

Secretive vs. Transparent Committees and the Group Reputation Effect

Gilat Levy¹

Abstract: In this paper I analyse the effect of transparency of the decision making process in committees on the decisions that are eventually taken. I focus on committees whose members are motivated by career concerns, so that each member tries to enhance his own reputation. When the decision making process is secretive, the individual votes of the committee members are not exposed to the public but only the final decision. Thus, individuals are evaluated according to the group's decision. I find that in such a case, a group reputation effect arises, which induces group members to comply with preexisting biases. For example, if the voting rule demands a supermajority to accept a reform, individuals vote more often against reforms and exacerbate the conservatism of the voting rule. When the decision making process becomes transparent and individual votes are observed, this effect disappears and such committees are then more likely to accept reforms.

¹Department of Economics, London School of Economics. Email: g.levy1@lse.ac.uk

1 Introduction

Many economic and political decisions are taken by groups of decision makers, i.e., by committees. Company boards, governments and monetary policy committees are some notable examples. There is however a clear line dividing committees into two types: the secretive and the transparent. For example, the minutes of the Federal Reserve Open Market Committee or the deliberations of the US Supreme Court Justices are published. But those of the European Central Bank and the EU Court of Justice, are hidden from the public eye. Only the final decisions of these institutions are publicly observed, but not the views of individual members. Another example is the contrast between legislatures and governments. Whereas most democratic parliaments hold open discussions, the executive branches of governments still keep their deliberation away from the public eye.

In many democratic countries however, committees go through a process of becoming more transparent. In particular, it is only a recent phenomenon that the Open Market Committee's minutes are published.² The trend is undoubtedly in this direction of more transparency, which is casually connected with more openness and more 'democratization'. In this paper I investigate the effect of transparency on the behaviour of committee members, and hence, on decision making in committees.

I focus the analysis on committees whose members are motivated by career concerns. Committee members such as members of governments, monetary committees or multi-judicial courts, are indeed likely to be concerned about their own promotion, re-election or prestige. Although by now we have a relatively good understanding of how individual decision makers behave when they are motivated by career concerns,³ we are still lacking an analysis of group decision making when the individuals members of the group have career concerns.

Moreover, in the presence of career concerns, transparency should indeed matter. In particular, if such careerist committee members use their vote to impress the public or other outsiders, then it is actually important whether committees' meetings are open to the public or not.

²These are published since 1993. The sunshine act of 1976 imposed publicity requirements on federal agencies in the US.

³The large literature following Holmström (1982) spans many types of decision makers (such as managers, financial advisers, politicians etc.) in a plethora of environments.

I therefore study the interaction of these reputation concerns with the decision process - be it transparent or secretive. I assume that each committee member wishes to accumulate reputation for being a high ability decision maker. That is, that he has accurate private information about the matter to be decided. Obviously then, when votes of the individual members of the committee can be observed, they can be used strategically by the committee members to affect their own reputation. I show here how reputation concerns induce committee members to vote strategically even when their individual votes are not observed by those they wish to impress. I can then use the analysis to assess how this affects decision making when committees change their procedure from a secretive process to a transparent one.

Some intuition would suggest that when a decision making process becomes transparent, then committee members would pander to public opinion, and would be less likely to accept radical suggestions or reforms. I show that this intuition is wrong when committee members are worried about their reputation as competent decision makers.⁴ Actually, the opposite effect arises and a switch to a transparent process implies that reforms are more likely to go through.

More specifically, I find that when the decision process in a committee is secretive, then committee members conform to preexisting biases in the decision making process. First, when the voting rule is biased against some decision so that it demands a supermajority in order to accept this decision, then individual votes are biased against this decision as well. Second, when the prior expectations are biased against a decision, then the individual votes are more biased against this decision in the secretive mechanism than in the transparent mechanism.

I isolate then a *group reputation effect*. This effect implies that when the mechanism is secretive, the committee members' votes exacerbate the conservatism of the voting rule or of the prior beliefs. This effect does not exist in the transparent case. The main result establishes then that when a decision making process in a committee changes to a transparent process, the committee members are more likely to accept the more radical decisions compared with the secretive mechanism. A status quo bias is therefore weaker under transparency.

To understand the intuition for the group reputation effect, let us first think of the case

⁴Other reputation concerns for agents may be to prove that they have some particular preferences.

of a transparent decision making process. Actually, in a transparent process, it turns out that there are no strategic group effects at all. That is, each individual can be evaluated only on the basis of his published recommendation. The group's decision, the voting rule, or others' recommendations, would not matter for his evaluation. It is therefore strategically equivalent to an individual reputation model. In such a case, each member will receive higher reputation when he or she makes the correct recommendation. This is a signal on ability because able experts are more likely to have accurate private information.

The more interesting case is that of a secretive committee. In this case, the reputation of any individual has a component of group reputation; outside evaluators cannot observe individual recommendations, and hence base their assessment on the group's decision, a common component to all members. The group's decision and thereby the voting rule and others' behaviour, become relevant for the strategic behaviour of the committee members.

The group reputation effect works in a particular direction though. Suppose that the voting rule is biased against decision A . That is, B is the status quo and A will be picked only if a large supermajority votes in its favour. Suppose that the committee indeed decides for A . In that case, outside evaluators would reasonably guess that it is highly likely that each individual had actually voted in favour of A , since many need to vote for it in order for it to be accepted. On the other hand, if the committee decides for B , these evaluators are in the dark. That is, there are many configurations of votes that would render B acceptable, and hence it is hard to extract information about individual votes.

This higher uncertainty about individual votes, which arises when the group's decision is B , is good for a committee member who cares about his reputation. In particular, it is good for the less able types. These types are the ones who are more worried about making the wrong recommendation (and thus acquiring bad reputation). They like uncertainty about individual votes because if the group's decision turns out to be wrong, they can shed the blame on others for voting in the wrong manner. Thus, when the voting rule is biased against A , committee members prefer that the committee decides for B . Consequently, committee members tend to vote for B more often (that is, for more of their types). More generally, committee members align with the voting rule so that the more biased is the voting rule against an action, the more biased are the individual votes against this action.

The effect described above arises only when individual votes are unobserved; with transparency, committee members cannot claim that others were the culprits. Thus, the group

reputation effect disappears when the process becomes transparent, and exiting procedural biases are diminished.

Similar intuition arises when the voting rule is not biased, as in the case of the simple majority rule, but the prior expectations are biased. That is, when the prior deems one particular decision as more likely to be correct. Again, when the committee decides for this likely decision, its members feel ‘safe’; since this decision is likely to be correct, it is likely that many other experts have voted in favour of it. Thus, even if it turns out to be the wrong decision, it is easier to hide one’s own responsibility for wrongdoing. The prior beliefs are therefore another source of a bias and experts’ votes would conform to this bias in the secretive case more than in the transparent case.

Note that the effect isolated here is over and above the obvious tendency of experts to follow the prior simply because it indicates what is the right course of action. Such a tendency - sometimes termed as ‘herding’ in the literature - arises also in the transparent case. But in the non transparent case, experts tend to follow the prior even more because of strategic group reputation considerations, namely, since it makes it easier to blame others for a failure. Thus, when the process becomes transparent, this group reputation effect is removed and the committee accepts more often the decision that goes against the prior expectations.

My paper identifies therefore an effect which is completely different from what previous literature on group decision making has found. In previous literature, it has been shown that agents are actually induced to vote more often in favour of the decision that the voting rule is biased against. Here, I show that the opposite behaviour arises, due to the group reputation effect. The reason is that previous literature has not analysed the case of agents being motivated by career concerns. In particular, in Feddersen and Pesendorfer (1997), Austen-Smith and Banks (1996), Persico (2004) and Austen-Smith and Feddersen (2003), agents have the (common) goal of attaining the correct decisions. In Eliaz, Ray and Razin (2004), agents have different ideological preferences (but again, no career concerns). I therefore show that voting in groups can have completely different consequences when committee members are motivated by career concerns.

There are many models which analyse *individual* decision making with career concerns (for example, Holmström (1982), Scharfstein and Stein (1990), Trueman (1994), and Levy (2004)). In our context, the ones that are most relevant are Prat (2003) and Avery and

Meyer (2003); these papers focus on the normative implications of the transparency of recommendations or actions of the careerist agents. Note that my paper asks positive and not normative questions; moreover, I study committees and not individuals and hence the analysis is completely different.

There are less than a handful of papers which combine group decision making with career concerns. A paper which analyses committee decision making with career concerns is Ottaviani and Sørensen (2000); however, they model sequential transparent voting processes, and thus no group reputation effects arise. Their model is then strategically equivalent to an individual decision making process. More closely related are Sibert (2003) and Stasavage (2004) which analyse behaviour in monetary policy committees and the effects of transparency. Both assume that agents care about acquiring a reputation for having some particular preferences, whereas in my analysis, agents acquire reputation for being able experts. Hence, a group reputation effect as in my analysis cannot arise.

Finally, a completely different analysis of group reputation involves moral hazard issues, as in the papers by Tirole (1996) and Bar-Isaac (2004). My analysis focuses on adverse selection.

The rest of the paper is organized as follows. In the next section I describe the model. Section 3 presents the equilibrium for the transparent case, and section 4 analyses the equilibrium for the secretive case. In section 5, I present the main result which compares the equilibrium results given the different procedures. I conclude by discussing some extensions in Section 6. The appendix has all proofs that are not in the text.

