State Manipulation and Asymptotic Inefficiency in a Dynamic Model of Monetary Policy*

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Abstract

One solution to the time-inconsistency problem of monetary policy rests on a simple folk theorem of repeated games which states that Pareto-efficient outcomes can be achieved if deviations are punished by a reversion to Nash equilibrium behavior, provided that players are sufficiently patient. We show in a dynamic version of a well-known monetary policy game that such "asymptotic efficiency" may not hold, as the presence of a state variable introduces an incentive for state manipulation. The solution may therefore not work, no matter how forward looking is the monetary authority.

Keywords: Monetary policy; time-inconsistency; credibility; unemployment persistence; dynamic games.

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1. Introduction

Following Kydland and Prescott (1977) and Calvo (1978), credibility of monetary policy has received enormous attention. These authors demonstrated the time-inconsistency of an ex ante optimal policy plan when policymakers have both the opportunity and the incentive to deviate from the plan after private-sector expectations are formed. In consequence, in these circumstances, the plan lacks credibility. The most celebrated example of this sort is the Barro and Gordon (1983) (BG) policy game wherein the monetary authority has the incentive to decrease unemployment through surprise inflation by exploiting a short-run Phillips-curve relationship. Under rational expectations, however, this incentive is anticipated and the economy ends up with an inflation bias. Despite its simplicity, this model highlights a basic problem, namely how can the monetary authority overcome time-inconsistency and secure credibility of efficient monetary policy?[1]

Since policy interactions are rarely of a one-shot nature, an appealing resolution of this problem acknowledges that if the immediate gain of deviation does not outweigh the (discounted) future costs associated with lost credibility, the optimal plan becomes credible. This conforms with well-known folk theorem results of infinitely repeated game theory which state that when players are sufficiently patient, efficient outcomes are attainable in a non-cooperative equilibrium.[2] BG also demonstrated this *asymptotic efficiency* using a particularly simple folk theorem where the loss of credibility following deviation reflects a reversion to the Nash equilibrium of the stage game — featuring the inflation bias — for a number of periods. This so-called *Nash punishment* has appeal for a number of reasons. It is simple and credible (players are willing to carry it out when required), and most importantly, in models where the "punishment" involves the private sector changing expectations, it is a natural "focal point" for expectations formation. It is therefore not surprising that the vast literature on folk theorem resolutions of time-inconsistency problems has mostly focused on the resulting *Nash-threats equilibria* (NTE). For example, al-Nowaihi and Levine (1994), Ball (1995), Ellis and Holden (1997), Herrendorf (1997), Horn and Persson (1988) all confirm, in variants of the BG model, asymptotic efficiency of NTE in monetary policy.[3]

It is the purpose of this paper to demonstrate that this widely applied resolution may not work if the basic BG model is extended in a natural and empirically relevant way. More

[1]See Persson and Tabellini (1999) for a recent survey of the literature that followed BG, and Ireland (1999) for recent econometric evidence for a BG based explanation of inflation in the US for the period 1960-97.

[2]See, e.g., Friedman (1971) and Fudenberg and Maskin (1986).

[3]The focus on NTE is not confined to the monetary credibility literature. It is a feature of most macroeconomic models of strategic interaction, where folk theorem arguments rationalize efficient equilibria.

specifically, we show that optimal monetary policy may *never* be achievable through Nash punishment, even if the monetary authority becomes arbitrarily patient. I.e., asymptotic *in*efficiency prevails. The extension we consider is to allow for unemployment persistence, which seems hardly questionable given the unemployment experience of Western economies in past decades (see, e.g., Layard *et al.*, 1995). Indeed, one could argue that given the autoregressive behavior of output and unemployment in Western economies, studying the static version of the BG model is inappropriate.

Unemployment persistence introduces a state variable (lagged unemployment) into the game which transforms it from a purely repeated game to a dynamic game. This change is crucial for folk theorem arguments. The reason is that when a state variable is present, players have an additional incentive to deviate from an agreed sequence of actions, apart from that pertaining to the immediate gain. In particular, through deviation, a player can affect the state variable so as to weaken the effect of the subsequent punishment. This is the incentive for *state manipulation* (cf. Dutta, 1995).

In our setting, the monetary authority has this incentive. To see this, note that in a dynamic game, the Nash punishment is reversion to the Markov-perfect equilibrium of the policy game for a number of periods. As shown by Lockwood and Philippopoulos (1994) the inflation bias in this Markov-perfect equilibrium is state-dependent in the sense that lower lagged unemployment implies a lower current inflation bias, and therefore a higher payoff in the Markov-perfect equilibrium.

Now suppose that the monetary authority deviates from an agreed level of inflation, thus reducing current unemployment. Due to persistence, this also reduces unemployment in subsequent periods, and thereby future inflation biases which, in effect, dampens the punishment. So, the authority clearly has an additional incentive to reduce current unemployment by increasing inflation in comparison with the conventional BG model without persistence. As we show, this state manipulation incentive may be strong enough to cause asymptotic inefficiency.[4]

Note that the above arguments imply that the incentive for state manipulation will be stronger, the less inflation-averse the authority is, as the cost of state manipulation — higher current inflation — then is lower. This also turns out to be the case, as our main result, Proposition 1, shows that asymptotic inefficiency occurs when the monetary authority is not too inflation averse. Hence, such an authority, no matter how patient it is, can never credibly reduce inflation to the socially optimal level.

[4]Moreover, we show by a numerical example that the monetary authority may not even get close to the optimal policy in a NTE as discounting becomes arbitrarily low, i.e., the magnitude of the asymptotic inefficiency may be quite significant.

Folk theorem resolutions have been rightfully criticized on a number of grounds.[5] Perhaps for these reasons, much recent research on monetary credibility has instead taken an "institutional approach" where various incentive schemes for independent central bankers are examined (this line of research originated with Rogoff, 1985). As argued by McCallum (1995), however, these delegation theories do not provide a satisfying resolution of credibility problems. The reason is that the government always has the incentive to change the delegation scheme after expectations have been formed. Hence, the credibility problem is merely "relocated" from the monetary policymaking stage to the delegation stage. The need for examining explicit precommitment technologies — like folk theorem resolutions — therefore remains. Some work on folk theorem resolutions in delegation models without unemployment persistence has already emerged, see al-Nowaihi and Levine (1996), Herrendorf (1998) and Jensen (1997), but all within purely repeated game frameworks.

In view of this, it is evident that our results should be of broad interest for the resolution of time-inconsistency problems in monetary policy. In general, our results provide an alternative reason (apart from the "expectations coordination" and multiplicity problems referred to in Footnote 5) why it may not be possible to achieve credible monetary policy via folk theorem mechanisms. In particular, our model thus predicts that patience (which could be interpreted a high degree of stability in policymaking, see, e.g., Cukierman, *et al.*, 1992) is no guarantee for credibility when unemployment persists. Hence, it seems that one cannot escape the simple fact that solving credibility problems must involve attacking them at their source, which in the BG model is the structural inefficiencies causing unemployment problems.[6]

The rest of the paper is organized as follows. The model and associated dynamic game are presented in Section 2 which also characterizes the dynamic equivalent of stage game equilibrium, Markov-perfect equilibrium. The properties of NTE are then presented in section 3 which contains our main result. Sections 4.1 and 4.2 study two important extensions of the basic model in order to assess the robustness of the main result. Section 5 concludes, and the Appendix contains some proofs.

[5] First, Nash punishment supports multiple policies ranging from the optimal to the inefficient stage-game Nash equilibrium policy. This multiplicity, of course, limits the model's predictive properties. A second, and related point, is how the private sector — presumably a pool of atomistic individuals — manages to coordinate expectations; not only concerning which NTE inflation rate will prevail, but also with respect to, for example, the duration of the potential punishment (this underlines the *ad hoc* property of the "focal point" argument mentioned in the main text). See Rogoff (1989) for a detailed discussion.

[6] Alternatively, one must search for equilibria building on more "severe punishments" than Nash (cf. Abreu, 1988). We discuss this issue further in the concluding section.

