On the Interplay of Informational Spillovers and Payoff Externalities*

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Abstract

Informational spillovers induce agents to outwait each other's actions in order to make more informed decisions. If waiting is costly we expect the best informed agent, who has the least to learn from other agents' decisions, to take the first action. In this paper we study the interplay between informational spillovers and a direct payoff externality. We show that when the payoff externality is positive or relatively weak, the above intuition is validated. On the contrary, if the externality is negative and strong the best informed agent has the most to gain from outwaiting the other.

1 Introduction

A fundamental insight in information economics is that, in a world where information is seldom perfect, one agent's actions often carry valuable information for other agents. Such informational spillovers are interesting both from a welfare perspective (like any externality, it gives rise to inefficiencies), and from a behavioral perspective, since agents take the informational value of their own actions as well as those of others into account.

In this paper we extend the study on informational spillovers to situations where agents' payoffs also are directly linked through their actions. This payoff externality could represent numerous real-life phenomena, such as pollution, movements in asset

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prices, or network effects. Indeed, we can think of few circumstances where informational spillovers are present but payoff externalities are not. The focus of the paper is how the externality, depending on its sign and size, affects the timing of actions.

We model a situation where two firms are about to enter a new product market. A firm must both decide where to position itself in product space (the firm's "niche"), as well as when to enter the market. Demand varies depending on which niche a firm chooses, but the exact demand structure is unknown. Both firms have some private information which they use - possibly together with the other firm's entry decision - to infer where demand is high. The payoff externality reflects how firms interact in the market: a negative externality means that products are strategic substitutes (which drives firms further apart in product space), a positive externality that products are strategic complements (which drives firms towards the same niche). Delay is costly so the presence of informational spillovers leads to a waiting game between the firms. Importantly, the only asymmetry between firms ex ante is the precision of their private signals. Our main result is that if products are close substitutes, the *worst* informed firm enters first in equilibrium.

In the studies on herd behavior by Banerjee (1992) and Bikhchandani, Hirschleifer and Welch (1992), agents make a binary decision in an exogenously given order. The outcome of the two alternatives is uncertain and each agent receives an imperfect signal of which alternative is the better. Agents' interests are fully aligned so the only impact of one agent's choice on another is the informational value his or her action provides. Zhang (1997) employs the same setting but endogenizes the decision order. Moreover, each agent's accuracy is now private knowledge so that any agent with less than perfect information could potentially benefit from observing another agent's action. By assuming delay to be costly, Zhang shows that the best informed agent takes the first action in the unique symmetric PBE. The intuition is of course that poorly informed agents (in expectation) have more to gain from observing others' decisions. In fact, this result was conjectured by Bikhchandani, Hirschleifer and Welch (1992).

Other papers that study waiting games with asymmetrically informed agents include Fudenberg and Tirole (1986), Hendricks and Kovenock (1989), Bolton and Farrell

(1990), and Gul and Lundholm (1995). However, none of these papers study the interplay of informational spillovers and direct externalities.[1] Fudenberg and Tirole (1986) study endogenous exit in a duopoly game. Firms differ with respect to their opportunity cost of exiting the market and this cost is private knowledge. By assuming that there is an ex ante positive probability that duopoly is profitable (unlike in the classic war of attrition), the authors derive a unique equilibrium where high-cost firms exit before low-cost ones. In Gul and Lundholm two agents receive a signal of the value of a project. Their task is to correctly estimate the size of the project, which always equals the sum of the two signals. The second agent will thus be able to forecast the project with certainty, hence a strong informational spillover is present. Due to discounting, a high signal – indicating high future profits – implies that delay is more costly. As a result, in the symmetric PBE types with higher signals make their estimates first.[2]

Less related papers include Shaked and Sutton (1982) and Judd (1985). Judd models a market with two product niches, where an incumbent has the opportunity to occupy both niches (to "crowd" the market) in order to preempt entry. Judd shows that if exit costs are low and the products are substitutes – though not *too* close substitutes – the incumbent refrains from crowding and allows entry. Shaked and Sutton study entry and quality choice in a vertical product market. They show that if entry is costly and the market only can sustain two products, exactly two firms enter in equilibrium and they choose distinct product qualities.

This paper is organized as follows. Section 2 sets up the model. Section 3 contains the results. We focus on a symmetric equilibrium where the firms' waiting strategy is strictly monotone and differentiable. These strategies are invertible, which makes it easy to characterize the information that transpires from the waiting game. Section 4

[1] Rob (1991) studies sequential entry in a market with demand uncertainty. In his model, like in the present, both informational spillovers and direct payoff externalities are present. However, in his model firms are ex ante (before entry) identical.

