

Degeneracy of choice and the spontaneous formation of trends

Luís M. A. Bettencourt

Santa Fe Institute, 1399 Hyde Park Rd., Santa Fe NM 87501

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Abstract

It is well documented that people do not always behave as rational agents. Particularly when making choices under limited information, or large uncertainty, certain general coping strategies emerge that involve the observation and qualified imitation of the choices of others. These strategies can lead to spontaneous herding, i.e. to the formation of social *trends*, large consensus that emerge out of self-interested individual choices. Social trends or fashions are spontaneous collective decisions made by large portions of a community, often without an apparent good reason. The spontaneous formation of trends provides a well documented mechanism for the spread of information across a population, the creation of culture and the self-regulation of social behavior. Here I introduce a dynamical model that captures the essence of trend formation and collapse. The resulting population dynamics alternates states of great diversity (large configurational entropy) with spontaneous herding, i.e. the dominance by a few trends. This behavior displays a kind of self-organized criticality, measurable through cumulants analogous to those used to study percolation. I also analyze the robustness of trend dynamics subject to external influences, such as population growth or contraction and in the presence of explicit information biases. The resulting population response gives insights about the fragility of public opinion in specific circumstances and suggests how it may be driven to produce social consensus or dissonance.

1 Introduction

Making choices about which social circles to join or evade is one of the most ubiquitous, important and difficult decisions facing each one of us every day. The basis for this difficulty is that our social environment is too complex for us to predict the detailed outcome of our actions. It is also a dynamical environment so that past experience may be a poor indicator of future events. As a result many of our most important choices must be made within a limited timespan and without full information.

These practical limitations make many different courses of action seem equally viable. This degeneracy can be described mathematically as a symmetry among all equally good choices at the individual level. This *choice degeneracy* is also familiar from information saturated environments, where useful information is not easily discriminated from noise, of times when information is accessible but not reliable, or may simply result from a choice not to form our own opinions.

How do we actually make choices in these difficult environments? In many cases we rely on the *actions* of others we know as the basis for our decisions. If their choices appear successful to us we may adopt them as our own. Whether this is a good or bad strategy depends then on how well informed our acquaintances are. In any case we are guaranteed not to do worse than most of the people that surround us, which may be all that matters. In this light the spontaneous emergence of collective behavior corresponds to a particular choice, made simultaneously by many agents, among others that are equally good - a spontaneous breaking of individual *choice symmetry* by the state of the population as a whole.

The strategy to base our decisions on the actions of others is very universal. The natural languages, for example, are rich in related aphorisms. Recently this decision making strategy has become the focus of a sizable literature in economics [1] and the social sciences [2]. Bikhchandani, Hirshleifer and Welch [3] (BHW) were, to my knowledge, the first to stress the universal nature of trends and fashions [4]. They also proposed a simple model of sequential choice in which they can develop [3, 5]. BHW called the widespread adoption of a particular decision an *information cascade*. The term refers to how a piece of information can spread quickly through the whole population as a trend. They also collected a vast literature, across many scientific disciplines in support of trend dynamics as an important social mechanism [2, 4].

In financial markets for example, where collecting quantitative data *a posteriori* is relatively easy, analysts [6] and mutual fund managers [7] are observed to follow each other's choices and recommendations. In elections it is well documented that opinion poles influence the decision process [8]. Cycles of innovation in industrial production [9] are determined in part by the success of one's industrial peers and/or by the fear of lagging behind or losing market share. Television programming displays similar patterns [10]. The incidence of crime or fraud [11], e.g. tax evasion, depends in part on the observation of others who may have gotten away with it.

There has also been a growing interest in related ideas [12, 13] in the literature of complex systems and critical phenomena. Close relatives of trend dynamics are models of flocking [14] and *herd behavior* [5, 15, 16]. The latter have been developed to model financial markets, the former to describe how groups of animals may coordinate their movements to form a flock and, more generally, the collective dynamics of self-propelled particles. Flocking requires the spontaneous breaking of rotational symmetry, which happens when the velocity of most moving agents aligns to form the familiar pattern

of organized collective motion.

In financial models of herd behavior agents are grouped together by given rules. Each of these groups or herds then makes collective decisions (buy/sell) in unison. Trend dynamics is somewhat different: each agent actually tries to stay ahead of the crowd, trading his position for another only if the latter offers greater promise. As we shall see below trend dynamics leads to *spontaneous* consensus even as all agents exercise free will. In this sense the class of models considered here will differ from those describing herd behavior, although some aspects of the emergent collective behavior may be similar.

Trend setting and trend following can often be thought of as a game, to be played many times by a large number of participants. For maximal advantage one would like to join a winning trend at the earliest possible opportunity, ride it to the height of its popularity, and leave before it collapses. However, because this strategy is shared by all players, it leads to some choices becoming very widespread. Once it is apparent that the same state is shared by all social advantage is lost and some agents are tempted to leave. At this point dominant trends become unstable to decay due to competition from new faster movements.

Thus we see qualitatively the two most important features of trend dynamics: the faster a trend is growing the greater its promise i.e. the more attractive it feels. Consequently, when a trend becomes widespread and cannot sustain its pace of growth, it loses its appeal. Then agents begin to look elsewhere for the next 'hot thing'. The purpose of the present paper is to capture the essence of trend formation and decay, starting at the individual level, in a population facing many competing choices. To do that I will construct simple models of agents interacting with each other and acting according to their (perceived) *individual* best interest.

Beyond analyzing the genesis of trends I will show that social behavior under trend dynamics is a prototypical complex system. A unifying property common to many complex systems is the interesting way in which their dynamics samples large dimensional configuration spaces. Here too we will see that the population is driven back and forth between states of order, or large configurational information, and disorder. Both these extremes are unstable so that the system spends a long time in between. This cycling between order and disorder, once time averaged, shows interesting analogies with ensemble averages of statistical systems in the vicinity of a critical point. Thus trend dynamics can display a specific sort of self-organized criticality [17].

Another interesting aspect of trend dynamics is its susceptibility to external influences. Because, as we shall see below, the system has no global stable fixed points its dynamics are extremely sensitive to the introduction or subtraction of new agents with particular preferences. These properties produce insights into the fragility or robustness of 'public opinion' in specific circumstances and tell us how it may be driven to generate social consensus or dissonance.

The remaining of this paper is organized as follows. In sec. 2 I describe the basic agent based model and several of its possible variations. In sec. 3 I construct quantities that give a global characterization of the dynamics. Firstly I will define an information (Shannon) entropy over population configurations. Secondly it is natural to define cumulants analogous to a percolation strength and susceptibility [18]. These quantities allow a comparison between time averages of population configurations and typical properties of critical phenomena. Section 4 is devoted to a more detailed analysis of

the dynamics. There I show how several macroscopic properties can be understood by simpler analytical expressions, without requiring knowledge of the whole population. Sec. 5 is devoted to some of the properties of the driven system where individuals are added to or subtracted from the population with specific trend preferences. This will show how trend dynamics can be driven by external influences. Finally in Sec. 6 I summarize the results and discuss other related problems and applications.

2 The model and its global dynamics

In this section I define the agent based model used in the remaining of this paper. I will also give a first characterization of the resulting dynamics by constructing quantities that capture their most important global properties.

2.1 Definition of the model and possible variants

I consider a population with N agents and L trends or labels. Every agent is characterized by a single label, denoting the group or trend he/she belongs to at a given time. All labels are equally good from the point of view of an isolated agent. For a discrete set of L labels this is a Z_L symmetry. This prescription implements the basic individual choice degeneracy discussed in the Introduction. Nevertheless, as we shall see, spontaneous collective choices will be generated and subsequently destroyed dynamically. The specification of a single label per agent can be easily relaxed, but would lead to more complicated (and arbitrary) dynamical rules.

