Mark-up Fluctuations and Fiscal Policy Stabilization in a Monetary Union:

Technical appendices not for publication

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Appendix

A. The underlying model in detail

A.1. Utilities and private consumption

There are two countries labeled H(ome) and F(oreign). These countries form a monetary union. The population of the union is a continuum of agents on the interval $[0,1]$. The population on the segment $[0,n)$ belongs to country H, while the population on $[n,1]$ belongs to country F. In period t, the utility of the representative household j living in country i is given by

$$U_t^j = \mathrm{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \left[U\left(C_s^j\right) + V\left(G_s^j\right) - v\left(y_s^j; \xi_s^i\right)\right], \qquad 0 < \beta < 1, \tag{A.1}$$

where C_s^j is consumption, G_s^j is per-capita public spending, and y_s^j is the amount of goods that household j produces. The functions U and V are strictly increasing and strictly concave, while v is increasing and strictly convex in y_s^j. Further, ξ_s^i is a disturbance affecting the disutility of work, which will throughout be interpreted as a productivity shock.[1]

The consumption index C^j is defined as

$$C^j \equiv \frac{\left(C_H^j\right)^n \left(C_F^j\right)^{1-n}}{n^n \left(1-n\right)^{1-n}}, \tag{A.2}$$

where C_H^j and C_F^j are the Dixit and Stiglitz (1977) indices of the sets of imperfectly substitutable goods produced in countries H and F, respectively:

$$C_H^j \equiv \left[\left(\frac{1}{n}\right)^{1/\sigma_t^H} \int_0^n c^j\left(h\right)^{\frac{\sigma_t^H-1}{\sigma_t^H}} dh\right]^{\frac{\sigma_t^H}{\sigma_t^H-1}}, \qquad C_F^j \equiv \left[\left(\frac{1}{1-n}\right)^{1/\sigma_t^F} \int_n^1 c^j\left(f\right)^{\frac{\sigma_t^F-1}{\sigma_t^F}} df\right]^{\frac{\sigma_t^F}{\sigma_t^F-1}}, \tag{A.3}$$

where $c^j(h)$ and $c^j(f)$ are j's consumption of Home- and Foreign-produced goods h and f, respectively, and $\sigma_t^H > 1$ and $\sigma_t^F > 1$ are stochastic elasticities of substitution across goods produced within a country, both with mean σ. The time variation in the elasticity of substitution will translate into time variation in the mark up, as we will see below.

[1] In the log-linearized model below, we consider only bounded fluctuations of at least order $\mathcal{O}(\|\xi\|)$, where ξ is the vector of all disturbances in the economies.

The price index of country i is given by $P^i = \left(P_H^i\right)^n \left(P_F^i\right)^{1-n}$ where

$$P_H^i = \left[\frac{1}{n}\int_0^n p^i(h)^{1-\sigma_t^H} dh\right]^{\frac{1}{1-\sigma_t^H}}, \qquad P_F^i = \left[\frac{1}{1-n}\int_n^1 p^i(f)^{1-\sigma_t^F} df\right]^{\frac{1}{1-\sigma_t^F}},$$

and where $p^i(h)$ and $p^i(f)$ are the prices in country i of the individual goods h and f produced in Home and Foreign, respectively. Because purchasing power parity holds, in the sequel, we will therefore drop the country superscript for prices. The terms of trade, T, is defined as the ratio of the price of a bundle of goods produced in country F and a bundle of goods produced in country H. That is, $T \equiv P_F/P_H$.

The allocation of resources over the various consumption goods takes place in three steps. The intertemporal trade-off, analyzed below, determines C^j. Given C^j, the household selects C_H^j and C_F^j so as to minimize total expenditure PC^j under restriction (A.2). This yields:

$$C_H^j = n\left(\frac{P_H}{P}\right)^{-1} C^j = nT^{1-n}C^j, \tag{A.4}$$

$$C_F^j = (1-n)\left(\frac{P_F}{P}\right)^{-1} C^j = (1-n)\,T^{-n}C^j. \tag{A.5}$$

Then, given C_H^j and C_F^j, the household optimally allocates spending over the individual goods by minimizing $P_H C_H^j$ and $P_F C_F^j$ under restriction (A.3). The implied demands for individual good h, produced in country H, and individual good f, produced in country F, are, respectively,

$$c^j(h) = \left(\frac{p(h)}{P_H}\right)^{-\sigma_t^H} T^{1-n}C^j, \qquad c^j(f) = \left(\frac{p(f)}{P_F}\right)^{-\sigma_t^F} T^{-n}C^j. \tag{A.6}$$

We assume that public spending is financed either by debt issuance or lump-sum taxation. Per capita public spending in countries H and F is given by the following indices, respectively:

$$G_t^H = \left[\frac{1}{n}\int_0^n g_t(h)^{\frac{\sigma_t^H-1}{\sigma_t^H}} dh\right]^{\frac{\sigma_t^H}{\sigma_t^H-1}}, \qquad G_t^F = \left[\frac{1}{1-n}\int_n^1 g_t(f)^{\frac{\sigma_t^F-1}{\sigma_t^F}} df\right]^{\frac{\sigma_t^F}{\sigma_t^F-1}}, \tag{A.7}$$

where $g(h)$ and $g(f)$ are public spending on individual goods h and f produced in Home and Foreign, respectively.

Minimization of $P_H G^H$ and $P_F G^F$ under restriction (A.7) yields the governments' demands for the individual goods h and f:

$$g_t(h) = \left(\frac{p_t(h)}{P_{H,t}}\right)^{-\sigma_t^H} G_t^H, \qquad g_t(f) = \left(\frac{p_t(f)}{P_{F,t}}\right)^{-\sigma_t^F} G_t^F. \tag{A.8}$$

Hence, combining (A.6) and (A.8), the total demands for the goods h and f are

$$y_t(h) = \left(\frac{p_t(h)}{P_{H,t}}\right)^{-\sigma_t^H} \left[T_t^{1-n} C_t^W + G_t^H\right], \qquad y_t(f) = \left(\frac{p_t(f)}{P_{F,t}}\right)^{-\sigma_t^F} \left[T_t^{-n} C_t^W + G_t^F\right], \tag{A.9}$$

where $C^W \equiv \int_0^1 C^j dj$, is aggregate consumption in the union.

Following Benigno and Benigno (2001), we assume that financial markets are complete both at the domestic and at the international level. Furthermore, each individual's initial holding of any type of asset is zero. These assumptions imply perfect consumption risk-sharing within each country and equalization of the marginal utilities of consumption between countries:

$$U_C\left(C_t^H\right) = U_C\left(C_t^F\right). \tag{A.10}$$

Hence, in the absence of exogenous disturbances to the marginal utility of consumption, $C_t^H = C_t^F = C_t^W$. Therefore, from now on, we ignore superscripts and denote consumption by C_t. Further, the Euler equation is

$$U_C(C_t) = (1 + R_t)\,\beta \mathrm{E}_t\left[U_C(C_{t+1})\,(P_t/P_{t+1})\right], \tag{A.11}$$

where R_t is the nominal interest rate on an internationally-traded nominal bond. The nominal interest rate is taken to be the union central bank's policy instrument. Finally, using the appropriate aggregators, aggregate demand in both countries is found as

$$Y_t^H = T_t^{1-n} C_t + G_t^H, \qquad Y_t^F = T_t^{-n} C_t + G_t^F. \tag{A.12}$$

A.2. Firms

Individual j is the monopolist provider of good j. The structure of price setting is assumed to be of the Calvo (1983) form. In each period, there is a fixed probability $(1 - \alpha^i)$ that producer j who resides in i can adjust his prices. This producer takes account of the fact that a change in the price of his product affects the demand for it. However, because he is infinitesimally small, he neglects any effects of his actions on aggregate variables. Hence, if individual j has the "chance" to reset his price in period t, he chooses his price, denoted

$\check{p}_t(j)$, to maximize

$$\mathrm{E}_t \sum_{k=0}^{\infty} \left(\alpha^i \beta\right)^k \left[\lambda_{t+k}^i \check{p}_t(j)\, y_{t,t+k}(j) - v\left(y_{t,t+k}(j); \xi_{t+k}^i\right)\right],$$

where $y_{t,t+k}(j)$ is given by (A.9), assuming that $\check{p}_t(j)$ still applies at $t+k$, and $\lambda_{t+k}^i \equiv U_C\left(C_{t+k}^i\right)/P_{t+k}$ is the marginal utility of nominal income. For a producer in country H this results in the following optimality condition:

$$\check{p}_t(h) = \frac{\mathrm{E}_t\left[\sum_{k=0}^{\infty}\left(\alpha^H\beta\right)^k \sigma_{t+k}^H v_y\left(y_{t,t+k}(h); \xi_{t+k}^H\right) y_{t,t+k}(h)\right]}{\mathrm{E}_t\left[\sum_{k=0}^{\infty}\left(\alpha^H\beta\right)^k \left(\sigma_{t+k}^H - 1\right)\lambda_{t+k}^H y_{t,t+k}(h)\right]}. \tag{A.13}$$

Realizing that, in equilibrium, each producer in a given country and a given period will set the same price when offered the chance to reset its price, one can show that

$$P_{H,t} = \left[\left(1-\alpha^H\right)\check{p}_t(h)^{1-\sigma_t^H} + \alpha^H P_{H,t-1}^{1-\sigma_t^H}\right]^{\frac{1}{1-\sigma_t^H}}, \tag{A.14}$$

$$P_{F,t} = \left[\left(1-\alpha^F\right)\check{p}_t(f)^{1-\sigma_t^F} + \alpha^F P_{F,t-1}^{1-\sigma_t^F}\right]^{\frac{1}{1-\sigma_t^F}}. \tag{A.15}$$

B. Inefficient steady-state equilibrium

We now derive the steady state, which is taken to be the equilibrium that is attained when prices are flexible and shocks are at the mean values, and when there are average monopolistic distortions.

Under flexible prices, (A.13) is replaced by

$$\check{p}_t(j) = \frac{\sigma_t^i}{\sigma_t^i - 1}\frac{v_y\left(y_{t,t}(j); \xi_t^i\right)}{\lambda_t^i}. \tag{B.1}$$

Because each agent in a given country chooses the same price, we have that $\check{p}_t(j) = P_{H,t}$ for all j living in Home, so that

$$U_C(C_t) = \frac{\sigma_t^H}{\sigma_t^H - 1} T_t^{1-n} v_y\left(T_t^{1-n} C_t + G_t^H; \xi_t^H\right), \tag{B.2}$$

and that $p_t(j) = P_{F,t}$ for all j living in Foreign, so that

$$U_C(C_t) = \frac{\sigma_t^F}{\sigma_t^F - 1} T_t^{-n} v_y\left(T_t^{-n} C_t + G_t^F; \xi_t^F\right). \tag{B.3}$$

Hence, the steady-state values for consumption and the terms of trade, conditional on

Home and Foreign public spending, follow upon setting the shocks σ_t and σ_t^* to their (common) mean σ and the other shocks to zero in (B.2) and (B.3).

Before we continue, we introduce some notation. Following Benigno (2003), we denote with a superscript "W" a world aggregate and with a superscript "R" a relative variable. Hence, for a generic variable X, define $X^W \equiv nX^H + (1-n)X^F$ and $X^R \equiv X^F - X^H$. Further, we introduce the following additional definitions, using an upperbar on a variable to denote its steady state in the presence of monopolistic distortions: $\rho \equiv -U_{CC}\left(\overline{C}\right)\overline{C}/U_C\left(\overline{C}\right) > 0$; $\rho_g \equiv -V_{GG}\left(\overline{G}\right)\overline{G}/V_G\left(\overline{G}\right) > 0$; $\eta \equiv v_{yy}\left(\overline{Y};0\right)\overline{Y}/v_y\left(\overline{Y};0\right) > 0$ (because of symmetry, $\overline{Y}^H = \overline{Y}^F \equiv \overline{Y}$); and S_t^i $(i = H,\ F)$ is defined such that $v_{y\xi}\left(\overline{Y};0\right)\xi_t^i = -\overline{Y}v_{yy}\left(\overline{Y};0\right)S_t^i$. Hence, S_t^i is proportional to the productivity shock. Finally, we denote by $0 < c_Y \equiv \bar{C}/\bar{Y} < 1$ the steady-state consumption share of output.

We consider a steady state in which the fiscal authorities coordinate their policies. Hence, the steady-state values for public spending follow upon maximizing over $\left\{G_s^H\right\}_{s=t}^{\infty}$ and $\left\{G_s^F\right\}_{s=t}^{\infty}$:

$$\mathrm{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \left\{ \begin{array}{c} n\left[U\left(C_s\right) + V\left(G_s^H\right) - v\left(Y_s^H;\xi_s^H\right)\right] \\ +\left(1-n\right)\left[U\left(C_s\right) + V\left(G_s^F\right) - v\left(Y_s^F;\xi_s^F\right)\right] \end{array} \right\}, \tag{B.4}$$

with ξ_s^H and ξ_s^F set to zero for all $s \geq t$ and taking into account the private sector first-order conditions in a steady state:

$$(1-\phi)\,U_C\left(C_t\right) = T_t^{1-n} v_y\left(T_t^{1-n}C_t + G_t^H, 0\right), \tag{B.5}$$

$$(1-\phi)\,U_C\left(C_t\right) = T_t^{-n} v_y\left(T_t^{-n}C_t + G_t^F, 0\right). \tag{B.6}$$

where

$$\phi \equiv 1/\sigma.$$

Setting the derivative of equation (B.4) with respect to G_t^H to zero, we have:

$$\begin{aligned} & \sum_{s=t}^{\infty} \beta^{s-t}\left[nU_C\left(C_s\right)\frac{\partial C_s}{\partial G_t^H} + (1-n)\,U_C\left(C_s\right)\frac{\partial C_s}{\partial G_t^H}\right] + nV_G\left(G_t^H\right) \\ & -\sum_{s=t}^{\infty} \beta^{s-t}\left\{nv_y\left(Y_s^H;0\right)\left[(1-n)\,T_s^{-n}C_s\frac{\partial T_s}{\partial G_t^H} + T_s^{1-n}\frac{\partial C_s}{\partial G_t^H}\right]\right\} - nv_y\left(Y_t^H;0\right) \\ & -\sum_{s=t}^{\infty} \beta^{s-t}\left\{(1-n)\,v_y\left(Y_s^F;0\right)\left[(-n)\,T_s^{-(n+1)}C_s\frac{\partial T_s}{\partial G_t^H} + T_s^{-n}\frac{\partial C_s}{\partial G_t^H}\right]\right\} \\ = \; & 0. \end{aligned}$$

This expression can be rewritten as:

$$
\begin{aligned}
& nU_C\left(C_t\right)\frac{\partial C_t}{\partial G_t^H}+(1-n)\,U_C\left(C_t\right)\frac{\partial C_t}{\partial G_t^H}+nV_G\left(G_t^H\right) \\
& -nv_y\left(Y_t^H;0\right)\left[(1-n)\,T_t^{-n}C_t\frac{\partial T_t}{\partial G_t^H}+T_t^{1-n}\frac{\partial C_t}{\partial G_t^H}\right]-nv_y\left(Y_t^H;0\right) \\
& -(1-n)\,v_y\left(Y_t^F;0\right)\left[(-n)\,T_t^{-(n+1)}C_t\frac{\partial T_t}{\partial G_t^H}+T_t^{-n}\frac{\partial C_t}{\partial G_t^H}\right] \\
= & \sum_{s=t+1}^{\infty}\beta^{s-t}\left\{n\,(1-n)\left[v_y\left(Y_s^H;0\right)-T_s^{-1}v_y\left(Y_s^F;0\right)\right]T_s^{-n}C_s\frac{\partial T_s}{\partial G_t^H}\right\} \\
& -\sum_{s=t+1}^{\infty}\beta^{s-t}\left[nU_C\left(C_s\right)\frac{\partial C_s}{\partial G_t^H}+(1-n)\,U_C\left(C_s\right)\frac{\partial C_s}{\partial G_t^H}\right] \\
& +\sum_{s=t+1}^{\infty}\beta^{s-t}\left[nv_y\left(Y_s^H;0\right)T_s^{1-n}+(1-n)\,v_y\left(Y_s^F;0\right)T_s^{-n}\right]\frac{\partial C_s}{\partial G_t^H}.
\end{aligned}
$$

Using that in steady state, $T_s=1$, $C_s=\overline{C}$, $Y_s^H=Y_s^F=\overline{Y}$, etc., $\forall s$, we have:

$$
\begin{aligned}
& nU_C\frac{\partial C_t}{\partial G_t^H}+(1-n)\,U_C\frac{\partial C_t}{\partial G_t^H}+nV_G-nv_y\left[(1-n)\,\overline{C}\frac{\partial T_t}{\partial G_t^H}+\frac{\partial C_t}{\partial G_t^H}\right] \\
& -nv_y-(1-n)\,v_y\left[-n\overline{C}\frac{\partial T_t}{\partial G_t^H}+\frac{\partial C_t}{\partial G_t^H}\right] \\
= & \sum_{s=t+1}^{\infty}\beta^{s-t}\left[v_y\left(\overline{Y},0\right)-U_C\left(\overline{C}\right)\right]\frac{\partial C_s}{\partial G_t^H},
\end{aligned}
$$

which, by (B.5) applied to the inefficient steady state, can be simplified further to:

$$
U_C\frac{\partial C_t}{\partial G_t^H}-v_y\frac{\partial C_t}{\partial G_t^H}+nV_G-nv_y=-\phi\sum_{s=t+1}^{\infty}\beta^{s-t}U_C\left(\overline{C}\right)\frac{\partial C_s}{\partial G_t^H}. \tag{B.7}
$$

From the first-order condition with respect to G_t^F, we derive a similar condition:

$$
U_C\frac{\partial C_t}{\partial G_t^F}-v_y\frac{\partial C_t}{\partial G_t^F}+(1-n)\,V_G-(1-n)\,v_y=-\phi\sum_{s=t+1}^{\infty}\beta^{s-t}U_C\left(\overline{C}\right)\frac{\partial C_s}{\partial G_t^F}. \tag{B.8}
$$

We now differentiate (B.5) with respect to G_t^H and G_t^F, respectively, and (B.6) with respect to G_t^H and G_t^F, respectively. This yields the following four conditions (where we already use the fact that we are evaluating at the inefficient steady state):

$$(1-\phi)\, U_{CC}\frac{\partial C_t}{\partial G_t^H} = (1-n)\, v_y \frac{\partial T_t}{\partial G_t^H} + v_{yy}\left[(1-n)\, \overline{C}\frac{\partial T_t}{\partial G_t^H} + \frac{\partial C_t}{\partial G_t^H} + 1\right], \quad \text{(B.9)}$$

$$(1-\phi)\, U_{CC}\frac{\partial C_t}{\partial G_t^F} = (1-n)\, v_y \frac{\partial T_t}{\partial G_t^F} + v_{yy}\left[(1-n)\, \overline{C}\frac{\partial T_t}{\partial G_t^F} + \frac{\partial C_t}{\partial G_t^F}\right], \quad \text{(B.10)}$$

$$(1-\phi)\, U_{CC}\frac{\partial C_t}{\partial G_t^H} = -n v_y \frac{\partial T_t}{\partial G_t^H} + v_{yy}\left[-n\overline{C}\frac{\partial T_t}{\partial G_t^H} + \frac{\partial C_t}{\partial G_t^H}\right], \quad \text{(B.11)}$$

$$(1-\phi)\, U_{CC}\frac{\partial C_t}{\partial G_t^F} = -n v_y \frac{\partial T_t}{\partial G_t^F} + v_{yy}\left[-n\overline{C}\frac{\partial T_t}{\partial G_t^F} + \frac{\partial C_t}{\partial G_t^F} + 1\right]. \quad \text{(B.12)}$$

Now, add n times (B.9) and $(1-n)$ times (B.11) to give:

$$(1-\phi)\, U_{CC}\frac{\partial C_t}{\partial G_t^H} = v_{yy}\left[\frac{\partial C_t}{\partial G_t^H} + n\right]. \quad \text{(B.13)}$$

Similarly, add n times (B.10) and $(1-n)$ times (B.12) to give:

$$(1-\phi)\, U_{CC}\frac{\partial C_t}{\partial G_t^F} = v_{yy}\left[\frac{\partial C_t}{\partial G_t^F} + (1-n)\right]. \quad \text{(B.14)}$$

We rewrite (B.13) and (B.14), to give, respectively:

$$\frac{\partial C_t}{\partial G_t^H} = \frac{n v_{yy}}{(1-\phi)\, U_{CC} - v_{yy}}, \quad \text{(B.15)}$$

$$\frac{\partial C_t}{\partial G_t^F} = \frac{(1-n)\, v_{yy}}{(1-\phi)\, U_{CC} - v_{yy}}. \quad \text{(B.16)}$$

We now also differentiate (B.5) in period $s > t$ with respect to G_t^H and G_t^F, respectively, and (B.6) in period $s > t$ with respect to G_t^H and G_t^F, respectively. This yields the following four conditions (where we already use the fact that we are evaluating at the inefficient steady state):

$$(1-\phi)\, U_{CC}\frac{\partial C_s}{\partial G_t^H} = (1-n)\, v_y \frac{\partial T_s}{\partial G_t^H} + v_{yy}\left[(1-n)\, \overline{C}\frac{\partial T_s}{\partial G_t^H} + \frac{\partial C_s}{\partial G_t^H}\right], \quad \text{(B.17)}$$

$$(1-\phi)\, U_{CC}\frac{\partial C_s}{\partial G_t^F} = (1-n)\, v_y \frac{\partial T_s}{\partial G_t^F} + v_{yy}\left[(1-n)\, \overline{C}\frac{\partial T_s}{\partial G_t^F} + \frac{\partial C_s}{\partial G_t^F}\right], \quad \text{(B.18)}$$

$$(1-\phi)\, U_{CC}\frac{\partial C_s}{\partial G_t^H} = -n v_y \frac{\partial T_s}{\partial G_t^H} + v_{yy}\left[-n\overline{C}\frac{\partial T_s}{\partial G_t^H} + \frac{\partial C_s}{\partial G_t^H}\right], \quad \text{(B.19)}$$

$$(1-\phi)\, U_{CC}\frac{\partial C_s}{\partial G_t^F} = -n v_y \frac{\partial T_s}{\partial G_t^F} + v_{yy}\left[-n\overline{C}\frac{\partial T_s}{\partial G_t^F} + \frac{\partial C_s}{\partial G_t^F}\right]. \quad \text{(B.20)}$$

Now, add n times (B.17) and $(1-n)$ times (B.19) to give:

$$(1-\phi)\,U_{CC}\frac{\partial C_s}{\partial G_t^H} = v_{yy}\frac{\partial C_s}{\partial G_t^H}. \tag{B.21}$$

We note that by (B.13) $(1-\phi)\,U_{CC} \neq -v_{yy}$. Hence, $\partial C_s/\partial G_t^H = 0$. Similarly, add n times (B.18) and $(1-n)$ times (B.20) to give:

$$(1-\phi)\,U_{CC}\frac{\partial C_s}{\partial G_t^F} = v_{yy}\frac{\partial C_s}{\partial G_t^F}. \tag{B.22}$$

Hence, $\partial C_s/\partial G_t^F = 0$.Hence, (B.7) and (B.8) become, respectively:

$$U_C\frac{\partial C_t}{\partial G_t^H} - v_y\frac{\partial C_t}{\partial G_t^H} + nV_G - nv_y = 0, \tag{B.23}$$

$$U_C\frac{\partial C_t}{\partial G_t^F} - v_y\frac{\partial C_t}{\partial G_t^F} + (1-n)\,V_G - (1-n)\,v_y = 0. \tag{B.24}$$

Using (B.5) in inefficient steady state, we can write (B.23) as:

$$\begin{aligned}
U_C\frac{\partial C_t}{\partial G_t^H} - (1-\phi)\,U_C\frac{\partial C_t}{\partial G_t^H} + nV_G - n(1-\phi)\,U_C &= 0,\\
\phi U_C\frac{\partial C_t}{\partial G_t^H} + nV_G - n(1-\phi)\,U_C &= 0,
\end{aligned}$$

$$\begin{aligned}
nV_G &= n(1-\phi)\,U_C - \phi U_C\frac{\partial C_t}{\partial G_t^H},\\
nV_G &= U_C\left[n(1-\phi) - \phi\frac{\partial C_t}{\partial G_t^H}\right],\\
V_G &= U_C\left[(1-\phi) - \frac{\phi v_{yy}}{(1-\phi)\,U_{CC} - v_{yy}}\right],
\end{aligned}$$

where we have substituted from (B.15). Further rewriting, we get

$$\begin{aligned}
V_G &= U_C\left[1 - \phi\frac{(1-\phi)\,U_{CC}}{(1-\phi)\,U_{CC} - v_{yy}}\right],\\
V_G &= U_C\left[1 - \phi\frac{(1-\phi)\,\rho U_C/\overline{C}}{(1-\phi)\,\rho U_C/\overline{C} + \eta v_y/\overline{Y}}\right],\\
V_G &= U_C\left[1 - \phi\frac{\rho v_y/\overline{C}}{\rho v_y/\overline{C} + \eta v_y/\overline{Y}}\right],\\
V_G &= U_C\left[1 - \phi\frac{\rho}{\rho + \eta c_Y}\right].
\end{aligned} \tag{B.25}$$

The route via (B.16) yields the same outcome.

C. Efficient steady state and flex-price equilibrium

Here, we derive an approximation to the efficient flexible price equilibrium. The efficient equilibrium obtains when there are no monopolistic distortions, which in this model framework can be represented by the case where producers have no market power, i.e., it corresponds to letting $\sigma_t^H \to \infty$ in (B.2) and $\sigma_t^F \to \infty$ in (B.3). We log linearize the efficient equilibrium around the associated *efficient steady state*. Further, we will refer to the outcomes of the variables in the efficient flex-price equilibrium as the *(stochastic) efficient rates.*

C.1. Efficient steady state

We denote the equilibrium values of variables in this steady state by a star superscript. Hence, the efficient steady-state values C^* and T^*, conditional on G^{*H} and G^{*F}, are implicitly defined by:

$$U_C(C^*) = (T^*)^{1-n} v_y\left((T^*)^{1-n} C^* + G^{*H}; 0\right),$$

and

$$U_C(C^*) = (T^*)^{-n} v_y\left((T^*)^{-n} C^* + G^{*F}; 0\right),$$

from which it follows that in a symmetric equilibrium $T^* = 1$. From (B.25) with $\phi = 0$ we obtain the steady-state values G^{*H} and G^{*F} for public spending as:

$$V_G\left(G^{*H}\right) = v_y\left(Y^{*H}; 0\right), \quad V_G\left(G^{*F}\right) = v_y\left(Y^{*F}; 0\right). \tag{C.1}$$

Because $Y^{*H} = Y^{*F} \equiv Y^*$, we have that $G^{*H} = G^{*F} \equiv G^*$. Finally, we obtain the efficient steady-state nominal (=real) interest rate from (A.11) as $1 + R^* = 1/\beta$.