[2] Importantly, Gul and Lundholm identify a phenomenon they call *anticipation*. Briefly put, as delay decreases monotonically with the signal, the first agent to make an estimate (in equilibrium) will rationally revise her expectation of the project's size downwards - simply because the other agent has not made an estimate so far. In the present model there is a phenomenon closely related to anticipation: in the symmetric equilibrium we investigate the second firm is able to infer the first firm's informational quality, which allows it to make a more precise estimate.

concludes and discusses some extensions. All proofs are found in the Appendix.

2 The Model

Two firms $i \in \{A, B\}$ will enter a horizontal product market. Each firm must make two decisions: it must choose a product design $\theta_i \in \Re$, and when to enter the market $t_i \in [0, \infty) = T$. Entry decisions are irreversible and the first firm's decision is observed by the other firm. Consider the following payoff function:

$$\pi_i(\theta_i, \theta_j, t_i) = -(\theta_i - \rho)^2 - \alpha(\theta_i - \theta_j)^2 - \delta t_i, \ i \neq j.$$

Profits depend on entry decisions in two ways. First, firm i's profit decreases quadratically with the distance between θ_i and ρ, where ρ is an unknown parameter. Firms receive an unbiased signal of ρ, such that $\rho_i = \rho + \epsilon_i$, where ϵ_i is a normally distributed r.v. with mean zero and variance v_i. Hence, each firm has two pieces of private information, a signal of the state and the precision of that signal. In words, ρ represents the (a priori) most profitable market niche. If θ_i is far from ρ the firm has chosen an unattractive product design, an event that is more likely the higher v_i is. We assume that variances are drawn from a non-atomic distribution $\Psi(v)$ with density function $\psi(v) > 0$ for all $v \in [0, \bar{v}]$, where $\bar{v}$ is finite. All draws are conditionally independent.

Second, both firms' profits vary quadratically with the distance between θ_A and θ_B, which captures the market interaction between firms. Parameter α characterizes how firms interact. If α is positive (negative), products are strategic complements (substitutes), since firms benefit from decreasing (increasing) the distance between θ_A and θ_B. If $\alpha = 0$, there is no direct externality and we have a case of pure informational spillovers. For example, think of a market where product-specific marketing also increases generic demand. If advertising costs are significant, firms should launch similar products so as to maximally exploit the spillovers from each other's marketing. If advertising costs are relatively small, or the spillovers from the other firm's marketing are insignificant, similar designs will only intensify competition so firms should differentiate their products.

Finally, firm i:s payoff decreases linearly in t_i, the time the firm enters the market.[3] This could for example reflect the fact that corporate resources are tied up as long as the decision is delayed, resources that could have been put to use elsewhere. Parameter $\delta > 0$ measures the degree of urgency: the higher is δ the more costly it is to delay the entry decision.

The Waiting Game

Since signals are unbiased the signal realization has no effect on the incentive to learn the other firm's information, and should therefore have no effect on the timing decision. This allows us to consider the entry decision (the choice of θ) and the timing decision (the choice of t) separately. Consider first the entry decision. There are two possibilities, either firm i enters as the leader or as the follower. Below, we impose sufficient conditions to ensure that the leader's and the follower's optimal decision, as a function of their available information, are unique. In turn, this allows us to characterize expected payoffs in terms of variances v_A and v_B only, which is done in the next section. For now, denote the leader's and follower's expected payoff (excluding delay) to be $L_i(v_i, v_j)$ and $F_i(v_i, v_j)$, respectively.

Consider now a firm's waiting strategy. Since delay is costly, once one firm has entered the other will follow as soon as possible. For simplicity we assume that there is no involuntary delay, so that both firms' delay is determined by the leader's choice.[4] Hence, it is sufficient to consider strategies that are conditioned on the fact that the other firm has not entered. Because Ψ is atomless, we may as usual restrict attention to

[3]The linear cost simplifies matters but is not necessary. As long as delay costs are separable, our results hold for all strictly increasing and differentiable functions $f(t_i)$. However, each function will give rise to a different stopping function (see Proposition 3). Our results do not extend easily to the case of multiplicative delay costs.

[4]This assumption simplifies matters since we do not have to be concerned with the "monopoly profits" the first firm would make before the other firm enters. However, the introduction of a period of monopoly profits would not change the results as long as these profits are not too large compared to overall profits.

"stopping time" strategies $s_i : V \to T$, $i = A, B$. That is, a (pure) waiting strategy is a mapping from a firm's variance to a nonnegative number: the time the firm will enter given that the other firm has not entered up to that moment. We restrict attention to equilibria where strategies are strictly monotone and differentiable. This simplifies the analysis in at least two important ways. First, since variances are drawn from a non-atomic distribution, the event that both firms enter at the same time is a set of probability measure zero, so that event can be ignored. Second, each strategy has an inverse function $\mathrm{s}_i^{-1}(\cdot)$ that maps each point in time to a unique variance. This means that when the first firm enters the other firm can infer its variance.