At each time every agent i contacts another j at random in his social circle. He compares the relative growth (or momentum) of their labels $p_i(t) = \Delta N_i(t)/N_i(t)$, where $N_i(t)$ is the number of agents in label i at time t and $\Delta N_i(t) = N_i(t) - N_i(t-1)$. If his label's momentum p_i is smaller than p_j he adopts the other agent's trend; otherwise he keeps his [19].

Moreover if the momentum of a given trend is smaller than a given value p_{crit} , agents leave their label for a brand new (empty) one. Thus p_{crit} parameterizes conformism; the threshold momentum below which staying in a slow moving trend becomes unbearable and taking the risk on a brand new thing is preferred. This parameter will play an important role in trend dynamics, as we shall see below. Here, I will assume, for simplicity, that all agents share the same value of p_{crit} .

A few extra remarks are in order:

i) the values of N and L can be made functions of time, reflecting population growth or decrease and the change in their realm of choices. Some of these scenarios will be explored in sec. 5.

ii) Each agent's social circle can be limited to subsets of the population; as it happens in reality in social networks [20]. These restrictions effectively condition the flow of information and make the choices of certain individuals more influential than those of others. These properties lead to fascinating dynamical effects, which can depend sensitively on the morphology of the network. Such issues require an adequate discussion of social networks and a parametric exploration of some of their structural freedoms. For this reason the influence of the structure of social networks on the dynamics of trends will be presented in a separate publication [21]. Below I shall

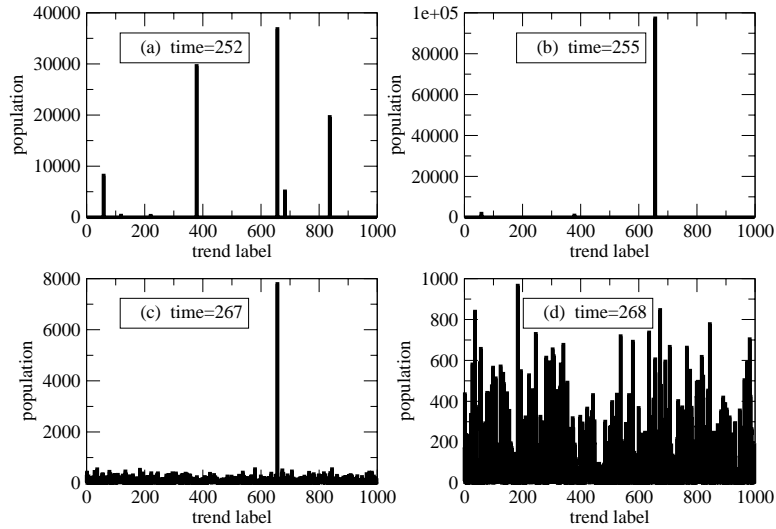


Figure 1: Snapshots of trend occupation numbers for an evolution with $N = 10^5$, $L = 10^3$ and $p_{\text{crit}} = 10^{-5}$. The system cycles between ordered states, characterized by the dominance of a few trends (a)-(b), their decay (c), and the emergence of many small competing labels (d), from which a few dominant ones emerge again and so on.

use an unstructured population, where each agent can contact any other with equal probability.

iii) The individual choice to change label can be made according to a more sophisticated probabilistic transition amplitude. This becomes essential if more parameters condition the choice or if agents do not act deterministically on the facts. Although these refinements may be necessary to model complex situations I shall not consider them in the context of the present paper.

iv) The value of p_{crit} is taken below as an input, a property shared by all agents. An interesting possibility is that its value may reflect a global sentiment that can change in time with the state of the population. Then p_{crit} should be computed self-consistently in time, e.g. conformism may decrease as agents perceive that most others belong to their trend or may rise during difficult times of population decrease.

Fig. 1 shows a typical evolution for a low value of $p_{\text{crit}} = 10^{-5}$. Clearly dominant trends alternate with periods of coexistence of many competing labels from which one eventually emerges to dominate and so on. The spontaneous formation of a large trend corresponds qualitatively to the phenomenon that BHW coined an *information cascade* as a label becomes widespread in the whole population. Here, however, the realm of choices is arbitrarily large and the population is not organized in a queue making sequential choices. This will allow us richer dynamics and closer similarities to systems familiar from statistical mechanics. Moreover the agents in the present model do not know about the previous choices of all others before them; they can merely compare the progress of their labels to those of their neighbors. Technically

the implementation used here is a Markov chain, where the state of the system at one time (its occupation numbers and their momenta) is determined (stochastically) from its state at the previous time. In this sense agents are not aware that they may be joining a large scale movement, they are just searching for the most promising choice in their realm of observation. The degeneracy of choice, built in at the individual level, persists as collective movements emerge dynamically because the winning trend can assume any of the L labels [22] with equal probability.

The dynamical structure of the system implies that there is no global stable fixed point. Demanding that a configuration be static implies $p_i = p_j = p$. If we insist that all $p_i = p$ are the same then $p = 0$ because of the overall conservation of individuals. This is a trivial static point. It can be realized in many ways (any arrangement of N individuals in L labels); most of which are close to the flat distribution, the most disordered state of the population. This fixed point is clearly unstable to any perturbation: if a single trend acquires positive momentum (and another negative from agent number conservation) the system will move away from the original configuration as all agents try to join the former and exit the latter.

A very particular instance of this fixed point is the asymptotic situation where $N_i \rightarrow N$, $p_i \rightarrow 0^+$ and $N_j \rightarrow 0$, $p_j \rightarrow 0^- \forall_{j \neq i}$, which corresponds to the growth of a single dominant trend at the expense of all others. This situation can be realized for each one of the L labels. A single fully occupied trend is the most ordered state the system can take. For it not to be *stable* it is necessary that $p_{\text{crit}} > 0$ - this is the first crucial role of p_{crit} . Otherwise, if $p_{\text{crit}} \leq 0$, agents have no desire to leave a static (or decaying) trend in the absence of faster growing competitors and the evolution freezes when the first dominant trend is formed.

2.2 Global characterization of trend dynamics

As we have already seen one of the features of the evolution at small p_{crit} is the formation of widespread trends, i.e. situations where one of the labels is substantially more populated than all the others. However, as it does so, the dominant trend must slow down and will eventually collapse into many small, fast moving trends, which proceed to compete for dominance, see Fig. 1. The instability of these states leads to characteristic cyclic (but aperiodic) dynamics. This section is dedicated to constructing global quantities inspired by analogies to statistical physics that capture these properties.

First, the qualitative sense that the population is alternatively in states of disorder (when many trends coexist) and order (where one emerges as dominant) can be captured by defining a Shannon entropy S , over the discrete set of labels

$$S = - \sum_{i=1}^L n_i \ln n_i, \quad (1)$$

where $0 \leq n_i \leq 1$ is the probability of finding an individual in label i . Thus by knowing n_i as a function of time we can measure the evolution of the total entropy of the system. S is not conserved because the evolution is not Hamiltonian.

