Ambiguity in Financial Markets: Herding and Contrarian Behaviour*

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May 6, 2005

Abstract

The paper studies the impact of ambiguity on history-dependant beahviour in the standard microstructure model of financial markets. We show that differences in ambiguity attitudes between market makers and traders can generate contrarian and herding behaviour in stock markets where assets are traded sequentially and trading prices are endogenously determined. We also show the mispricing can be only short-term, and in the long-run market is efficient in the sense that the market price aggregates information without distortions.

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*We would like to acknowledge helpful comments from seminar participants at the department of Economics, University of Exeter. We are particularly grateful to Panicos Demetriades, Simon Grant and Jürgen Eichberger for their comments.

Keywords: Ambiguity, Choquet Expected Utility, Generalized Bayesian update, Optimism, Herding, Contrarian behavior

JEL Classification: D81, G1

1 Introduction: Ambiguity and Herd behaviour in Financial Markets

Herding in financial markets has been explored extensively in the literature in recent years[1].Why does herding occur? It might be due to many factors: conformity, imitation of investment strategies (of fund managers) being rewarded, informational externality (imperfect information).[2] The majority of the existing literature has focused on rational herding which can be catalogued as: informational herding, reputation-based and compensation-based herding. In this paper we confine our attention to informational herding in financial markets.

There are generally two types of models of informational herding with two conclusions. The first takes price (cost) as exogenous and fixed throughout from the beginning of the period. So a subsequent decision, to buy or to sell, will not alter the cost. In such conditions it has been demonstrated that a herd will occur eventually at some point in time with a high probability. The other model takes price to be determined endogenously with traders disclosing their private information on

[1]Bikhchandani and Sharma (2001) provides an overview of the recent theoretical and empirical research on this topic.

[2]There are also some studies on payoff externality as the reason for bank runs but not as an important cause of other herds.

fundamentals via their actions at each trading round. Eventually the price reflects fundamentals and there is no mis-pricing. Markets present strong-form information efficiency in the long run and herding will never occur. Both models are open to criticism. In regard to the former, the exogenous price assumption seems so inappropriate for the modelling of financial markets, in which prices of securities can change often and rapidly. For the latter model, numerous empirical and theoretical enquiries have challenged its results. Herding behaviour is such a widespread phenomenon in financial markets therefore asset prices often fail to disclose sufficient or any private information on market fundamentals.

In recent years, there have been many attempts to explain herding in the second framework with different rationales. Challenges remain, however, for example, in the majority of these literatures, agents process information rationally and there is no room to explain contagion of emotions such as panic or frenzy which are often the hallmarks of herding behaviour. It thus seems worth considering a once-prominent question in the herding literature: Are asset markets driven by "animal spirits", where investors behave like imitative lemmings? We will provide one answer to that question by analyzing a simple herding model where full rationality is relaxed somewhat by considering agents' attitudes to informational ambiguity.

In our paradigm, trades are assumed to occur sequentially and trading prices are determined endogenously at each trade. Neither market makers nor traders know the value of assets exactly. Market makers form their expected value of assets on the basis of public observed information. Traders receive private information that enables them to update public beliefs thereby enabling them to form their expected value of assets by using the most up-to-date information. At first we assume that it is only traders who perceive ambiguity in either public or private information.[3] That is, we assume asymmetric ambiguity between market makers and traders. Thus we model market makers and traders' beliefs in different ways. We show that this simple difference can change the conclusions about herding reached in previous literature; especially, it can generate the other type of history-dependent behaviour, contrarian behaviour, when traders trade against market trends. We subsequently relax this assumption to investigate the case where both market makers and traders are ambiguous about the information at their disposal. We then demonstrate that herding can occur in specific situations. Thus ambiguity can cause herding in circumstances where it would not be possible otherwise. Moreover, we generally show that such behaviour only leads to short-run mis-pricing in financial markets: those markets exhibit informational efficiency in the long-run.

We use ambiguity to denote types of uncertainty where the relevant probabilities are imperfectly known or even unknown (See Knight (1921)). The presence of ambiguity may be due to unfamiliarity with decision-problems or lack of confidence in the likelihood of events. In particular, the assumption in this paper is that agents are able to formulate probability estimates but are not completely confident about them. We believe that such situations are not uncommon in financial markets. Ac-

[3]We do not find the "Harsanyi doctrine", that economic agents having the same information neccessarily should have the same beliefs, too compelling in our model. We do, however think that to hold this assumption makes the model more easily interpretable. The justification for our assumptions is provided in later sections.

cording to a now extensive body of experimental evidence, of which the Ellsberg Paradox is probably the best-known, ambiguity can have a significant impact on decisions. Unfortunately, the modelling of beliefs in terms of conventional probabilities via subjective expected utility theory cannot predict correctly agents' behaviour.

Knight (1921) maintained that agents differ in their attitudes to ambiguity. The majority of people are ambiguity-averse, behaving more cautiously when probabilities are undefined, while a significant minority of individuals appear to be the opposite, being ambiguity-loving (See the experimental evidence in Camerer and Weber (1992)). Consequently, numerous alternative decision-making theories to *expected utility theory* (henceforth EUT) have been proposed for modelling behaviour in situations of ambiguity. One such, which has gained extensive popularity in the literature, is *Choquet expected utility* (henceforth CEU), which was first axiomized by Schmeidler (1989). In CEU, agents' beliefs are represented as capacities (non-additive subjective probabilities) and they make decisions by maximizing the expected value of a utility with respect to their non-additive beliefs (the expectation is expressed as a Choquet integral, Choquet (1953-4)). In this theory, agents are ambiguity-averse if they put more weight on bad outcomes than EU maximizers, while they are ambiguity-loving if they put more weight on good outcomes.

In this paper, we restrict attention to a special case, namely, CEU with respect to neo-additive capacities (Chateauneuf, Eichberger, and Grant (2002)), where agents'expectation may be represented as a weighted average of the expected value of utility, the maximum value of utility and the minimum value of utility. This is expressed as,

$$\lambda M(w) + \gamma m(w) + (1 - \gamma - \lambda) E_{\pi} u(w) \quad (1)$$

where $M(w)$ (resp. $m(w)$) is maximum (resp. minimum)value of, say, trading an asset, in our model, and $E_{\pi} u(w)$ is expected value of trading an asset. We assume that when they make their trading decisions traders perceive ambiguity, due to imperfect knowledge of the true value of assets and their uncertainty about the information at their disposal. Therefore, we model traders' beliefs as CEU with respect to *neo-additive* capacities and investigate the effects of their attitudes to ambiguity on their trading behaviour. One point worthy of note, we define ambiguity-averse behaviour as *pessimism* when traders put more weight on the possibly low value of an asset, and ambiguity-loving behaviour as *optimism* where agents put more weight on the possibly good value of an asset. The implications are consistent through the updating beliefs from private signals.

In this environment, the challenge is to model how traders update their beliefs, their neo-additive capacities, as new information is received. Formally, Bayes' rule is still well-defined even if beliefs are not additive. However Gilboa and Schmeidler (1993) show that the application of Bayes' rule to non-additive beliefs corresponds to 'optimistic updating' in the sense that new information gained is always regarded as good news. It seems, therefore, to conflict with the assumption that players are ambiguity-averse. In fact, there have been a number of proposals for updating CEU preferences (see, for instance, Gilboa and Schmeidler (1993), Kelsey (1995)[4] and

[4]In this literature, the 'Dempster-Shafer rule' is discussed to update non-additive beliefs, which is shown as a pessimistic updating rule.

Eichberger, Grant, and Kelsey (2003)), but there is no consensus on the updating rule for non-additive beliefs. Instead, it has been agreed that different updating rules are appropriate for different circumstances.[5] The rules, nevertheless, all coincide, as noted above, with Bayesian updating when beliefs are additive. In line with our assumption that traders can be either ambiguity-averse or ambiguity-loving, we use the updating rule proposed in Eichberger, Grant, and Kelsey (2003), namely, Generalized Bayesian Updating rule (henceforth GBU), which we will outline in section 2.

