Tipping the balances of a small world: Spontaneous herding and social structure

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Abstract

Recent progress in the large scale mapping of social networks is opening new quantitative windows into the structure of human societies. These networks are largely the result of how we access and utilize information. Here I show that a universal decision mechanism under uncertainty, where we base our choices on the actions of others, can explain much of their structure. Such collective social arrangements emerge from successful strategies to handle information flow at the individual level. They include the formation of closely-knit communities and the emergence of well-connected individuals. The latter can command the following of others while only exercising ordinary judgment.

In recent years there has been growing interest in the quantitative structure of human societies. It has emerged that we are part of heterogeneous networks or graphs [1, 2, 3, 4], sets of links that connect each one of us to all our acquaintances. Not all people are alike: some live almost isolated, most belong to distinguishable communities [1, 5] and a small fraction of the population is made up of exceptionally well connected individuals [6]. Social networks have the remarkable property that one can reach anyone else through a very small number of connections - the famous six degrees of separation [7, 8].

These findings beg important questions: Why are social networks invariably clustered in communities? Why are there individuals with such different connectivity? Answering these puzzles requires tying the morphology of social networks to their function [2, 9, 10]. Similar problems occur in the study of other complex networks, for example, dealing with gene and protein-protein interactions [11, 12, 13], metabolism [14, 15], ecosystems [16, 17] (foodwebs) and neural activity. Thus understanding the simultaneous robustness and adaptability of these complex systems in the light of their function is a general problem at the forefront of the current scientific agenda across many disciplines [9].

The difficulty of this approach consists in defining the function of each of these complex networks in a way that captures their essence and simultaneously permits quantitative progress. Clearly many details of social behavior, in particular, appear too rich and our understanding of them remains too qualitative to fall in this class. There are however important well documented exceptions.

A familiar situation is having to choose between seemingly equivalent options, at least given the amount of information and time at our disposal [18]. In practice many of our decisions fall in this class. This leads to a degeneracy of choice, typical also of situations when relevant information is difficult to discriminate from too much noise, or when it cannot be trusted. In these situations we often rely on the observation of the actions of others we know, as the basis for our decisions [19, 20, 21, 22, 23]. This strategy has two important advantages: we can be sure not to do worse than most of the people we know and, in addition, we may actually join a winning trend early and profit from it.

Recently this type of discriminating imitation has become the focus of an extensive empirical literature in economy [19] and the social sciences. Bikhchandani, Hirshleifer and Welch [20, 21] collected a vast amount of empirical evidence that establishes the universal importance of the choices of others in influencing our own and were able to model this phenomenon in simple terms. They dubbed the formation of the trend or fad that often results an *information cascade*; a process whereby sequential individual choices propagate a piece of information through the entire population [22]. This phenomenon is also often liked (qualitatively) to the spread of an epidemic [10, 22]. Interestingly, information cascades lead to the spontaneous formation of large consensus where there are a priori no individual preferences.

Here I use an implementation of these ideas [23] consisting of a population of N agents, facing a choice among L labels. These numbers can be varied arbitrarily. At each time step individuals compare the relative growth rate of their label to that of one of their immediate acquaintances', chosen at random. If the latter's growth rate (the trend's relative momentum) is greater the agent switches to his neighbor's trend; otherwise he keeps his. The model has one additional ingredient: if a trend's growth slows down individuals may decide to take a risk in something new (an empty label). This effect is modeled by $p_{\rm crit}$, the relative growth rate below which non-conformism sets in. Here $p_{\rm crit} < 1/N$, which results in population wide trends or cascades [23]. References to other related dynamical implementations [22] (including models of herding) and additional discussion are given elsewhere [23].

Typical dynamics [23] are characterized by cycles alternating population disorder, when many different trends coexist, and order, when most of the population falls into the same label. Both collective states of order and disorder are dynamically unstable making the evolution very sensitive to chance events [20, 21, 23]. As a result it becomes very difficult in practice for an external observer to profit from the reckoning that agents are following trends, especially when the number of choices becomes large.

