Investment Dynamics with Common and Private Values*

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August 5, 2005

Abstract

We characterize equilibrium in a dynamic investment game with two-dimensional signals, where each firm observes its idiosyncratic cost of investment and a signal correlated with common investment returns. Investment-cost cutoffs for type-1 firms (with the high common-value signal) and type-0 firms (with the low common-value signal) are shown to satisfy a simple equation that does not depend on the cost distribution. However, "reversals" are possible, where beliefs about investment returns are decreasing in the number of firms that invest in a given round. For some large markets, there is an initial surge in investment, nearly revealing the state of the economy, and outcomes are (almost) efficient. For other large markets, there is a positive probability of no investment, even when the return is high and all firms would stand to profit.

*This material is based upon work supported by the NSF under Grant No. SES-0417352. Any opinions, findings and conclusions or recommendations expressed are those of the authors and do not necessarily reflect the views of the NSF. We thank the John Glenn Institute at Ohio State for their support. We are particularly grateful to Asen Ivanov for identifying problems in an earlier draft. This paper has benefitted from comments by seminar participants at ITAM, IUPUI, University of Michigan, The Ohio State University, The Second World Congress in Game Theory (Marseilles, France), Tel Aviv University, Vanderbilt University, and Washington University. E-mail: levin.36@osu.edu, peck.33@osu.edu.

1. Introduction

An enduring debate in Economics is whether markets efficiently aggregate the information of individual agents, and whether efficient allocations are the result. The stakes are high. If markets are informationally efficient and lead to efficient allocations, the government should not interfere with the workings of markets. On the other hand, if markets are not informationally efficient, then active government intervention could improve welfare. For example, when the economy is in recession but the investment climate has improved, that information might be dispersed among many firms who observe only a small piece of the overall picture. Firms with favorable information might postpone investment until they are confident that other firms share their assessment, thereby prolonging a recession. Properly formulated investment subsidy programs would have the potential to improve economic welfare.

This paper addresses the issue of information aggregation and allocative efficiency in a dynamic setting, where firms receive two signals and then face a sequence of decisions about whether or not to invest. One signal is correlated with the aggregate state of the economy, which in our context is the unknown return on investment shared by all firms. We assume that this "common value" signal can take one of two values, and call a firm receiving the favorable signal a type-1 firm and a firm receiving the unfavorable signal a type-0 firm. The other signal is the cost of undertaking the investment, which is firm specific and independent of the costs faced by other firms. Observing the investment decisions of other firms could be used to improve inference about the aggregate state, but firms must disentangle whether another firm invests because it receives a favorable signal about investment returns or simply has a low cost.

The closest papers in the literature are the articles by Chamley and Gale (1994) and Chamley (2004), who analyze models in which the investment cost is a fixed constant, so that the signal is one-dimensional. Chamley and Gale (1994) find that there is a unique symmetric perfect Bayesian equilibrium, and that the equilibrium is inefficient. There is a positive probability that little or no investment occurs, even when the number of firms approaches infinity and investment is profitable for everyone. Indeed, firms are no better off than in the static game, in which firms must invest without learning anything about other firms' information. Chamley (2004) introduces a distribution of beliefs about the investment return, resulting from private information. Pure-strategy equilibrium is characterized, the possibility of multiple equilibrium is demonstrated, and the case of a large number of agents is studied.

We introduce a richer informational environment than Chamley and Gale (1994) or Chamley (2004). Signals are two-dimensional, reflecting information about a firm's private cost of investment, and information about the common investment return. Introducing two-dimensional signals is important for several reasons. First, it makes the inference problem more realistic and interesting. An investor can be a type-1 firm, whose signal about the aggregate state of the economy is favorable. Alternatively, an investor can be a type-0 firm, whose signal about the aggregate state of the economy is unfavorable, but whose investment cost is low. Second, two-dimensional signals allow for a phenomenon that is completely new to this literature, which we call a "reversal." The higher the number of firms that invest in round 0, the higher the posterior probability each firm assigns to the high investment return. However, after some histories, it is possible that the higher the number of firms that invest, the *lower* the posterior probability each firm assigns to the high investment return. The reason behind reversals, that a type-0 firm can be more likely to invest than a type-1 firm, is quite intuitive and nonpathelogical. After some histories, the marginal type-0 firm can have a much lower cost than the marginal type-1 firm, which encourages investment by type-0 firms. This effect helps to offset the fact that type-1 firms have more favorable information about investment returns.

The third reason for introducing two-dimensional signals is that it allows us to contribute to the debate about informational efficiency of markets. While introducing heterogeneous costs might make information aggregation less efficient, due to the more difficult inference problem, it might make information aggregation more efficient, if firms with a range of private cost realizations strictly desire to invest immediately. This initial surge of investment could be highly informative.[1] Gul and Lundholm (1995) attribute any inefficiencies to the fact that investment must be all or nothing, rather than varying with the strength of the signal. Their argument is that one can

[1] Chamley (2004) shows similar asymptotic possibilities in a model with one-dimensional signals.

invert the investment function to recover the investor's type. However, with two-dimensional signals, continuous investment choices would not be fully revealing, so this environment seems ideal for exploring the deeper causes of inefficient information aggregation.[2]

Analyzing models with two-dimensional signals is notoriously difficult. Our equilibrium characterization requires the development of intricate arguments to handle two dimensions, while simple arguments could handle one dimension. However, the model is surprisingly well behaved in some respects. Proposition 2 provides a general existence result and a sharp characterization of equilibrium. For histories after which both types invest with positive probability, we provide a simple formula relating the investment costs of the marginal type-0 firm and the marginal type-1 firm. Surprisingly, this relationship does not depend on the discount factor or the distribution function for investment costs.

Our model is related to some of the papers on herd behavior and information cascades. Bikhchandani, Hirshleifer, and Welch (1992) and Banerjee (1992) consider a model in which each investor, in an exogenously determined sequence, faces an investment decision after privately observing a signal related to investment yields. Our model differs in the crucial timing dimension. Instead of facing an exogenously given single opportunity to invest, our firms endogenously choose when to invest, if at all. It is this ability to delay investment and free ride on the information provided by others that leads to lower and later investment. Chari and Kehoe (2004) endogenize the timing of investment, but in each period exactly one firm receives a signal, and in their equilibrium the exogenous sequence of firms receiving signals plays a prominent role.[3]

We want to distinguish our approach from the large literature on multiple equi-

[2]We assume that firms are risk neutral and can invest at most once, so any firm that invests would invest to the maximal extent possible. However, with risk averse firms and more than one signal, we conjecture that equilibrium would involve interior investment choices that do not fully reveal the investor's type. See also Chari and Kehoe (2004).

[3]Caplin and Leahy (1994) consider a model in which firms receive signals in each period, and are allowed to suspend and restart investment. In Jeitchko and Taylor (2001), investors privately observe the successes and failures of their investments, and update their beliefs about the unknown probability of success in order to decide whether or not to continue the investment. Investment returns depend on the overall level of investment as well as the success parameter, and at some point a coordination avalanche occurs. See also Morris and Shin (1998) and Baliga and Sjostrom (2002) for static games exhibiting this contagion effect.

librium and Keynesian coordination failure or trading externalities, based on the importance of self-fulfilling expectations of the *actions* that other agents are taking.[4] To emphasize the role of information and inference, we entirely eliminate the effect of one firm's actions on another firm's payoffs. We imagine that firms are not directly competing with each other, so that profitability depends on the aggregate shock and not the expansion decisions of other firms.[5] The delay and possible collapse of investment arising in our model is reminiscent of the Keynesian story of self-fulfilling pessimism.[6]

In section 2, we present the model, provide some preliminary results, and provide a general existence/characterization result (Proposition 2). After histories allowing an interior solution, a simple relationship is derived, relating the type-0 and the type-1 investment cutoffs. We provide an example of "reversals," in which more investment in round 0 is good news about investment returns, but more investment is bad news after some histories. Section 3 considers asymptotic results, as the number of firms approaches infinity. Some policy implications are discussed in section 4. Concluding remarks are offered in section 5. Proofs are in the Appendix.

2. The Model

There are $n \geq 2$ risk-neutral firms or potential investors, and each firm privately observes a signal correlated with the return on investment common to all investors. Letting Z denote the investment return and X_i denote the "common value" signal of firm i, we assume that $Z \in \{0, 1\}$ and $X_i \in \{0, 1\}$. We also assume that the

[4] See Diamond (1982), Bryant (1983,1987), Milgrom and Roberts (1990), Cooper and John (1988), and Jones and Manuelli (1992).

[5] The investment decision in our model is related to the decision to enter a market. See Dixit and Shapiro (1986), Fudenberg and Tirole (1985), Vettas (2000), and in particular, papers that incorporate private information by Bolton and Farrell (1990) and Levin and Peck (2003). The present paper introduces common values, which allows us to interpret signals as information about demand. Another major difference is that a firm's revenues do not depend on the number of entrants.

[6] Keynes (General Theory, p. 210) argues that pessimism causes consumers to reduce their demand, a sort of inaction. The reduction in consumption demand is not combined with an order for future consumption. Thus, firms could be deterred from investing, justifying the pessimism. Similarly, in our model, when a firm with a strong signal does not invest, the fact that the firm might be willing to invest in the future is not revealed to the market.

unconditional expected return is 0.5, and that signals are independent, conditional on Z. The accuracy of the signal is given by the parameter, $\alpha \in [\frac{1}{2}, 1]$:

$$pr(Z = 0 \mid X_i = 0) = pr(Z = 1 \mid X_i = 1) = \alpha.$$

When we have $\alpha = \frac{1}{2}$, common-value signals have no information content at all, and when we have $\alpha = 1$, a common-value signal fully reveals the aggregate state. Thus, the parameter α effectively captures the informativeness of the common-value signal, X_i. We call a firm that has received the high signal, $X_i = 1$, a type-1 firm and a firm that has received the low signal, $X_i = 0$, a type-0 firm.

Each firm i also privately observes a second signal, representing the idiosyncratic cost of undertaking the investment, c_i. We assume that c_i is independent of all other variables, and distributed according to the continuous and strictly increasing distribution function F, defined over the support, $[\underline{c}, \overline{c}]$. Assume that we have $0 \leq \underline{c} < 1 \leq \overline{c}$. The structure of signals is common knowledge.

Impatience is measured by the discount factor, $\delta < 1$. If firm i has cost c_i and the state is Z, its profits are zero if it does not invest, and $\delta^t(Z - c_i)$ if it invests in round t. We now describe the game. First, each firm observes its signals, (X_i, c_i). In each round, starting with round 0, each firm observes the history of play, and firms not yet invested simultaneously decide whether to invest. More formally, for $t = 0, 1, ...$, denote the action of firm i in round t as $e_i^t \in \{0, 1\}$, where the action, 0, represents no change in status (either not yet invested or invested in a previous round) and the action, 1, represents investing in that round. We assume that once a firm invests it remains invested. Let k^t denote the number of firms who invest in round t, $k^t = \sum_{i=1}^n e_i^t$, and denote the history of length t as $h^{t-1} = (k^0, k^1, ..., k^{t-1})$. We will sometimes denote the history, h^t, as (h^{t-1}, k^t). Let h denote the set of histories of any length, including the null history observed in round 0. A strategy for firm i is a mapping from signal realizations and histories into a decision of whether to invest, satisfying the restriction that a firm can change its investment status at most once.

Our solution concept is symmetric Bayesian Nash equilibrium. Refinements are not needed, because beliefs off the equilibrium path play no role in the analysis. If, after some history, h^{t-1}, $k^t = 0$ is off the equilibrium path, then it must be the case that all remaining firms are investing with probability one. After a deviation by firm

i not to invest, the beliefs of other firms are irrelevant, since they have invested and have nothing more to do. If, after some history, h^{t-1}, $k^t = k > 0$ is off the equilibrium path, then it must be the case that no firm is investing. After a deviation by firm i to invest, its payoff is determined independent of the future play of the game, so the beliefs of other firms are irrelevant to the decision to deviate. The following lemma is standard in the literature, and greatly simplifies the analysis, by showing that equilibrium is characterized by cutoff investment costs, such that any firm with investment cost below the cutoff will invest, if it has not already done so.

Lemma 1: *Suppose that F is continuous over the nondegenerate support, $[\underline{c}, \overline{c}]$. Then any Bayesian Nash equilibrium has the interval property. For any history, h^{t-1}, that arises with positive probability in the equilibrium, there are functions, $\beta_0(h^{t-1})$ and $\beta_1(h^{t-1})$, such that a type-0 firm (not previously invested) invests in round t if and only if $c_i \leq \beta_0(h^{t-1})$ holds, and a type-1 firm (not previously invested) invests in round t if and only if $c_i \leq \beta_1(h^{t-1})$ holds.*

We now characterize equilibrium and provide some welfare results for the general model. One might conjecture that the expected asset return, conditional on the history h^{t-1}, is always weakly increasing in k^t. We show, below, that this conjecture is false, in general. We show in Proposition 1 that expected asset returns are either increasing in k^t for both types, or decreasing in k^t for both types. Proposition 2 shows that equilibrium exists, satisfies the *one-step property*, and exhibits a simple relationship between $\beta_0(h^{t-1})$ and $\beta_1(h^{t-1})$ that is independent of the discount factor or cost distribution.