The value of n_i leading to the highest entropy S_{max} is the flat distribution

$$n_i = \frac{1}{L}, \quad \forall_i. \quad (2)$$

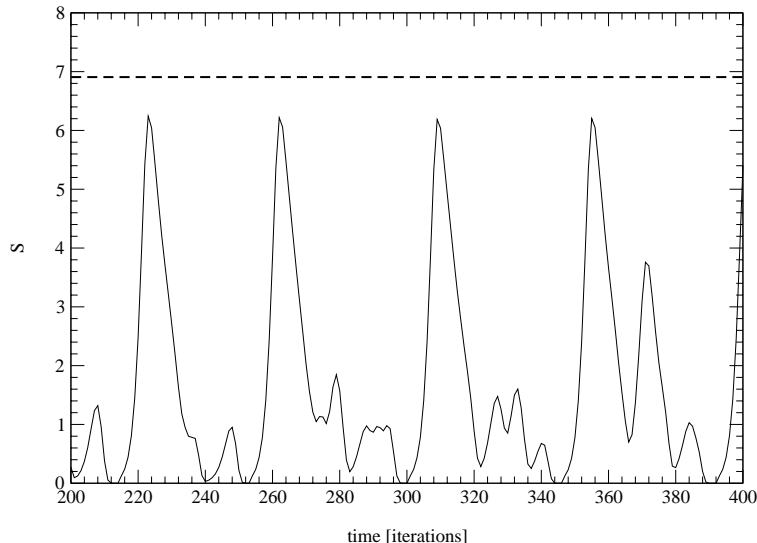


Figure 2: The behavior of the total entropy S , through several cycles of trend formation and decay, for a population with $N = 10^5$ agents, $L = 10^3$ labels and $p_{\text{crit}} = 10^{-5}$. The dashed line shows the maximal entropy $S_{\text{max}} = \ln(L) \simeq 6.91$, corresponding to the flat distribution.

Then the entropy $S_{\text{max}} = \ln(L)$. In contrast for one single trend containing the whole population $S_{\text{min}} = 0$. Dynamically we expect the system to alternate between these two asymptotic states, at least at low p_{crit} . Fig. 2 shows several cycles of growth and decay in the entropy S , for $p_{\text{crit}} = 10^{-5}$, $N = 10^5$ and $L = 10^3$.

We have already anticipated that the dynamics of trends, in analogy with many other complex systems, may display certain forms of self-organized criticality. I now define quantities that allow us to diagnose such behavior. The simplest and most paradigmatic critical phenomenon is percolation [18]. As in other second order transitions the critical point is associated with the divergence of a characteristic macroscopic length, together with several other susceptibilities, associated with the system's response to macroscopic stresses.

Statistical distributions in the vicinity of the critical point can be defined by two critical exponents (and by dimensionality). For percolation these two exponents can be obtained from the behavior of two independent ensemble averages, usually the percolation strength P_c (the "size" of the largest correlated cluster) and a percolation susceptibility S_c , defined by the sum of squares of the cluster sizes with the largest cluster subtracted. Criticality is the onset of the formation of a spanning cluster and is associated with a sharp increase of P_c and a peak (in the finite volume) in S_c . In the thermodynamic limit, where the number of degrees of freedom tends to infinity (the infinite volume limit), S_c usually diverges with some characteristic critical exponent.

In the context of our model each agent can exist in one of L states. Thus a percolation strength P_c can be defined as the fraction of the population in the largest trend

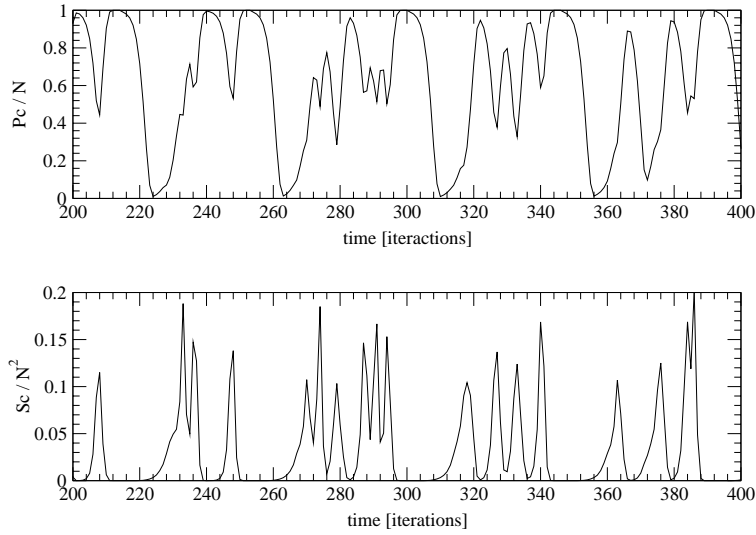


Figure 3: The behavior of the percolation strength P_c (upper panel) divided by N , and the percolation susceptibility S_c (lower panel), divided by N^2 , for the evolution of Fig. 1. P_c cycles through states of maximal occupancy $P_c = 1$ and very low occupancy $P_c \simeq 0$. The intermediate state, at the onset of the formation of a dominant term, is characterized by a sharp maximum of S_c , signaling critical behavior.

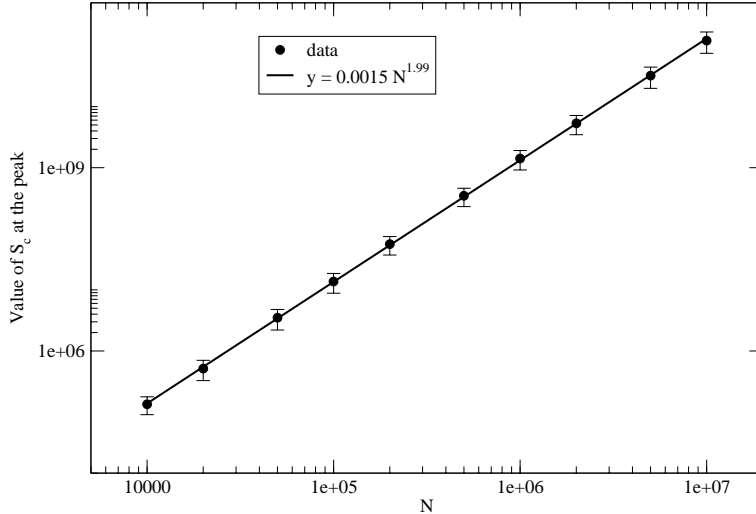


Figure 4: The average value of S_c at the peak, as the number of agents $N \rightarrow \infty$, at fixed $N/L = 100$. S diverges with N^γ with an exponent $\gamma \simeq 2$ (here $\gamma = 1.99 \pm 0.01$).

$P_c = \max(N_i)$. The percolation susceptibility S_c is then defined as

$$S_c = \left(\sum_{i=1}^L N_i^2 \right) - P_c^2. \quad (3)$$

A limit analogous to the thermodynamic limit in statistical systems can be taken by letting the number of agents N tend to infinity while also increasing the number of labels L such that the ratio of agents to labels N/L stays constant.

Fig. 3 show the evolution of P_c and S_c for the example of Fig. 2. We see that when a label emerges as the dominant trend P_c grows rapidly, and S_c goes through a sharp maximum. Fig. 4 shows the variation of the peak of S_c with N , at fixed $N/L = 100$. The behavior of S_c 's peak *vs.* N shows a divergence as $N \rightarrow \infty$, making a good case for the analogy between the dynamical spontaneous symmetry breaking of choice symmetry described here and critical phenomena.

3 Boom and bust: the rise and fall of trends

As we have seen in the previous section the global motion of the system can be characterized by a few simple quantities for which we have some intuition from statistical physics. In this section I analyze trend dynamics in more detail and derive semi-analytic reduced descriptions for some of their properties.

I start with the decay of a dominant trend. This is a simple process because it involves the transfer of agents from the main label to all others and, at least initially, can be understood without taking into account the detailed dynamics of the latter.

