetic.

c) Further assume, as in figure 3.B.5 of the book, that Lars' preferences are strictly convex. Show that Lars' demand function $x(p, \mathrm{w})$ and indirect utility function $v(p, \mathrm{w})$ are homogeneous of degree one in the wealth, i.e. that $x(p, \alpha\mathrm{w}) = \alpha x(p, \mathrm{w})$ and $v(p, \alpha\mathrm{w}) = \alpha v(p, \mathrm{w})$ for every $\alpha > 0$.

d) Argue that Karl's preferences of Exercise 4 are homothetic.