2. The Model

2.1. The dynamic game

The model is a dynamic version of the BG model as developed by Lockwood and Philippopoulos (1994). There are two agents, an authority responsible for monetary policy (the MA), and the private sector (PS), which sets the nominal wage. The interaction between these two evolves over an infinite number of periods, $t = 1, 2, ..., \infty$. Within a period, the order of events is as follows. First, the PS sets the (log of) the nominal wage, w_t. Then, having observed w_t, the MA sets the (log of) the price level p_t hence determining the real wage. Finally, the log of employment ℓ_t is determined from the labor demand relationship $\ell_t = -\beta(w_t - p_t)$, $\beta > 0$.

It is well known that in this setting, at t the actions of the MA and PS can be taken to be actual and expected rates of inflation π_t, π_t^e, respectively. The relationship between these actions and (un)employment at time t depends on the PS's objectives in wage-setting. We assume the following about these objectives. At t, the PS has an employment target $\hat{\ell}_t$, i.e., its per-period loss is $(\ell_t - \hat{\ell}_t)^2$. Following, e.g., Blanchard and Summers (1986), the target is assumed to be a convex combination of (log of) the labor force, n, and last period's employed, ℓ_{t-1} (the "insiders"): $\hat{\ell}_t = \rho\ell_{t-1} + (1-\rho)n$. A positive weight $0 < \rho < 1$ on ℓ_{t-1} indicates "insider power," i.e., that insiders have a disproportionate influence on wage-setting. The PS will then set the nominal wage to minimize $(\ell_t - \hat{\ell}_t)^2$, given the labor demand schedule and its expectation of the price level, p_t^e, i.e., $w_t = p_t^e - \hat{\ell}_t/\beta$. Inserting this wage back into the labor demand schedule, then enables us to write unemployment, $u_t \equiv n - \ell_t$ as

$$u_t = \rho u_{t-1} - \beta(\pi_t - \pi_t^e), \quad t = 1, 2..., \infty, \quad u_0 > 0 \text{ given}, \tag{1}$$

where $\pi_t \equiv p_t - p_{t-1}$, $\pi_t^e \equiv p_t^e - p_{t-1}$. Note that ρ parameterizes the persistence of unemployment.

It remains to specify the preferences of the two agents over sequences of unemployment, inflation, and expected inflation, $(u_t, \pi_t, \pi_t^e)_{t=1}^{\infty}$. As usual in this literature these are expressed in terms of quadratic loss functions. The loss of the MA is:

$$L^{MA} = \frac{1}{2}(1-\delta)\sum_{t=1}^{\infty}\delta^{t-1}\left[u_t^2 + \lambda\pi_t^2\right]. \tag{2}$$

This function implies that the MA in any period dislikes either unemployment or inflation different from zero (including some particular optimal value for the inflation rate would not add anything to our analysis). Parameter $\lambda > 0$ quantifies the MA's relative concern for inflation and unemployment objectives, i.e., it is a measure of its degree of inflation aversion. The discount factor is $0 < \delta < 1$. Finally, the loss of the PS is as follows. Using (1), its per-period loss can be written $(\ell_t - \hat{\ell}_t)^2 = \beta^2(\pi_t - \pi_t^e)^2$, so ignoring the constant β^2, we get

$$L^{PS} = \frac{1}{2}(1-\delta)\sum_{t=1}^{\infty}\delta^{t-1}(\pi_t - \pi_t^e)^2. \tag{3}$$

Equations (1)-(3) fully describe the discrete-time dynamic monetary policy game with state variable u_{t-1}.[7] Note that the specification of L^{PS} implies that the best response of the PS to π_t at t is $\pi_t^e = \pi_t$. This rational expectations property is assumed to hold in any equilibrium to be considered, and it is then clear by (3) that in equilibrium $L^{PS} = 0$. The interesting differences across equilibria will therefore be in terms of inflation performance (in that respect our model does not differ from the purely repeated game), so we can focus on the behavior of the MA, and for simplicity we define $L^{MA} \equiv L$.

2.2. Inflation rules

In what follows, we want to examine which sequences of inflation rates can be sustained as a NTE. We restrict attention to *proportional inflation rules* where inflation is a time-invariant, linear function of the state variable, i.e., $\pi_t = \pi u_{t-1}$ where π characterizes the rule.[8] In a rational expectations equilibrium it follows from (1) that $u_t = \rho u_{t-1}$, and the equilibrium loss to the MA in any period t from an inflation rule π, given state u_{t-1}, is therefore, by (2), given as:

$$L^r(\pi; u_{t-1}) = \frac{1}{2}(1-\delta)\sum_{t=1}^{\infty}\delta^{t-1}\left[u_t^2 + \lambda\pi_t^2\right] = \frac{1}{2}(1-\delta)\frac{\rho^2 + \lambda\pi^2}{1-\delta\rho^2}u_{t-1}^2. \tag{4}$$

It follows immediately from (4) that the optimal inflation rule for the MA is $\pi = 0$, which is not surprising in a rational expectations model where equilibrium unemployment evolves independently of actual inflation. Hence, $L^r(0; u_{t-1})$ is the *efficient loss* for the MA.

But as is well known, behavior leading to this loss may not be credible due to the MA's incentives to reduce unemployment by raising inflation once inflation expectations have been set. Given the infinite horizon of the model, conventional folk theorem arguments of purely repeated games suggest that $\pi = 0$ can be credible (supported in a NTE), and we

[7] As is by now well known, different preferences of the MA and PS are not necessary to generate a time-inconsistency problem in monetary policy. See, e.g., Ireland (1997) or Neiss (1999) for micro-founded models with a benevolent monetary authority where an inflation bias also arises.

[8] As the focus of this paper is on the zero inflation rule, this is without loss of generality as this rule is certainly proportional.

are therefore mainly interested in establishing the conditions for which this may or may not be true in this dynamic model. In order to do this, we turn to the specification of the punishment phase of the game.

2.3. Markov-perfect equilibrium

In the purely repeated game version of the model, the punishment in the NTE is a reversion to the perfect Nash equilibrium of the stage game for T periods. In a dynamic game, however, there is no well-defined stage game, but the natural corresponding Nash punishment is reversion to Markov-perfect Nash equilibrium (MPE) behavior for T periods.[9,10] Note that we, by definition, rule out any coordination problems in expectations formation and we are thereby in agreement with Stokey (1989) in assuming that the form and length of punishment by the private sector following deviant behavior of the MA is an exogenous characteristic of private sector agents. Our ensuing results on potential asymptotic inefficiency will therefore not rest on the various forms of coordination failures in the private sector's behavior.

We now consider the MPE of a T-period game, starting in period t, with initial level of unemployment u_{t-1}. Equilibrium behavior of the MA is described by a reaction function $\pi_t(\pi_t^e)$ defined as

$$\pi_t(\pi_t^e, u_{t-1}) = \underset{\pi_t}{\operatorname{argmin}} \left\{ \frac{1}{2} (1-\delta) \left[u_t^2 + \lambda \pi_t^2\right] + \delta L_{T-1}(u_t) \quad \text{s.t. } (1),\ \pi_t^e \text{ given} \right\}, \quad T > 1. \tag{5}$$

That is, $\pi_t(\pi_t^e, u_{t-1})$ is the value of π_t that minimizes the present value of losses to the MA, given that losses from $t+1$ onwards are $L_{T-1}(u_t)$, where $L_{T-1}(u_t)$ is the present value of future losses to the MA in a $T-1$-period MPE, given u_t. Due to the linear-quadratic structure of the model, we can take $L_T(u_t)$ to be quadratic:

$$L_T(u_t) = \frac{\gamma_T}{2} u_t^2, \tag{6}$$

where $\gamma_T \geq 0$ is a parameter to be determined. We can now find the conditions characterizing the MPE. It is helpful to proceed in two stages. First, find equilibrium behavior conditional on γ_T. Secondly, solve for γ_T.

[9] This is also the position of Cave (1987) in a dynamic game of resource extraction. He refers to the MPE as the *status quo ante*, and argues that it has a natural credibility as a disagreement point in an underlying bargain (one which, of course, is only implicit in our model). This argument is strengthened by the fact that the MPE in our model is unique (see below) and therefore is a good prediction of the consequences of disagreement.