C.2. Derivation of relationships between efficient and inefficient steady states

We derive $c^* \equiv -\ln\left(\overline{C}/C^*\right)$ and $g^* \equiv -\ln\left(\overline{G}/G^*\right)$, which we shall use in the sequel. To this end, recall the steady-state relations derived earlier:

$$(1-\phi) U_C\left(\overline{C}\right) = v_y\left(\overline{C} + \overline{G}; 0\right), \tag{C.2}$$

$$V_G\left(\overline{G}\right) = U_C\left(\overline{C}\right)\left[1 - \phi\frac{\rho}{\rho + \eta c_Y}\right], \tag{C.3}$$

remembering that $\overline{T} = 1$ and $\overline{G}^H = \overline{G}^F = \overline{G}^W \equiv \overline{G}$. Take a first-order Taylor approximation to (C.2) evaluated around the efficient steady state:

$$\begin{aligned} -\phi + \rho c^* &= -\eta \left[c_Y c^* + (1 - c_Y) g^* \right] \Leftrightarrow \\ \eta (1 - c_Y) g^* + (\rho + \eta c_Y) c^* &= \phi, \end{aligned} \tag{C.4}$$

where we have used that $C^*/Y^* = c_Y + \mathcal{O}(\|\xi\|)$, because ϕ is $\mathcal{O}(\|\xi\|)$. Also, take a first-order Taylor approximation of (C.3) around the efficient steady state:

$$\begin{aligned} \rho_g g^* &= -\phi \frac{\rho}{\rho + \eta c_Y} + \rho c^* \Leftrightarrow \\ g^* &= \frac{\rho}{\rho_g} \left[-\frac{\phi}{\rho + \eta c_Y} + c^* \right], \end{aligned} \tag{C.5}$$

Substitute this into (C.4), to give:

$$\begin{aligned} \eta (1 - c_Y) \frac{\rho}{\rho_g} \left[-\frac{\phi}{\rho + \eta c_Y} + c^* \right] + (\rho + \eta c_Y) c^* &= \phi \Leftrightarrow \\ -\eta (1 - c_Y) \rho \phi + \eta \rho (1 - c_Y)(\rho + \eta c_Y) c^* + \rho_g (\rho + \eta c_Y)^2 c^* &= \phi \rho_g (\rho + \eta c_Y) \Leftrightarrow \end{aligned}$$

$$\begin{aligned} (\rho + \eta c_Y) \left[\eta \rho (1 - c_Y) + \rho_g (\rho + \eta c_Y) \right] c^* &= \phi \left[\rho \left(\eta + \rho_g \right) + \eta c_Y \left(\rho_g - \rho \right) \right] \Leftrightarrow \\ (\rho + \eta c_Y) \left[\rho \left(\eta + \rho_g \right) + \eta c_Y \left(\rho_g - \rho \right) \right] c^* &= \phi \left[\rho \left(\eta + \rho_g \right) + \eta c_Y \left(\rho_g - \rho \right) \right] \Leftrightarrow \\ c^* &= \frac{\phi}{\rho + \eta c_Y} \Leftrightarrow \\ \phi &= (\rho + \eta c_Y) c^*. \end{aligned}$$

Hence, we have

$$c^* = \phi / (\rho + \eta c_Y), \tag{C.6}$$

$$g^* = 0. \tag{C.7}$$

C.3. Efficient flexible price equilibrium

Log-linearizing (B.5), with $\sigma_t^H / \left(\sigma_t^H - 1 \right) = 1$ imposed around the efficient steady state, we have:

$$-\rho \widetilde{C}_t = (1 - n) \widetilde{T}_t + \eta \left[(1 - n) c_Y \widetilde{T}_t + c_Y \widetilde{C}_t + (1 - c_Y) \widetilde{G}_t^H \right] - \eta S_t^H, \tag{C.8}$$

and an analogous equation for the Foreign country:

$$-\rho\widetilde{C}_t = -n\widetilde{T}_t + \eta\left[-nc_Y\widetilde{T}_t + c_Y\widetilde{C}_t + (1-c_Y)\,\widetilde{G}_t^F\right] - \eta S_t^F. \tag{C.9}$$

Taking a weighted average (with weights n and $1-n$) of these equations, we obtain $-\rho\widetilde{C}_t = \eta\left[c_Y\widetilde{C}_t + (1-c_Y)\,\widetilde{G}_t^W\right] - \eta S_t^W$. Hence,

$$\widetilde{C}_t = \frac{\eta}{\rho+\eta c_Y}\left[S_t^W - (1-c_Y)\,\widetilde{G}_t^W\right]. \tag{C.10}$$

Subtracting (C.9) from (C.8) we obtain $0 = \widetilde{T}_t + \eta\left[c_Y\widetilde{T}_t - (1-c_Y)\,\widetilde{G}_t^R\right] + \eta S_t^R$ and thus

$$\widetilde{T}_t = \frac{\eta}{1+\eta c_Y}\left[(1-c_Y)\,\widetilde{G}_t^R - S_t^R\right]. \tag{C.11}$$

Further, because $\widetilde{Y}_t^H = \left[(1-n)\,(T^*)^{1-n}\,C^*\widetilde{T}_t + (T^*)^{1-n}\,C^*\widetilde{C}_t + G^{*H}\widetilde{G}_t^H\right]/Y^{*H}$, we can also write (C.8) as $-\rho\widetilde{C}_t = (1-n)\,\widetilde{T}_t + \eta\widetilde{Y}_t^H - \eta S_t^H$ and (C.9) as $-\rho\widetilde{C}_t = -n\widetilde{T}_t + \eta\widetilde{Y}_t^F - \eta S_t^F$. Taking a weighted average (with weights n and $1-n$) of these two equations, we then obtain

$$-\rho\widetilde{C}_t = \eta\widetilde{Y}_t^W - \eta S_t^W. \tag{C.12}$$

Combining this with (C.10), we find that:

$$\widetilde{Y}_t^W = \frac{\eta c_Y}{\rho+\eta c_Y}S_t^W + \frac{\rho\,(1-c_Y)}{\rho+\eta c_Y}\widetilde{G}_t^W. \tag{C.13}$$

We solve now for $\widetilde{G}_t^H$ and $\widetilde{G}_t^F$, thereby completing the solution of the efficient flex-price equilibrium. Above we found the steady state values for public spending as the solutions to (C.1). Further, we have that

$$V_G\left(G_t^H\right) = v_y\left(Y_t^H;\xi_t^H\right). \tag{C.14}$$

To show this, set the derivative of (B.4) with respect to G_t^H to zero:

$$\begin{aligned}
&\mathrm{E}_t\sum_{s=t}^{\infty}\beta^{s-t}\left[nU_C\left(C_s\right)\frac{\partial C_s}{\partial G_t^H} + (1-n)\,U_C\left(C_s\right)\frac{\partial C_s}{\partial G_t^H}\right] + nV_G\left(G_t^H\right)\\
&-\mathrm{E}_t\sum_{s=t}^{\infty}\beta^{s-t}\left\{nv_y\left(Y_s^H;\xi_s^H\right)\left[(1-n)\,T_s^{-n}C_s\frac{\partial T_s}{\partial G_t^H} + T_s^{1-n}\frac{\partial C_s}{\partial G_t^H}\right]\right\} - nv_y\left(Y_t^H;\xi_t^H\right)\\
&-\mathrm{E}_t\sum_{s=t}^{\infty}\beta^{s-t}\left\{(1-n)\,v_y\left(Y_s^F;\xi_s^F\right)\left[-nT_s^{-(n+1)}C_s\frac{\partial T_s}{\partial G_t^H} + T_s^{-n}\frac{\partial C_s}{\partial G_t^H}\right]\right\} = 0.
\end{aligned} \tag{C.15}$$

Using (B.2) with $\sigma_t^H \to \infty$, (B.3) with $\sigma_t^F \to \infty$ and (A.10), we have that $T_s v_y\left(Y_s^H;\xi_s^H\right) = v_y\left(Y_s^F;\xi_s^F\right)$, for all $s \geq t$. Hence, (C.15) becomes:

$$\begin{aligned}
& \mathrm{E}_t \sum_{s=t}^{\infty} \beta^{s-t}\left[nU_C\left(C_s\right)\frac{\partial C_s}{\partial G_t^H} + (1-n)\,U_C\left(C_s\right)\frac{\partial C_s}{\partial G_t^H}\right] \\
& -\mathrm{E}_t \sum_{s=t}^{\infty} \beta^{s-t} n v_y\left(Y_s^H;\xi_s^H\right)\left\{\begin{array}{c} \left[(1-n)\,T_s^{-n} C_s \frac{\partial T_s}{\partial G_t^H} + T_s^{1-n}\frac{\partial C_s}{\partial G_t^H}\right] \\ -(1-n)\,v_y\left(Y_s^H,\xi_s^H\right)\left[(-n)\,T_s^{-n} C_s \frac{\partial T_s}{\partial G_t^H} + T_s^{1-n}\frac{\partial C_s}{\partial G_t^H}\right] \end{array}\right\} \\
& +nV_G\left(G_t^H\right) - nv_y\left(Y_t^H;\xi_t^H\right) \\
= \; & \mathrm{E}_t \sum_{s=t}^{\infty} \beta^{s-t}\left\{nU_C\left(C_s\right)\frac{\partial C_s}{\partial G_t^H} + (1-n)\,U_C\left(C_s\right)\frac{\partial C_s}{\partial G_t^H} - v_y\left(Y_s^H,\xi_s^H\right)T_s^{1-n}\frac{\partial C_s}{\partial G_t^H}\right\} \\
& +nV_G\left(G_t^H\right) - nv_y\left(Y_t^H;\xi_t^H\right),
\end{aligned}$$

which is equal to

$$\begin{aligned}
& \mathrm{E}_t \sum_{s=t}^{\infty} \beta^{s-t}\left\{\begin{array}{c} nU_C\left(C_s\right)\frac{\partial C_s}{\partial G_t^H} + (1-n)\,U_C\left(C_s\right)\frac{\partial C_s}{\partial G_t^H} \\ -v_y\left(Y_s^H;\xi_s^H\right)T_s^{1-n}\left[n\frac{\partial C_s}{\partial G_t^H} + (1-n)\frac{\partial C_s}{\partial G_t^H}\right] \end{array}\right\} \\
& +nV_G\left(G_t^H\right) - nv_y\left(Y_t^H;\xi_t^H\right) \\
= \; & \mathrm{E}_t \sum_{s=t}^{\infty} \beta^{s-t}\left\{n\left[U_C\left(C_s\right) - T_s^{1-n} v_y\left(Y_s^H;\xi_s^H\right)\right]\frac{\partial C_s}{\partial G_t^H}\right\} \\
& +\mathrm{E}_t \sum_{s=t}^{\infty} \beta^{s-t}\left\{(1-n)\left[U_C\left(C_s\right) - T_s^{1-n} v_y\left(Y_s^H;\xi_s^H\right)\right]\frac{\partial C_s}{\partial G_t^H}\right\} \\
& +nV_G\left(G_t^H\right) - nv_y\left(Y_t^H;\xi_t^H\right),
\end{aligned}$$

which is equal to

$$\begin{aligned}
& \mathrm{E}_t \sum_{s=t}^{\infty} \beta^{s-t}\left\{n\left[U_C\left(C_s\right) - T_s^{1-n} v_y\left(Y_s^H;\xi_s^H\right)\right]\frac{\partial C_s}{\partial G_t^H}\right\} \\
& +\mathrm{E}_t \sum_{s=t}^{\infty} \beta^{s-t}\left\{(1-n)\left[U_C\left(C_s\right) - T_s^{-n} v_y\left(Y_s^F;\xi_s^F\right)\right]\frac{\partial C_s}{\partial G_t^H}\right\} \\
& +nV_G\left(G_t^H\right) - nv_y\left(Y_t^H;\xi_t^H\right) \\
= \; & nV_G\left(G_t^H\right) - nv_y\left(Y_t^H;\xi_t^H\right),
\end{aligned}$$

where in the final step we have used again (B.2) with $\sigma_t^H \to \infty$ and (B.3) with $\sigma_t^F \to \infty$.

We log linearize (C.14) and find $-\rho_g \widetilde{G}_t^H = \eta\left[(1-n)\,c_Y \widetilde{T}_t + c_Y \widetilde{C}_t + (1-c_Y)\,\widetilde{G}_t^H\right] -$

ηS_t^H, from which we obtain

$$\widetilde{G}_t^H = \frac{\eta}{\rho_g + \eta(1-c_Y)}\left[S_t^H - c_Y\left((1-n)\widetilde{T}_t + \widetilde{C}_t\right)\right]. \tag{C.16}$$

For Foreign spending we similarly find

$$\widetilde{G}_t^F = \frac{\eta}{\rho_g + \eta(1-c_Y)}\left[S_t^F - c_Y\left(-n\widetilde{T}_t + \widetilde{C}_t\right)\right]. \tag{C.17}$$

Together with (C.10) and (C.11), we then have four equations in four unknowns: $\widetilde{G}_t^H$, $\widetilde{G}_t^F$, $\widetilde{T}_t$ and $\widetilde{C}_t$. These equations are solved to yield

$$\widetilde{C}_t = \frac{\eta\rho_g}{\rho\left[\rho_g + \eta(1-c_Y)\right] + \eta c_Y \rho_g} S_t^W, \tag{C.18}$$

$$\widetilde{G}_t^W = \frac{\eta\rho}{\rho\left[\rho_g + \eta(1-c_Y)\right] + \eta c_Y \rho_g} S_t^W, \tag{C.19}$$

$$\widetilde{G}_t^R = \frac{\eta}{\rho_g(1+\eta c_Y) + \eta(1-c_Y)} S_t^R, \tag{C.20}$$

$$\widetilde{T}_t = -\frac{\eta\rho_g}{\rho_g(1+\eta c_Y) + \eta(1-c_Y)} S_t^R. \tag{C.21}$$

The above expressions have been derived as follows. Using (C.16) and (C.17), we get $\widetilde{G}_t^R = \frac{\eta}{\rho_g+\eta(1-c_Y)}\left(S_t^R + c_Y\widetilde{T}_t\right)$. By substituting this into (C.11), one then recovers (C.21). Next, combining (C.16) and (C.17) with weights n and $(1-n)$, respectively, yields $\widetilde{G}_t^W = \frac{\eta}{\rho_g+\eta(1-c_Y)}\left(S_t^W - c_Y\widetilde{C}_t\right)$. Combining this with (C.10), and solving, give (C.19). Substituting (C.19) back into (C.10) and working out yield (C.18).

Finally, assuming that the inflation rate in the flex-price equilibrium is zero, we derive the efficient nominal rate of interest from (A.11) as

$$\widetilde{R}_t = \rho \mathrm{E}_t\left(\widetilde{C}_{t+1} - \widetilde{C}_t\right). \tag{C.22}$$

D. The model under sticky prices

Log linearizing (A.11) and using (C.22), it is straightforward to derive (D.3) below, where for a generic variable X, $\widehat{X} = \ln\left(X/\overline{X}\right)$. Log linearizing (A.12), we derive (D.4) and (D.5), below, and by log linearizing the definition of the terms of trade, $T \equiv P_F/P_H$, we obtain (D.8). Most computationally intensive is the derivation of the Phillips curves, (D.6) and (D.7) below, which we provide now.

We can rewrite (A.13), for $i = H$ and $j = h$, as

$$0 = \mathrm{E}_t \sum_{k=0}^{\infty} \left(\alpha^H \beta\right)^k \left\{ \left[\lambda_{t+k}^H p_t(h) + v_y\left(y_{t,t+k}(h);\xi_{t+k}^H\right)\right] y_{t,t+k}(h) \right\}.$$

After substituting for λ_{t+k}^H we obtain

$$\mathrm{E}_t \sum_{k=0}^{\infty} \left(\alpha^H \beta\right)^k \left\{ \left[\begin{array}{c} \left(\sigma_{t+k}^H - 1\right) U_C\left(C_{t+k}\right) \dfrac{p_t(h)}{P_{H,t+k}} T_{t+k}^{n-1} \\ -\sigma_{t+k}^H v_y\left(y_{t,t+k}(h), \xi_{t+k}^H\right) \end{array} \right] y_{t,t+k}(h) \right\} = 0. \tag{D.1}$$

To log linearize this condition around the steady state, we first need to log linearize $y_{t,t+k}(h)$: taking logarithmic changes on both sides of (A.9) and evaluating around the steady state, we obtain:

$$\begin{aligned}
\mathrm{d}\ln y_{t,t+k}(h) &= \mathrm{d}\ln\left(\frac{p_t(h)}{P_{H,t+k}}\right)^{-\sigma_{t+k}^H} + \mathrm{d}\ln\left[T_{t+k}^{1-n} C_{t+k} + G_{t+k}^H\right] \\
\hat{y}_{t,t+k}(h) &= -\sigma\left[\frac{\overline{P}_H \mathrm{d}p_t(h) - \overline{P}_H \mathrm{d}P_{H,t+k}}{\overline{P}_H^2}\right] + \frac{\bar{C}(1-n)\mathrm{d}T_{t+k} + \mathrm{d}C_{t+k} + \mathrm{d}G_{t+k}^H}{\overline{Y}} \\
\hat{y}_t(h) &= -\sigma\left(\hat{p}_t(h) - \hat{P}_{H,t+k}\right) + c_Y\left((1-n)\hat{T}_{t+k} + \hat{C}_{t+k}\right) + (1-c_Y)\hat{G}_{t+k}^H \\
\hat{y}_t(h) &= -\sigma\hat{p}_{t,t+k}(h) + c_Y\left((1-n)\hat{T}_{t+k} + \hat{C}_{t+k}\right) + (1-c_Y)\hat{G}_{t+k}^H.
\end{aligned}$$

Using this expression, the log-linearized version of (D.1) around the steady state is:

$$0 = \mathrm{E}_t \sum_{k=0}^{\infty} \left(\alpha^H \beta\right)^k \left\{ \begin{array}{c} \widehat{p}_{t,t+k} - (1-n)\widehat{T}_{t+k} - \rho\widehat{C}_{t+k} + \frac{1}{\sigma-1}\widehat{\sigma}_{t+k}^H \\ -\eta\left[-\sigma\widehat{p}_{t,t+k} + c_Y\left((1-n)\widehat{T}_{t+k} + \widehat{C}_{t+k}\right) + (1-c_Y)\widehat{G}_{t+k}^H - S_{t+k}^H\right] \end{array} \right\},$$

where $\widehat{p}_{t,t+k} \equiv \ln\left(p_t(h)/P_{H,t+k}\right)$. We rewrite this expression, using that $\widehat{p}_{t,t+k} = \widehat{p}_{t,t} - \sum_{s=1}^k \pi_{t+s}^H$, as:

$$\begin{aligned}
\frac{\widehat{p}_{t,t}}{1-\alpha^H\beta} &= \mathrm{E}_t \sum_{k=0}^{\infty} \left(\alpha^H \beta\right)^k \left\{ \begin{array}{c} \frac{1+\eta c_Y}{1+\eta\sigma}(1-n)\widehat{T}_{t+k} + \frac{\rho+\eta c_Y}{1+\eta\sigma}\widehat{C}_{t+k} \\ +\frac{1}{1-\sigma}\frac{1}{1+\eta\sigma}\widehat{\sigma}_{t+k}^H + \frac{\eta}{1+\eta\sigma}\left((1-c_Y)\widehat{G}_{t+k}^H - S_{t+k}^H\right) \end{array} \right\} \\
&\quad + \mathrm{E}_t \sum_{k=0}^{\infty} \left(\alpha^H \beta\right)^k \left[\sum_{s=1}^{k} \pi_{t+s}^H\right].
\end{aligned}$$

Log-linearizing (A.14), we obtain $\widehat{p}_{t,t} = \frac{\alpha^H}{1-\alpha^H}\pi_t^H$, which we use to simplify the previous expression:

$$\frac{\pi_t^H}{1-\alpha^H\beta}\frac{\alpha^H}{1-\alpha^H} = \mathrm{E}_t\sum_{k=0}^{\infty}\left(\alpha^H\beta\right)^k\left\{\begin{array}{c}\frac{1+\eta c_Y}{1+\eta\sigma}\left(1-n\right)\widehat{T}_{t+k}+\frac{\rho+\eta c_Y}{1+\eta\sigma}\widehat{C}_{t+k}\\+\frac{1}{1-\sigma}\frac{1}{1+\eta\sigma}\widehat{\sigma}_{t+k}^H+\frac{\eta}{1+\eta\sigma}\left(\left(1-c_Y\right)\widehat{G}_{t+k}^H-S_{t+k}^H\right)\end{array}\right\}$$
$$+\mathrm{E}_t\sum_{k=1}^{\infty}\left(\alpha^H\beta\right)^k\frac{\pi_{t+k}^H}{1-\alpha^H\beta}.$$

Finally, we then obtain

$$\pi_t^H=\frac{\left(1-\alpha^H\beta\right)\left(1-\alpha^H\right)}{\alpha^H}\left[\begin{array}{c}\frac{1+\eta c_Y}{1+\eta\sigma}\left(1-n\right)\widehat{T}_t+\frac{\rho+\eta c_Y}{1+\eta\sigma}\widehat{C}_t+\frac{\eta(1-c_Y)}{1+\eta\sigma}\widehat{G}_t^H\\+\frac{1}{1-\sigma}\frac{1}{1+\eta\sigma}\widehat{\sigma}_t^H-\frac{\eta}{1+\eta\sigma}S_t^H\end{array}\right]+\beta\mathrm{E}_t\pi_{t+1}^H. \tag{D.2}$$

Combine (C.12) and (C.13) to find that $\widetilde{C}_t=\frac{\eta}{\rho+\eta c_Y}S_t^W-\frac{\eta(1-c_Y)}{\rho+\eta c_Y}\widetilde{G}_t^W$. Using this expression and (C.11), it is straightforward to show that $-\left(1+\eta c_Y\right)\left(1-n\right)\widetilde{T}_t-\left(\rho+\eta c_Y\right)\widetilde{C}_t-\eta\left(1-c_Y\right)\widetilde{G}_t^H=-\eta S_t^H$. Hence, (D.2) can be rewritten as (D.6) below. In an analogous fashion we derive (D.7) below.

Summarizing, the log-linearized system under sticky prices is given by

$$\mathrm{E}_t\left(\widehat{C}_{t+1}-\widetilde{C}_{t+1}\right)=\left(\widehat{C}_t-\widetilde{C}_t\right)+\rho^{-1}\left[\left(\widehat{R}_t-\widetilde{R}_t\right)-\mathrm{E}_t\left(\pi_{t+1}^W\right)\right], \tag{D.3}$$

$$\widehat{Y}_t^H = c_Y\left[\left(1-n\right)\widehat{T}_t+\widehat{C}_t\right]+\left(1-c_Y\right)\widehat{G}_t^H, \tag{D.4}$$

$$\widehat{Y}_t^F = c_Y\left[-n\widehat{T}_t+\widehat{C}_t\right]+\left(1-c_Y\right)\widehat{G}_t^F, \tag{D.5}$$

$$\pi_t^H = \beta\mathrm{E}_t\pi_{t+1}^H+\kappa_T^H\left(1-n\right)\left(\widehat{T}_t-\widetilde{T}_t\right)+\kappa_C^H\left(\widehat{C}_t-\widetilde{C}_t\right)+\kappa_G^H\left(\widehat{G}_t^H-\widetilde{G}_t^H\right)+u_t^H, \tag{D.6}$$

$$\pi_t^F = \beta\mathrm{E}_t\pi_{t+1}^F-\kappa_T^F n\left(\widehat{T}_t-\widetilde{T}_t\right)+\kappa_C^F\left(\widehat{C}_t-\widetilde{C}_t\right)+\kappa_G^F\left(\widehat{G}_t^F-\widetilde{G}_t^F\right)+u_t^F, \tag{D.7}$$

$$\left(\widehat{T}_t-\widetilde{T}_t\right)=\left(\widehat{T}_{t-1}-\widetilde{T}_{t-1}\right)+\pi_t^R-\left(\widetilde{T}_t-\widetilde{T}_{t-1}\right), \tag{D.8}$$

which is the system (1)-(6) in the paper, where

$$\begin{aligned} \kappa_T^H &\equiv \kappa^H (1+\eta c_Y), & \kappa_T^F \equiv \kappa^F (1+\eta c_Y), \\ \kappa_C^H &\equiv \kappa^H (\rho+\eta c_Y), & \kappa_C^F \equiv \kappa^F (\rho+\eta c_Y), \\ \kappa_G^H &\equiv \kappa^H \eta (1-c_Y), & \kappa_G^F \equiv \kappa^F \eta (1-c_Y), \end{aligned}$$

with

$$\kappa^H \equiv \frac{\left(1-\alpha^H\beta\right)\left(1-\alpha^H\right)}{\alpha^H (1+\eta\sigma)}, \qquad \kappa^F \equiv \frac{\left(1-\alpha^F\beta\right)\left(1-\alpha^F\right)}{\alpha^F (1+\eta\sigma)},$$

and where

$$u_t^H \equiv \kappa^H \tfrac{1}{1-\sigma}\widehat{\sigma}_t^H, \qquad u_t^F \equiv \kappa^F \tfrac{1}{1-\sigma}\widehat{\sigma}_t^F,$$

refer to inflation variations caused by fluctuations in producers' market power. For *any* of the cases with *equal rigidities* considered in the sequel, we define:

$$\begin{aligned} \alpha &\equiv \alpha^H = \alpha^F, & \kappa \equiv \kappa^H = \kappa^F, \\ \kappa_T &\equiv \kappa_T^H = \kappa_T^F, & \kappa_C \equiv \kappa_C^H = \kappa_C^F, \\ \kappa_G &\equiv \kappa_G^H = \kappa_G^F. \end{aligned}$$

E. Derivation of the micro-founded loss function

Here, we derive the utility-based loss function. The per-period average utility flows of the households belonging to countries H and F, respectively, are:

$$w_t^H = U(C_t) + V\left(G_t^H\right) - \frac{1}{n}\int_0^n v\left(y_t(h);\xi_t^H\right)\mathrm{d}h, \tag{E.1}$$

$$w_t^F = U(C_t) + V\left(G_t^F\right) - \frac{1}{1-n}\int_n^1 v\left(y_t(f);\xi_t^F\right)\mathrm{d}f. \tag{E.2}$$

The welfare criterion of the authorities (the common central bank and the coordinating fiscal authorities) is a population weighted average of households' utilities:

$$W^C = \mathrm{E}_0 \sum_{j=0}^{\infty} \beta^j \left[n w_{t+j}^H + (1-n)\, w_{t+j}^F\right]. \tag{E.3}$$

We start by making computations for Home. The computations for Foreign are analogous and, therefore, not shown explicitly. After this, we combine the expressions for Home and Foreign to obtain W^C.

E.1. The term $U\left(C_t^H\right)$

Take a second-order expansion of $U\left(C_t\right)$ around the steady-state value $\overline{C}$:

$$U\left(C_t\right) = U\left(\overline{C}\right) + U_C\left(C_t - \overline{C}\right) + \frac{1}{2}U_{CC}\left(C_t - \overline{C}\right)^2 + \mathcal{O}\left(\|\xi\|^3\right), \tag{E.4}$$

where $\mathcal{O}\left(\|\xi\|^3\right)$ stands for terms of third or higher order (remember that all variables are, in equilibrium, functions of the shock vector, which exhibits bounded fluctuations of order $\|\xi\|$). Note that a second-order log-expansion of C_t^H around $\overline{C}$ yields:

$$C_t = \overline{C}\left[1 + \widehat{C}_t + \frac{1}{2}\widehat{C}_t^2\right] + \mathcal{O}\left(\|\xi\|^3\right). \tag{E.5}$$

Substitute (E.5) into (E.4) to give:

$$U\left(C_t\right) = U\left(\overline{C}\right) + U_C\overline{C}\left[\widehat{C}_t + \tfrac{1}{2}\widehat{C}_t^2\right] + \tfrac{1}{2}U_{CC}\overline{C}^2\left(\widehat{C}_t\right)^2 + \mathcal{O}\left(\|\xi\|^3\right),$$

and thus

$$\begin{aligned}
U\left(C_t\right) &= U_C\overline{C}\left[\widehat{C}_t + \tfrac{1}{2}\widehat{C}_t^2\right] + \tfrac{1}{2}U_{CC}\overline{C}^2\left(\widehat{C}_t\right)^2 + \text{t.i.p.} + \mathcal{O}\left(\|\xi\|^3\right) \Rightarrow \\
U\left(C_t\right) &= U_C\overline{C}\left[\widehat{C}_t + \tfrac{1}{2}\widehat{C}_t^2 + \tfrac{1}{2}\frac{U_{CC}\overline{C}}{U_C}\left(\widehat{C}_t\right)^2\right] + \text{t.i.p.} + \mathcal{O}\left(\|\xi\|^3\right) \Rightarrow \\
U\left(C_t\right) &= U_C\overline{C}\left[\widehat{C}_t + \tfrac{1}{2}\left(1-\rho\right)\widehat{C}_t^2\right] + \text{t.i.p.} + \mathcal{O}\left(\|\xi\|^3\right),
\end{aligned} \tag{E.6}$$

where "t.i.p." stands for "terms independent of policy."

E.2. The term $V\left(G_t^H\right)$

We approximate in an analogous way $V\left(G_t^H\right)$. This yields:

$$V\left(G_t^H\right) = V_G\overline{G}\left[\widehat{G}_t^H + \frac{1}{2}\left(1-\rho_g\right)\left(\widehat{G}_t^H\right)^2\right] + \text{t.i.p.} + \mathcal{O}\left(\|\xi\|^3\right). \tag{E.7}$$

Using (B.25) and assuming that ϕ is of at least order $\mathcal{O}\left(\|\xi\|\right)$, we can write(E.7) as:

$$V\left(G_t^H\right) = U_C\overline{G}\left[\left(1 - \phi\frac{\rho}{\rho + \eta c_Y}\right)\widehat{G}_t^H + \frac{1}{2}\left(1-\rho_g\right)\left(\widehat{G}_t^H\right)^2\right] + \text{t.i.p.} + \mathcal{O}\left(\|\xi\|^3\right), \tag{E.8}$$

having used that $-\phi\left(\widehat{G}_t^H\right)^2$ is of order at least $\mathcal{O}\left(\|\xi\|^3\right)$.

E.3. The term $v\left(y_t\left(h\right);\xi_t^H\right)$

Similarly, we take a second-order Taylor expansion of $v\left(y_t\left(h\right);\xi_t^H\right)$ around a steady state where $y_t\left(h\right)=\overline{Y}$ for each h and at each date t, and where $\xi_t^H=0$ at each date t. We obtain:

$$\begin{aligned} v\left(y_t\left(h\right);\xi_t^H\right) &= v\left(\overline{Y};0\right)+v_y\left(y_t\left(h\right)-\overline{Y}\right)+v_\xi\xi_t^H+\tfrac{1}{2}v_{yy}\left(y_t\left(h\right)-\overline{Y}\right)^2 \\ &\quad +v_{y\xi}\xi_t^H\left(y_t\left(h\right)-\overline{Y}\right)+\tfrac{1}{2}\left(\xi_t^H\right)'v_{\xi\xi}\xi_t^H+\mathcal{O}\left(\|\xi\|^3\right). \end{aligned}$$

Then note that a second-order logarithmic expansion of $y_t\left(h\right)$ gives:

$$y_t\left(h\right)=\overline{Y}\left[1+\widehat{y}_t\left(h\right)+\tfrac{1}{2}\widehat{y}_t\left(h\right)^2\right]+\mathcal{O}\left(\|\xi\|^3\right).$$

Using this expression, we simplify

$$v\left(y_t\left(h\right);\xi_t^H\right)=v_y y_t\left(h\right)+\frac{1}{2}v_{yy}\left(y_t\left(h\right)-\overline{Y}\right)^2+v_{y\xi}\xi_t^H y_t\left(h\right)+\text{t.i.p.}+\mathcal{O}\left(\|\xi\|^3\right)$$

to

$$v\left(y_t\left(h\right);\xi_t^H\right)=v_y\overline{Y}\left[\widehat{y}_t\left(h\right)+\frac{1}{2}\widehat{y}_t\left(h\right)^2+\frac{1}{2}\frac{v_{yy}\overline{Y}}{v_y}\widehat{y}_t\left(h\right)^2+\frac{v_{y\xi}}{v_y}\xi_t^H\widehat{y}_t\left(h\right)\right]+\text{t.i.p.}+\mathcal{O}\left(\|\xi\|^3\right),$$

or

$$v\left(y_t\left(h\right);\xi_t^H\right)=v_y\overline{Y}\left[\widehat{y}_t\left(h\right)+\frac{1+\eta}{2}\widehat{y}_t\left(h\right)^2+\frac{v_{y\xi}}{v_y}\xi_t^H\widehat{y}_t\left(h\right)\right]+\text{t.i.p.}+\mathcal{O}\left(\|\xi\|^3\right).$$

Then, recalling the definition $v_{y\xi}\xi_t^H=-\overline{Y}v_{yy}S_t^H$ for S_t^H, we finally arrive at

$$v\left(y_t\left(h\right);\xi_t^H\right)=v_y\overline{Y}\left[\widehat{y}_t\left(h\right)+\frac{1+\eta}{2}\widehat{y}_t\left(h\right)^2-\eta S_t^H\widehat{y}_t\left(h\right)\right]+\text{t.i.p.}+\mathcal{O}\left(\|\xi\|^3\right). \quad \text{(E.9)}$$

Recall that $\left(1-\phi\right)U_C\left(\overline{C}\right)=v_y\left(\overline{Y};0\right)$. Hence, using this, we can write (E.9) as:

$$v\left(y_t\left(h\right);\xi_t^H\right)=U_C\overline{Y}\left[\left(1-\phi\right)\widehat{y}_t\left(h\right)+\frac{1+\eta}{2}\widehat{y}_t\left(h\right)^2-\eta S_t^H\widehat{y}_t\left(h\right)\right]+\text{t.i.p.}+\mathcal{O}\left(\|\xi\|^3\right),$$

where we have used that $\phi\widehat{y}_t\left(h\right)^2$ and $\phi S_t^H\widehat{y}_t\left(h\right)$ are of order at least $\mathcal{O}\left(\|\xi\|^3\right)$.