Before characterizing this equilibrium, we introduce some notation. Let the number of firms that have invested during rounds 0 through $t-1$ be denoted as $n(h^{t-1})$, given by $n(h^{t-1}) = \sum_{\tau=0}^{t-1} k^\tau$. Let H denote the following ratio of probabilities[7]

$$H = \frac{pr(h^{t-1} \mid Z = 0)}{pr(h^{t-1} \mid Z = 1)}.$$

Let $P_0(h^{t-1})$ denote the probability that we have $Z = 1$, given the history, h^{t-1}, and

[7] Implicit in these conditional probabilities are the investment cutoffs chosen in rounds 0 through $t-1$, and the fact that the firm considering this history has not yet invested. For simplicity, we suppress the dependence of H on the history, h^{t-1}.

given that a type-0 firm has not yet invested. Let $P_1(h^{t-1})$ denote the probability that we have $Z = 1$, given the history, h^{t-1}, and given that a type-1 firm has not yet invested. Using Bayes' rule, we have

$$P_0(h^{t-1}) = \frac{1}{1 + \frac{H\alpha}{(1-\alpha)}} \quad \text{and} \tag{2.1}$$

$$P_1(h^{t-1}) = \frac{1}{1 + \frac{H(1-\alpha)}{\alpha}} \tag{2.2}$$

Define the probability that a type-0 (respectively, type-1) firm invests in round t, after the history h^{t-1}, by

$$q_0(h^{t-1}) = \frac{F(\beta_0(h^{t-1})) - F(\beta_0(h^{t-2}))}{1 - F(\beta_0(h^{t-2}))}, \tag{2.3}$$

$$q_1(h^{t-1}) = \frac{F(\beta_1(h^{t-1})) - F(\beta_1(h^{t-2}))}{1 - F(\beta_1(h^{t-2}))}. \tag{2.4}$$

From (2.3) and (2.4), we see that finding the investment cost cutoffs for round t is equivalent to finding the probabilities that a firm will invest in round t (having not yet invested). Define $K^*(h^{t-1}, q_0, q_1)$ to be the set containing all values of k^t for which investment is unprofitable in round $t+1$ for the marginal type-1 firm. Thus, we have

$$K^*(h^{t-1}, q_0, q_1) = \{k : P_1(h^{t-1}, k^t = k; q_0(h^{t-1}) = q_0, q_1(h^{t-1}) = q_1) < \beta_1(h^{t-1})\}.$$

Similarly, for type-0, define

$$K^{**}(h^{t-1}, q_0, q_1) = \{k : P_0(h^{t-1}, k^t = k; q_0(h^{t-1}) = q_0, q_1(h^{t-1}) = q_1) < \beta_0(h^{t-1})\}.$$

We also define

$$Q(h^{t-1}, k, q_0, q_1) \equiv \frac{pr(k^t = k \mid h^{t-1}, Z = 0, q_0(h^{t-1}) = q_0, q_1(h^{t-1}) = q_1)}{pr(k^t = k \mid h^{t-1}, Z = 1, q_0(h^{t-1}) = q_0, q_1(h^{t-1}) = q_1)}. \tag{2.5}$$

The ratio, $Q(h^{t-1}, k, q_0, q_1)$, represents the likelihood that a firm that has not yet invested observes k firms invest in round t, given that the state is low, relative to the likelihood of observing k firms invest in round t, given that the state is high. This likelihood ratio depends on the history, h^{t-1}, and depends on the probabilities that a type-1 firm and a type-0 firm invest after h^{t-1}.

Application of Bayes' rule, and making explicit the dependence of $P_0(h^t)$ and $P_1(h^t)$ on $q_0(h^{t-1})$ and $q_1(h^{t-1})$, yields

$$P_0(h^{t-1},k;q_0,q_1) = \frac{1}{1+\frac{\alpha}{1-\alpha}HQ(h^{t-1},k,q_0,q_1)} \quad \text{and} \tag{2.6}$$

$$P_1(h^{t-1},k;q_0,q_1) = \frac{1}{1+\frac{1-\alpha}{\alpha}HQ(h^{t-1},k,q_0,q_1)} \tag{2.7}$$

Proposition 1: *If $q_1(h^{t-1}) > q_0(h^{t-1})$ holds, then $P_0(h^{t-1},k;q_0,q_1)$ and $P_1(h^{t-1},k;q_0,q_1)$ are strictly increasing in k. If $q_1(h^{t-1}) < q_0(h^{t-1})$ holds, then $P_0(h^{t-1},k;q_0,q_1)$ and $P_1(h^{t-1},k;q_0,q_1)$ are strictly decreasing in k. In other words, more investment in round t implies a higher (lower) posterior probability that the state is high, if and only if a type-1 firm invests in round t with a higher (lower) probability than a type-0 firm.*

We now return to the general model. The expected profit that a type-1 firm with cost c_i saves in round t, due to the option of *not* investing in round $t+1$ when $k^t = k$ occurs, is defined by

$$\begin{aligned}\theta_k(h^{t-1},q_0,q_1,c_i) &= pr(Z=1,k^t=k \mid h^{t-1},X_i=1,q_0(h^{t-1})=q_0,q_1(h^{t-1})=q_1)[c_i-1]\\ +pr(Z &= 0,k^t=k \mid h^{t-1},X_i=1,q_0(h^{t-1})=q_0,q_1(h^{t-1})=q_1)[c_i].\end{aligned}$$

Using Bayes' rule, this equation can be simplified to

$$\begin{aligned}\theta_k(h^{t-1},q_0,q_1,c_i) &= [c_i-1]\frac{pr(k^t=k \mid h^{t-1},Z=1,q_0(h^{t-1})=q_0,q_1(h^{t-1})=q_1)}{1+\frac{H(1-\alpha)}{\alpha}} +\\ &\quad [c_i]\frac{pr(k^t=k \mid h^{t-1},Z=0,q_0(h^{t-1})=q_0,q_1(h^{t-1})=q_1)}{1+\frac{\alpha}{H(1-\alpha)}}.\end{aligned} \tag{2.8}$$

For a type-0 firm, denote the profit savings as $\eta_k(h^{t-1},q_1,q_2,c_i)$, defined by

$$\begin{aligned}\eta_k(h^{t-1},q_0,q_1,c_i) &= pr(Z=1,k^t=k \mid h^{t-1},X_i=0,q_0(h^{t-1})=q_0,q_1(h^{t-1})=q_1)[c_i-1]\\ +pr(Z &= 0,k^t=k \mid h^{t-1},X_i=0,q_0(h^{t-1})=q_0,q_1(h^{t-1})=q_1)[c_i].\end{aligned}$$

Using Bayes' rule, this equation can be simplified to

$$\eta_k(h^{t-1}, q_0, q_1, c_i) = [c_i - 1]\frac{pr(k^t = k \mid h^{t-1}, Z = 1, q_0(h^{t-1}) = q_0, q_1(h^{t-1}) = q_1)}{1 + \frac{H\alpha}{1-\alpha}} + [c_i]\frac{pr(k^t = k \mid h^{t-1}, Z = 0, q_0(h^{t-1}) = q_0, q_1(h^{t-1}) = q_1)}{1 + \frac{1-\alpha}{H\alpha}}. \quad (2.9)$$

Proposition 2: *Under our maintained assumptions, there exists an equilibrium characterized as follows. After a history, h^{t-1}, in which investment is unprofitable for a type-1 firm with cost $\beta_1(h^{t-2})$, investment ceases. If investment is profitable and the previous type-0 cutoff is at $\underline{c}$, $\beta_0(h^{t-2}) = \underline{c}$, then there may be a corner solution, with $q_0(h^{t-1}) = 0$* and *$q_1(h^{t-1})$* solving

$$\left(P_1(h^{t-1}) - \beta_1(h^{t-1})\right)\left(\frac{1-\delta}{\delta}\right) = \sum_{k \in K^*(h^{t-1},0,q_1)} \theta_k(h^{t-1}, 0, q_1, \beta_1(h^{t-1})) \text{ and}$$

$$\left(P_0(h^{t-1}) - \underline{c}\right)\left(\frac{1-\delta}{\delta}\right) < \sum_{k \in K^{**}(h^{t-1},0,q_1)} \eta_k(h^{t-1}, 0, q_1, \underline{c}).$$

Otherwise, type-1 firms and type-0 firms invest with positive probability, we have $K^*(h^{t-1}, q_0, q_1) = K^{**}(h^{t-1}, q_0, q_1)$, and the investment probabilities solve

$$\left(P_1(h^{t-1}) - \beta_1(h^{t-1})\right)\left(\frac{1-\delta}{\delta}\right) = \sum_{k \in K^*(h^{t-1},q_0,q_1)} \theta_k(h^{t-1}, q_0, q_1, \beta_1(h^{t-1})) \text{ and} \quad (2.10)$$

$$\left(P_0(h^{t-1}) - \beta_0(h^{t-1})\right)\left(\frac{1-\delta}{\delta}\right) = \sum_{k \in K^{**}(h^{t-1},q_0,q_1)} \eta_k(h^{t-1}, q_0, q_1, \beta_0(h^{t-1})). \quad (2.11)$$

Furthermore, whenever we have $\beta_0(h^{t-1}) > \underline{c}$, investment cutoffs are related according to

$$\frac{\beta_0(h^{t-1})}{1 - \beta_0(h^{t-1})} = \frac{\beta_1(h^{t-1})}{1 - \beta_1(h^{t-1})}\frac{(1-\alpha)^2}{\alpha^2}. \quad (2.12)$$

Proposition 2 characterizes an equilibrium. Starting with a history at which both types invest with positive probability, all histories that are profitable for the lowest cost uninvested type-1 firm are profitable for the lowest cost uninvested type-0 firm, and vice versa. Both cutoffs then shift together, until investment becomes unprofitable and ceases. The investment probabilities of type-0 and type-1 firms simultaneously solve (2.10) and (2.11), which impose the condition that these marginal firms are indifferent between investing in round t and investing in round $t+1$, if and only if investment in round $t+1$ is profitable. It is remarkable that equilibrium investment cutoffs are related according to equation (2.12), irrespective of the distribution of costs and the discount factor. On the other hand, if $\underline{c}$ is sufficiently large but less than α, then there is an initial phase of the equilibrium, in which a type 0 firm with cost $\underline{c}$ strictly prefers to delay investment. Proposition 2 provides a way to compute an equilibrium, history by history, if the number of firms is not too large. When the number of firms is fairly large, computing the entrire equilibrium might not be practical, but Proposition 2 shows the way to compute the equilibrium cutoffs for the first few rounds.

The proof of Proposition 2 overcomes several technical difficulties that arise only when signals are multi-dimensional. With one-dimensional signals, there is a single indifference equation determining the cutoff investor after any history. In our model, there are two cutoffs that must be determined jointly. Establishing that there are investment probabilities of type-0 and type-1 firms that simultaneously solve (2.10) and (2.11), or possibly a corner solution in which one of the types invests with probability zero, occupies a large portion of the proof. The other difficulty lies in establishing the one-step property. With one-dimensional signals, there is only one cutoff type of firm after any history, and if that firm decides to wait, then it becomes more favorably inclined to invest than any other remaining firm. Therefore, this firm had better invest in the next round whenever investment remains profitable, because no other firm will invest before the most favorably inclined firm invests. However, in our model, there is a cutoff type-0 firm and a cutoff type-1 firm. It is conceivable that the type-1 firm with cost $\beta_1(h^{t-1})$, rather than comparing investment in round t with investment in round $t+1$ when it is profitable, can receive even higher profits by waiting until round $t+2$. This would be possible if a type-0 firm (with lower investment cost!) might invest in round $t+1$. Fortunately, for the equilibrium constructed in Proposition 2, the one-step property holds. Otherwise, (2.10) and (2.11) would no longer determine equilibrium investment cutoffs.

In evaluating the welfare of firms, one benchmark for comparison is the *static* *game*, in which firms decide whether to invest in round 0 or never invest. The increase in welfare of the equilibrium allocation over the allocation from the static game represents the social benefit of learning. Another potentially interesting benchmark is the outcome of the "*rigid-timing*" game, in which firms face a once-and-for-all decision whether to invest in a randomly-determined sequence, observing the decisions of firms ahead of it in the queue. This rigid timing model, with exogenous timing of the investment decision, is usually assumed in the herding literature. This benchmark allows us to measure the advantage or disadvantage of being able to choose when to invest.