[10] In a MPE, current actions (π_t, π_t^e) are constrained only to depend on the history of play though the current value of the state, u_{t-1}; see, e.g., Fudenberg and Tirole (1991) for a more rigorous definition.

Performing the minimization in (5), we obtain inflation as a function of the state variable and expected inflation. Using that $\pi_t^e = \pi_t$, cf. Section 2.1, we then derive the MA's equilibrium behavior, conditional on γ_{T-1}, as

$$\pi_t = \pi_t(u_{t-1}) = \frac{\beta}{\lambda} y_{T-1} \rho u_{t-1}, \quad T > 1, \tag{7}$$

where $y_{T-1} \equiv 1 + \gamma_{T-1}\delta/(1-\delta) > 0$. Note that when $u_{t-1} > 0$, $\pi_t > 0$, i.e., the MPE is characterized by the familiar inflation bias due to the MA's *ex post* incentive to produce "surprise" inflation as in the non-dynamic BG model. Note that in this dynamic model, the bias is *state-dependent* as it increases with u_{t-1}. This is because higher unemployment inherited from the past increases the marginal benefit of an inflation surprise, thus leading to a higher equilibrium inflation rate. The second stage is to solve for y_T (and thus γ_T). In appendix A it is demonstrated that whenever

$$(1-\rho^2)^2 > 4\frac{\beta^2\rho^2}{\lambda} \tag{A1}$$

holds, a *unique* solution to y_T exists for all δ. We therefore assume (A1) in what follows. Moreover, if a mild requirement on π is satisfied (one that does not bind when $\pi = 0$; the case we will be most interested in), the solution to y_T is unique and bounded for all δ, T.

3. Nash-threats equilibria

Now we examine in detail which inflation rules can be supported by Nash punishments, i.e., be part of a NTE. Clearly, a rule π is a NTE rule if it does not pay for the MA to deviate from it, given that a deviation at t is punished by reversion to a MPE for T periods, followed by reversion to the rule at $t+T+1$ and onwards. To characterize such rules, the minimal loss from deviation must be calculated, and it suffices to consider the minimal loss from a one-shot deviation at t. We can express this minimal loss as a function of the inflation rule and the state variable as follows:

$$\begin{aligned} L^d(\pi; u_{t-1}) \quad = \quad & \min_{\varphi_t} \left\{ \frac{1}{2}(1-\delta)\left[u_t^2 + \lambda\varphi_t^2\right] + \delta L_T(u_t) + \delta^{T+1} L^r(\pi; \rho^T u_t) \right. \\ & \left. \text{s.t. } u_t = \rho u_{t-1} + \beta(\pi u_{t-1} - \varphi_t) \right\}, \end{aligned} \tag{8}$$

where we have used the fact that a deviation at t will imply unemployment of $\rho^T u_t$ at $t+T$. Carrying out the minimization in (8), using (4) and (6), we find after some calculations (available upon request) that

$$L^d(\pi;u_{t-1}) = \frac{1}{2}(1-\delta)\frac{\lambda\beta^2\hat{y}_T}{\lambda+\beta^2\hat{y}_T}\left(\frac{\rho}{\beta}+\pi\right)^2 u_{t-1}^2, \tag{9}$$

$$\hat{y}_T \equiv y_T + \frac{(\delta\rho^2)^{T+1}+\delta^{T+1}\rho^{2T}\lambda\pi^2}{1-\delta\rho^2} > 0. \tag{10}$$

So, $\pi^{NTE} \in \Pi$ is a NTE rule if the loss from the rule does not exceed the loss from deviation at any period of the game (i.e., at any value of the state variable), i.e., iff

$$L^r(\pi^{NTE};u_{t-1}) \leq L^d(\pi^{NTE};u_{t-1}), \qquad u_{t-1}=\rho^{t-1}u_0, \text{ all } t=1,2,... \tag{11}$$

Using (4) and (9), simplifying and cancelling terms in (11), allows us to rewrite the condition for π to be a NTE rule as:

$$\Lambda^r(\pi^{NTE}) \equiv \frac{\rho^2+\lambda\left(\pi^{NTE}\right)^2}{1-\delta\rho^2} \leq \frac{\lambda\beta^2\hat{y}_T}{\lambda+\beta^2\hat{y}_T}\left(\frac{\rho}{\beta}+\pi^{NTE}\right)^2 \equiv \Lambda^d(\pi^{NTE}) \tag{12}$$

With this simplification, we can state our main result:

Proposition 1. *Suppose that the MA is not too inflation averse, $\lambda < \beta^2$. Then, no matter how long the punishment period, and no matter how little discounting, any NTE inflation rule is uniformly bounded above zero, i.e., for some $\varepsilon > 0$, $\pi^{NTE} > \varepsilon$, all δ, all T.*

Proof. See Appendix B. ■

The proposition shows that no matter how long the punishment period is and no matter the degree of patience of the MA, then the zero inflation rule cannot be attained through Nash punishments. As a corollary, the MA's loss can never approach the efficient loss. So not even in the case where $\delta \to 1$ will zero inflation be credible. The presence of such asymptotic inefficiency is strikingly different to the results of BG and others (cf. the references in the Introduction), where the efficient inflation rule is attainable (at least) asymptotically as $\delta \to 1$ for some periods of Nash punishment.

As stressed in the Introduction, the failure of Nash punishments in our model is caused by the dynamic nature of the game and the associated state manipulation incentive. In particular, in this game, if the MA deviates at t, this has consequences not only for unemployment at t as in the conventional non-dynamic case, but it will also affect the loss during the punishment phase, $L_T(u_t)$, as current unemployment is next period's state variable. And since $L_T(u_t)$ is an increasing function of u_t, cf. (6) [because the inflation bias in MPE increases, cf. (7)], the MA has a state manipulation incentive to reduce unemployment at t, in addition to the incentive to deviate for immediate gain. As this

Parameter deviation	None	$\beta = 0.3$	$\beta = 0.1$	$\rho = 0.6$	$\rho = 0.4$	$\lambda = 1.5$	$\lambda = 0.5$
$\underline{\pi}^{NTE}/\pi^{MPE}$	0.48	0.45	0.49	0.25	0.66	0.49	0.46

The ratio of lowest NTE inflation rule to the MPE inflation rule for a baseline parameter set, $\beta = 0.2$, $\rho = 0.5$, $\lambda = 1$, $\delta \to 1$, $T \to \infty$, as well as deviations.

Table 1: Lowest NTE inflation rule vs. MPE inflation rule

incentive becomes stronger to more the MA cares about the future, it follows that a higher value of δ increases the temptation to deviate as well as the punishment. This is in contrast to the purely repeated game where the temptation to deviate is independent of δ. The proposition thus shows that if $\lambda < \beta^2$, the state manipulation incentive is sufficiently strong such that the punishment *never* deters deviation.

This condition has a natural interpretation. To reduce u_t, inflation has to be increased at a marginal cost proportional to λ. Moreover, from (1), the benefit in terms of reduced unemployment, from a given increase in inflation is proportional to β^2. Our result thus says that if the marginal cost of state manipulation is low enough relative to the benefit, the incentive for state manipulation dominates the loss from punishment even as $\delta \to 1$.

It should be noted, however, that in the proof we have not used any knowledge about the relationship between the model's parameters and $\widehat{y}_T$; indeed the condition $\lambda < \beta^2$ is only a *sufficient* condition for asymptotic inefficiency. To emphasize this, we present a numerical example which also serves the purpose of quantifying the difference between the smallest NTE inflation rule, denoted $\underline{\pi}^{NTE}$, and the MPE inflation rule, denoted π^{MPE}. This example therefore reveals whether the MA is "close" or "far" from attaining the socially optimal rate of inflation of zero, as it quantifies the magnitude of the maximal inflation reduction through Nash punishments. The example is constructed so as to give Nash punishments as much deterrent force as possible: we consider the case of $T \to \infty$ and $\delta \to 1$.[11] Table 1 shows the ratio $\underline{\pi}^{NTE}/\pi^{MPE}$ for a baseline parameter set as well as for some deviations.