Suppose that firm i draws variance v_i and that the firms use strategies $s_i(v)$ and $s_j(v)$. Firm i:s expected payoff can then be written $(i \neq j)$

$$\hat{\pi}_i(s_i, s_j, v_i) =$$

$$\int\limits_{v_j \in \{V : s_i(v_i) < s_j(v_j)\}} (L_i(v_i, v_j) - \delta s_i(v_i)) d\Psi dv_j + \int\limits_{v_j \in \{V : s_i(v_i) > s_j(v_j)\}} (F_i(v_i, v_j) - \delta s_j(v_j)) d\Psi dv_j. \qquad (1)$$

Let μ_i be firm i:s posterior over firm j:s variance. A strategy profile $s = \{s_i, s_j\}$ and a belief system $\mu = \{\mu_i, \mu_j\}$ constitute a perfect Bayesian equilibrium if $\hat{\pi}_i$ and $\hat{\pi}_j$ are maximized given μ and the other firm's strategy, and μ is consistent with s in terms of Bayesian updating.[5] In the following we shall focus on the symmetric PBE, which is natural given the ex ante symmetry between firms. That is, we impose the condition

[5] Formally, let maps $S_i^V : V_i \to S_i$ be a firm's set of pure strategies in the "expanded game" (Fudenberg and Tirole 1996), i.e., before v_i is realized. The strategy profile $s = \{s_i, s_j\}$ is a perfect Bayesian equilibrium if for each firm,

$$s_i(\cdot) \in \arg\max_{\acute{s}_i \in S_i^V} \int\limits_{v_i} \int\limits_{v_j} \hat{\pi}_i(\acute{s}_i(v), s_j(v), (v_i, v_j)) \left(\psi(v)\right)^2 dv_i dv_j, \; i \neq j.$$

$$s_A(v) = s_B(v), \quad \forall v \in V.$$

3 Results

3.1 Entry Decisions

We now characterize the leader's and follower's expected payoff as a function of their variances. In order to guarantee interior equilibria we must put a lower bound on α, i.e., we have to assume that products are not too close substitutes. If they were, there would be an equilibrium where firms ignored their private information and resorted to "maximum differentiation", which, in fact, would result in infinite profits. Alternatively, the bound below ensures that the leader actually uses its private information.[6]

Assumption 1 $\alpha > \frac{\sqrt{5}-3}{2} \approx -0.38$.

Lemma 1 *Suppose firm A is the leader. Let m denote B's conditional expectation of ρ (on observing θ_A). Under Assumption 1, in a perfect Bayesian equilibrium firm A sets*

$$\theta_A = \rho_A, \tag{2}$$

and firm B sets

$$\theta_B = \frac{m + \alpha\theta_A}{1+\alpha}. \tag{3}$$

Proof. See the Appendix. ■

[6] See Neven (1985) for a location game where the outcome is maximum differentiation.

The expressions when B is the leader are analogous. The expression in (3) captures the follower's trade-off between the informational value the leader's entry provides and the externality it imposes. If α is positive (the marketing spillover dominates the effects of competition) the follower will choose a design closer to the leader's as compared with its expectation of ρ. If α is negative the follower will choose a design less similar to the leader's. The fact that the leader chooses its design according to its signal is important: thereby the follower has de facto access to both signals. Since the follower also infers the leader's variance, its expectation of ρ is simply a linear combination of ρ_A and ρ_B. Expected payoffs therefore take simple expressions.

Lemma 2 *Excluding delay costs, firm A:s expected payoff from being the leader and the follower is, respectively,*

$$L_A = -v_A - \frac{\alpha v_A^2}{(1+\alpha)^2(v_A + v_B)}, \tag{4}$$

and

$$F_A = -\frac{v_B\left(v_B\alpha + (1+\alpha)v_A\right)}{(1+\alpha)\left(v_A + v_B\right)}. \tag{5}$$

Proof. See the Appendix. ■

The corresponding payoffs for firm B are analogous. To reiterate, under Assumption 1 the leader's expected payoff decreases with its variance ($\frac{\partial L_A}{\partial v_A} < 0$). However, how the leader's payoff varies with the follower's variance depends on the payoff externality. Intuitively, the less informed the following firm is, the more inclined it will be to imitate the leader's choice, rather than to trust its own information. Poorly informed followers therefore (on average) choose positions that are closer to the leader. Hence, if products are substitutes the leader benefits from having a well informed follower ($\frac{\partial L_A}{\partial v_B} < 0$).

The latter effect is key to our main result, namely that if products are substitutes the well-informed firm may have the stronger incentive to wait.