This last expression should be integrated over the Home population, to find

$$\begin{aligned}
&\frac{1}{n}\int_0^n v\left(y_t(h);\xi_t^H\right)\mathrm{d}h \\
=\ & U_C\overline{Y}\left((1-\phi)\,\mathrm{E}_h\widehat{y}_t(h)+\tfrac{1+\eta}{2}\left[\mathrm{Var}_h\widehat{y}_t(h)+\left[\mathrm{E}_h\widehat{y}_t(h)\right]^2\right]-\eta S_t^H\mathrm{E}_h\widehat{y}_t(h)\right)+\text{t.i.p.}+\mathcal{O}\left(\|\xi\|^3\right).
\end{aligned}$$

We then take a second-order log-expansion of the aggregator Y_t^H to obtain:

$$\begin{aligned}
\widehat{Y}_t^H &= \mathrm{E}_h\widehat{y}_t(h)+\tfrac{1}{2}\tfrac{\sigma_t^H-1}{\sigma_t^H}\mathrm{Var}_h\widehat{y}_t(h)+\mathcal{O}\left(\|\xi\|^3\right) \\
&= \mathrm{E}_h\widehat{y}_t(h)+\tfrac{1}{2}\tfrac{\sigma-1}{\sigma}\mathrm{Var}_h\widehat{y}_t(h)+\mathcal{O}\left(\|\xi\|^3\right),
\end{aligned}$$

where we observe that

$$\tfrac{\sigma_t^H-1}{\sigma_t^H}=\tfrac{\sigma-1}{\sigma}\left\{1+\ln\left[\left(\tfrac{\sigma_t^H-1}{\sigma_t^H}\right)/\left(\tfrac{\sigma-1}{\sigma}\right)\right]+\left(\ln\left[\left(\tfrac{\sigma_t^H-1}{\sigma_t^H}\right)/\left(\tfrac{\sigma-1}{\sigma}\right)\right]\right)^2\right\}+\mathcal{O}\left(\|\xi\|^3\right),$$

and $\ln\left[\left(\tfrac{\sigma_t^H-1}{\sigma_t^H}\right)/\left(\tfrac{\sigma-1}{\sigma}\right)\right]$ is of order at least $\mathcal{O}(\|\xi\|)$, so that

$$\tfrac{\sigma_t^H-1}{\sigma_t^H}\mathrm{Var}_h\widehat{y}_t(h)=\tfrac{\sigma-1}{\sigma}\mathrm{Var}_h\widehat{y}_t(h)+\mathcal{O}\left(\|\xi\|^3\right).$$

Insert the implied value for $\mathrm{E}_h\widehat{y}_t(h)$ into the previous expression:

$$\begin{aligned}
&\frac{1}{n}\int_0^n v\left(y_t(h);\xi_t^H\right)\mathrm{d}h \\
=\ & U_C\overline{Y}\left((1-\phi)\,\widehat{Y}_t^H-\tfrac{1}{2}\tfrac{\sigma-1}{\sigma}\mathrm{Var}_h\widehat{y}_t(h)+\tfrac{1+\eta}{2}\left[\mathrm{Var}_h\widehat{y}_t(h)+\left(\widehat{Y}_t^H\right)^2\right]-\eta S_t^H\widehat{Y}_t^H\right) \\
& +\text{t.i.p.}+\mathcal{O}\left(\|\xi\|^3\right) \\
=\ & U_C\overline{Y}\left[(1-\phi)\,\widehat{Y}_t^H+\tfrac{1+\eta}{2}\left(\widehat{Y}_t^H\right)^2-\tfrac{1}{2}\left[\tfrac{\sigma-1}{\sigma}-1-\eta\right]\mathrm{Var}_h\widehat{y}_t(h)-\eta S_t^H\widehat{Y}_t^H\right]+\text{t.i.p.}+\mathcal{O}\left(\|\xi\|^3\right) \\
=\ & U_C\overline{Y}\left[(1-\phi)\,\widehat{Y}_t^H+\tfrac{1+\eta}{2}\left(\widehat{Y}_t^H\right)^2+\tfrac{1}{2}\left[\sigma^{-1}+\eta\right]\mathrm{Var}_h\widehat{y}_t(h)-\eta S_t^H\widehat{Y}_t^H\right]+\text{t.i.p.}+\mathcal{O}\left(\|\xi\|^3\right).
\end{aligned}$$

E.4. Combining the terms

Combining (E.6), (E.8) and the previous expression, the relevant Home welfare criterion is

$$
\begin{aligned}
w_t^H &= U_C\overline{C}\left[\widehat{C}_t + \frac{1}{2}(1-\rho)\widehat{C}_t^2\right] \\
&+U_C\overline{G}\left[\left(1-\phi\frac{\rho}{\rho+\eta c_Y}\right)\widehat{G}_t^H + \frac{1}{2}(1-\rho_g)\left(\widehat{G}_t^H\right)^2\right] \\
&-U_C\overline{Y}\left[(1-\phi)\widehat{Y}_t^H + \tfrac{1+\eta}{2}\left(\widehat{Y}_t^H\right)^2 + \tfrac{1}{2}\left[\sigma^{-1}+\eta\right]\mathrm{Var}_h\widehat{y}_t(h) - \eta S_t^H\widehat{Y}_t^H\right] \\
&+\text{t.i.p.} + \mathcal{O}\left(\|\xi\|^3\right),
\end{aligned}
$$

or

$$
\begin{aligned}
w_t^H &= U_C\overline{C}\left\{\left[\widehat{C}_t + \frac{1}{2}(1-\rho)\widehat{C}_t^2\right]\right. \\
&+\frac{1-c_Y}{c_Y}\left[\left(1-\phi\left(\tfrac{\rho}{\rho+\eta c_Y}\right)\right)\widehat{G}_t^H + \frac{1}{2}(1-\rho_g)\left(\widehat{G}_t^H\right)^2\right] \\
&\left.-\frac{1}{c_Y}\left[(1-\phi)\widehat{Y}_t^H + \tfrac{1+\eta}{2}\left(\widehat{Y}_t^H\right)^2 + \tfrac{1}{2}\left[\sigma^{-1}+\eta\right]\mathrm{Var}_h\widehat{y}_t(h) - \eta S_t^H\widehat{Y}_t^H\right]\right\} \\
&+\text{t.i.p.} + \mathcal{O}\left(\|\xi\|^3\right).
\end{aligned}
$$

For Foreign, we similarly find:

$$
\begin{aligned}
w_t^F &= U_C\overline{C}\left\{\left[\widehat{C}_t + \frac{1}{2}(1-\rho)\widehat{C}_t^2\right]\right. \\
&+\frac{1-c_Y}{c_Y}\left[\left(1-\phi\frac{\rho}{\rho+\eta c_Y}\right)\widehat{G}_t^F + \frac{1}{2}(1-\rho_g)\left(\widehat{G}_t^F\right)^2\right] \\
&\left.-\frac{1}{c_Y}\left[(1-\phi)\widehat{Y}_t^F + \tfrac{1+\eta}{2}\left(\widehat{Y}_t^F\right)^2 + \tfrac{1}{2}\left[\sigma^{-1}+\eta\right]\mathrm{Var}_f\widehat{y}_t(f) - \eta S_t^F\widehat{Y}_t^F\right]\right\} \\
&+\text{t.i.p.} + \mathcal{O}\left(\|\xi\|^3\right).
\end{aligned}
$$

Now, take a weighted average of w_t^H and w_t^F with weights n and $1-n$, respectively:

$$\begin{aligned}
w_t &= U_C\overline{C}\left\{\left[\widehat{C}_t + \tfrac{1}{2}(1-\rho)\widehat{C}_t^2\right]\right. \\
&\quad + \frac{1-c_Y}{c_Y}\left[\left(1-\phi\frac{\rho}{\rho+\eta c_Y}\right)\widehat{G}_t^W + \tfrac{1}{2}(1-\rho_g)\left(n\left(\widehat{G}_t^H\right)^2 + (1-n)\left(\widehat{G}_t^F\right)^2\right)\right] \\
&\quad \left. - \frac{1}{c_Y}\left[\begin{array}{c} (1-\phi)\widehat{Y}_t^W + \frac{1+\eta}{2}\left(n\left(\widehat{Y}_t^H\right)^2 + (1-n)\left(\widehat{Y}_t^F\right)^2\right) \\ +\frac{1}{2}\left[\sigma^{-1}+\eta\right]\left[n\mathrm{Var}_h\widehat{y}_t(h) + (1-n)\mathrm{Var}_f\widehat{y}_t(f)\right] \\ -\eta n S_t^H\widehat{Y}_t^H - \eta(1-n)S_t^F\widehat{Y}_t^F \end{array}\right]\right\} \\
&\quad + \text{t.i.p.} + \mathcal{O}\left(\|\xi\|^3\right).
\end{aligned} \tag{E.10}$$

E.4.1. Expansion of $\widehat{Y}_t$

Before continuing, we expand $\widehat{Y}_t^H$. Define the function $W\left(Y_t^H\right) \equiv \widehat{Y}_t^H = \ln\left(Y_t^H/\overline{Y}\right)$. We approximate this as:

$$\begin{aligned}
\widehat{Y}_t^H &= W\left(\overline{Y}\right) + W'\left(\overline{Y}\right)\left(Y_t^H - \overline{Y}\right) + \frac{1}{2}W''\left(\overline{Y}\right)\left(Y_t^H - \overline{Y}\right)^2 + \mathcal{O}\left(\|\xi\|^3\right) \\
&= 0 + \left(\frac{Y_t^H - \overline{Y}}{\overline{Y}}\right) - \frac{1}{2}\left(\frac{Y_t^H - \overline{Y}}{\overline{Y}}\right)^2 + \mathcal{O}\left(\|\xi\|^3\right) \\
&= \frac{T_t^{1-n}C_t + G_t^H - \left(\overline{T}^{1-n}\overline{C} + \overline{G}\right)}{\overline{Y}} - \frac{1}{2}\left[\frac{T_t^{1-n}C_t + G_t^H - \left(\overline{T}^{1-n}\overline{C} + \overline{G}\right)}{\overline{Y}}\right]^2 + \mathcal{O}\left(\|\xi\|^3\right) \\
&= \frac{T_t^{1-n}C_t - \overline{T}^{1-n}\overline{C}}{\overline{Y}} + \frac{G_t^H - \overline{G}}{\overline{Y}} - \frac{1}{2}\left[\frac{T_t^{n-1}C_t - \overline{T}^{1-n}\overline{C}}{\overline{Y}} + \frac{G_t^H - \overline{G}}{\overline{Y}}\right]^2 + \mathcal{O}\left(\|\xi\|^3\right).
\end{aligned} \tag{E.11}$$

Now define $Z(T_t, C_t) \equiv T_t^{1-n}C_t$. Taking a second-order Taylor expansion of $Z(T_t, C_t)$ around the point $\left(\overline{T}, \overline{C}\right)$ gives:

$$\begin{aligned}
Z(T_t, C_t) &= Z\left(\overline{T}, \overline{C}\right) + Z_T\left(T_t - \overline{T}\right) + \tfrac{1}{2}Z_{TT}\left(T_t - \overline{T}\right)^2 + Z_C\left(C_t - \overline{C}\right) \\
&\quad + \tfrac{1}{2}Z_{CC}\left(C_t - \overline{C}\right)^2 + Z_{TC}\left(T_t - \overline{T}\right)\left(C_t - \overline{C}\right) + \mathcal{O}\left(\|\xi\|^3\right) \\
&= \overline{T}^{1-n}\overline{C} + (1-n)\overline{T}^{-n}\overline{C}\left(T_t - \overline{T}\right) - \tfrac{1}{2}(1-n)n\overline{T}^{-(n+1)}\bar{C}\left(T_t - \overline{T}\right)^2 \\
&\quad + \overline{T}^{1-n}\left(C_t - \overline{C}\right) + (1-n)\overline{T}^{-n}\left(T_t - \overline{T}\right)\left(C_t - \overline{C}\right) + \mathcal{O}\left(\|\xi\|^3\right) \\
&= \overline{T}^{1-n}\overline{C} + (1-n)\overline{T}^{1-n}\overline{C}\left(\frac{T_t - \overline{T}}{\overline{T}}\right) - \tfrac{1}{2}(1-n)n\overline{T}^{1-n}\overline{C}\left(\frac{T_t - \overline{T}}{\overline{T}}\right)^2 \\
&\quad + \overline{T}^{1-n}\overline{C}\left(\frac{C_t - \overline{C}}{\overline{C}}\right) + (1-n)\overline{T}^{1-n}\overline{C}\left(\frac{T_t - \overline{T}}{\overline{T}}\right)\left(\frac{C_t - \overline{C}}{\overline{C}}\right) + \mathcal{O}\left(\|\xi\|^3\right).
\end{aligned}$$

Hence,

$$\begin{aligned}
& \frac{T_t^{1-n}C_t - \overline{T}^{1-n}\overline{C}}{\bar{Y}} \\
= \; & (1-n)\, c_Y \left(\frac{T_t - \overline{T}}{\overline{T}}\right) - \tfrac{1}{2}(1-n)\, n c_Y \left(\frac{T_t - \overline{T}}{\overline{T}}\right)^2 \\
& + c_Y \left(\frac{C_t - \overline{C}}{\overline{C}}\right) + (1-n)\, c_Y \left(\frac{T_t - \overline{T}}{\overline{T}}\right)\left(\frac{C_t - \overline{C}}{\overline{C}}\right) + \mathcal{O}\left(\|\xi\|^3\right).
\end{aligned}$$

Into this expression, substitute:

$$\begin{aligned}
C_t &= \overline{C}\left(1 + \widehat{C}_t + \frac{1}{2}\widehat{C}_t^2\right) + \mathcal{O}\left(\|\xi\|^3\right), \\
T_t &= \overline{T}\left(1 + \widehat{T}_t + \frac{1}{2}\widehat{T}_t^2\right) + \mathcal{O}\left(\|\xi\|^3\right),
\end{aligned}$$

so that the right-hand side becomes:

$$\begin{aligned}
& (1-n)\, c_Y \left(\widehat{T}_t + \tfrac{1}{2}\widehat{T}_t^2\right) - \tfrac{1}{2}(1-n)\, n c_Y \left(\widehat{T}_t + \tfrac{1}{2}\widehat{T}_t^2\right)^2 \\
& + c_Y \left(\widehat{C}_t + \tfrac{1}{2}\widehat{C}_t^2\right) + (1-n)\, c_Y \left(\widehat{T}_t + \tfrac{1}{2}\widehat{T}_t^2\right)\left(\widehat{C}_t + \tfrac{1}{2}\widehat{C}_t^2\right) + \mathcal{O}\left(\|\xi\|^3\right) \\
= \; & (1-n)\, c_Y \widehat{T}_t + c_Y \widehat{C}_t + \tfrac{1}{2}(1-n)\, c_Y \widehat{T}_t^2 \\
& - \tfrac{1}{2}(1-n)\, n c_Y \widehat{T}_t^2 + \tfrac{1}{2} c_Y \widehat{C}_t^2 + (1-n)\, c_Y \widehat{T}_t \widehat{C}_t + \mathcal{O}\left(\|\xi\|^3\right) \\
= \; & (1-n)\, c_Y \widehat{T}_t + c_Y \widehat{C}_t + \tfrac{1}{2}(1-n)^2 c_Y \widehat{T}_t^2 + \tfrac{1}{2} c_Y \widehat{C}_t^2 + (1-n)\, c_Y \widehat{T}_t \widehat{C}_t + \mathcal{O}\left(\|\xi\|^3\right).
\end{aligned}$$

Using that

$$G_t^H = \overline{G}\left(1 + \widehat{G}_t^H + \frac{1}{2}\left(\widehat{G}_t^H\right)^2\right) + \mathcal{O}\left(\|\xi\|^3\right),$$

we can write:

$$\begin{aligned}
\left(\frac{Y_t^H - \overline{Y}}{\overline{Y}}\right) &= c_Y \left[(1-n)\, \widehat{T}_t + \widehat{C}_t + \tfrac{1}{2}(1-n)^2 \widehat{T}_t^2 + \tfrac{1}{2}\widehat{C}_t^2 + (1-n)\, \widehat{T}_t \widehat{C}_t\right] \\
& \quad + (1-c_Y)\left(\widehat{G}_t^H + \tfrac{1}{2}\left(\widehat{G}_t^H\right)^2\right) + \mathcal{O}\left(\|\xi\|^3\right).
\end{aligned}$$

Hence:

$$
\begin{aligned}
& \left(\frac{Y_t^H-\overline{Y}}{\overline{Y}}\right)^2 \\
= \; & c_Y^2\left[(1-n)\widehat{T}_t+\widehat{C}_t+\tfrac{1}{2}(1-n)^2\widehat{T}_t^2+\tfrac{1}{2}\widehat{C}_t^2+(1-n)\widehat{T}_t\widehat{C}_t\right]^2 \\
& +2c_Y(1-c_Y)\left[(1-n)\widehat{T}_t+\widehat{C}_t+\tfrac{1}{2}(1-n)^2\widehat{T}_t^2+\tfrac{1}{2}\widehat{C}_t^2+(1-n)\widehat{T}_t\widehat{C}_t\right]\left[\widehat{G}_t^H+\frac{1}{2}\left(\widehat{G}_t^H\right)^2\right] \\
& +(1-c_Y)^2\left[\widehat{G}_t^H+\frac{1}{2}\left(\widehat{G}_t^H\right)^2\right]^2+\mathcal{O}\left(\|\xi\|^3\right) \\
= \; & c_Y^2\left[(1-n)^2\widehat{T}_t^2+\widehat{C}_t^2+2(1-n)\widehat{T}_t\widehat{C}_t\right]+(1-c_Y)^2\left(\widehat{G}_t^H\right)^2 \\
& +2c_Y(1-c_Y)(1-n)\widehat{T}_t\widehat{G}_t^H+2c_Y(1-c_Y)\widehat{C}_t\widehat{G}_t^H+\mathcal{O}\left(\|\xi\|^3\right).
\end{aligned}
$$

Hence,

$$
\begin{aligned}
& \left(\frac{Y_t^H-\overline{Y}}{\overline{Y}}\right)-\frac{1}{2}\left(\frac{Y_t^H-\overline{Y}}{\overline{Y}}\right)^2 \\
= \; & c_Y\left[(1-n)\widehat{T}_t+\widehat{C}_t\right]+(1-c_Y)\widehat{G}_t^H+\tfrac{1}{2}(1-n)^2c_Y\widehat{T}_t^2+\tfrac{1}{2}c_Y\widehat{C}_t^2+(1-n)c_Y\widehat{T}_t\widehat{C}_t \\
& +\tfrac{1}{2}(1-c_Y)\left(\widehat{G}_t^H\right)^2-\tfrac{1}{2}c_Y^2(1-n)^2\widehat{T}_t^2-\tfrac{1}{2}c_Y^2\widehat{C}_t^2-(1-n)c_Y^2\widehat{T}_t\widehat{C}_t-\tfrac{1}{2}(1-c_Y)^2\left(\widehat{G}_t^H\right)^2 \\
& -c_Y(1-c_Y)(1-n)\widehat{T}_t\widehat{G}_t^H-c_Y(1-c_Y)\widehat{C}_t\widehat{G}_t^H+\mathcal{O}\left(\|\xi\|^3\right).
\end{aligned}
$$

or

$$
\begin{aligned}
& \widehat{Y}_t^H=\left(\frac{Y_t^H-\overline{Y}}{\overline{Y}}\right)-\frac{1}{2}\left(\frac{Y_t^H-\overline{Y}}{\overline{Y}}\right)^2+\mathcal{O}\left(\|\xi\|^3\right) \\
= \; & \left[c_Y\left((1-n)\widehat{T}_t+\widehat{C}_t\right)+(1-c_Y)\widehat{G}_t^H\right] \\
& +\tfrac{1}{2}(1-n)^2c_Y(1-c_Y)\widehat{T}_t^2+\tfrac{1}{2}c_Y(1-c_Y)\widehat{C}_t^2+\tfrac{1}{2}c_Y(1-c_Y)\left(\widehat{G}_t^H\right)^2 \\
& +(1-n)c_Y(1-c_Y)\widehat{T}_t\widehat{C}_t-c_Y(1-c_Y)(1-n)\widehat{T}_t\widehat{G}_t^H \\
& -c_Y(1-c_Y)\widehat{C}_t\widehat{G}_t^H+\mathcal{O}\left(\|\xi\|^3\right).
\end{aligned}
$$

In a similar way we derive the corresponding expression for the foreign country:

$$
\begin{aligned}
\widehat{Y}_t^F &= \left(\frac{Y_t^F - \overline{Y}}{\overline{Y}}\right) - \frac{1}{2}\left(\frac{Y_t^F - \overline{Y}}{\overline{Y}}\right)^2 + \mathcal{O}\left(\|\xi\|^3\right) \\
= \; & \left[c_Y\left(-n\widehat{T}_t + \widehat{C}_t\right) + (1-c_Y)\,\widehat{G}_t^F\right] \\
& + \tfrac{1}{2}n^2 c_Y (1-c_Y)\,\widehat{T}_t^2 + \tfrac{1}{2}c_Y(1-c_Y)\,\widehat{C}_t^2 + \tfrac{1}{2}c_Y(1-c_Y)\left(\widehat{G}_t^F\right)^2 \\
& - n c_Y (1-c_Y)\,\widehat{T}_t\widehat{C}_t + c_Y(1-c_Y)\,n\widehat{T}_t\widehat{G}_t^F \\
& - c_Y(1-c_Y)\,\widehat{C}_t\widehat{G}_t^F + \mathcal{O}\left(\|\xi\|^3\right).
\end{aligned}
$$

E.4.2. Continuation of approximation

To further work out the approximation of the welfare loss function, it is useful to compute some numbers, before actually making the substitutions. We have:

$$
\begin{aligned}
\widehat{Y}_t^W &= n\widehat{Y}_t^H + (1-n)\,\widehat{Y}_t^F \\
&= \left[c_Y\widehat{C}_t + (1-c_Y)\,\widehat{G}_t^W\right] + \tfrac{1}{2}n(1-n)\,c_Y(1-c_Y)\,\widehat{T}_t^2 \\
& \quad + \tfrac{1}{2}c_Y(1-c_Y)\left[n\left(\widehat{C}_t - \widehat{G}_t^H\right)^2 + (1-n)\left(\widehat{C}_t - \widehat{G}_t^F\right)^2\right] \\
& \quad + c_Y(1-c_Y)\,n(1-n)\,\widehat{T}_t\widehat{G}_t^R + \mathcal{O}\left(\|\xi\|^3\right).
\end{aligned}
$$

Hence,

$$
\begin{aligned}
(1-\phi)\,\widehat{Y}_t^W &= (1-\phi)\left[c_Y\widehat{C}_t + (1-c_Y)\,\widehat{G}_t^W\right] + \tfrac{1}{2}n(1-n)\,c_Y(1-c_Y)\,\widehat{T}_t^2 \\
& \quad + \tfrac{1}{2}c_Y(1-c_Y)\left[n\left(\widehat{C}_t - \widehat{G}_t^H\right)^2 + (1-n)\left(\widehat{C}_t - \widehat{G}_t^F\right)^2\right] \\
& \quad + c_Y(1-c_Y)\,n(1-n)\,\widehat{T}_t\widehat{G}_t^R + \mathcal{O}\left(\|\xi\|^3\right),
\end{aligned}
$$

again using that ϕ is of order at least $\mathcal{O}(\|\xi\|)$. Further,

$$
\begin{aligned}
\left(\widehat{Y}_t^H\right)^2 &= \left[c_Y\left((1-n)\,\widehat{T}_t + \widehat{C}_t\right) + (1-c_Y)\,\widehat{G}_t^H\right]^2 + \mathcal{O}\left(\|\xi\|^3\right), \\
\left(\widehat{Y}_t^F\right)^2 &= \left[c_Y\left(-n\widehat{T}_t + \widehat{C}_t\right) + (1-c_Y)\,\widehat{G}_t^F\right]^2 + \mathcal{O}\left(\|\xi\|^3\right).
\end{aligned}
$$

Hence,

$$
\begin{aligned}
& n\left(\widehat{Y}_t^H\right)^2+(1-n)\left(\widehat{Y}_t^F\right)^2 \\
= \ & nc_Y^2\left[(1-n)\widehat{T}_t+\widehat{C}_t\right]^2+(1-n)c_Y^2\left[-n\widehat{T}_t+\widehat{C}_t\right]^2+ \\
& 2nc_Y(1-c_Y)\left[(1-n)\widehat{T}_t+\widehat{C}_t\right]\widehat{G}_t^H+2(1-n)c_Y(1-c_Y)\left[-n\widehat{T}_t+\widehat{C}_t\right]\widehat{G}_t^F \\
& +n(1-c_Y)^2\left(\widehat{G}_t^H\right)^2+(1-n)(1-c_Y)^2\left(\widehat{G}_t^H\right)^2+\mathcal{O}\left(\|\xi\|^3\right) \\
= \ & nc_Y^2\left[(1-n)^2\widehat{T}_t^2+2(1-n)\widehat{T}_t\widehat{C}_t+\widehat{C}_t^2\right]+(1-n)c_Y^2\left[n^2\widehat{T}_t^2-2n\widehat{T}_t\widehat{C}_t+\widehat{C}_t^2\right] \\
& +2c_Y(1-c_Y)\left[\widehat{C}_t\widehat{G}_t^W-n(1-n)\widehat{T}_t\widehat{G}_t^R\right] \\
& +n(1-c_Y)^2\left(\widehat{G}_t^H\right)^2+(1-n)(1-c_Y)^2\left(\widehat{G}_t^H\right)^2+\mathcal{O}\left(\|\xi\|^3\right).
\end{aligned}
$$

Hence,

$$
\begin{aligned}
& n\left(\widehat{Y}_t^H\right)^2+(1-n)\left(\widehat{Y}_t^F\right)^2 \\
= \ & n(1-n)c_Y^2\widehat{T}_t^2+c_Y^2\widehat{C}_t^2+2c_Y(1-c_Y)\left[\widehat{C}_t\widehat{G}_t^W-n(1-n)\widehat{T}_t\widehat{G}_t^R\right] \\
& +(1-c_Y)^2\left[n\left(\widehat{G}_t^H\right)^2+(1-n)\left(\widehat{G}_t^F\right)^2\right]+\mathcal{O}\left(\|\xi\|^3\right) \\
= \ & n(1-n)c_Y^2\widehat{T}_t^2-2c_Y(1-c_Y)n(1-n)\widehat{T}_t\widehat{G}_t^R+ \\
& n\left[c_Y^2\widehat{C}_t^2+2c_Y(1-c_Y)\widehat{C}_t\widehat{G}_t^H+(1-c_Y)^2\left(\widehat{G}_t^H\right)^2\right] \\
& (1-n)\left[c_Y^2\widehat{C}_t^2+2c_Y(1-c_Y)\widehat{C}_t\widehat{G}_t^F+(1-c_Y)^2\left(\widehat{G}_t^F\right)^2\right]+\mathcal{O}\left(\|\xi\|^3\right).
\end{aligned}
$$

Further,

$$
\begin{aligned}
S_t^H\widehat{Y}_t^H &= S_t^H\left[c_Y\left((1-n)\widehat{T}_t+\widehat{C}_t\right)+(1-c_Y)\widehat{G}_t^H\right]+\mathcal{O}\left(\|\xi\|^3\right), \\
S_t^F\widehat{Y}_t^F &= S_t^F\left[c_Y\left(-n\widehat{T}_t+\widehat{C}_t\right)+(1-c_Y)\widehat{G}_t^F\right]+\mathcal{O}\left(\|\xi\|^3\right).
\end{aligned}
$$

Hence,

$$
\begin{aligned}
& \eta\left[nS_t^H\widehat{Y}_t^H+(1-n)S_t^F\widehat{Y}_t^F\right] \\
= \ & \eta nS_t^H\left[c_Y\left((1-n)\widehat{T}_t+\widehat{C}_t\right)+(1-c_Y)\widehat{G}_t^H\right]+ \\
& \eta(1-n)S_t^F\left[c_Y\left(-n\widehat{T}_t+\widehat{C}_t\right)+(1-c_Y)\widehat{G}_t^F\right]+\mathcal{O}\left(\|\xi\|^3\right) \\
= \ & -\eta c_Y(1-n)n\widehat{T}_tS_t^R+\eta c_Y\widehat{C}_tS_t^W \\
& +\eta(1-c_Y)\left[nS_t^H\widehat{G}_t^H+(1-n)S_t^F\widehat{G}_t^F\right]+\mathcal{O}\left(\|\xi\|^3\right).
\end{aligned}
$$