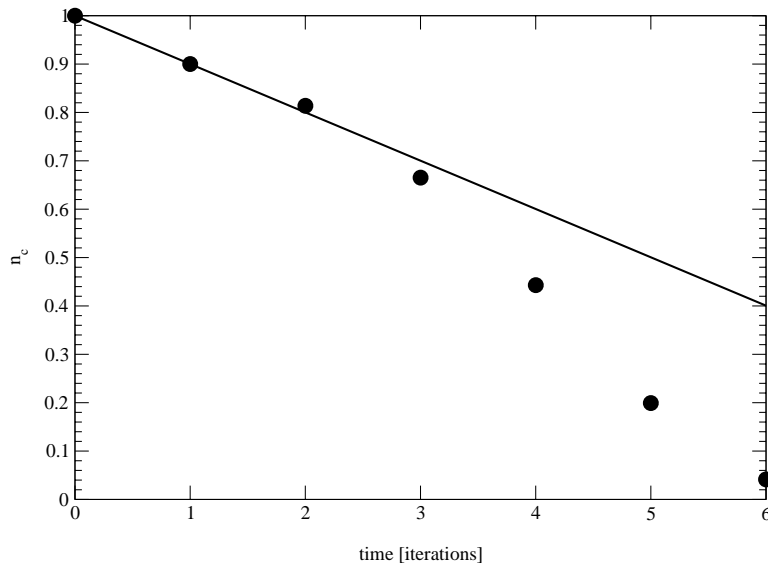


Figure 5: The early decay of the dominant trend for an evolution with $N = 10^4$ and $L = 10^3$. Initially the decay proceeds by agents spontaneously seeking new, empty labels and is given approximately by $n_c(t + 1) = n_c(t) - (L - 1)$, shown as the solid line (see text).

As discussed in sec. 2 the dominant cluster will only decay if some individuals choose to leave it, even though there may not be any alternative faster growing label available at that time. For the detailed implementation I choose that the decision to abandon a trend is made when its momentum becomes smaller than a certain value p_{crit} . When $p_i < p_{\text{crit}}$ each agent makes a trial search of label space at random; if an empty label is found it is adopted. Then the first few steps in the decay of the main cluster result in $n_c(t + 1) = n_c(t) - (L - 1)$, see Fig. 5.

As labels become filled many small fast growing trends are formed and the usual momentum comparison between agents becomes the dominant dynamical force. Then the occupation number distribution will be characterized by the dominant trend with negative momentum and all other much smaller trends, initially with large positive momentum. In these circumstances the decay of the main cluster is dictated by the probability of an individual belonging to it to find another individual outside. This process it is approximately described by $n_c(t + 1) = n_c(t) - p_{\text{out}}n_c(t)$, where p_{out} is the probability of finding an individual *outside* the main cluster, $p_{\text{out}} = \sum_{k \neq c} n_k$.

Figure 6 shows that this expectation fits the decay of the main cluster extremely well, even at relatively late times. This description breaks down as other trends become similar in size to the original dominant one.

The subsequent growth of trends is more difficult to analyze because it involves the comparison between many different competitors. Note, however, that as dominant trends emerge the system undergoes an effective reduction of its phase space. This is because many labels become empty. The evolution of the probability to find an empty

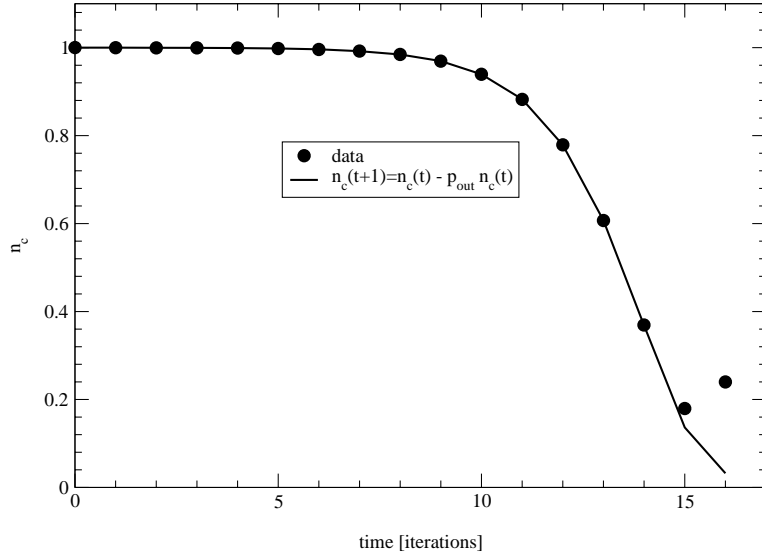


Figure 6: The decay of the largest cluster for $N = 10^5$ agents and $L = 10$. The line shows the initial prediction given by $n_c(t + 1) = n_c(t) - p_{\text{out}} n_c(t)$. The last point shows a new cluster that has overtaken the former dominant one in size

label P_0 is reasonably well described by

$$P_0(t + 1) = P_0(t) + \left[\frac{1 - P_0(t)}{2} \right] P_0(t), \quad (4)$$

see fig. 7 for a comparison to data. Eq. (4) is based on the simple expectation that at each time step half of the filled trends become empty. Because of this dynamical *thinning* of the number of available labels the late evolution of trend formation becomes fairly simple as it is characterized only by a few choices.

It is useful to consider some very simple cases. If only two trends are present, a typical *endgame* situation, the outcome is invariably that the faster growing one always wins, even if it is initially smaller. In fact, because of the total agent number is conserved they cannot both grow. This may not always reflect the real world of e.g. brand or political party competition, situations that can in many instances be dominated by choosing between two alternatives. In reality in these cases the relative growth of mutual competitors has become closely monitored. Intervention in the form of e.g. publicity campaigns is used to change agents perceptions leading to more complicated decision making processes, beyond the scope of the present model.

If three or more trends are present more interesting situations are possible. A typical situation that we explored above is that one of the labels is decaying and feeding the growth of others. In this situation the decaying cluster functions as a source that allows other trends to grow simultaneously. The fate of the latter is largely determined at the time when this source is extinguished, and the remaining labels need to start competing for population.

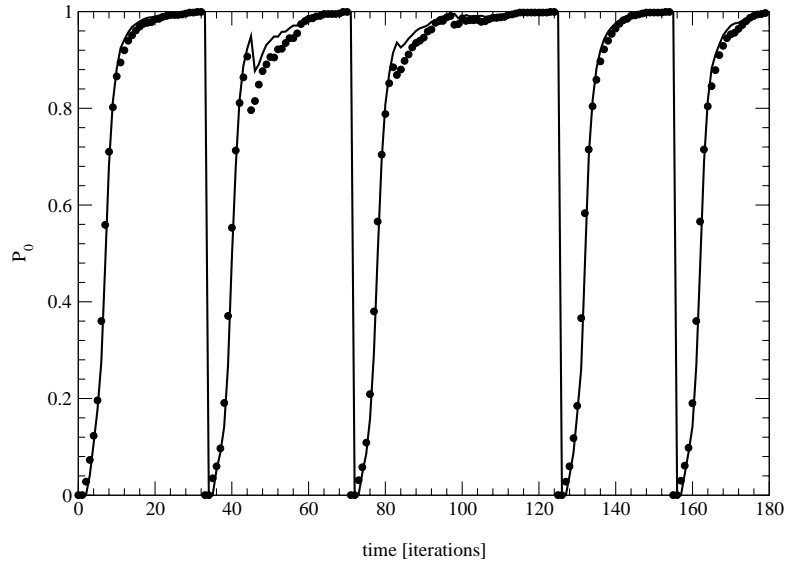


Figure 7: The evolution of the probability of a label being empty P_0 (data points), through several dynamical cycles with $N = 10^4$, $L = 10^3$ and $p_{\text{crit}} = 10^{-5}$. As dominant trends emerge the number of competing labels decreases, resulting in an effective dynamical reduction of label choices. All labels are subsequently temporarily re-populated as large clusters collapse. The evolution of P_0 is well described by Eq. (4) (solid line).