Corollary: *For the equilibrium characterized in Proposition 2, expected profits, conditional on (X_i, c_i), are weakly higher than expected profits in the static game. If there is a positive probability of investment in round 0, then expected profits are strictly higher than in the static game for type-1 firms with $c_i > \beta_1$, and for type-0 firms with $c_i > \beta_0$.*

Remark 1: In Chamley and Gale (1994), all firms have the same cost, and only firms with the favorable signal (type-1 firms in our terminology) have an opportunity to invest. Thus, all firms deciding whether to invest are identical. Chamley and Gale find that there is a unique symmetric mixed-strategy equilibrium, and that ex ante welfare is the same as in the static game, with only one round of investing. This strong inefficiency result disappears when we have heterogeneity, including Chamley's (2004) model with one-dimensional and heterogeneous types. The Corollary indicates that all firms that do not invest in round 0 receive strictly higher profits, in the equilibrium to the flexible-timing game characterized in Proposition 2, than they would receive in the static game. These firms benefit from the ability to learn from market activity.

Remark 2: The ex ante welfare comparison between the flexible-timing game and the rigid-timing game is ambiguous. An advantage of the flexible-timing game is that firms with the most favorable signals invest first, so that firms who benefit most from observing market activity have an opportunity to delay their decision to invest. A disadvantage of the flexible-timing game is that some type-1 firms, for whom investment is profitable, delay their investment decision and thereby delay their favorable information from being observed by the market. In Example 1 below, welfare is higher in the flexible-timing game. If we were to change the parameters, however, welfare could easily be higher in the rigid-timing game. For example, consider a game with two firms, $n = 2$, a discount factor close to one, and investment costs uniformly distributed over $[\alpha - \varepsilon, \alpha]$, for small positive ε. In the flexible-timing game, the probability of investment in round 0 is close to zero, so welfare is close to zero. In the rigid-timing game, the first firm to act will invest if and only if it is of type 1, so the second firm will receive significant expected profits if it is also a type-1 firm.

Example 1: Consider the following example, with parameters, $n = 2$, $\alpha = .75$, and $\delta = 1$, and a cost distribution that is uniform on $[0, 1]$. Table 1 shows the unique symmetric equilibrium of our "flexible timing" game. For the purpose of comparison, Table 1 also shows the unique equilibrium of the rigid timing game usually studied in the herding literature.

	flexible timing game	rigid timing game
β_0	0.17913	0.25
β_1	0.66261	0.75
$\beta_0(1)$	0.37576	0.35714
$\beta_1(1)$	0.84417	0.83333
$\beta_0(0)$	no further investment	0.16667
$\beta_1(0)$	no further investment	0.64286
profit (ex ante)	0.16054	0.15848

Table 1: $n = 2, \ \delta = 1, \ \alpha = .75, \ c_i \sim U[0, 1]$

The fact that β_0 and β_1 are lower for the flexible timing game than for the rigid timing game illustrates the incentive to delay investment, due to the option value of not investing in round 1 if the other firm did not invest in round 0. A type-0 firm with cost $c_i \in (0.17913, 0.25)$ would receive positive profit by investing in round 0, but profit is higher by waiting until round 1. Similar reasoning applies to a type-1 firm with cost, $c_i \in (0.66261, 0.75)$. The cutoffs for investing in round 1, after observing the other firm invest in round 0, are higher for the flexible timing game than the rigid timing game. The reason is that there is a stronger inference that the other firm is a type-1 firm in the flexible timing game than in the rigid timing game, because β_1/β_0 is higher. Indeed, our simple example illustrates the role heterogeneous costs play in diluting the information gathered from another firm's investment. Suppose a firm observes its rival invest in round 0, and could infer that the rival is type-1. Then the hypothetical cutoffs for investment in round 1 would be $\widetilde{\beta}_0(1) = .50$ and $\widetilde{\beta}_1(1) = .90$. The actual values are significantly lower, reflecting the fact that investment by the rival is a noisy indicator that the rival is a type-1 firm. While the rival's investment is surely good news about the aggregate state, a firm must take into account the possibility that the rival is a type-0 firm with low investment cost.

Choosing the limiting discount factor, $\delta = 1$, allows a clean comparison of welfare in the two games, since forcing one of the firms to delay investment in the rigid timing game is not itself a source of inefficiency. Rather, any inefficiency that arises when a type-1 firm delays investment is due to the fact that the other firm cannot benefit from that information. Ex ante profit is higher for the flexible timing game, so the gains from endogenous sorting outweigh the loss due to strategic delay for this example. To put these profit values into perspective, ex ante profit in the static game (where no learning is possible) is 0.15625, and ex ante profit would be 0.25 if signals were perfectly accurate ($\alpha = 1$). The market clearly benefits from the opportunity to learn.

Reversals

We define a *reversal* to be an equilibrium in which more investment in round 0 is good news, but after some histories, more investment in later rounds is actually bad news. Reversals are *impossible* in the rigid timing game or in the flexible timing game with deterministic costs. The reason that reversals are possible in the flexible timing game with multi-dimensional signals is that, following a certain history, a type-0 firm is more likely to invest than a type-1 firm. The following example of a reversal has three firms, and if exactly one firm invests in round 1, expected revenues increase. However, after one firm invests in round 0, type-0 firms are more likely than type-1 firms to invest in round 1, so investment in round 1 is bad news. Choosing an example in which firms are infinitely impatient simplifies the calculations enormously, but the qualitative features of the example continue to hold for small positive δ.

Example 2: Consider the following example, with $\alpha = .75$, $\delta = 0$, $n = 3$, and the distribution function given by,

$$F(c_i) = \frac{c_i}{5} \qquad \text{for } c_i \leq 5$$

The distribution function is uniform over the interval, $[0, 5]$.[8]

Because firms are infinitely impatient, they will invest at the first profitable opportunity. Therefore, the equilibrium cutoffs are given by

$$\begin{aligned} \beta_0(h^{t-1}) &= P_0(h^{t-1}) \text{ and} \\ \beta_1(h^{t-1}) &= P_1(h^{t-1}) \end{aligned}$$

whenever k^{t-1} represents good news, in the sense that $P_0(h^{t-1}) > P_0(h^{t-2})$ and $P_1(h^{t-1}) > P_1(h^{t-2})$ hold.[9] If k^{t-1} represents bad news, then there is no more investment. Table 2 presents our computations of the equilibrium cutoffs for the first few rounds.

	expected revenue and cutoff for investment		probability of investing	
history	$P_0(h^{t-1})$	$P_1(h^{t-1})$	$q_0(h^{t-1})$	$q_1(h^{t-1})$
null	.25	.75	.05	.15
(0)	no more investment		0	0
(1)	.344488	.825472	.019892	.017758
(2)	.480769	.892857	.048583	.033613
$(1,0)$	.344733	.825628	.0000526	.0000374
$(1,1)$	no more investment		0	0

Table 2

From Table 2, we see why reversals can occur in equilibrium. More investment in round 0 is good news, because firms that invest are more likely to be type 1.[10] If no

[8] Example 2 can easily be altered to have $\overline{c} = 1$. Example 2 is equivalent to an example in which we have $F(c_i) = \frac{c_i}{5}$ for $c_i \leq 0.9$, and $F(c_i) = \frac{41c_i}{5} - \frac{36}{5}$ for $0.9 < c_i \leq 1$.

[9] Proposition 1 shows that good news for a type-0 firm is good news for a type-1 firm, and vice versa.

[10] We see that, for a type-i firm ($i = 0, 1$), we have $q_i(2) > q_i(1) > q_i(0)$ and $P_i(2) > P_i(2) > P_i(2)$.

one invests in round 0, this is bad news, and there is no further investment. If one firm invests in round 0, this is good news, as indicated by the fact that expected revenue increases for both types. Notice, however, that after one firm invests in round 0, a type-0 firm is more likely to invest than a type-1 firm. Therefore, after the history $(1, 1)$, the firm that invested in round 1 is much more likely to be a type-0 firm. While the investment in round 0 is good news, the investment in round 1 is bad news, and investment ceases. On the other hand, if no one invests in round 1, this is good news, and we have a reversal. Notice that, if two firms invest in round 0, a type-0 firm is also more likely to invest in round 1 than a type-1 firm. Expected revenue is higher after the history $(2, 0)$ than the history $(2, 1)$, but since there is only one remaining uninvested firm in round 1, obviously that firm does not learn anything from market activity in round 1.

In Example 2, following a history of the form, $(1, 0, 0, ...)$, there is always a positive probability of investment by a second firm, although this probability quickly approaches zero. Each round in which there is no investment leads firms to slightly increase their posterior probability of the good state. This scenario is reminiscent of a war of attrition, although here there is no strategic interaction between firms.[11] If a second firm ever invests, this is bad news for the remaining firm, and investment ceases.

3. The Model with Many Firms

In the limit, as $n \to \infty$, the law of large numbers implies that the aggregate state could be known with certainty if the firms were to pool their information. How efficient will the investment market be at aggregating this information? Because we have a large market, there will be many firms with costs near the lower bound, so $\underline{c}$ plays an important role in characterizing the equilibrium. Proposition 3 characterizes the round 0 investment cutoffs for the limiting equilibria, as $n \to \infty$. Proposition 4 addresses overall equilibrium welfare.

[11] Reversals, in which more investment switches from being good news to being bad news, is reminiscent of the phenomenon documented by Park and Smith (2003), in which timing games can switch from a "preemptive explosion" to a war of attrition.

Proposition 3: *Fix α, δ, $\underline{c}$, $\overline{c}$, and the continuous and strictly increasing distribution function, F. Consider a sequence of economies, indexed by n, and let (β_0^n, β_1^n) be equilibrium investment cutoffs in round 0 for the economy with n firms. Consider a convergent subsequence, where $(\beta_0^n, \beta_1^n) \to (\beta_0^*, \beta_1^*)$. Then we have the following exhaustive possibilities:*

(1) Parameters satisfy $\underline{c} > \alpha$ [Region 1 in Figure 1], and cutoffs satisfy $\beta_0^* = \beta_1^* = \underline{c}$,

(2) Parameters satisfy $\frac{\alpha(1-\delta)}{1-\alpha\delta} < \underline{c} < \alpha$ [Region 2 in Figure 1], and cutoffs satisfy $\beta_0^* = \beta_1^* = \underline{c}$,

(3) Parameters satisfy $\frac{(1-\alpha)(1-\delta)}{1-(1-\alpha)\delta} < \underline{c} < \frac{\alpha(1-\delta)}{1-\alpha\delta}$ [Region 3 in Figure 1], and cutoffs satisfy $\beta_0^* = \underline{c}$ and $\beta_1^* = \frac{\alpha(1-\delta)}{1-\alpha\delta}$,

(4) Parameters satisfy $\underline{c} < \frac{(1-\alpha)(1-\delta)}{1-(1-\alpha)\delta}$ [Region 4 in Figure 1], and cutoffs satisfy $\beta_0^* = \frac{(1-\alpha)(1-\delta)}{1-(1-\alpha)\delta}$ and $\beta_1^* = \frac{\alpha(1-\delta)}{1-\alpha\delta}$,

We now discuss the equilibria characterized in Proposition 3, leaving the more complicated part (2) for last. In region 1, for all firms, the cost exceeds the expected return, so no one would be willing to be the first to invest. Since no one invests in round 0, no further inference is made, and investment never occurs. This equilibrium is inefficient, because when investment returns are high, $Z = 1$, investment is profitable for all firms (ex post), yet no investment takes place. We also have no investment in the rigid timing game. For equilibria corresponding to part (3), type-0 firms do not invest in round 0, and type-1 firms invest with probability $F(\beta_1^n)$. By the law of large numbers, the limiting fraction of firms that invest in round 0 is $\alpha F(\beta_1^*)$ if the state is high, $Z = 1$, and the limiting fraction of firms that invest in round 0 is $(1-\alpha)F(\beta_1^*)$ if the state is low, $Z = 0$. Thus, activity in round 0 reveals the state, thereby justifying the hypothesized investment behavior in round 0. The outcome is nearly first-best efficient.[12] For equilibria corresponding to part (4), the limiting equilibrium cutoffs

[12] The only departures from first-best efficiency are that (i) some firms might invest in round 0 in the low state, and (ii) some firms might delay their investment until round 1 in the high state. However, if it takes one round for a fully informed planner to communicate the state to the firms, it is impossible to improve on the equilibrium. By contrast, the rigid timing game outcome is inefficient even ignoring delays (assuming $\underline{c} > 0$), because a finite sequence of noninvestment can stop all investment forever.

for round 0 are

$$\beta_0^* = \frac{(1-\alpha)(1-\delta)}{1-(1-\alpha)\delta} \text{ and } \beta_1^* = \frac{\alpha(1-\delta)}{1-\alpha\delta}.$$

By the law of large numbers, the limiting fraction of firms that invest in round 0 is $\alpha F(\beta_1^*) + (1-\alpha)F(\beta_0^*)$ if the state is high, $Z = 1$, and the limiting fraction of firms that invest in round 0 is $(1-\alpha)F(\beta_1^*) + \alpha F(\beta_0^*)$ if the state is low, $Z = 0$. Thus, activity in round 0 reveals the state, thereby justifying the hypothesized investment behavior in round 0. Again, the equilibrium is nearly first-best efficient, while the rigid-timing game can exhibit inefficient herding.