We can now start to make substitutions into (E.10). First, substitute the expression for $\widehat{Y}_t^W$ and observe that the linear terms cancel. Thus, we have:

$$\begin{aligned}
&\frac{w_t}{U_C\overline{C}} \\
= \; & -\tfrac{1-c_Y}{c_Y}\phi\frac{\rho}{\rho+\eta c_Y}\widehat{G}_t^W + \tfrac{1}{2}(1-\rho)\widehat{C}_t^2 \\
& +\tfrac{1-c_Y}{2c_Y}\left(1-\rho_g\right)\left[n\left(\widehat{G}_t^H\right)^2 + (1-n)\left(\widehat{G}_t^F\right)^2\right] \\
& -\tfrac{1}{2c_Y}A_t - \tfrac{1}{2c_Y}\tfrac{1+\eta\sigma}{\sigma}\left[n\text{Var}_h\widehat{y}_t(h) + (1-n)\text{Var}_f\widehat{y}_t(f)\right] + \text{t.i.p.} + \mathcal{O}\left(\|\xi\|^3\right), \quad \text{(E.12)}
\end{aligned}$$

where

$$\begin{aligned}
A_t \equiv \; & -2\phi\left[c_Y\widehat{C}_t + (1-c_Y)\widehat{G}_t^W\right] \\
& +n(1-n)c_Y(1-c_Y)\widehat{T}_t^2 + 2n(1-n)c_Y(1-c_Y)\widehat{T}_t\widehat{G}_t^R \\
& +c_Y(1-c_Y)\left[n\left(\widehat{C}_t-\widehat{G}_t^H\right)^2 + (1-n)\left(\widehat{C}_t-\widehat{G}_t^F\right)^2\right] \\
& +(1+\eta)n(1-n)c_Y^2\widehat{T}_t^2 - 2(1+\eta)c_Y(1-c_Y)n(1-n)\widehat{T}_t\widehat{G}_t^R \\
& +(1+\eta)n\left[c_Y^2\widehat{C}_t^2 + 2c_Y(1-c_Y)\widehat{C}_t\widehat{G}_t^H + (1-c_Y)^2\left(\widehat{G}_t^H\right)^2\right] \\
& +(1+\eta)(1-n)\left[c_Y^2\widehat{C}_t^2 + 2c_Y(1-c_Y)\widehat{C}_t\widehat{G}_t^F + (1-c_Y)^2\left(\widehat{G}_t^F\right)^2\right] \\
& -2\eta\left[nS_t^H\widehat{Y}_t^H + (1-n)S_t^F\widehat{Y}_t^F\right].
\end{aligned}$$

Substitute this back into (E.12), to give:

$$\begin{aligned}
\frac{w_t}{U_C\overline{C}} = \; & \phi\widehat{C}_t + \phi\frac{\eta(1-c_Y)}{\rho+\eta c_Y}\widehat{G}_t^W \\
& +\tfrac{1}{2}(1-\rho)\widehat{C}_t^2 + n\tfrac{1-c_Y}{2c_Y}\left(1-\rho_g\right)\left(\widehat{G}_t^H\right)^2 - n\tfrac{1-c_Y}{2}\left(\widehat{C}_t-\widehat{G}_t^H\right)^2 \\
& +(1-n)\tfrac{1-c_Y}{2c_Y}\left(1-\rho_g\right)\left(\widehat{G}_t^F\right)^2 - (1-n)\tfrac{1-c_Y}{2}\left(\widehat{C}_t-\widehat{G}_t^F\right)^2 \\
& -n\tfrac{1+\eta}{2c_Y}\left[c_Y^2\widehat{C}_t^2 + (1-c_Y)^2\left(\widehat{G}_t^H\right)^2 + 2c_Y(1-c_Y)\widehat{C}_t\widehat{G}_t^H\right] \\
& -(1-n)\tfrac{1+\eta}{2c_Y}\left[c_Y^2\widehat{C}_t^2 + 2c_Y(1-c_Y)\widehat{C}_t\widehat{G}_t^F + (1-c_Y)^2\left(\widehat{G}_t^F\right)^2\right] \\
& -\tfrac{1}{2}n(1-n)(1-c_Y)\widehat{T}_t^2 - n(1-n)(1-c_Y)\widehat{T}_t\widehat{G}_t^R \\
& -\tfrac{1}{2}(1+\eta)n(1-n)c_Y\widehat{T}_t^2 + (1+\eta)(1-c_Y)n(1-n)\widehat{T}_t\widehat{G}_t^R \\
& +\tfrac{\eta}{c_Y}\left[nS_t^H\widehat{Y}_t^H + (1-n)S_t^F\widehat{Y}_t^F\right] \\
& -\tfrac{1}{2c_Y}\tfrac{1+\eta\sigma}{\sigma}\left[n\text{Var}_h\widehat{y}_t(h) + (1-n)\text{Var}_f\widehat{y}_t(f)\right] + \text{t.i.p.} + \mathcal{O}\left(\|\xi\|^3\right).
\end{aligned}$$

Hence,

$$\begin{aligned}
\frac{w_t}{U_C \overline{C}} &= \phi \widehat{C}_t + \phi \frac{\eta \left(1 - c_Y\right)}{\rho + \eta c_Y} \widehat{G}_t^W \\
&+ \tfrac{1}{2} \left[1 - \rho - c_Y \left(1 + \eta\right)\right] \widehat{C}_t^2 + n \tfrac{1 - c_Y}{2 c_Y} \left[1 - \rho_g - \left(1 - c_Y\right)\left(1 + \eta\right)\right] \left(\widehat{G}_t^H\right)^2 \\
&+ \left(1 - n\right) \tfrac{1 - c_Y}{2 c_Y} \left[1 - \rho_g - \left(1 - c_Y\right)\left(1 + \eta\right)\right] \left(\widehat{G}_t^F\right)^2 \\
&- \left(1 + \eta\right)\left(1 - c_Y\right) \widehat{C}_t \widehat{G}_t^W - n \tfrac{1 - c_Y}{2} \left(\widehat{C}_t - \widehat{G}_t^H\right)^2 - \left(1 - n\right) \tfrac{1 - c_Y}{2} \left(\widehat{C}_t - \widehat{G}_t^F\right)^2 \\
&- \tfrac{1}{2} n \left(1 - n\right)\left(1 + \eta c_Y\right) \widehat{T}_t^2 + \eta n \left(1 - n\right)\left(1 - c_Y\right) \widehat{T}_t \widehat{G}_t^R \\
&+ \tfrac{\eta}{c_Y} \left[n S_t^H \widehat{Y}_t^H + \left(1 - n\right) S_t^F \widehat{Y}_t^F\right] \\
&- \tfrac{1}{2 c_Y} \tfrac{1 + \eta \sigma}{\sigma} \left[n \mathrm{Var}_h \widehat{y}_t\left(h\right) + \left(1 - n\right) \mathrm{Var}_f \widehat{y}_t\left(f\right)\right] + \text{t.i.p.} + \mathcal{O}\left(\left\|\xi\right\|^3\right).
\end{aligned}$$

Hence,

$$\begin{aligned}
\frac{w_t}{U_C \overline{C}} &= \phi \widehat{C}_t + \phi \frac{\eta \left(1 - c_Y\right)}{\rho + \eta c_Y} \widehat{G}_t^W \\
&+ \tfrac{1}{2} \left[1 - \rho - c_Y \left(1 + \eta\right)\right] \widehat{C}_t^2 + n \tfrac{1 - c_Y}{2 c_Y} \left[1 - \rho_g - \left(1 - c_Y\right)\left(1 + \eta\right)\right] \left(\widehat{G}_t^H\right)^2 \\
&+ \left(1 - n\right) \tfrac{1 - c_Y}{2 c_Y} \left[1 - \rho_g - \left(1 - c_Y\right)\left(1 + \eta\right)\right] \left(\widehat{G}_t^F\right)^2 \\
&- \left(1 + \eta\right)\left(1 - c_Y\right) \widehat{C}_t \widehat{G}_t^W - n \tfrac{1 - c_Y}{2} \left(\widehat{C}_t - \widehat{G}_t^H\right)^2 - \left(1 - n\right) \tfrac{1 - c_Y}{2} \left(\widehat{C}_t - \widehat{G}_t^F\right)^2 \\
&- \tfrac{1}{2} n \left(1 - n\right)\left(1 + \eta c_Y\right) \widehat{T}_t^2 + \eta n \left(1 - n\right)\left(1 - c_Y\right) \widehat{T}_t \widehat{G}_t^R \\
&+ \eta S_t^W \widehat{C}_t - \eta \left(1 - n\right) n \widehat{T}_t S_t^R + \tfrac{\eta}{c_Y} \left(1 - c_Y\right) \left[n S_t^H \widehat{G}_t^H + \left(1 - n\right) S_t^F \widehat{G}_t^F\right] \\
&- \tfrac{1}{2 c_Y} \tfrac{1 + \eta \sigma}{\sigma} \left[n \mathrm{Var}_h \widehat{y}_t\left(h\right) + \left(1 - n\right) \mathrm{Var}_f \widehat{y}_t\left(f\right)\right] + \text{t.i.p.} + \mathcal{O}\left(\left\|\xi\right\|^3\right).
\end{aligned}$$

Hence,

$$\begin{aligned}
\frac{w_t}{U_C \overline{C}} &= \phi \widehat{C}_t + \phi \frac{\eta \left(1 - c_Y\right)}{\rho + \eta c_Y} \widehat{G}_t^W \\
&- \tfrac{1}{2} \left(\rho + \eta\right) \widehat{C}_t^2 + \tfrac{1}{2} \left(1 - c_Y\right)\left(1 + \eta\right) \widehat{C}_t^2 \\
&- n \tfrac{1 - c_Y}{2 c_Y} \left(\rho_g + \eta\right) \left(\widehat{G}_t^H\right)^2 + n \tfrac{1}{2} \left(1 - c_Y\right)\left(1 + \eta\right) \left(\widehat{G}_t^H\right)^2 \\
&- \left(1 - n\right) \tfrac{1 - c_Y}{2 c_Y} \left(\rho_g + \eta\right) \left(\widehat{G}_t^F\right)^2 + \left(1 - n\right) \tfrac{1}{2} \left(1 - c_Y\right)\left(1 + \eta\right) \left(\widehat{G}_t^F\right)^2 \\
&- \left(1 + \eta\right)\left(1 - c_Y\right) \widehat{C}_t \widehat{G}_t^W - n \tfrac{1 - c_Y}{2} \left(\widehat{C}_t - \widehat{G}_t^H\right)^2 - \left(1 - n\right) \tfrac{1 - c_Y}{2} \left(\widehat{C}_t - \widehat{G}_t^F\right)^2 \\
&- \tfrac{1}{2} n \left(1 - n\right)\left(1 + \eta c_Y\right) \widehat{T}_t^2 + \eta n \left(1 - n\right)\left(1 - c_Y\right) \widehat{T}_t \widehat{G}_t^R \\
&+ \eta S_t^W \widehat{C}_t - \eta \left(1 - n\right) n \widehat{T}_t S_t^R + \tfrac{\eta}{c_Y} \left(1 - c_Y\right) \left[n S_t^H \widehat{G}_t^H + \left(1 - n\right) S_t^F \widehat{G}_t^F\right] \\
&- \tfrac{1}{2 c_Y} \tfrac{1 + \eta \sigma}{\sigma} \left[n \mathrm{Var}_h \widehat{y}_t\left(h\right) + \left(1 - n\right) \mathrm{Var}_f \widehat{y}_t\left(f\right)\right] + \text{t.i.p.} + \mathcal{O}\left(\left\|\xi\right\|^3\right).
\end{aligned}$$

Hence,

$$\begin{aligned}
\frac{w_t}{U_C\overline{C}} &= \phi\widehat{C}_t + \phi\frac{\eta(1-c_Y)}{\rho+\eta c_Y}\widehat{G}_t^W \\
&-\tfrac{1}{2}(\rho+\eta)\widehat{C}_t^2 + n\tfrac{1}{2}(1-c_Y)(1+\eta)\left(\widehat{C}_t-\widehat{G}_t^H\right)^2 - n\tfrac{1-c_Y}{2}\left(\widehat{C}_t-\widehat{G}_t^H\right)^2 \\
&+(1-n)\tfrac{1}{2}(1-c_Y)(1+\eta)\left(\widehat{C}_t-\widehat{G}_t^F\right)^2 - (1-n)\tfrac{1-c_Y}{2}\left(\widehat{C}_t-\widehat{G}_t^F\right)^2 \\
&-n\tfrac{1-c_Y}{2c_Y}\left(\rho_g+\eta\right)\left(\widehat{G}_t^H\right)^2 - (1-n)\tfrac{1-c_Y}{2c_Y}\left(\rho_g+\eta\right)\left(\widehat{G}_t^F\right)^2 \\
&-\tfrac{1}{2}n(1-n)(1+\eta c_Y)\widehat{T}_t^2 + \eta n(1-n)(1-c_Y)\widehat{T}_t\widehat{G}_t^R \\
&+\eta S_t^W\widehat{C}_t - \eta(1-n)n\widehat{T}_tS_t^R + \tfrac{\eta}{c_Y}(1-c_Y)\left[nS_t^H\widehat{G}_t^H + (1-n)S_t^F\widehat{G}_t^F\right] \\
&-\tfrac{1}{2c_Y}\tfrac{1+\eta\sigma}{\sigma}\left[n\mathrm{Var}_h\widehat{y}_t(h) + (1-n)\mathrm{Var}_f\widehat{y}_t(f)\right] + \text{t.i.p.} + \mathcal{O}\left(\|\xi\|^3\right).
\end{aligned}$$

Hence,

$$\begin{aligned}
\frac{w_t}{U_C\overline{C}} &= \phi\widehat{C}_t + \phi\frac{\eta(1-c_Y)}{\rho+\eta c_Y}\widehat{G}_t^W \\
&-\tfrac{1}{2}(\rho+\eta)\widehat{C}_t^2 - \tfrac{1-c_Y}{2c_Y}\left(\rho_g+\eta\right)\left[n\left(\widehat{G}_t^H\right)^2 + (1-n)\left(\widehat{G}_t^F\right)^2\right] \\
&+\tfrac{1}{2}(1-c_Y)\eta\left[n\left(\widehat{C}_t-\widehat{G}_t^H\right)^2 + (1-n)\left(\widehat{C}_t-\widehat{G}_t^F\right)^2\right] \\
&-\tfrac{1}{2}n(1-n)(1+\eta c_Y)\widehat{T}_t^2 + \eta n(1-n)(1-c_Y)\widehat{T}_t\widehat{G}_t^R \\
&+\eta S_t^W\widehat{C}_t - \eta(1-n)n\widehat{T}_tS_t^R + \tfrac{\eta}{c_Y}(1-c_Y)\left[nS_t^H\widehat{G}_t^H + (1-n)S_t^F\widehat{G}_t^F\right] \\
&-\tfrac{1}{2c_Y}\tfrac{1+\eta\sigma}{\sigma}\left[n\mathrm{Var}_h\widehat{y}_t(h) + (1-n)\mathrm{Var}_f\widehat{y}_t(f)\right] + \text{t.i.p.} + \mathcal{O}\left(\|\xi\|^3\right).
\end{aligned}$$

Now, express everything in terms of gaps:

$$
\begin{aligned}
&\frac{w_t}{U_C\overline{C}} = \phi\widehat{C}_t + \phi\frac{\eta\left(1-c_Y\right)}{\rho+\eta c_Y}\widehat{G}_t^W \\
&-\tfrac{1}{2}\left(\rho+\eta\right)\left(\widehat{C}_t-\widetilde{C}_t\right)^2 - \tfrac{1-c_Y}{2c_Y}\left(\rho_g+\eta\right)\left[n\left(\widehat{G}_t^H-\widetilde{G}_t^H\right)^2+\left(1-n\right)\left(\widehat{G}_t^F-\widetilde{G}_t^F\right)^2\right] \\
&+\tfrac{1}{2}\left(1-c_Y\right)\eta\left[n\left(\widehat{C}_t-\widetilde{C}_t-\left(\widehat{G}_t^H-\widetilde{G}_t^H\right)\right)^2+\left(1-n\right)\left(\widehat{C}_t-\widetilde{C}_t-\left(\widehat{G}_t^F-\widetilde{G}_t^F\right)\right)^2\right] \\
&-\tfrac{1}{2}n\left(1-n\right)\left(1+\eta c_Y\right)\left(\widehat{T}_t-\widetilde{T}_t\right)^2+\eta n\left(1-n\right)\left(1-c_Y\right)\left(\widehat{T}_t-\widetilde{T}_t\right)\left(\widehat{G}_t^R-\widetilde{G}_t^R\right) \\
&-\left(\rho+\eta\right)\widehat{C}_t\widetilde{C}_t - n\tfrac{1-c_Y}{c_Y}\left(\rho_g+\eta\right)\widehat{G}_t^H\widetilde{G}_t^H-\left(1-n\right)\tfrac{1-c_Y}{c_Y}\left(\rho_g+\eta\right)\widehat{G}_t^F\widetilde{G}_t^F \\
&+n\left(1-c_Y\right)\eta\widehat{C}_t\widetilde{C}_t-n\left(1-c_Y\right)\eta\widehat{C}_t\widetilde{G}_t^H-n\left(1-c_Y\right)\eta\widetilde{C}_t\widehat{G}_t^H+n\left(1-c_Y\right)\eta\widetilde{G}_t^H\widehat{G}_t^H \\
&+\left(1-n\right)\left(1-c_Y\right)\eta\widehat{C}_t\widetilde{C}_t-\left(1-n\right)\left(1-c_Y\right)\eta\widehat{C}_t\widetilde{G}_t^F-\left(1-n\right)\left(1-c_Y\right)\eta\widetilde{C}_t\widehat{G}_t^F \\
&+\left(1-n\right)\left(1-c_Y\right)\eta\widetilde{G}_t^F\widehat{G}_t^F \\
&-n\left(1-n\right)\left(1+\eta c_Y\right)\widehat{T}_t\widetilde{T}_t+\eta n\left(1-n\right)\left(1-c_Y\right)\widetilde{T}_t\widehat{G}_t^R+\eta n\left(1-n\right)\left(1-c_Y\right)\widehat{T}_t\widetilde{G}_t^R \\
&+\eta S_t^W\widehat{C}_t-\eta\left(1-n\right)n\widehat{T}_tS_t^R+\tfrac{\eta}{c_Y}\left(1-c_Y\right)\left[nS_t^H\widehat{G}_t^H+\left(1-n\right)S_t^F\widehat{G}_t^F\right] \\
&-\tfrac{1}{2c_Y}\tfrac{1+\eta\sigma}{\sigma}\left[n\mathrm{Var}_h\widehat{y}_t\left(h\right)+\left(1-n\right)\mathrm{Var}_f\widehat{y}_t\left(f\right)\right]+\text{t.i.p.}+\mathcal{O}\left(\left\|\xi\right\|^3\right),
\end{aligned}
$$

where products of the exogenous natural levels of variables have been put into the "t.i.p.". Simplify the previous expression:

$$\begin{aligned}
&\frac{w_t}{U_C\overline{C}} \\
= \;& \phi\widehat{C}_t + \phi\frac{\eta\left(1-c_Y\right)}{\rho+\eta c_Y}\widehat{G}_t^W \\
& -\tfrac{1}{2}\left(\rho+\eta c_Y\right)\left(\widehat{C}_t-\widetilde{C}_t\right)^2 - n\tfrac{1-c_Y}{2c_Y}\left[\rho_g+\eta\left(1-c_Y\right)\right]\left(\widehat{G}_t^H-\widetilde{G}_t^H\right)^2 \\
& -\left(1-n\right)\tfrac{1-c_Y}{2c_Y}\left[\rho_g+\eta\left(1-c_Y\right)\right]\left(\widehat{G}_t^F-\widetilde{G}_t^F\right)^2 - n\left(1-c_Y\right)\eta\left(\widehat{C}_t-\widetilde{C}_t\right)\left(\widehat{G}_t^H-\widetilde{G}_t^H\right) \\
& -\left(1-n\right)\left(1-c_Y\right)\eta\left(\widehat{C}_t-\widetilde{C}_t\right)\left(\widehat{G}_t^F-\widetilde{G}_t^F\right) \\
& -\tfrac{1}{2}n\left(1-n\right)\left(1+\eta c_Y\right)\left(\widehat{T}_t-\widetilde{T}_t\right)^2 + \eta n\left(1-n\right)\left(1-c_Y\right)\left(\widehat{T}_t-\widetilde{T}_t\right)\left(\widehat{G}_t^R-\widetilde{G}_t^R\right) \\
& \\
& -\left(\rho+\eta c_Y\right)\widehat{C}_t\widetilde{C}_t - \left(1-c_Y\right)\eta\widehat{C}_t\widetilde{G}_t^W - \left(1-c_Y\right)\eta\widetilde{C}_t\widehat{G}_t^W \\
& -n\tfrac{1-c_Y}{c_Y}\left[\rho_g+\eta\left(1-c_Y\right)\right]\widehat{G}_t^H\widetilde{G}_t^H - \left(1-n\right)\tfrac{1-c_Y}{c_Y}\left[\rho_g+\eta\left(1-c_Y\right)\right]\widehat{G}_t^F\widetilde{G}_t^F \\
& \\
& -n\left(1-n\right)\left(1+\eta c_Y\right)\widehat{T}_t\widetilde{T}_t + \eta n\left(1-n\right)\left(1-c_Y\right)\widetilde{T}_t\widehat{G}_t^R + \eta n\left(1-n\right)\left(1-c_Y\right)\widehat{T}_t\widetilde{G}_t^R \\
& \\
& +\eta S_t^W\widehat{C}_t - \eta\left(1-n\right)n\widehat{T}_tS_t^R + \tfrac{\eta}{c_Y}\left(1-c_Y\right)\left[nS_t^H\widehat{G}_t^H + \left(1-n\right)S_t^F\widehat{G}_t^F\right] \\
& -\tfrac{1}{2c_Y}\tfrac{1+\eta\sigma}{\sigma}\left[n\mathrm{Var}_h\widehat{y}_t\left(h\right) + \left(1-n\right)\mathrm{Var}_f\widehat{y}_t\left(f\right)\right] + \text{t.i.p.} + \mathcal{O}\left(\|\xi\|^3\right),
\end{aligned}$$

We can rewrite the latest expression further as:

$$\begin{aligned}
&\tfrac{w_t}{U_C\overline{C}} \\
= \;& \phi\widehat{C}_t + \phi\left[\tfrac{\eta\left(1-c_Y\right)}{\rho+\eta c_Y}\right]\widehat{G}_t^W + \\
& -\tfrac{1}{2}\left(\rho+\eta c_Y\right)\left(\widehat{C}_t-\widetilde{C}_t\right)^2 - n\tfrac{1-c_Y}{2c_Y}\left[\rho_g+\eta\left(1-c_Y\right)\right]\left(\widehat{G}_t^H-\widetilde{G}_t^H\right)^2 \\
& -\left(1-n\right)\tfrac{1-c_Y}{2c_Y}\left[\rho_g+\eta\left(1-c_Y\right)\right]\left(\widehat{G}_t^F-\widetilde{G}_t^F\right)^2 - n\left(1-c_Y\right)\eta\left(\widehat{C}_t-\widetilde{C}_t\right)\left(\widehat{G}_t^H-\widetilde{G}_t^H\right) \\
& -\left(1-n\right)\left(1-c_Y\right)\eta\left(\widehat{C}_t-\widetilde{C}_t\right)\left(\widehat{G}_t^F-\widetilde{G}_t^F\right) \\
& -\tfrac{1}{2}n\left(1-n\right)\left(1+\eta c_Y\right)\left(\widehat{T}_t-\widetilde{T}_t\right)^2 + \eta n\left(1-n\right)\left(1-c_Y\right)\left(\widehat{T}_t-\widetilde{T}_t\right)\left(\widehat{G}_t^R-\widetilde{G}_t^R\right) \\
& +A_{CW,t}\widehat{C}_t + A_{GH,t}\widehat{G}_t^H + A_{GF,t}\widehat{G}_t^F + A_{T,t}\widehat{T}_t \\
& -\tfrac{1}{2c_Y}\tfrac{1+\eta\sigma}{\sigma}\left[n\mathrm{Var}_h\widehat{y}_t\left(h\right) + \left(1-n\right)\mathrm{Var}_f\widehat{y}_t\left(f\right)\right] + \text{t.i.p.} + \mathcal{O}\left(\|\xi\|^3\right),
\end{aligned}$$

where

$$
\begin{aligned}
A_{CW,t} &= -(\rho+\eta c_Y)\widetilde{C}_t-(1-c_Y)\eta\widetilde{G}_t^W+\eta S_t^W \\
&= \eta S_t^W-(\rho+\eta)\widetilde{C}_t+(1-c_Y)\eta\left(\widetilde{C}_t-\widetilde{G}_t^W\right), \\
A_{GH,t} &= -n(1-c_Y)\eta\widetilde{C}_t-n\tfrac{1-c_Y}{c_Y}\left[\rho_g+\eta(1-c_Y)\right]\widetilde{G}_t^H \\
&\quad -\eta n(1-n)(1-c_Y)\widetilde{T}_t+\tfrac{\eta}{c_Y}(1-c_Y)nS_t^H, \\
A_{GF,t} &= -(1-n)(1-c_Y)\eta\widetilde{C}_t-(1-n)\tfrac{1-c_Y}{c_Y}\left[\rho_g+\eta(1-c_Y)\right]\widetilde{G}_t^F \\
&\quad +\eta n(1-n)(1-c_Y)\widetilde{T}_t+\tfrac{\eta}{c_Y}(1-c_Y)(1-n)S_t^F, \\
A_{T,t} &= -n(1-n)(1+\eta c_Y)\widetilde{T}_t+\eta n(1-n)(1-c_Y)\widetilde{G}_t^R-\eta(1-n)nS_t^R.
\end{aligned}
$$

We shall now evaluate out these coefficients $A_{j,t}$. However, before doing so, we make use (C.18) and (C.19), so that

$$
\begin{aligned}
\widetilde{C}_t-\widetilde{G}_t^W &= \frac{\eta\rho_g}{\rho\left[\rho_g+\eta(1-c_Y)\right]+\eta c_Y\rho_g}S_t^W-\frac{\eta\rho}{\rho\left[\rho_g+\eta(1-c_Y)\right]+\eta c_Y\rho_g}S_t^W \\
&= \frac{\eta(\rho_g-\rho)}{\rho\left[\rho_g+\eta(1-c_Y)\right]+\eta c_Y\rho_g}S_t^W,
\end{aligned}
$$

and

$$
\begin{aligned}
&-\frac{1}{c_Y}(\rho_g+\eta)\widetilde{G}_t^W-\eta\left(\widetilde{C}_t-\widetilde{G}_t^W\right)+\frac{\eta}{c_Y}S_t^W \\
=&\ -\frac{1}{c_Y}(\rho_g+\eta)\frac{\eta\rho}{\rho\left[\rho_g+\eta(1-c_Y)\right]+\eta c_Y\rho_g}S_t^W \\
&-\eta\frac{\eta(\rho_g-\rho)}{\rho\left[\rho_g+\eta(1-c_Y)\right]+\eta c_Y\rho_g}S_t^W+\frac{\eta}{c_Y}S_t^W \\
=&\ -\left[\frac{1}{c_Y}\frac{\eta\rho(\rho_g+\eta)}{\rho\left[\rho_g+\eta(1-c_Y)\right]+\eta c_Y\rho_g}+\frac{\eta^2(\rho_g-\rho)}{\rho\left[\rho_g+\eta(1-c_Y)\right]+\eta c_Y\rho_g}-\frac{\eta}{c_Y}\right]S_t^W \\
=&\ -\frac{1}{c_Y}\frac{\eta}{\rho\left[\rho_g+\eta(1-c_Y)\right]+\eta c_Y\rho_g} \\
&\times\left[\rho(\rho_g+\eta)+c_Y\eta(\rho_g-\rho)-\rho\left[\rho_g+\eta(1-c_Y)\right]-\eta c_Y\rho_g\right]S_t^W \\
=&\ -\frac{1}{c_Y}\frac{\eta}{\rho\left[\rho_g+\eta(1-c_Y)\right]+\eta c_Y\rho_g}\left[\rho(\rho_g+\eta)-c_Y\eta\rho-\rho\left[\rho_g+\eta(1-c_Y)\right]\right]S_t^W \\
=&\ -\frac{1}{c_Y}\frac{\eta}{\rho\left[\rho_g+\eta(1-c_Y)\right]+\eta c_Y\rho_g}\left[\rho(\rho_g+\eta)-\rho\left[\rho_g+\eta\right]\right]S_t^W \\
=&\ 0.
\end{aligned}
$$