The remainder of the paper is organized as follows. In section 2, we review some basic principles of ambiguity theory, In section 3 we introduce GBU for neo-additive capacities. Section 4 describes the model and definitions of herding/contrarian behaviour. We solve our model in section 5, demonstrating the possibility of contrarian and herd behaviour. Section 6 offers some concluding observations. The appendix contains the (simple, but lengthy) proof of Proposition 5.1.

2 Modelling Ambiguity

We now outline the CEU model for single person decisions when there is ambiguity. We begin with the general concepts of capacities and the Choquet Integral; then turn our attention to CEU with respect to neo-additive capacities. Throughout the paper, the following notation is adopted.

Notation. We consider a finite set of states of nature S. A subset E, of S will be referred to as an event. The set of possible outcomes or consequences is denoted by

[5] Eichberger and Kelsey (1996) conclude that there is no 'correct' updating rule for non-additive beliefs and argue that the updating rule should depend on the application.

X. An act is a function from S to X. The set of all acts is denoted by $A(S)$.

(1) Capacities and Choquet Expected Utility (CEU)

A capacity generalizes the notion of probability and assigns non-additive weights to subsets of S. Formally, we have:

Definition 2.1 *A capacity on S is a real-valued function $v : \mathcal{P}(S) \to \mathbb{R}$ where $\mathcal{P}(S)$ is the set of all subsets of S, which satisfies the following properties:*

1. *$E, F \subset S, E \subseteq F$ implies $v(E) \leqslant v(F)$, monotonicity*

2. *$v(\varnothing) = 0$ and $v(S) = 1$, normalization*

The capacity is called convex if $v(E)+v(F) \leq v(E \cup F)+v(E \cap F)$ and concave when $\leq$ is replaced by $\geq$. For CEU preferences, a convex (resp. concave) capacity represents pessimistic (resp. optimistic) attitudes to ambiguity(for detailed discussion see Wakker (2001)). The neo-additive capacity allows both optimistic and pessimistic attitudes.[6]

Definition 2.2 *An expectation of utility (u) with respect to a capacity (v) is defined as the Choquet Expected Utility, $\int udv = \sum_{k=1}^{r} u(s_k) \cdot [v(S_k) - v(S_{k-1})]$, where $S_k = \{s_1, s_2, ...s_k\}$ and $s_0 = \varnothing$.*

The Choquet integral is like an expectation as it is a weighted sum of utilities. The weight assigned to a state depends on how the outcome is ranked.[7]

[6] Note it is supported by experimental evidence, eg., Kilka and Weber (2001).

[7] Gilboa (1987), Schmeidler (1989) and Sarin and Wakker (1992) provide axiomatisations for CEU preferences. Wakker (2001) characterise capacities representing ambiguity-averse or pessimistic attitudes of a decision maker.

Because it has too many free parameters, the general CEU model can be mathematically difficult to apply to economic problems. For example, with r states, a general capacity is described by 2^r parameters. Fortunately, CEU with respect to the special case of neo-additive capacities, provides a solution that is more easily applied and intuitively explained. Unlike the general CEU model, CEU with respect to neo-additive capacities only over-weights the best and worst outcomes. Its axiomatic foundation can be found in Chateauneuf, Eichberger, and Grant (2002). We briefly give the conceptual explanation here.

(2) Choquet integral with a neo-additive capacity

Definition 2.3 *For a pair of real numbers λ, γ, such that $\lambda \geqslant 0, \gamma \geqslant 0,$ and $\lambda + \gamma \leqslant 1$ and a given probability $\pi(E)$, a neo-additive capacity is defined as:*

$$v(E) = \begin{cases} 1 & \text{for } E = S \\ \lambda + (1 - \lambda - \gamma)\pi(E) & \text{for } \varnothing \subsetneqq E \subsetneqq S \\ 0 & \text{for } E = \varnothing \end{cases}$$

It is readily seen that a neo-additive capacity is a convex combination of an additive capacity and capacities on two extreme outcomes, one is complete ignorance with objective probability of 1 and one is complete ambiguity with objective probability of 0. We have noted that pessimism refers to the beliefs which overweight the bad outcomes and optimism refers to the beliefs which overweight the good outcomes. Therefore, a neo-additive capacity represents pure optimism if $\gamma = 0$, and pure pessimism if $\lambda = 0$.

Definition 2.4 *The Choquet expected value of a real valued function* $f : S \to \mathbb{R}$ *with respect to a neo-additive capacity* v *is defined as:*

$$CEU(v) = \int f dv = \gamma \cdot \inf_{s \in S}(f) + \lambda \cdot \sup_{s \in S}(f) + (1 - \lambda - \gamma) E_{\pi}(f). \tag{2}$$

Thus, the payoff of any strategy is given by a weighted average of the expected payoff and maximal, minimal payoffs for given S. The proof can be found in Chateauneuf, Eichberger, and Grant (2002).

Intuitively, CEU with respect to neo-additive capacities describes a situation in which agents believe the events predicted by EU with the additive probabilities. However, they lack confidence in this prediction. In part they react to this in an optimistic way measured by λ and in part in a pessimistic fashion measured by γ. We can, therefore, interpret the additive part of CEU, $E_{\pi}(f)$, as the agent's belief and $(1 - \gamma - \lambda)$ as his(her) degree of confidence in that belief. We refer to the parameters λ and γ, respectively, as degree of optimism and pessimism[8].

3 Generalised Bayesian Update (GBU)

To apply CEU with neo-additive beliefs to a dynamic process, it is necessary to model how agents update their beliefs upon the arrival of new information. We adopt GBU proposed in Eichberger, Grant, and Kelsey (2003). The change to GBU guarantees

[8] The following examples relate CEU to some more familiar decision rules:

1. if $\lambda = 0$, preferences have the maximin form and are extremely pessimistic;
2. if $\gamma = 0$, preferences exhibit the maximal degree of optimism;
3. if $\gamma + \lambda = 1$, preferences coincide with the Hurwicz criterion (see Hurwicz (1951))

that the updated preferences can be represented again by a Choquet integral. Also we will show that the updated neo-additive belief is itself still a neo-additive capacity. The neo-additive belief updating rule is defined as follows.

Definition 3.1 *For any event $F \subseteq S$, define capacities v_F^0, v_F^1 as follows:*

$$v_F^0(E) = \begin{cases} 0, & \text{if } E \cap F \neq F \\ 1, & \text{if } E \cap F = F \end{cases}, \qquad v_F^1(E) = \begin{cases} 0, & \text{if } E \cap F = \varnothing \\ 1, & \text{if } E \cap F \neq \varnothing \end{cases}.$$

Definition 3.2 *If F is observed, the updated neo-additive capacity of E is,*

$$v_F(E|\pi, \lambda, \gamma) = [1 - \delta(F) \cdot (\lambda + \gamma)] \pi_F(E) + \delta(F)(\lambda v_F^1(E) + \gamma v_F^0(E))$$

where,

$$\delta(F) = \frac{1}{(1 - \lambda - \gamma)\pi(F) + (\lambda + \gamma)}.$$

Definition 3.3 *The Choquet expected utility with respect to a conditional neo-additive capacity is defined as,*

$$CEU(f|v_F(E|\pi, \delta)) = [1 - \delta(F)(\gamma + \lambda)] E_{\pi|F}(f) + \delta(F)(\lambda \sup f + \gamma \inf f).$$

Note, the Choquet expected value of a random variable with respect to any conditional neo-additive capacity is well defined even if the conditioning event is an ex ante zero probability event, provided $\lambda > 0$ or $\gamma > 0$. Especially, the CEU of a random variable conditional on a zero probability event is simply the weighted average of the best and worst elements that can obtain on that event. i.e., when $\pi(F) = 0, \delta(F) = \frac{1}{(\lambda+\gamma)}$, the degree of confidence $1 - \delta(F) \cdot (\lambda + \gamma)$ is zero on a zero probability event, so no weight is given to the additive probability part of the neo-additive capacity. The updated conditional belief CEU becomes,

$$CEU\left(f|v_F\left(E|\pi,\delta\right)\right)=\frac{\lambda}{\lambda+\gamma}\sup f+\frac{\gamma}{\lambda+\gamma}\inf f. \tag{3}$$

It is also observed $\delta\left(F\right)$ is in general greater than 1. We say that, with all $0<\pi\left(F\right)<1$, the gaining of signal will increase the ambiguity level. If and only if event F happens with certainty, receiving of the signal will not increase the ambiguity perceived. When $\pi(F)=1, \delta\left(F\right)\cdot(\lambda+\gamma)=\lambda+\gamma$, the conditional CEU becomes the same as unconditional CEU.