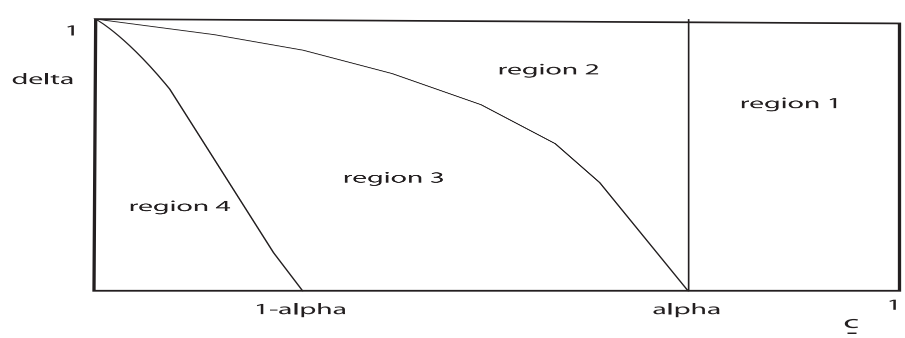

Figure 1

The more difficult and interesting case occurs when we consider equilibria described in part (2) of Proposition 3. Limiting investment cutoffs satisfy $\beta_0^* = \beta_1^* = \underline{c}$, so the probability that any particular firm invests in round 0 approaches zero. However, if firms are certain that there will be no investment, a type-1 firm with cost close to $\underline{c}$ should invest in round 0. This issue has been treated rigorously in the literature, for the case of one-dimensional signals, and the same logic applies here as well. For sufficiently large n, type-0 firms strictly prefer not to invest in round 0, $\beta_0^n = \underline{c}$. The cutoff for investment by type-1 firms is approaching $\underline{c}$, but the probability of $k^t = 0$ has a well-defined limit that is between zero and one. Thus, observing only one or two firms invest in round 0 could be good news, for all values of n.

When δ approaches one and n approaches infinity, the limiting probability that no firm ever invests, given that we are in the high state, $Z = 1$, can be computed as $\left(\frac{\alpha(1-\underline{c})}{(1-\alpha)\underline{c}}\right)^{-\alpha/(2\alpha-1)}$.The limiting probability that no firm invests in the low state can be computed as $\left(\frac{(1-\underline{c})(1-\alpha)}{\alpha\underline{c}}\right)^{(1-\alpha)/(2\alpha-1)}$. (Derivations are available from the authors.) These probabilities of no investment are significant. For example, if we have $\delta \to 1$, $n \to \infty$, $\underline{c} = 1/2$, and $\alpha = 2/3$, the probability of no investment in the good and bad states, respectively, are 1/4 and 1/2.

When we observe enough investment in round 0 to keep the process moving, investment could cease at some point in the future. What can be said about investment beyond round 0, in the good state and in the bad state, and what are the welfare implications?

Proposition 4: *Fix α, δ, $\underline{c}$, $\overline{c}$, and the continuous and strictly increasing distribution function, F. Consider a sequence of economies, indexed by n, and let $W^n(0, c_i)$ and $W^n(1, c_i)$ be equilibrium profits, conditional on being a type-0 or type-1 firm with investment cost c_i, for the economy with n firms. Consider a convergent subsequence, where $(W^n(0, c_i), W^n(1, c_i)) \to (W^*(0, c_i), W^*(1, c_i))$. Then we have:*

(1) In region 1, $W^*(0, c_i) = W^*(1, c_i) = 0$,

(2) In region 2,

$$\begin{aligned}
\delta\left(\frac{\alpha-\underline{c}}{1-\underline{c}}\right)\left(\frac{1-\alpha}{\alpha}\right)(1-c_i) &\leq W^*(0, c_i) \leq \frac{1}{\delta}\left(\frac{\alpha-\underline{c}}{1-\underline{c}}\right)\left(\frac{1-\alpha}{\alpha}\right)(1-c_i), \\
\delta\left(\frac{\alpha-\underline{c}}{1-\underline{c}}\right)(1-c_i) &\leq W^*(1, c_i) \leq \frac{1}{\delta}\left(\frac{\alpha-\underline{c}}{1-\underline{c}}\right)(1-c_i),
\end{aligned}$$

(3) In region 3,

$$\begin{aligned}
W^*(0, c_i) &= \delta(1-\alpha)(1-c_i), \\
W^*(1, c_i) &= \delta\alpha(1-c_i) \quad \text{for } c_i \geq \frac{\alpha(1-\delta)}{1-\alpha\delta}, \\
W^*(1, c_i) &= \alpha - c_i \quad \text{for } c_i < \frac{\alpha(1-\delta)}{1-\alpha\delta},
\end{aligned}$$

(4) In region 4,

$$
\begin{aligned}
W^*(0, c_i) &= \delta(1-\alpha)(1-c_i), \quad \text{for } c_i \geq \frac{(1-\alpha)(1-\delta)}{1-(1-\alpha)\delta}, \\
W^*(0, c_i) &= 1-\alpha-c_i, \quad \text{for } c_i < \frac{(1-\alpha)(1-\delta)}{1-(1-\alpha)\delta}, \\
W^*(1, c_i) &= \delta\alpha(1-c_i) \quad \text{for } c_i \geq \frac{\alpha(1-\delta)}{1-\alpha\delta}, \\
W^*(1, c_i) &= \alpha - c_i \quad \text{for } c_i < \frac{\alpha(1-\delta)}{1-\alpha\delta}.
\end{aligned}
$$

The intuition for Proposition 4 is that, once the investment cutoffs are above $\underline{c}$, the fraction of firms that have invested will reveal the state. In region 1, we have no investment. In regions 3 and 4, firms that do not invest in round 0 invest in round 1 in the high state, and do not invest in the low state. In region 2, whether or not a positive fraction of firms invest depends delicately on the exact realizations of investment costs for the type-1 firms with the lowest costs. Basically, $P_1(h^{t-1})$ and $P_0(h^{t-1})$ replace α and $1-\alpha$ in determining which region the economy moves into. During the round in which the economy first moves into region 3 or region 4, a positive fraction of firms invest, but the state is not yet known. Therefore, there can be overinvestment as well as underinvestment. There is a positive probability that investment never takes off in the high state, and there is a positive probability that a positive fraction of firms invest in the low state. For this reason, it is difficult to characterize welfare when the economy begins in region 2. However, the welfare bounds we establish in part (2) are extremely accurate when δ is close to 1. For δ arbitrarily close to 1, equilibrium profits are exactly determined, and the probability of overinvestment is zero.

Suppose that we have $\delta \simeq 1$. Then there is no chance of overinvestment, where a positive fraction of firms invest in the bad state. Consider what happens to welfare as we vary $\underline{c}$. For $\underline{c} = 0$, we have $W^*(0, c_i) = (1-\alpha)(1-c_i)$ and $W^*(1, c_i) = \alpha(1-c_i)$, which implies that we achieve the first-best. Firms receive the profits they would receive if they acted with full knowledge of the state. For small $\underline{c}$, the outcome is nearly first-best efficient. The chance of investment collapse (the fraction investing in the good state is zero) is small. As $\underline{c}$ rises, the chance of investment collapse rises, but if the fraction investing becomes positive, all firms will know that the state is high and invest.

Back to the general case, notice that the characterization in Proposition 4 does not assume anything about the distribution, F, except that it is continuous and strictly increasing! The only feature of F that affects equilibrium profits is the lower support, $\underline{c}$. If we start in region 3 or 4, and consider any firm, then changing the distribution in any way that lowers $\underline{c}$ will have no effect on the firm's expected profits. However, if we start in region 2, then lowering $\underline{c}$ will increase the firm's expected profits. If we start in region 1, then lowering $\underline{c}$ enough to move the economy into another region will increase the firm's expected profits; if we remain in region 1, then there is no effect.

Let us summarize the efficiency properties of large markets. Suppose all firms would delay investment if waiting allows firms to learn the state (region 1 or region 2). Then equilibrium is inefficient, free riding limits information flow and leads to underinvestment. However, equilibrium yields higher welfare than the static game in region 2. When some firms receive favorable enough signals that they would not delay investment, even if waiting would allow them to learn the state (region 3 or region 4), then the market aggregates information efficiently. Even though a single firm has a blunt instrument for conveying information, invest or not, a market with a large number of firms can be highly informative. Our asymptotic results show similarities to Chamley (2004), who considers a model with one-dimensional types.

4. Policy Implications

In this section we consider the welfare implications of subsidies or taxes on investment.[13] One might think that a small subsidy would improve welfare, by increasing information flow as a result of the increased incentive to invest. This is true in Cham-

[13] Of course, more sophisticated mechanisms could achieve complete information outcomes, by having firms report their types and giving firms suggestions about whether to invest. However, such mechanisms seem inconsistent with markets, and would lead to difficulties outside the scope of the present analysis. Moreover, our point in this section is that simple investment policies are likely to be ineffective or worse. Investment incentives should be targeted, to encourage information flow.

ley and Gale (1994), with one-dimensional types.[14] However, the issue is far more subtle with two-dimensional types, because the beneficial effect of encouraging investment by type-1 firms is offset by the harmful effect of encouraging investment by type-0 firms. Let s denote a subsidy given to investors during the round in which they invest (a tax if s is negative). For the case of two firms, the effect of a subsidy on welfare is ambiguous. For $c_i \sim U[0,1]$, $\alpha = 0.75$, $\delta = 0.9$, it turns out that a small investment subsidy is welfare diminishing! On the other hand, if we choose a distribution putting more mass on higher costs, $F(c) = c^3$, then a small subsidy encourages more investment by type-1 firms than type-0 firms, leading to higher welfare.[15]

We now consider the optimal permanent subsidy for arbitrarily large economies.

Proposition 5: *Consider an arbitrarily large economy, and assume $\bar{c} \leq 1$. The optimal subsidy, s^*, is given by*

$$s^* = \max[0, \underline{c} - \frac{\alpha(1-\delta)}{1-\alpha\delta}]. \tag{4.1}$$

From Proposition 5, we see that the optimal subsidy for large economies is either zero, or a targeted subsidy that induces an arbitrarily small fraction of firms to invest in round 0. The purpose of the subsidy is not to generate a lot of investment for its own sake, but to generate enough investment to release a lot of information to the market. Because the subsidy considered in Proposition 5 is permanent, then when the investment return is high, all firms eventually invest and receive the subsidy. However, it is possible to design a temporary subsidy only received by those who invest in round 0, yielding the same welfare as (4.1), but with a per capita subsidy payment arbitrarily close to zero. The optimal temporary subsidy for a large economy is $\max[0, \underline{c}(1-\alpha\delta) - \alpha(1-\delta)]$.[16]

[14] A small subsidy increases welfare in the pure common value case of our model as well. Although type-0 firms are allowed to invest, the equilibrium is such that type-1 firms mix and type-0 firms do not invest initially.

[15] Details available from the authors. Based on differentiating the welfare function with respect to s, we conjecture that subsidies are always welfare diminishing for the uniform distribution, but have not been able to sign the enormous expression.

[16] Investing in round 0 yields a type-1 firm with cost $\underline{c}$ expected profits of $\alpha - \underline{c} + s$. Waiting, and learning the state, yields expected profits of $\alpha\delta(1-\underline{c})$. The (limiting) optimal subsidy makes a type-1 firm with cost $\underline{c}$ indifferent between investing in round 0 and waiting.

5. Conclusions

A reasonable argument is that firms could credibly announce their signals to the market, thereby avoiding the inefficiencies arising in our model due to limited information flow. Similar criticisms could be made to much of the literatures on herding and coordination failures. We would respond to this argument on two levels. First, our analysis benefits from the useful abstraction of eliminating any strategic interaction between firms' investment decisions. Given that a firm invests, its profits depend only on the aggregate state and not on how many other firms invest. However, it remains to be seen whether firms would willingly reveal their favorable information to competing firms.[17] Second, it is costly to send and to receive communication, especially in large markets. In a model that includes a communication decision as well as an investment decision, there may be an incentive for firms to free-ride by not incurring the communication costs. As the size of the market increases, the communication cost might rise, and the information content of a single firm's communication might fall.

Our assumption of conditional independence implies that, for large economies, the state could be known for certain if firms pool their information. Future work will extend the model to information structures in which the state cannot be known with certainty. One idea is to replicate the economy, so that there are n classes of firms, with r identical firms in each class receiving identical signals. As r approaches infinity, the aggregate information possessed by all firms remains constant. A special feature of the replication economy is that firms know that there is a tie for who has the lowest cost, leading to complicated mixed-strategy equilibria, in spite of the continuous investment cost distribution. We are currently exploring this and other information structures.

A macroeconomic interpretation of the model is that the economy is in recession,

[17] Although investment by competing firms might be harmful to investment returns, the opposite might be true. In line with the coordination failure literature, investment by other firms could stimulate economic activity and increase investment returns. Here, the question is whether a firm would want to admit that it has a weak signal.

but the investment climate might have improved. In equilibrium, firms with favorable signals might delay their investment, and there is a positive probability that no one invests, even if the investment climate has improved and the recession should be over. Although this is the implication of the theory, notice that firms with low investment costs are almost indifferent between investing in round 0 and waiting, and that common knowledge of rationality might be a strong assumption in practice. Some type-1 firms might instead see profitable opportunities and invest, as in Keynes' notion of *animal spirits*. The fascinating point here is that this urge to invest can actually improve the informativeness of markets, thereby improving economic efficiency! This phenomenon would be interesting to test experimentally.