Hence,

$$
\begin{aligned}
& A_{CW,t} \\
= \; & \eta S_t^W - (\rho+\eta)\widetilde{C}_t + (1-c_Y)\eta\left(\widetilde{C}_t - \widetilde{G}_t^W\right) \\
= \; & \eta S_t^W - (\rho+\eta)\frac{\eta\rho_g}{\rho\left[\rho_g+\eta(1-c_Y)\right]+\eta c_Y\rho_g}S_t^W + (1-c_Y)\eta\frac{\eta\left(\rho_g-\rho\right)}{\rho\left[\rho_g+\eta(1-c_Y)\right]+\eta c_Y\rho_g}S_t^W \\
= \; & \eta\left[1-\frac{(\rho+\eta)\rho_g}{\rho\left[\rho_g+\eta(1-c_Y)\right]+\eta c_Y\rho_g}+\frac{(1-c_Y)\eta\left(\rho_g-\rho\right)}{\rho\left[\rho_g+\eta(1-c_Y)\right]+\eta c_Y\rho_g}\right]S_t^W \\
= \; & \frac{\eta}{\rho\left[\rho_g+\eta(1-c_Y)\right]+\eta c_Y\rho_g} \\
& \times\left[\rho\left[\rho_g+\eta(1-c_Y)\right]+\eta c_Y\rho_g-(\rho+\eta)\rho_g+(1-c_Y)\eta\left(\rho_g-\rho\right)\right]S_t^W \\
= \; & \frac{\eta}{\rho\left[\rho_g+\eta(1-c_Y)\right]+\eta c_Y\rho_g}\left[\rho\rho_g+\eta c_Y\rho_g-(\rho+\eta)\rho_g+(1-c_Y)\eta\rho_g\right]S_t^W \\
= \; & 0.
\end{aligned}
$$

and, noting that $\widetilde{G}_t^H = \widetilde{G}_t^W - (1-n)\widetilde{G}_t^R$, $S_t^H = S_t^W - (1-n)S_t^R$ and $\widetilde{T}_t = -\rho_g\widetilde{G}_t^R$:

$$
\begin{aligned}
\tfrac{1}{n}A_{GH,t} = \; & -(1-c_Y)\eta\widetilde{C}_t - \tfrac{1-c_Y}{c_Y}\left[\rho_g+\eta(1-c_Y)\right]\left[\widetilde{G}_t^W-(1-n)\widetilde{G}_t^R\right] \\
& +\eta\rho_g(1-n)(1-c_Y)\widetilde{G}_t^R + \tfrac{\eta}{c_Y}(1-c_Y)\left[S_t^W-(1-n)S_t^R\right] \\
= \; & -\frac{1-c_Y}{c_Y}\left(\rho_g+\eta\right)\widetilde{G}_t^W - (1-c_Y)\eta\left(\widetilde{C}_t-\widetilde{G}_t^W\right)+\frac{\eta(1-c_Y)}{c_Y}S_t^W \\
& +\tfrac{1-c_Y}{c_Y}\left[\rho_g+\eta(1-c_Y)\right](1-n)\widetilde{G}_t^R + \eta\rho_g(1-n)(1-c_Y)\widetilde{G}_t^R \\
& -\tfrac{\eta}{c_Y}(1-c_Y)(1-n)S_t^R \\
= \; & 0+\frac{(1-n)(1-c_Y)}{c_Y}\left\{\left[\left(\rho_g+\eta(1-c_Y)\right)+\eta\rho_g c_Y\right]\widetilde{G}_t^R-\eta S_t^R\right\} \\
= \; & 0+0=0,
\end{aligned}
$$

and, noting that $\widetilde{G}_t^F = \widetilde{G}_t^W + n\widetilde{G}_t^R$, $S_t^F = S_t^W + nS_t^R$ and $\widetilde{T}_t = -\rho_g\widetilde{G}_t^R$:

$$
\begin{aligned}
\tfrac{1}{1-n}A_{GF,t} = \; & -(1-c_Y)\eta\widetilde{C}_t - \tfrac{1-c_Y}{c_Y}\left[\rho_g+\eta(1-c_Y)\right]\left[\widetilde{G}_t^W+n\widetilde{G}_t^R\right] \\
& -\eta\rho_g n(1-c_Y)\widetilde{G}_t^R + \tfrac{\eta}{c_Y}(1-c_Y)\left(S_t^W+nS_t^R\right) \\
= \; & 0-\tfrac{n(1-c_Y)}{c_Y}\left[\left[\rho_g+\eta(1-c_Y)\right]\widetilde{G}_t^R+\eta\rho_g c_Y\widetilde{G}_t^R-\eta S_t^R\right] \\
= \; & 0+0=0.
\end{aligned}
$$

and, noting that $\widetilde{T}_t = -\rho_g \widetilde{G}_t^R$:

$$
\begin{aligned}
A_{T,t} &= -n(1-n)(1+\eta c_Y)\widetilde{T}_t + \eta n(1-n)(1-c_Y)\widetilde{G}_t^R - \eta(1-n)nS_t^R \\
&= -n(1-n)\left[(1+\eta c_Y)\widetilde{T}_t - \eta(1-c_Y)\widetilde{G}_t^R + \eta S_t^R\right] \\
&= n(1-n)\left[(1+\eta c_Y)\rho_g\widetilde{G}_t^R + \eta(1-c_Y)\widetilde{G}_t^R - \eta S_t^R\right] \\
&= 0+0.
\end{aligned}
$$

Hence, all the $A_{j,t}$ terms are zero, and we have in conclusion:

$$
\begin{aligned}
\frac{w_t}{U_C\overline{C}} &= \phi\widehat{C}_t + \phi\frac{\eta(1-c_Y)}{\rho+\eta c_Y}\widehat{G}_t^W \\
&-\tfrac{1}{2}(\rho+\eta c_Y)\left(\widehat{C}_t - \widetilde{C}_t\right)^2 - n\tfrac{1-c_Y}{2c_Y}\left[\rho_g + \eta(1-c_Y)\right]\left(\widehat{G}_t^H - \widetilde{G}_t^H\right)^2 \\
&-(1-n)\tfrac{1-c_Y}{2c_Y}\left[\rho_g + \eta(1-c_Y)\right]\left(\widehat{G}_t^F - \widetilde{G}_t^F\right)^2 \\
&-n(1-c_Y)\eta\left(\widehat{C}_t - \widetilde{C}_t\right)\left(\widehat{G}_t^H - \widetilde{G}_t^H\right) - (1-n)(1-c_Y)\eta\left(\widehat{C}_t - \widetilde{C}_t\right)\left(\widehat{G}_t^F - \widetilde{G}_t^F\right) \\
&-\tfrac{1}{2}n(1-n)(1+\eta c_Y)\left(\widehat{T}_t - \widetilde{T}_t\right)^2 + \eta n(1-n)(1-c_Y)\left(\widehat{T}_t - \widetilde{T}_t\right)\left(\widehat{G}_t^R - \widetilde{G}_t^R\right) \\
&-\tfrac{1}{2c_Y}\tfrac{1+\eta\sigma}{\sigma}\left[n\mathrm{Var}_h\widehat{y}_t(h) + (1-n)\mathrm{Var}_f\widehat{y}_t(f)\right] + \text{t.i.p.} + \mathcal{O}\left(\|\xi\|^3\right).
\end{aligned}
$$

Using (C.6) and (C.7), we can write:

$$
\begin{aligned}
&\frac{w_t}{U_C\overline{C}} = (\rho+\eta c_Y)c^*\widehat{C}_t + \eta(1-c_Y)c^*\widehat{G}_t^W \\
&-\tfrac{1}{2}(\rho+\eta c_Y)\left(\widehat{C}_t - \widetilde{C}_t\right)^2 - n\tfrac{1-c_Y}{2c_Y}\left[\rho_g + \eta(1-c_Y)\right]\left(\widehat{G}_t^H - \widetilde{G}_t^H\right)^2 \\
&-(1-n)\tfrac{1-c_Y}{2c_Y}\left[\rho_g + \eta(1-c_Y)\right]\left(\widehat{G}_t^F - \widetilde{G}_t^F\right)^2 - n(1-c_Y)\eta\left(\widehat{C}_t - \widetilde{C}_t\right)\left(\widehat{G}_t^H - \widetilde{G}_t^H\right) \\
&-(1-n)(1-c_Y)\eta\left(\widehat{C}_t - \widetilde{C}_t\right)\left(\widehat{G}_t^F - \widetilde{G}_t^F\right) \\
&-\tfrac{1}{2}n(1-n)(1+\eta c_Y)\left(\widehat{T}_t - \widetilde{T}_t\right)^2 + \eta n(1-n)(1-c_Y)\left(\widehat{T}_t - \widetilde{T}_t\right)\left(\widehat{G}_t^R - \widetilde{G}_t^R\right) \\
&-\tfrac{1}{2c_Y}\tfrac{1+\eta\sigma}{\sigma}\left[n\mathrm{Var}_h\widehat{y}_t(h) + (1-n)\mathrm{Var}_f\widehat{y}_t(f)\right] + \text{t.i.p.} + \mathcal{O}\left(\|\xi\|^3\right),
\end{aligned}
$$

and thus

$$\frac{w_t}{U_C \overline{C}} = -\tfrac{1}{2}\left(\rho + \eta c_Y\right)\left(\widehat{C}_t - \widetilde{C}_t - c^*\right)^2 - n\tfrac{1-c_Y}{2c_Y}\left[\rho_g + \eta\left(1-c_Y\right)\right]\left(\widehat{G}_t^H - \widetilde{G}_t^H\right)^2$$
$$-\left(1-n\right)\tfrac{1-c_Y}{2c_Y}\left[\rho_g + \eta\left(1-c_Y\right)\right]\left(\widehat{G}_t^F - \widetilde{G}_t^F\right)^2 - n\left(1-c_Y\right)\eta\left(\widehat{C}_t - \widetilde{C}_t - c^*\right)\left(\widehat{G}_t^H - \widetilde{G}_t^H\right)$$
$$-\left(1-n\right)\left(1-c_Y\right)\eta\left(\widehat{C}_t - \widetilde{C}_t - c^*\right)\left(\widehat{G}_t^F - \widetilde{G}_t^F\right)$$
$$-\tfrac{1}{2}n\left(1-n\right)\left(1+\eta c_Y\right)\left(\widehat{T}_t - \widetilde{T}_t\right)^2 + \eta n\left(1-n\right)\left(1-c_Y\right)\left(\widehat{T}_t - \widetilde{T}_t\right)\left(\widehat{G}_t^R - \widetilde{G}_t^R\right)$$
$$-\tfrac{1}{2c_Y}\tfrac{1+\eta\sigma}{\sigma}\left[n\mathrm{Var}_h\widehat{y}_t\left(h\right) + \left(1-n\right)\mathrm{Var}_f\widehat{y}_t\left(f\right)\right] + \text{t.i.p.} + \mathcal{O}\left(\|\xi\|^3\right),$$

where we have put terms involving $c^*\widetilde{C}_t$, $c^*\widetilde{G}_t^H$ and $c^*\widetilde{G}_t^F$ into "t.i.p.". Furthermore, we have written out $\eta\left(1-c_Y\right)c^*\widehat{G}_t^W = \eta\left(1-c_Y\right)c^*\left[n\widehat{G}_t^H + \left(1-n\right)\widehat{G}_t^F\right]$.

The final step is to derive $\mathrm{Var}_h\widehat{y}_t\left(h\right)$ and $\mathrm{Var}_f\widehat{y}_t\left(f\right)$. We have that

$$\begin{aligned}\mathrm{var}_h\left[\log y_t\left(h\right)\right] &= \left(\sigma_t^H\right)^2\mathrm{var}_h\left[\log p_t\left(h\right)\right] \\ &= \sigma^2\mathrm{var}_h\left[\log p_t\left(h\right)\right] + \mathcal{O}\left(\|\xi\|^3\right).\end{aligned}$$

We have

$$\begin{aligned}\mathrm{var}_h\left[\log p_t\left(h\right)\right] &= \mathrm{var}_h\left[\log p_t\left(h\right) - \bar{p}_{t-1}\right] = \mathrm{E}_h\left[\log p_t\left(h\right) - \bar{p}_{t-1}\right]^2 - \left(\Delta\bar{p}_t\right)^2 \\ &= \alpha^H\mathrm{E}_h\left[\log p_{t-1}\left(h\right) - \bar{p}_{t-1}\right]^2 + \left(1-\alpha^H\right)\left[\log\tilde{p}_t\left(h\right) - \bar{p}_{t-1}\right]^2 - \left(\Delta\bar{p}_t\right)^2 \\ &= \alpha^H\mathrm{var}_h\left[\log p_{t-1}\left(h\right)\right] + \left(1-\alpha^H\right)\left[\log\tilde{p}_t\left(h\right) - \bar{p}_{t-1}\right]^2 - \left(\Delta\bar{p}_t\right)^2,\end{aligned}$$

where

$$\bar{p}_t \equiv \mathrm{E}_h\left[\log p_t\left(h\right)\right].$$

Further,

$$\bar{p}_t - \bar{p}_{t-1} = \left(1-\alpha^H\right)\left[\log\tilde{p}_t\left(h\right) - \bar{p}_{t-1}\right].$$

Hence,

$$\mathrm{var}_h\left[\log p_t\left(h\right)\right] = \alpha^H\mathrm{var}_h\left[\log p_{t-1}\left(h\right)\right] + \tfrac{\alpha^H}{1-\alpha^H}\left(\Delta\bar{p}_t\right)^2.$$

Using

$$\bar{p}_t = \log P_{H,t} + \mathcal{O}\left(\|\xi\|^2\right),$$

we have:

$$\mathrm{var}_h\left[\log p_t\left(h\right)\right] = \alpha^H\mathrm{var}_h\left[\log p_{t-1}\left(h\right)\right] + \tfrac{\alpha^H}{1-\alpha^H}\left(\pi_t^H\right)^2 + \mathcal{O}\left(\|\xi\|^3\right).$$

Hence,

$$\begin{aligned}
\mathrm{var}_h\left[\log p_t\left(h\right)\right] &= \left(\alpha^H\right)^{t+1}\mathrm{var}_h\left[\log p_{-1}\left(h\right)\right] + \sum_{s=0}^{t}\left(\alpha^H\right)^{t-s}\tfrac{\alpha^H}{1-\alpha^H}\left(\pi_s^H\right)^2 + \mathcal{O}\left(\|\xi\|^3\right) \\
&= \sum_{s=0}^{t}\left(\alpha^H\right)^{t-s}\tfrac{\alpha^H}{1-\alpha^H}\left(\pi_s^H\right)^2 + \text{t.i.p.} + \mathcal{O}\left(\|\xi\|^3\right),
\end{aligned}$$

and thus

$$\sum_{t=0}^{\infty}\beta^t\mathrm{var}_h\left[\log p_t\left(h\right)\right] = d^H\sum_{t=0}^{\infty}\beta^t\left(\pi_t^H\right)^2 + \text{t.i.p.} + \mathcal{O}\left(\|\xi\|^3\right),$$

where

$$d^H \equiv \frac{\alpha^H}{\left(1-\alpha^H\beta\right)\left(1-\alpha^H\right)}.$$

Similarly, we derive for Foreign:

$$\sum_{t=0}^{\infty}\beta^t\mathrm{var}_f\left[\log p_t\left(f\right)\right] = d^F\sum_{t=0}^{\infty}\beta^t\left(\pi_t^F\right)^2 + \text{t.i.p.} + \mathcal{O}\left(\|\xi\|^3\right),$$

where

$$d^F \equiv \frac{\alpha^F}{\left(1-\alpha^F\beta\right)\left(1-\alpha^F\right)}.$$

Hence, ignoring terms independent of policy as well as terms of order $\mathcal{O}\left(\|\xi\|^3\right)$ or higher, the second-order welfare approximation is given by:

$$\sum_{t=0}^{\infty}\beta^t\mathrm{E}_0\left[w_t^C\right],$$

where

$$\begin{aligned}
&\frac{w_t^C}{U_C\overline{C}} = -\tfrac{1}{2}\left(\rho+\eta c_Y\right)\left(\widehat{C}_t-\widetilde{C}_t-c^*\right)^2 - n\tfrac{1-c_Y}{2c_Y}\left[\rho_g+\eta\left(1-c_Y\right)\right]\left(\widehat{G}_t^H-\widetilde{G}_t^H\right)^2 \\
&-\left(1-n\right)\tfrac{1-c_Y}{2c_Y}\left[\rho_g+\eta\left(1-c_Y\right)\right]\left(\widehat{G}_t^F-\widetilde{G}_t^F\right)^2 - \left(1-c_Y\right)\eta\left(\widehat{C}_t-\widetilde{C}_t-c^*\right)\left(\widehat{G}_t^W-\widetilde{G}_t^W\right) \\
&-\tfrac{1}{2}n\left(1-n\right)\left(1+\eta c_Y\right)\left(\widehat{T}_t-\widetilde{T}_t\right)^2 + \eta n\left(1-n\right)\left(1-c_Y\right)\left(\widehat{T}_t-\widetilde{T}_t\right)\left(\widehat{G}_t^R-\widetilde{G}_t^R\right) \\
&-\tfrac{1}{2c_Y}\tfrac{1+\eta\sigma}{\sigma}\left[n\sigma^2 d^H\left(\pi_t^H\right)^2 + \left(1-n\right)\sigma^2 d^F\left(\pi_t^F\right)^2\right].
\end{aligned}$$

Hence,

$$w_t^C = \frac{1}{2} U_C \overline{C} \left(1+\eta\sigma\right) \sigma / c_Y *$$
$$\left\{ \begin{array}{c} -\frac{c_Y(\rho+\eta c_Y)}{(1+\eta\sigma)\sigma} \left(\widehat{C}_t - \widetilde{C}_t - c^*\right)^2 - \frac{n(1-c_Y)\left[\rho_g+\eta(1-c_Y)\right]}{(1+\eta\sigma)\sigma} \left(\widehat{G}_t^H - \widetilde{G}_t^H\right)^2 \\ -\frac{(1-n)(1-c_Y)\left[\rho_g+\eta(1-c_Y)\right]}{(1+\eta\sigma)\sigma} \left(\widehat{G}_t^F - \widetilde{G}_t^F\right)^2 - \frac{2c_Y(1-c_Y)\eta}{(1+\eta\sigma)\sigma} \left(\widehat{C}_t - \widetilde{C}_t - c^*\right) \left(\widehat{G}_t^W - \widetilde{G}_t^W\right) \\ -\frac{n(1-n)c_Y(1+\eta c_Y)}{(1+\eta\sigma)\sigma} \left(\widehat{T}_t - \widetilde{T}_t\right)^2 + \frac{2\eta n(1-n)c_Y(1-c_Y)}{(1+\eta\sigma)\sigma} \left(\widehat{T}_t - \widetilde{T}_t\right) \left(\widehat{G}_t^R - \widetilde{G}_t^R\right) \\ -\left[n d^H \left(\pi_t^H\right)^2 + (1-n)\, d^F \left(\pi_t^F\right)^2\right] \end{array} \right\}$$

Observing that $\left[n d^H + (1-n)\, d^F\right] = \left[n/\kappa^H + (1-n)/\kappa^F\right] / (1+\eta\sigma)$, we can write

$$w_t^C = \frac{1}{2} U_C \overline{C} \left[n/\kappa^H + (1-n)/\kappa^F\right] \sigma / c_Y *$$
$$\left\{ \begin{array}{c} -\frac{c_Y(\rho+\eta c_Y)}{[n/\kappa^H+(1-n)/\kappa^F]\sigma} \left(\widehat{C}_t - \widetilde{C}_t - c^*\right)^2 - \frac{n(1-c_Y)\left[\rho_g+\eta(1-c_Y)\right]}{[n/\kappa^H+(1-n)/\kappa^F]\sigma} \left(\widehat{G}_t^H - \widetilde{G}_t^H\right)^2 \\ -\frac{(1-n)(1-c_Y)\left[\rho_g+\eta(1-c_Y)\right]}{[n/\kappa^H+(1-n)/\kappa^F]\sigma} \left(\widehat{G}_t^F - \widetilde{G}_t^F\right)^2 - \frac{2c_Y(1-c_Y)\eta}{[n/\kappa^H+(1-n)/\kappa^F]\sigma} \left(\widehat{C}_t - \widetilde{C}_t - c^*\right) \left(\widehat{G}_t^W - \widetilde{G}_t^W\right) \\ -\frac{n(1-n)c_Y(1+\eta c_Y)}{[n/\kappa^H+(1-n)/\kappa^F]\sigma} \left(\widehat{T}_t - \widetilde{T}_t\right)^2 + \frac{2\eta n(1-n)c_Y(1-c_Y)}{[n/\kappa^H+(1-n)/\kappa^F]\sigma} \left(\widehat{T}_t - \widetilde{T}_t\right) \left(\widehat{G}_t^R - \widetilde{G}_t^R\right) \\ -\left[\lambda_{\pi^H} \left(\pi_t^H\right)^2 + \lambda_{\pi^F} \left(\pi_t^F\right)^2\right] \end{array} \right\},$$

where

$$\lambda_{\pi^H} \equiv \frac{n d^H}{n d^H + (1-n)\, d^F} = \frac{n/\kappa^H}{n/\kappa^H + (1-n)/\kappa^F}, \quad \lambda_{\pi^F} = 1 - \lambda_{\pi^H}.$$

Ignoring an irrelevant proportionality factor, the associated *loss* function is given by

$$L = \sum_{t=0}^{\infty} \beta^t \mathrm{E}_0 \left[L_t\right], \tag{E.13}$$

where

$$L_t = \left\{ \begin{array}{c} \lambda_C \left(\widehat{C}_t - \widetilde{C}_t - c^*\right)^2 + n\lambda_G \left(\widehat{G}_t^H - \widetilde{G}_t^H\right)^2 \\ +(1-n)\,\lambda_G \left(\widehat{G}_t^F - \widetilde{G}_t^F\right)^2 + 2\lambda_{CG} \left(\widehat{C}_t - \widetilde{C}_t - c^*\right) \left(\widehat{G}_t^W - \widetilde{G}_t^W\right) \\ +\lambda_T \left(\widehat{T}_t - \widetilde{T}_t\right)^2 - 2\lambda_{TG} \left(\widehat{T}_t - \widetilde{T}_t\right) \left(\widehat{G}_t^R - \widetilde{G}_t^R\right) \\ +\lambda_{\pi^H} \left(\pi_t^H\right)^2 + \lambda_{\pi^F} \left(\pi_t^F\right)^2 \end{array} \right\}, \tag{E.14}$$

where

$$\begin{aligned}
\lambda_C &\equiv \frac{c_Y(\rho+\eta c_Y)/\sigma}{n/\kappa^H+(1-n)/\kappa^F}, \quad \lambda_G \equiv \frac{(1-c_Y)\left[\rho_g+\eta(1-c_Y)\right]/\sigma}{n/\kappa^H+(1-n)/\kappa^F}, \\
\lambda_{CG} &\equiv \frac{c_Y(1-c_Y)\eta/\sigma}{n/\kappa^H+(1-n)/\kappa^F}, \\
\lambda_T &\equiv \frac{n(1-n)c_Y(1+\eta c_Y)/\sigma}{n/\kappa^H+(1-n)/\kappa^F}, \quad \lambda_{TG} \equiv \frac{n(1-n)\eta c_Y(1-c_Y)/\sigma}{n/\kappa^H+(1-n)/\kappa^F}.
\end{aligned}$$

An alternative representation, which expresses the loss function exclusively in terms of underlying parameters, follows by multiplying the above weights by $\left[n/\kappa^H+(1-n)/\kappa^F\right]\sigma$:

$$L_t/\Lambda = \left\{ \begin{array}{c}
c_Y(\rho+\eta c_Y)\left(\widehat{C}_t-\widetilde{C}_t-c^*\right)^2 \\
+n(1-c_Y)\left[\rho_g+\eta(1-c_Y)\right]\left(\widehat{G}_t^H-\widetilde{G}_t^H\right)^2 \\
+(1-n)(1-c_Y)\left[\rho_g+\eta(1-c_Y)\right]\left(\widehat{G}_t^F-\widetilde{G}_t^F\right)^2 \\
+n(1-n)c_Y(1+\eta c_Y)\left(\widehat{T}_t-\widetilde{T}_t\right)^2 \\
+2c_Y(1-c_Y)\eta\left(\widehat{C}_t-\widetilde{C}_t-c^*\right)\left(\widehat{G}_t^W-\widetilde{G}_t^W\right) \\
-2n(1-n)c_Y(1-c_Y)\eta\left(\widehat{T}_t-\widetilde{T}_t\right)\left(\widehat{G}_t^R-\widetilde{G}_t^R\right) \\
+\frac{n\sigma}{\kappa^H}\left(\pi_t^H\right)^2+\frac{(1-n)\sigma}{\kappa^F}\left(\pi_t^F\right)^2
\end{array} \right\},$$

which is expression (8) in the paper, where $\Lambda \equiv \left[n/\kappa^H+(1-n)/\kappa^F\right]\sigma$.

F. Derivation of (9)-(12)

Take a weighted average with weights n and $1-n$ of (D.6) and (D.7):

$$\begin{aligned}
\pi_t^W &= \beta \mathrm{E}_t\pi_{t+1}^W+(1+\eta c_Y)n(1-n)\left(\kappa^H-\kappa^F\right)\left(\widehat{T}_t-\widetilde{T}_t\right) \\
&\quad +(\rho+\eta c_Y)\left[n\kappa^H+(1-n)\kappa^F\right]\left(\widehat{C}_t-\widetilde{C}_t\right)+n\kappa^H\eta(1-c_Y)\left(\widehat{G}_t^H-\widetilde{G}_t^H\right) \\
&\quad +(1-n)\eta(1-c_Y)\left(\widehat{G}_t^F-\widetilde{G}_t^F\right)+u_t^W.
\end{aligned}$$

With equal rigidities we can write:

$$\pi_t^W=\beta E_t\pi_{t+1}^W+\kappa\left[(\rho+\eta c_Y)\left(\widehat{C}_t-\widetilde{C}_t\right)+\eta(1-c_Y)\left(\widehat{G}_t^W-\widetilde{G}_t^W\right)\right]+u_t^W.$$

Further, writing out L_t in (E.14) yields:

$$L_t=-2\left[\lambda_C\left(\widehat{C}_t-\widetilde{C}_t\right)+\lambda_{CG}\left(\widehat{G}_t^W-\widetilde{G}_t^W\right)\right]c^*+\text{ t.i.p.}+L_t^S,$$

where "t.i.p." is a term independent of policy, namely $\lambda_C (c^*)^2$ and where

$$\begin{aligned} L_t^S &= \lambda_C \left(\widehat{C}_t - \widetilde{C}_t\right)^2 + \lambda_T \left(\widehat{T}_t - \widetilde{T}_t\right)^2 + n\lambda_G \left(\widehat{G}_t^H - \widetilde{G}_t^H\right)^2 \\ &\quad + (1-n)\lambda_G \left(\widehat{G}_t^F - \widetilde{G}_t^F\right)^2 + \lambda_{\pi^H} \left(\pi_t^H\right)^2 + \lambda_{\pi^F} \left(\pi_t^F\right)^2 \\ &\quad + 2\lambda_{CG} \left(\widehat{C}_t - \widetilde{C}_t\right)\left(\widehat{G}_t^W - \widetilde{G}_t^W\right) - 2\lambda_{TG} \left(\widehat{T}_t - \widetilde{T}_t\right)\left(\widehat{G}_t^R - \widetilde{G}_t^R\right). \end{aligned} \tag{F.1}$$

Define $\Omega \equiv 2\frac{c_Y/\sigma}{n/\kappa^H + (1-n)/\kappa^F}$. We observe that $2\lambda_C = (\rho + \eta c_Y)\,\Omega$, $2\lambda_{CG} = \eta (1 - c_Y)\,\Omega$ and $2\lambda_G = \frac{1-c_Y}{c_Y}\left[\rho_g + \eta (1 - c_Y)\right]\Omega$, so that we write:

$$L_t = -(\Omega/\kappa)\left(\pi_t^W - \beta \mathrm{E}_t \pi_{t+1}^W - u_t^W\right) c^* + \text{ t.i.p.} + L_t^S.$$

Hence,

$$\sum_{t=0}^{\infty} \beta^t \mathrm{E}_0 [L_t] = -(\Omega/\kappa)\, c^* \sum_{t=0}^{\infty} \beta^t \mathrm{E}_0 \left(\pi_t^W - \beta \mathrm{E}_t \pi_{t+1}^W - u_t^W\right) + \text{ t.i.p.} + \sum_{t=0}^{\infty} \beta^t \mathrm{E}_0 \left[L_t^S\right].$$

Using that $\sum_{t=0}^{\infty} \beta^t \mathrm{E}_0 \left(\pi_t^W - \beta \pi_{t+1}^W - u_t^W\right) = \left(\pi_0^W - \beta \mathrm{E}_0 \pi_1^W - u_0^W\right) + \beta \left(\mathrm{E}_0 \pi_1^W - \beta \mathrm{E}_0 \pi_2^W\right) + \ldots = \pi_0^W - \sum_{t=0}^{\infty} \beta^t \mathrm{E}_0 u_t^W$, we can write this last expression as:

$$\sum_{t=0}^{\infty} \beta^t \mathrm{E}_0 [L_t] = -(\Omega/\kappa)\, c^* \left(\pi_0^W - \sum_{t=0}^{\infty} \beta^t \mathrm{E}_0 u_t^W\right) + \text{ t.i.p.} + \sum_{t=0}^{\infty} \beta^t \mathrm{E}_0 \left[L_t^S\right],$$

which is equation (9) in the paper.