More generally, the more unlikely (in terms of the additive 'prior' π) is the event, the less confidence (the lower is $1-\delta\left(F\right)\cdot(\lambda+\gamma)$) the individual has in the 'additive part of the theory' and the more weight (the greater $\delta\left(F\right)$ is) he places on 'extreme' outcomes (in proportion to his relative degree of optimism, λ, versus his relative degree of pessimism, γ). Relatively a consistent signal $(F=E)$ reduces confidence less than an inconsistent signal $(F\neq E)$ does.

To intuitively explain, although the arrival of enough public information will improve informational efficiency in decisions, the arrival of a signal on public disclosure may, paradoxically, make decisions worse. As argued in Bikhchandani, Hirshleifer, and Welch (1992), aggregating the information of fewer individuals, additional information can encourage individuals to fall into a cascade sooner, so there is no presumption that the signal will improve decisions. For a similar reason, the ability of individuals to observe past actions with noise, or the ability to observe payoff outcomes in addition to past actions, can make decisions worse on average. As Hirshleifer and Teoh (2001) stated: "a little knowledge is a dangerous thing."[9]

The updating rule is not linear, which complicates its application considerably. Therefore we apply it here to the simplest version of a trading model in order to obtain insights into its implications.

4 The Model

4.1 Description

We consider a trading model which retains the basic features of the model considered by Bikhchandani, Hirshleifer, and Welch (1992) (hereafter BHW as often referred in herding literatures). Briefly, the model is as follows.

Market Mechanism. The market is for a risky asset exchanged for money among market makers (MM henceforth) and traders. The liquidation value of the asset is denoted by $w = W + \varepsilon$ and W is the true value and is restricted to be $\{0, 1\}$. Information about the true value W arrives slowly in the market. Trading occurs sequentially and one trader is randomly selected in each period. There is an infinite sequence of traders indexed by $t = 0, 1, 2, ...$ MMs set the trading price at the beginning of each trading period then interact with the selected trader. At any given period t, the selected trader can buy or sell a unit of an asset at the market price, then leave the market after trading.

[9] The ability to learn by observing predecessors can make the decisions of followers noisier by reducing their incentives to collect (perhaps more accurate) information themselves (Cao and Hirshleifer, 1997).

Market Makers. Consider now MMs. As argued in Chari and Kehoe (2003), including market makers is a convenient way of modelling trade between informed and uninformed traders. In the model, all trades occur between traders and market makers. There is an infinite number of risk-neutral market makers, who set prices in a competitive fashion. Market makers make money from the uninformed (idiosyncratic) traders but lose money to the informed traders, thereby making zero expected profits. MMs (who do not receive private signals about the value of the asset) set the market price equal to their expected value of the asset according to the past trading history. For the sake of simplicity, we assume that MMs do not try to distinguish between types of traders. For instance, when a trader buys from the MM, MM does not know (or try to infer) whether the buyer is well-informed (with a "Bullish" signal, defined later) and is following the signal, or is herd buying, or is an idiosyncratic trader (see explanation later). Accordingly, we ignore the bid-ask spread and consider the same trading price for either purchase or sale. To abstract from the existence of the bid-ask spread helps us to interpret our results. In equilibrium, it is claimed that competition among MMs ensures prices yield zero expected profits.

Traders. In the model, traders must choose whether to buy or to sell the specific asset, and their actions depend on their expected value of the asset. There are two broad types of traders, idiosyncratic and standard. Idiosyncratic traders trade for different reasons, regardless of the price and any other information, which guarantees that the equilibrium of our model always has trade. Their decisions or payoffs are not our concerns here. Informed traders receive private information and are assumed to maximize expected profit at the market maker's expense. In the following, we use traders as a shorthand for informed traders and focus on their behaviours.

In the game, traders get information from two sources: they receive private signals and they observe the price of the asset which reflects others' previous investment decisions. Events in each period t are such that a trader first receives a signal then investment decisions are made. Note that not trading is never optimal unless we introduce transaction costs because traders always have an informational advantage over the market maker. The private signals x concern the value of the risky asset w and are independent, imperfect and randomly distributed to one. There are two possible values for x, $x \in \{h, l\}$, which implies that the value of the asset is high(h) or low(l). The signals are informative and symmetric in the sense that where $P(x \neq w) = p < \frac{1}{2}$, i.e., $(p(x = l|w = 1) = p(x = h|w = 0) = p < 1/2)$. Intuitively, signal l is more likely when $w = 0$ and it can be interpreted as a "Bearish" signal. Similarly, $x = h$ can be interpreted as a "Bullish" signal. Moreover, we have $E[w|x = l] < E[w] < E[w|x = h]$.

Public belief. Traders act sequentially and observe H_t, the history of actions up until time t. We define $\pi_1^t = P(w = 1|H_t)$ as the public belief at time t, that is, the probability that the value is high, conditional on the public history H_t. For any given trader's action $f \in F = \{s, b\}$, where s represents selling and b as buying, the public beliefs update according to the Bayes' rule. We say that a trader's action f is informative when it affects the public belief: $P(w = 1|H_t, f) \neq \pi_1^t$. Finally we define H_t as a positive history (resp. a negative history) if $\pi_t > \pi_0$ (resp. $\pi_t < \pi_0$).

We denote $\hat{c}$ as the cost of a unit of the asset, which is also its market price. The cost of the asset, set by the Market Maker (MM), reflects all publicly available information: $\hat{c} = E[w|H_t] = P(w=1|H_t) = \pi_1^t$. Initially, we assume that MM only observes the public information and sets the price based on Bayes' updated public belief. Later in the paper, we change this assumption by incorporating MM's ambiguity attitudes as well.

Private belief. An informed trader receives a private signal in addition to the public information π_1^t, therefore (s)he forms his(her) private belief by updating the public information using the private signal. The difference between our model and previous models arises from our assumption here that, unlike MM, the trader has beliefs represented as neo-additive capacities and updates his(her) beliefs using the GBU rather than Bayes' rule. We suppose traders only know the cost to be π_1^t but are not completely confident about the prices, because they do not observe all trades in the market as perfectly as MMs.[10] They are also not completely confident about its correctness. Therefore individual traders process all such information within their ambiguity attitudes. Their beliefs capture the extent to which they trust information revealed either through markets or by their private information.

Decision rule. We assume that there is always a minimal amount of "useful" information in the market. As long as past trading does not identify the value perfectly, then there is strictly positive probability that some trader has an assessed value that differs from a MM's by a nontrivial amount. The choice made by a trader depends on whether the expected value of buying or selling a unit of the asset is greater than

[10] Tallon (May, 1998) argued that agents do not fully trust the information revealed by the price system with "noise traders". Their beliefs are non-additive and they behave as if they are maximizing a non-additive expected utility.

$\hat{c}$, the expected value of the asset held by the MM.

4.2 The Definition of Herd and Contrarian behaviour

We adopt the same definition of herding and contrarian behaviour as Avery and Zemsky (1998) (Henceforth AZ). In AZ, an informed trader who buys in period t engages in herd behaviour if three conditions are met. First, this trader receives a "Bearish" signal. Second, there is a positive history of trades H_t, $\pi_t > \pi_0$, that is, the pattern of trading causes an increase in the price set by MM. Third, knowing both conditions above, the trader buys.