As is clear from the table, in all the reported cases, asymptotic inefficiency is not trivial. The largest possible inflation reduction from the MPE is 75 percent, but for most parameter configurations the reduction is only around 50 percent. So Proposition 1 does not describe inefficiency of negligible quantitative importance. Moreover, as the simulations indicate, we can have asymptotic inefficiency even when $\lambda > \beta^2$, which emphasizes that the condition stated in the proposition is merely sufficient, not necessary.

[11] This also simplifies computation of MPE since $\lim_{T\to\infty} \widehat{y}_T = \lim_{T\to\infty} y_T = y^-$ (cf. (10) and Section 2.3) which is independent of π. Then, (12) reduces to comparing two quadratic expressions in π, and when evaluating (12) with equality, one obtains two roots corresponding, respectively, to the lowest π in NTE and the highest, which will be the MPE rule.

4. Extensions

The particular economic model used in the previous sections, i.e., the unemployment equation, (1), can be extended in at least two directions, both of which have been extensively studied in the literature of the non-dynamic versions of this policy game. This section considers these two extensions in turn. In doing so, we can evaluate the robustness of the main result of the previous section. Moreover, the first extension allows a more immediate comparison with the purely repeated game than is possible with the simple game analyzed in Sections 2 and 3.

4.1. Long-run disagreements

A special feature of our model is that the wage-setters are assumed to care about a target level of employment, which is a convex combination of the labor force and last period's employment level. Although this is a standard assumption in the literature on "insiders and outsiders," it is also rather special. The reason is that in this setting, as $t \to \infty$, the employment target of the wage-setters approximates the MA's target of no unemployment. So, asymptotically, there is no "disagreement" between wage-setters and the MA over the appropriate level of unemployment. In other words, the long run natural rate of unemployment is considered optimal by the MA. Although this *could* be justified as a plausible long-run scenario, it implies that without persistence, the model does not reduce to the standard BG model where disagreements last forever. Instead, it collapses to a special (and uninteresting) case where wage-setters in any period aim for the same level of unemployment as the MA (zero). Then there will be no incentives for the MA to inflate, and the time-inconsistency problem disappears. Hence, direct comparison with the well-understood non-dynamic case is somewhat problematic. We therefore extend the model so as to capture a long-run disagreement over unemployment between the two players. This will further emphasize the role of unemployment persistence in causing asymptotic inefficiency as well as reveal an unfavorable trade-off between credibility of monetary policy and long-run unemployment problems.

A long-run disagreement is incorporated by changing (1) to:[12]

$$u_t = (1-\rho)u_n + \rho u_{t-1} - \beta(\pi_t - \pi_t^e), \quad t = 1, 2..., \infty, \quad u_n > 0, \quad u_0 > 0 \text{ given.} \tag{13}$$

[12] The process (13) can, e.g., be rationalized by a model where wage setters also have a real wage target that is incompatible with full employment, i.e., the target wage is too high, see, e.g., Lockwood *et al.* (1998). Alternatively, the process arises if wage-setters have an employment target covering only some part of the total labor force (say, union members).

Clearly, in a rational expectations equilibrium, as $t \to \infty$, unemployment now converges to u_n which thereby measures the difference between the wage-setters' long-run unemployment target and the MA's target of zero. The question is now how this affect our results on asymptotic inefficiency in the associated dynamic game.

To set the scene for our answer, note first that if $\rho = 0$, the dynamic game with state equation (13) reduces to a standard purely repeated game of the type studied by BG. In this case, the MPE is simply the Nash equilibrium of the stage game which is easily calculated to be $\pi_t = \pi_t^e = (\beta/\lambda)\, u_n$. The punishment triggered by deviation from a rule π is therefore $(\beta/\lambda)\, u_n$ for T periods. In this case, it is possible to prove that *one-period* punishments suffice to enforce the zero inflation rule when δ is large enough:

Proposition 2. *In the purely repeated game, $\rho = 0$, with unemployment given by (13), then whatever the length of the punishment period $(T \geq 1)$ there exists a $\overline{\delta} < 1$ such that for all $\delta \geq \overline{\delta}$, $\pi = 0$ is a NTE rule.*

Proof. See Appendix C. ■

Without any dynamic link between periods, there is, of course, no incentive for state manipulation, as the future punishment will be independent of current actions. So, as the proposition shows, without state manipulation, Nash punishments are very effective: even the threat of reverting to the MPE for just one period is sufficient to deter deviation from optimal monetary policy if the MA is sufficiently patient.

We now show that with even a small amount of persistence, this picture becomes dramatically different. Specifically, we prove that if $\rho > 0$, for a fixed punishment period, no matter how long, there exists a value of u_n small enough so that zero inflation may not be a NTE outcome, for any value of δ. Formally:

Proposition 3. *Consider the dynamic game, $\rho > 0$, with unemployment given by (13), and suppose that the MA is not too inflation averse, $\lambda < \beta^2$. For any given T, if u_n/u_0 is small enough, then any NTE inflation rule is bounded above zero, uniformly in the discount factor, i.e., for some $\varepsilon > 0$, $\pi^{NTE} > \varepsilon$, all δ.*

Proof. See Appendix D. ■

So, in the case of long-run disagreements over the appropriate unemployment rate we can also have asymptotic inefficiency, i.e., optimal monetary policy may not be part of a NTE for any length of punishment, even if the MA does not discount the future at all. The intuition is qualitatively the same as in the case without any long-run disagreement. The dynamic structure of the game implies an incentive for state manipulation, which may not be deterred by Nash punishments even for $\delta \to 1$. This is in stark contrast with the purely repeated game where asymptotic efficiency *always* applies, cf. Proposition 2.

As seen from Proposition 3, we need one more condition in order to have asymptotic inefficiency in comparison with the case made by Proposition 1; namely that u_n/u_0 must be small. This has an immediate and intuitive explanation. First note that when unemployment follows (13) instead of (1), the MPE inflation rate will not just depend on u_{t-1} (state-dependent inflation bias). It now includes a constant part in u_n reflecting a permanent inflation bias due to the long-run disagreement over unemployment. This permanent bias is, of course, increasing in u_n, and can by its nature not be affected through state manipulation. Hence, the higher permanent bias relative to the state-dependent bias, quantified by u_{t-1}, the smaller is the MA's scope for state manipulation, and the more deterrent are Nash punishments. So, asymptotic inefficiency due to state manipulation occurs when u_n is small relative to u_{t-1}. A necessary condition for this is that u_n is small relative to u_0. (Along the equilibrium path, if $u_0 > u_n$, then $u_0 > u_{t-1} > u_n$, $t > 1$, so $u_n/u_{t-1} < \varepsilon$, some t, implies $u_n/u_0 < \varepsilon$. So, u_n/u_0 small is necessary.)

This condition reveals a rather unfavorable trade-off in policymaking. Imagine that u_n is a variable which the authorities can control, say, through some labor market policy. Then, removing long-run distortions, i.e., setting $u_n = 0$ may be good for unemployment, but detrimental for credibility of monetary policy as revealed by Proposition 1. Setting u_n at some relative high value will obviously be unfavorable in terms of unemployment, but as revealed by Proposition 3, it may be conducive for credibility of optimal monetary policy. Alternatively, if u_n cannot be considered a choice variable, the result implies that only economies where current unemployment conditions are sufficiently favorable (a low value of u_0) will be capable of pursuing a credible low inflation policy — a feature which does not seem entirely at odds with what is observed in reality.

4.2. Supply shocks and state-contingent rules

We also consider another natural extension of (1), namely the introduction of supply shocks. With a supply shock, equation (1) becomes

$$u_t = \rho u_{t-1} - \beta(\pi_t - \pi_t^e) + \varepsilon_t, \quad t = 1, 2..., \infty, \quad u_0 > 0 \text{ given}, \tag{14}$$

where ε_t is an i.i.d. shock with mean zero and variance σ^2 which hits the economy after inflation expectations have been formed. We furthermore assume that ε_t is bounded. Otherwise, as is well-known, cooperative agreements may break down even in purely repeated games as particularly extreme realizations of the shock cause the gain from deviation to be insurmountable (see, e.g., Canzoneri and Henderson, 1988).