6. Appendix: Proofs

Proof of Lemma 1. A type-1 firm that invests in round t receives expected profit, $pr(Z = 1 \mid h^{t-1}, X_i = 1) - c_i$. Suppose the firm does not invest, and instead chooses continuation strategy, s_i. The equilibrium strategies of the other firms and the history, h^{t-1}, determine the expected profit from the continuation strategy, which can be written as $R_1(h^{t-1}, s_i) - \varphi_1(h^{t-1}, s_i)c_i$, where $R_1(h^{t-1}, s_i)$ denotes expected discounted revenue of a type-1 firm and $\varphi_1(h^{t-1}, s_i)c_i$ denotes expected discounted investment cost of a type-1 firm, given h^{t-1} and s_i. If, given the history, h^{t-1}, a type-1 firm with cost c_i invests in round t, it follows that

$$pr(Z = 1 \mid h^{t-1}, X_i = 1) - R_1(h^{t-1}, s_i) \geq [1 - \varphi_1(h^{t-1}, s_i)]c_i \tag{6.1}$$

holds for all continuation strategies, s_i. From (6.1), and the fact that $\varphi_1(h^{t-1}, s_i) < 1$ holds, it follows that (6.1) holds as a strict inequality for all $c_i' < c_i$ and all continuation strategies, s_i. Therefore, if a type-1 firm with cost c_i invests in round t, a type-1 firm with a lower cost also invests in round t, unless it has already invested. An identical argument applies to type-0 firms. Because F is continuous and the support is nondegenerate, the probability that a firm's cost is exactly $\beta_0(h^{t-1})$ or $\beta_1(h^{t-1})$ is zero, so assuming that firms with these cutoff costs invest is without loss of generality. This establishes the interval property of equilibrium. ■

Proof of Proposition 1. We will derive and simplify expressions for the numerator and denominator of $Q(h^{t-1}, k, q_0, q_1)$. These expressions are complicated by the fact that, given the state, we do not know the number of type-1 firms who have not yet invested at the beginning of round t. Let $\overline{\alpha}_0$ denote the probability that a firm is of type-0, given that the state is low, given the history, h^{t-1}, and given that the firm has not invested by the beginning of round t. Similarly, let $\overline{\alpha}_1$ denote the probability that a firm is of type-1, given that the state is high, given the history, h^{t-1}, and given that the firm has not invested by the beginning of round t. From Bayes' rule, we have

$$\overline{\alpha}_0 = pr(X_j = 0 \mid h^{t-1}, Z = 0, j \text{ not invested}) = \frac{1}{1 + \frac{1-\alpha}{\alpha}\left[\frac{1-F(\beta_1(h^{t-2})}{1-F(\beta_0(h^{t-2})}\right]}, \quad (6.2)$$

$$\overline{\alpha}_1 = pr(X_j = 1 \mid h^{t-1}, Z = 1, j \text{ not invested}) = \frac{1}{1 + \frac{1-\alpha}{\alpha}\left[\frac{1-F(\beta_0(h^{t-2})}{1-F(\beta_1(h^{t-2})}\right]}. \quad (6.3)$$

The following probabilities are conditional on firm i not having invested before round t. Let $\overline{n}$ denote the number of firms that have not yet invested before round t, not including firm i. Of these firms, let κ denote the number of type-1 firms, and let k_1 denote the number of these type-1 firms that invest in round t. Then we can write:

$$pr(k^t = k \mid h^{t-1}, Z = 0) = \sum_{k_1=0}^{k}\sum_{\kappa=k_1}^{\overline{n}}\left[\begin{array}{c} \binom{\overline{n}}{\kappa}\overline{\alpha}_0^{\overline{n}-\kappa}(1-\overline{\alpha}_0)^{\kappa}\left[\binom{\kappa}{k_1}q_1^{k_1}(1-q_1)^{\kappa-k_1}\right] \\ \left[\binom{\overline{n}-\kappa}{k-k_1}q_0^{k-k_1}(1-q_0)^{\overline{n}-\kappa-k+k_1}\right]\end{array}\right] \quad (6.4)$$

and

$$pr(k^t = k \mid h^{t-1}, Z = 1) = \sum_{k_1=0}^{k}\sum_{\kappa=k_1}^{\overline{n}}\left[\begin{array}{c} \binom{\overline{n}}{\kappa}\overline{\alpha}_1^{\kappa}(1-\overline{\alpha}_1)^{\overline{n}-\kappa}\left[\binom{\kappa}{k_1}q_1^{k_1}(1-q_1)^{\kappa-k_1}\right] \\ \left[\binom{\overline{n}-\kappa}{k-k_1}q_0^{k-k_1}(1-q_0)^{\overline{n}-\kappa-k+k_1}\right]\end{array}\right]. \quad (6.5)$$

Equations (6.4) and (6.5) can be simplified as follows:

$$pr(k^t = k \mid h^{t-1}, Z = 0) = \binom{\overline{n}}{k}A^k(1-A)^{n-k} \quad \text{and} \quad (6.6)$$

$$pr(k^t = k \mid h^{t-1}, Z = 1) = \binom{\overline{n}}{k}B^k(1-B)^{n-k}, \quad (6.7)$$

where $A \equiv \overline{\alpha}_0 q_0 + (1-\overline{\alpha}_0)q_1$ and $B \equiv (1-\overline{\alpha}_1)q_0 + \overline{\alpha}_1 q_1$. From (2.5), (6.6), and (6.7), we have

$$Q(h^{t-1}, k, q_0, q_1) = \left(\frac{1-A}{1-B}\right)^{\overline{n}} \left(\frac{A(1-B)}{B(1-A)}\right)^{k}. \tag{6.8}$$

Equations (2.6) and (2.7) imply that $P_0(h^{t-1}, k; q_0, q_1)$ and $P_1(h^{t-1}, k; q_0, q_1)$ are increasing in k if and only if $Q(h^{t-1}, k, q_0, q_1)$ is decreasing in k. From (6.8), it follows that $Q(h^{t-1}, k, q_0, q_1)$ is decreasing in k if and only if $B > A$ holds, which is equivalent to the condition,

$$(\overline{\alpha}_0 + \overline{\alpha}_1 - 1)(q_1 - q_0) > 0. \tag{6.9}$$

Algebraic manipulation of equations (6.2) and (6.3) establishes that $\overline{\alpha}_0 + \overline{\alpha}_1 - 1$ must be positive, and the result follows. ■

Proof of Proposition 2. Define $G_0(h^{t-1}, q_0, q_1)$ and $G_1(h^{t-1}, q_0, q_1)$ by

$$\begin{aligned} G_1(h^{t-1}, q_0, q_1) &= \left(P_1(h^{t-1}) - \beta_1(h^{t-1})\right)\left(\frac{1-\delta}{\delta}\right) \\ &\quad - \sum_{k \in K^*(h^{t-1}, q_0, q_1)} \theta_k(h^{t-1}, q_0, q_1, \beta_1(h^{t-1})) \quad \textit{and} \qquad (6.10) \\ G_0(h^{t-1}, q_0, q_1) &= \left(P_0(h^{t-1}) - \beta_0(h^{t-1})\right)\left(\frac{1-\delta}{\delta}\right) \\ &\quad - \sum_{k \in K^{**}(h^{t-1}, q_0, q_1)} \eta_k(h^{t-1}, q_0, q_1, \beta_0(h^{t-1})). \qquad (6.11) \end{aligned}$$

In (6.10) and (6.11), we treat $\beta_0(h^{t-1})$ as a function of q_0 and $\beta_1(h^{t-1})$ as a function of q_1, based on (2.3) and (2.4). These expressions represent the net advantage of investing in round t, as opposed to investing in round $t+1$ if and only if investment remains profitable. We first establish that, after any history, there exist q_0 and q_1 such that one of the following conditions holds: (i) $G_0(h^{t-1}, q_0, q_1) = G_1(h^{t-1}, q_0, q_1) = 0$, (ii) $G_0(h^{t-1}, 0, q_1) < 0$ and $G_1(h^{t-1}, 0, q_1) = 0$, (iii) $G_0(h^{t-1}, q_0, 0) = 0$ and $G_1(h^{t-1}, q_0, 0) < 0$, (iv) $G_0(h^{t-1}, 0, 0) < 0$ and $G_1(h^{t-1}, 0, 0) < 0$. We next establish that these investment probabilities determine an equilibrium.

It is easy to see that $\theta_k(h^{t-1}, q_0, q_1, \beta_1(h^{t-1}))$ and $\eta_k(h^{t-1}, q_0, q_1, \beta_1(h^{t-1}))$ are continuous functions of q_0 and q_1. Suppose (q_0, q_1) is a point of discontinuity in

$K^*(h^{t-1}, q_0, q_1)$, in which k is dropped from the set. This can only occur when $\theta_k(h^{t-1}, q_0, q_1, \beta_1(h^{t-1})) = 0$ holds. Therefore, $G_1(h^{t-1}, q_0, q_1)$ is continuous in (q_0, q_1). An identical argument holds for $G_0(h^{t-1}, q_0, q_1)$. Suppose that a type-1 firm with cost $\beta_1(h^{t-2})$ or a type-0 firm with cost $\beta_0(h^{t-2})$ would receive strictly positive expected profits by investing in round t. Let $\widehat{q}_1$ denote the value of q_1 for which a type-1 firm with the corresponding cost, $\beta_1(h^{t-1})$ based on (2.4), receives zero expected profits by investing in round t, $P_1(h^{t-1}) - \beta_1(h^{t-1}) = 0$. Similarly, let $\widehat{q}_0$ denote the value of q_0 for which a type-0 firm with the corresponding cost, $\beta_0(h^{t-1})$ based on (2.3), receives zero expected profits by investing in round t, $P_0(h^{t-1}) - \beta_0(h^{t-1}) = 0$.[18]

Case 1: $\widehat{q}_1 > \widehat{q}_0$

We know that $G_1(h^{t-1},\ \widehat{q}_1,\ \widehat{q}_1) = 0$ holds, because nothing is learned by market activity in round t, so a type-1 firm with cost $\widehat{q}_1$ receives zero expected profits by investing or waiting. We must have $G_1(h^{t-1},\ q_0,\ \widehat{q}_1) < 0$ for all $q_0 < \widehat{q}_1$, because investing in round t yields zero profits, but observing all remaining firms invest in round t increases the posterior probability of $Z = 1$, so there is a positive option value of waiting. We must have $G_1(h^{t-1},\ q, q) > 0$ for all $q < \widehat{q}_1$, because nothing is learned by market activity in round t, and investment is profitable for a type-1 firm with cost q. In Figure 2, $G_1 < 0$ holds along the entire segment with height $\widehat{q}_1$, between the vertical axis and the 45° line. Also, $G_1 > 0$ holds along the 45° line below the point $(\widehat{q}_1, \widehat{q}_1)$. By continuity, the $G_1 = 0$ manifold must go from the point, $(\widehat{q}_1, \widehat{q}_1)$ to somewhere on the vertical axis, lying below the segment, $q_1 = \widehat{q}_1$ and above the 45° line. Denote the vertical intercept of the $G_1 = 0$ manifold by $\widetilde{q}_1$, which satisfies $G_1(h^{t-1}, 0, \widetilde{q}_1) = 0$.[19]

[18] If investment is unprofitable for a type-i firm, let $\widehat{q}_i = 0$.

[19] Our conjecture is that the vertical intercept is unique, but otherwise let $\widetilde{q}_1$ be the intercept of the $G_1 = 0$ manifold that first reached along the path from the point $(\widehat{q}_1, \widehat{q}_1)$.

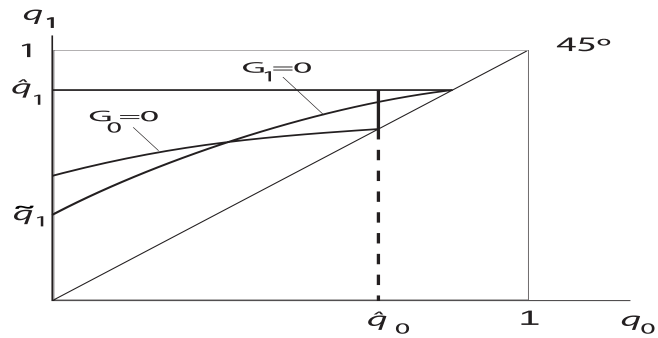

Figure 2

Clearly we must either have $G_0(h^{t-1}, 0, \widetilde{q}_1) < 0$ and $G_1(h^{t-1}, 0, \widetilde{q}_1) = 0$, or else $G_0(h^{t-1}, 0, \widetilde{q}_1) \geq 0$ and $G_1(h^{t-1}, 0, \widetilde{q}_1) = 0$. If the latter is true, then we have $G_0(h^{t-1}, 0, \widetilde{q}_1) \geq 0$ and $G_0(h^{t-1}, \widehat{q}_1, \widehat{q}_1) < 0$ (which follows from $\widehat{q}_1 > \widehat{q}_0$), so by continuity, there must be some point on the $G_1 = 0$ manifold for which we also have $G_0(h^{t-1}, q_0, q_1) = 0$.