To explore the effects of the underlying network morphology on the dynamics I generate

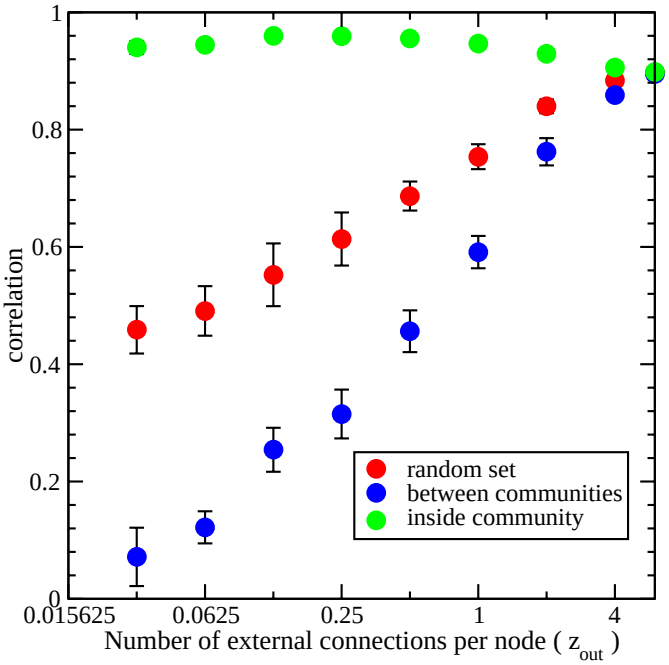

Figure 1: The correlation between two halves of the same community (green), halves of distinct communities (blue) and a set of individuals with random connections (red) for $N = 256$, $L = 1000$, $z = 8$, divided in 4 communities (see text). The correlation inside a community is always close to 1. The correlation and synchronization of choices between distinct communities is low for small $z_{\rm out}$, becoming higher as the number of connections between them increases. Individuals with random connections display intermediate correlation. For high $z_{\rm out}$ the original communities merge together. Error bars denote standard deviations over a set of 20 network realizations and many cascade cycles.

directed, binary artificial social networks as small world graphs [1, 24]. These are random graphs with clustering: N individuals are represented as nodes, each with an an expected number of connections z given in terms of a distribution $P(z)$. Clustering is produced by dividing the population into communities, each characterized by an average higher degree of internal connections per node $z_{\rm in}$ than external $z_{\rm out}$ connections ($z = z_{\rm in} + z_{\rm out}$).

In addition to measure choice correlations between parts of the population it is useful to define a label state vector

$$\vec{v}_c = \left(N_1^c, N_2^c, \ldots, N_L^c\right)/A_c, \qquad A_c = \sqrt{\sum_{i=1}^{L} \left(N_i^c\right)^2}, \tag{1}$$

where N_i^c denotes the number of individuals in label i belonging to group c. The natural inner product

$$\langle \vec{v}_a | \vec{v}_b \rangle = \left(A_a A_b\right)^{-1} \sum_{i=1}^{L} \left(N_i^a N_i^b\right) \tag{2}$$

is a (positive definite) measure of the correlation between different groups, see Fig.1.

Fig. 1 shows the correlation of choices between several subsets of the population, within a community, between two distinct communities and for a control set of individuals with random