2 The Model

I describe now the decision making process in a committee. I make some simplifying assumptions to facilitate the analysis. First, I analyse the smallest size of a committee which can yield interesting results in terms of different voting rules, that is, a three member committee (the results, as can be seen from the proof and the general intuition, can be extended to larger committees). Second, I focus the analysis on agents with career concerns only. The results are robust when we assume that experts are also genuinely interested in the committee's decision on top of their career concerns (see section 6).

Consider therefore a three-member committee that needs to decide between two alternatives, A and B . Each member i , $i \in \{1, 2, 3\}$, receives information on a random variable

$w_i \in \{a_i, b_i\}$. Each of these random variable could be interpreted as determining a dimension of the problem, for example, a different criterion according to which A and B are evaluated. Thus, each member is responsible for evaluating the choices on a different dimension (as is the case for example in committees in which members represent different business units in the same company, or in governments in which each minister holds a different portfolio).

I assume that these dimensions are not correlated, i.e., the w_i 's are independently distributed. This is done for exposition purposes, as it allows me to isolate the group reputation effect. The results are robust when one allows for correlation in the information of the committee members (see the discussion in section 6).

I assume therefore that expert i receives a signal $s_i \in \{a_i, b_i\}$, such that $\Pr(s_i = w_i | w_i) = t_i$ where t_i is uniformly distributed on $[\frac{1}{2}, 1]$ for each expert (independently) and is private information for expert i . The talent of an expert is therefore measured by t_i ; the more talented is expert i , the more accurate is his information. The prior probability is $\Pr(w_i = b_i) = q_i$. For simplicity, I assume that $q_i = q > \frac{1}{2}$ for all i .

All members vote simultaneously. Their vote/message is denoted by $m_i \in \{a, b\}$, which therefore indicates whether they support A or B .

A *voting rule* is denoted by $x \in (1, 2, 3)$. A voting rule x implies that if $\#\{i | m_i = a\} \geq x$, then A is taken. When $x = 3$, the voting rule is *A-unanimity*, i.e., all need to approve A for it to be accepted. When $x = 2$, the voting rule is *majority* (two votes are enough to support either action) and when $x = 1$, the voting rule is *B-unanimity* (one vote is enough to approve A). Denote the decision of the committee by $d \in \{A, B\}$.

An additional agent is the evaluator, denoted by E . If the committee is formed of politicians, the evaluator can represent the public who assesses how competent is each politician. If the committee is composed of different unit managers in the firm, the evaluator can represent the shareholders, who decide whom to promote.

The evaluator updates his beliefs about t_i for each expert i , given the uniform prior on each. E observes w_i (after the decision had been taken) for all i . In addition he observes the decision d and knows the voting rule x . Finally, if the committee's meeting is transparent, he also observes the votes m_i for $i \in \{1, 2, 3\}$. If the committee's decision making process is secretive, he does not observe m_i for all i . On the basis of his information, E updates his beliefs on the talent of each expert. Denote the posterior expectations that the evaluator has on the type t_i of expert i by τ_i .

Each expert's objective is to maximize $E(\tau_i)$.⁵ Thus, committee members are interested in an external evaluation of their own individual ability (as opposed to a situation in which they may be concerned with impressing their fellow committee members). As usual in career concerns models, I don't attribute any utility function to the evaluator but simply assume that he rationally updates his beliefs. This can be derived from a motivation to promote the most able experts.

In equilibrium, the evaluator's beliefs rely on Bayes rule whenever possible, and each expert sends the vote which maximizes $E(\tau_i)$, given his own information and the strategies of other experts. I focus on informative equilibria, i.e., equilibria in which each expert's votes are sometimes responsive to their signals, and ignore 'mirror' equilibria in which the meaning of the signals is reversed.

To summarize, the timing of the game is as follows:

1. The states w_i for $i \in \{1, 2, 3\}$ are realized and each expert i learns $s_i \in \{a, b\}$.
2. Each expert sends $m_i \in \{a, b\}$, given $\{s_i, t_i\}$.
3. The decision of the committee is $d = A$ if $\{\#\{i | m_i = a\} \geq x$ and $d = B$ otherwise.
4. E updates beliefs on t_i for all i , given the decision of the committee d , w_i for all i , and if the mechanism is transparent, also m_i for all i .

3 Transparent committees

Suppose first that the evaluator observes the personal recommendation of each expert, i.e., the voting mechanism is transparent. But then, the evaluator would only assess the talent of expert i based on m_i - the recommendation of expert i , and w_i - the relevant state of the world for expert i . In particular, the committee's decision d , the voting rule x , and the behaviour of other experts are of no consequence for the assessment of the evaluator and hence do not affect the equilibrium behaviour of expert i . Thus, each expert accumulates *individual reputation* in the transparent mechanism. To solve for the equilibrium, we can then focus on a generic expert and follow previous literature on individual career concerns⁶ (thus, for the remaining of this section, I drop the index i).

I now introduce the notation and the equilibrium solution method of the transparent case. This will also prove useful for the analysis of the secretive case. First, it is easy to

⁵All the results hold if instead of receiving utility of τ_i , an expert would receive utility of $V(\tau_i)$, where V is an increasing function, as long as V is not 'too concave'.

⁶See for example Levy (2004), Trueman (1994).

establish that in any informative equilibrium, an expert recommends a only if he believes that the probability that the state is indeed a is high enough. Let us define the following:

$$v(q, s, t) \equiv \Pr(w = a|q, s, t) = \begin{cases} \frac{(1-q)(1-t)}{qt+(1-q)(1-t)} & \text{if } s = b; \\ \frac{(1-q)t}{q(1-t)+(1-q)t} & \text{if } s = a. \end{cases}$$

Thus, in equilibrium, an expert uses a cutoff point $v(q, s^*, t^*)$, such that he recommends a if and only if $v(q, s, t) \geq v(q, s^*, t^*)$.⁷ Denote this cutoff in short as $v^*(q)$. As mentioned above, this cutoff does not depend on the voting rule and hence is the same for all x .

The evaluator E , using Bayesian updating, forms his expectations $\tau_v(m, w)$ about the ability of the expert, given some conjecture of the expert's strategy of a cutoff point v , and the observations of m and w . Thus, for example, $\tau_v(a, a)$ denotes the posterior expectations over the type t of the agent when $m = a$ and $w = a$ and E conjectures the cutoff v , $\tau_v(b, a)$ is the reputation of the expert when $m = b$ and $w = a$, and so on. In equilibrium, the conjecture of E about v has to be correct.

We now have all the ingredients necessary for solving for the equilibrium cutoff. In equilibrium, the type $v^*(q)$ - that is, the type (s^*, t^*) for which $\Pr(w = a|q, s^*, t^*) = v^*(q)$ - must be indifferent between recommending b or a . This type knows that if he recommends a , he receives the reputation $\tau_{v^*}(a, a)$ with probability $v^*(q)$ and $\tau_{v^*}(a, b)$ with probability $1 - v^*(q)$. The following equation has therefore to hold:

$$v^*(q)\tau_{v^*}(a, a) + (1 - v^*(q))\tau_{v^*}(a, b) = v^*(q)\tau_{v^*}(b, a) + (1 - v^*(q))\tau_{v^*}(b, b) \Leftrightarrow \quad (1)$$

$$v^*(q)(\tau_{v^*}(a, a) - \tau_{v^*}(b, a)) = (1 - v^*(q))(\tau_{v^*}(b, b) - \tau_{v^*}(a, b)) \quad (2)$$

The equilibrium is characterized in the following Proposition:

Proposition 1 (i) *There is a unique informative equilibrium in which an expert recommends $m_i = b$ if $v(q, s, t) \leq v^*(q)$ and recommends $m_i = a$ otherwise.* (ii) *In the equilibrium, the reputation of an expert is higher when he makes the correct recommendation, i.e., $\tau_{v^*}(a, a) \geq \tau_{v^*}(a, b)$ and $\tau_{v^*}(b, b) \geq \tau_{v^*}(b, a)$.* (iii) *In the equilibrium, the reputation of an expert is higher when he recommends for A , i.e., $\tau_{v^*}(a, a) \geq \tau_{v^*}(b, b)$ and $\tau_{v^*}(a, b) \geq \tau_{v^*}(b, a)$.*

A first thing to note is that an informative equilibrium exists even though the expert only cares for his reputation and does not have a genuine interest in the decision itself.

⁷Recall that I ignore 'mirror' equilibria. It is also easy to show that there are no other perverse equilibria.

The reason is that in an informative equilibrium, an expert is indeed rewarded with higher reputation for making the correct recommendation. Thus, he has an interest to make a good use of his private signal. This also implies that the higher is the prior probability q that B is the correct course of action, the more an expert votes for B , that is, for more of his types.

However, a second important point is that for all q , there is a strictly positive ex ante probability that an expert votes for A (as well as for B). In particular, even if the prior is heavily biased towards B , the expert is still recommending for A with a strictly positive probability (that is, for an interval of types which does not shrink to zero measure with q). The reason is, as the Proposition states, that the expert is rewarded - in reputation terms - when he contradicts the prior and recommends for A . This increases reputation since it ‘proves’ that the experts’ own private information is rather accurate, or at least, more accurate than the prior.

We can now put the experts together (note that all behave in the same way as characterized in Proposition 1) and register the implications of the above for the decision making by the committee.

Corollary 1 (*Group decision making with transparency*): (i) For any voting rule x , all experts recommend a if they believe that the probability that A is the right decision is above $v^*(q)$. (ii) For all q and x , each decision, A or B , is taken with an ex ante probability that is bounded away from zero. (iii) The decision $d=A$ is taken more often when x is smaller.