3.2 Timing

When considering their waiting strategy firms strike a trade-off between expected delay costs and the possibility of being the follower instead of the leader. (This possibility is not necessarily of positive value, but in the symmetric equilibrium it is never negative.) Consider firm A. Conditional on B's strategy and its own variance v_A, A chooses an optimal entry time, t. Equivalently, A can choose the variance v that, given B's strategy, corresponds to t. For example, suppose firm B uses an increasing strategy, $s_B(v)$. Using the payoff expressions (4) and (5) in the maximization problem (1) gives the v which must maximize the sum of

$$-\int_{v}^{\bar{v}} \left(v_A + \frac{\alpha v_A^2}{(1+\alpha)^2 (v_A + v_B)} + \delta \mathrm{s}_B(v) \right) \psi(v_B) \mathrm{d}v_B \tag{6}$$

and

$$-\int_{0}^{v} \left(\frac{v_B \left(v_B \alpha + (1+\alpha) v_A \right)}{(1+\alpha)\left(v_A + v_B \right)} + \delta \mathrm{s}_B(v_B) \right) \psi(v_B) \mathrm{d}v_B. \tag{7}$$

The integral in (6) is firm A's expected payoff from being the leader, which happens if A picks a v that is smaller than v_B. Likewise, the integral in (7) is the expected payoff from being the follower, which happens when v is larger than v_B. The case of decreasing strategies is completely analogous. We are now ready to present our main result. The proposition uses the following definition:

Definition 1 $\alpha^* = \frac{\sqrt{13}-5}{4} \approx -.35$.

Proposition 1

(a) If $\alpha > \alpha^*$ *the best informed firm enters first. The equilibrium stopping function is*

$$s(v) = \frac{(2\alpha + 1)}{2\delta (1 + \alpha)^2} \left[\bar{v} \ln \bar{v} - v - \bar{v} \ln(\bar{v} - v) \right].$$

(b) If $\alpha < \alpha^*$ *the worst informed firm enters first. The equilibrium stopping function is*

$$\mathrm{s}(v) = \frac{(2\alpha + 1)}{2\delta (1 + \alpha)^2} (\bar{v} - v).$$

(c) If $\alpha = \alpha^*$*, both equilibria are possible.*

Proof. See the Appendix. ■

When products are complements or weak substitutes the time a firm is prepared to wait is increasing in its variance – much like in Zhang (1997). Here, Farrell and Saloner's (1986) "penguin effect" dominates: firms wait because they hope to get more information about the unknown demand parameter ρ. Moreover, the function is exponential so poorly informed firms are prepared to wait a disproportionately longer time than better informed ones. The stopping function for case (a) is illustrated in Figure 1.

Figure 1 about here.

As α is reduced, i.e., as products become closer and closer substitutes, a well informed firm's incentive to outwait a poorly informed firm becomes relatively stronger. The intuition is as follows. When products are substitutes the follower always chooses a niche too close to the leader's - from the leader's point of view. Moreover, the worse informed the follower is relative to the leader, the more the follower relies on the leader's choice. Hence, a poorly informed follower imposes a larger externality on the leader than a well-informed one, and more so the lower is α.

When α passes below α^*, a well-informed firm's incentive to wait becomes stronger than that of a poorly informed firm's. For this parameter region, the negative externality a poorly informed firm imposes as a follower, through its "penguin-like" behavior, is larger than the informational spillover a well informed firm would generate as a leader. Alternatively, when competition is sufficiently harmful (or marketing spillovers are sufficiently small) a well informed firm gains more from enjoying "monopoly" in a good market niche than what a poorly informed firm loses from choosing a bad niche.[7] The stopping function for case (b) is illustrated in Figure 2.

Figure 2 about here.

Delay

We conclude by some remarks on delay. Inspection of the stopping functions in Proposition 1 gives that delay is proportional to the factor

$$\frac{(2\alpha+1)}{2\delta\left(1+\alpha\right)^2}. \tag{8}$$

Note that expected delay decreases geometrically with δ and thus, that delay *costs* are independent of the degree of urgency. Differentiating (8) w.r.t. α gives

$$-\frac{\alpha}{\delta\left(1+\alpha\right)^3}.$$

[7] Note that the decision to delay only reveals something about the firm's informational quality (v_i), not its information about market properties (ρ_i). Hence, unlike in Mailath (1993) the (potential) second-mover advantage does not disappear because of the endogeneous timing.

Ceteris paribus, the longest delay occurs when $\alpha = 0$. Whenever an externality is present, the following firm to some extent chooses a niche based on the leader's choice, rather than according to market information. Hence, the stronger the externality – whether positive or negative – the less important (relatively) becomes product design per se. This reduces the incentive to observe the other firm's decision, and decreases delay.[8] Note from (8) that delay appears to go to zero as α goes to $-.5$. However, Proposition 1 presumes that the leader enters in accordance with its signal, which is no longer optimal if $\alpha < -.38$ (the limit in Assumption 1).