Case 2: $\widehat{q}_1 \leq \widehat{q}_0$

By the same reasoning used in case 1, there is a $G_0 = 0$ manifold running from the point, $(\widehat{q}_0, \widehat{q}_0)$ to somewhere on the horizontal axis, lying to the left of the segment, $q_0 = \widehat{q}_0$ and below the 45° line.[20] Denote the horizontal intercept of the $G_0 = 0$ manifold by $\widetilde{q}_0$, which satisfies $G_0(h^{t-1}, \widetilde{q}_0, 0) = 0$ (see Figure 3). Clearly we must either have $G_0(h^{t-1}, \widetilde{q}_0, 0) = 0$ and $G_1(h^{t-1}, \widetilde{q}_0, 0) < 0$, or else $G_0(h^{t-1}, \widetilde{q}_0, 0) = 0$ and $G_1(h^{t-1}, \widetilde{q}_0, 0) \geq 0$. If the latter is true, then we have $G_1(h^{t-1}, \widetilde{q}_0, 0) \geq 0$ and $G_0(h^{t-1}, \widehat{q}_1, \widehat{q}_1) < 0$ (which follows from $\widehat{q}_1 < \widehat{q}_0$), so by continuity, there must be some point on the $G_0 = 0$ manifold for which we also have $G_1(h^{t-1}, q_0, q_1) = 0$.

[20] If we have $\widehat{q}_1 = \widehat{q}_0 = 0$, so that investment is unprofitable for all remaining firms, then $G_0(h^{t-1}, 0, 0) < 0$ and $G_1(h^{t-1}, 0, 0) < 0$ must hold.

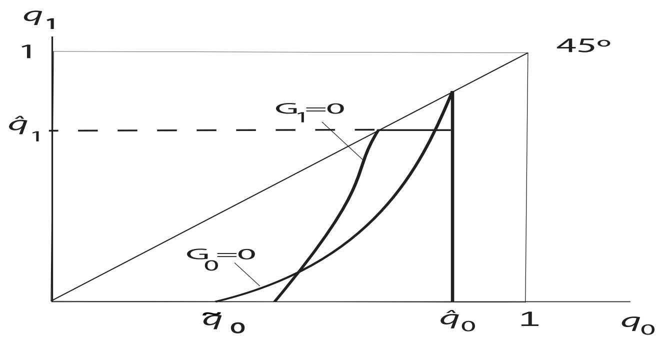

Figure 3

From Proposition 2, there exist k^* and k^{**} such that we have either $K^*(h^{t-1}, q_0, q_1) = \{k^t : k^t < k^*\}$ and $K^{**}(h^{t-1}, q_0, q_1) = \{k^t : k^t < k^{**}\}$, or $K^*(h^{t-1}, q_0, q_1) = \{k^t : k^t > k^*\}$ and $K^{**}(h^{t-1}, q_0, q_1) = \{k^t : k^t > k^{**}\}$. The following argument will consider the case in which we have $K^*(h^{t-1}, q_0, q_1) = \{k^t : k^t < k^*\}$ and $K^{**}(h^{t-1}, q_0, q_1) = \{k^t : k^t < k^{**}\}$, but the argument for the other case is identical. Multiplying (6.10) by $[\alpha + (1-\alpha)H]G_1(h^{t-1}, q_0, q_1)$, and using (2.1) and (2.8), we see that $G_1(h^{t-1}, q_0, q_1)$ is positive (respectively, zero) if and only if

$$\left(\frac{1-\delta}{\delta}\right)[\alpha(1-\beta_1(h^{t-1})) - (1-\alpha)H\beta_1(h^{t-1})] -$$
$$(1-\alpha)H\beta_1(h^{t-1})pr(k^t < k^* \mid Z=0) + \alpha(1-\beta_1(h^{t-1}))pr(k^t < k^* \mid Z=1)$$

is positive (respectively, zero). Now define

$$\begin{aligned} Y_1(h^{t-1}) &= \frac{(1-\alpha)H\beta_1(h^{t-1})}{\alpha(1-\beta_1(h^{t-1}))} \quad \text{and} \\ Q^* &= \frac{pr(k^t < k^* \mid Z=0)}{pr(k^t < k^* \mid Z=1)}. \end{aligned}$$

It follows that $G_1(h^{t-1}, q_0, q_1)$ is positive (respectively, zero) if and only if

$$\left(\frac{1-\delta}{\delta}\right)\left[1 - Y_1(h^{t-1})\right] - \left[Y_1(h^{t-1})Q^* - 1\right] pr(k^t < k^* \mid Z=1) \tag{6.12}$$

is positive (respectively, zero). Defining

$$\begin{aligned} Y_0(h^{t-1}) &= \frac{\alpha H\beta_0(h^{t-1})}{(1-\alpha)(1-\beta_0(h^{t-1}))} \quad \text{and} \\ Q^{**} &= \frac{pr(k^t < k^{**} \mid Z=0)}{pr(k^t < k^{**} \mid Z=1)}, \end{aligned}$$

a similar argument implies that $G_0(h^{t-1}, q_0, q_1)$ is positive (respectively, zero) if and only if

$$\left(\frac{1-\delta}{\delta}\right)\left[1 - Y_0(h^{t-1})\right] - \left[Y_0(h^{t-1})Q^{**} - 1\right] pr(k^t < k^{**} \mid Z=1) \tag{6.13}$$

is positive (respectively, zero).

Claim 1: $G_1(h^{t-1}, q_0, q_1) = G_0(h^{t-1}, q_0, q_1) = 0$ implies that $k^* = k^{**}$, and therefore, $Q^* = Q^{**}$, hold.

Proof of Claim 1: Suppose instead that $k^* < k^{**}$ holds (without loss of generality). Then we have $[Y_0(h^{t-1})Q^{**} - 1]\, pr(k^t < k^{**} \mid Z = 1) = Y_0(h^{t-1})pr(k^t < k^{**} \mid Z = 0) - pr(k^t < k^{**} \mid Z = 1) = [Y_0(h^{t-1})Q^* - 1]pr(k^t < k^* \mid Z = 1) + [Y_0(h^{t-1})Q(h^{t-1}, k^*, q_0, q_1) - 1]pr(k^t = k^* \mid Z = 1) + \cdots + [Y_0(h^{t-1})Q(h^{t-1}, k^{**} - 1, q_0, q_1) - 1]pr(k^t = k^{**} - 1 \mid Z = 1)$. For all $k^t < k^{**}$, investment is unprofitable for a type-0 firm with cost $\beta_0(h^{t-1})$. From (2.6), it follows that $Y_0(h^{t-1})Q(h^{t-1}, k^t, q_0, q_1) -$

$1 > 0$ holds for $k^t = k^*, ..., k^{**} - 1$. Therefore, we have

$$\begin{aligned} \left(\frac{1-\delta}{\delta}\right)\left[1 - Y_0(h^{t-1})\right] &= \left[Y_0(h^{t-1})Q^{**} - 1\right] pr(k^t < k^{**} \mid Z = 1) \\ &> [Y_0(h^{t-1})Q^* - 1] pr(k^t < k^* \mid Z = 1). \end{aligned} \tag{6.14}$$

For $k^* \leq k^t < k^{**}$, investment is profitable for a type-1 firm with cost $\beta_1(h^{t-1})$. From (2.7), it follows that $Y_1(h^{t-1})Q(h^{t-1}, k^t, q_0, q_1) - 1 < 0$ holds for $k^t = k^*, ..., k^{**} - 1$, and therefore, that $Y_0(h^{t-1}) > Y_1(h^{t-1})$ holds. Therefore, we have

$$\begin{aligned} \left(\frac{1-\delta}{\delta}\right)\left[1 - Y_1(h^{t-1})\right] &= \left[Y_1(h^{t-1})Q^* - 1\right] pr(k^t < k^* \mid Z = 1) \\ &< [Y_0(h^{t-1})Q^* - 1] pr(k^t < k^* \mid Z = 1). \end{aligned} \tag{6.15}$$

Comparing (6.14) and (6.15), we conclude

$$\left(\frac{1-\delta}{\delta}\right)\left[1 - Y_0(h^{t-1})\right] > \left(\frac{1-\delta}{\delta}\right)\left[1 - Y_1(h^{t-1})\right],$$

which implies $Y_1(h^{t-1}) > Y_0(h^{t-1})$, a contradiction.

This establishes the fact that $G_1(h^{t-1}, q_0, q_1) = G_0(h^{t-1}, q_0, q_1) = 0$ implies that $k^* = k^{**}$, and therefore, $Q^* = Q^{**}$, hold. From (6.12), (6.13), and the fact that $Q^* > 1$ must hold, simple algebra establishes $Y_0(h^{t-1}) = Y_1(h^{t-1})$ and therefore, (2.12).

Claim 2: $G_1(h^{t-2}, q_0(h^{t-2}), q_1(h^{t-2})) = G_0(h^{t-2}, q_0(h^{t-2}), q_1(h^{t-2})) = 0$ implies $G_0(h^{t-1}, q_0, q_1) = 0$ if and only if $G_1(h^{t-1}, q_0, q_1) = 0$.

Proof of Claim 2: Suppose instead that we have $G_0(h^{t-1}, q_0, q_1) = 0$ and $G_1(h^{t-1}, q_0, q_1) < 0$. This implies

$$\left(\frac{1-\delta}{\delta}\right)\left[1 - Y_0(h^{t-1})\right] = \left[Y_0(h^{t-1})Q^{**} - 1\right] pr(k^t < k^{**} \mid Z = 1) \text{ and} \tag{6.16}$$

$$\left(\frac{1-\delta}{\delta}\right)\left[1 - Y_1(h^{t-2})\right] < \left[Y_1(h^{t-2})Q^* - 1\right] pr(k^t < k^* \mid Z = 1). \tag{6.17}$$

Suppose $k^* \geq k^{**}$ holds. Therefore, the right side of (6.17) equals

$$\begin{aligned} \left[Y_1(h^{t-2})Q^{**} - 1\right] pr(k^t &< k^{**} \mid Z = 1) + \\ [Y_1(h^{t-2})Q(h^{t-1}, k^{**}, q_0, q_1) - 1]pr(k^t &= k^{**} \mid Z = 1) + \cdots \\ +[Y_1(h^{t-2})Q(h^{t-1}, k^* - 1, q_0, q_1) - 1]pr(k^t &= k^* - 1 \mid Z = 1). \end{aligned} \tag{6.18}$$

Since realizations of k^t below k^* are unprofitable for a type-1 firm with cost $\beta_1(h^{t-2})$, all terms in (6.18) are positive, from which we conclude

$$\left[Y_1(h^{t-2})Q^* - 1\right] pr(k^t < k^* \mid Z = 1) > \left[Y_1(h^{t-2})Q^{**} - 1\right] pr(k^t < k^{**} \mid Z = 1). \tag{6.19}$$

By a similar argument, we can show that $k^* < k^{**}$ also implies (6.19). From (6.16), (6.19), and $Q^{**} > 1$, we can show $Y_1(h^{t-2}) > Y_0(h^{t-1})$, which implies

$$\frac{\beta_0(h^{t-1})}{1 - \beta_0(h^{t-1})} < \frac{\beta_1(h^{t-2})}{1 - \beta_1(h^{t-2})} \frac{(1-\alpha)^2}{\alpha^2}.$$

From the hypothesis of the claim, we have

$$\frac{\beta_0(h^{t-2})}{1 - \beta_0(h^{t-2})} = \frac{\beta_1(h^{t-2})}{1 - \beta_1(h^{t-2})} \frac{(1-\alpha)^2}{\alpha^2}.$$

Therefore, $\beta_0(h^{t-2}) > \beta_0(h^{t-1})$, a contradiction. An identical argument holds if we suppose that $G_0(h^{t-1}, q_0, q_1) < 0$ and $G_1(h^{t-1}, q_0, q_1) = 0$ holds. This establishes Claim 2.

Claim 3: $G_1(h^{t-2}, q_0(h^{t-2}), q_1(h^{t-2})) = 0$ and $G_0(h^{t-2}, q_0(h^{t-2}), q_1(h^{t-2})) < 0$ implies $G_0(h^{t-1}, q_0, q_1) = 0$ only if $G_1(h^{t-1}, q_0, q_1) = 0$.

Proof of Claim 3: Suppose instead that we have $G_0(h^{t-1}, q_0, q_1) = 0$ and $G_1(h^{t-1}, q_0, q_1) < 0$. By the same reasoning as that given in the proof of Claim 2, we reach a contradiction.