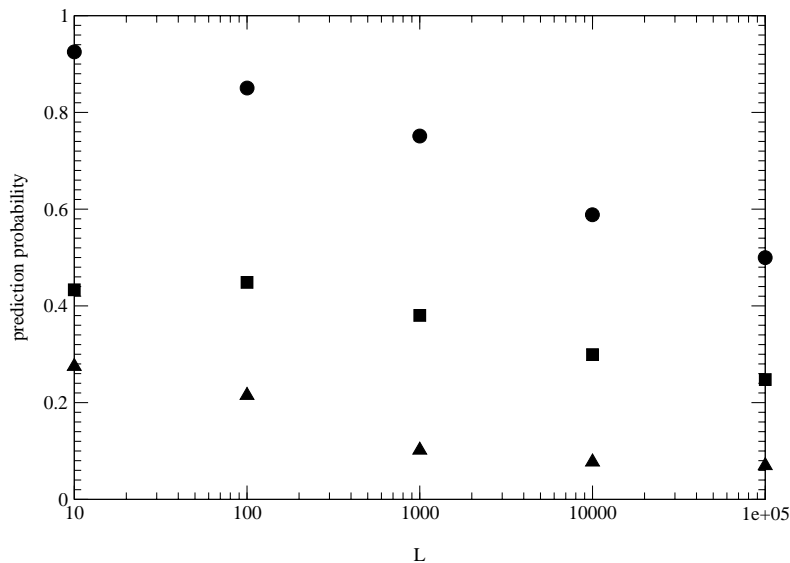


Figure 8: The probability that the largest secondary cluster (squares), the fastest cluster (triangles) or the cluster with the highest product of momentum and size (circles) at the particular time when a dominant trend disappears becomes the new dominant trend, for a population with $N = 10^3$, $p_{\text{crit}} = 10^{-5}$ and varying number of labels. As the number of labels increases calling the next winning trend with confidence becomes nearly impossible by any of these criteria.

Is it possible, at this particular, time to predict the next winning trend? There is a delicate balance between the visibility of a trend and its momentum. For example it is not true that the fastest growing trend becomes dominant and neither does the largest (secondary) cluster. In most cases the fastest moving trends are the smallest, which suffer from lack of visibility (i.e. it is unlikely to find an agent belonging to it). On the other hand a secondary large trend will not become dominant if the result of many inquires find it the laggard. Thus becoming the next winner requires being both relatively large, being fast and some good portion of luck. Fig. 8 shows how well several of these criteria fare at predicting the winner.

The results of Fig. 8 show that even if one is aware that a whole population is following trends it is difficult to call a winner early enough to profit from the realization. In fact, the most successful criterion shown in fig. 8, picking the trend with the largest product of size and momentum at the critical time, offers odds comparable to tossing a coin when L becomes large. Moreover this strategy requires precise knowledge of the state of the whole population at a very particular time, which is in general difficult (and expensive) to obtain.

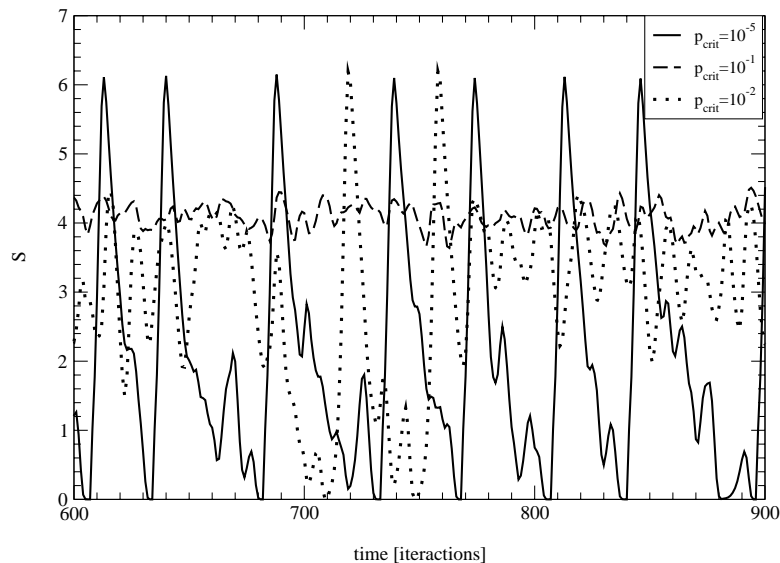


Figure 9: The dependence of the time evolution of the total entropy S on p_{crit} the conformist parameter for $N = 10^5$ and $L = 10^3$. p_{crit} controls the amplitude and regularity of the trend cycling. For low enough $p_{\text{crit}} \sim 1/N$ the motion is clearly cyclic alternating totally dominant trends with quasi-random distributions. As p_{crit} is increased the extremes become less pronounced and the motion less cyclic.

4 Time averages and distributions: analogies to critical phenomena

I have up to this point analyzed some of the general dynamical properties of the model introduced in sec. 2. As we have seen there are no static stable solutions, and the population cycles between ordered states, where there is one dominant trend, and periods where many similar sized trends coexist and compete for dominance. In this section I discuss some of the time-averaged properties of these distributions, over many cycles of growth and collapse. Much like microcanonical time averages can coincide with canonical ensemble expectation values, time averages of complex systems give us a statistical measure of their most likely states. It is usually these averages that are compared to analogous quantities (over ensembles) in critical phenomena in order to demonstrate that the system displays self-organized criticality [17].

Before analyzing time averages of distributions it is important to understand better another role of the conformism parameter p_{crit} . Fig. 9 shows how p_{crit} determines the amplitude of the motion between order and disorder. Evolutions with low values of $p_{\text{crit}} \leq N^{-1}$ reach absolute order (where one single trend engulfs the whole population) alternating with very disordered states. As p_{crit} is increased the amplitude decreases and the system spends more and more time in intermediate states. These are neither extremely ordered nor disordered but rather somewhere in between, as can be seen in

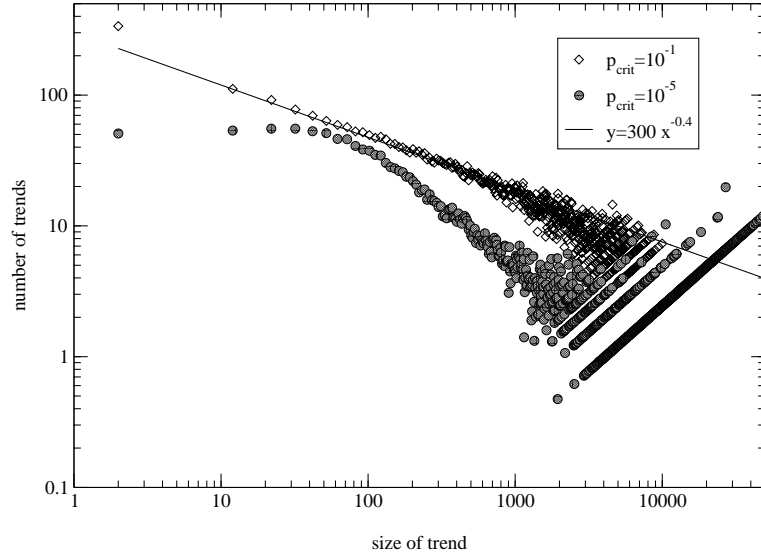


Figure 10: The time-averaged trend size distribution for $N = 10^5$, $L = 10^3$ and two values of $p_{\text{crit}} = 10^{-1}$, 10^{-5} . p_{crit} controls the relative times spent by the system in each extreme configuration (trend dominance or disorder). For high enough values of p_{crit} the distribution becomes a power law with a small exponent, here $\alpha \sim 0.4$. For small p_{crit} the distribution is distorted by an abundance of both small and large trends.

an entropy plot, Fig. 9.