Figure 3 about here.

Note that with a positive payoff externality, delay decreases relatively slowly as the externality grows stronger. Hence, firms suffer substantial delay costs despite there being large gains from sharing information. This is not very realistic: at some point these gains will induce firms to overcome any potential coordination costs. In particular, if firms can engage in cheap talk they can reach the first-best solution whenever $\alpha \geq 0$ by revealing their private information and entering without delay (at the same location).

4 Conclusion

Informational spillovers induce agents to outwait each other in order to make more informed decisions themselves. If delay is costly the presence of spillovers leads to a classic war of attrition between agents. Zhang (1997) showed that if agents have different informational precisions, the best informed agent takes the first action in a symmetric equilibrium. In this paper we combine informational spillovers with a direct

[8] Though not modeled here, the presence of more than two firms should strengthen the effect of the payoff externality. Suppose that the leader in our model instead was followed by n firms of the same type. The higher is n, the more beneficial it becomes to generate an informational spillover if products are complements, and the more costly it becomes if products are substitutes. This effect is similar to that of magnifying α, which reduces delay. As a consequence, adding more firms should also increase the threshhold α^*, requiring less intense competition for the least informed firm to enter first.

payoff externality. Still, the only difference between agents ex ante is the quality of their private information.

The addition of the direct externality has two effects on the waiting game. First, it reduces delay per se. The stronger is the externality – whether positive or negative – the smaller becomes the (relative) importance of being well informed. This attenuates the second-mover advantage and decreases delay. Interestingly, the externality may have a more qualitative effect. When the externality is negative and very strong, it turns out that poorly informed agents take action before well-informed ones. The intuition is that poorly informed agents mimic the behavior of others to a larger extent. Hence, as a follower they impose a larger negative externality on the leader than do well-informed agents. If the externality is sufficiently strong this effect outweighs informational concerns, which makes well-informed agents wait the longest.

We have illustrated this mechanism as an entry game between two firms. In this context the direct externality has a straightforward interpretation as a measure of the strategic complementarity/substitutability between products. However, the model should apply to any situation where informational spillovers and payoff externalities co-exist. For example, the agents could be investors in the stock market. A trading decision has a direct effect on the price of the asset in question, but also reveals something about the investor's private information or expectations. Will a purchase trigger other investors to buy or sell the stock? Gamblers in betting markets with moving odds face a similar situation. As a political application, consider candidates choosing what policy platform to adopt on a complex issue. Not only does a candidate want to endorse policies that appeal to a large share of the electorate, he may also be anxious to represent a policy that stands out from those of other politicians. Hence, the order in which politicians take stands may depend on how well informed they are as well as how badly they need publicity. It may be important to recognize that, in some circumstances, the politicians who choose policies first are those with the least knowledge, and that the sooner a politician decides, the less he knows.

5 Appendix

For ease of exposition, we let $v_A = a$ and $v_B = b$ in the entire appendix. Let the cumulative distributions $\mathrm{G}(\rho)$ and $\mathrm{H}(\rho_j)$ denote firm i:s posterior of ρ and ρ_j $(i \neq j)$, respectively.

Proof of Lemma 1.

Given θ_A, firm B solves the following problem:

$$\underset{\theta_B}{Max} \int_{\rho} \left[-(\theta_B - \rho)^2 - \alpha(\theta_B - \theta_A)^2 \right] d\mathrm{G}(\rho).$$

Let $m = E[\rho \mid \rho_B, \theta_A]$ denote B:s expectation of ρ. The first-order condition reads

$$-2\theta_B(1+\alpha) + 2m + 2\alpha\theta_A = 0. \tag{A1}$$

As long as $\alpha > -1$, the LHS of (A1) is everywhere decreasing in θ_B so that the first-order condition gives a global maximum. Rearrange (A1) to get

$$\theta_B = \frac{m + \alpha\theta_A}{1+\alpha},$$

which proves the second part of the lemma. Anticipating this, the leader (firm A) solves

$$\underset{\theta_A}{Max} \iint_{\rho_B, \rho} \left[-(\theta_A - \rho)^2 - \alpha(\theta_A - \frac{m + \alpha\theta_A}{1+\alpha})^2 \right] d\mathrm{G}(\rho) d\mathrm{H}(\rho_B).$$