Construct the equilibrium values of $q_0(h^{t-1})$ and $q_1(h^{t-1})$ as follows. After any history, h^{t-1}, if we have $\widehat{q}_0 = 0$ and $\widehat{q}_1 = 0$, then investment ceases. If we have $\widehat{q}_1 > \widehat{q}_0$, we find investment probabilities that either solve $G_0 = 0$ and $G_1 = 0$ (above the 45° line), or $G_0 < 0$, $q_0 = 0$, and $G_1 = 0$ (above the 45° line). If we have $\widehat{q}_1 \leq \widehat{q}_0$, we find investment probabilities that solve $G_0 = 0$ and $G_1 = 0$ below the 45° line or $G_0 = 0$, $q_1 = 0$, and $G_1 < 0$ (below the 45° line). However, in round 0, either investment is unprofitable for all firms, or else we have $\widehat{q}_1 > \widehat{q}_0$, in which case $G_1 = 0$ must hold. If we have $G_1 = 0$ and $G_0 = 0$ in round t, then by Claim 2, either investment ceases or both types invest in round $t+1$ with positive probability. If we have $G_1 = 0$ and $G_0 < 0$ in round t, then by Claim 3, either investment ceases or type-1 firms invest in round $t+1$ with positive probability. Thus, the only corner solutions involve investment ceasing, or an initial phase of the game in which type-1 firms invest with positive probability and type-0 firms strictly prefer to wait.

The above construction guarantees that no firm has a profitable deviation to invest after a history in which it should remain uninvested. Also, no firm that is supposed to invest after any history, h^{t-1}, has a profitable deviation in which it never invests after round $t+1$. If we can demonstrate that no other deviations are profitable, then the one-step property holds, and we are done.

Claim 4: The one-step property holds.

Proof of Claim 4 (by induction): Suppose that there is a history, h^{t-1}, with **two** uninvested firms, where a type-1 firm has a profitable deviation not to invest, and let ε denote the gain in profits. Then a type-1 firm with cost $\beta_1(h^{t-1})$ has a profitable deviation that involves **not** investing in round $t+1$, after some profitable realizations of k^t. Let us denote one such realization by $\overline{k}^t$. If $\overline{k}^t > 0$ holds, then all other firms have invested, and there in nothing to be gained by waiting. However, investment is profitable, a contradiction. Thus, $\overline{k}^t = 0$ holds. Consider the type-1 firm with cost $\beta_1(h^{t-1}, 0)$, who is indifferent between investing in round $t+1$, and investing in round $t+2$ following every profitable k^{t+1}. Since this firm is indifferent, the firm with the lower cost, $\beta_1(h^{t-1})$, must strictly prefer to invest after $(h^{t-1}, 0)$ than after every profitable k^{t+1}. Thus, the type-1 firm with cost $\beta_1(h^{t-1})$ has a profitable deviation that involves **not** investing in round $t+2$, after some profitable realizations of k^{t+1}. Let us denote one such realization by $\overline{k}^{t+1}$. By the above argument, we have $\overline{k}^{t+1} = 0$. Therefore, for any positive integer T, the deviation must involve not investing until after round $t+T$. Because $\delta < 1$ holds, the gain in profits is below ε for sufficiently large T, a contradiction.

Suppose the one-step property holds whenever the number of uninvested firms is

$j-1$ or fewer, but there is a history, h^{t-1}, with j uninvested firms, where a type-1 firm has a profitable deviation not to invest, and let ε denote the gain in profits. Then a type-1 firm with cost $\beta_1(h^{t-1})$ has a profitable deviation that involves **not** investing in round $t+1$, after some profitable realizations of k^t. Let us denote one such realization by $\overline{k}^t$. If $\overline{k}^t > 0$ holds, then consider the type-1 firm with cost $\beta_1(h^{t-1}, \overline{k}^t)$, who is indifferent between investing in round $t+1$, and investing in round $t+2$ following every profitable k^{t+1}. Since this firm is indifferent, the firm with the lower cost, $\beta_1(h^{t-1})$, must strictly prefer to invest after $(h^{t-1}, \overline{k}^t)$ than after every profitable k^{t+1}. Since there are $j-1$ or fewer uninvested firms, the firm with cost, $\beta_1(h^{t-1})$, must strictly prefer to invest after $(h^{t-1}, \overline{k}^t)$ than follow any other continuation strategy, a contradiction. Thus, $\overline{k}^t = 0$ holds, and the deviation involves **not** investing in round $t+2$, after some profitable realizations of k^{t+1}. Let us denote one such realization by $\overline{k}^{t+1}$. By the above argument, we must have $\overline{k}^{t+1} = 0$. Therefore, for any T, a type-1 firm with cost $\beta_1(h^{t-1})$ has a deviation following h^{t-1} whose gain is bounded below the profit from not investing for T consecutive rounds, then investing if profitable. Because $\delta < 1$ holds, the gain in profits is below ε for sufficiently large T, a contradiction. ■

Proof of Proposition 3. Consider parameters in region 1. Even a type-1 firm with cost $\underline{c}$ would receive negative expected profits by investing, so no firm invests in round 0. Therefore, nothing is learned from past behavior, so there is no investment in subsequent rounds.

Consider parameters in region 2. Suppose we have $\beta_1^* > \beta_0^* \geq \underline{c}$. For sufficiently large n, the law of large numbers implies the following. If we have $Z = 1$, with probability arbitrarily close to one, the fraction of firms investing in round 0 is arbitrarily close to $\alpha F(\beta_1^*) + (1-\alpha)F(\beta_0^*)$. If we have $Z = 0$, with probability arbitrarily close to one, the fraction of firms investing in round 0 is arbitrarily close to $\alpha F(\beta_0^*) + (1-\alpha)F(\beta_1^*)$. Because these fractions are different, firms can infer the true state from round 0 activity, with probability arbitrarily close to one. A type-1 firm with cost c receives expected profits of $\alpha - c$ by investing in round 0, but would receive arbitrarily close to $\delta\alpha(1-c)$ by waiting until round 1. Because $\frac{\alpha(1-\delta)}{1-\alpha\delta} < \underline{c}$ holds, it follows that all type-1 firms should wait until round 1, contradicting $\beta_1^* > \underline{c}$. If we have $\beta_0^* > \beta_1^* \geq \underline{c}$, once again firms can infer the state from round 0 activity, and we reach the same contradiction.

Suppose we have $\beta_0^* = \beta_1^* > \underline{c}$. For all $\varepsilon > 0$, there exists N such that $n > N$ implies $\beta_1^n < \beta_1^* + \varepsilon$ and $\beta_0^n > \beta_0^* - \varepsilon$. Therefore, if firm i is type-1 with cost $c_i = \beta_1^* + \varepsilon$, it prefers not to invest in round 0, choosing instead some other strategy, s_i^n. Thus, we have

$$\alpha - \beta_1^* - \varepsilon < \alpha\pi(s_i^n, \beta_1^* + \varepsilon \mid Z = 1) + (1 - \alpha)\pi(s_i^n, \beta_1^* + \varepsilon \mid Z = 0),$$

which implies

$$\alpha - \beta_1^* - \varepsilon < \alpha\pi(s_i^n, \beta_1^* \mid Z = 1) + (1 - \alpha)\pi(s_i^n, \beta_1^* \mid Z = 0) + \varepsilon, \tag{6.20}$$

where $\pi(s, c \mid Z = z)$ denotes the discounted expected profits for a firm with investment cost c, playing the strategy s, given that the state is z.[21] If firm i is type-0 with cost $c_i = \beta_0^* - \varepsilon$, it prefers to invest in round 0, rather than choosing the strategy, s_i^n. Thus, we have

$$1 - \alpha - \beta_0^* + \varepsilon > (1 - \alpha)\pi(s_i^n, \beta_0^* - \varepsilon \mid Z = 1) + \alpha\pi(s_i^n, \beta_0^* - \varepsilon \mid Z = 0),$$

which implies

$$1 - \alpha - \beta_0^* + \varepsilon > (1 - \alpha)\pi(s_i^n, \beta_0^* \mid Z = 1) + \alpha\pi(s_i^n, \beta_0^* \mid Z = 0) - \varepsilon. \tag{6.21}$$

From (6.20), (6.21), and $\beta_0^* = \beta_1^*$, we have

$$0 < (2\alpha - 1)\left[\pi(s_i^n, \beta_1^* \mid Z = 1) - \pi(s_i^n, \beta_1^* \mid Z = 0) - 1\right] + 4\varepsilon. \tag{6.22}$$

Because $\pi(s_i^n, \beta_1^* \mid Z = 1) < \delta(1 - \beta_1^*)$ and $\pi(s_i^n, \beta_1^* \mid Z = 0) > -\delta\beta_1^*$ must hold, it follows that the term in brackets in (6.22) is negative. Therefore, for sufficiently small ε, we have a contradiction. The remaining cases are: (i) $c_i = \beta_1^* + \varepsilon$ (which is impossible), $\beta_1^* = \overline{c}$ (which is also impossible), or $\beta_0^* = \beta_1^* = \underline{c}$.

Consider parameters in region 3. Suppose we have $\beta_1^* \neq \beta_0^*$. By the law-of-large-numbers argument given above for region 2, for sufficiently large n, a firm not

[21] Since there are no histories to observe and we are conditioning on the true state, firm i's common value signal, X_i, provides no additional information about expected revenues. Therefore, $\pi(s, c \mid Z = z)$ is independent of a firm's common value signal.

investing in round 0 will be able to infer the true state arbitrarily precisely. Based on the parameter range for region 3, all type-0 firms would rather learn the state in round 1 than invest in round 0. A type-1 firm with cost c_i would rather invest in round 0 than learn the state in round 1 if and only if we have $c_i < \frac{\alpha(1-\delta)}{1-\alpha\delta}$. Therefore, we must have $\beta_0^* = \underline{c}$ and $\beta_1^* = \frac{\alpha(1-\delta)}{1-\alpha\delta}$.

Suppose we have $\beta_0^* = \beta_1^* = \underline{c}$. For a type-1 firm with cost, $c_i < \frac{\alpha(1-\delta)}{1-\alpha\delta}$, investing in round 0 strictly dominates any other strategy, so we cannot have $\beta_1^* = \underline{c}$.

Suppose we have $\overline{c} > \beta_0^* = \beta_1^* > \underline{c}$. By the identical argument given above for region 2, we reach a contradiction. The remaining possibility for region 3 is $\beta_0^* = \beta_1^* = \overline{c}$, which is impossible.

Consider parameters in region 4. Suppose we have $\beta_1^* \neq \beta_0^*$. By the law-of-large-numbers argument given above for region 2, for sufficiently large n, a firm not investing in round 0 will be able to infer the true state arbitrarily precisely. Based on the parameter range for region 4, a type-0 firm with cost c_i would rather invest in round 0 than learn the state in round 1 if and only if we have $c_i < \frac{(1-\alpha)(1-\delta)}{1-(1-\alpha)\delta}$. A type-1 firm with cost c_i would rather invest in round 0 than learn the state in round 1 if and only if we have $c_i < \frac{\alpha(1-\delta)}{1-\alpha\delta}$. Therefore, we must have $\beta_0^* = \frac{(1-\alpha)(1-\delta)}{1-(1-\alpha)\delta}$ and $\beta_1^* = \frac{\alpha(1-\delta)}{1-\alpha\delta}$.

Suppose we have $\beta_0^* = \beta_1^* = \underline{c}$. For a type-1 firm with cost, $c_i < \frac{\alpha(1-\delta)}{1-\alpha\delta}$, investing in round 0 strictly dominates any other strategy, so we cannot have $\beta_1^* = \underline{c}$.

Suppose we have $\overline{c} > \beta_0^* = \beta_1^* > \underline{c}$. By the identical argument given above for region 2, we reach a contradiction. The remaining possibility for region 4 is $\beta_0^* = \beta_1^* = \overline{c}$, which is impossible. ■

Proof of Proposition 4. Part (1) is obvious. Suppose the parameters are in region 3, and consider a type-1 firm with investment cost below β_1^*. Then Proposition 3 implies that for sufficiently large n, this firm will invest in round 0, in which case $W^*(1, c_i) = \alpha - c_i$ holds. All other firms will not invest in round 0, for sufficiently large n, due to the assumption, $\overline{c} \geq 1$. If other firms invest in round 1 if and only if we have $k^0/n \geq F(\beta_1^*)/2$, then by the law of large numbers, the probability of investing in the low state converges to zero, and the probability of investing in the high state converges to one. Since the probability of the high state is α for a type-1 firm and $1-\alpha$ for a type-0 firm, part (3) of Proposition 8 follows. The same argument applies to region 4, except that the firms that do not invest in round 0 should invest in round 1 if and only if we have $k^0/n \geq [F(\beta_0^*) + (F(\beta_1^*)]/2$.