4 Secretive committees

We now consider the case in which the decision making process in the committee is non-transparent, that is, E does not observe the individual votes m_i for all i . But note that the evaluator does observe the decision of the committee, d . This decision allows the evaluator to extract some information about what possibly could individual experts say in the committee’s meeting. An obvious example is when the voting rule is *A-unanimity*; when three supportive votes are needed in order to accept A , then if indeed the committee’s decision is $d = A$, the evaluator knows with certainty that each expert voted for A .

More generally, in the secretive case, the reputation of the individual has a component of *group reputation*; this component is based on the decision of the committee, which is common for all experts. This implies that each expert becomes now - indirectly - interested

in the decision d . But since the committee's decision becomes important, then the voting rule comes into play as well. In other words, each expert's equilibrium behaviour would now depend on the voting rule x and on other experts' behaviour. Thus, as opposed to the transparent case, the committee's decision making process creates a strategic interaction between its members.

To further the analysis, I impose the following. First, as usual in strategic voting games, I focus only on the interesting equilibria in which experts do not use weakly dominated strategies. In other words, they vote as if they are pivotal. Second, since the experts are ex ante symmetric, I analyse the symmetric equilibrium in which all experts use the same strategy.

We can now analyse the equilibrium. As in the transparent case, each expert would use a unique cutoff point. Denote the cutoff point as a function of the voting rule by $v^x(q)$. Recall that we have defined $\tau_v(m_i, w_i)$; this is the reputation of expert i given a message m_i , the state w_i , and the conjecture of the evaluator about some cutoff v . This was a useful tool in the analysis of the transparent case, in which the evaluator observes m_i . But it is still useful - in the secretive case, the evaluator would simply form some beliefs regarding what expert i had recommended. In other words, given some group decision d , an expert knows that he is perceived to have voted for a with some probability (and hence have reputation of $\tau_v(a, w_i)$), and to have voted for b in the remaining probability (and hence have reputation of $\tau_v(b, w_i)$).

Let us then define these probabilities. Recall that only (d, w_i, x) are observed by the evaluator. Therefore, let $\alpha_v(d, w_i, x)$ denote the expected probability (from the point of view of expert i) that the evaluator would believe that $m_i = a$ for expert i , given the decision d , the state w_i , the voting rule x and some conjecture of a cutoff strategy v for all members. For example, $\alpha_v(A, a, x)$ is the probability with which expert i is perceived to have recommended a , given the decision $d = A$ and $w_i = a$. Similarly, $\alpha_v(B, a, x)$ is the probability with which expert i is perceived to have recommended a , given the decision $d = B$ and $w_i = a$, and so on.

Hence, the reputation of a committee member i , given some decision d and state w_i , can be expressed as:

$$\alpha_v(d, w_i, x)\tau_v(a, w_i) + (1 - \alpha_v(d, w_i, x))\tau_v(b, w_i)$$

The component of group reputation – the dependence of the individual reputation on the decision of the group d – is reflected in this formulation only through $\alpha_v(d, w_i, x)$. These α probabilities will then play an important role in the analysis; it is also easy to see how they depend on the voting rule. For example, $\alpha_v(A, w_i, 3) = 1$ and $\alpha_v(B, w_i, 1) = 0$.

Given the above, we can write the equilibrium condition for a voting rule x . An expert votes as if he is pivotal. The equilibrium condition has to equate the expected utility of expert i from $d = A$ and from $d = B$ precisely at the cutoff point $v^x(q)$. Recall also that $v^x(q)$ denotes the belief of the expert that the state is a . The equilibrium condition is then stated below.

$$\begin{aligned}
& v^x(q)(\alpha_{v^x}(A, a, x)\tau_{v^x}(a, a) + (1 - \alpha_{v^x}(A, a, x))\tau_{v^x}(b, a)) \\
& + (1 - v^x(q))(\alpha_{v^x}(A, b, x)\tau_{v^x}(a, b) + (1 - \alpha_{v^x}(A, b, x))\tau_{v^x}(b, b)) \\
= & v^x(q)(\alpha_{v^x}(B, a, x)\tau_{v^x}(a, a) + (1 - \alpha_{v^x}(B, a, x))\tau_{v^x}(b, a)) \\
& + (1 - v^x(q))(\alpha_{v^x}(B, b, x)\tau_{v^x}(a, b) + (1 - \alpha_{v^x}(B, b, x))\tau_{v^x}(b, b))
\end{aligned}$$

After re-arranging, this becomes:

$$\begin{aligned}
& v^x(q)(\alpha_{v^x}(A, a, x) - \alpha_{v^x}(B, a, x))(\tau_{v^x}(a, a) - \tau_{v^x}(b, a)) \\
= & (1 - v^x(q))(\alpha_{v^x}(A, b, x) - \alpha_{v^x}(B, b, x))(\tau_{v^x}(b, b) - \tau_{v^x}(a, b))
\end{aligned} \tag{3}$$

As an indication of what will follow, we can contrast the above equilibrium equation (3) with the equilibrium equation of the transparent case, (2). One can then easily see that the comparison between these cases would amount to an analysis of the values of the different α probabilities. These probabilities would therefore give rise to the group reputation effect, which, as we will show later on, induce experts to vote conservatively and exacerbate the biases of the voting rule or the prior beliefs.

But first, given (3), we can characterize the equilibrium behaviour of the experts in the secretive mechanism:

Proposition 2 *For any voting rule $x \in \{1, 2, 3\}$, there exists a unique symmetric equilibrium with a cutoff $v^x(q)$ such that each expert sends $m = a$ if $v(q, s, t) \geq v^x(q)$ and sends $m = b$ otherwise. For all q and x , the ex ante probability that any action A or B is taken, is bounded away from zero.*

The equilibrium in the secretive mechanism maintains therefore the same features of that of the transparent mechanism. The cutoff point, though, depends on the voting rule x ; by analysing the equilibrium characteristics we can therefore ask whether the cutoff point is higher - that is, experts vote more often for B - when the voting rule is biased against B or for B . In other words, whether experts counter or align their votes with the voting rule. Moreover, the Proposition indicates that it is easy to compare between the transparent and the secretive mechanism because they only differ in the value of the cutoff points. We can then ask how do the cutoff points $v^x(q)$ compare to the one of the transparent case, $v^*(q)$. We now proceed to answer these questions.

5 Transparency and Reputation

I will now show how secretive and transparent committees tend to take different decisions, and that in particular, secretive committees are actually more ‘conservative’. The main difference between the two committee types is the group reputation effect, which I now study. First, a useful Lemma which characterizes the probabilities as functions of the decision d and the state w_i .

5.1 A useful lemma

Below I show that expert i is perceived as more likely to have recommended a when the decision of the committee is indeed $d = A$, than when it is $d = B$, for any of his states w_i , and that given any decision of the committee, expert i is perceived as more likely to recommend a , when his state is $w_i = a$ than when his state is $w_i = b$.

Lemma 1 *For any cutoff point v and voting rule x : (i) $\alpha_v(A, a, x) \geq \alpha_v(B, a, x)$ and $\alpha_v(A, b, x) \geq \alpha_v(B, b, x)$; (ii) $\alpha_v(A, a, x) \geq \alpha_v(A, b, x)$ and $\alpha_v(B, a, x) \geq \alpha_v(B, b, x)$.*

Proof of Lemma 1:

To see how the α probabilities are constructed, let us consider an expert i and let $w = \{w_j, w_h\}$ for $j, h \neq i$ denote the vector of states of the world for the other experts, whereas Ω denotes all such possible states w . The distribution function over w is the joint distribution over the different $\{w_j, w_h\}$. The expert knows that w would be known to the evaluator at the time of his assessment, and hence for each state w , one can construct $\alpha_v^w(d, w_i, x)$. This is the probability with which the evaluator believes that $m_i = a$, given the state $w = \{w_j, w_h\}$ for experts j and h , the state w_i of expert i , the decision d , the

voting rule x , and when all experts use the same cutoff point, v .

When an individual changes the group's decision, i.e., when he is pivotal, he knows that there must be $x - 1$ votes of a , and $3 - (x - 1)$ votes of b , for $x \in \{1, 2, 3\}$. This pivotal event allows him to update his probability distribution over $w \in \Omega$, so that (where *piv* stands for pivotal):

$$\alpha_v(d, w_i, x) = \sum_{w \in \Omega} \Pr(w | \text{piv}, x, v) \alpha_v^w(d, w_i, x).$$

Note however that $\Pr(w | \text{piv}, x, v)$ does not depend on d or w_i . Thus, to prove the Lemma, it is sufficient to prove that for any $w \in \Omega$, $\alpha_v^w(A, a, x) \geq \alpha_v^w(B, a, x)$, $\alpha_v^w(A, b, x) \geq \alpha_v^w(B, b, x)$, $\alpha_v^w(A, a, x) \geq \alpha_v^w(A, b, x)$ and $\alpha_v^w(B, a, x) \geq \alpha_v^w(B, b, x)$.