Formally, we define herd behaviour as:

Definition 4.1 *A trader with private information x engages in herd behaviour at time t if he buys when $E_x^0(w) < E^0(w) < E^t(w)$ or if he sells when $E_x^0(w) > E^0(w) > E^t(w)$; and buying (or selling) is strictly preferred to other actions.*

Thus, a trader engages in a buy (resp. sell) herd if (s)he is initially inclined to sell (resp. buy) on the basis of his(her) private information $E_x^0(w) < E^0(w)$ (resp. $E_x^0(w) > E^0(w)$), but actually reverses his(her) inclination to follow the observed positive (resp. negative) trading history $E^0(w) < E^t(w)$ (resp. $E^0(w) > E^t(w)$). This buying behaviour happens if and only if traders have $E_x^t(w) > E^t(w) = \widehat{c}$ instead of $E_x^t(w) < E^t(w) = \widehat{c}$. In other words, the trader's signal constitutes "Bearish" information, causing him(her) to reduce his(her) assessment of the asset's value. Yet, with an observed positive trading history, (s)he must adjusts his(her) expectation in a way to agree more with the market trend on the good value of the asset, therefore buying in spite of the "Bearish" signal. Such behaviour has been modelled in AZ model, but criticized and labelled by Chari and Kehoe (2003) as one of "waves of optimism and pessimism" rather than of herd behaviour. We, however, maintain that such a label is suitable to explain a herd, and suggest our model possibly justifies that contention more explicitly than does the AZ model. AZ suggests the occurrence of history-dependent behaviour when there is multiple dimensions of uncertainty and sufficiently poor private information[11] Here, with more generalized ambiguity[12] and types of private signal, we also suggest the presence of such history-dependent behaviour. Such behaviour arises when traders assess information available in a way to incorporate their ambiguity attitudes towards the value of the asset. For instance, when traders are sufficiently optimistic in their valuation of the asset, they will buy even when their private signal is negative.

There are, of course, many other ways to define a herd for different purposes. As mentioned in AZ, Vives (1996) defines herding as a socially inefficient reliance on public information and mainly concern informational efficiency. We will comment later on the extent to which herd behaviour defined here can lead to distortions and inefficiencies.

As the other form of history-dependent behaviour, contrarian behaviour is the

[11] AZ shows that traders change the value of $E_x^t(w)$ by weighting more public information rather than private information with the presence of multiple dimensions of uncertianty and poor private information. Also, markets respond slower than traders. Therefore the buying behaviour occurs with the condition $E_x^t(w) > E^{t-1}(w) = \widehat{c}$ satisfied.

[12] We understand the first-dimension of uncertainty (value uncertainty) in AZ as risk, the second and third dimensions (event and composition uncertainty) as ambiguity.

converse of herding. With contrarian behaviour traders ignore their private information about value of the asset and trade against the trading trend. Such behaviour is also commonly observed in markets.

By analogy with herding behaviour, we provide a formal definition of contrarian behaviour.

Definition 4.2 *A trader with private information x engages in contrarian behaviour at time t if either he buys when $E_x^0(w) < E^0(w)$ and $E^0(w) > E^t(w)$, or he sells when $E_x^0(w) > E^0(w)$ and $E^0(w) < E^t(w)$.*

Compared to AZ's definition, we describe the same behaviour without requiring an additional condition[13]. In AZ, contrarian behaviour only occurs if signals are sufficiently imprecise. Here we show that contrarian behaviour can arise in the same context as herd behaviour.

5 CEU Updating and Trading Analysis

5.1 Results on Herding and Contrarian Behaviour

Now, we study how differences in the reactions of the MM and traders to information can generate different trading behaviour. First, we assume that the MM and the trader have different forms of beliefs, their expected values of the risky asset differ too. The MM's beliefs are represented traditionally by probabilities and (s)he is

[13]This is because we always hold that, $E_x^t(w) < E^t(w)$ for contrarian buying behavior and $E_x^t(w) > E^t(w)$ for contrarian selling behavior.

an expected-utility maximizer. However, a trader's beliefs are represented as neo-additive capacities and (s)he maximizes Choquet expected utilities conditional on his(her) private signal as well as the information revealed by the price market system. Generally speaking, we work thereafter under the assumption that the MM is risk and ambiguity neutral whereas traders are risk-neutral and may be ambiguity-loving or ambiguity-averse[14].

As explained before, the MM's belief is the same as the public belief and his(her) expected value of the asset is the market price, $\hat{c} = E_M [w|H_t] = P(w = 1|H_t) = \pi_1^t$. The subscript M represents the MM.

Now we model the trader's belief and behaviour.

It follows from Bayes' rule that the updated belief after the signal[15] is:

$$\pi(w|x) = \pi_x(w) = \begin{cases} \pi(w=1|x=1) = \frac{(1-p)\pi_1^t}{(1-p)\pi_1^t + p\left(1-\pi_1^t\right)} = \pi_{x_1}(w) \\ \pi(w=1|x=0) = \frac{p\pi_1^t}{p\pi_1^t + (1-p)\left(1-\pi_1^t\right)} = \pi_{x_0}(w) \\ \pi(w=0|x=1) \\ \pi(w=0|x=0). \end{cases}$$

By allowing for the ambiguity attitudes, we represent the trader's updated belief after the signal as a neo-additive capacity. Denote by $v_x(w)$ the conditional neo-additive belief of w given x, then:

[14]Decamps and LOVO (2002) has relaxed the assumption of risk-neutrality and suggested the presence of history-dependent behaviors and long-run informational inefficiency in financial markets. The other assumption, a fixed limit size of trade per period is introduced. However we suspect the behaviour descriped in the paper is "spurious herding", bacause it only happens when trading for maintaining components of traders' inventory coincides with the market trends. .

[15]With $w = 0$, there is no need to figure out $\pi_1(0)$ and $\pi_0(0)$.

Definition 5.1 *The conditional belief of w given x is the function,* $v_x(w|\pi,\lambda,\gamma) = (1-\delta(x)\cdot(\lambda+\gamma))\,\pi_x(w) + \delta(x)\left(\lambda\mu_x^1(w) + \gamma\mu_x^0(w)\right)$

where

$$\delta(x) = \frac{1}{(1-\lambda-\gamma)\,\pi(x) + (\lambda+\gamma)}.$$

Without the signal x, the trader's (T) CEU of the asset's value would be,

$$CEU_T(f|v(w|\pi,\lambda,\gamma)) = (1-\lambda-\gamma)\,E_\pi(f) + \lambda \sup f + \gamma \inf f\ ; \quad (4)$$

$$CEU_T(f|v(w|\pi,\lambda,\gamma)) = (1-\lambda-\gamma)\,\pi_1^t + \lambda. \quad (5)$$

Correspondingly, the CEU with respect to the conditional neo-additive capacity $v_x(w|\pi,\lambda,\gamma)$ is given as,

$$CEU_T(f|v_x(w|\pi,\lambda,\gamma)) = (1-\delta(x)(\gamma+\lambda))\,E_{\pi|x}(f) + \delta(x)(\lambda \sup f + \gamma \inf f)\,. \quad (6)$$

Now we show that this diversity can generate different trading behaviours in the market in different situations.

(1) *No signal and no ambiguity.*

Suppose there is no private information and no ambiguity in the market, then $\gamma = \lambda = 0$, equation (6) becomes $CEU_T(f|v(w|\pi,\lambda,\gamma)) = EU_T(f|w.\pi) = \pi_1^t = EU_M(w,\pi)$. In this case, illustrated in Figure 1, there is no trade on the information. Given our assumptions, this case is excluded from further consideration.

