Problem Set 12

Solve before the classes May 20–22.

Exercise 1

Let z denote the aggregate excess demand in an economy with L commodities. Throughout this exercise, remember that z satisfies Walras' Law and is homogeneous of degree 0.

(a) Assume that z satisfies the gross substitute property. The book mentions on page 613, that the following is then true: For every price pair p and p':

$$z(p) = 0 \text{ and } z(p') \neq 0 \text{ implies } p \cdot z(p') > 0. \tag{17.F.3}$$

Verify the book's claim in the special case $L = 2$.

(b) Interpret (17.F.3) as the weak axiom (def. 17.F.1) holding at equilibrium prices.

(c) Assume that z satisfies (17.F.3). Prove that the set of equilibrium prices is convex.

(d) Prove that (17.F.3) gives that $Dz(p)$ is negative semidefinite when p is an equilibrium price vector.

Exercise 2

In a 2-commodity economy, the price of good 2 is normalized to 1. The price of good 1 is now called p. The aggregate excess demand for good 1 is, for all $p > 0$,

$$z(p) = \log\left(\frac{1+p}{2p}\right) - \frac{4C}{(1+p)^2} + C.$$

log denotes the natural logarithm with derivative $1/x$. Recall that log satisfies $\log(x/y) = \log(x) - \log(y)$. Assume that $C < \log(2)$. Equilibrium is given by the equation $z(p) = 0$.

(a) Verify that $\lim_{p\to 0} z(p) = +\infty$ and $\lim_{p\to\infty} z(p) = C - \log(2) < 0$.

(b) Show that $p = 1$ gives an equilibrium, for any $C < \log(2)$.

(c) Find the derivative $z'(p)$. Show that $z'(1) \geq 0$ when $C \geq 1/2$. Show generally, that $p > 0$ solves $z'(p) = 0$ if and only if $p^2 + (2 - 8C)p + 1 = 0$.

(d) A quadratic equation $p^2 + bp + c = 0$ can be solved if $d = b^2 - 4c \geq 0$, and its solutions are then $p = (-b \pm \sqrt{d})/2$. Show that the equation $z'(p) = 0$ has $d = 64C(C - 1/2)$. Observe that $\sqrt{d} < |b|$, and show that $z'(p) = 0$ can be solved with $p > 0$ if and only if $C \geq 1/2$.

(e) Graph $z(p)$ in each of the three cases $C < 1/2$, $C = 1/2$, and $1/2 < C < \log(2)$. Identify the number of the economy's equilibria in each of the three cases.

Exercise 3

Consider an economy with 2 commodities: time for work/leisure and a consumption good. A firm has the capacity to transform time into the good using a convex production set. Every worker has given endowments $(T_i, c_i) \gg 0$, and owns a fixed share of the firm. All workers have rational, continuous, strictly convex and strongly monotone preferences over (t, c). The price of the good is normalized to 1.

a) Argue in abstract, that this economy could possess multiple equilibria.

b) Focus on the labor market. In case of multiple equilibria, the excess demand function cannot be a monotone function of wage. Must this non-monotonicity originate in the labor demand or labor supply?

c) Assume that the economy is regular, and that it possesses three equilibria. Compare the three equilibria in terms of real wages, total labor supplied, and profits. Are all three equilibria Pareto optimal?