We now proceed by showing that the MA cannot achieve its most preferred inflation rule as an outcome of the NTE, no matter how high δ. Due to the model's linear-quadratic structure, we can restrict attention to rules of the form $\pi_t(\varepsilon_t) = \pi u_{t-1} + s\varepsilon_t$, where s is a stabilization coefficient. Clearly, whatever s, $\pi = 0$ is optimal for the MA, so we still focus on $\pi = 0$ in what follows. Given $\pi = 0$, what is the efficient stabilization coefficient with unemployment dynamics? Let E_t be the expectation conditional on $\{\varepsilon_\tau\}_{\tau=1}^{t}$. Then, the loss of the rule from the beginning of period t is given by

$$\begin{aligned} \bar{L}^r(s; u_{t-1}) &= \frac{1}{2}(1-\delta)\,\mathrm{E}_{t-1}\sum_{\tau=t}^{\infty}\delta^{\tau-t}\left[u_\tau^2 + \lambda\pi_\tau^2\right], \\ &= \frac{1}{2}(1-\delta)\frac{\rho^2}{1-\delta\rho^2}u_{t-1}^2 + \frac{1}{2}\left[\frac{1}{1-\delta\rho^2}(1-\beta s)^2 + \lambda s^2\right]\sigma^2. \end{aligned} \qquad (15)$$

It follows from (15) that the s minimizing $\bar{L}^r(s; u_{t-1})$ is $s^r \equiv \beta/\left[\beta^2 + \lambda(1-\delta\rho^2)\right]$. Note that this is increasing in persistence as a given shock to unemployment then has more severe consequences for future unemployment thereby necessitating a stronger policy response (for more discussion, see Lockwood *et al.*, 1998).

We now ask whether the optimal rule $\pi_t(\varepsilon_t) = s^r\varepsilon_t$ achievable as the outcome of a NTE for some δ. The following proposition shows that this may not be the case no matter what the value of δ, and is thereby a generalization of Proposition 1 to the case of supply shocks.

Proposition 4. *Let unemployment be given by (14), and suppose that the MA is not too inflation averse, $\lambda < \beta^2$. For any given T, there is a σ/u_0 small enough, such that the inflation rule $\pi_t(\varepsilon_t) = s^r\varepsilon_t$ is not an NTE rule, no matter what the discount factor.*

Proof. See Appendix E. ■

So, in the presence of supply shocks, we may also have asymptotic inefficiency of monetary policy. But just as the introduction of a long-run disagreement over unemployment rates introduced an additional requirement for asymptotic inefficiency, so does the introduction of shocks. In comparison with Proposition 1, we see from Proposition 4 that σ/u_0 must be small. This also has an intuitive explanation. The MPE inflation rate will contain a stabilization component which is different from $s^r\varepsilon_t$. More specifically, stabilization will be too high in the MPE as the MA has an additional incentive to reduce the impact of a shock when it knows that the equilibrium features a future inflation bias which depends on current stabilization efforts. In consequence, inflation becomes too volatile in comparison with the optimal rule (cf. Lockwood *et al.*, 1998).

It is important to note that this inefficient response is a function of the shocks hitting the economy, not of the particular level of unemployment, i.e., the particular value of the state. Therefore, the MA cannot manipulate this future "excess inflation volatility" through deviation. Hence, the higher the variability of shocks, σ^2, the more deterrent is the MPE as a punishment in expected value. So it follows that asymptotic inefficiency due to state manipulation occurs when σ is small relative to u_0 as the latter quantifies the state-dependent inflation bias, and thus the incentive for state manipulation.

5. Concluding comments

In this paper we have demonstrated that a simple — and widely applied — folk theorem of repeated games may not apply in the conventional monetary policy game put forth by BG, when we amend it with unemployment persistence. The presence of dynamics is shown to generate an incentive for state manipulation. This additional incentive, not present in purely repeated games, may result in failure to sustain the efficient outcome as an NTE even when players do not discount the future at all.

We have focused on NTE, where the punishment is reversion to a MPE of the game. This has been motivated on the grounds that NTE has been predominant in most of the literature on folk theorem resolutions of the monetary credibility problem. It is, however, as briefly mentioned in the Introduction, conceivable that there exist more severe punishments that allow zero inflation as an equilibrium outcome as $\delta \to 1$ (cf. Abreu, 1988).[13] The folk theorem for stochastic games (of which our dynamic games are special cases) provided by Dutta (1995) suggests that this may indeed be the case, and our results therefore indicate that one must seek for such severe punishments if credibility of monetary policy should always be attainable for low discounting.[14]

In purely repeated games of the BG type, Rogoff (1989) and Stokey (1991) have examined credibility through more severe punishment behavior. But these analyses mainly

[13]Recent work by, e.g., Chang (1998) and Ireland (1997) demonstrate methods (related to the "Sustainable Plans" concept of Chari and Kehoe, 1990) to characterize the entire set of time-consistent monetary policy equilibria in micro-founded models. They generally find the efficient equilibria in these sets. These models, however, do not contain dynamic state variables like the unemployment rate in our model. The incentives for state manipulation by the policymaker(s) are therefore not an issue.

[14]Note, however, that Dutta's theorem does not apply directly to our model. First, the theorem is confined to games with bounded action spaces. Secondly, it considers strictly individually rational payoffs, i.e., payoffs strictly better than min-max payoffs. Such payoffs, however, does not exist for PS in our game as it can never achieve more than its min-max payoff (because this is zero — the best possible payoff, or, lowest possible loss).

served the purpose of demonstrating that credibility could then be attained even with high discounting. They were not necessitated by a failure of asymptotic efficiency of NTE. We are not aware of similar analyses for models with unemployment persistence, and we also agree with Rogoff (1989) who "do not regard severe punishment equilibria as being particular plausible in the present context" (p. 243). (Stokey, 1989, has similar reservations within other macroeconomic model frameworks.) Nevertheless, an important topic for future research is to identify simple and sensible precommitment technologies that support optimal monetary policy in dynamic models where state manipulation causes failure of the Nash punishment on which the macroeconomic policy games literature has been focused for the past decades.

Appendix

A. Solving y_T

For the purpose of solving for y_T, note that along the equilibrium path, the following recursion equation must hold:

$$L_T(u_{t-1}) = \frac{1}{2}(1-\delta)\left[u_t^2 + \lambda\pi_t^2\right] + \delta L_{T-1}(u_t), \quad T > 1, \tag{A.1}$$

with $u_t = \rho u_{t-1}$ and $\pi_t = \pi_t(u_{t-1})$. Equating coefficients in u_{t-1}^2 on both sides of (A.1), yields the Riccati difference equation in y_T:

$$y_T = 1 + \frac{\beta^2}{\lambda}\delta\rho^2 y_{T-1}^2 + \delta\rho^2 y_{T-1}, \quad T > 1. \tag{A.2}$$

When determining a boundary condition on (A.2), note that we are interested in the MPE insofar as it provides a punishment. So, for the final period of play, i.e., for the 1-period game with state variable u_{t-1}, the discounted future loss is $\delta L^r(\pi; u_t)$, and optimal policy therefore minimizes $\frac{1}{2}(1-\delta)\left[u_t^2 + \lambda\pi_t^2\right] + \delta L^r(\pi; u_t)$ s.t. (1), π_t^e given. From this problem we obtain the MPE of the 1-period game as

$$\pi_t|_{T=1} = \frac{\beta}{\lambda}\left(1 + \delta\frac{\rho^2 + \lambda\pi^2}{1-\delta\rho^2}\right)\rho u_{t-1}. \tag{A.3}$$

Using (A.3) with $u_t = \rho u_{t-1}$ gives $L_1(u_{t-1}) = \frac{1}{2}(1-\delta)\left[1 + \frac{\beta^2}{\lambda}\left(1 + \delta\frac{\rho^2+\lambda\pi^2}{1-\delta\rho^2}\right)^2\right]\rho^2 u_{t-1}^2$, from which we find $\gamma_1 = (1-\delta)\left[1 + \frac{\beta^2}{\lambda}\left(1 + \delta\frac{\rho^2+\lambda\pi^2}{1-\delta\rho^2}\right)^2\right]\rho^2$, implying the boundary condition:

$$y_1 = 1 + \delta \left[1 + \frac{\beta^2}{\lambda} \left(1 + \delta \frac{\rho^2 + \lambda \pi^2}{1 - \delta \rho^2} \right)^2 \right] \rho^2 > 0. \tag{A.4}$$