(2) *With signal but no ambiguity.*

In this case, the trader's expected value of the asset is respectively as $E[w|x=l]$ when (s)he receives a "Bearish" signal and $E[w|x=h]$ when a "Bullish" signal is received. Given our assumptions $E[w|x=l] < E[w] < E[w|x=h]$, we conclude

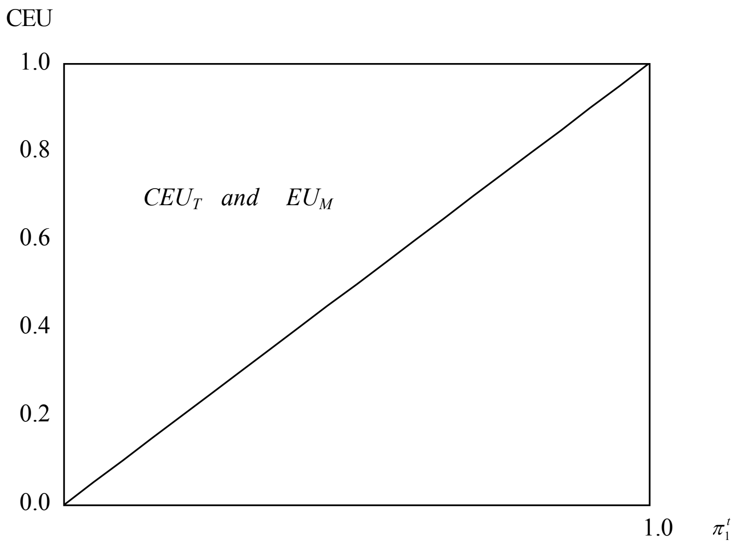

Figure 1: No signal and no ambiguity

that there is no herding or contrarian behaviour, and traders always trade following their private signals. In effect, a trader buys (sells) if (s)he receives a "Bullish" ("Bearish") signal. This is illustrated in Figure2. The concavity of $CEU_{T,h}(w)$ and the convexity of $CEU_{T,l}(w)$ are demonstrated in the proof of Proposition 5.1 below. Additionally, we observe that both of those equations have identical magnitudes at, respectively, $\pi_1^t = 0$ and $\pi_1^t = 1$

(3) *With signal and asymmetric ambiguity.*

This is the complete version of the model which we described above. In this case, expected values of the asset, respectively, for the MM, the trader with a "Bearish" signal and the trader with a "Bullish" signal are:

$$EU_M\left[w|H_t\right] = \hat{c} = P\left(w = 1|H_t\right) = \pi_1^t, \tag{7}$$

$$CEU_{T,x_l}\left(w\right) = \left[1 - \delta\left(x\right)\left(\lambda + \gamma\right)\right] \cdot E_{\pi|x_l} + \delta\left(x\right)\lambda, \tag{8}$$

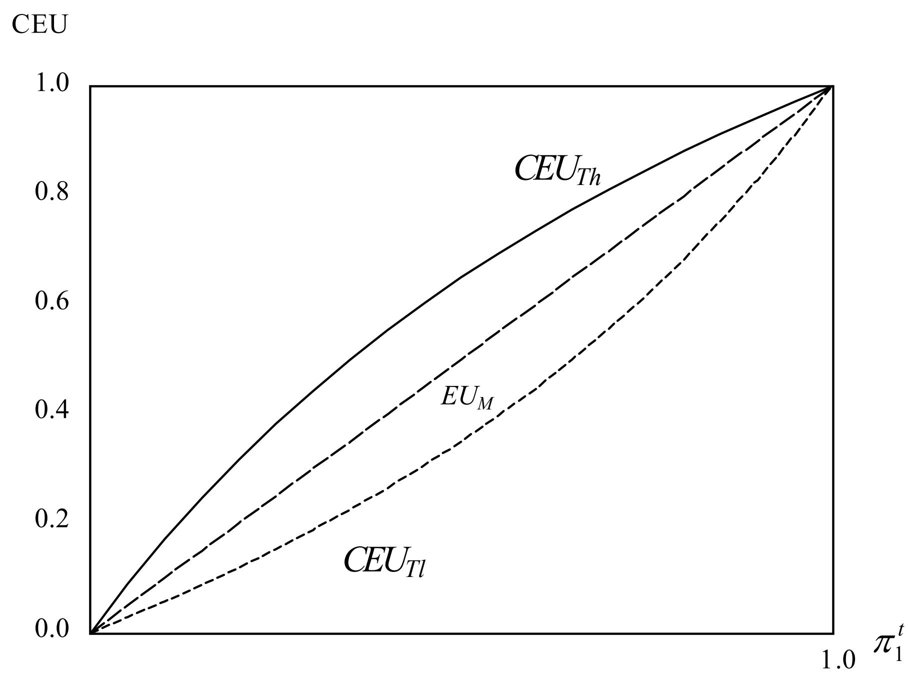

Figure 2: With Signal and no ambiguity

$$CEU_{T,x_h}(w) = [1 - \delta(x)(\lambda+\gamma)] \cdot E_{\pi|x_h} + \delta(x)\lambda. \tag{9}$$

"Bearish" signal is represented as x_l, which indicates selling as the optimal action (the signal implies that assets have low value thus are overpriced in the market); the "Bullish" signal is represented as x_h, which indicates buying as the optimal action. In equation (8) and equation (9),

$$E_{\pi|x_l} = \frac{p \cdot \pi_1^t}{p \cdot \pi_1^t + (1-p) \cdot (1-\pi_1^t)}, \tag{10}$$

$$E_{\pi|x_h} = \frac{(1-p) \cdot \pi_1^t}{(1-p) \cdot \pi_1^t + p \cdot (1-\pi_1^t)}, \tag{11}$$

$$\delta(x) = \frac{1}{(1-\lambda-\gamma) \cdot \pi(x) + (\lambda+\gamma)}. \tag{12}$$

Given these valuations of the asset's price, what types of behaviour can be evidenced in the market? For given amount of ambiguity, $\delta(x)(\lambda+\gamma)$, and particular levels of pessimism γ and optimism λ, we can illustrate the possibilities in Figure 3 (since the CEU equations for the trader must start at identical, non-zero, intercepts on the vertical axis). Consider then $CEU_{T_l}(\equiv CEU_{T,x_l}(w))$ taken in conjuction with EU_M. In the interval $[0, \pi^*]$, we see that contrarian buy behaviour will arise, if we explain that over the interval the *market history* is one where prices have been relatively low or have been falling in recent times. Likewise, when the "Bullish" signal is received the trader will engage in contrarian selling in the interval $[\pi^{**}, 1]$; taking the *market history* as one where prices have been in this relatively high range or rising over it. A trader with good signal will ignore both his own signal and the positive market trend to sell the asset instead. The fact that contrarian trading behaviours occur in different scenarios has supports from a popular traders' aphorism in the real world: " Buy on the rumor and sell on the news ". We draw these last observations together and prove them formally in the following Proposition.

Proposition 5.1 *For given γ, λ for traders, and market makers with no ambiguity, there exist π^*, π^{**} such that:*

(a). If $\pi \in [\pi^*, \pi^{**}]$, then trades are informative.

(b). If $\pi \in [0, \pi^*]$, then contrarian buying behaviour occurs with positive probability.

(c). If $\pi \in [\pi^{**}, 1]$, then contrarian selling behaviour occurs with positive probability.

Proof: See Appendix.