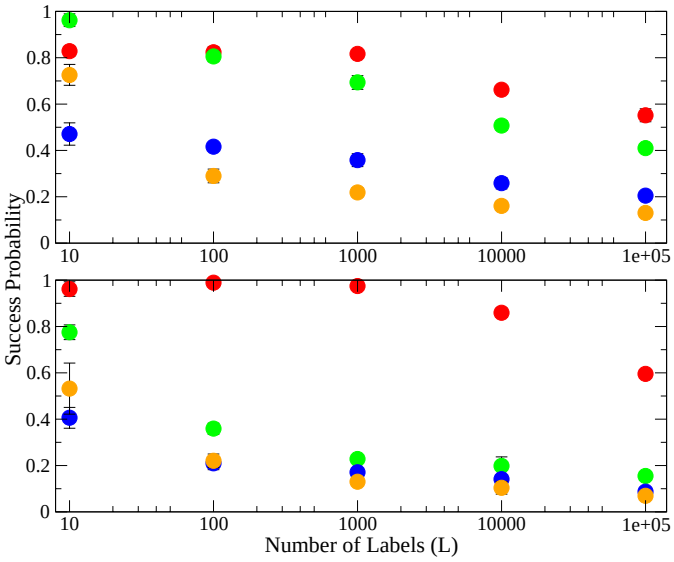

Figure 2: The success rate of several criteria for predicting the next winning trend at the time when the dominant movement decays. The next winning trend is not easily determined as the largest secondary trend (blue), the fastest growing (orange) or even the trend with the largest product of size and momentum (green). The best predictor is the choice of the hub (red), particularly as the number of choices L becomes large. The upper panel refers to lower hub visibility (his input is considered on average by each individual with probability $p_{\rm hub} = 1/8$, each time), the lower panel to higher visibility ($p_{\rm hub} = 1/2$). Error bars are as in Fig. 1.

connections. It is clear why it is a good defensive strategy to belong to a tightly knit collective: communities are *islands of coherence in choice space*. Thanks to the large redundancy of personal connections inside the community the coherence of local information is preserved over time and personal deviations inside the group are small compared to those to the outside. This remains true even if a few individuals or connections are lost.

Comforting as it may be to keep up with our neighbors it may actually be better to be a step ahead. As we discussed above this is a tall order, even if one is fully informed of the state of the whole population. Figure 2 shows the success rate of several criteria attempting to predict the emerging new trend at the particularly important time when a former dominant movement collapses, i.e. when it becomes as large as the largest secondary trend. All criteria based on the full knowledge of the state of population at this particular time (the largest secondary trend, the fastest growing one or the trend with the largest product of momentum and size) are far from good and become very poor for large number of competing choices L.

Interestingly there is a simple alternative solution - it relies on connections, not reasoning or information about the specific merits of a given choice. First I examine this scenario by introducing a new well-connected individual into the population, a network hub, as in Fig. 3. The hub bases his/her decisions, like any other agent, on the state of an average number of other individuals $\bar{z}$, but his choices can be seen by everybody else. More explicitly we construct the hub as an individual with typical out-connectivity (the individuals that influence his/her choice), but with very large in-connectivity. What is particular about the evolution is illustrated in Figs. 2 and 4: the hub is exceptionally good at picking the next winning trend early, before it becomes dominant.

However the hub is, by construction, neither better informed nor animated by superior decision making. This apparent paradox is easily dispelled: the hub's actions are very visible