Then, when the voting rule is x , we can compute the probability that the group takes the action A , given that expert i recommends a or b :

$$\Pr(d = A | m_i = a, x, w, v) = \sum_{l=x-1}^2 \Pr(\text{exactly } l \text{ experts recommend } a | w, v) \quad (4)$$

and

$$\Pr(d = A | m_i = b, x, w, v) = \sum_{l=x}^2 \Pr(\text{exactly } l \text{ experts recommend } a | w, v) \quad (5)$$

For brevity, let $d_{m_i} = \Pr(d = A | m_i, x, w, v)$ and $m_{w_i} = \Pr(m_i = a | w_i, v)$. Then, by Bayesian updating:

$$\begin{aligned} \alpha_v^w(A, w_i, x) &= \frac{m_{w_i} d_a}{m_{w_i} d_a + (1 - m_{w_i}) d_b} \\ \alpha_v^w(B, w_i, x) &= \frac{m_{w_i} (1 - d_a)}{m_{w_i} (1 - d_a) + (1 - m_{w_i}) (1 - d_b)}. \end{aligned} \quad (6)$$

However, it is easy to see from (4) and (5) that $d_a > d_b$. This, with (6), immediately implies that $\alpha_v^w(A, a, x) \geq \alpha_v^w(B, a, x)$ and $\alpha_v^w(A, b, x) \geq \alpha_v^w(B, b, x)$. Moreover, it is clear that $m_a > m_b$; given a cutoff point strategy, an expert is more likely to vote a when $w_i = a$ than when $w_i = b$. From (6), it then immediately follows that $\alpha_v^w(B, a, x) \geq \alpha_v^w(B, b, x)$ and $\alpha_v^w(A, a, x) \geq \alpha_v^w(A, b, x)$. \square

5.2 The Group Reputation effect

Using Lemma 1, we can now analyse how decision making in committees changes when the deliberation process becomes transparent. Do experts support an action more or less often

when their recommendations are public compared to when they remain a secret?

Let us start with the *A-unanimity* rule, $x = 3$. In this case, all experts need to support A for it to be accepted. When $d = A$, it is easy to see that the secretive mechanism becomes ‘transparent’ since it is obvious that all need to vote for A for it to be accepted. On the other hand, when the committee decides for B , then there is still uncertainty regarding individual votes in the secretive mechanism. In particular, it is still quite likely that an individual has voted for A even if the decision is B .

Such uncertainty is good for the less able types who don’t have much information and are therefore more worried about being perceived as making the wrong recommendation and acquiring bad reputation as a result. For these types the uncertainty about their votes associated with a group decision of B allows them to shed the blame on others for a wrong decision. In particular, if $d = B$ but $w_i = a$, expert i cannot be fully blamed for the ‘wrong’ decision since actually it is quite likely that expert i made the correct recommendation, i.e., supporting A when $w_i = a$. This possibility, of taking the blame off one’s shoulders, cannot happen if the decision is A , or in the transparent case, when no such uncertainty arises.

Thus, relative to the transparent mechanism, an expert is perceived as more likely to make the correct recommendation - that is, recommendation a in state a - when the decision is B rather than A . This means that more types are inclined to vote for B when the process is secretive and $x = 3$, so that $v^*(q) < v^3(q)$.

To see the formal argument, note that for all v , when $x = 3$, the following holds:

$$\alpha_v(A, a, x) - \alpha_v(B, a, x) < \alpha_v(A, b, x) - \alpha_v(B, b, x)$$

because $\alpha_v(A, a, x) = \alpha_v(A, b, x) = 1$ for $x = 3$ and by Lemma 1, $\alpha_v(B, b, x) < \alpha_v(B, a, x)$ for all x . Also, the equilibrium condition for the transparent case is:

$$v^*(q)(\tau_{v^*}(a, a) - \tau_{v^*}(b, a)) = (1 - v^*(q))(\tau_{v^*}(b, b) - \tau_{v^*}(a, b))$$

Thus, when $x = 3$:

$$\begin{aligned} & v^*(q)(\alpha_{v^*}(A, a, x) - \alpha_{v^*}(B, a, x))(\tau_{v^*}(a, a) - \tau_{v^*}(b, a)) \\ & < (1 - v^*(q))(\alpha_{v^*}(A, b, x) - \alpha_{v^*}(B, b, x))(\tau_{v^*}(b, b) - \tau_{v^*}(a, b)) \end{aligned}$$

In other words, at the cutoff point $v^*(q)$, the expected utility from $d = B$ is higher than that from $d = A$ when $x = 3$. This along with the equilibrium uniqueness implies

that experts will tend to support B more often in the secretive mechanism than in the transparent mechanism when $x = 3$, so that $v^*(q) < v^3(q)$.

The analogous analysis holds for $x = 1$, the rule of B -*unanimity*. In such a case, the voting rule is biased towards A so that experts are induced to vote more often for A , so that $v^1(q) < v^*(q)$. We can then already conclude that a switch to a transparent process will have consequences on decision making which would depend on the voting rule used by the committee.

But now it also becomes clear that the more a voting rule is biased against an action, the more often an individual votes against this action in the secretive process, i.e., that $v^1(q) < v^2(q) < v^3(q)$ for any q . The higher is x , the more uncertainty there is about individual votes when the decision is B . Thus, individual experts will be considered as more likely to make the correct recommendation when the committee decides for B . Thus, more types will indeed vote for B the higher is x . Similar to our analysis above, one can look at the ratio

$$\frac{\alpha_v(A, a, x) - \alpha_v(B, a, x)}{\alpha_v(A, b, x) - \alpha_v(B, b, x)}$$

which represents the relative likelihood of being perceived as making the correct recommendation (that is, a in state a relative to a in state b), when the group decision shifts from B to A . This ratio decreases in x as the appendix shows. This implies that individuals' gain from shifting the decision from B to A becomes lower with x or in other words, that they essentially prefer to vote for B when x is higher.

Thus, the *group reputation effect* encourages experts to align their votes with the voting rule; the more supportive votes the voting rule demands in order to pick an action, the less often each committee members votes for this action. Obviously, with transparency, this effect does not arise since no one can blame others for making the wrong recommendation. Thus, the effect of procedural biases, such as voting rules which demand supermajority to accept a particular action, is weakened when the process becomes transparent.

But not all voting rules are biased - majority rule, in particular, is not biased towards any decision (two votes in favour of any are enough to implement it). How does the committee changes its decision if the process becomes transparent when the voting rule is majority? This is where the prior q finally plays a role in the analysis. Let $x = 2$, and think first of $q = \frac{1}{2}$. In this case, given the fully symmetric model, the evaluator's possible assessments are the same when $d = A$ and when $d = B$. Since there is no differential learning from

the group’s decision, the model becomes analogous to the transparent one and hence when $q = \frac{1}{2}$, then $v^*(q) = v^2(q)$.

However, when the prior q is high enough and points towards B as the likely state, this ‘breaks’ the symmetry. In particular, since B is more likely to be correct, it is more likely that experts vote for B , simply because their signals and their messages are informative. But then if a group decision of B proved to be the wrong decision from the point of view of expert i , i.e., when $w_i = a$, then this expert can blame others for voting for B (a likely possibility) and ‘claim’ that he actually voted for A (also likely given that $w_i = a$). He can therefore boost his own reputation as the one who made the correct recommendation when he votes for B . This induces agents to recommend B more often in the secret mechanism, so that $v^*(q) < v^2(q)$. Note that this effect is over and above the obvious tendency of experts to vote more often for B when the prior supports B . Such a ‘herding’ effect exists also in the transparent case. But in the secretive case, experts tend to follow the prior even more because of strategic group reputation considerations.

We therefore establish that:

Proposition 3 (i) $v^1(q) < v^2(q) < v^3(q)$; (ii) $v^1(q) < v^*(q) < v^3(q)$, and for high enough values of q , also $v^*(q) < v^2(q)$.⁸

More generally, we can think of both the prior q and the voting rule x as (somewhat interchangeable) sources of *bias* in the non transparent case. The voting rule is a procedural bias. For example, consider a constitutional law which requires that any change to an existing law can be approved only if a supermajority of votes are in favour. This can lead to a status quo bias. Its effect however, is diminished when discussions are transparent so that committees become more likely to accept reforms. The prior creates another bias, of beliefs. Once the committee’s deliberations are transparent however, they would tend to decide more often against the popular views or public opinions. Corollary 2 summarizes the main results of the paper.

Corollary 2 (*Group reputation, voting rules and transparency*):

(i) *When the mechanism changes from secretive to transparent, then the committee accepts more often the action that the voting rule is biased against (demands larger support*

⁸ Analytically I can show that $v^*(q) < v^2(q)$ only for high values of q but I have also checked numerically that $v^*(q) < v^2(q)$ for all other values of q .

in order to approve it).

(ii) When the voting rule is majority rule (i.e., a non-biased rule), then when the mechanism changes from secretive to transparent, the committee accepts more often the action that goes against the initial prior belief when the prior is sufficiently high.

(iii) Under both mechanisms, the transparent and the secretive, an action is accepted more often when the voting rule demands a weaker support to approve it. However, when there is a change in the voting rule, it has a larger effect when the mechanism is secretive than when it is transparent.

As a final comment, note that the analysis in this paper is a positive analysis. Obviously, we can use its findings to ask normative questions. For example, given some preference ordering (which presumably weighs the different dimensions and criteria), one can find what is the optimal voting rule and, whether the committee's decision making process should be transparent or not.

6 Discussion

I conclude by discussing some extensions of the model, to show its robustness as well as to point at avenues for future research.

6.1 Correlated information

In the model, the members of the committee have information which is uncorrelated. This assumption allows us to distinguish the effect of group reputation from another effect that exists in group decision making, namely the 'pivotal effect'. Recall that each expert makes a recommendation conditional on being pivotal. When the experts' information is correlated, each expert learns additional information about the state of the world given the event that he is pivotal. This information may induce the expert to bias his recommendations.