We note that this is straightforward. All that is required is to: (a) prove the

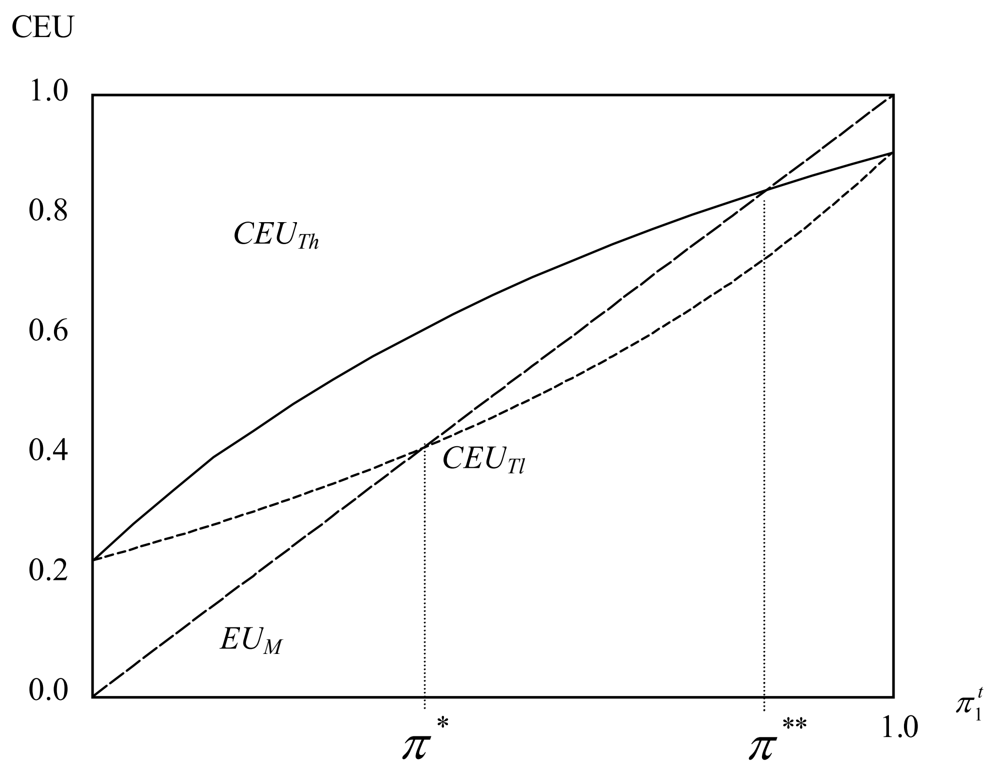

Figure 3: With signal and asymmetric ambiguity

concavity of the trader's CEU equation for the receipt of the high signal and the convexity of that for the receipt of the low signal; (b) show that both slopes lie between 0 and 1; and, (c) note that the trader's $CEUs$ have identical values, at $\pi_1^t = 0$, which exceed the intercept of EU_M; and that the trader's $CEUs$ have identical magnitudes at $\pi_1^t = 1$, which can be less than the concomitant level of EU_M, depending upon the ambiguity parameters, λ and γ. Consequently, the CEU equations for the trader can intersect EU_M, as illustrated on Figure 3.

Why does contrarian behaviour emerge as soon as there is ambiguity in the mind of the trader? When the trader has an unambiguous view of the true value of the asset and, hence has perfect belief/confidence in the his(her) up-dated market expectation of that value (as in Figure 2), when, for example, (s)he receives a high signal, his(her) expected value (namely, $CEU_{T,xh}(w)$) must exceed $\pi_1^t = EU_M$ within the unit interval; since in those conditions, $CEU_{T,xh}(w) = E_{\pi|x_h} > \pi_1^t$. Consequently, the trader will always buy upon receiving a high private signal. Mutatis mutandis, (s)he will always sell upon receiving a low private signal.

Once the trader holds ambiguous opinions about his(her) up-date of the market's expectations upon receiving either signal, however, (s)he will have less than complete confidence or belief in that up-dated value. Her(his) assessment of the value of the asset will then be based (as it were, truly) on its CEU and, for example, given at least some degree of optimism, will exceed the market valuation of the asset (π_1^t) at zero and near zero values of that valuation (as in Figure 3). As the market value (π_1^t) rises, $CEU_{T,xh}(w)$ will fall below $\pi_1^t = CEU_M$, as π_1^t approaches unity, irrespective of the absolute/relative values of the two degrees of ambiguity. That divergence between

$CEU_{T,xh}(w)$ and π_1^t= CEU_M must increase proportionally as π_1^t approaches unity. Consequently, despite having received a signal such that his(her) expected valuation of the asset exceeds π_1^t, the concomitant doubt about the consequent up-dated valuation, causes the individual to adjust the value ($CEU_{T,xh}(w)$, here) downwards increasingly on a rising market, such that it implies that the best strategy is to sell the asset against the market trend. The trader engages in contrarian selling, doubting the wisdom of the market; or strictly, doubting the wisdom of slavishly following market sentiment.

Following on from Proposition 5.1, we can state this further proposition:

Proposition 5.2 *Optimism increases the price range over which contrarian buying behaviour occurs; pessimism increases the price range over which contrarian selling behaviour occurs.*

Proof: First, we note that z decreases and $\lambda\delta(x)$ increases with an increase in λ, where $z = [1 - \delta(x)(\lambda + \gamma)]$; since from the definitions of z and $\delta(x)$ it follows immediately that:

$$\frac{\partial z}{\partial \lambda} = -\pi(x)\delta^2 < 0; \quad \frac{\partial(\lambda\delta)}{\partial \lambda} = \delta(1 - \lambda\delta) > 0.$$

Now we focus on the impact of an increase in λ on the low-signal CEU, since the following can be adopted *mutatis mutandis* for the impact of an increase in γ on the high signal CEU.

When the $CEU_{T,xl}(w)$ equation intersects the 45^0 line (the equation for the EU of the MM), $CEU_{T,xl}(w) = \pi_1^t \equiv \pi^*$. Hence,

$$\pi_1^{*2}(2p - 1) + [(1 - p) - z_1 p + \lambda_1\delta_1(1 - 2p)]\pi_1^* - \lambda_1\delta_1(1 - p) = 0, \qquad (13)$$

$$\pi_2^{*2}\left(2p-1\right)+\left[\left(1-p\right)-z_2p+\lambda_2\delta_2\left(1-2p\right)\right]\pi_2^*-\lambda_2\delta_2\left(1-p\right)=0. \qquad (14)$$

In equations (13) and (14) $\pi_1^t \equiv \pi^*$, with the subscripts 1 and 2 denoting its magnitude for two values of λ and the concomitant values of z and δ. Suppose that $\lambda_1 < \lambda_2$. Given the partial derivatives of z and $\lambda\delta$ with respect to λ, we find that the coefficient on π_2^* exceeds that on π_1^* and the absolute value of the intercept term in equation (14) also exceeds its counterpart in equation (13). It then follows that π_2^* itself must exceed π_1^*. We further note that given that $p < 1/2$, and $0 < z < 1$ by construction, the two values of π^* generated by each of the equations (13) and (14) will be positive; with the lower one being the relevant one since the CEU equations intersect the 45^0 line at only one point in the $[0, 1]$ interval.■

This has a simple intuitive explanation: no matter what incentive traders have to engage in contrarian behaviour, sufficient optimism about the value of the asset will encourage more contrarian buying and discourage contrarian selling. Sufficient pessimism has just the opposite effects. In Figure 3, with a higher value of λ, both curves $CEU_{T,x_h}(w)$ and $CEU_{T,x_l}(w)$ will shift upwards, have a reduced slope and generally result in higher values of π^*, π^{**}. Conversely, a higher γ will shift both curves $CEU_{T,x_h}(w)$ and $CEU_{T,x_l}(w)$ downwards and produce lower values of π^*, π^{**}. In other words, as γ increases the interval $[0, \pi^*]$ become narrower, implying that contrarian buying occurs less and, the interval $[\pi^{**}, 1]$ increases, so that the price range over which contrarian selling occurs also increases.

Contrarian behaviour alone is feasible under the current assumptions: herding cannot occur. The reason is simple, no matter how traders update their belief and how ambiguous they are, the CEU-expected value of the asset is never below zero or above 1. Therefore, to understand herding in the market, we need to modify our assumptions. What we do here is to introduce symmetric ambiguity between market makers and traders. That is, like the traders, market makers are ambiguous to the publicly available information so that their beliefs are non-additive as well.

(4) *With signal and symmetric ambiguity*

At the beginning of the trade at $t = 0$, there is no history, therefore, we assume no ambiguity, it is easy to see we have $E^0_{x_l}(w) < E^0(w)$. Suppose now at time t we have a positive history, $E^0(w) < E^t(w)$, and the trader receives a private signal. If markets are informationally efficient in the sense that there is no ambiguity, as shown in case (2), we will always hold the inequality $E_{\pi|x_l} < \pi_1^t < E_{\pi|x_h}$. This implies that herd behaviour will never appear in such markets[16]. However, we show here that the conclusion can be overturned if we allow both market makers and traders to perceive ambiguity about information available in the market. Market makers still only receive public information but they set the price based on that information as well taking ambiguity attitudes into account, which we assume to be the same for all the MMs. Therefore, an MM's expected value of the asset is:

$$CEU_M\ (w) = (1 - \gamma_M - \lambda_M) \cdot \pi_1^t + \lambda_M. \tag{15}$$

As previously, the trader has a private signal x, this expected value of the asset:

$$CEU_{T,x}\,(w) = [1 - \delta\,(x)\,(\lambda_T + \gamma_T)] \cdot E_{\pi|x} + \delta\,(x)\,\lambda_T. \tag{16}$$

Considering these two expected values of the asset, we have two following propositions formally.

[16]That is also a result derived in AZ paper.

Proposition 5.3 *When both Market Makers and traders perceive ambiguity, for given γ_M, λ_M and γ_T, λ_T, there exist π^*, π^{**} such that:*

(a). If $\pi \in [\pi^*, \pi^{**}]$, then trades are informative.

