

Problem Set 10

Solve before the classes May 6–8.

Exercise 1

Consider an exchange economy with two goods and two consumers, whose utility functions are

$$u_1(x_{11}, x_{21}) = x_{11} - 100e^{-x_{21}/10} \quad \text{and} \quad u_2(x_{12}, x_{22}) = x_{22} - 100e^{-x_{12}/10}.$$

Initial endowments are $\omega_1 = (40, 0)$ and $\omega_2 = (0, 50)$.

a) Allow, with quasi-linear preferences, that consumer 1 has consumption set $\mathbb{R} \times \mathbb{R}_+$, and calculate consumer 1's demand at given prices $(p_1, p_2) \gg 0$ and wealth $w_1 > 0$.

b) When $w_1 = p \cdot \omega_1 = 40p_1$, it can be proved that $w_1 > 10p_2 \log(10p_1/p_2)$ for any prices $(p_1, p_2) \gg 0$. This precludes negative demand for good 1, so verify that consumer 1's demand function is

$$x_1(p) = \begin{cases} (40, 0) & \text{if } p_2 > 10p_1 \\ (40 - 10\frac{p_2}{p_1} \log(\frac{10p_1}{p_2}), 10 \log(\frac{10p_1}{p_2})) & \text{if } p_2 \leq 10p_1. \end{cases}$$

c) Likewise write down consumer 2's demand function $x_2(p)$. Simply use that the utility function is like consumer 1's, only with goods one and two interchanged. Again, it can be proved that negative demand is precluded when $w_2 = p\omega_2 = 50p_2$.

d) Aiming to find equilibria, let $p_2 = 1$ and allow $p_1 > 0$ to vary freely. Find $z_1(p_1, 1)$, the aggregate excess demand for good 1 as a function of p_1 .

e) Show that

$$\frac{\partial z_1(p_1, 1)}{\partial p_1} = -\frac{10}{p_1} - \frac{10}{p_1^2} + \frac{10}{p_1^2} \log(10p_1).$$

f) Show that $p_1 = 1$ gives an equilibrium. Show $\partial z_1(1, 1)/\partial p_1 > 0$ (using $\log(10) > 2$).

g) Argue from a figure like 17.C.1, that there exists at least two further equilibria.

Exercise 2 (similar to an exam question January 2001)

Consider a two-goods exchange economy with two consumers. Both have consumption set \mathbb{R}_+^2 . Consumer 1's preferences are represented by the utility function $u_1(x_{11}, x_{21}) = \min\{x_{11}^2/16, x_{21}\}$. Consumer 2 has Leontief preferences represented by the utility function $u_2(x_{12}, x_{22}) = \min\{x_{12}, x_{22}\}$. Initial endowments are $\omega_1 = (3, 11)$ and $\omega_2 = (16, 5)$.

a) Draw consumer 2's offer curve for all non-negative prices in an (x_{11}, x_{21}) -diagram. Be careful in considering the demand where $p_1/p_2 = 0$ and where $p_2/p_1 = 0$.

b) Like a), now for consumer 1. Remember that the preferences are different from 2's.

c) Draw an Edgeworth box for this economy.

d) Place the offer curves in the Edgeworth box. Show that they intersect where consumer 1 obtains $(4, 1)$, or $(12, 9)$, or a bundle on the line from $(4\sqrt{11}, 11)$ to $(14, 11)$.

e) Find price vectors corresponding to the equilibrium allocations found in d).