The pivotal effect works in the opposite direction compared with the group reputation effect. In particular, when the voting rule demands a larger consensus in order to implement some action, the pivotal effect actually encourages experts to vote for this action. To see this, consider the *A-unanimity* rule ($x = 3$). When an expert is pivotal, he learns that the two other experts have voted a . Since their information is correlated, the expert believes that the state a is more likely. But to enhance reputation, recommendations have to be correct. Thus, the expert is indeed encouraged to vote a (whereas the group reputation

effect would encourage individuals to vote for b in this case).⁹

In a general model which allows for correlation of information, the results described in this paper are robust even with the additional pivotal effect. Consider for example a model in which with some probability the dimensions on which the experts glean information about are fully correlated, whereas with the remaining probability the states of the world on these dimensions are uncorrelated. It is then easy to see that the results are maintained as long as the degree or probability of correlation is not too high.

6.2 Decision concerns

Experts may be genuinely concerned for the committee to take the ‘right’ decision. Since there is no normative analysis in this paper, the easiest way to think of decision concerns is to assume that each expert would benefit when the decision of the committee accords with the true state of the world on the dimension he is responsible to. Thus, each expert, on top of his utility from reputation, could receive some utils (in an additive manner) if his state of the world identifies with the group decision.

When experts vote taking into consideration both their career concerns and their decision concerns, they do so on the basis of being pivotal. Thus, decision concerns would induce experts to recommend what they believe is the right decision. This implies that both for the transparent and the secretive mechanism, and for all voting rules, the cutoff point would move towards the ‘efficient’ one. It is however evident that this additional element of decision concerns affects all voting rules in a similar way and all the results would be maintained for small enough degree of decision concerns.

6.3 Large committees

I have analysed the smallest possible committee of three members. The intuition provided in the analysis is general and should extend for larger committees. However, it is probably the case that individuals serving in very large decision bodies do not have career concerns. The analysis in this paper is therefore more suitable to investigation of small decision making bodies, such as monetary policy committees or governments.

⁹In related models, this effect arises when agents care for the decision per se and try to attain the right decision (see Austen-Smith and Banks (1996) or Federssen and Pesendorfer (1998)). In my model it arises because career concerns also induce agents to try and make the correct decision since this results in higher reputation.

However, the model analysed here can also shed some light as to why career concerns may diminish in large committees. To see this, think of agents who also have decision concerns, and suppose that the voting rule is majority. In such a case, when the committee becomes very large, then it becomes harder for the evaluator to learn about each individual recommendation from the committee's decision. In other words, the evaluator does not learn any different information from the decision being A or B , since any particular individual was not likely to affect it. Individuals would then know that their vote does not affect their reputation. As a result, career concerns would diminish relative to decision concerns when the committee grows larger.

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Appendix

A. Notations and definitions.

Recall that $v(q) = v(q, s, t)$ and is derived in the following way:

$$v(q) = \begin{cases} v(q, a, t) = \frac{(1-q)t}{(1-q)t+q(1-t)} & \text{when } s = a, \\ v(q, b, t) = \frac{(1-q)(1-t)}{(1-q)(1-t)+qt} & \text{when } s = b. \end{cases} \quad (7)$$

Given (7), I sometimes refer to t as the generic cutoff point and sometimes to v , depending on convenience and when no confusion occurs. Let v^* or $v^*(q)$ denote the equilibrium cutoff point in the transparent case and let v^x or $v^x(q)$ denote the equilibrium cutoff point in the non-transparent case for $x \in \{1, 2, 3\}$. Given a voting rule x and some cutoff point v , let

$$\Gamma(t; q, x) = \frac{\alpha_v(A, b, x) - \alpha_v(B, b, x)}{\alpha_v(A, a, x) - \alpha_v(B, a, x)}$$

B. Proof of Proposition 1, Proposition 2 and Proposition 3.

Proof of Proposition 1: See Levy (2004). ■

Proof of Proposition 2: Recall that the equilibrium condition is:

$$\begin{aligned} & v^x(q)(\alpha_{v^x}(A, a, x) - \alpha_{v^x}(B, a, x))(\tau_{v^x}(a, a) - \tau_{v^x}(b, a)) \\ &= (1 - v^x(q))(\alpha_{v^x}(A, b, x) - \alpha_{v^x}(B, b, x))(\tau_{v^x}(b, b) - \tau_{v^x}(a, b)) \end{aligned}$$

By Lemma 1, $\alpha_{v^x}(A, a, x) - \alpha_{v^x}(B, a, x) > 0$ and $\alpha_{v^x}(A, b, x) - \alpha_{v^x}(B, b, x) > 0$ for all q and x . Then the equilibrium existence follows from the same arguments as in the transparent case. The fact that the equilibrium cutoff point is such that the probability that each expert to recommend the project is bounded away from zero, follows from the same argument as in the transparent case. This also implies that the probability that the committee accepts the project is bounded away from zero. We also have more information about the cutoff points, in particular for the case of $x = 3$; the cutoff point must admit $s^x = a$. To see this, note that the equilibrium cutoff $v^3(q)$ satisfies:

$$\frac{v^3(q)(\tau_{v^3}(a, a) - \tau_{v^3}(b, a))}{(1 - v^3(q))(\tau_{v^3}(a, b) - \tau_{v^3}(b, b))} = \frac{\alpha_{v^3}(a, b) - \alpha_{v^3}(b, b)}{\alpha_{v^3}(a, a) - \alpha_{v^3}(b, a)} \quad (8)$$

At $s = b$, the left-hand-side is smaller than 1. For $x = 3$, by Lemma 1 and the fact that $\alpha_v(A, b, x) = \alpha_v(A, a, x) = 1$, the right-hand-side is larger than 1 for all $s \in \{a, b\}$. Hence, it must be that the cutoff point satisfies $v^x(q) = v(q, s^x, t^x)$ is such that $s^x = a$ when $x = 3$.

It is left to show that the equilibrium cutoff point is unique. Since the cutoff point for the transparent case is unique, to show that the cutoff point for the non transparent case is unique it is enough to show that for all x , $\frac{\alpha_v(A, b, x) - \alpha_v(B, b, x)}{\alpha_v(A, a, x) - \alpha_v(B, a, x)}$ decreases in v :

Lemma A1. $\frac{\alpha_v(A, b, x) - \alpha_v(B, b, x)}{\alpha_v(A, a, x) - \alpha_v(B, a, x)}$ decreases in v .

Proof: see section C.

This concludes the proof of Proposition 2. ■

Proof of Proposition 3:

At the cutoff of an equilibrium, $v^x(q)$ or $v^*(q)$, an expert is indifferent between recommending a or b (i.e., between $d = A$ and $d = B$ since he is pivotal), whereas for all higher v 's, the experts prefer to induce a . Thus, to show for example that $v^1(q) < v^2(q)$, given the uniqueness of equilibrium, it is enough to show that when $x = 2$, at the cutoff point $v^1(q)$, an expert prefers to recommend b , and hence $v^2(q) > v^1(q)$. In other words, when $x = 2$, if the evaluator believes that all expert use $v^1(q)$ and that the other experts apart from i use $v^1(q)$, then the utility of expert i at $v^1(q)$ is strictly higher from recommending b . I use this feature in the proofs of parts (i) and (ii) below.

Proof of part (i): Note that in equilibrium, the only element that depends on the voting rule x is $\alpha_v(d, w_i, x)$. To prove this part of the proposition, it is therefore useful to show the following:

Lemma A2. $\frac{\alpha_v(A, b, x) - \alpha_v(B, b, x)}{\alpha_v(A, a, x) - \alpha_v(B, a, x)}$ increases in x .

Proof: see section C.

Then, consider the cutoff point $v^2(q)$. I then have to show that at $x = 3$,

$$\begin{aligned} & v^2(q)(\alpha_{v,2}(A, a, 3) - \alpha_{v,2}(B, a, 3))(\tau_{v,2}(a, a) - \tau_{v,2}(b, a)) \\ & < (1 - v^2(q))(\alpha_{v,2}(A, b, 3) - \alpha_{v,2}(B, b, 3))(\tau_{v,2}(b, b) - \tau_{v,2}(a, b)). \end{aligned}$$

At $v^2(q)$, when $x = 2$, the above holds with equality (recall that only the are a function of x). However, since by Lemma A2, $\frac{\alpha_v(A, b, x) - \alpha_v(B, b, x)}{\alpha_v(A, a, x) - \alpha_v(B, a, x)}$ increases in x , the above holds. Thus,

$v^2(q) < v^3(q)$. Finally, for $x = 1$, I have to show that

$$\begin{aligned} & v^2(q)(\alpha_{v^2}(A, a, 1) - \alpha_{v^2}(B, a, 1))(\tau_{v^2}(a, a) - \tau_{v^2}(b, a)) \\ & > (1 - v^2(q))(\alpha_{v^2}(A, b, 1) - \alpha_{v^2}(B, b, 1))(\tau_{v^2}(b, b) - \tau_{v^2}(a, b)). \end{aligned}$$

but using the equilibrium condition at $x = 2$ and Lemma A2, this holds and therefore $v^1(q) < v^2(q)$.