From these considerations about the role of p_{crit} we expect that time averages of distributions with small p_{crit} will include regions of configuration space with both more ordered and disordered states, those with higher p_{crit} will be peaked in between. This potentially leads to different time averaged distributions, as seen in Fig. 10, which shows the number distribution of trend sizes.

Another interesting quantity is the size distribution of population fluxes, defined as the number of individuals leaving or joining a trend per unit time. This is shown in Fig. 11. The flux size distribution is essentially a dynamical process and shows much more robustness against changes of p_{crit} . A related quantity is the number distribution of individuals entering or leaving a trend after a test agent.

The number of individuals that join or leave after a given agent is a quantity that carries an important meaning. Think again of trend dynamics as a race, a game among agents. Ideally an agent wants to join a winning trend early in its development and ride it until it is dominant; he also wants to leave before it starts decaying and start off the next winner.

This strategy is simply that of being followed by the maximum number of others and follow the least. This is desirable in many circumstances e.g. in (speculative) financial markets. Here, however, the cost of joining a trend does not increase with the number of agents already in it. Thus the present model can only hope to describe speculative

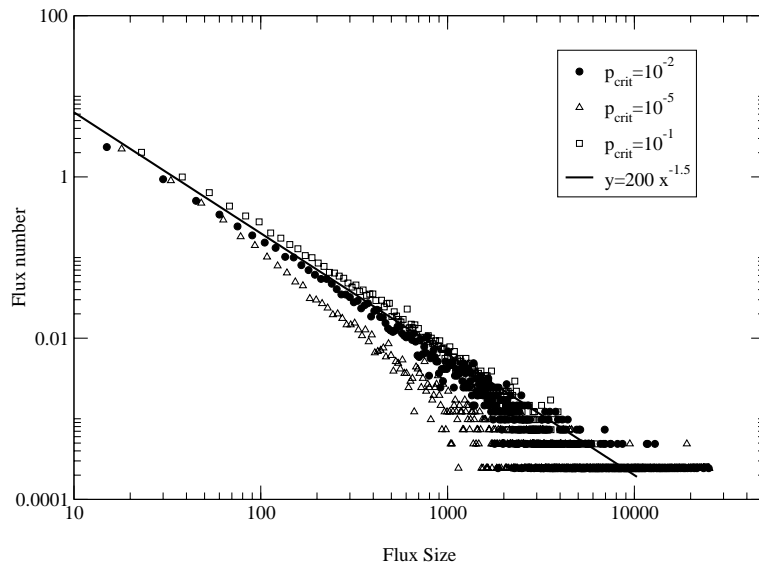


Figure 11: The size distribution of population fluxes for the example of Fig. 10. The flux distributions shows significantly more robust power law behavior against variation of p_{crit} than the size distribution of trends.

bubbles in financial markets as long as price is no object for most agents.

Fig. 12 shows that over many cycles of trend growth and decay an individual is on average followed by as many others as those he follows. More interestingly the second moment of the distribution measures the possible fluctuations around zero gain or loss. If the game of following trends is played only a few times it will give a measure of the possible losses or gains incurred by a typical agent. Fig. 13 shows a detail of the distribution of gains. Although displaying finite population effects for very large numbers it is clear that the main distribution is very flat, making it possible for an individual playing the game only a few times to experience spectacular gains and/or losses.

5 Trend dynamics of open systems: changing populations and external information biases

So far I have considered the dynamics of trends over closed populations, characterized by a fixed number of agents N and trends L . It is interesting to generalize this closed system to an open one, and examine the effects of adding or subtracting new individuals with particular preferences. This is the subject of this section.

For purposes of illustration I consider a linear population growth law, i.e.

$$N(t+1) = N(t) + R, \tag{5}$$

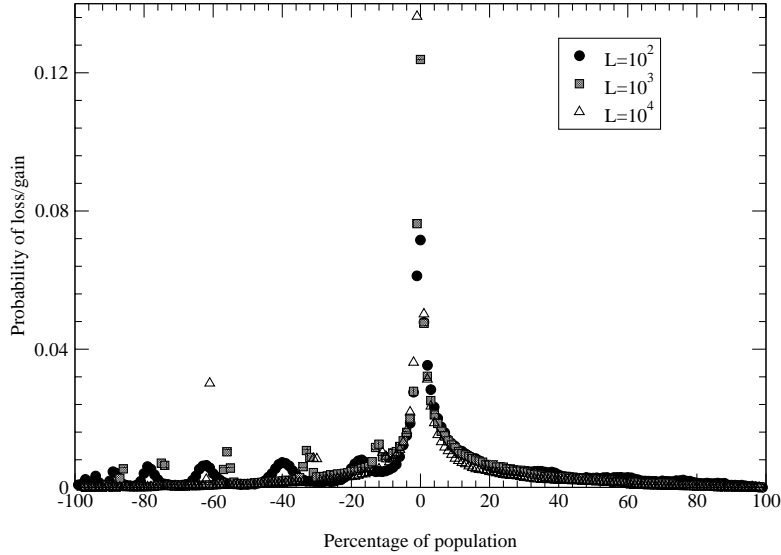


Figure 12: The number distribution of individuals entering (positive) and leaving (negative) a trend after an agent has joined it for $N = 10^4$, $p_{\text{crit}} = 10^{-5}$ and several L . On average an agent is followed by as many agents as those he follows. See also Fig. 13.

where R is the number of individuals joining the populations at each time step. The exact form of Eq. (5) is not particularly important for our discussion.

The influence of the new individuals on a population undergoing trend dynamics depends sensitively on their trend preferences. First I investigate the effect of letting the new agents enter the population at a random trend, an "open minded youth". Note that this type of driving force, once time averaged, does not break the choice symmetry of the closed system (any trend may still become dominant). It does however introduce a *disordering* external effect on the dynamics, much like driving a statistical system with white noise.

Fig. 14 shows the evolution of the total entropy under these circumstances, for several values of R . The introduction of new agents at random, even in a population with small $p_{\text{crit}} = 10^{-5}$ and at small rates, leads to dramatic results. The distinctive cycling between order and disorder becomes less well defined even for the smallest $R = 1$. The lifetime of the largest trends is reduced as they suffer from stiffer competition from smaller movements. These receive the strongest enhancement of their relative numbers and momentum due to the injection of new agents. Curiously the highest entropy states of the population are also suppressed, at least for small R . These states were formerly produced in the early decay of a very dominant trend. For large R the average state of the population remains fairly disordered, but still far from the random distribution, which is presumably reached as $R \rightarrow \infty$.