(b). If $\pi \in [0, \pi^*]$, then herd selling behaviour occurs with positive probability.

(c). If $\pi \in [\pi^{**}, 1]$, then herd buying behaviour occurs with positive probability.

Proof: Given that our objective is simply to demonstrate that herd behaviour is possible, we assume that $\delta(x)\lambda_T < \lambda_M$. Given equations (15) and (16) the result is a situation illustrated in Figure 4[9]. We concentrate on the $CEU_{T,l(x)}(w)$ function. What has to be established is the possibility that the function can intersect that of $CEU_M(w)$ as some value such as π^{**} on Figure 4 in the interval $[0, 1]$. To do so it is only necessary to show that there will be particular ambiguity parameters for which the maximum value of $CEU_M\ (w)$ will be lower than that of $CEU_{T,l(x)}(w)$. At $\pi_1^t = 1$, the maximum value of the former is $1-\gamma_M$ and of the latter it is $1-\delta(x)\lambda_T$. It is then apparent that it is possible for both $(1-\delta(x)\lambda_T) > 1-\gamma_M$ and $\delta(x)\lambda_T < \lambda_M$ to hold simultaneously; $CEU_{T,l(x)}(w)$ and $CEU_M\ (w)$ must then intersect at some π^{**}. It follows immediately, given the relative properties of the functions $CEU_{T,l(x)}(w)$ and $CEU_{T,h(x)}(w)$, already established, that a point such as π^* must also exist.[10] ■

[9] In deriving the equations for Figure 4 we used: $\lambda_M = 0.25; \gamma_M = 0.2; \lambda_T = 0.1; \gamma_T = 0.04185$;and, $\pi(x) = 0.35$.

[10] At the general level, of course, the exact magnitude, and existence, of π^{**}, for given ambiguity parameters will depend upon $\pi(x)$ in $\delta(x)$. In the extreme case, when $\pi(x)$ approaches zero it is the maximum minimorum of $CEU_{T,x}(w)$ at $\pi_1^t = 1$ that will arise, since $\delta(x)$ approaches its maximum: $CEU_{T,l(x)}(w)$ will then equal $(\lambda_T)/(\lambda_T + \gamma_T)$. In that situation, the initial values of the ambiguity parameters can still result in that magnitude's exceeding $1 - \gamma_M$; but it can be demonstrated that

Proposition 5.4 *When, for given ambiguity on behalf of market makers, traders become more optimistic or less pessimistic about the market trend when there is a history of rising (falling) prices, the price range over which herd buying (selling) will occur will increase (decrease).*

Proof: Consider a given situation such as that illustrated in Figure 4. Let there be a ceteris paribus increase in the degree of optimism of the trader (an increase in λ_T), such that $\delta(x)\lambda_T < \lambda_M$ still holds. From the proof of Proposition 5.2, the intercept and slope of $CEU_{T,l(x)}(w)$ will increase and fall, respectively; and its maximum value will remain unaltered. The consequence will be a lower value of π^{**}; that is, the range of prices over which buy herding will occur will increase. Simultaneously, the intercept, slope and maximum value of $CEU_{T,h(x)}(w)$ will change in the same manner as they do for $CEU_{T,l(x)}(w)$; consequently, π^* will also fall, implying that the price range over which herd selling will occur will be reduced. Now, consider the case of a ceteris paribus reduction in the trader's pessimism (γ_T): this will increase the intercept, the slope and maximum value of both $CEU_{T,l(x)}(w)$ and $CEU_{T,h(x)}(w)$, leading again to a reduction in both $\pi^*$and π^{**}. Accordingly, there will again be an increase (a reduction) in the price range over which the trader will engage in herd buying (selling) behaviour. ■

We normally assume that traders with low signals usually have lower valuation of the asset and engage in selling. Traders with high signals usually value the asset higher than the market and engage in buying. However, through comparing the $CEUs$

the intercept of CEU_M (w) would have to be lower than that of $CEU_{T,l(x)}(w)$, when the former could lie below the latter.

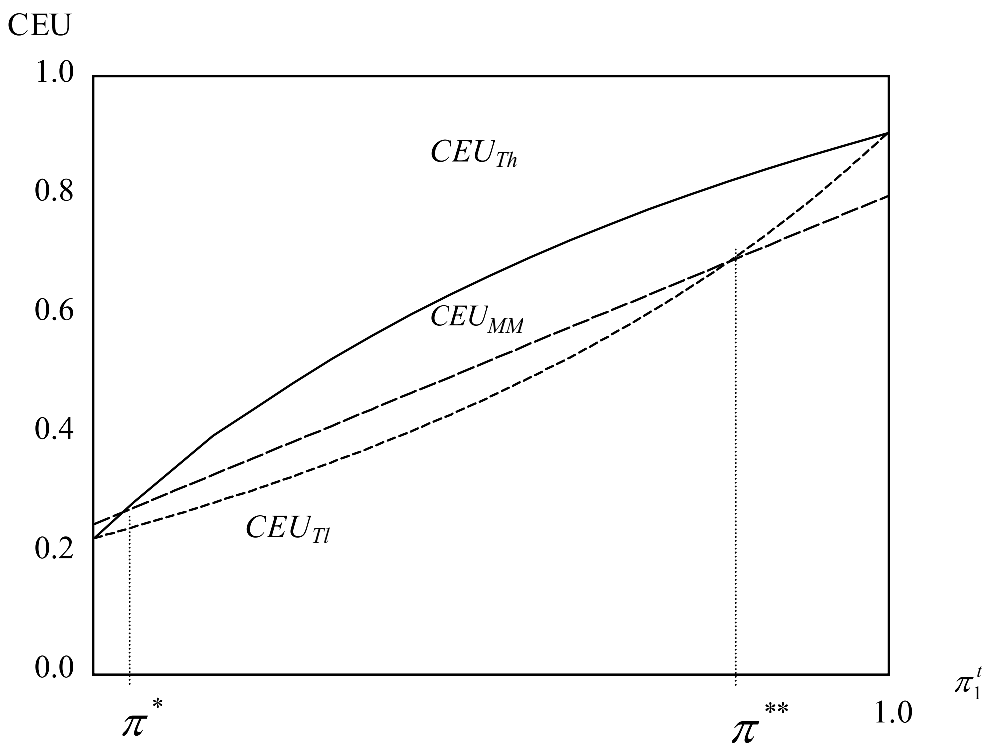

Figure 4: With signal and symmetric ambiguity

of market makers and traders, we see that traders may abandon their own signals and follow others even when they are doubtful that the others are definitely right, but in comparison they are more doubtful about their own information. Specifically, sufficiently optimistic traders with a low signal will engage in herd buying after market prices reach a certain level, i.e., $\pi_1^t > \pi^{**}$. Conversely, sufficiently pessimistic traders with high signals will engage in herd selling when market prices are relatively low, i.e., $\pi_1^t < \pi^*$. (note: $\pi^* < \pi^{**}$). This is consistent with real observation. The more optimistic or the less pessimistic traders become, compared with the market makers, and the higher is the price of the asset, the more likely are traders to engage in herd buying. Conversely, the more pessimistic are traders than the market, or the more pessimistic they are than market makers, and the lower is the asset's market price, the more likely are traders to engage in herd selling.

5.2 Herd behaviour and Price Bubbles

The possible consequences of herd behaviour are, of course, excess volatility and, especially, price bubbles. Recent literatures on the latter has concentrated on the classic, fully rational models of securities market price formation therefore have difficulties to explain. Further regarding market information efficiency, conclusions are mixed[17]. In our framework we can see that herd buying (resp. selling) caused by sufficient optimism (resp. pessimism) can lead to extreme price effects, but only in

[17] For example, Vives (1996) and Decamps and LOVO (2002) suggested informational inefficiency and implied price distortions in the long-run. However AZ paper suggested the short-run mispricing assets but long-run efficient markets, which our results agree with.

the short-run and not in the long-run.