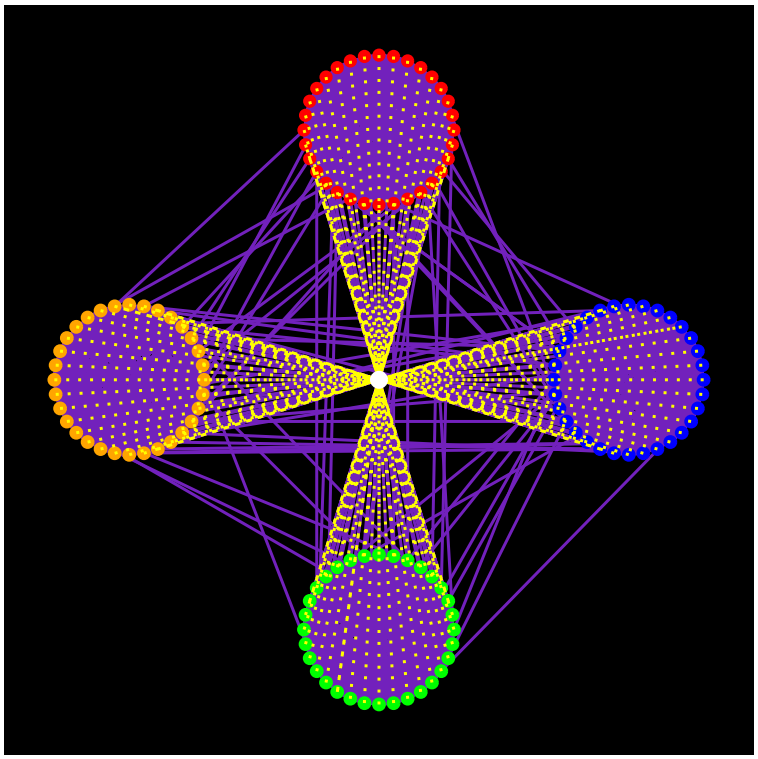

Figure 3: An example of a (binary) social network with $N = 128$, $z = 4$, divided into 4 communities (red, blue, green, orange) and with a hub (central node). Here the state of the hub is seen by all individuals with probability $p_{\text{hub}} = 0.25$ each time, but has input from $z = 4$ individuals. As such the actions of the hub are very visible but not better informed.

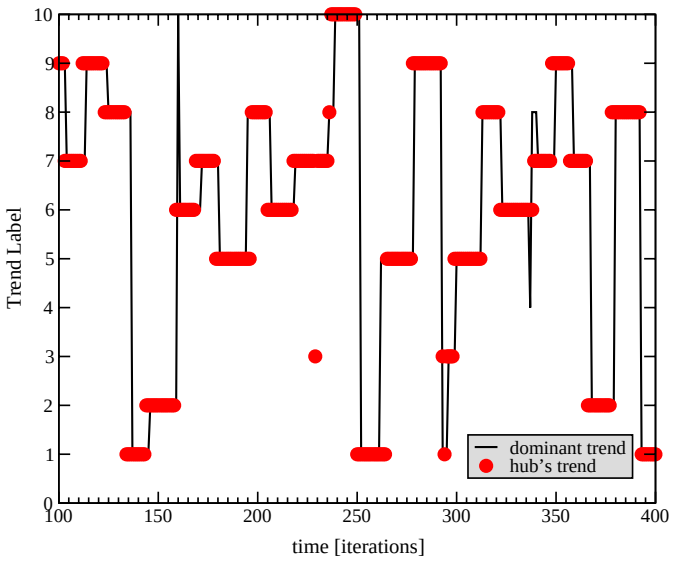

Figure 4: Fig. 4 Evolution of the dominant trend (solid line) and the hub's trend (red circles) for $L = 10$. The hub invariably picks the next dominant trend correctly and early, qualities that reinforce his social role as a bellwether.

to others. Any reasonable decision on his/her part (the adoption of any growing label) has a large probability of being immediately followed by many and thus to *make* the winning trend. Specifically if the average out-connectivity of any node is $\bar{z}$, and the hub has connections to $N_{\rm hub}$ individuals, then the probability that the hub's choice is considered *directly* by any individual is

$$p_{\rm hub} = \frac{N_{\rm hub}}{N\ \bar{z}}. \tag{3}$$

In practice the pull that the largest hub exerts on the population is larger as the influence of its choice is also felt indirectly, through the propagation of its choices through its neighbors.

The strong influence exerted by nodes with large in-connectivity on the choice of the collective is independent of the underlying community structure and is enhanced for larger populations (larger N), as long as $p_{\rm hub}(N)$ is such that $d(Np_{\rm hub})/dN > 0$. Thus, it is popularity (in degree), not knowledge or reasoning, that leads to the most successful strategy in an environment characterized by strong choice degeneracy.