Proof of part (ii): The case of $v^*(q) < v^3(q)$ is in the text. I now have to show that $v^*(q) < v^2(q)$ for high enough q . It is enough to show that at the cutoff point v^* , then at $x = 2$, the utility from $d = B$ is higher, i.e.:

$$\begin{aligned} & v^*(q)(\alpha_{v^*}(A, a, 2) - \alpha_{v^*}(B, a, 2))(\tau_{v^*}(a, a) - \tau_{v^*}(b, a)) \\ & < (1 - v^*(q))(\alpha_{v^*}(A, b, 2) - \alpha_{v^*}(B, b, 2))(\tau_{v^*}(b, b) - \tau_{v^*}(a, b)) \end{aligned}$$

for large enough q . However, given the equilibrium condition (2), it is therefore enough to show that

$$\frac{\alpha_{v^*}(A, b, 2) - \alpha_{v^*}(B, b, 2)}{\alpha_{v^*}(A, a, 2) - \alpha_{v^*}(B, a, 2)} > 1$$

for large enough q . Consider $q = 1$ and the expression for $\Gamma(t; q, 2)$ (see the proof of step 1 in Lemma A2 below). Then for all t , the above amounts to:

$$\Gamma(t; 1, 2) = \frac{m_b(1-m_b)(m_a(1-m_b)^2+(1-m_a)(1-m_b^2))(1-(m_a(1-m_b)^2+(1-m_a)(1-m_b^2)))}{m_a(1-m_a)(m_b(1-m_b)^2+(1-m_b)(1-m_b^2))(1-(m_b(1-m_b)^2+(1-m_b)(1-m_b^2)))}.$$

Given that $s = a$ when $x = 2$, then $m_a = 1 - t^2$ and $m_b = (1 - t)^2$. Moreover, the type t at this cutoff is bounded for all q because the probability that each decision is accepted is bounded away from zero. Denote this bound by \hat{t} . The bound is easy to compute and thus it is left to check that for all $t \leq \hat{t}$, $\Gamma(t; 1, 2) > 1$ which indeed holds. The case for $v^1(q) < v^*(q)$ is as follows. I have to show that at $v^*(q)$,

$$\begin{aligned} & v^*(q)(\alpha_{v^*}(A, a, 1) - \alpha_{v^*}(B, a, 1))(\tau_{v^*}(a, a) - \tau_{v^*}(b, a)) \\ & > (1 - v^*(q))(\alpha_{v^*}(A, b, 1) - \alpha_{v^*}(B, b, 1))(\tau_{v^*}(b, b) - \tau_{v^*}(a, b)) \end{aligned}$$

Plugging (2), noting that $\alpha_{v^*}(B, a, 1) = \alpha_{v^*}(B, b, 1) = 0$ (because if an expert recommends a it cannot be that $d = B$), and using Lemma 1 which states that $\alpha_{v^*}(A, a, x) > \alpha_{v^*}(A, b, x)$, establishes the above so that $v^1(q) < v^*(q)$. This completes the proof of part (ii).

This completes the proof of Proposition 3. ■

C. Proofs of Lemma A1 and Lemma A2.

Proof of Lemma A1:

In particular, I will show that when $s = a$, then $\Gamma(t; q, x)$ decreases in t and that when $s = b$, then $\Gamma(t; q, x)$ increases in t , which proves the Lemma. Let us consider the case for $x = 3$. Then, since $\alpha_v(A, b, 3) = \alpha_v(A, a, 3) = 1$, the expression $\frac{\alpha_v(A, b, 3) - \alpha_v(B, b, 3)}{\alpha_v(A, a, 3) - \alpha_v(B, a, 3)}$ becomes:

$$\frac{1 - \alpha_{v3}(B, b, 3)}{1 - \alpha_{v3}(B, a, 3)} = \frac{\sum_{w \in \Omega} \Pr(w) \Pr(piv|w, 3, v)(1 - \alpha_v^w(B, b, 3))}{\sum_{w \in \Omega} \Pr(w) \Pr(piv|w, 3, v)(1 - \alpha_v^w(B, a, 3))}, \quad (9)$$

where Ω is the set of possible states for the other experts. Let these possible states be denoted by $w_1 = \{b, b\}$, $w_2 = \{b, a\}$, $w_3 = \{a, b\}$ and $w_4 = \{a, a\}$. Note that $\Pr(w_1) = q^2$, $\Pr(w_2) = \Pr(w_3) = q(1 - q)$ and $\Pr(w_4) = (1 - q)^2$. Also, w_2 and w_3 are strategically equivalent given the symmetry of the experts and I therefore treat them as one state, w_2 , and assume that this state occurs with probability $2q(1 - q)$.

Recall that $m_a = \Pr(m_i = a|w_i = a, v)$ and $m_b = \Pr(m_i = a|w_i = b, v)$ for all experts i . Given that $s^x = a$ at $x = 3$, this implies that

$$m_a = \int_t^1 2z dz = 1 - t^2;$$

$$m_b = \int_t^1 2(1 - z) dz = (1 - t)^2$$

for some cutoff $t \in [.5, 1]$. Note that

$$m_a > m_b.$$

Recall also that $d_a = \Pr(d = a|m_i = a, x, w, v)$ and $d_b = \Pr(d = a|m_i = b, x, w, v)$. Also, from (6) in the text,

$$\alpha_v^w(b, w_i, x) = \frac{m_{w_i}(1 - d_a)}{m_{w_i}(1 - d_a) + (1 - m_{w_i})(1 - d_b)}.$$

This allows us to derive the expressions for the relevant w_i 's :

$$\Pr(piv|w_1, 3, v^3) = m_b^2; \Pr(piv|w_2, 3, v^3) = m_b m_a; \Pr(piv|w_4, 3, v^3) = m_a^2; \quad (10)$$

$$\alpha_v^{w_1}(B, b, 3) = \frac{m_b(1 - m_b^2)}{m_b(1 - m_b^2) + (1 - m_b)}; \alpha_v^{w_1}(B, a, 3) = \frac{m_a(1 - m_b^2)}{m_a(1 - m_b^2) + (1 - m_a)};$$

$$\alpha_v^{w_2}(B, b, 3) = \frac{m_b(1 - m_b m_a)}{m_b(1 - m_b m_a) + (1 - m_b)}; \alpha_v^{w_2}(B, a, 3) = \frac{m_a(1 - m_b m_a)}{m_a(1 - m_b m_a) + (1 - m_a)};$$

$$\alpha_v^{w_3}(B, b, 3) = \frac{m_b(1 - m_a^2)}{m_b(1 - m_a^2) + (1 - m_b)}; \alpha_v^{w_3}(B, a, 3) = \frac{m_a(1 - m_a^2)}{m_a(1 - m_a^2) + (1 - m_a)}.$$

And therefore, when $x = 3$ then $\Gamma(t; q, 3)$ becomes:

$$\frac{1 - \alpha_v(B, b, 3)}{1 - \alpha_v(B, a, 3)} = \frac{\frac{q^2 m_b^2 (1 - m_b)}{1 - m_b^3} + \frac{2q(1-q)m_b m_a (1 - m_b)}{1 - m_b^2 m_a} + \frac{(1-q)^2 m_a^2 (1 - m_b)}{1 - m_b m_a^2}}{\frac{q^2 m_b^2 (1 - m_a)}{1 - m_b^2 m_a} + \frac{2q(1-q)m_b m_a (1 - m_a)}{1 - m_b m_a^2} + \frac{(1-q)^2 m_a^2 (1 - m_a)}{1 - m_a^3}} \quad (11)$$

An important result is that for any $i > j$, it holds that

$$\frac{1 - \alpha_v^{w_i}(B, b, 3)}{1 - \alpha_v^{w_i}(B, a, 3)} < \frac{1 - \alpha_v^{w_j}(B, b, 3)}{1 - \alpha_v^{w_j}(B, a, 3)} \quad (12)$$

To see why it is true, consider for example $i = 2$ and $j = 1$, then:

$$\frac{\frac{(1-m_b)}{1-m_b^2 m_a}}{\frac{(1-m_a)}{1-m_b m_a^2}} < \frac{\frac{(1-m_b)}{1-m_b^3}}{\frac{(1-m_a)}{1-m_b^2 m_a}} \Leftrightarrow \frac{1 - m_b m_a^2}{1 - m_b^2 m_a} < \frac{1 - m_b^2 m_a}{1 - m_b^3} \Leftrightarrow 0 < (m_a - m_b)^2.$$

Given that $s^x = a$ for $x = 3$, I therefore need to show that $\Gamma(t; q, 3)$ decreases in t . To show this, I take the derivative of (9) with respect to t . Its sign has the sign of

$$\frac{\sum_{w \in \Omega} \Pr(w) \frac{\partial}{\partial t} \Pr(\text{piv}|w, 3, v)(1 - \alpha_v^w(B, b, 3))}{\sum_{w \in \Omega} \Pr(w) \Pr(\text{piv}|w, 3, v)(1 - \alpha_v^w(B, b, 3))} - \frac{\sum_{w \in \Omega} \Pr(w) \frac{\partial}{\partial t} \Pr(\text{piv}|w, 3, v)(1 - \alpha_v^w(B, a, 3))}{\sum_{w \in \Omega} \Pr(w) \Pr(\text{piv}|w, 3, v)(1 - \alpha_v^w(B, a, 3))} \quad (13)$$

To prove the Lemma, I will show that the sign of (13) is negative. After re-arranging, it is enough to show that for any i and j , for $i, j \in \{1, 2, 4\}$:

$$(1 - \alpha_v^{w_j}(B, a, 3)) \frac{\partial \Pr(\text{piv}|w_i, 3, v)(1 - \alpha_v^{w_i}(B, b, 3))}{\partial t} < (1 - \alpha_v^{w_j}(B, b, 3)) \frac{\partial \Pr(\text{piv}|w_i, 3, v)(1 - \alpha_v^{w_i}(B, a, 3))}{\partial t} \quad (14)$$