Straightforward calculation shows that (A.2) possesses real stationary points if and only if

$$(1 - \delta\rho^2)^2 > 4 \frac{\beta^2 \delta \rho^2}{\lambda}, \tag{A.5}$$

in which case it has two stationary points, $y^+ > y^- > 0$. As our analysis involves changes in the discount factor, (A.5) must hold for *all* values of δ, i.e., (A1) must hold. Now, if (A.5) holds, analysis of the phase diagram of (A.2) indicates that y_T converges monotonically to y^- as $T \to \infty$, as long as $y_1 < y^+$ (otherwise, $y_T \to \infty$ in case of $y_1 > y^+$ or $y_T = y^+$, all T, if $y_1 = y^+$). However, $y_1 < y^+$ is equivalent to saying that the loss from $\pi_t|_{T=1}$ is less than the loss from the MPE inflation rule (7) where $y_T = y^+$. So, we confine attention to inflation rules $\pi \in \Pi \equiv \{\pi \,|\, \pi_t|_{T=1} < (\beta/\lambda)\, y^+ \rho u_{t-1}\}$ in order to ensure that y_T is positive, unique and bounded for all T. For $\pi = 0$, however, this restriction does not to bind.[15]

B. Proof of Proposition 1

To prove the proposition, we only need to establish

$$\Lambda^r(0) - \Lambda^d(0) \geq b, \quad \text{all } T,\ \delta, \tag{B.1}$$

for some $b > 0$.[16] For then, (12) is violated at $\pi^{NTE} = 0$, which means that a NTE rule is bounded away from zero. To establish (B.1), we proceed as follows. First, the assumption $\lambda < \beta^2$ is sufficient to ensure

$$\frac{\beta^2}{1 - \delta\rho^2} - \lambda \geq \beta^2 - \lambda \geq \hat{b}, \quad \text{all } \delta,$$

for some $\hat{b} > 0$. Simple manipulation then implies

[15]Note first that $y^+ = [(1 - \delta\rho^2) + \sqrt{D}] / \left(2\beta^2\delta\rho^2/\lambda\right)$ with $D \equiv \left(1 - \delta\rho^2\right)^2 - 4\beta^2\delta\rho^2/\lambda$. For $\pi = 0$, we have, using (A.3), that $\pi_t|_{T=1} < (\beta/\lambda)\, y^+ \rho u_{t-1}$ is equivalent of $\left(1 - \delta\rho^2\right)^{-1} < y^+$. Using the expression for y^+, this becomes $2\beta^2\delta\rho^2/\lambda - \left(1 - \delta\rho^2\right)^2 < \left(1 - \delta\rho^2\right)\sqrt{D}$, which always holds as the left hand side is negative, cf. (A.5).

[16]For this condition to meaningful we must, of course, require that $\Lambda^r(0)$ and $\Lambda^d(0)$ are bounded for all T, δ. From (12), $\Lambda^r(0)$ obviously meets this requirement. Using the definition of $\widehat{y}_T$ in (10), it follows that $\Lambda^d(0)$ is bounded if $y_T|_{\pi=0}$ is bounded. But this is clearly the case as $\pi = 0 \in \Pi$. For by the construction of Π, $y_T|_{\pi=\pi^{NTE}}$ is bounded for all T, δ if $\pi^{NTE} \in \Pi$; cf. Appendix A.

$$\frac{\rho^2}{1-\delta\rho^2} - \frac{\beta^2 \hat{y}_T}{\lambda+\beta^2\hat{y}_T}\frac{\lambda\rho^2}{\beta^2} \geq \frac{\rho^2}{1-\delta\rho^2} - \frac{\lambda\rho^2}{\beta^2} \geq \hat{b}\frac{\rho^2}{\beta^2}, \quad \text{all } T,\ \delta,$$

since $\hat{y}_T > 0$, all T, δ, cf. (10). Setting $b = \hat{b}\frac{\rho^2}{\beta^2}$ we then conclude that

$$\frac{\rho^2}{1-\delta\rho^2} - \frac{\lambda\beta^2\hat{y}_T}{\lambda+\beta^2\hat{y}_T}\frac{\rho^2}{\beta^2} \geq b > 0, \quad \text{all } T, \delta,$$

which by use of the definitions of Λ^r and Λ^d, cf. (12), establishes (B.1).

C. Proof of Proposition 2

It clearly suffices to prove the proposition for $T = 1$. Now, the loss from the zero-inflation rule is $L^r = \frac{1}{2}u_n^2$. The per-period loss in the MPE is $\frac{1}{2}(1-\delta)\left(1+\beta^2/\lambda\right)u_n^2$, and finally, the one-period loss from deviation from the zero-inflation rule is easily calculated to be $\frac{1}{2}(1-\delta)\lambda/\left(\lambda+\beta^2\right)u_n^2$. So, the overall deviation loss with a one-period punishment is

$$L^d = \frac{1}{2}(1-\delta)\frac{\lambda}{\lambda+\beta^2}u_n^2 + \delta\frac{1}{2}(1-\delta)\frac{\lambda+\beta^2}{\lambda}u_n^2 + \delta^2\frac{1}{2}u_n^2.$$

We have that the zero-inflation rule is a NTE rule iff $L^d \geq L^r$. Using the above formulae for L^r and L^d this condition is

$$\frac{1}{2}(1-\delta)\frac{\lambda}{\lambda+\beta^2}u_n^2 + \delta\frac{1}{2}(1-\delta)\frac{\lambda+\beta^2}{\lambda}u_n^2 \geq (1-\delta)\frac{1}{2}(1+\delta)u_n^2,$$

which easily reduces to $\delta \geq \lambda/\left(\lambda+\beta^2\right)$. So, letting $\overline{\delta} \equiv \lambda/\left(\lambda+\beta^2\right) < 1$, $L^d \geq L^r$ holds for $\delta \geq \overline{\delta}$, as required.

D. Proof of Proposition 3

When (13) applies, the equilibrium loss from the rule $\pi = 0$ is given by

$$\widetilde{L}^r(0; u_{t-1}) = \frac{1}{2}(1-\delta)\sum_{t=1}^{\infty}\delta^{t-1}u_t^2 = c_0 + c_1 u_{t-1} + c_2\frac{u_{t-1}^2}{2}, \tag{D.1}$$

where $c_0 \equiv \frac{1}{2}(1-\delta)\left(\frac{1}{1-\delta} + \frac{\rho^2}{1-\delta\rho^2} - \frac{2\rho}{1-\delta\rho}\right)u_n^2 > 0$, $c_1 \equiv (1-\delta)\frac{\rho(1-\rho)}{(1-\delta\rho)(1-\delta\rho^2)}u_n > 0$, and $c_2 \equiv (1-\delta)\frac{\rho^2}{1-\delta\rho^2} > 0$. The MPE is somewhat different when unemployment follows (13) instead of (1), in the sense that the MA's loss from a T-period MPE is now given by

$$\widetilde{L}_T(u_t) = \gamma_{0,T} + \gamma_{1,T}u_t + \gamma_{2,T}\frac{u_t^2}{2}. \tag{D.2}$$

We show, in the technical appendix available upon request, that $\gamma_{0,T}$ has a solution of the following form:

$$\gamma_{0,T} = \kappa_{0,T}(1-\delta)u_n^2, \tag{D.3}$$

where κ_{0T} is bounded in δ for any given T.