We note that for any generic variable X, the following holds:

$$n\left(X^H\right)^2 + (1-n)\left(X^F\right)^2 = \left(X^W\right)^2 + n(1-n)\left(X^R\right)^2. \tag{F.2}$$

Using this, we can rewrite L_t^S as:

$$L_t^S = L_t^W + n(1-n)\, L_t^R, \tag{F.3}$$

which is equation (10) in the paper, where

$$L_t^W = \lambda_C^W \left(\widehat{C}_t - \widetilde{C}_t\right)^2 + \lambda_G^W \left(\widehat{G}_t^W - \widetilde{G}_t^W\right)^2 + \left(\pi_t^W\right)^2 + 2\lambda_{CG}^W \left(\widehat{C}_t - \widetilde{C}_t\right)\left(\widehat{G}_t^W - \widetilde{G}_t^W\right), \tag{F.4}$$

which is equation (11) in the paper, and

$$L_t^R = \lambda_T^R \left(\widehat{T}_t - \widetilde{T}_t\right)^2 + \left(\pi_t^R\right)^2 + \lambda_G^R \left(\widehat{G}_t^R - \widetilde{G}_t^R\right)^2 - 2\lambda_{TG}^R \left(\widehat{T}_t - \widetilde{T}_t\right)\left(\widehat{G}_t^R - \widetilde{G}_t^R\right), \tag{F.5}$$

which is equation (12) in the paper, where

$$\lambda_C^W \equiv \frac{\kappa_C c_Y}{\sigma}, \quad \lambda_G^W \equiv \frac{\kappa_G\left[\rho_g + \eta\left(1-c_Y\right)\right]}{\eta\sigma}, \quad \lambda_{CG}^W \equiv \frac{\kappa_G c_Y}{\sigma},$$
$$\lambda_T^R \equiv \frac{\kappa_T c_Y}{\sigma}, \quad \lambda_G^R \equiv \frac{\kappa_G\left[\rho_g + \eta\left(1-c_Y\right)\right]}{\eta\sigma}, \quad \lambda_{TG}^R \equiv \frac{\kappa_G c_Y}{\sigma}.$$

G. Optimal commitment policies with equal rigidities

With equal rigidities, (D.6) and (D.7) become, respectively:

$$\pi_t^H = \beta \mathrm{E}_t \pi_{t+1}^H + (1-n)\kappa_T\left(\widehat{T}_t - \widetilde{T}_t\right) + \kappa_C\left(\widehat{C}_t - \widetilde{C}_t\right) + \kappa_G\left(\widehat{G}_t^H - \widetilde{G}_t^H\right) + u_t^H \tag{G.1}$$
$$\pi_t^F = \beta \mathrm{E}_t \pi_{t+1}^F - n\kappa_T\left(\widehat{T}_t - \widetilde{T}_t\right) + \kappa_C\left(\widehat{C}_t - \widetilde{C}_t\right) + \kappa_G\left(\widehat{G}_t^F - \widetilde{G}_t^F\right) + u_t^F. \tag{G.2}$$

To solve for the optimal policies under commitment we set up the relevant Lagrangian (see, e.g., Woodford, 1999):

$$\begin{aligned}\mathcal{L} = \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \Big\{ & L_t^S \\
& +2\phi_{1,t}\left[\pi_t^H - \beta\pi_{t+1}^H - \kappa_T(1-n)\left(\widehat{T}_t - \widetilde{T}_t\right) - \kappa_C\left(\widehat{C}_t - \widetilde{C}_t\right) - \kappa_G\left(\widehat{G}_t^H - \widetilde{G}_t^H\right) - u_t^H\right] \\
& +2\phi_{2,t}\left[\pi_t^F - \beta\pi_{t+1}^F + \kappa_T n\left(\widehat{T}_t - \widetilde{T}_t\right) - \kappa_C\left(\widehat{C}_t - \widetilde{C}_t\right) - \kappa_G\left(\widehat{G}_t^F - \widetilde{G}_t^F\right) - u_t^F\right] \\
& +2\phi_{3,t}\left[\left(\widehat{T}_t - \widetilde{T}_t\right) - \left(\widehat{T}_{t-1} - \widetilde{T}_{t-1}\right) - \pi_t^F + \pi_t^H + \left(\widetilde{T}_t - \widetilde{T}_{t-1}\right)\right]\Big\},\end{aligned}$$

where $2\phi_{1,t}$, $2\phi_{2,t}$, and $2\phi_{3,t}$ are the multipliers on (G.1), (G.2), and (D.8), respectively, and L_t^S is given by (F.1). Optimizing over $\widehat{C}_t - \widetilde{C}_t$, $\widehat{T}_t - \widetilde{T}_t$, π_t^H, π_t^F, $\widehat{G}_t^H - \widetilde{G}_t^H$, and $\widehat{G}_t^F - \widetilde{G}_t^F$

yields the following six necessary first-order conditions for $t \geq 1$,

$$\lambda_C \left(\widehat{C}_t - \widetilde{C}_t\right) + \lambda_{CG} \left(\widehat{G}_t^W - \widetilde{G}_t^W\right) - \phi_{1,t}\kappa_C - \phi_{2,t}\kappa_C = 0, \tag{G.3}$$

$$\lambda_T \left(\widehat{T}_t - \widetilde{T}_t\right) - \lambda_{TG} \left(\widehat{G}_t^R - \widetilde{G}_t^R\right) - \phi_{1,t}\kappa_T (1-n) + \phi_{2,t}\kappa_T n + \phi_{3,t} - \beta\phi_{3,t+1} = 0, \tag{G.4}$$

$$\lambda_{\pi^H}\pi_t^H + \phi_{1,t} - \phi_{1,t-1} + \phi_{3,t} = 0, \tag{G.5}$$

$$\lambda_{\pi^F}\pi_t^F + \phi_{2,t} - \phi_{2,t-1} - \phi_{3,t} = 0, \tag{G.6}$$

$$n\lambda_G \left(\widehat{G}_t^H - \widetilde{G}_t^H\right) + n\lambda_{CG} \left(\widehat{C}_t - \widetilde{C}_t\right) + \lambda_{TG} \left(\widehat{T}_t - \widetilde{T}_t\right) - \phi_{1,t}\kappa_G = 0, \tag{G.7}$$

$$(1-n)\lambda_G \left(\widehat{G}_t^F - \widetilde{G}_t^F\right) + (1-n)\lambda_{CG} \left(\widehat{C}_t - \widetilde{C}_t\right) - \lambda_{TG} \left(\widehat{T}_t - \widetilde{T}_t\right) - \phi_{2,t}\kappa_G = 0. \tag{G.8}$$

Use the values of the loss function parameters to get

$$(\kappa_C c_Y/\sigma) \left(\widehat{C}_t - \widetilde{C}_t\right) + (\kappa_G c_Y/\sigma) \left(\widehat{G}_t^W - \widetilde{G}_t^W\right) - \left(\phi_{1,t} + \phi_{2,t}\right)\kappa_C = 0,$$

$$\begin{aligned} &(\kappa_T n (1-n) c_Y/\sigma) \left(\widehat{T}_t - \widetilde{T}_t\right) - (\kappa_G n (1-n) c_Y/\sigma) \left(\widehat{G}_t^R - \widetilde{G}_t^R\right) \\ &-\phi_{1,t}\kappa_T (1-n) + \phi_{2,t}\kappa_T n + \phi_{3,t} - \beta\phi_{3,t+1} = 0, \end{aligned}$$

$$n\pi_t^H + \phi_{1,t} - \phi_{1,t-1} + \phi_{3,t} = 0,$$

$$(1-n)\pi_t^F + \phi_{2,t} - \phi_{2,t-1} - \phi_{3,t} = 0,$$

$$\begin{aligned} &n \left(\kappa_G \left[\rho_g/\eta + (1-c_Y)\right]/\sigma\right) \left(\widehat{G}_t^H - \widetilde{G}_t^H\right) + n (\kappa_G c_Y/\sigma) \left(\widehat{C}_t - \widetilde{C}_t\right) \\ &+ (\kappa_G n (1-n) c_Y/\sigma) \left(\widehat{T}_t - \widetilde{T}_t\right) - \phi_{1,t}\kappa_G = 0, \end{aligned}$$

$$\begin{aligned} &(1-n) \left(\kappa_G \left[\rho_g/\eta + (1-c_Y)\right]/\sigma\right) \left(\widehat{G}_t^F - \widetilde{G}_t^F\right) + (1-n) (\kappa_G c_Y/\sigma) \left(\widehat{C}_t - \widetilde{C}_t\right) \\ &- (\kappa_G n (1-n) c_Y/\sigma) \left(\widehat{T}_t - \widetilde{T}_t\right) - \phi_{2,t}\kappa_G = 0. \end{aligned}$$

Hence,

$$\left(c_Y/\sigma\right)\left(\widehat{C}_t-\widetilde{C}_t\right)+\left(\frac{\kappa_G}{\kappa_C}c_Y/\sigma\right)\left(\widehat{G}_t^W-\widetilde{G}_t^W\right)-\left(\phi_{1,t}+\phi_{2,t}\right)=0, \tag{G.9}$$

$$\begin{aligned}&\left(\kappa_T n\left(1-n\right)c_Y/\sigma\right)\left(\widehat{T}_t-\widetilde{T}_t\right)-\left(\kappa_G n\left(1-n\right)c_Y/\sigma\right)\left(\widehat{G}_t^R-\widetilde{G}_t^R\right)\\&-\phi_{1,t}\kappa_T\left(1-n\right)+\phi_{2,t}\kappa_T n+\phi_{3,t}-\beta\phi_{3,t+1}=0,\end{aligned} \tag{G.10}$$

$$n\pi_t^H+\phi_{1,t}-\phi_{1,t-1}+\phi_{3,t}=0, \tag{G.11}$$

$$\left(1-n\right)\pi_t^F+\phi_{2,t}-\phi_{2,t-1}-\phi_{3,t}=0, \tag{G.12}$$

$$\begin{aligned}&n\left(\left[\rho_g/\eta+\left(1-c_Y\right)\right]/\sigma\right)\left(\widehat{G}_t^H-\widetilde{G}_t^H\right)+n\left(c_Y/\sigma\right)\left(\widehat{C}_t-\widetilde{C}_t\right)\\&+\left(n\left(1-n\right)c_Y/\sigma\right)\left(\widehat{T}_t-\widetilde{T}_t\right)-\phi_{1,t}=0,\end{aligned} \tag{G.13}$$

$$\begin{aligned}&\left(1-n\right)\left(\left[\rho_g/\eta+\left(1-c_Y\right)\right]/\sigma\right)\left(\widehat{G}_t^F-\widetilde{G}_t^F\right)+\left(1-n\right)\left(c_Y/\sigma\right)\left(\widehat{C}_t-\widetilde{C}_t\right)\\&-\left(n\left(1-n\right)c_Y/\sigma\right)\left(\widehat{T}_t-\widetilde{T}_t\right)-\phi_{2,t}=0.\end{aligned} \tag{G.14}$$

Adding the last two conditions gives

$$\left(\left[\rho_g/\eta+\left(1-c_Y\right)\right]/\sigma\right)\left(\widehat{G}_t^W-\widetilde{G}_t^W\right)+\left(c_Y/\sigma\right)\left(\widehat{C}_t-\widetilde{C}_t\right)=\phi_{1,t}+\phi_{2,t}.$$

Combine this with the first equation:

$$\left(c_Y/\sigma\right)\left(\widehat{C}_t-\widetilde{C}_t\right)+\left(\frac{\kappa_G}{\kappa_C}c_Y/\sigma\right)\left(\widehat{G}_t^W-\widetilde{G}_t^W\right)=\phi_{1,t}+\phi_{2,t},$$

to get

$$\begin{aligned}&\left(\left[\rho_g/\eta+\left(1-c_Y\right)\right]/\sigma\right)\left(\widehat{G}_t^W-\widetilde{G}_t^W\right)+\left(c_Y/\sigma\right)\left(\widehat{C}_t-\widetilde{C}_t\right)\\=\;&\left(c_Y/\sigma\right)\left(\widehat{C}_t-\widetilde{C}_t\right)+\left(\frac{\kappa_G}{\kappa_C}c_Y/\sigma\right)\left(\widehat{G}_t^W-\widetilde{G}_t^W\right)\Rightarrow\end{aligned}$$

$$\left(\left[\rho_g/\eta+\left(1-c_Y\right)\right]\right)\left(\widehat{G}_t^W-\widetilde{G}_t^W\right)=\left(\frac{\kappa_G}{\kappa_C}c_Y\right)\left(\widehat{G}_t^W-\widetilde{G}_t^W\right),$$

from which it follows that

$$\widehat{G}_t^W-\widetilde{G}_t^W=0,$$

unless

$$\left(\left[\rho_g/\eta+(1-c_Y)\right]\right)=\frac{\kappa_G}{\kappa_C}c_Y \Leftrightarrow$$

$$\rho_g/\eta+(1-c_Y)=\frac{\eta\,(1-c_Y)}{(\rho+\eta c_Y)}c_Y \Leftrightarrow$$

$$\rho_g\,(\rho+\eta c_Y)/\eta+(1-c_Y)\,(\rho+\eta c_Y)=\eta\,(1-c_Y)\,c_Y \Leftrightarrow$$

$$\rho_g\rho/\eta+\rho_g c_Y+(1-c_Y)\,(\rho+\eta c_Y-\eta c_Y)=0 \Leftrightarrow$$

$$\rho_g/\eta+\rho_g c_Y/\rho+(1-c_Y)=0,$$

which is never the case. I.e., world government spending gap is closed under the optimal plan.

Adding the third and the fourth equation yields

$$\pi_t^W+\left(\phi_{1,t}+\phi_{2,t}\right)-\left(\phi_{1,t-1}+\phi_{2,t-1}\right)=0,$$

and, therefore, by (G.9)

$$\pi_t^W=-\left(c_Y/\sigma\right)\left[\left(\widehat{C}_t-\widetilde{C}_t\right)-\left(\widehat{C}_{t-1}-\widetilde{C}_{t-1}\right)\right].$$

We now turn to the characterization of relative variables. The equations (G.13) and (G.14) can be rearranged to (by multiplying the first by $(1-n)$ and multiplying the second by n and then subtracting the first from the second)

$$\begin{aligned}&(1-n)\,n\left(\left[\rho_g/\eta+(1-c_Y)\right]/\sigma\right)\left(\widehat{G}_t^R-\widetilde{G}_t^R\right)\\&-\left(n\,(1-n)\,c_Y/\sigma\right)\left(\widehat{T}_t-\widetilde{T}_t\right)-n\phi_{2,t}+(1-n)\,\phi_{1,t}=0.\end{aligned} \tag{G.15}$$

From the “inflation equations” (G.11) and (G.12) we get

$$(1-n)\,n\pi_t^R+n\left(\phi_{2,t}-\phi_{2,t-1}\right)-(1-n)\left(\phi_{1,t}-\phi_{1,t-1}\right)-\phi_{3,t}=0 \Leftrightarrow$$

$$(1-n)\,n\pi_t^R+n\phi_{2,t}-(1-n)\,\phi_{1,t}-n\phi_{2,t-1}+(1-n)\,\phi_{1,t-1}-\phi_{3,t}=0.$$

Therefore,

$$\begin{aligned}&(1-n)\,n\pi_t^R+(1-n)\,n\left(\left[\rho_g/\eta+(1-c_Y)\right]/\sigma\right)\left(\widehat{G}_t^R-\widetilde{G}_t^R\right)-\left(n\,(1-n)\,c_Y/\sigma\right)\left(\widehat{T}_t-\widetilde{T}_t\right)\\&-(1-n)\,n\left(\left[\rho_g/\eta+(1-c_Y)\right]/\sigma\right)\left(\widehat{G}_{t-1}^R-\widetilde{G}_{t-1}^R\right)+\left(n\,(1-n)\,c_Y/\sigma\right)\left(\widehat{T}_{t-1}-\widetilde{T}_{t-1}\right)=\phi_{3,t},\end{aligned}$$

$$\begin{aligned}&(1-n)\,n\pi_t^R+(1-n)\,n\left(\left[\rho_g/\eta+(1-c_Y)\right]/\sigma\right)\left[\left(\widehat{G}_t^R-\widetilde{G}_t^R\right)-\left(\widehat{G}_{t-1}^R-\widetilde{G}_{t-1}^R\right)\right]\\&-(n\,(1-n)\,c_Y/\sigma)\left[\left(\widehat{T}_t-\widetilde{T}_t\right)-\left(\widehat{T}_{t-1}-\widetilde{T}_{t-1}\right)\right]=\phi_{3,t},\end{aligned}$$

$$\begin{aligned}&\pi_t^R+\left(\left[\rho_g/\eta+(1-c_Y)\right]/\sigma\right)\left[\left(\widehat{G}_t^R-\widetilde{G}_t^R\right)-\left(\widehat{G}_{t-1}^R-\widetilde{G}_{t-1}^R\right)\right]\\&-(c_Y/\sigma)\left[\left(\widehat{T}_t-\widetilde{T}_t\right)-\left(\widehat{T}_{t-1}-\widetilde{T}_{t-1}\right)\right]=\phi_{3,t},\end{aligned}$$

or, by use of (D.8),

$$\begin{aligned}&\pi_t^R+\left(\left[\rho_g/\eta+(1-c_Y)\right]/\sigma\right)\left[\left(\widehat{G}_t^R-\widetilde{G}_t^R\right)-\left(\widehat{G}_{t-1}^R-\widetilde{G}_{t-1}^R\right)\right]\\&-(c_Y/\sigma)\left[\pi_t^R-\left(\widetilde{T}_t-\widetilde{T}_{t-1}\right)\right]=\phi_{3,t},\end{aligned}$$

which becomes

$$\begin{aligned}&\pi_t^R\,(1-c_Y/\sigma)+\left(\left[\rho_g/\eta+(1-c_Y)\right]/\sigma\right)\left[\left(\widehat{G}_t^R-\widetilde{G}_t^R\right)-\left(\widehat{G}_{t-1}^R-\widetilde{G}_{t-1}^R\right)\right]\\&+(c_Y/\sigma)\left(\widetilde{T}_t-\widetilde{T}_{t-1}\right)=\phi_{3,t}.\end{aligned}$$

Now examine

$$\begin{aligned}&(\kappa_T n\,(1-n)\,c_Y/\sigma)\left(\widehat{T}_t-\widetilde{T}_t\right)-(\kappa_G n\,(1-n)\,c_Y/\sigma)\left(\widehat{G}_t^R-\widetilde{G}_t^R\right)\\&-\phi_{1,t}\kappa_T\,(1-n)+\phi_{2,t}\kappa_T n+\phi_{3,t}-\beta\phi_{3,t+1}=0\Leftrightarrow\end{aligned}$$

$$\begin{aligned}&(n\,(1-n)\,c_Y/\sigma)\left(\widehat{T}_t-\widetilde{T}_t\right)-\left(\frac{\kappa_G}{\kappa_T}n\,(1-n)\,c_Y/\sigma\right)\left(\widehat{G}_t^R-\widetilde{G}_t^R\right)\\&-\phi_{1,t}\,(1-n)+\phi_{2,t}n+\frac{\phi_{3,t}-\beta\phi_{3,t+1}}{\kappa_T}=0.\end{aligned}$$

We find $n\phi_{2,t}-(1-n)\,\phi_{1,t}$ from (G.15)

$$\begin{aligned}&(1-n)\,n\left(\left[\rho_g/\eta+(1-c_Y)\right]/\sigma\right)\left(\widehat{G}_t^R-\widetilde{G}_t^R\right)-(n\,(1-n)\,c_Y/\sigma)\left(\widehat{T}_t-\widetilde{T}_t\right)\\=\;&n\phi_{2,t}-(1-n)\,\phi_{1,t},\end{aligned}$$

to get:

$$
\begin{aligned}
& \left(n\left(1-n\right)c_Y/\sigma\right)\left(\widehat{T}_t-\widetilde{T}_t\right)-\left(\frac{\kappa_G}{\kappa_T}n\left(1-n\right)c_Y/\sigma\right)\left(\widehat{G}_t^R-\widetilde{G}_t^R\right) \\
& +\left(1-n\right)n\left(\left[\rho_g/\eta+\left(1-c_Y\right)\right]/\sigma\right)\left(\widehat{G}_t^R-\widetilde{G}_t^R\right) \\
& -\left(n\left(1-n\right)c_Y/\sigma\right)\left(\widehat{T}_t-\widetilde{T}_t\right)+\frac{\phi_{3,t}-\beta\phi_{3,t+1}}{\kappa_T} \\
= \; & 0
\end{aligned}
$$

$$
\left(1-n\right)n\left[\left(\left[\rho_g/\eta+\left(1-c_Y\right)\right]/\sigma\right)-\frac{\kappa_G}{\kappa_T}\left(c_Y/\sigma\right)\right]\left(\widehat{G}_t^R-\widetilde{G}_t^R\right)+\frac{\phi_{3,t}-\beta\phi_{3,t+1}}{\kappa_T}=0.
$$

Using that

$$
\frac{\kappa_G}{\kappa_T}=\frac{\eta\left(1-c_Y\right)}{\left(1+\eta c_Y\right)},
$$

we get

$$
\frac{\left(1-n\right)n}{\sigma}\left[\rho_g/\eta+\left(1-c_Y\right)-\frac{\eta\left(1-c_Y\right)c_Y}{\left(1+\eta c_Y\right)}\right]\left(\widehat{G}_t^R-\widetilde{G}_t^R\right)+\frac{\phi_{3,t}-\beta\phi_{3,t+1}}{\kappa_T}=0,
$$

and then

$$
\phi_{3,t}=\beta\phi_{3,t+1}-\frac{\kappa_T\left(1-n\right)n}{\sigma}\left[\frac{\rho_g/\eta+c_Y\rho_g+\left(1-c_Y\right)}{\left(1+\eta c_Y\right)}\right]\left(\widehat{G}_t^R-\widetilde{G}_t^R\right),
$$

Hence,

$$
\phi_{3,t}=-\frac{\kappa_T\left(1-n\right)n}{\sigma}\left[\frac{\rho_g/\eta+c_Y\rho_g+\left(1-c_Y\right)}{1+\eta c_Y}\right]\sum_{i=0}^{\infty}\beta^i\left(\widehat{G}_{t+i}^R-\widetilde{G}_{t+i}^R\right).
$$

To sum up, we have

$$
\widehat{G}_t^W-\widetilde{G}_t^W=0,
$$

$$
\pi_t^W=-\left(c_Y/\sigma\right)\left[\left(\widehat{C}_t-\widetilde{C}_t\right)-\left(\widehat{C}_{t-1}-\widetilde{C}_{t-1}\right)\right],
$$

$$
\begin{aligned}
& \pi_t^R\left(1-c_Y/\sigma\right)+\left(\left[\rho_g/\eta+\left(1-c_Y\right)\right]/\sigma\right)\left[\left(\widehat{G}_t^R-\widetilde{G}_t^R\right)-\left(\widehat{G}_{t-1}^R-\widetilde{G}_{t-1}^R\right)\right] \\
& +\left(c_Y/\sigma\right)\left(\widetilde{T}_t-\widetilde{T}_{t-1}\right)=\phi_{3,t},
\end{aligned}
$$

$$
\phi_{3,t}=-\frac{\kappa_T\left(1-n\right)n}{\sigma}\left[\frac{\rho_g/\eta+c_Y\rho_g+\left(1-c_Y\right)}{1+\eta c_Y}\right]\sum_{i=0}^{\infty}\beta^i\left(\widehat{G}_{t+i}^R-\widetilde{G}_{t+i}^R\right),
$$

which is the system (16)-(19) in the paper. Together with the Phillips curves the system determines the six endogenous variables $\left(\widehat{C}_t-\widetilde{C}_t\right)$, $\left(\widehat{G}_t^H-\widetilde{G}_t^H\right)$, $\left(\widehat{G}_t^F-\widetilde{G}_t^F\right)$, π_t^H, π_t^F

and $\phi_{3,t}$.

H. Optimal policies under discretion and equal rigidities

We observe that (G.1) and (G.2) can restated in terms of world and relative variables exclusively:

$$\pi_t^W = \beta \mathrm{E}_t \pi_{t+1}^W + \kappa_C \left(\widehat{C}_t - \widetilde{C}_t \right) + \kappa_G \left(\widehat{G}_t^W - \widetilde{G}_t^W \right) + u_t^W, \tag{H.1}$$

$$\pi_t^R = \beta \mathrm{E}_t \pi_{t+1}^R - \kappa_T \left(\widehat{T}_t - \widetilde{T}_t \right) + \kappa_G \left(\widehat{G}_t^R - \widetilde{G}_t^R \right) + u_t^R. \tag{H.2}$$

The problem is to minimize the stream of L_t^S as given by (F.3), subject to the constraints (H.1), (H.2) and (D.8). Since the nominal interest rate can be adjusted freely at no loss, we do not treat equation (D.3) as a constraint, but assume that the consumption gap is treated as the monetary policy instrument directly, which together with the world government spending gap and the relative government spending gap forms the full set of policy instruments.

Having realized this, part of the discretionary optimization becomes simple; namely the choice of world consumption and world government spending. Notice that these variables do not affect the relative inflation rate, and nor do they affect the terms of trade directly; cf. (H.2) and (D.8). Equally important, the variables enter the loss function additively separable from the terms of trade and relative government spending. Hence, the optimal choice of the consumption gap and world government spending gap can be cast as a problem of minimizing the discounted sum of L_t^S as given by (F.3), taking as given the path of relative inflation rates and the terms of trade, subject to (H.1). This can be labelled as the "world part" of the problem. One can then independently of this determine the optimal relative spending gap as the one that minimizes the discounted sum of L_t^S given by (F.3), taking as given the path of world government spending, the world inflation rate and the consumption gap, and where the minimization is subject to (H.2) and (D.8). This can be labelled as the "relative part" of the problem. We now turn to solving these two parts.

H.1. "The world part"

The "world part" of the problem reduces to a sequence of static optimization problems of the form

$$\min_{\left(\widehat{C}_t - \widetilde{C}_t\right), \left(\widehat{G}_t^W - \widetilde{G}_t^W\right)} L_t \qquad \text{s.t. (H.1)} \tag{H.3}$$

taking as given $\mathrm{E}_t\pi^W_{t+1}$, as the period t consumption gap or government spending gap have no dynamic implications.

Substitute (H.1) into (F.3). Then, the necessary and sufficient first-order conditions to (H.3) are:

$$(\kappa_C c_Y/\sigma)\left(\widehat{C}_t - \widetilde{C}_t\right) + \kappa_C \pi^W_t + (\kappa_G c_Y/\sigma)\left(\widehat{G}^W_t - \widetilde{G}^W_t\right) = 0 \quad \text{(H.4)}$$

$$\left(\kappa_G\left[\rho_g/\eta + (1-c_Y)\right]/\sigma\right)\left(\widehat{G}^W_t - \widetilde{G}^W_t\right) + \kappa_G \pi^W_t + (\kappa_G c_Y/\sigma)\left(\widehat{C}_t - \widetilde{C}_t\right) = 0 \quad \text{(H.5)}$$

Reducing these equations slightly, reveals the following:

$$\begin{aligned}
(c_Y/\sigma)\left(\widehat{C}_t - \widetilde{C}_t\right) + \pi^W_t + \left(\frac{\kappa_G c_Y}{\kappa_C \sigma}\right)\left(\widehat{G}^W_t - \widetilde{G}^W_t\right) &= 0 \\
\left(\left[\rho_g/\eta + (1-c_Y)\right]/\sigma\right)\left(\widehat{G}^W_t - \widetilde{G}^W_t\right) + \pi^W_t + (c_Y/\sigma)\left(\widehat{C}_t - \widetilde{C}_t\right) &= 0
\end{aligned}$$

Hence, world government spending follows as

$$\widehat{G}^W_t - \widetilde{G}^W_t = 0,$$

which is the equation preceding equation (20) in the main text, and, hence,[2]

$$\pi^W_t = -\frac{c_Y}{\sigma}\left(\widehat{C}_t - \widetilde{C}_t\right),$$

which is equation (20) of the main text.

H.2. The "relative part"

The "relative part" of the discretionary optimization problem involves, as mentioned, the choice of relative government spending. For this purpose, it is important to acknowledge that this choice only affect the relative inflation rate and the terms of trade. As these terms enter additively in (F.3) (and relative government spending only enters multiplicatively with the terms of trade), the problem "reduces" to one of minimizing the discounted sum

[2] The generality of this solution requires that

$$\frac{\kappa_G \xi_c}{\kappa_C \sigma} \neq \left[\rho_g/\eta + (1-\xi_c)\right]/\sigma,$$

which is easy to confirm.

of

$$
\begin{aligned}
L_t^R &= (\kappa_T n(1-n) c_Y/\sigma)\left(\widehat{T}_t - \widetilde{T}_t\right)^2 \\
&\quad + n(1-n)\left(\kappa_G\left[\rho_g/\eta + (1-c_Y)\right]/\sigma\right)\left(\widehat{G}_t^R - \widetilde{G}_t^R\right)^2 + n(1-n)\left(\pi_t^R\right)^2 \\
&\quad - 2(\kappa_G n(1-n) c_Y/\sigma)\left(\widehat{T}_t - \widetilde{T}_t\right)\left(\widehat{G}_t^R - \widetilde{G}_t^R\right),
\end{aligned}
$$

analogous to

$$
\begin{aligned}
\widetilde{L}_t^R &= (\kappa_T c_Y/\sigma)\left(\widehat{T}_t - \widetilde{T}_t\right)^2 + \left(\kappa_G\left[\rho_g/\eta + (1-c_Y)\right]/\sigma\right)\left(\widehat{G}_t^R - \widetilde{G}_t^R\right)^2 \\
&\quad + \left(\pi_t^R\right)^2 - 2(\kappa_G c_Y/\sigma)\left(\widehat{T}_t - \widetilde{T}_t\right)\left(\widehat{G}_t^R - \widetilde{G}_t^R\right)
\end{aligned}
$$

subject to (H.2) and (D.8). This problem does not correspond to a sequence of one-period problems, as the choice of $\left(\widehat{G}_t^R - \widetilde{G}_t^R\right)$ affects π_t^R, and thus $\left(\widehat{T}_t - \widetilde{T}_t\right)$ with direct loss implications through the next period's terms of trade (by the dynamics of (D.8)].