If after a period of growth the population ceases to increase, i.e. if the drive is switched off at some late time, the dynamics quickly resumes its cyclic pattern of

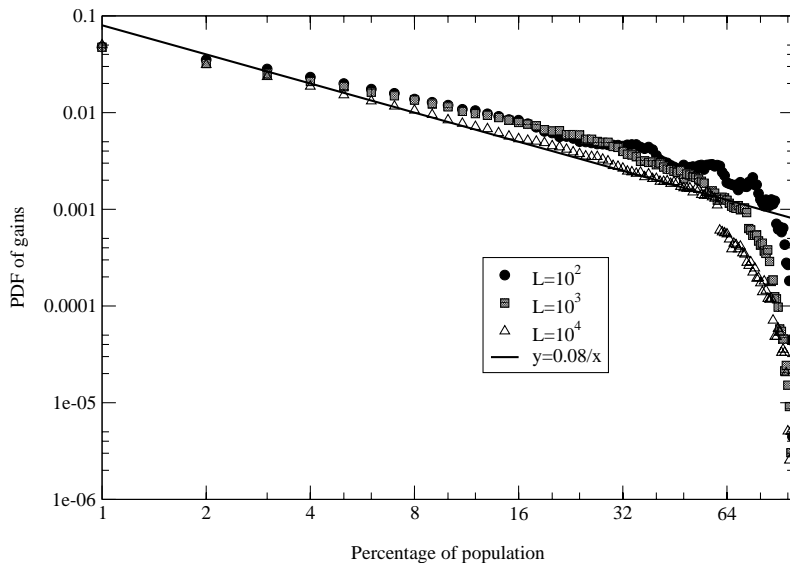


Figure 13: Detail of Fig. 12 for agents entering a trend. The tail of the distribution is fat, leading to large higher moments. The distribution is eventually cutoff for large numbers by finite N effects.

dominant trend formation and decay, see Fig 15, although now involving many more agents

Alternatively new agents may not be introduced at random. This constitutes a systematic ordering effect that explicitly breaks trend choice symmetry. Because the evolution is characterized by a sequence of instabilities the dynamics are extremely sensitive to the preferences of the new agents. If new agents are introduced in a specific label then it becomes invariably the dominant trend, even at the smallest $R = 1$. Similarly if the new agents prefer a *subset* of the total labels, these become all the dominant trends. Remaining trends gather at best a small fraction of the population and only at times when a dominant trend decays.

Finally I consider the effects of a decreasing population. The simplest and most natural situation is that individuals are eliminated from the population at random, according to Eq. (5), now with $R < 0$. This operation actually has no effect on most aspects of the dynamics. This is because the change in momentum of each trend due to population losses is on average proportional to trend size (the probability that the individual is in that trend). Thus the change in momentum of a trend i due to random population loss is $\Delta p_i = R n_i / (N n_i) = R/N$, the same for all trends. We conclude (and observe explicitly in the dynamics) that eliminating individuals at random preserves the momentum hierarchy among trends and consequently does not change the dynamics.

The exception to this rule is the decay of the single dominant trend. I postulated that only if the trend momentum is slow enough (but still positive) will agents leave it to start something new. However, due to population loss, the largest trend may

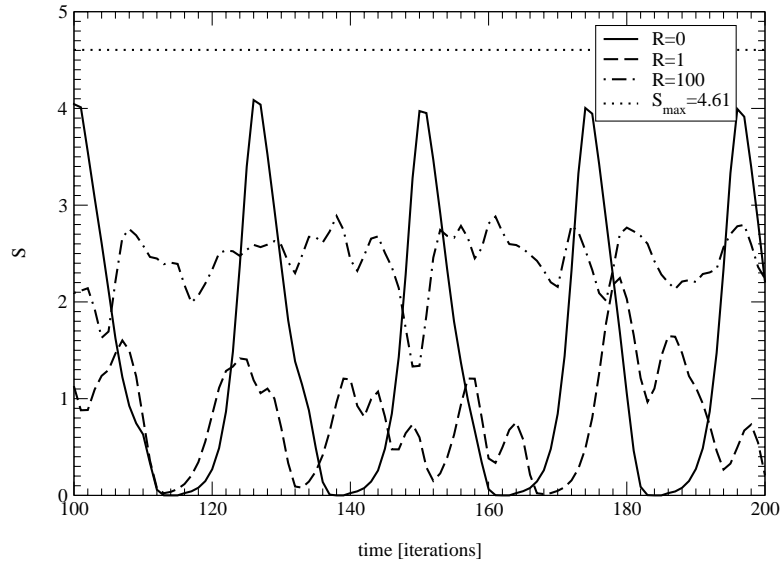


Figure 14: The evolution of the total entropy S for growing populations at different rates R for $N(t=0) = 5 \times 10^4$ and $p_{\text{crit}} = 10^{-5}$. New agents enter the population at labels chosen at random. This leads to the disappearance of the cyclic nature of the motion and in general to higher entropy states, particularly for large R .

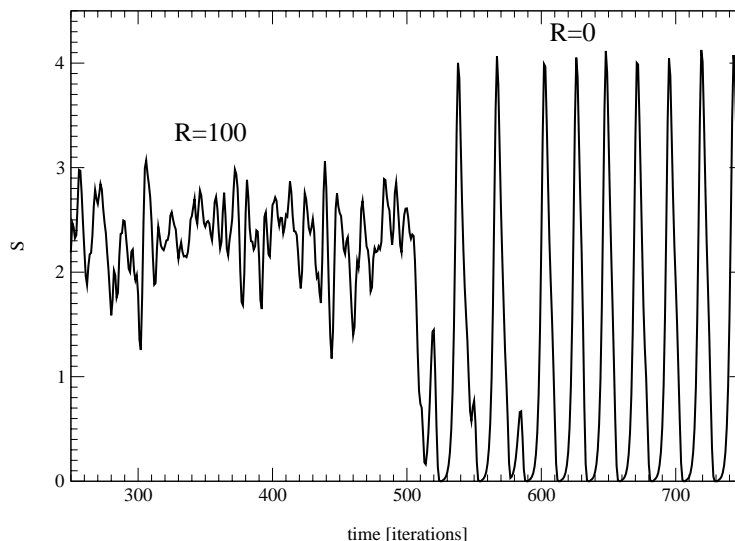


Figure 15: The evolution of the total entropy for the growing population of Fig. 14 with $R = 100$. The motion returns to its cyclic characteristic pattern once the inflow of new agents ceases (here at $t = 500$).

never come sufficiently close to $p = 0$ (it would in a continuous time approximation). Then it is possible that all remaining agents become stuck in a single decaying trend. This happens at a characteristic time $t \simeq N/R - p_{\text{crit}}^{-1}$, obtained from comparing the momentum due to population losses alone to p_{crit} , starting at $t = 0$ with N agents. For small p_{crit} the evolution freezes as the first large trend is formed.

We see that random population growth or decay have quite different consequences. A growing population with no particular preference at birth makes spontaneous consensus rarer, more unstable and encompassing fewer agents. Population growth with particular preferences, on the other hand, explicitly breaks the spontaneous choice symmetry of the closed model and reduces the space of possible large trends to those preferred by the new agents. To combat this tendency the existing (older) elements of the population must not follow trends. Finally the effects of population decrease are generally more benign. The important exception is that if a consensus occurs at a time of population decrease in a conformist population agents may become stuck in a sinking trend. The same effect can take place if conformism rose with population loss and would have devastating consequences if diversity were necessary for eventual population recovery.

6 Conclusions and outlook

In this paper I introduced a simple agent based model describing how coherent social choices - trends - arise spontaneously and how these movements eventually slow down

and disappear. Trends, fashions, fads are all well documented mechanisms for the creation of shared knowledge (culture) and triggers for behavior change and its self-regulation in human societies. Examples range from the most mundane (fads in popular culture) to the most important (setting cultural norms, fraud/crime control).

The dynamical pattern of trend formation and decay developed in this paper is particularly clear in populations with a high degree of conformism (small p_{crit}), where individuals are slow to leave sluggish trends in order to start something new. Then the population goes through cycles alternating states of order, dominated by a single label, and disorder where many small trends compete for population. If conformism is lower the size and frequency of occurrence of dominant trends becomes smaller. As a result the state of the population remains somewhere halfway in between order and disorder and is characterized by scale invariant (power law) distributions - the dynamics oscillates about a state of criticality. The amplitude of fluctuations around this state is also determined by the conformism parameter p_{crit} . The passage through criticality is made apparent by the construction of familiar quantities from statistical mechanics, analogous to percolation cumulants.