Now, the loss from deviation from $\pi = 0$ is defined exactly as in (8) above, i.e.:

$$\begin{aligned} \widetilde{L}^d(0; u_{t-1}) &= \min_{\varphi_t} \left\{ \frac{1}{2}(1-\delta)\left[u_t^2 + \lambda\varphi_t^2\right] + \delta \widetilde{L}_T(u_t) + \delta^{T+1}\widetilde{L}^r(0; u_{t+T}) \right. \\ &\qquad \left. \text{s.t. } u_t = (1-\rho)u_n + \rho u_{t-1} - \beta\varphi_t \right\}, \end{aligned}$$

where we note that $u_{t+T} = \left(1-\rho^T\right)u_n + \rho^T u_t$. Using (D.1), (D.2), and the definition of c_2, this is rewritten as

$$\begin{aligned} \widetilde{L}^d(0; u_{t-1}) &= \frac{1}{2}(1-\delta)\min_{\varphi_t} \left\{ \widetilde{y}_T u_t^2 + \lambda\varphi_t^2 + \frac{2\eta_T}{1-\delta}u_t \right. \\ &\qquad \left. \text{s.t. } u_t = (1-\rho)u_n + \rho u_{t-1} - \beta\varphi_t \right\} + A_T, \end{aligned} \tag{D.4}$$

where $\widetilde{y}_T \equiv 1+\delta\gamma_{2,T}/(1-\delta)+(\delta\rho^2)^{T+1}/(1-\delta\rho^2)$, $\eta_T \equiv \delta\gamma_{1,T}+\delta^{T+1}\rho^T\left[c_1 + \left(1-\rho^T\right)c_2 u_n\right]$, and $A_T \equiv \delta\gamma_{0,T} + \delta^{T+1}\left[c_0 + c_1\left(1-\rho^T\right)u_n + \left(1-\rho^T\right)^2 \frac{c_2}{2}u_n^2\right]$. Consider now a *suboptimal* deviation, namely one setting $u_t = 0$. This requires $\beta\varphi_t = (1-\rho)u_n + \rho u_{t-1}$. Using (D.4), the associated loss, denoted $\widetilde{L}^{sd}(0; u_{t-1})$, is recovered as

$$\widetilde{L}^{sd}(0; u_{t-1}) = \frac{1}{2}(1-\delta)\frac{\lambda}{\beta^2}\left[(1-\rho)u_n + \rho u_{t-1}\right]^2 + A_T. \tag{D.5}$$

Clearly, $\widetilde{L}^{sd}(0; u_{t-1}) \geq \widetilde{L}^d(0; u_{t-1})$. Thus, a sufficient condition proving the proposition is $\widetilde{L}^r(0; u_0) > \widetilde{L}^{sd}(0; u_0)$. Using (D.1), (D.5), and the definition of A_T, this requires

$$\begin{aligned} c_0 + c_1 u_0 + c_2\frac{u_0^2}{2} &> \frac{1}{2}(1-\delta)\frac{\lambda}{\beta^2}\left((1-\rho)u_n + \rho u_0\right)^2 \\ &\quad + \delta\gamma_{0,T} + \delta^{T+1}\left(c_0 + c_1\left(1-\rho^T\right)u_n + \left(1-\rho^T\right)^2\frac{c_2}{2}u_n^2\right). \end{aligned}$$

Dividing by $\frac{1}{2}(1-\delta)$ on both sides and rearranging, this becomes [by use of (D.3) and the definitions of c_1 and c_2]:

$$\begin{aligned} &\left(1-\delta^{T+1}\right)\left(\frac{1}{1-\delta} + \frac{\rho^2}{1-\delta\rho^2} - \frac{2\rho}{1-\delta\rho}\right)u_n^2 \\ &+2(1-\rho)\left(\frac{1}{(1-\delta\rho)(1-\delta\rho^2)} - \frac{\lambda}{\beta^2}\right)u_n u_0 + \left(\frac{1}{1-\delta\rho^2} - \frac{\lambda}{\beta^2}\right)\rho^2 u_0^2 \end{aligned} \tag{D.6}$$

$$> \left\{ \frac{\lambda}{\beta^2} (1-\rho)^2 + 2\kappa_{0,T} + \delta^{T+1} \left(1-\rho^T\right) \rho \left[\frac{2(1-\rho)}{(1-\delta\rho)(1-\delta\rho^2)} + \frac{\left(1-\rho^T\right)\rho}{1-\delta\rho^2} \right] \right\} u_n^2.$$

Clearly, as $\frac{1}{1-\delta} + \frac{\rho^2}{1-\delta\rho^2} - \frac{2\rho}{1-\delta\rho} > 0$, a sufficient condition for the left hand side to be positive is that $\lambda < \beta^2$ as assumed. Moreover, the left hand side is increasing in u_0 while the right hand side is increasing in u_n. Finally, since the expression in curly brackets on the right hand side is bounded in δ for any given T, we can always choose u_n/u_0 small enough such that (D.6) is satisfied for any δ as claimed.

E. Proof of Proposition 4

It suffices to show that the rule cannot be a NTE for any δ under the conditions stated at t when $\varepsilon_t = 0$. Now, if $\varepsilon_t = 0$, the loss of sticking to rule $\pi_t(\varepsilon_t) = s^r\varepsilon_t$ is $\bar{L}^r(s^r; u_{t-1})\big|_{\varepsilon_t=0} = \frac{1}{2}(1-\delta)\rho^2 u_{t-1}^2 + \delta\bar{L}^r(s^r; \rho u_{t-1})$ where, using (15) and the definition of s^r,

$$\bar{L}^r(s^r; u_{t-1}) = \frac{1}{2}(1-\delta)\frac{\rho^2}{1-\delta\rho^2}u_{t-1}^2 + \frac{1}{2}\frac{\lambda}{\beta^2 + \lambda(1-\delta\rho^2)}\sigma^2. \tag{E.1}$$

Hence,

$$\bar{L}^r(s^r; u_{t-1})\Big|_{\varepsilon_t=0} = \frac{1}{2}(1-\delta)\frac{\rho^2}{1-\delta\rho^2}u_{t-1}^2 + \frac{\delta}{2}\frac{\lambda\sigma^2}{\beta^2 + \lambda(1-\delta\rho^2)}\sigma^2. \tag{E.2}$$

What is the payoff to defection from the optimal rule $\pi_t(\varepsilon_t) = s^r\varepsilon_t$? First, the MPE is only changed slightly by the introduction of stochastic shocks in the sense that the MA's expected loss from a T-period MPE is given by:

$$\bar{L}_T(u_{t-1}) = \frac{\gamma_T}{2}u_{t-1}^2 + \frac{\theta_T}{2}\sigma^2, \tag{E.3}$$

where γ_T is determined by (A.2), and where θ_T has a solution of the following form (this is demonstrated in the technical appendix available upon request):

$$\theta_T = \mu_T(1-\delta), \tag{E.4}$$

where μ_T is bounded in δ for any given T. So, the loss to deviation at t when $\varepsilon_t = 0$ is

$$\bar{L}^d(s^r; u_{t-1})\Big|_{\varepsilon_t=0} = \min_{\varphi_t}\left\{\frac{1}{2}(1-\delta)\left[u_t^2 + \lambda\varphi_t^2\right] + \delta\bar{L}_T(u_t) + \delta^{T+1}\mathrm{E}_t\bar{L}^r(s^r; u_{t+T}) \right.$$
$$\left. \text{s.t. } u_t = \rho u_{t-1} - \beta\varphi_t) \right\}. \tag{E.5}$$

From (E.1) we find:

$$\begin{aligned}
\mathrm{E}_t\bar{L}^r(s^r;u_{t+T}) &= \frac{1}{2}(1-\delta)\frac{\rho^2}{1-\delta\rho^2}\mathrm{E}_t u_{t+T}^2 + \frac{1}{2}\frac{\lambda}{\beta^2+\lambda(1-\delta\rho^2)}\sigma^2 \qquad \text{(E.6)} \\
&= \frac{1}{2}(1-\delta)\frac{\rho^2}{1-\delta\rho^2}\left[\rho^T u_t^2 + \rho^2\frac{1-\rho^T}{1-\rho}\sigma^2\right] + \frac{1}{2}\frac{\lambda}{\beta^2+\lambda(1-\delta\rho^2)}\sigma^2.
\end{aligned}$$