In a perfect Bayesian equilibrium, firm B must have the correct expectation of ρ_A. Suppose therefore, without loss of generality, that B:s expectation of ρ is a linear combination of the two signals, i.e., $m = \lambda\rho_A + (1-\lambda)\rho_B$ for some $\lambda \in [0,1]$. Firm A:s expectation of ρ is simply ρ_A. Firm A:s first-order condition then reads

$$-2\theta_A + 2\rho_A - \frac{\alpha}{(1+\alpha)^2}\left[2\theta_A - 2\int_{\rho_B}(\lambda\rho_A + (1-\lambda)\rho_B)dG(\rho_B)\right] = 0. \qquad \text{(A2)}$$

Both estimators are unbiased so $E[\rho_B] = \rho_A$. (A2) becomes

$$-2\left(\theta_A - \rho_A\right)\left[1 + \frac{\alpha}{(1+\alpha)^2}\right] = 0. \qquad \text{(A3)}$$

If the expression in square brackets is positive, the derivative is everywhere decreasing in θ_A and (A3) gives a global maximum. This occurs as long as $\alpha > \frac{\sqrt{5}-3}{2}$, i.e., as long as Assumption 1 is satisfied. The solution is, naturally, to set $\theta_A = \rho_A$.

Proof of Lemma 2.

Consider first the case when A is the follower. By Lemma 1, $\theta_B = \rho_B$, so upon observing B:s entry decision and its own signal ρ_A, firm A's posterior distribution over ρ is normal with expected value

$$m = \frac{b\rho_A + a\rho_B}{a+b},$$

and variance

$$w = \frac{ab}{a+b}.$$

Conditional on observing θ_B, A's expected payoff is

$$\int_{\rho} \left[-(\theta_A - \rho)^2 - \alpha(\theta_B - \theta_A)^2\right] d\mathrm{G}(\rho).$$

Substitute for $E[\rho]$ and $E[\rho^2]$ and the equilibrium expressions for θ_A and θ_B and extend the expression by $(1+\alpha)$ to get

$$\frac{-(w+m^2)(1+\alpha) + 2\alpha m \rho_B + m^2 - \alpha \rho_B^2}{1+\alpha}.$$

Substituting for m and w and extending by $(a+b)^2$ gives

$$b\frac{-ab - \alpha ab - a^2 - \alpha a^2 - \alpha b \rho_A^2 + 2\alpha b \rho_A \rho_B - \alpha b \rho_B^2}{(a+b)^2 (1+\alpha)}.$$

We want the "unconditional" expectation of this (i.e., before A observes θ_B). Since the two estimators are unbiased and conditionally independent, we have that, conditional on ρ_A, $E[\rho_B] = \rho_A$ and $E[\rho_B^2] = \rho_A^2 + a + b$, $\forall \rho_A$. Hence, we have

$$\int_{\rho_B} \left(b\frac{-ab - \alpha ab - a^2 - \alpha a^2 - \alpha b \rho_A^2 + 2\alpha b \rho_A \rho_B - \alpha b \rho_B^2}{(a+b)^2 (1+\alpha)} \right) d\mathrm{H}(\rho_B)$$

$$= b\frac{-ab - \alpha ab - a^2 - \alpha a^2 - \alpha b \rho_A^2 + 2b\alpha \rho_A^2 - b\alpha[\rho_A^2 + a + b]}{(a+b)^2 (1+\alpha)}$$

$$= \frac{-b\,(b\alpha + (1+\alpha)a)}{(1+\alpha)\,(a+b)},$$

which proves the second part of the lemma. Now suppose A is the leader. In equilibrium, its expected payoff is (ignoring delay costs)

$$\int_{\rho_B}\int_{\rho} [-(\theta_A - \rho)^2 - \alpha(\theta_A - \rho_B)^2] d\mathrm{G}(\rho) d\mathrm{H}(\rho_B)$$

$$= -a - \alpha \int_{\rho_B} \left(\rho_A - \left(\frac{b\rho_A + a\rho_B}{(a+b)(1+\alpha)} + \frac{\alpha\rho_A}{1+\alpha} \right) \right)^2 d\mathrm{H}(\rho_B).$$

Extend the integral by $(1+\alpha)(a+b)^2$ and rearrange to get

$$= -a - \frac{\alpha}{(1+\alpha)^2(a+b)^2} \int_{\rho_B} \left[\rho_A^2 a^2 - 2\rho_A \rho_B a^2 + \rho_B^2 a^2 \right] d\mathrm{H}(\rho_B).$$

Finally, substitute for $E[\rho_B^2]$ to get

$$= -a - \frac{\alpha a^2}{(1+\alpha)^2(a+b)}.$$

Proof of Proposition 1.

(a) $\alpha > \alpha^*$. Suppose that firm B uses an increasing strategy $\mathrm{s}(v)$ so that firm A:s posterior over B:s variance at time t, given that no firm has entered, ranges over $[\mathrm{s}^{-1}(t), \bar{v}]$.