The fundamental instabilities characteristic of both asymptotic states of absolute order and absolute disorder make the dynamics extremely susceptible to external influences, such as the preferences of new individuals. Simple as the model is it suggests that variety can be sustained in an open population following trends only if new agents (youth) remain unprejudiced. On the other hand diversity of choices will be quenched if their attention spans only a subset of all pre-existing possibilities. To prevent this loss of diversity it is then necessary that older agents stick to their own preferences and do not follow trends. Population loss at random on the other hand does not affect most aspects of trend dynamics but may lead to a scenario where the state of the population is frozen in a single decaying trend if conformism is too high. The maintenance of diversity in this situation requires that agents continue to take risks in the face of adversity.

The spontaneous breaking of choice symmetry inherent to trend dynamics illustrates a simple but important point about decision making strategies. In the present model there is no extremum principle that makes the choice of a winning trend predictable. Instead a consensus is built out of accidental elements of the dynamics and, paradoxically, the desire of individual agents to stay ahead of the crowd. Thus a trend that finds itself large and fast growing at the right moment will have a sort of first striker's advantage. This leads to accidental winners that may not be optimal solutions in the long run, a phenomenon familiar to technological and (presumably) biological development.

The most important consequence of the difficulty to predict winning trends is that, although it may be apparent to an external observer that everyone is following trends, it is extremely difficult to take advantage of the phenomenon for individual profit. This interesting aspect of the problem will be discussed further in a future publication [21]. There I will consider how trend dynamics can be formulated as a game played for social advantage over a typical social network. It will then become apparent that some of morphological characteristics of social networks are not accidental, but rather may follow from our collective desire to access socially relevant information as early as possible and from the necessity to keep up with our friends and neighbors.

References

- [1] In Economics the importance of trends was probably first expressed in print by J. M. Keynes, *The general theory of Employment, Interest and Money* (Macmillan, London, 1936).
- [2] See e.g. the discussion and references in C. Cornell and S. Cohn, *Am. J. Sociol.* **101**, 366 (1995).
- [3] S. Bikhchandani, D. Hirshleifer and I. Welch, *J. Polit. Econ.* **100**, 992-1026 (1992); *J. Econ. Perspect.* **12**, 151 (1998).
- [4] The reader is encouraged to pursue the few hundred publications that cite Ref. [3] across several fields. For a few highlights taken from this set and beyond see [2, 6, 7, 8, 9, 10, 11].
- [5] For early arguments and analysis see K. D. West, *J. Financ.* **43**, 639 (1988) and references therein. For early models of herding in financial markets see R. Topol, *Econ. J.* **101**, 768 (1991); A. Bannerjee, *Q. J. Econ.* **107**, 797 (1992); *Rev. Econ. Stud.* **60**, 309 (1993); A. Orléan, *J. Econ. Behav. Organ.* **28**, 257 (1995); T. Lux, *Econ. J.* **105**, 881 (1995).
- [6] B. Trueman, *Rev. Financ. Stud.* **7**, 97 (1994); I. Welch, *J. Financ. Econ.* **58**, 369 (2000); H. Rao, H. R. Greve, G. F. Davis, *Admin. Sci. Quart.* **46**, 502 (2001).
- [7] D. S. Sharfstein and J. C. Stein, *Am. Econ. Rev.* **80**, 465 (1990); M. Grinblatt, S. Titman, and R. Wermers, *Am. Econ. Rev.* **85**, 1088 (1995).
- [8] R. D. McKelvey and P. C. Ordeshook, *J. Econ. Theory* **36**, 55 (1985); L. M. Bartels, *Presidential Primaries and the Dynamics of Public Choice* (Princeton Univ. Press, Princeton NJ, 1988); A. Cukierman, *Asymmetric Information and the Electoral Momentum of Public Opinion Polls* (Princeton University Press, Princeton, 1989) and references therein.
- [9] R. J. Gilbert and M. Lieberman, *Rand J. Econ.* **18**, 17 (1987); W. Pesendorfer, *Am. Econ. Rev.* **85**, 771 (1995).
- [10] R. E. Kennedy, *J. Ind. Econ.* **50**, 57 (2002).
- [11] S. M. Sheffrin and R. K. Triest, in *Why People pay taxes* Ed. J. Slemrod (1992); E. Glaeser, B. Sacerdote and J. Scheikman, *Q. J. Econ.* **111**, 507 (1996).
- [12] For other models of social decision making inspired by concepts of Statistical Physics see D. Chowdhury, D. Stauffer, *E. Phys. J. B* **8**, 477 (1999); D. Stauffer, N. Jan, *Int. J. Mod. Phys. C* **11**, 147 (2000); K. Sznajd-Weron, J. Sznajd, *Int. J. Mod. Phys. C* **11**, 1157 (2000); D. Stauffer, A. O. Sousa, S. M. de Oliveira, *Int. J. Mod. Phys. C* **11**, 1239 (2000); R. Florian, S. Galam, *E. Phys. J. B* **16**, 189 (2000); S. Solomon, G. Weisbuch, L. de Arcangelis, N. Jan, D. Stauffer, *Physica A* **277**, 239 (2000).
- [13] R. Donangelo, K. Sneppen, *Physica A* **276**, 572 (2000); R. Donangelo, A. Hansen, K. Sneppen, S. R. Souza, *Physica A* **287**, 539 (2000).
- [14] J. Toner, Y. Tu, *Phys. Rev. Lett.* **75**, 4326 (1995); *ibid.* **80**, 4819 (1998); *Phys. Rev. E* **58**, 4828 (1998); A. S. Mikhailov and D. H. Zanette, *Phys. Rev. E* **60**, 4571 (1999); A. Czirók, A.-L. Barabási, and T. Vicsek, *Phys. Rev. Lett.* **82**, 209 (1999).

- [15] More recent models of herd behavior in financial markets inspired by statistical physics are R. Cont and J.-P. Bouchaud, *Macroecon. Dyn.* **4**, 170 (2000); T. Lux and M. Marchesi, *Nature (London)* **397**, 498 (1999); V. M. Eguiluz and M. G. Zimmermann, *Phys. Rev. Lett.* **85**, 5659 (2000).
- [16] For more extensive analysis and further exploration of the class of models introduced in [15] see D.-F. Zheng, G. J. Rodgers, P. M. Hui, R. D'Hulst, *Physica A* **303**, 176 (2002); G. J. Rodgers, D.-F. Zheng, *Physica A* **308**, 375 (2002); D.-F. Zheng, P. M. Hui, K. F. Yip, N. F. Johnson, *E. Phys. J. B* **27**, 213 (2002); F. Wagner, *cond-mat/0207280*.
- [17] P. Bak, C. Tang, and K. Wiesenfeld, *Phys. Rev. Lett.* **59**, 381 (1987); see also P. Bak, *How nature works: the science of self-organized criticality* (Copernicus, New York, NY, USA 1996) and references therein.
- [18] See e.g. D. Stauffer and A. Aharony, *Introduction to percolation theory* (Taylor & Francis, London, UK, 1992); K. Binder and D. W. Heermann, *Monte Carlo simulation in Statistical Physics* (Springer-Verlag, Heidelberg, Germany 1997).
- [19] Clearly the same dynamics results from the comparison between agents of any growing function of their trend's momentum.
- [20] For recent reviews of the structure of social networks see D. J. Watts, *Small worlds* (Princeton University Press, Princeton, 1999); M. E. J. Newman, *J. Stat. Phys.* **101**, 819 (2000).
- [21] L. M. A. Bettencourt, *Trend dynamics and the structure of social networks*, in preparation.
- [22] Animations of the evolution of the trend occupation numbers over many cycles of trend formation and decay can be seen at <http://www.mit.edu/~lmbett>.