The period t problem is therefore solved by dynamic programming with past period's terms-of-trade gap as the state variable. I.e., the problem is characterized by the recursion

$$
\begin{aligned}
& V\left(\widehat{T}_{t-1} - \widetilde{T}_{t-1}\right) \\
= & \min_{\left(\widehat{G}_t^R - \widetilde{G}_t^R\right)} \mathrm{E}_{t-1}\left\{(\kappa_T c_Y/\sigma)\left(\widehat{T}_t - \widetilde{T}_t\right)^2 + \left(\kappa_G\left[\rho_g/\eta + (1-c_Y)\right]/\sigma\right)\left(\widehat{G}_t^R - \widetilde{G}_t^R\right)^2\right. \\
& \left. + \left(\pi_t^R\right)^2 - 2(\kappa_G c_Y/\sigma)\left(\widehat{T}_t - \widetilde{T}_t\right)\left(\widehat{G}_t^R - \widetilde{G}_t^R\right) + \beta V\left(\widehat{T}_t - \widetilde{T}_t\right)\right\},
\end{aligned}
$$

where V is the "value" function, and where the minimization is subject to (H.2) and (D.8). Now, combine these constraints to

$$
\begin{aligned}
\pi_t^R &= \beta \mathrm{E}_t \pi_{t+1}^R - \kappa_T\left[\left(\widehat{T}_{t-1} - \widetilde{T}_{t-1}\right) + \pi_t^R - \left(\widetilde{T}_t - \widetilde{T}_{t-1}\right)\right] \\
&\quad + \kappa_G\left(\widehat{G}_t^R - \widetilde{G}_t^R\right) + u_t^R.
\end{aligned}
$$

To proceed with the solution, assume that the relevant driving variables of the system of relative variables, $\widetilde{T}_t - \widetilde{T}_{t-1}$ and u_t^R, both follow $AR(1)$ processes.[3] Therefore we conjecture that the solution to the relative variables will be linear functions of the state and driving

[3] As $\widetilde{T}_t - \widetilde{T}_{t-1}$ and u_t^R both are linear functions of the underlying national shocks (productivity and mark-up shocks, respectively), we could also have assumed that these shocks followed $AR(1)$ (or more general) processes and formulated the conjecture in terms of the state and all these shocks. This, however, would make the exposition more messy, without affecting the characterization of optimal relative spending gaps we present in the main text; cf. below.

variables. I.e., we conjecture that

$$\pi_t^R = -b_1\left(\widehat{T}_{t-1} - \widetilde{T}_{t-1}\right) + b_2\left(\widetilde{T}_t - \widetilde{T}_{t-1}\right) + b_3 u_t^R, \tag{H.6}$$

where b_1, b_2 and b_3 are unknown coefficients to be determined. By use of (H.6) one obtains relative inflation as

$$\begin{aligned}\pi_t^R &= -b_1\beta\left(\widehat{T}_t - \widetilde{T}_t\right) + \beta \mathrm{E}_t\left[b_2\left(\widetilde{T}_{t+1} - \widetilde{T}_t\right) + b_3 u_{t+1}^R\right] \\ &\quad -\kappa_T\left[\left(\widehat{T}_{t-1} - \widetilde{T}_{t-1}\right) + \pi_t^R - \left(\widetilde{T}_t - \widetilde{T}_{t-1}\right)\right] + \kappa_G\left(\widehat{G}_t^R - \widetilde{G}_t^R\right) + u_t^R,\end{aligned}$$

$$\begin{aligned}\pi_t^R &= -b_1\beta\left[\left(\widehat{T}_{t-1} - \widetilde{T}_{t-1}\right) + \pi_t^R - \left(\widetilde{T}_t - \widetilde{T}_{t-1}\right)\right] + \beta \mathrm{E}_t\left[b_2\left(\widetilde{T}_{t+1} - \widetilde{T}_t\right) + b_3 u_{t+1}^R\right] \\ &\quad -\kappa_T\left[\left(\widehat{T}_{t-1} - \widetilde{T}_{t-1}\right) + \pi_t^R - \left(\widetilde{T}_t - \widetilde{T}_{t-1}\right)\right] + \kappa_G\left(\widehat{G}_t^R - \widetilde{G}_t^R\right) + u_t^R,\end{aligned}$$

$$\begin{aligned}\pi_t^R\left(1 + b_1\beta + \kappa_T\right) &= -\left(b_1\beta + \kappa_T\right)\left[\left(\widehat{T}_{t-1} - \widetilde{T}_{t-1}\right) - \left(\widetilde{T}_t - \widetilde{T}_{t-1}\right)\right] \\ &\quad +\kappa_G\left(\widehat{G}_t^R - \widetilde{G}_t^R\right) + u_t^R + \beta \mathrm{E}_t\left[b_2\left(\widetilde{T}_{t+1} - \widetilde{T}_t\right) + b_3 u_{t+1}^R\right],\end{aligned}$$

$$\begin{aligned}\pi_t^R &= -\frac{b_1\beta + \kappa_T}{1 + b_1\beta + \kappa_T}\left[\left(\widehat{T}_{t-1} - \widetilde{T}_{t-1}\right) - \left(\widetilde{T}_t - \widetilde{T}_{t-1}\right)\right] \\ &\quad +\frac{\kappa_G}{1 + b_1\beta + \kappa_T}\left(\widehat{G}_t^R - \widetilde{G}_t^R\right) + \frac{u_t^R + \beta \mathrm{E}_t\left[b_2\left(\widetilde{T}_{t+1} - \widetilde{T}_t\right) + b_3 u_{t+1}^R\right]}{1 + b_1\beta + \kappa_T},\end{aligned} \tag{H.7}$$

and the terms-of-trade gap as

$$\begin{aligned}\widehat{T}_t - \widetilde{T}_t &= \left(\widehat{T}_{t-1} - \widetilde{T}_{t-1}\right) \\ &\quad -\frac{b_1\beta + \kappa_T}{1 + b_1\beta + \kappa_T}\left[\left(\widehat{T}_{t-1} - \widetilde{T}_{t-1}\right) - \left(\widetilde{T}_t - \widetilde{T}_{t-1}\right)\right] \\ &\quad +\frac{\kappa_G}{1 + b_1\beta + \kappa_T}\left(\widehat{G}_t^R - \widetilde{G}_t^R\right) + \frac{u_t^R + \beta \mathrm{E}_t\left[b_2\left(\widetilde{T}_{t+1} - \widetilde{T}_t\right) + b_3 u_{t+1}^R\right]}{1 + b_1\beta + \kappa_T} \\ &\quad -\left(\widetilde{T}_t - \widetilde{T}_{t-1}\right),\end{aligned}$$

and, thus,

$$\widehat{T}_t - \widetilde{T}_t = \frac{1}{1+b_1\beta+\kappa_T}\left[\left(\widehat{T}_{t-1}-\widetilde{T}_{t-1}\right)-\left(\widetilde{T}_t-\widetilde{T}_{t-1}\right)\right] + \frac{\kappa_G}{1+b_1\beta+\kappa_T}\left(\widehat{G}_t^R-\widetilde{G}_t^R\right)+\frac{u_t^R+\beta \mathrm{E}_t\left[b_2\left(\widetilde{T}_{t+1}-\widetilde{T}_t\right)+b_3 u_{t+1}^R\right]}{1+b_1\beta+\kappa_T}. \quad \text{(H.8)}$$

One can then insert (H.7) and (H.8) into the value function and obtain an unconstrained minimization problem. The first-order condition for optimal $\widehat{G}_t^R - \widetilde{G}_t^R$ is

$$\begin{aligned}
&(\kappa_T c_Y/\sigma)\left(\widehat{T}_t-\widetilde{T}_t\right)\frac{\partial\left(\widehat{T}_t-\widetilde{T}_t\right)}{\partial\left(\widehat{G}_t^R-\widetilde{G}_t^R\right)}+\left(\kappa_G\left[\rho_g/\eta+(1-c_Y)\right]/\sigma\right)\left(\widehat{G}_t^R-\widetilde{G}_t^R\right)\\
&+\left(\pi_t^R\right)\frac{\partial \pi_t^R}{\partial\left(\widehat{G}_t^R-\widetilde{G}_t^R\right)}-(\kappa_G c_Y/\sigma)\left(\widehat{T}_t-\widetilde{T}_t\right)\\
&-(\kappa_G c_Y/\sigma)\left(\widehat{G}_t^R-\widetilde{G}_t^R\right)\frac{\partial\left(\widehat{T}_t-\widetilde{T}_t\right)}{\partial\left(\widehat{G}_t^R-\widetilde{G}_t^R\right)}+\frac{1}{2}\beta V'\left(\widehat{T}_t-\widetilde{T}_t\right)\frac{\partial\left(\widehat{T}_t-\widetilde{T}_t\right)}{\partial\left(\widehat{G}_t^R-\widetilde{G}_t^R\right)}\\
=\;& 0,
\end{aligned}$$

or,

$$\begin{aligned}
&(\kappa_T c_Y/\sigma)\left(\widehat{T}_t-\widetilde{T}_t\right)\frac{\kappa_G}{1+b_1\beta+\kappa_T}+\left(\kappa_G\left[\rho_g/\eta+(1-c_Y)\right]/\sigma\right)\left(\widehat{G}_t^R-\widetilde{G}_t^R\right)\\
&+\left(\pi_t^R\right)\frac{\kappa_G}{1+b_1\beta+\kappa_T}-(\kappa_G c_Y/\sigma)\left(\widehat{T}_t-\widetilde{T}_t\right)-(\kappa_G c_Y/\sigma)\left(\widehat{G}_t^R-\widetilde{G}_t^R\right)\frac{\kappa_G}{1+b_1\beta+\kappa_T}\\
&+\frac{1}{2}\beta V'\left(\widehat{T}_t-\widetilde{T}_t\right)\frac{\kappa_G}{1+b_1\beta+\kappa_T}\\
=\;& 0. \qquad \text{(H.9)}
\end{aligned}$$

Differentiating the value function with respect to $\left(\widehat{T}_{t-1}-\widetilde{T}_{t-1}\right)$ yields:

$$\begin{aligned}
\frac{1}{2}V'\left(\widehat{T}_{t-1}-\widetilde{T}_{t-1}\right) &= \left(\kappa_T c_Y/\sigma\right)\left(\widehat{T}_t-\widetilde{T}_t\right)\frac{\partial\left(\widehat{T}_t-\widetilde{T}_t\right)}{\partial\left(\widehat{T}_{t-1}-\widetilde{T}_{t-1}\right)} \\
&+\left(\kappa_G\left[\rho_g/\eta+(1-c_Y)\right]/\sigma\right)\left(\widehat{G}_t^R-\widetilde{G}_t^R\right)\frac{\partial\left(\widehat{G}_t^R-\widetilde{G}_t^R\right)}{\partial\left(\widehat{T}_{t-1}-\widetilde{T}_{t-1}\right)} \\
&+\left(\pi_t^R\right)\frac{\partial\pi_t^R}{\partial\left(\widehat{T}_{t-1}-\widetilde{T}_{t-1}\right)}-\left(\kappa_G c_Y/\sigma\right)\left(\widehat{T}_t-\widetilde{T}_t\right)\frac{\partial\left(\widehat{G}_t^R-\widetilde{G}_t^R\right)}{\partial\left(\widehat{T}_{t-1}-\widetilde{T}_{t-1}\right)} \\
&-\left(\kappa_G c_Y/\sigma\right)\left(\widehat{G}_t^R-\widetilde{G}_t^R\right)\frac{\partial\left(\widehat{T}_t-\widetilde{T}_t\right)}{\partial\left(\widehat{T}_{t-1}-\widetilde{T}_{t-1}\right)} \\
&+\frac{1}{2}\beta V'\left(\widehat{T}_t-\widetilde{T}_t\right)\frac{\partial\left(\widehat{T}_t-\widetilde{T}_t\right)}{\partial\left(\widehat{T}_{t-1}-\widetilde{T}_{t-1}\right)}. \qquad \text{(H.10)}
\end{aligned}$$

By the Envelope Theorem, we eliminate all terms involving $\partial\left(\widehat{G}_t^R-\widetilde{G}_t^R\right)/\partial\left(\widehat{T}_{t-1}-\widetilde{T}_{t-1}\right)$ [the explicit ones and those implicitly appearing in $\partial\left(\widehat{T}_t-\widetilde{T}_t\right)/\partial\left(\widehat{T}_{t-1}-\widetilde{T}_{t-1}\right)$ and $\partial\pi_t^R/\partial\left(\widehat{T}_{t-1}-\widetilde{T}_{t-1}\right)$] to get:

$$\begin{aligned}
\frac{1}{2}V'\left(\widehat{T}_{t-1}-\widetilde{T}_{t-1}\right) &= \left(\kappa_T c_Y/\sigma\right)\left(\widehat{T}_t-\widetilde{T}_t\right)\frac{1}{1+b_1\beta+\kappa_T}-\pi_t^R\frac{b_1\beta+\kappa_T}{1+b_1\beta+\kappa_T} \\
&-\left(\kappa_G c_Y/\sigma\right)\left(\widehat{G}_t^R-\widetilde{G}_t^R\right)\frac{1}{1+b_1\beta+\kappa_T} \\
&+\frac{1}{2}\beta V'\left(\widehat{T}_t-\widetilde{T}_t\right)\frac{1}{1+b_1\beta+\kappa_T}. \qquad \text{(H.11)}
\end{aligned}$$

Multiply on both sides by κ_G to get

$$\begin{aligned}
\frac{\kappa_G}{2}V'\left(\widehat{T}_{t-1}-\widetilde{T}_{t-1}\right) &= \left(\kappa_T c_Y/\sigma\right)\left(\widehat{T}_t-\widetilde{T}_t\right)\frac{\kappa_G}{1+b_1\beta+\kappa_T}-\pi_t^R\kappa_G\frac{b_1\beta+\kappa_T}{1+b_1\beta+\kappa_T} \\
&-\left(\kappa_G c_Y/\sigma\right)\left(\widehat{G}_t^R-\widetilde{G}_t^R\right)\frac{\kappa_G}{1+b_1\beta+\kappa_T} \\
&+\frac{1}{2}\beta V'\left(\widehat{T}_t-\widetilde{T}_t\right)\frac{\kappa_G}{1+b_1\beta+\kappa_T},
\end{aligned}$$

and add this to (H.9):

$$
\begin{aligned}
& (\kappa_T c_Y/\sigma)\left(\widehat{T}_t-\widetilde{T}_t\right)\frac{\kappa_G}{1+b_1\beta+\kappa_T}+\left(\kappa_G\left[\rho_g/\eta+(1-c_Y)\right]/\sigma\right)\left(\widehat{G}_t^R-\widetilde{G}_t^R\right)+ \\
& \left(\pi_t^R\right)\frac{\kappa_G}{1+b_1\beta+\kappa_T}-(\kappa_G c_Y/\sigma)\left(\widehat{T}_t-\widetilde{T}_t\right)-(\kappa_G c_Y/\sigma)\left(\widehat{G}_t^R-\widetilde{G}_t^R\right)\frac{\kappa_G}{1+b_1\beta+\kappa_T} \\
& +\frac{1}{2}\beta V'\left(\widehat{T}_t-\widetilde{T}_t\right)\frac{\kappa_G}{1+b_1\beta+\kappa_T}+\frac{\kappa_G}{2}V'\left(\widehat{T}_{t-1}-\widetilde{T}_{t-1}\right) \\
= \; & (\kappa_T c_Y/\sigma)\left(\widehat{T}_t-\widetilde{T}_t\right)\frac{\kappa_G}{1+b_1\beta+\kappa_T}-\pi_t^R\kappa_G\frac{b_1\beta+\kappa_T}{1+b_1\beta+\kappa_T} \\
& -(\kappa_G c_Y/\sigma)\left(\widehat{G}_t^R-\widetilde{G}_t^R\right)\frac{\kappa_G}{1+b_1\beta+\kappa_T}+\frac{1}{2}\beta V'\left(\widehat{T}_t-\widetilde{T}_t\right)\frac{\kappa_G}{1+b_1\beta+\kappa_T},
\end{aligned}
$$

which simplifies to

$$
\begin{aligned}
& \left(\kappa_G\left[\rho_g/\eta+(1-c_Y)\right]/\sigma\right)\left(\widehat{G}_t^R-\widetilde{G}_t^R\right)+\left(\pi_t^R\right)\frac{\kappa_G}{1+b_1\beta+\kappa_T} \\
& -(\kappa_G c_Y/\sigma)\left(\widehat{T}_t-\widetilde{T}_t\right)+\frac{\kappa_G}{2}V'\left(\widehat{T}_{t-1}-\widetilde{T}_{t-1}\right) \\
= \; & -\pi_t^R\kappa_G\frac{b_1\beta+\kappa_T}{1+b_1\beta+\kappa_T},
\end{aligned}
$$

from which one gets

$$
\frac{1}{2}V'\left(\widehat{T}_{t-1}-\widetilde{T}_{t-1}\right)=(c_Y/\sigma)\left(\widehat{T}_t-\widetilde{T}_t\right)-\left(\left[\rho_g/\eta+(1-c_Y)\right]/\sigma\right)\left(\widehat{G}_t^R-\widetilde{G}_t^R\right)-\pi_t^R
$$

Forward this one period, and use it in (H.9) to eliminate the derivative of the value function:

$$
\begin{aligned}
& (\kappa_T c_Y/\sigma)\left(\widehat{T}_t-\widetilde{T}_t\right)\frac{\kappa_G}{1+b_1\beta+\kappa_T}+\left(\kappa_G\left[\rho_g/\eta+(1-c_Y)\right]/\sigma\right)\left(\widehat{G}_t^R-\widetilde{G}_t^R\right) \\
& +\left(\pi_t^R\right)\frac{\kappa_G}{1+b_1\beta+\kappa_T}-(\kappa_G c_Y/\sigma)\left(\widehat{T}_t-\widetilde{T}_t\right)-(\kappa_G c_Y/\sigma)\left(\widehat{G}_t^R-\widetilde{G}_t^R\right)\frac{\kappa_G}{1+b_1\beta+\kappa_T} \\
& +\frac{\beta\kappa_G}{1+b_1\beta+\kappa_T}\left[(c_Y/\sigma)\,\mathrm{E}_t\left(\widehat{T}_{t+1}-\widetilde{T}_{t+1}\right)-\left(\left[\rho_g/\eta+(1-c_Y)\right]/\sigma\right)\mathrm{E}_t\left(\widehat{G}_{t+1}^R-\widetilde{G}_{t+1}^R\right)-\mathrm{E}_t\pi_{t+1}^R\right] \\
= \; & 0,
\end{aligned}
$$

and thus

$$
\begin{aligned}
& \frac{c_Y\kappa_T}{1+b_1\beta+\kappa_T}\left(\widehat{T}_t-\widetilde{T}_t\right)+\left[\rho_g/\eta+(1-c_Y)\right]\left(\widehat{G}_t^R-\widetilde{G}_t^R\right) \\
& +\frac{\sigma}{1+b_1\beta+\kappa_T}\pi_t^R-c_Y\left(\widehat{T}_t-\widetilde{T}_t\right)-\frac{c_Y\kappa_G}{1+b_1\beta+\kappa_T}\left(\widehat{G}_t^R-\widetilde{G}_t^R\right) \\
& +\frac{\beta}{1+b_1\beta+\kappa_T}\left[c_Y\mathrm{E}_t\left(\widehat{T}_{t+1}-\widetilde{T}_{t+1}\right)-\left[\rho_g/\eta+(1-c_Y)\right]\mathrm{E}_t\left(\widehat{G}_{t+1}^R-\widetilde{G}_{t+1}^R\right)-\sigma\mathrm{E}_t\pi_{t+1}^R\right] \\
= \; & 0,
\end{aligned}
$$

or,

$$
\begin{aligned}
& -\frac{c_Y (1+b_1\beta)}{1+b_1\beta+\kappa_T}\left(\widehat{T}_t-\widetilde{T}_t\right)+\left(\mu-\frac{c_Y\kappa_G}{1+b_1\beta+\kappa_T}\right)\left(\widehat{G}_t^R-\widetilde{G}_t^R\right)+\frac{\sigma}{1+b_1\beta+\kappa_T}\pi_t^R \\
& +\frac{\beta}{1+b_1\beta+\kappa_T}\left[c_Y\mathrm{E}_t\left(\widehat{T}_{t+1}-\widetilde{T}_{t+1}\right)-\mu\mathrm{E}_t\left(\widehat{G}_{t+1}^R-\widetilde{G}_{t+1}^R\right)-\sigma\mathrm{E}_t\pi_{t+1}^R\right] \\
= & \ 0,
\end{aligned}
$$

with

$$
\mu \equiv \rho_g/\eta+(1-c_Y).
$$

This is further reduced to

$$
\begin{aligned}
& -c_Y(1+b_1\beta)\left(\widehat{T}_t-\widetilde{T}_t\right)+\left(\mu\left[1+b_1\beta+\kappa_T\right]-c_Y\kappa_G\right)\left(\widehat{G}_t^R-\widetilde{G}_t^R\right)+\sigma\pi_t^R \\
& +\beta\left[c_Y\mathrm{E}_t\left(\widehat{T}_{t+1}-\widetilde{T}_{t+1}\right)-\mu\mathrm{E}_t\left(\widehat{G}_{t+1}^R-\widetilde{G}_{t+1}^R\right)-\sigma\mathrm{E}_t\pi_{t+1}^R\right] \\
= & \ 0.
\end{aligned}
$$

This equation is equation (21) of the main text, and will together with (H.7) and (H.8) provide solutions for the paths for $\left(\widehat{G}_t^R-\widetilde{G}_t^R\right)$, $\left(\widehat{T}_t-\widetilde{T}_t\right)$ and π_t^R as functions of the state and $\left(\widetilde{T}_t-\widetilde{T}_{t-1}\right)$ and u_t^R. Given the assumption about the stochastic properties of $\left(\widetilde{T}_t-\widetilde{T}_{t-1}\right)$ and u_t^R the solution can be characterized by the method of undertermined coefficients. The coefficients found in this step will be functions of the unknown parameters b_1, b_2 and b_3. These are then finally identified by equating the coefficients in the solution for π_t^R with those in the conjecture.

Note that indeed only the undetermined coefficient to the state variable appears in the characterization of the solution of the system of relative variables as given by equation (21). Hence, had we replaced (H.6), by a linear conjecture which depended on the state and the underlying shocks [and assumed that these shocks were $AR(1)$ or more general processes], we would have arrived at the same characterization of optimal relative spending gaps as equation (21) of the main text. The reason is that the impact of government spending changes on the inflation differential and the terms-of-trade-gap only depends on the undetermined coefficient on the state variable. The coefficients on the shocks do therefore not affect the first-order condition or the envelope condition [see equations (H.9) and (H.11)]. We can therefore without loss of analytical generality arrive at equation (21) with our parsimonious conjecture (H.6) as claimed in Footnote 3.

I. Optimal monetary policy with constrained fiscal policy under equal rigidities

For convenience, we write (G.1) and (G.2) out as

$$\pi_t^H = \beta \mathrm{E}_t \pi_{t+1}^H + \kappa (1-n)(1+\eta c_Y)\left(\widehat{T}_t - \widetilde{T}_t\right) + \kappa(\rho + \eta c_Y)\left(\widehat{C}_t - \widetilde{C}_t\right) + u_t^H, \quad \text{(I.1)}$$

$$\pi_t^F = \beta \mathrm{E}_t \pi_{t+1}^F - \kappa n (1+\eta c_Y)\left(\widehat{T}_t - \widetilde{T}_t\right) + \kappa(\rho + \eta c_Y)\left(\widehat{C}_t - \widetilde{C}_t\right) + u_t^F. \quad \text{(I.2)}$$

The loss function is given by (F.1) with the coefficients given by (because rigidities equal in the two countries):

$$\begin{aligned}
\lambda_C &\equiv \kappa c_Y (\rho + \eta c_Y)/\sigma, \quad \lambda_T \equiv \kappa n (1-n) c_Y (1+\eta c_Y)/\sigma, \\
\lambda_G &\equiv \kappa (1-c_Y)\left[\rho_g + \eta (1-c_Y)\right]/\sigma, \quad \lambda_{CG} = \kappa c_Y (1-c_Y)\eta/\sigma, \\
\lambda_{TG} &\equiv \kappa n (1-n) \eta c_Y (1-c_Y)/\sigma, \\
\lambda_{\pi^H} &\equiv n, \quad \lambda_{\pi^F} = 1-n.
\end{aligned}$$

I.1. Characterization of optimal policies under precommitment

To solve for the optimal policies under commitment we set up the relevant Lagrangian (see, e.g., Woodford, 1999):

$$\begin{aligned}
\mathcal{L} = \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \Big\{ & L_t^S \\
& + 2\phi_{1,t}\left[\pi_t^H - \beta\pi_{t+1}^H - \kappa(1-n)(1+\eta c_Y)\left(\widehat{T}_t - \widetilde{T}_t\right) - \kappa(\rho+\eta c_Y)\left(\widehat{C}_t - \widetilde{C}_t\right) - u_t^H\right] \\
& + 2\phi_{2,t}\left[\pi_t^F - \beta\pi_{t+1}^F + \kappa n(1+\eta c_Y)\left(\widehat{T}_t - \widetilde{T}_t\right) - \kappa(\rho+\eta c_Y)\left(\widehat{C}_t - \widetilde{C}_t\right) - u_t^F\right] \\
& + 2\phi_{3,t}\left[\left(\widehat{T}_t - \widetilde{T}_t\right) - \left(\widehat{T}_{t-1} - \widetilde{T}_{t-1}\right) - \pi_t^F + \pi_t^H + \left(\widetilde{T}_t - \widetilde{T}_{t-1}\right)\right]\Big\},
\end{aligned}$$

where $2\phi_{1,t}$, $2\phi_{2,t}$, and $2\phi_{3,t}$ are the multipliers on (I.1), (I.2), and (D.8), respectively. Optimizing over $\widehat{C}_t^W - \widetilde{C}_t^W$, $\widehat{T}_t - \widetilde{T}_t$, π_t^H and π_t^F, yields the following four necessary first-order conditions for $t \geq 1$,

$$\lambda_C\left(\widehat{C}_t - \widetilde{C}_t\right) - \phi_{1,t}\kappa(\rho+\eta c_Y) - \phi_{2,t}\kappa(\rho+\eta c_Y) = 0, \quad \text{(I.3)}$$

$$\lambda_T\left(\widehat{T}_t - \widetilde{T}_t\right) - \phi_{1,t}\kappa(1-n)(1+\eta c_Y) + \phi_{2,t}\kappa n(1+\eta c_Y) + \phi_{3,t} - \beta\phi_{3,t+1} = 0, \quad \text{(I.4)}$$

$$n\pi_t^H + \phi_{1,t} - \phi_{1,t-1} + \phi_{3,t} = 0, \quad \text{(I.5)}$$

$$(1-n)\pi_t^F + \phi_{2,t} - \phi_{2,t-1} - \phi_{3,t} = 0. \quad \text{(I.6)}$$

Use that $\lambda_C \equiv \kappa c_Y (\rho + \eta c_Y)/\sigma$ to simplify (I.3):

$$\frac{c_Y}{\sigma}\left(\widehat{C}_t - \widetilde{C}_t\right) - \phi_{1,t} - \phi_{2,t} = 0. \tag{I.7}$$

Use that $\lambda_T \equiv \kappa n (1-n) c_Y (1 + \eta c_Y)/\sigma$ to simplify (I.4):

$$n(1-n)\frac{c_Y}{\sigma}\left(\widehat{T}_t - \widetilde{T}_t\right) - \phi_{1,t}(1-n) + \phi_{2,t} n + \frac{1}{\kappa(1+\eta c_Y)}\left(\phi_{3,t} - \beta\phi_{3,t+1}\right) = 0. \tag{I.8}$$

Add (I.5) and (I.6) to get

$$\pi_t^W = -\phi_{1,t} + \phi_{1,t-1} - \phi_{2,t} + \phi_{2,t-1}. \tag{I.9}$$

Combine (I.7) and (I.9) to get

$$\pi_t^W = -\frac{c_Y}{\sigma}\left[\left(\widehat{C}_t - \widetilde{C}_t\right) - \left(\widehat{C}_{t-1} - \widetilde{C}_{t-1}\right)\right]. \tag{I.10}$$

By taking an appropriately weighted average of the two Phillips curves, (I.1), and (I.2), one gets a "world" Phillips curve given by

$$\pi_t^W = \beta \mathrm{E}_t \pi_{t+1}^W + \kappa(\rho + \eta c_Y)\left(\widehat{C}_t - \widetilde{C}_t\right) + u_t^W. \tag{I.11}$$

Note that (I.10) and (I.11) provide solutions for $\left(\widehat{C}_t - \widetilde{C}_t\right)$ and π_t^W.

Then note that an expression for the inflation differential, π_t^R can be obtained from (I.1) and (I.2):

$$\begin{aligned}
\pi_t^R &= \beta \mathrm{E}_t \pi_{t+1}^F - \kappa n (1+\eta c_Y)\left(\widehat{T}_t - \widetilde{T}_t\right) + \kappa(\rho+\eta c_Y)\left(\widehat{C}_t - \widetilde{C}_t\right) + u_t^F \\
&\quad - \beta \mathrm{E}_t \pi_{t+1}^H - \kappa(1-n)(1+\eta c_Y)\left(\widehat{T}_t - \widetilde{T}_t\right) - \kappa(\rho+\eta c_Y)\left(\widehat{C}_t - \widetilde{C}_t\right) - u_t^H \\
&= \beta \mathrm{E}_t \pi_{t+1}^R - \kappa(1+\eta c_Y)\left(\widehat{T}_t - \widetilde{T}_t\right) + u_t^R.
\end{aligned} \tag{I.12}$$

It follows that (I.12) and (D.8) provide solutions for $\left(\widehat{T}_t - \widetilde{T}_t\right)$ and π_t^R.

Hence, with solutions for $\left(\widehat{C}_t - \widetilde{C}_t\right)$, π_t^W, $\left(\widehat{T}_t - \widetilde{T}_t\right)$ and π_t^R one can readily get local inflation rates as $\pi_t^H = \pi_t^W - (1-n)\pi_t^R$ and $\pi_t^F = \pi_t^W + n\pi_t^R$.