Consider herd buying as an example. From the last section, we know traders make their investment decision according to the sign of :$CEU_{T,x_l}(w) - CEU_M(w)$, namely:

$$\left(E_{\pi|x_l} - \pi_1^t\right) + \lambda_M\left(\pi_1^t - 1\right) + \gamma_M \cdot \pi_1^t + \delta(x)\lambda_T\left(1 - E_{\pi|x_l}\right) - \delta(x) \cdot \gamma_T \cdot E_{\pi|x_l}. \quad (17)$$

Remember we have been assuming that above π^{**} for any value of the MM's (current) price, π_1^t, the market has experienced rising prices in the immediate past. Now, suppose that it is impossible for the MM to distinguish whether traders are herding or are trading in response to their own information: the MM will increase the price further to capitalize on the buying trend. This implies that we will have increased π_1^t, as well as $E_{\pi|x_l}$ in equation (17). However, because of the receipt by the trader of the "low signal", x_l, $E_{\pi|x_l}$ increases at slower speed than π_1^t, and the absolute value of $E_{\pi|x_l} - \pi_1^t$ is an increasing function of π_1^t. Intuitively, traders are slow to respond because they have more information. Now, for a given pair of γ_M and λ_M, eventually herd buying will be impeded by a sufficiently high price and a sufficiently large spread of $E_{\pi|x_l} - \pi_1^t$ which, even with λ_T equal to 1 and γ_T equal to zero, exceeds the trader's expectation. Therefore, the herd is halted, the price naturally has to drop, and so the market crashes.

The longevity of any bubble and the speed at which the market subsequently crashes, will depend upon whether or not the market trader adjusts his ambiguity parameters as the boom progresses. The intensity and time-span of, say, rising asset prices, could themselves prompt adjustments to the degree of ambiguity felt by the market trader about the future price. For example, the trader's optimism (pessimism) parameter might eventually decline (rise); whether this will be gradually or otherwise will, clearly, depend on a number of factors including the level of the asset's latest price and the time-span over which it has risen. The trader will come to doubt eventually whether the rising market price is sustainable, and so his(her) high private signal will be discounted, and his(her) low private signal will now be accepted. At some point sooner than when the traders' optimism and pessimism parameters are constant, the trader's CEU of the asset will fall below the market's and the market maker's expectation; assuming, as previously, that the market maker continues to increase the asset's price to capitalize on what (s)he imagines is a continual buying trend. The trader then does not continue to herd, (s)he sells the asset rather than buys it, and so the price falls, as will.

In sum, herd buying can produce an unsustainable run-up in price that eventually results in a crash. Accordingly assets can be mispriced in the short-run but the subsequent correction in the market can produce information efficiency in the long-run.

6 Concluding remarks

We have re-examined herding behaviour in a financial market where trade is sequential and prices of assets are endogenously determined. To investigate the effects of ambiguity in financial markets, we modelled agents' beliefs as neo-additive capacities and their preference as CEU. We have demonstrated that different patterns of behaviour can be generated by introducing ambiguity and ambiguity attitudes. In particular, we first assumed asymmetric ambiguity between the MM and traders and proved that contrarian behaviours will appear in the markets. That is, if MMs do not perceive ambiguity to information available in the market but traders do, then it is optimal for traders to trade against the market trend in certain circumstances. In general, optimism helps contrarian buying and pessimism helps contrarian selling. We then investigated the case where both the MM and traders are ambiguous to information and show that herd buying (resp. selling) can arise and their extent will be caused by the degree of optimism (resp. pessimism) of the traders. Moreover, we suggested that herding only appears in the short-run and price bubbles become possible due to the market impetus engendered by the ambiguity attitudes of traders and the MM. However, it can be contended that the price mechanism assures that choices are efficient and herd behaviour is impeded in the long-run. We also suggested that market efficiency does not mean perfect foresight, so we expect an analyst's forecasts and market prices to be "wrong" ex post in the presence of ambiguity.

Appendix

Proof of Proposition 5.1:

We first prove that the $CEU_{T,xl}(w)$ is a concave function with slope less than 1 for the interval $\pi_1^t \in [0,1]$, $CEU_{T,xh}(w)$ is a convex function with slope less than 1 for the interval $\pi_1^t \in [0,1]$.

Suppose there is negative history observed in the market and the trader receives a negative signal x_l, so $\pi(x_l) = 1 - p > 1/2$.

$$\begin{aligned} CEU_{T,xl}(w) &= [1 - \delta(x)(\lambda + \gamma)] \cdot E_{\pi|x_l} + \delta(x)\lambda, \quad (18) \\ CEU_{T,xl}(w) &= z \cdot \frac{p \cdot \pi_1^t}{p \cdot \pi_1^t + (1-p) \cdot (1 - \pi_1^t)} + \delta(x)\lambda, \quad (19) \\ \text{where } z &= [1 - \delta(x)(\lambda + \gamma)]. \quad (20) \end{aligned}$$

Taking the partial derivative of equation (18) with respect to π_1^t we have:

$$\frac{\partial CEU_{T,xl}(w)}{\partial \pi_1^t} = z \cdot \frac{p(1-p)}{[p \cdot \pi_1^t + (1-p)(1-\pi_1^t)]^2}. \quad (21)$$

Since $0 < z = \frac{(1-\lambda-\gamma)\pi(x)}{(1-\lambda-\gamma)\pi(x)+\lambda+\gamma} < 1$,and given the assumption that $p < 1/2$, it follows that equation (21) is positive; and it is straightforward to see that besides having $z < 1$, we also have the term $\frac{p(1-p)}{\left[p \cdot \pi_1^t + (1-p)\left(1-\pi_1^t\right)\right]^2} < 1$ too. Thus we have that $\frac{dCEU_{T,xl}(w)}{d\pi_1^t} < 1$.

From equation (18), we get,

$$\frac{\partial^2 CEU_{T,xl}(w)}{\partial \pi_1^{t2}} = \frac{2p(p-1)(2p-1)[p \cdot \pi_1^t + (1-p)(1-\pi_1^t)]}{[p \cdot \pi_1^t + (1-p)(1-\pi_1^t)]^4}. \quad (22)$$

Accordingly, under the assumption that $p < 1/2$, the sign of the equation above is the sign of :

$$\left[p \cdot \pi_1^t + (1-p)\left(1-\pi_1^t\right)\right] > 0. \quad (23)$$

Therefore the function $CEU_{T,xl}(w)$ is convex.

Similarly, it is easy to see that we have, $\frac{\partial CEU_{T,xh}(w)}{\partial \pi_1^t} < 1$, and $\frac{\partial^2 CEU_{T,xh}(w)}{\partial \pi_1^{t2}} < 0$, so that $CEU_{T,xh}(w)$ is a concave function.

$$\begin{aligned} CEU_{T,x_h}(w) &= [1 - \delta(x)(\lambda + \gamma)] \cdot E_{\pi|x_h} + \delta(x)\lambda, \quad (24) \\ CEU_{T,x_h}(w) &= [1 - \delta(x)(\lambda + \gamma)] \cdot \frac{(1-p) \cdot \pi_1^t}{(1-p) \cdot \pi_1^t + p \cdot (1 - \pi_1^t)} + \delta(x)\lambda. \end{aligned}$$

Clearly, for given values of γ and λ, $CEU_{T,x_h}(w)$ is always greater than $CEU_{T,xl}(w)$ for all intermediate values of $\pi_1^t \in (0,1)$,and identical at the boundaries with $\pi_1^t = [0,1]$.

Further, it shows that, at $\pi_1^t = 1$,

$$CEU_{T,xl}(w) = [1 - \delta(x)\gamma] \cdot z + \delta(x)\lambda \cdot (1-z) < z + (1-z) = 1, \tag{25}$$

and at $\pi_1^t = 1$,

$$CEU_{T,x_h}(w) = (1-p) + \delta(x)\gamma \cdot (p-1) + p \cdot \delta(x)\lambda < (1-p) + p = 1. \tag{26}$$

These conditions imply that $CEU_{T,xl}(w)$ intersects the 45^0 degree line once at lower π_1^t, say π^*, than $CEU_{T,xh}(w)$ which intersects the 45^0 degree line at higher π_1^t, say, π^{**}. Therefore below π^* "contrarian buying" behaviour occurs with a negative history and a negative signal; "contrarian selling" behaviour occurs above a market price of π^{**} with a positive history and a positive signal.■

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