Firm A chooses v to maximize

$$-\int_{v}^{\bar{v}} \left(a + \frac{\alpha a^2}{(1+\alpha)^2(a+b)} + \delta s(v)\right) \psi(b)db$$

$$-\int_{0}^{v} \left(\frac{b\,(b\alpha + (1+\alpha)a)}{(1+\alpha)\,(a+b)} + \delta s(b)\right) \psi(b)db.$$

The first-order condition w.r.t. v reads

$$\frac{3\alpha a^2 + a^2 + a^2\alpha^2 - v^2\alpha - v^2\alpha^2}{(1+\alpha)^2\,(a+v)} - \delta s'(v)[\bar{v} - v] = 0. \tag{A4}$$

Firm B naturally solves the analogous problem. Following Gul and Lundholm's (1995) style of proof, if there is a symmetric PBE, (A4) is satisfied for all $v = a$. In other words, in equilibrium both firms must find it optimal to use the strategy that they postulate the other firm uses. Let us confirm that (A4) indeed yields the maximum. The simplest way of doing this is to differentiate (A4) w.r.t. a instead of v. If the resulting second-order condition is *positive* at $v = a$, we know that (A4) gives a maximum. Differentiating (A4) w.r.t a gives

$$\frac{3\alpha a^2 + 6a\alpha v + a^2 + 2av + a^2\alpha^2 + 2a\alpha^2 v + v^2\alpha + v^2\alpha^2}{(1+\alpha)^2\,(a+v)^2}.$$

Setting $v = a$ gives

$$\frac{10\alpha + 3 + 4\alpha^2}{4\,(1+\alpha)^2}. \tag{A5}$$

As long as $\alpha > \alpha^*$, (A5) is positive. Further, in the increasing equilibrium we have the boundary condition that s(0) = 0. Otherwise a firm with variance zero would suffer a positive delay cost yet enter first almost surely. Setting $v = a$ in (A4) gives

$$\frac{a(2\alpha+1)}{2\left(1+\alpha\right)^2\left[\bar{v}-a\right]} = \delta s'(a).$$

Integrate and use the boundary condition to get case (a) of the proposition.

(b) $\alpha < \alpha^*$. Suppose instead that firm B uses a decreasing strategy so that firm A's posterior over B:s variance at time t ranges over $[0, s^{-1}(t)]$. Firm A chooses v to maximize

$$-\int_0^v \left[a + \frac{\alpha a^2}{(1+\alpha)^2(a+b)} + \delta s(v)\right]\psi(b)db$$

$$-\int_v^{\bar{v}} \left[\frac{b\left(b\alpha + (1+\alpha)a\right)}{(1+\alpha)\left(a+b\right)} + \delta s(b)\right]\psi(b)db.$$

The first-order condition reads

$$-\left(\frac{a^2 + 3\alpha a^2 + a^2\alpha^2 - v^2\alpha - \alpha^2 v^2}{\left(1+\alpha\right)^2\left(a+v\right)}\right) - \delta s'(v)v = 0. \tag{A6}$$

In analogy with case (a), (A6) gives a maximum as long as $\alpha < \alpha^*$. Setting $v = a$ in (A6) gives

$$-\frac{(1+2\alpha)}{2\left(1+\alpha\right)^2} = \delta s'(a).$$

In the decreasing equilibrium we have the boundary condition $s(\bar{v}) = 0$. Using this proves case (b) of the proposition. Finally, if $\alpha = \alpha^*$, the second derivative is exactly zero so that both equilibria are possible.

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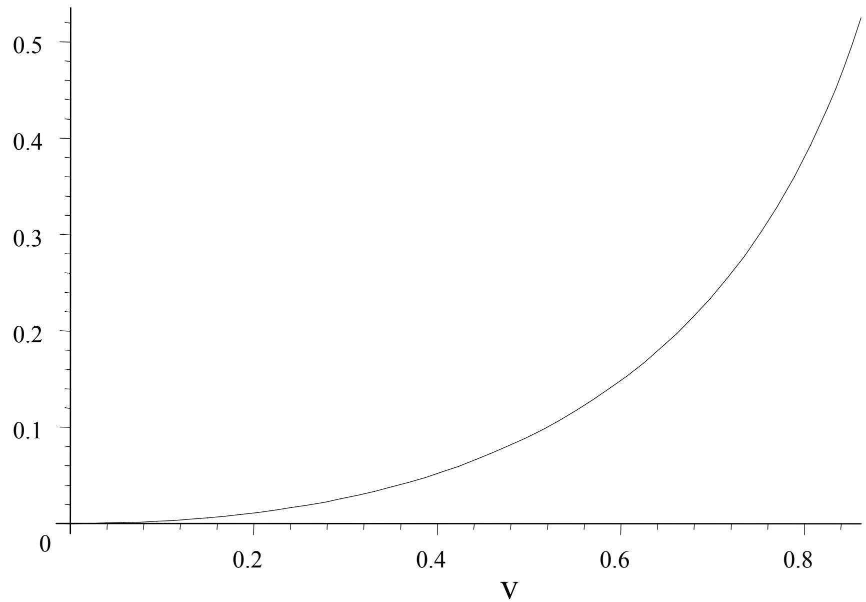

Figure 1. Stopping time as a function of variance when products are weak substitutes (v = 1, δ = 1, α = -0.2).