Given memory the hub's successes reinforce his position and (apparent) foresight. Each correct 'prediction' encourages others to heed his/her choices and follow at the next opportunity. This reinforces the hub's popularity, allowing him/her to pick the next winning trend with greater certainty and so on: the process is self-reinforcing. It also naturally leads to a specific form of preferential attachment [25], where the most connected node - the best trend predictor - is preferred. Thus, under choice degeneracy, one should expect the appearance of well-connected, very visible individuals as a social network evolves.

The statements above about the coherence of choice, resulting from community structure, and of predictability, due to the presence of nodes with large in-degree, were so far illustrated for a particular class of networks. More general networks, with qualitatively different degree distributions, give similar results. Figure shows the probability that the choice of the largest hub (highest in-degree) corresponds to the largest trend, compared to the prediction from the label of the largest, fastest moving trend, and that of the fastest trend. It also shows how the effects of tighter community structure, captured by the clustering coefficient of a given node, imply greater choice coherence between that node and its neighbors.

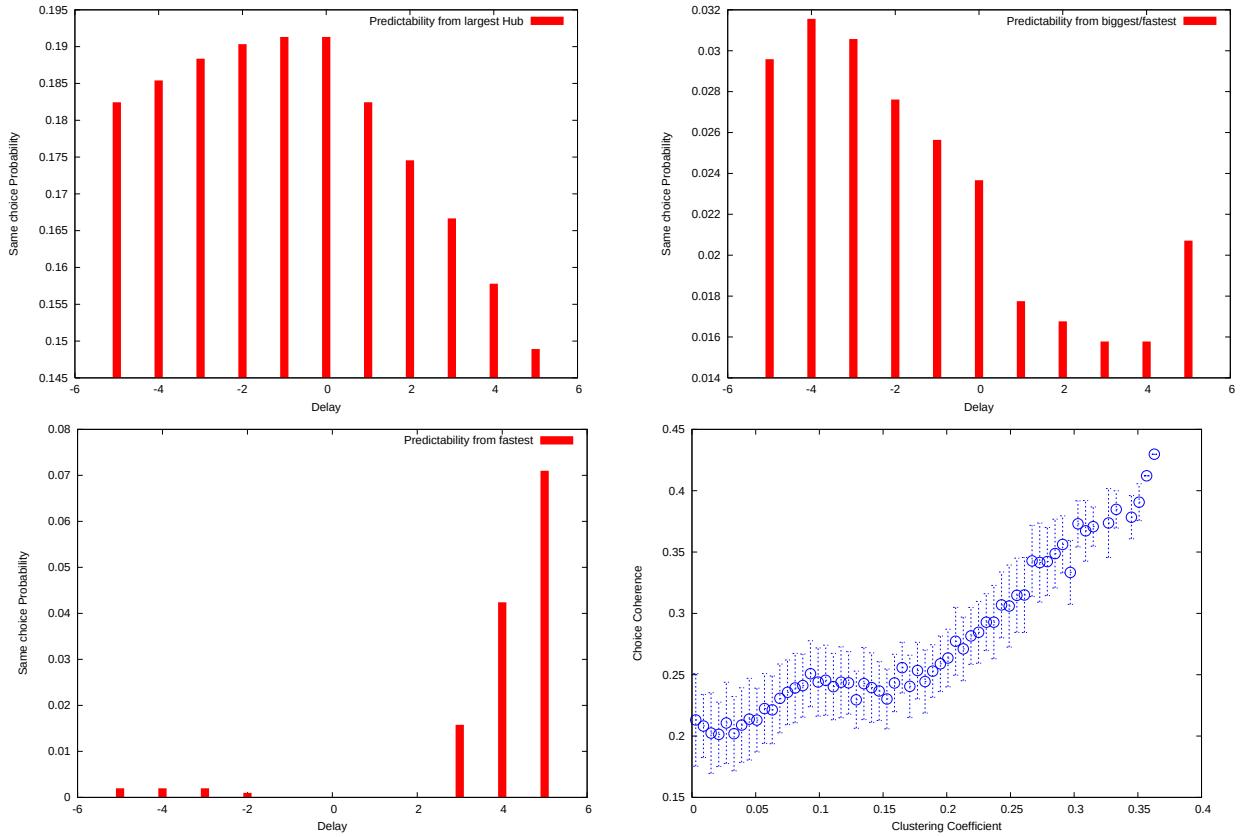

Figure 5: The probability of coincidence between the largest trend and the choice of the largest hub (top left), the largest, fastest moving trend (top right) and the fastest label (bottom left), for a network with a power law (exponent $\alpha = 2$) in-degree distribution. Different delays correspond to varying shifts in time, so that e.g. a delay of -1, indicates the probability that the trend predicts the largest trend in the next time step. The most connected hub continues to be the best predictor of the largest trend, even in spite of a smaller probability (here $p_{\text{hub}} = 0.09$). The value of p_{hub} gives a lower bound on its prediction capability, for delay < 0. The positive correlation (bottom right) between clustering coefficient (a measure of tightness of community structure) and the probability of equal choice between a node and its neighbors, for a network with power law distributed in-degree with exponent $\alpha = 3$.

Nodes with an exceptionally large connectivity are a common property of other complex networks, including scale-free graphs [26] describing e.g. WWW, power-grids and the large scale features of protein-protein interactions *in vivo*. It has been pointed [27, 28] out that such networks' utmost fragility is due to the loss of these key nodes. Trend dynamics shows how this fragility may only be apparent in social networks. Hubs are common nodes, only its in-degree is exceptionally large. I argued above that there is a fundamental instability for a common individual to be promoted