I.1.1. Deriving the solutions for $\left(\widehat{C}_t - \widetilde{C}_t\right)$ and π_t^W

Substitute the expression for π_t^W given by (I.10) into (I.11):

$$\begin{aligned}
& -\frac{c_Y}{\sigma}\left[\left(\widehat{C}_t - \widetilde{C}_t\right) - \left(\widehat{C}_{t-1} - \widetilde{C}_{t-1}\right)\right] \\
= & -\frac{\beta c_Y}{\sigma}\mathrm{E}_t\left[\left(\widehat{C}_{t+1} - \widetilde{C}_{t+1}\right) - \left(\widehat{C}_t - \widetilde{C}_t\right)\right] + \kappa\left(\rho + \eta c_Y\right)\left(\widehat{C}_t - \widetilde{C}_t\right) + u_t^W,
\end{aligned}$$

which yields as second-order expectational difference equation in $\widehat{C}_t - \widetilde{C}_t$:

$$\begin{aligned}
\mathrm{E}_t\left(\widehat{C}_{t+1} - \widetilde{C}_{t+1}\right) = & \left(1 + \beta^{-1}\left(1 + \frac{\kappa\sigma\left(\rho + \eta c_Y\right)}{c_Y}\right)\right)\left(\widehat{C}_t - \widetilde{C}_t\right) \\
& -\beta^{-1}\left(\widehat{C}_{t-1} - \widetilde{C}_{t-1}\right) + \frac{\sigma}{\beta c_Y}u_t^W,
\end{aligned}$$

or,

$$\begin{aligned}
\mathrm{E}_t\left(\widehat{C}_{t+1} - \widetilde{C}_{t+1}\right) = & \left(1 + \beta^{-1}\left(1 + \frac{\kappa\sigma\left(\rho + \eta c_Y\right)}{c_Y}\right)\right)\left(\widehat{C}_t - \widetilde{C}_t\right) \\
& -\beta^{-1}\left(\widehat{C}_{t-1} - \widetilde{C}_{t-1}\right) + \frac{\sigma n}{\beta c_Y}u_t^H + \frac{\sigma\left(1-n\right)}{\beta c_Y}u_t^F, \qquad \text{(I.13)}
\end{aligned}$$

This is solved by the methods of undetermined coefficients by conjecturing a solution of the form:

$$\widehat{C}_t - \widetilde{C}_t = \chi^C\left(\widehat{C}_{t-1} - \widetilde{C}_{t-1}\right) - \varphi^{UH}u_t^H - \varphi^{UF}u_t^F. \qquad \text{(I.14)}$$

Forward (I.14) one period and take period t expectations:

$$\mathrm{E}_t\left(\widehat{C}_{t+1} - \widetilde{C}_{t+1}\right) = \chi^C\left(\widehat{C}_t - \widetilde{C}_t\right) - \varphi^{UH}\mathrm{E}_t u_{t+1}^H - \varphi^{UF}\mathrm{E}_t u_{t+1}^F.$$

Assume that shocks follow $AR(1)$ processes with persistence parameters γ^{UH} and γ^{UF}, respectively. We then get

$$\mathrm{E}_t\left(\widehat{C}_{t+1} - \widetilde{C}_{t+1}\right) = \chi^C\left(\widehat{C}_t - \widetilde{C}_t\right) - \varphi^{UH}\gamma^{UH}u_t^H - \varphi^{UF}\gamma^{UF}u_t^F,$$

which combined with (I.13) gives

$$\begin{aligned}
& \chi^C\left(\widehat{C}_t - \widetilde{C}_t\right) - \varphi^{UH}\gamma^{UH}u_t^H - \varphi^{UF}\gamma^{UF}u_t^F \\
= & \left[1 + \beta^{-1}\left(1 + \frac{\kappa\sigma\left(\rho + \eta c_Y\right)}{c_Y}\right)\right]\left(\widehat{C}_t - \widetilde{C}_t\right) \\
& -\beta^{-1}\left(\widehat{C}_{t-1} - \widetilde{C}_{t-1}\right) + \frac{\sigma n}{\beta c_Y}u_t^H + \frac{\sigma\left(1-n\right)}{\beta c_Y}u_t^F,
\end{aligned}$$

or,

$$
\begin{aligned}
\widehat{C}_t - \widetilde{C}_t &= -\frac{\beta^{-1}}{\chi^C - \left[1+\beta^{-1}\left(1+\frac{\kappa\sigma\left(\rho+\eta c_Y\right)}{c_Y}\right)\right]}\left(\widehat{C}_{t-1} - \widetilde{C}_{t-1}\right) \\
&+\frac{\frac{\sigma n}{\beta c_Y}+\varphi^{UH}\gamma^{UH}}{\chi^C - \left[1+\beta^{-1}\left(1+\frac{\kappa\sigma\left(\rho+\eta c_Y\right)}{c_Y}\right)\right]} u_t^H \\
&+\frac{\frac{\sigma(1-n)}{\beta c_Y}+\varphi^{UF}\gamma^{UF}}{\chi^C - \left[1+\beta^{-1}\left(1+\frac{\kappa\sigma\left(\rho+\eta c_Y\right)}{c_Y}\right)\right]} u_t^F.
\end{aligned}
$$

So, the undetermined coefficients must satisfy

$$
\begin{aligned}
\chi^C &= -\frac{\beta^{-1}}{\chi^C - \left[1+\beta^{-1}\left(1+\frac{\kappa\sigma\left(\rho+\eta c_Y\right)}{c_Y}\right)\right]}, \\
-\varphi^{UH} &= \frac{\frac{\sigma n}{\beta c_Y}+\varphi^{UH}\gamma^{UH}}{\chi^C - \left[1+\beta^{-1}\left(1+\frac{\kappa\sigma\left(\rho+\eta c_Y\right)}{c_Y}\right)\right]}, \\
-\varphi^{UF} &= \frac{\frac{\sigma(1-n)}{\beta c_Y}+\varphi^{UF}\gamma^{UF}}{\chi^C - \left[1+\beta^{-1}\left(1+\frac{\kappa\sigma\left(\rho+\eta c_Y\right)}{c_Y}\right)\right]}.
\end{aligned}
$$

Hence, χ^C solves the polynomial

$$
\left(\chi^C\right)^2 - \left[1+\beta^{-1}\left(1+\frac{\kappa\sigma\left(\rho+\eta c_Y\right)}{c_Y}\right)\right]\chi^C + \beta^{-1} = 0.
$$

Of the two real roots, one is higher than one and one root is lower than one. Only, the solution associated with the lower root is therefore consistent with a non-explosive rational expectations equilibrium. We find

$$
0 < \chi^C = \frac{1+\beta^{-1}\left(1+\frac{\kappa\sigma\left(\rho+\eta c_Y\right)}{c_Y}\right) - \sqrt{\left(1+\beta^{-1}\left(1+\frac{\kappa\sigma\left(\rho+\eta c_Y\right)}{c_Y}\right)\right)^2 - 4\beta^{-1}}}{2} < 1.
$$

Subsequently we find

$$
-\varphi^{UH} = \frac{\frac{\sigma n}{\beta c_Y}+\varphi^{UH}\gamma^{UH}}{\chi^C - \left(1+\beta^{-1}\left(1+\frac{\kappa\sigma\left(\rho+\eta c_Y\right)}{c_Y}\right)\right)},
$$

$$-\varphi^{UH}\left(1+\frac{\gamma^{UH}}{\chi^{C}-\left(1+\beta^{-1}\left(1+\frac{\kappa\sigma\left(\rho+\eta c_Y\right)}{c_Y}\right)\right)}\right)=\frac{\frac{\sigma n}{\beta c_Y}}{\chi^{C}-\left(1+\beta^{-1}\left(1+\frac{\kappa\sigma\left(\rho+\eta c_Y\right)}{c_Y}\right)\right)},$$

$$\varphi^{UH}=-\frac{\sigma n/\left(\beta c_Y\right)}{\chi^{C}-\left(1+\beta^{-1}\left(1+\frac{\kappa\sigma\left(\rho+\eta c_Y\right)}{c_Y}\right)\right)+\gamma^{UH}}.$$

This is simplified, as we know from the above polynomial that

$$\chi^{C}-\left(1+\beta^{-1}\left(1+\frac{\kappa\sigma\left(\rho+\eta c_Y\right)}{c_Y}\right)\right)=-\frac{\beta^{-1}}{\chi^{C}},$$

so

$$\begin{aligned}\varphi^{UH} &= -\frac{\sigma n/\left(\beta c_Y\right)}{\gamma^{UH}-\beta^{-1}/\chi^{C}}\\ &= \frac{\sigma\chi^{C}}{c_Y\left(1-\chi^{C}\beta\gamma^{UH}\right)}n>0.\end{aligned}$$

Likewise, φ^{F} is found as

$$\varphi^{UF}=\frac{\sigma\chi^{C}}{c_Y\left(1-\chi^{C}\beta\gamma^{UF}\right)}\left(1-n\right)>0.$$

World inflation then follows from (I.10) as

$$\begin{aligned}\pi_t^{W} &= -\frac{c_Y}{\sigma}\left[\chi^{C}\left(\widehat{C}_{t-1}-\widetilde{C}_{t-1}\right)-\varphi^{UH}u_t^{H}-\varphi^{UF}u_t^{F}-\left(\widehat{C}_{t-1}-\widetilde{C}_{t-1}\right)\right]\\ &= \frac{c_Y}{\sigma}\left(1-\chi^{C}\right)\left(\widehat{C}_{t-1}-\widetilde{C}_{t-1}\right)+\frac{c_Y}{\sigma}\varphi^{UH}u_t^{H}+\frac{c_Y}{\sigma}\varphi^{UF}u_t^{F}.\end{aligned}$$

To sum up, the closed-form solutions for the precommitment consumption gap and world inflation, which we discuss in the main text, are

$$\begin{aligned}\widehat{C}_t-\widetilde{C}_t &= \chi^{C}\left(\widehat{C}_{t-1}-\widetilde{C}_{t-1}\right)-\varphi^{UH}u_t^{H}-\varphi^{UF}u_t^{F}\\ \pi_t^{W} &= \frac{c_Y}{\sigma}\left(1-\chi^{C}\right)\left(\widehat{C}_{t-1}-\widetilde{C}_{t-1}\right)+\frac{c_Y}{\sigma}\varphi^{UH}u_t^{H}+\frac{c_Y}{\sigma}\varphi^{UF}u_t^{F},\end{aligned}$$

with

$$0<\chi^{C}\equiv\frac{1+\beta^{-1}\left(1+\frac{\kappa\sigma\left(\rho+\eta c_Y\right)}{c_Y}\right)-\sqrt{\left(1+\beta^{-1}\left(1+\frac{\kappa\sigma\left(\rho+\eta c_Y\right)}{c_Y}\right)\right)^{2}-4\beta^{-1}}}{2}<1,$$

$$\varphi^{UH} \equiv \frac{\sigma\chi^C}{c_Y\left(1-\chi^C\beta\gamma^{UH}\right)}n > 0,$$
$$\varphi^{UF} \equiv \frac{\sigma\chi^C}{c_Y\left(1-\chi^C\beta\gamma^{UF}\right)}(1-n) > 0.$$

I.1.2. Deriving the solutions for $\widehat{T}_t$ and π_t^R

We proceed as in the previous subsubsection. Use (D.8) in (I.12) to eliminate π_t^R and get a second-order expectational difference equation in the term of trade. Note, however, that it is convenient only to solve for $\widehat{T}_t$, as one then avoids dealing with the lagged natural rate of the terms of trade. Hence, one uses the relationship

$$\pi_t^R = \widehat{T}_t - \widehat{T}_{t-1},$$

to obtain

$$\widehat{T}_t - \widehat{T}_{t-1} = \beta \mathrm{E}_t\left(\widehat{T}_{t+1} - \widehat{T}_t\right) - \kappa\left(1+\eta c_Y\right)\left(\widehat{T}_t - \widetilde{T}_t\right) + u_t^R,$$

and thus

$$\begin{aligned} \mathrm{E}_t\widehat{T}_{t+1} &= \left[1+\beta^{-1}\left(1+\kappa\left(1+\eta c_Y\right)\right)\right]\widehat{T}_t - \beta^{-1}\widehat{T}_{t-1} \\ &\quad -\beta^{-1}\kappa\left(1+\eta c_Y\right)\widetilde{T}_t - \beta^{-1}\left(u_t^F - u_t^H\right). \end{aligned} \tag{I.15}$$

Remember that

$$\widetilde{T}_t = -\Gamma\left(S_t^F - S_t^H\right),$$
$$\Gamma \equiv \frac{\eta\rho_g}{\rho_g\left(1+\eta c_Y\right) + \eta\left(1-c_Y\right)},$$

with

$$S_t^i = \gamma^{Si} S_{t-1}^i + \mu_{t,i}^S, \quad i = H, F.$$

We now conjecture that (I.15) has the following solution:

$$\widehat{T}_t = \chi^T\widehat{T}_{t-1} - \omega^{SF}S_t^F + \omega^{SH}S_t^H + \omega^{UF}u_t^F - \omega^{UH}u_t^H.$$

Forward it one period and take expectations:

$$\mathrm{E}_t\widehat{T}_{t+1} = \chi^T\widehat{T}_t - \omega^{SF}\gamma^{SF}S_t^F + \omega^{SH}\gamma^{SH}S_t^H + \omega^{UF}\gamma^{UF}u_t^F - \omega^{UH}\gamma^{UH}u_t^H.$$

Combine it with the difference equation to get:

$$\begin{aligned}
& \chi^T \widehat{T}_t - \omega^{SF}\gamma^{SF} S_t^F + \omega^{SH}\gamma^{SH} S_t^H + \omega^{UF}\gamma^{UF} u_t^F - \omega^{UH}\gamma^{UH} u_t^H \\
= & \left(1+\beta^{-1}\left(1+\kappa\left(1+\eta c_Y\right)\right)\right)\widehat{T}_t - \beta^{-1}\widehat{T}_{t-1} \\
& +\beta^{-1}\kappa\left(1+\eta c_Y\right)\Gamma S_t^F - \beta^{-1}\kappa\left(1+\eta c_Y\right)\Gamma S_t^H - \beta^{-1}\left(u_t^F - u_t^H\right).
\end{aligned}$$

so as to get

$$\begin{aligned}
\widehat{T}_t = & -\frac{\beta^{-1}}{\chi^T - \left(1+\beta^{-1}\left(1+\kappa\left(1+\eta c_Y\right)\right)\right)}\widehat{T}_{t-1} \\
& +\frac{\beta^{-1}\kappa\left(1+\eta c_Y\right)\Gamma + \omega^{SF}\gamma^{SF}}{\chi^T - \left(1+\beta^{-1}\left(1+\kappa\left(1+\eta c_Y\right)\right)\right)} S_t^F - \frac{\beta^{-1}\kappa\left(1+\eta c_Y\right)\Gamma + \omega^{SH}\gamma^{SH}}{\chi^T - \left(1+\beta^{-1}\left(1+\kappa\left(1+\eta c_Y\right)\right)\right)} S_t^H \\
& -\frac{\beta^{-1} + \omega^{UF}\gamma^{UF}}{\chi^T - \left(1+\beta^{-1}\left(1+\kappa\left(1+\eta c_Y\right)\right)\right)} u_t^F + \frac{\beta^{-1} + \omega^{UH}\gamma^{UH}}{\chi^T - \left(1+\beta^{-1}\left(1+\kappa\left(1+\eta c_Y\right)\right)\right)} u^H.
\end{aligned}$$

So the undetermined coefficients are determined by

$$\begin{aligned}
\chi^T &= -\frac{\beta^{-1}}{\chi^T - \left(1+\beta^{-1}\left(1+\kappa\left(1+\eta c_Y\right)\right)\right)}, \\
\omega^{SF} &= -\frac{\beta^{-1}\kappa\left(1+\eta c_Y\right)\Gamma + \omega^{SF}\gamma^{SF}}{\chi^T - \left(1+\beta^{-1}\left(1+\kappa\left(1+\eta c_Y\right)\right)\right)}, \\
\omega^{SH} &= -\frac{\beta^{-1}\kappa\left(1+\eta c_Y\right)\Gamma + \omega^{SH}\gamma^{SH}}{\chi^T - \left(1+\beta^{-1}\left(1+\kappa\left(1+\eta c_Y\right)\right)\right)}, \\
\omega^{UF} &= -\frac{\beta^{-1} + \omega^{UF}\gamma^{UF}}{\chi^T - \left(1+\beta^{-1}\left(1+\kappa\left(1+\eta c_Y\right)\right)\right)}, \\
\omega^{UH} &= -\frac{\beta^{-1} + \omega^{UH}\gamma^{UH}}{\chi^T - \left(1+\beta^{-1}\left(1+\kappa\left(1+\eta c_Y\right)\right)\right)}.
\end{aligned}$$

As in the previous subsection we find χ^T as

$$0 < \chi^T = \frac{1+\beta^{-1}\left(1+\kappa\left(1+\eta c_Y\right)\right) - \sqrt{\left(1+\beta^{-1}\left(1+\kappa\left(1+\eta c_Y\right)\right)\right)^2 - 4\beta^{-1}}}{2} < 1.$$

Using that

$$\chi^T\beta = -\frac{1}{\chi^T - \left(1+\beta^{-1}\left(1+\kappa\left(1+\eta c_Y\right)\right)\right)},$$

we simplify the identification of the remainder parameters as

$$\begin{aligned}
\omega^{SF} &= \chi^T\beta\left(\beta^{-1}\kappa\left(1+\eta c_Y\right)\Gamma+\omega^{SF}\gamma^{SF}\right),\\
\omega^{SH} &= \chi^T\beta\left(\beta^{-1}\kappa\left(1+\eta c_Y\right)\Gamma+\omega^{SH}\gamma^{SH}\right),\\
\omega^{UF} &= \chi^T\beta\left(\beta^{-1}+\omega^{UF}\gamma^{UF}\right),\\
\omega^{UH} &= \chi^T\beta\left(\beta^{-1}+\omega^{UH}\gamma^{UH}\right).
\end{aligned}$$

So we get

$$\begin{aligned}
\omega^{SF} &= \frac{\chi^T\kappa\left(1+\eta c_Y\right)\Gamma}{1-\chi^T\beta\gamma^{SF}}, \qquad \omega^{SH}=\frac{\chi^T\kappa\left(1+\eta c_Y\right)\Gamma}{1-\chi^T\beta\gamma^{SH}},\\
\omega^{UF} &= \frac{\chi^T}{1-\chi^T\beta\gamma^{UF}}, \qquad \omega^{UH}=\frac{\chi^T}{1-\chi^T\beta\gamma^{UH}}.
\end{aligned}$$

We note that

$$\begin{aligned}
\pi_t^R &= \widehat{T}_t-\widehat{T}_{t-1}\\
&= \chi^T\widehat{T}_{t-1}-\omega^{SF}S_t^F+\omega^{SH}S_t^H+\omega^{UF}u_t^F-\omega^{UH}u_t^H-\widehat{T}_{t-1}\\
&= -\left(1-\chi^T\right)\widehat{T}_{t-1}-\omega^{SF}S_t^F+\omega^{SH}S_t^H+\omega^{UF}u_t^F-\omega^{UH}u_t^H.
\end{aligned}$$

To repeat, the closed-form solutions for $\widehat{T}_t$ and π_t^R, which we discuss in the main text, are given by

$$\widehat{T}_t=\chi^T\widehat{T}_{t-1}-\omega^{SF}S_t^F+\omega^{SH}S_t^H+\omega^{UF}u_t^F-\omega^{UH}u_t^H, \tag{I.16}$$

and

$$\pi_t^R=-\left(1-\chi^T\right)\widehat{T}_{t-1}-\omega^{SF}S_t^F+\omega^{SH}S_t^H+\omega^{UF}u_t^F-\omega^{UH}u_t^H, \tag{I.17}$$

with

$$0<\chi^T=\frac{1+\beta^{-1}\left(1+\kappa\left(1+\eta c_Y\right)\right)-\sqrt{\left(1+\beta^{-1}\left(1+\kappa\left(1+\eta c_Y\right)\right)\right)^2-4\beta^{-1}}}{2}<1,$$

$$\begin{aligned}
\omega^{SF} &= \frac{\chi^T\kappa\left(1+\eta c_Y\right)\Gamma}{1-\chi^T\beta\gamma^{SF}}, \qquad \omega^{SH}=\frac{\chi^T\kappa\left(1+\eta c_Y\right)\Gamma}{1-\chi^T\beta\gamma^{SH}},\\
\omega^{UF} &= \frac{\chi^T}{1-\chi^T\beta\gamma^{UF}}, \qquad \omega^{UH}=\frac{\chi^T}{1-\chi^T\beta\gamma^{UH}}.
\end{aligned}$$

We can then finally find the solution for the local inflation rates as

$$\begin{aligned}
\pi_t^H &= \pi_t^W - (1-n)\,\pi_t^R \\
&= \frac{c_Y}{\sigma}\left(1-\chi^C\right)\left(\widehat{C}_{t-1}-\widetilde{C}_{t-1}\right)+\frac{c_Y}{\sigma}\varphi^{UH}u_t^H+\frac{c_Y}{\sigma}\varphi^{UF}u_t^F \\
&\quad -(1-n)\left[-\left(1-\chi^T\right)\widehat{T}_{t-1}-\omega^{SF}S_t^F+\omega^{SH}S_t^H+\omega^{UF}u_t^F-\omega^{UH}u_t^H\right],
\end{aligned}$$

and

$$\begin{aligned}
\pi_t^F &= \pi_t^W + n\pi_t^R \\
&= \frac{c_Y}{\sigma}\left(1-\chi^C\right)\left(\widehat{C}_{t-1}-\widetilde{C}_{t-1}\right)+\frac{c_Y}{\sigma}\varphi^{UH}u_t^H+\frac{c_Y}{\sigma}\varphi^{UF}u_t^F \\
&\quad +n\left[-\left(1-\chi^T\right)\widehat{T}_{t-1}-\omega^{SF}S_t^F+\omega^{SH}S_t^H+\omega^{UF}u_t^F-\omega^{UH}u_t^H\right].
\end{aligned}$$

I.2. Characterization of optimal monetary policies under discretion

Under discretion, monetary policy cannot affect expected future variables (given the absence of endogenous persistence). Hence, it will at date t take $\mathrm{E}_t\pi_{t+1}^H$ and $\mathrm{E}_t\pi_{t+1}^F$ as given. Hence, its optimization implies a sequence of static optimization problems. Furthermore, as the terms of trade is not affected by the consumption gap, the terms of trade will follow the same path as under the precommitment solution.

When setting $\left(\widehat{C}_t-\widetilde{C}_t\right)$, the central bank solves

$$\min\ \lambda_C^2\left(\widehat{C}_t-\widetilde{C}_t\right)+\lambda_T\left(\widehat{T}_t-\widetilde{T}_t\right)^2+\lambda_{\pi^H}\left(\pi_t^H\right)^2+\lambda_{\pi^F}\left(\pi_t^F\right)^2,$$

subject to (I.1) and (I.2). Inserting these directly into the loss function results in an unconstrained minimization problem, and the first-order condition is

$$\begin{aligned}
&\lambda_C\left(\widehat{C}_t-\widetilde{C}_t\right) \\
&+\lambda_{\pi^H}\kappa\left(\rho+\eta c_Y\right)\left(\beta\mathrm{E}_t\pi_{t+1}^H+\kappa(1-n)\left(1+\eta c_Y\right)\left(\widehat{T}_t-\widetilde{T}_t\right)+\kappa\left(\rho+\eta c_Y\right)\left(\widehat{C}_t-\widetilde{C}_t\right)+u_t^H\right) \\
&+\lambda_{\pi^F}\kappa\left(\rho+\eta c_Y\right)\left(\beta\mathrm{E}_t\pi_{t+1}^F-\kappa n\left(1+\eta c_Y\right)\left(\widehat{T}_t-\widetilde{T}_t\right)+\kappa\left(\rho+\eta c_Y\right)\left(\widehat{C}_t-\widetilde{C}_t\right)+u_t^F\right) \\
=\ & 0. \qquad \text{(I.18)}
\end{aligned}$$

Now apply the values of λ_C, λ_{π^H} and λ_{π^F} under symmetry:

$$\begin{aligned}
& (\kappa c_Y (\rho + \eta c_Y)/\sigma) \left(\widehat{C}_t - \widetilde{C}_t\right) \\
& + n\kappa (\rho + \eta c_Y) \left(\beta \mathrm{E}_t \pi^H_{t+1} + \kappa (1-n)(1+\eta c_Y) \left(\widehat{T}_t - \widetilde{T}_t\right) + \kappa (\rho + \eta c_Y) \left(\widehat{C}_t - \widetilde{C}_t\right) + u^H_t\right) \\
& + (1-n) \kappa (\rho + \eta c_Y) \left(\beta \mathrm{E}_t \pi^F_{t+1} - \kappa n (1+\eta c_Y) \left(\widehat{T}_t - \widetilde{T}_t\right) + \kappa (\rho + \eta c_Y) \left(\widehat{C}_t - \widetilde{C}_t\right) + u^F_t\right) \\
= & \; 0.
\end{aligned}$$

This reduces to

$$\begin{aligned}
& (c_Y/\sigma) \left(\widehat{C}_t - \widetilde{C}_t\right) \\
& + n \left(\beta \mathrm{E}_t \pi^H_{t+1} + \kappa (1-n)(1+\eta c_Y) \left(\widehat{T}_t - \widetilde{T}_t\right) + \kappa (\rho + \eta c_Y) \left(\widehat{C}_t - \widetilde{C}_t\right) + u^H_t\right) \\
& + (1-n) \left(\beta \mathrm{E}_t \pi^F_{t+1} - \kappa n (1+\eta c_Y) \left(\widehat{T}_t - \widetilde{T}_t\right) + \kappa (\rho + \eta c_Y) \left(\widehat{C}_t - \widetilde{C}_t\right) + u^F_t\right) \\
= & \; 0,
\end{aligned}$$

and, then, by (I.1) and (I.2) again, to

$$\pi^W_t = -\frac{c_Y}{\sigma} \left(\widehat{C}_t - \widetilde{C}_t\right).$$

I.2.1. Deriving the solutions for $\left(\widehat{C}_t - \widetilde{C}_t\right)$ and π^W_t

To obtain solutions we combine

$$\pi^W_t = -\frac{c_Y}{\sigma} \left(\widehat{C}_t - \widetilde{C}_t\right),$$

and

$$\pi^W_t = \beta \mathrm{E}_t \pi^W_{t+1} + \kappa (\rho + \eta c_Y) \left(\widehat{C}_t - \widetilde{C}_t\right) + u^W_t,$$

Eliminate the consumption gap from the Phillips curve:

$$\begin{aligned}
\pi^W_t &= \beta \mathrm{E}_t \pi^W_{t+1} - \frac{\kappa \sigma (\rho + \eta c_Y)}{c_Y} \pi^W_t + u^W_t, \\
\pi^W_t [1 + \kappa \sigma (\rho + \eta c_Y)/c_Y] &= \beta \mathrm{E}_t \pi^W_{t+1} + u^W_t,
\end{aligned}$$

or,

$$\mathrm{E}_t \pi^W_{t+1} = \beta^{-1} [1 + \kappa \sigma (\rho + \eta c_Y)/c_Y] \pi^W_t - \beta^{-1} \left[n u^H_t + (1-n) u^F_t\right].$$

Conjecture a solution of the form

$$\pi_t^W = \varphi^{WH} u_t^H + \varphi^{WF} u_t^F,$$

where φ^{WH} and φ^{WF} are to be determined. Forward the conjecture and take period t expectations:

$$\mathrm{E}_t \pi_{t+1}^W = \varphi^{WH} \gamma^{UH} u_t^H + \varphi^{WF} \gamma^{UF} u_t^F.$$

We then get

$$\begin{aligned}
& \varphi^{WH} \gamma^{UH} u_t^H + \varphi^{WF} \gamma^{UF} u_t^F \\
= \; & \beta^{-1} \left[1 + \kappa\sigma\left(\rho + \eta c_Y\right)/c_Y\right] \left(\varphi^{WH} u_t^H + \varphi^{WF} u_t^F\right) \\
& - \beta^{-1} \left[n u_t^H + (1-n) u_t^F\right],
\end{aligned}$$

which identify the unknown coefficients according to

$$\begin{aligned}
\varphi^{WH} \gamma^{UH} &= \beta^{-1} \left[1 + \kappa\sigma\left(\rho + \eta c_Y\right)/c_Y\right] \varphi^{WH} - \beta^{-1} n, \\
\varphi^{WF} \gamma^{UF} &= \beta^{-1} \left[1 + \kappa\sigma\left(\rho + \eta c_Y\right)/c_Y\right] \varphi^{WF} - \beta^{-1} (1-n).
\end{aligned}$$

It thus follows that

$$\varphi^{WH} = -\frac{\beta^{-1} n}{\gamma^{UH} - \beta^{-1} \left[1 + \kappa\sigma\left(\rho + \eta c_Y\right)/c_Y\right]},$$

or, more conveniently,

$$\begin{aligned}
\varphi^{WH} &= \frac{n}{\kappa\sigma\left(\rho + \eta c_Y\right)/c_Y + 1 - \beta\gamma^{UH}} > 0, \\
\varphi^{WF} &= \frac{1-n}{\kappa\sigma\left(\rho + \eta c_Y\right)/c_Y + 1 - \beta\gamma^{UF}} > 0.
\end{aligned}$$

To sum up, the solutions for world inflation and the consumption gap, which we discuss in the main text, are

$$\pi_t^W = \varphi^{WH} u_t^H + \varphi^{WF} u_t^F,$$

and

$$\widehat{C}_t - \widetilde{C}_t = -\frac{\sigma}{c_Y} \varphi^{WH} u_t^H - \frac{\sigma}{c_Y} \varphi^{WF} u_t^F,$$

where

$$\varphi^{WH} = \frac{n}{\kappa\sigma\left(\rho+\eta c_Y\right)/c_Y + 1 - \beta\gamma^{UH}} > 0,$$
$$\varphi^{WF} = \frac{1-n}{\kappa\sigma\left(\rho+\eta c_Y\right)/c_Y + 1 - \beta\gamma^{UF}} > 0.$$

I.2.2. Deriving the solutions for $\widehat{T}_t$ and π_t^R

As we notice in the main text, the solutions for $\widehat{T}_t$ and π_t^R are independent of the monetary regime and, therefore, again given by (I.16) and (I.17), respectively.

We then get the local inflation rates as

$$\begin{aligned}\pi_t^H &= \pi_t^W - (1-n)\,\pi_t^R \\ &= \varphi^{WH}u_t^H + \varphi^{WF}u_t^F \\ &\quad - (1-n)\left[-\left(1-\chi^T\right)\widehat{T}_{t-1} - \omega^{SF}S_t^F + \omega^{SH}S_t^H + \omega^{UF}u_t^F - \omega^{UH}u_t^H\right],\end{aligned}$$

and

$$\begin{aligned}\pi_t^F &= \pi_t^W + n\pi_t^R \\ &= \varphi^{WH}u_t^H + \varphi^{WF}u_t^F \\ &\quad + n\left[-\left(1-\chi^T\right)\widehat{T}_{t-1} - \omega^{SF}S_t^F + \omega^{SH}S_t^H + \omega^{UF}u_t^F - \omega^{UH}u_t^H\right].\end{aligned}$$

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