Problem Set 1

Solve before the classes 11–13/2.

Exercise 1

Problem 1.B.4 of the book.

Exercise 2

Problem 1.C.1 of the book.

Exercise 3

In Birte's world there are only two goods. Her demand satisfies Walras' law. At the price pair $p = (2, 4)$ she demands $x = (1, 2)$. At the price pair $p' = (6, 3)$ she demands $x' = (2, 1)$. Show that Birte's demand violates the weak axiom.

Exercise 4

Problem 1.C.2 of the book. In order to prove the equivalence of two statements, first show that one implies the other, and then that the other implies the first.

Clarify that the formal statement of the problem can be read as follows: "Assume that x and y are both members of two budget sets, that x is an acceptable alternative in one budget set, and that y is an acceptable alternative in the other budget set. Then both x and y must be acceptable alternatives in both budget sets."

Exercise 5

In Arne's world there are three goods. Arne's demand satisfies Walras' law. He once faced the price vector $p = (4, 2, 4)$ and demanded $x = (1, 2, 10)$. At some other occasion, prices were $p' = (1, 1, 1)$ and Arne demanded $x' = (2, 2, 2)$. Calculate $(p' - p) \cdot (x' - x)$ and sign it (positive or negative). Does the given information tell you that Arne's demand violates the weak axiom? How does the information accord with Definition 2.F.1 of the weak axiom?

Exercise 6

This exercise refers to the book's mathematical appendix M.A (pages 926–7).

a) Consider the functions $g, h : \mathbb{R}^3 \mapsto \mathbb{R}$ given by $g(x_1, x_2, x_3) = x_1x_2 + x_2x_3$ and $h(x_1, x_2, x_3) = x_1x_2 + x_1x_3$. Calculate $g(1, 1, 1)$, $h(1, 1, 1)$, $Dg(1, 1, 1)$ and $Dh(1, 1, 1)$. Now, let the function $f : \mathbb{R}^3 \mapsto \mathbb{R}$ be defined by $f(x) = g(x)h(x)$. Calculate $g(x)h(x)$ in order to find an expression for $f(x)$. Use this expression to find $f(1, 1, 1)$ and $Df(1, 1, 1)$. Now verify, that your calculations fit result (M.A.2) of page 927 in the book.

b) Consider the functions $g, h : \mathbb{R}^2 \mapsto \mathbb{R}^2$ given by $g(x_1, x_2) = (x_1x_2^2, x_1^2x_2)$ and $h(x_1, x_2) = (x_1+x_2, x_1)$. Calculate $g(1, 2)$, $h(1, 2)$, $Dg(1, 2)$ and $Dh(1, 2)$. Now, let the function $f : \mathbb{R}^2 \mapsto \mathbb{R}$ be defined by $f(x) = g(x) \cdot h(x)$. Calculate $g(x) \cdot h(x)$ in order to find an expression for $f(x)$. Use this expression to find $f(1, 2)$ and $Df(1, 2)$. Now verify, that your calculations fit result (M.A.3) of page 927 in the book.