So, using (E.3), (E.4) and (E.6), we can re-write (E.5) as:

$$\begin{aligned}
\bar{L}^d\left(s^r;u_{t-1}\right)\Big|_{\varepsilon_t=0} &= \frac{1}{2}(1-\delta)\min_{\varphi_1}\left\{\hat{y}_T u_t^2 + \lambda\varphi_t^2 \text{ s.t. } u_t = \rho u_{t-1} - \beta\varphi_t\right\} \\
&+\frac{\delta}{2}\left[\mu_T(1-\delta) + \delta^T\frac{(1-\delta)\rho^4}{1-\delta\rho^2}\frac{(1-\rho^T)}{(1-\rho)} + \delta^T\frac{\lambda}{\beta^2+\lambda(1-\delta\rho^2)}\right]\sigma^2.
\end{aligned}$$

Following the steps leading to (9) (available upon request), we then obtain

$$\begin{aligned}
\bar{L}^d\left(s^r;u_{t-1}\right)\Big|_{\varepsilon_t=0} &= \frac{1}{2}(1-\delta)\frac{\lambda\beta^2\hat{y}_T}{\lambda+\beta^2\hat{y}_T}\frac{\rho^2}{\beta^2}u_{t-1}^2 \qquad \text{(E.7)} \\
&+\frac{\delta}{2}\left[\mu_T(1-\delta) + \delta^T\frac{(1-\delta)\rho^4}{1-\delta\rho^2}\frac{(1-\rho^T)}{(1-\rho)} + \delta^T\frac{\lambda}{\beta^2+\lambda(1-\delta\rho^2)}\right]\sigma^2.
\end{aligned}$$

All we now need to show is that $\bar{L}^r\left(s^r;u_0\right)\Big|_{\varepsilon_t=0} > \bar{L}^d\left(s^r;u_0\right)\Big|_{\varepsilon_t=0}$ for all δ. Using (E.1) and (E.7), this is found to require

$$\begin{aligned}
\frac{1}{2}(1-\delta)\frac{\rho^2}{1-\delta\rho^2}u_0^2 + \frac{\delta}{2}\frac{\lambda}{\beta^2+\lambda(1-\delta\rho^2)}\sigma^2 &> \frac{1}{2}(1-\delta)\frac{\lambda\beta^2\hat{y}_T}{\lambda+\beta^2\hat{y}_T}\frac{\rho^2}{\beta^2}u_0^2 \\
+\frac{\delta}{2}\left[\mu_T(1-\delta) + \delta^T\frac{(1-\delta)\rho^4}{1-\delta\rho^2}\frac{(1-\rho^T)}{(1-\rho)} + \delta^T\frac{\lambda}{\beta^2+\lambda(1-\delta\rho^2)}\right]\sigma^2.
\end{aligned}$$

Dividing by $\frac{1}{2}(1-\delta)$ on both sides, rearranging, and using the definitions of Λ^r and Λ^d, (12), this becomes:

$$\left[\Lambda^r(0)-\Lambda^d(0)\right]u_0^2 > \delta\left[\mu_T + \delta^T\frac{\rho^4}{1-\delta\rho^2}\frac{(1-\rho^T)}{(1-\rho)} + \frac{(1-\delta^T)}{(1-\delta)}\frac{\lambda}{\beta^2+\lambda(1-\delta\rho^2)}\right]\sigma^2. \qquad \text{(E.8)}$$

So, using that when $\lambda < \beta^2$, $\Lambda^r(0) - \Lambda^d(0) > 0$ (cf. the proof of Proposition 1), it follows that the left hand side is positive and increasing in u_0. The right hand side is positive and increasing in σ, and since the expression in square brackets is bounded in δ for any given T, we can always choose σ/u_0 small enough such that (E.8) is satisfied as claimed.

References

al-Nowaihi, A. and P. L. Levine, 1994, Can Reputation Resolve the Monetary Policy Credibility Problem?, Journal of Monetary Economics 33, 355-80.

al-Nowaihi, A. and P. L. Levine, 1996, Independent but Accountable: Walsh Contracts and the Credibility Problem, Global Economic Institutions W.P. Series, No. 11.

Abreu, D., 1988, On the Theory of Infinitely Repeated Games with Discounting, Econometrica 56, 383-396.

Ball, L., 1995, Time-Consistent Policy and Persistent Changes in Inflation, Journal of Monetary Economics 36, 329-350.

Barro, R. J. and D. B. Gordon, 1983, Rules, Discretion, and Reputation in a Model of Monetary Policy, Journal of Monetary Economics 12, 101-121.

Blanchard, O. J., and Summers, L. H., 1986, Hysteresis and the European Unemployment Problem, NBER Macroeconomics Annual 1, 15-78.

Canzoneri, M. B. and D. W. Henderson, 1988, Is Sovereign Policymaking Bad?, Carnegie-Rochester Conference Series on Public Policy 28, 93-140.

Cave, J., 1987, Long-term Competition in a Dynamic Game: The Cold Fish War, Rand Journal of Economics 18, 596-610.

Chang, R., 1998, Credible Monetary Policy in an Infinite Horizon Model: Recursive Approaches, Journal of Economic Theory 81, 431-461.

Chari, V. V. and P. Kehoe, 1990, Sustainable Plans, Journal of Political Economy 98, 783-802.

Cukierman, A., S. Edwards and G. Tabellini, 1992, Seignorage and Political Instability, American Economic Review 82, 537-555.

Dutta, P. K., 1995, A Folk Theorem for Stochastic Games, Journal of Economic Theory 66, 1-32.

Ellis, C. J. and S. Holden, 1997, Optimal Contract Length in a Reputational Model of Monetary Policy, European Economic Review 41, 227-43.

Friedman, J. W., 1971, A Non-cooperative Equilibrium for Supergames, Review of Economic Studies 38, 1-12.

Fudenberg, D. and E. Maskin, 1986, The Folk Theorem in Repeated Games with Discounting or with Incomplete Information, Econometrica 54, 533-554.

Fudenberg, D. and J. Tirole, 1991, Game Theory (Cambridge: The MIT Press).

Herrendorf, B., 1997, Importing Credibility through Exchange Rate Pegging, The Economic Journal 107, 687-694.

Herrendorf, B., 1998, Inflation Targeting as a Way of Precommitment, Oxford Economic

Papers, 50, 431-448.

Horn, H. and T. Persson, 1988, Exchange Rate Policy, Wage Formation and Credibility, European Economic Review 32, 1621-1636.

Ireland, P. N., 1997, Sustainable Monetary Policies, Journal of Economic Dynamics and Control 22, 87-108.

Ireland, P. N., 1999, Does the Time-Consistency Problem Explain the Behavior of Inflation in the United States? Journal of Monetary Economics 44, 279-291.

Jensen, H., 1997, Credibility of Optimal Monetary Delegation, American Economic Review 87, 911-920.

Kydland, F. E. and E. C. Prescott, 1977, Rules Rather than Discretion: The Inconsistency of Optimal Plans, Journal of Political Economy 85, 473-492.

Layard, R., S. Nickell and R. Jackman, 1995, The Unemployment Crisis (Oxford: Oxford University Press).

Lockwood, B., M. Miller and L. Zhang, 1998, Delegating Monetary Policy when Unemployment Persists, Economica 65, 327-345.

Lockwood, B. and A. Philippopoulos, 1994, Insider Power, Unemployment Dynamics and Multiple Inflation Equilibria, Economica 61, 59-77.

McCallum, B. T., 1995, Two Fallacies Concerning Central-Bank Independence, American Economic Review, Papers and Proceedings 85, 207-211.

Neiss, K. S., 1999, Discretionary Inflation in a General Equilibrium Model, Journal of Money, Credit, and Banking 31, 357-374.

Persson, T. and G. Tabellini, 1999, Political Economics and Macroeconomic Policy, forthcoming in J. Taylor and M. Woodford, eds., Handbook of Macroeconomics, Vol. 1C (Amsterdam: North-Holland).

Rogoff, K., 1985, The Optimal Degree of Commitment to an Intermediate Monetary Target, Quarterly Journal of Economics 100, 1169-1189.

Rogoff, K., 1989, Reputation, Coordination, and Monetary Policy, in R. J. Barro, ed., Modern Business Cycle Theory (Cambridge, Mass.: Harvard University Press), 236-264.

Stokey, N. L., 1989, Reputation and Time Consistency, American Economic Review, Papers and Proceedings 79, 134-139.

Stokey, N. L., 1991, Credible Public Policy, Journal of Economic Dynamics and Control 15, 627-56.