Problem Set 11

Solve before the classes May 13–15.

Exercise 1

Exercise 4.C.11 in Mas-Colell, Whinston and Green. Observe in part (a) that the demands are non-negative if $\mathrm{w} > 8$ and $p = (1,1)$. In parts (b) and (c), feel free to assume $\mathrm{w} > 8$ and $p = (1,1)$. On pages 114–115, the book provides a related discussion of Slutsky matrices.

Exercise 2

Exercise 4.D.6 in Mas-Colell, Whinston and Green.

Exercise 3

In chapter 17, unlike in chapter 4, the income of the consumer is an endogenous function of the price vector. Let us study this situation. Consider a production economy with I consumers and J firms. Consumer i is assumed to have a uniquely defined demand function $x_i(p, \mathrm{w}_i)$, satisfying homogeneity of degree zero, Walras' Law, and the weak axiom (as in chapter 3). Firm j is supposed to have a well-defined supply function $y_j(p)$ (as in chapter 5) and profit function $\pi_j(p)$. Consumer i has the endogenous income $\mathrm{w}_i = p \cdot \omega_i + \sum_{j=1}^{J} \theta_{ij}\pi_j(p)$. Following the book's equation (17.B.3), let us define the production inclusive excess demand function as

$$z(p) = \sum_{i=1}^{I} x_i(p, p \cdot \omega_i + \sum_{j=1}^{J} \theta_{ij}\pi_j(p)) - \sum_{i=1}^{I} \omega_i - \sum_{j=1}^{J} y_j(p).$$

a) Prove that z is homogeneous of degree zero, and satisfies Walras' Law that $p \cdot z(p) = 0$.

b) Express $Dz(p)$ using the derivatives of the demand and supply functions. Using Hotelling's Lemma (Prop. 5.C.1 (vi)), and the definition of the Slutsky matrix $S_i(p, \mathrm{w}_i)$, you should arrive at

$$Dz(p) = \sum_{i=1}^{I} \left[S_i(p, \mathrm{w}_i) + D_w x_i(p, \mathrm{w}_i) \left(\omega_i^T + \sum_{j=1}^{J} \theta_{ij} y_j^T(p) - x_i^T(p, \mathrm{w}_i) \right) \right] - \sum_{j=1}^{J} Dy_j(p).$$

c) Assume that the consumers' preferences are on the Gorman form as in Proposition 4.B.1. We noticed in the proof of Prop. 4.B.1, that then $D_w x_i(p, \mathrm{w}_i) = -(1/b(p))Db(p)$, the same for all consumers. Use this to simplify the above expression as

$$Dz(p) = \sum_{i=1}^{I} S_i(p, \mathrm{w}_i) - (1/b(p))Db(p) \sum_{i=1}^{I} \left(\omega_i^T + \sum_{j=1}^{J} \theta_{ij} y_j^T(p) - x_i^T(p, \mathrm{w}_i) \right) - \sum_{j=1}^{J} Dy_j(p).$$

d) If p is an equilibrium price, $z(p) = 0$. Use this fact, and $\sum_{i=1}^{I} \theta_{ij} = 1$, to conclude that at equilibrium, $Dz(p) = \sum_{i=1}^{I} S_i(p, \mathrm{w}_i) - \sum_{j=1}^{J} Dy_j(p)$. Recall that $S_i(p, \mathrm{w}_i)$ and $-Dy_j(p)$ are symmetric and negatively semi-definite, and conclude that $Dz(p)$ is symmetric and negatively semi-definite.