to this position. Because social connections are rearranged on much faster time scales than nodes [29], upon loss of a hub a new one can quickly develop from another node and the structural integrity of the network will be preserved after a short transient. Moreover the addition of a second well-connected node dealing with the same information reduces the predictability of emerging trends, unless the two hubs work in tandem (as would happen under specific types of assortative mixing [29], another property of social networks) and so forth. It is however perfectly natural for separate hubs to coexist if they relate to different social dimensions [30], i.e. if they deal with different types of information. In this way the large scale structure of human societies, when averaged over time and social dimensions may be characterized by many hubs with varying reaches and interdependencies. These properties may lead to the emergence of interesting scale-invariances in large social networks associated with decision making and information flow.

Observing the actions of others is a universal simple mechanism that allows us to handle imperfect information in our complex social environment to make difficult decisions. We can protect ourselves from the tyranny of fashions by associating into tightly knit communities or we may try to set trends by influencing the choices of others through our social connections. Here I showed that these successful individual strategies lead to dynamically stable social arrangements, which coincide with some of the most notable observed structures of social networks. Trend dynamics breaks the degeneracy of our individual choices and leads to the spontaneous formation of collective movements. Whenever concerted social action is more productive than the sum of individual efforts hubs may become the social mechanism that facilitates the creation of consensus most promptly and predictably.

Acknowledgments

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References

[1] D. J. Watts and S. H. Strogatz, Collective dynamics of small world networks, Nature **393**, 440-442 (1998).

[2] D. J. Watts, Small worlds (Princeton University Press, Princeton University Press, Princeton, 1999).

[3] L. A. N. Amaral, A. Scala, M. Barthelemy, H. E. Stanley, Classes of small-world networks, Proc. Natl. Acad. Sci. U.S.A. **97**, 11149-11152 (2000).

[4] E. M. Jin, M. Girvan, M. E. J. Newman, Structure of growing social networks, Phys. Rev. E **64**, 046132 (2001).

[5] M. Girvan and M. E. J. Newman, Community Structure in social and biological networks, Proc. Natl. Acad. Sci. USA **99**, 8271-8276 (2002).

[6] M. Gladwell, The tipping point: How little things can make a big difference (Little, Brown and Company, Boston, 2000).

[7] S. Milgram, The small world problem, Psychol. Today **2**, 60-67 (1967).

[8] I. Pool and M. Kochen, Contact and influence, Social Networks **1**, 1-48 (1978).