Suppose the parameters are in region 2. For the equilibrium of the economy with n firms, let $E^{t,\varepsilon}$ be the event that a type-1 firm with investment cost $\underline{c} + \varepsilon$ invests in round t, and define $T_1^n(1,\varepsilon) = \sum_{t=1}^{\infty} \delta^t pr(Z = 1 \text{ and } E^{t,\varepsilon} \mid X_i = 1)$, define $T_1^n(0,\varepsilon) = \sum_{t=1}^{\infty} \delta^t pr(Z = 0 \text{ and } E^{t,\varepsilon} \mid X_i = 1)$, define $T_0^n(1,\varepsilon) = \sum_{t=1}^{\infty} \delta^t pr(Z = 1$ and $E^{t,\varepsilon} \mid X_i = 0)$, and define $T_0^n(0,\varepsilon) = \sum_{t=1}^{\infty} \delta^t pr(Z = 0 \text{ and } E^{t,\varepsilon} \mid X_i = 0)$.

Clearly, $W^n(1, c_i)$ must be continuous and decreasing in c_i, because otherwise some type-1 firm would have a profitable deviation to imitate the strategy chosen by a type-1 firm with nearby investment cost. For all $\varepsilon_1 > 0$, there exists $\varepsilon_2 > 0$, such that $\varepsilon < \varepsilon_2$ implies

$$W^n(1, \underline{c}) - W^n(1, \underline{c} + \varepsilon) < \varepsilon_1. \tag{6.23}$$

We have $W^n(1, \underline{c}) = \alpha - \underline{c}$ (they invest in round 0) and $W^n(1, \underline{c} + \varepsilon) = T_1^n(1,\varepsilon)(1 - \underline{c} - \varepsilon) - T_1^n(0,\varepsilon)(\underline{c} + \varepsilon)$. Imposing $\varepsilon < \min[\varepsilon_1, \varepsilon_2]$, it follows from (6.23) that we have

$$\alpha - \underline{c} - T_1^n(1,\varepsilon)(1 - \underline{c}) + T_1^n(0,\varepsilon)(\underline{c}) < 2\varepsilon_1. \tag{6.24}$$

Claim: For all c_i and all $\varepsilon > 0$, there exists N such that $n > N$ implies

$$W^n(1, c_i) \geq \delta T_1^n(1,\varepsilon)(1 - c_i) - \varepsilon_1. \tag{6.25}$$

Proof of Claim: Consider a type-1 firm with investment cost c_i, which waits until the round after a type-1 firm with cost $\underline{c}+\varepsilon$ would invest. That is, we are considering histories such that $\beta_1^n(h^{t-1}) \geq \underline{c}+\varepsilon$. By the law of large numbers, for sufficiently large n, with probability arbitrarily close to one, the cumulated investment in state 1 is arbitrarily close to $\alpha F(\beta_1^n(h^{t-1}))+(1-\alpha)F(\beta_0^n(h^{t-1}))$, and the cumulated investment in state 0 is arbitrarily close to $(1-\alpha)F(\beta_1^n(h^{t-1})) + \alpha F(\beta_0^n(h^{t-1}))$. A type-1 firm with cost $\beta_1^n(h^{t-1})$ is indifferent between investing in round t and some continuation strategy, s, in which the probability of eventual investment is less than one. (This fact follows from $\delta < 1$ and our assumption, $\overline{c} \geq 1$, which guarantees that there are type-1 firms with costs above $\beta_1^n(h^{t-1})$ that do not invest.) However, a type-0 firm with the same cost, $\beta_1^n(h^{t-1})$, must strictly prefer the continuation strategy, s, because $P_1(h^{t-1}) > P_0(h^{t-1})$. Thus, we conclude that $\beta_0^n(h^{t-1}) < \beta_1^n(h^{t-1})$ must hold. By investing in round $t+1$ if and only if we have $\beta_1^n(h^{t-1}) \geq \underline{c} + \varepsilon$ and

$$\frac{\sum_{\tau=1}^{t} k^\tau}{n} \geq \frac{F(\beta_0^n(h^{t-1})) + F(\beta_1^n(h^{t-1}))}{2},$$

the probability of investing in the high state (when we have $\beta_1^n(h^{t-1}) \geq \underline{c} + \varepsilon$) is arbitrarily close to one, and the probability of investing in the low state (when we have $\beta_1^n(h^{t-1}) \geq \underline{c} + \varepsilon$) is arbitrarily close to zero. Thus, adopting this strategy yields expected profit arbitrarily close to $\delta T_1^n(1,\varepsilon)(1-c_i)$, thereby establishing the Claim.

From (6.24), and the fact that $T_1^n(0,\varepsilon) \geq 0$, we have

$$T_1^n(1,\varepsilon) \geq \frac{\alpha - \underline{c}}{1-\underline{c}} - \frac{2\varepsilon_1}{1-\underline{c}}. \tag{6.26}$$

From (6.25) and (6.26), we have

$$W^n(1,c_i) \geq \delta\left[\frac{\alpha-\underline{c}}{1-\underline{c}}\right](1-c_i) - \delta\left[\frac{2\varepsilon_1}{1-\underline{c}}\right](1-c_i) - \varepsilon_1. \tag{6.27}$$

Letting $c_i = \underline{c}$ hold in (6.25), we have $\delta T_1^n(1,\varepsilon)(1-\underline{c}) - \varepsilon_1 \leq \alpha - \underline{c}$, and using (6.24), this becomes $\delta T_1^n(1,\varepsilon)(1-\underline{c}) - \varepsilon_1 < T_1^n(1,\varepsilon)(1-\underline{c}) - T_1^n(0,\varepsilon)(\underline{c}) + 2\varepsilon_1$, implying

$$T_1^n(0,\varepsilon)(\underline{c}) < T_1^n(1,\varepsilon)(1-\underline{c})(1-\delta) + 3\varepsilon_1. \tag{6.28}$$

A type-1 firm with cost c_i cannot possibly do better than to decide whether or not to invest during the same round that firm $(1, \underline{c}+\varepsilon)$ invests, but with full knowledge of the state. Therefore, we have

$$W^n(1,c_i) \leq T_1^n(1,\varepsilon)(1-c_i). \tag{6.29}$$

Since firm $(1,\underline{c})$ weakly prefers to invest in round 0, rather than mimic the strategy of firm $(1, \underline{c} + \varepsilon)$, we have $\alpha - \underline{c} \geq T_1^n(1, \varepsilon)(1 - \underline{c}) - T_1^n(0, \varepsilon)(\underline{c})$. Thus, we have

$$T_1^n(1, \varepsilon)(1 - \underline{c}) \leq \alpha - \underline{c} + T_1^n(0, \varepsilon)(\underline{c}). \tag{6.30}$$

Using (6.28) and (6.30), we have $T_1^n(1, \varepsilon)(1 - \underline{c}) \leq \alpha - \underline{c} + T_1^n(1, \varepsilon)(1 - \underline{c})(1 - \delta) + 3\varepsilon_1$, from which we have

$$T_1^n(1, \varepsilon) \leq \frac{\alpha - \underline{c}}{\delta(1 - \underline{c})} + \frac{3\varepsilon_1}{\delta(1 - \underline{c})}. \tag{6.31}$$

From (6.29) and (6.31), we conclude

$$W^n(1, c_i) \leq \left[\frac{\alpha - \underline{c}}{\delta(1 - \underline{c})}\right](1 - c_i) + \frac{3\varepsilon_1(1 - c_i)}{\delta(1 - \underline{c})}. \tag{6.32}$$

For type-0 firms, a simple calculation yields

$$\begin{aligned} T_0^n(1, \varepsilon) &= \frac{1 - \alpha}{\alpha} T_1^n(1, \varepsilon) \quad \text{and} \\ T_0^n(0, \varepsilon) &= \frac{\alpha}{1 - \alpha} T_1^n(0, \varepsilon). \end{aligned}$$

The law-of-large-numbers argument given in the Claim also establishes

$$\begin{aligned} W^n(0, c_i) &\geq \delta T_0^n(1, \varepsilon)(1 - c_i) - \varepsilon_1, \text{ implying} \\ W^n(0, c_i) &\geq \delta \frac{1 - \alpha}{\alpha} T_1^n(1, \varepsilon)(1 - c_i) - \varepsilon_1. \end{aligned} \tag{6.33}$$

From (6.26) and (6.33), it follows that

$$W^n(0, c_i) \geq \delta \left[\frac{1 - \alpha}{\alpha}\right]\left[\frac{\alpha - \underline{c}}{1 - \underline{c}}\right](1 - c_i) - \delta \left[\frac{1 - \alpha}{\alpha}\right]\left[\frac{2\varepsilon_1}{1 - \underline{c}}\right](1 - c_i) \tag{6.34}$$

holds. A type-0 firm with cost c_i cannot possibly do better than to decide whether or not to invest during the same round that firm $(1, \underline{c} + \varepsilon)$ invests, but with full knowledge of the state. Therefore, we have

$$W^n(0, c_i) \leq T_0^n(1, \varepsilon)(1 - c_i) = .\frac{1 - \alpha}{\alpha} T_1^n(1, \varepsilon)(1 - c_i). \tag{6.35}$$

Combining (6.31) and (6.35), we have

$$W^n(0, c_i) \leq \left[\frac{1-\alpha}{\alpha}\right]\left[\frac{\alpha - \underline{c}}{\delta(1-\underline{c})}\right](1-c_i) + \left[\frac{1-\alpha}{\alpha}\right]\frac{3\varepsilon_1(1-c_i)}{\delta(1-\underline{c})}. \tag{6.36}$$

Because inequalities (6.29), (6.32), (6.34), and (6.36) hold for all ε_1, the results for region 2 follow. ■

Proof of Proposition 5. To verify (4.1), notice that the subsidy lowers a firm's effective cost from c_i to $c_i - s$. The equilibrium for the subsidized economy is identical to the equilibrium of a translated economy, with no subsidy and costs given by the effective costs. First consider an economy in which the initial parameters lie in region 1 or region 2 of Figure 1. By choosing $s^* = \underline{c} - \frac{\alpha(1-\delta)}{1-\alpha\delta}$, the translated economy is on the boundary between region 2 and region 3. The fraction of firms that invest in round 0 is zero, but the amount of investment in round 0 reveals the aggregate state (in the limit). From Proposition 4, a type-1 firm with investment cost c_i generates welfare of $\delta\alpha(1-c_i)$. For any subsidy, $s < s^*$, the fraction of firms investing in round 0 is zero, but the amount of investment in round 0 is not fully revealing. Therefore, the welfare generated by a type-1 firm with investment cost c_i is lower with s than with s^*. For any subsidy, $s > s^*$, the fraction of firms that invest in round 0 is positive, and the amount of investment in round 0 reveals the aggregate state. A type-1 firm with $c_i < s - s^* + \underline{c}$ generates welfare of $\alpha - c_i$, and a type-1 firm with $c_i \geq s - s^* + \underline{c}$ generates welfare of $\delta\alpha(1-c_i)$. Since the initial parameters are in region 1 or region 2, we have $\delta\alpha(1-c_i) > \alpha - c_i$ for all c_i, so conditional on being a type-1 firm, welfare is higher with the subsidy s^* than with the subsidy s. A similar argument applies to type-0 firms.

Next consider an economy in which the initial parameters lie in region 3 or region 4. By choosing $s^* = 0$, a type-1 firm with $c_i < \frac{\alpha(1-\delta)}{1-\alpha\delta}$ generates welfare of $\alpha - c_i$, and a type-1 firm with $c_i \geq \frac{\alpha(1-\delta)}{1-\alpha\delta}$ generates welfare of $\delta\alpha(1-c_i)$. Now consider a subsidy, $s \neq 0$, such that the translated economy remains in region 3 or region 4. A type-1 firm with $c_i < s + \frac{\alpha(1-\delta)}{1-\alpha\delta}$ generates welfare of $\alpha - c_i$, and a type-1 firm with $c_i \geq s + \frac{\alpha(1-\delta)}{1-\alpha\delta}$ generates welfare of $\delta\alpha(1-c_i)$. Thus, if s is positive, the net welfare effect of the subsidy is to cause type-1 firms with $c_i \in (\frac{\alpha(1-\delta)}{1-\alpha\delta}, \frac{\alpha(1-\delta)}{1-\alpha\delta} + s)$ to invest in round 0; because they were not investing in round 0 without the subsidy, we have

$\delta\alpha(1-c_i) > \alpha - c_i$ for all c_i in this interval, so conditional on being a type-1 firm, welfare is higher with the subsidy $s^* = 0$ than with the subsidy s. If s is negative, the net welfare effect of the subsidy is to cause type-1 firms with $c_i \in (\frac{\alpha(1-\delta)}{1-\alpha\delta} + s, \frac{\alpha(1-\delta)}{1-\alpha\delta})$ *not* to invest in round 0; because they were investing in round 0 without the subsidy, we have $\delta\alpha(1-c_i) < \alpha - c_i$ for all c_i in this interval, so conditional on being a type-1 firm, welfare is higher with the subsidy $s^* = 0$ than with the subsidy s. Finally, if s is such that the translated economy is in region 1 or region 2, then there is a welfare loss from causing some type-1 firms not to invest in round 0, and there is an additional welfare loss from the fact that the amount of investment in round 0 does not reveal the aggregate state. A similar argument applies to type-0 firms. $\square$

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