To see why (14) is true, I derive $\frac{\partial \Pr(\text{piv}|w_i, 3, v)(1 - \alpha_v^{w_i}(B, b, 3))}{\partial t}$ and $\frac{\partial \Pr(\text{piv}|w_i, 3, v)(1 - \alpha_v^{w_i}(B, a, 3))}{\partial t}$ for all i . Given (11), I find the derivatives of the different expressions:

$$\begin{aligned} \frac{\partial \Pr(\text{piv}|w_1, 3, v)(1 - \alpha_v^{w_1}(B, b, 3))}{\partial t} &= -2(1-t) \frac{2m_b - 3m_b^2 + m_b^4}{(1-m_b^3)^2}; \\ \frac{\partial \Pr(\text{piv}|w_1, 3, v)(1 - \alpha_v^{w_1}(B, a, 3))}{\partial t} &= -2(1-t) \frac{2m_b - 2m_b m_a}{(1-m_b^2 m_a)^2} - 2t \frac{m_b^4 - m_b^2}{(1-m_b^2 m_a)^2}; \\ \frac{\partial \Pr(\text{piv}|w_2, 3, v)(1 - \alpha_v^{w_2}(B, b, 3))}{\partial t} &= -2(1-t) \frac{m_a - 2m_b m_a + m_b^2 m_a^2}{(1-m_b^2 m_a)^2} - 2t \frac{m_b - m_b^2}{(1-m_b^2 m_a)^2}; \\ \frac{\partial \Pr(\text{piv}|w_2, 3, v)(1 - \alpha_v^{w_2}(B, a, 3))}{\partial t} &= -2(1-t) \frac{m_a - m_a^2}{(1-m_b m_a^2)^2} - 2t \frac{m_b - 2m_b m_a + m_b^2 m_a^2}{(1-m_b m_a^2)^2}; \\ \frac{\partial \Pr(\text{piv}|w_3, 3, v)(1 - \alpha_v^{w_3}(B, b, 3))}{\partial t} &= -2(1-t) \frac{m_a^4 - m_a^2}{(1-m_b m_a^2)^2} - 2t \frac{2m_a - 2m_b m_a}{(1-m_b m_a^2)^2}; \\ \frac{\partial \Pr(\text{piv}|w_3, 3, v)(1 - \alpha_v^{w_3}(B, a, 3))}{\partial t} &= -2t \frac{2m_a - 3m_a^2 + m_a^4}{(1-m_a^3)^2} \end{aligned}$$

Thus, for all $i \in \{1, 2, 4\}$:

$$\frac{\partial \Pr(\text{piv}|w_i, 3, v)(1 - \alpha_v^{w_i}(B, b, 3))}{\partial t} < 0$$

But given (12), to show (14), it is enough to show that for all $i \in \{1, 2, 4\}$:

$$\frac{\left| \frac{\partial \Pr(\text{piv}|w_i, 3, v)(1 - \alpha_v^{w_i}(B, b, 3))}{\partial t} \right|}{\left| \frac{\partial \Pr(\text{piv}|w_i, 3, v)(1 - \alpha_v^{w_i}(B, a, 3))}{\partial t} \right|} > \frac{1 - \alpha_v^{w_1}(B, b, 3)}{1 - \alpha_v^{w_1}(B, a, 3)} = \frac{(1 - m_b)}{1 - m_b^3} = \frac{(1 - m_a)}{1 - m_b^2 m_a} \quad (15)$$

Then:

$$\begin{aligned} \frac{|\Pr(\text{piv}|w_1, 3, v)(1 - \alpha_v^{w_1}(B, b, 3))|}{|\Pr(\text{piv}|w_1, 3, v)(1 - \alpha_v^{w_1}(B, a, 3))|} &= \frac{2(1-t) \frac{2m_b - 3m_b^2 + m_b^4}{(1 - m_b^3)^2}}{2(1-t) \frac{2m_b - 2m_b m_a}{(1 - m_b^2 m_a)^2} + 2t \frac{m_b^4 - m_b^2}{(1 - m_b^2 m_a)^2}}; \\ \frac{|\Pr(\text{piv}|w_2, 3, v)(1 - \alpha_v^{w_2}(B, b, 3))|}{|\Pr(\text{piv}|w_2, 3, v)(1 - \alpha_v^{w_2}(B, a, 3))|} &= \frac{2(1-t) \frac{m_a - 2m_b m_a + m_b^2 m_a^2}{(1 - m_b^2 m_a)^2} + 2t \frac{m_b - m_b^2}{(1 - m_b^2 m_a)^2}}{2(1-t) \frac{m_a - m_a^2}{(1 - m_b m_a^2)^2} + 2t \frac{m_b - 2m_b m_a + m_b^2 m_a^2}{(1 - m_b m_a^2)^2}}; \\ \frac{|\Pr(\text{piv}|w_3, 3, v)(1 - \alpha_v^{w_3}(B, b, 3))|}{|\Pr(\text{piv}|w_3, 3, v)(1 - \alpha_v^{w_3}(B, a, 3))|} &= \frac{2(1-t) \frac{m_a^4 - m_a^2}{(1 - m_b m_a^2)^2} + 2t \frac{2m_a - 2m_b m_a}{(1 - m_b m_a^2)^2}}{2t \frac{2m_a - 3m_a^2 + m_a^4}{(1 - m_a^3)^2}}. \end{aligned}$$

It is easy to see that $\frac{|\Pr(\text{piv}|w_2, 3, v)(1 - \alpha_v^{w_2}(B, b, 3))|}{|\Pr(\text{piv}|w_2, 3, v)(1 - \alpha_v^{w_2}(B, a, 3))|} > \frac{|\Pr(\text{piv}|w_1, 3, v)(1 - \alpha_v^{w_1}(B, b, 3))|}{|\Pr(\text{piv}|w_1, 3, v)(1 - \alpha_v^{w_1}(B, a, 3))|}$ and therefore to show (15), it is left to show the following two inequalities:

$$\frac{2(1-t) \frac{m_a^4 - m_a^2}{(1 - m_b m_a^2)^2} + 2t \frac{2m_a - 2m_b m_a}{(1 - m_b m_a^2)^2}}{2t \frac{2m_a - 3m_a^2 + m_a^4}{(1 - m_a^3)^2}} > \frac{(1 - m_b)}{1 - m_b^3} \quad \text{and} \quad \frac{2(1-t) \frac{2m_b - 3m_b^2 + m_b^4}{(1 - m_b^3)^2}}{2(1-t) \frac{2m_b - 2m_b m_a}{(1 - m_b^2 m_a)^2} + 2t \frac{m_b^4 - m_b^2}{(1 - m_b^2 m_a)^2}} > \frac{(1 - m_b)}{1 - m_b^3}.$$

Given that $m_a = 1 - t^2$ and $m_b = (1 - t)^2$, these are functions of $t \in [.5, 1]$ only and a routine calculation shows that these are indeed positive.¹⁰ This proves that $\Gamma(t; q, 3)$ decreases in t for $x = 3$ or that $\frac{\alpha_v(A, b, 3) - \alpha_v(B, b, 3)}{\alpha_v(A, a, 3) - \alpha_v(B, a, 3)}$ decreases in v . The analogous results for $x = 1$ and $x = 2$ are derived in a similar way and omitted for brevity.

This concludes the proof of Lemma A1. ■

Proof of Lemma A2:

I will prove a restricted version of the Lemma which is enough for our purpose; in particular, I will show that $\Gamma(t; q, x)$ increases in x when the cutoff point satisfies $s = a$. In Proposition 2, I showed that the cutoff point when $x = 3$ must satisfy $s = a$. Below I will show that the cutoff point when $x = 2$ must satisfy $s = a$. Thus, for the purpose of Proposition 3, it is sufficient to prove that $\Gamma(t; q, x)$ increases in x when the cutoff point satisfies $s = a$; it proves that $v^1 < v^2 < v^3$ if all cutoff points satisfy $s = a$, and if v^1 is such that $s = b$, then moreover $v^1 < v^2$ since v^2 is such that $s = a$.

¹⁰For example, the second expression becomes $-(t^2 - 2t + 3) \frac{(-5t + 4 + 5t^3 - 4t^4 + t^5)^2}{(2 - 10t + 10t^2 - 5t^3 + t^4)(t^4 - 4t^3 + 7t^2 - 6t + 3)^2} - \frac{-5t + 4 + 5t^3 - 4t^4 + t^5}{t(t^4 - 4t^3 + 7t^2 - 6t + 3)} \iff \frac{t(-t^2 + 2t - 3)(-5t + 4 + 5t^3 - 4t^4 + t^5)}{(2 - 10t + 10t^2 - 5t^3 + t^4)(t^4 - 4t^3 + 7t^2 - 6t + 3)} > 1$ which is easy to establish.

Step 1: When $t = q = \frac{1}{2}$ and $x = 2$, then $\Gamma(\frac{1}{2}; \frac{1}{2}, 2) = 1$.

Proof of step 1: Consider

$$\frac{\alpha_v(A, b, x) - \alpha_v(B, b, x)}{\alpha_v(A, a, x) - \alpha_v(B, a, x)} = \frac{\sum_{w \in \Omega} \Pr(w) \Pr(\text{piv}|w, x, v)(\alpha_v^w(A, b, x) - \alpha_v^w(B, b, x))}{\sum_{w \in \Omega} \Pr(w) \Pr(\text{piv}|w, x, v^x)(\alpha_v^w(A, a, x) - \alpha_v^w(B, a, x))},$$

when $x = 2$.