[9] L. H. Hartwell, J. J. Hopfield, S. Leibler, A. W. Murray, From molecular to modular cell biology, Nature **402**, 47-52 (1999).

[10] S. H. Strogatz, Exploring complex networks, Nature **410**, 268-276 (2001).

[11] H. Jeong, S. P. Mason, Z. N. Oltvai, A.-L. Barabási, Lethality and centrality in protein networks, Nature **411**, 41-42 (2001).

[12] S. Maslov and K. Sneppen, Specificity and Stability in Topology of Protein Networks, Science **296**, 910-913 (2002).

[13] A.-C. Gavin, *et al.* Functional organization of the yeast proteome by systematic analysis of protein complexes, Nature **415**, 141-147 (2002).

[14] H. Jeong, B. Tombor, R. Albert, Z. N. Oltvai, A.-L. Barabási, The large-scale organization of metabolic networks, Nature **407**, 651-654 (2000).

[15] D. A. Fell and A. Wagner, The small world of metabolism, Nature Biotechnology **18**, 1121-1122 (2000).

[16] S. L. Pimm, J. H. Lawton, J. E. Cohen, Food web patterns and their consequences, Nature **350**, 669-674 (1991).

[17] R. J. Williams and N. D. Martinez, Simple rules yield complex food webs, Nature **404**, 180-183 (2000).

[18] This degeneracy of choice was first formalized by J. Bernouilli, Ars Conjectandi (Thurnisiorum, Basil, 1713) as 'the principle of insufficient reason'; later it was also known as 'the principle of indifference', J. M. Keynes, A Treatise on probability (Macmillan, London, 1921). It is also a well-known criterion in the context of the theory of decision making under uncertainty, prescribing that a decision should be reached under the assumption of a uniform probability distribution over 'states of nature'.

[19] The importance of the decisions of others in setting our own choices was first articulated in print by J. M. Keynes in The general theory of employment, interest and money (Macmillan, London, 1936), pages 155-156.

[20] S. Bikhchandani, D. Hirshleifer, I. Welch, A theory of fads, fashion, custom, and cultural change as information cascades, J. Pol. Economy **100**, 992-1026 (1992).

[21] S. Bikhchandani, D. Hirshleifer, I. Welch, Learning from the behavior of others: conformity, fads and information cascades, J. Econ. Perspectives **12**, 151-170 (1998).

[22] See also D. J. Watts, A simple model of global cascades on random networks, Proc. Natl. Acad. Sci. U.S.A. **99**, 5766-5771 (2002), which explores similar dynamics over random networks.

[23] L. M. A. Bettencourt, From boom to bust and back again: The complex dynamics of trends and fashions, (available at http://arXiv.org/abs/cond-mat/0212267).

[24] M. E. J. Newman, Models of the small world, a review, J. Stat. Phys. **101**, 819-841 (2000).

[25] A.-L. Barabási and R. Albert, Emergence of scaling in random networks, Science **286** 509-512 (1999).

[26] R. Albert and A.-L. Barabási, Statistical mechanics of complex networks, Rev. Mod. Phys. **74**, 47-97 (2002).

[27] R. Albert, H. Jeong, A.-L. Barabási, Error and attack tolerance of complex networks, Nature **406**, 378-381 (1999).

[28] D. S. Callaway, M. E. J. Newman, S. H. Strogatz, D. J. Watts, Network robustness and fragility: Percolation on random graphs, Phys. Rev. Lett. **85**, 5468-5471 (2000).

[29] M. E. J. Newman, Mixing patterns in networks: Empirical results and models, (available at http://arXiv.org/abs/cond-mat/0209450); M. E. J. Newman and M. Girvan, Mixing patterns and community structure in networks, (available at http://arXiv.org/abs/cond-mat/0210146).

[30] D. J. Watts, P. S. Dodds, M. E. J. Newman, Identity and search in social networks, Science **296**, 1302-1305 (2002).