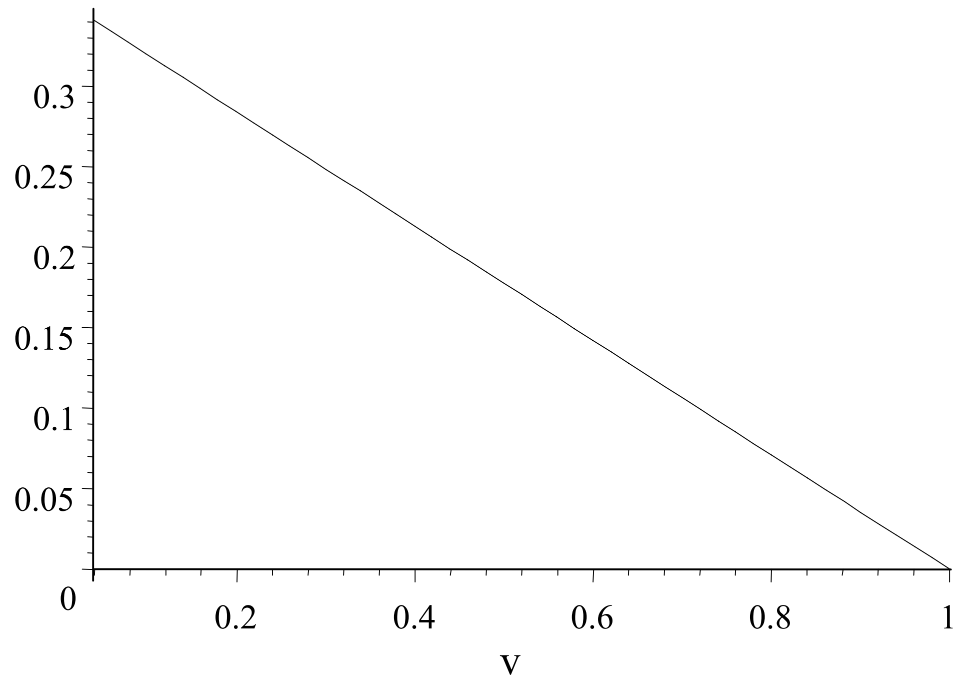

Figure 2. Stopping time as a function of variance when products are strong substitutes (v = 1, δ = 1, α = -0.35)

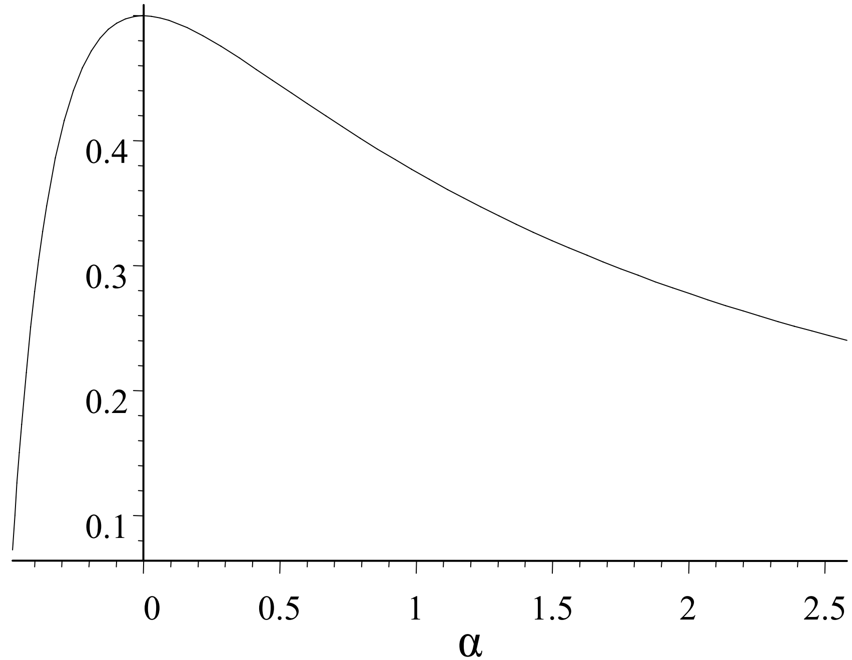

Figure 3. Expected delay as a function of α.