One can then, using Bayesian updating, derive the following expressions:

$$\begin{aligned} \Pr(\text{piv}|w_1, 2, v)(\alpha_v^{w_1}(A, b, 2) - \alpha_v^{w_1}(B, b, 2)) &= \frac{(2m_b(1-m_b))^2 m_b(1-m_b)}{(m_b(1-m_b)^2 + (1-m_b)(1-m_b^2))(1-(m_b(1-m_b)^2 + (1-m_b)(1-m_b^2)))}; \\ \Pr(\text{piv}|w_1, 2, v)(\alpha_v^{w_1}(A, a, 2) - \alpha_v^{w_1}(B, a, 2)) &= \frac{(2m_b(1-m_b))^2 m_a(1-m_a)}{(m_a(1-m_b)^2 + (1-m_a)(1-m_b^2))(1-(m_a(1-m_b)^2 + (1-m_a)(1-m_b^2)))}; \\ \Pr(\text{piv}|w_2, 2, v)(\alpha_v^{w_2}(A, b, 2) - \alpha_v^{w_2}(B, b, 2)) &= \frac{(m_b(1-m_a) + m_a(1-m_b))^2 m_b(1-m_b)}{(m_b(1-m_b)(1-m_a) + (1-m_b)(1-m_b m_a))(1-(m_b(1-m_b)(1-m_a) + (1-m_b)(1-m_b m_a)))}; \\ \Pr(\text{piv}|w_2, 2, v)(\alpha_v^{w_2}(A, a, 2) - \alpha_v^{w_2}(B, a, 2)) &= \frac{(m_b(1-m_a) + m_a(1-m_b))^2 m_a(1-m_a)}{(m_a(1-m_b)(1-m_a) + (1-m_a)(1-m_b m_a))(1-(m_a(1-m_b)(1-m_a) + (1-m_a)(1-m_b m_a)))}; \\ \Pr(\text{piv}|w_4, 2, v)(\alpha_v^{w_4}(A, b, 2) - \alpha_v^{w_4}(B, b, 2)) &= \frac{(2m_a(1-m_a))^2 m_b(1-m_b)}{(m_b(1-m_a)^2 + (1-m_b)(1-m_a^2))(1-(m_b(1-m_a)^2 + (1-m_b)(1-m_a^2)))}; \\ \Pr(\text{piv}|w_4, 2, v)(\alpha_v^{w_4}(A, a, 2) - \alpha_v^{w_4}(B, a, 2)) &= \frac{(2m_a(1-m_a))^2 m_a(1-m_a)}{(m_a(1-m_a)^2 + (1-m_a)(1-m_a^2))(1-(m_a(1-m_a)^2 + (1-m_a)(1-m_a^2)))}. \end{aligned}$$

Note that when $t = \frac{1}{2}$, then $m_a = \frac{3}{4} = 1 - m_b = \frac{1}{4}$. This, along with the definitions in (6), allows us to find that $\Pr(\text{piv}|w_1, 2, v)(\alpha_v^{w_1}(A, b, 2) - \alpha_v^{w_1}(B, b, 2)) = \Pr(\text{piv}|w_4, 2, v)(\alpha_v^{w_4}(A, a, 2) - \alpha_v^{w_4}(B, a, 2))$, $\Pr(\text{piv}|w_2, 2, v)(\alpha_v^{w_2}(A, b, 2) - \alpha_v^{w_2}(B, b, 2)) = \Pr(\text{piv}|w_2, 2, v)(\alpha_v^{w_2}(A, a, 2) - \alpha_v^{w_2}(B, a, 2))$ and $\Pr(\text{piv}|w_4, 2, v)(\alpha_v^{w_4}(A, b, 2) - \alpha_v^{w_4}(B, b, 2)) = \Pr(\text{piv}|w_1, 2, v)(\alpha_v^{w_1}(A, a, 2) - \alpha_v^{w_1}(B, a, 2))$. When $q = \frac{1}{2}$, then $\Pr(w_1) = \Pr(w_4)$. This then implies that $\frac{\alpha_v(A, b, 2) - \alpha_v(B, b, 2)}{\alpha_v(A, a, 2) - \alpha_v(B, a, 2)} = 1$. \square

Step 2: $\Gamma(t; q, x)$ increases in q for all x .

Proof of step 2: As in the proof of Lemma A1, I focus on the case of $x = 3$ and the other cases follow from similar calculations. In the proof of Lemma A1, I find that for $x = 3$, then

$$\frac{1 - \alpha_v(B, b, 3)}{1 - \alpha_v(B, a, 3)} = \frac{\frac{q^2 m_b^2 (1-m_b)}{1-m_b^3} + \frac{2q(1-q)m_b m_a (1-m_b)}{1-m_b^2 m_a} + \frac{(1-q)^2 m_a^2 (1-m_b)}{1-m_b m_a^2}}{\frac{q^2 m_b^2 (1-m_a)}{1-m_b^2 m_a} + \frac{2q(1-q)m_b m_a (1-m_a)}{1-m_b m_a^2} + \frac{(1-q)^2 m_a^2 (1-m_a)}{1-m_a^3}}$$

Taking a derivative w.r.t. q , its sign is positive if

$$\begin{aligned} &2q^2 m_b^3 m_a \left[\frac{(1-m_b)(1-m_a)}{1-m_b^3} \frac{1-m_a}{1-m_b m_a^2} - \frac{(1-m_b)(1-m_a)}{1-m_b^2 m_a} \frac{1-m_a}{1-m_b^2 m_a} \right] + \\ &2q(1-q) m_b^2 m_a^2 \left[\frac{(1-m_b)(1-m_a)}{1-m_b^3} \frac{1-m_a}{1-m_a^3} - \frac{(1-m_b)(1-m_a)}{1-m_b m_a^2} \frac{1-m_a}{1-m_b^2 m_a} \right] + \\ &2(1-q)^2 m_b m_a^3 \left[\frac{(1-m_b)(1-m_a)}{1-m_b^2 m_a} \frac{1-m_a}{1-m_a^3} - \frac{(1-m_b)(1-m_a)}{1-m_b m_a^2} \frac{1-m_a}{1-m_b m_a^2} \right] \end{aligned}$$

But each brackets in the expression is positive - each expression in the brackets is (see (10)):

$$(1 - \alpha_v^{w_i}(B, b, 3))(1 - \alpha_v^{w_j}(B, a, 3)) - (1 - \alpha_v^{w_j}(B, b, 3))(1 - \alpha_v^{w_i}(B, a, 3))$$

for some $i < j$, and it was established in Lemma A1 (see (12)), that for any $i < j$,

$$\frac{1 - \alpha_v^{w_j}(B, b, 3)}{1 - \alpha_v^{w_j}(B, a, 3)} < \frac{1 - \alpha_v^{w_i}(B, b, 3)}{1 - \alpha_v^{w_i}(B, a, 3)}.$$

This implies that $\Gamma(t; q, 3)$ increases in q for $x = 3$. \square

Step 3: In equilibrium, the cutoff point for $x = 2$, must admit $s = a$.

Proof of step 3: Recall that (8) has to hold. The left-hand-side is smaller than 1 for all x when $s = b$. When $x = 2$, then when $t = \frac{1}{2}$, by step 1, $\Gamma(\frac{1}{2}; \frac{1}{2}, 2) = 1$, and by step 2, $\Gamma(\frac{1}{2}; q, 2) > 1$. By Lemma A1, $\Gamma(t; q, 2)$ decreases in v , or in other words, it decreases in t when $s = a$ and increases in t whenever $s = b$. This implies that $\Gamma(t; q, 2) > 1$ whenever $s = b$ so that (8) cannot hold. Thus, the equilibrium cutoff point must admit $s = a$. \square

Step 4: When $s = a$, then $\Gamma(t; 1, 1) < \Gamma(t; \frac{1}{2}, 2)$ and $\Gamma(t; 1, 2) < \Gamma(t; \frac{1}{2}, 3)$.

Proof of step 4: I now provide the expression for $\Gamma(t; q, 1) = \frac{\alpha_v(A, b, x)}{\alpha_v(A, a, x)}$, which is:

$$\frac{q^2 \frac{(1-m_b)^2 m_b}{m_b + (1-m_b)(1-(1-m_b)^2)} + 2q(1-q) \frac{(1-m_b)(1-m_a)m_b}{m_b + (1-m_b)(1-(1-m_b)(1-m_a))} + (1-q)^2 \frac{(1-m_a)^2 m_b}{m_b + (1-m_b)(1-(1-m_a)^2)}}{q^2 \frac{(1-m_b)^2 m_a}{m_a + (1-m_a)(1-(1-m_b)^2)} + 2q(1-q) \frac{(1-m_b)(1-m_a)m_a}{m_a + (1-m_a)(1-(1-m_b)(1-m_a))} + (1-q)^2 \frac{(1-m_a)^2 m_a}{m_a + (1-m_a)(1-(1-m_a)^2)}}$$

In Lemma A1, I provide the expression for $\Gamma(t; q, 3)$ whereas in step 1, I provide the expression for $x = 2$. When $s = a$, then $m_a = 1 - t^2$ and $m_b = (1 - t)^2$. The expressions for $\Gamma(t; q, x)$ simplify significantly when we plug the extreme parameter values $q = 1$ and $q = .5$, and it is then easy to verify that the statement holds. \square

Steps 2 and 4 prove therefore that when $s = a$, then for a given t and q , $\Gamma(t; q, 1) < \Gamma(t; q, 2) < \Gamma(t; q, 3)$. This completes the proof of Lemma 2. \blacksquare