

All questions must be answered

In the evaluation, the three main questions will be weighted equally

Henrik Jensen

Department of Economics, University of Copenhagen

August 2008

MONETARY ECONOMICS: MACRO ASPECTS

SUGGESTED ANSWERS TO AUGUST 20 EXAM

QUESTION 1:

Evaluate whether the following statements are true or false. Explain your answers.

- (i) When choosing between the nominal interest rate and the money supply as the policy instrument, money demand shocks play no role in the original Poole model of optimal instrument choice.

FALSE. It is indeed the relative variability of money demand shocks and goods demand shocks that determine which operating procedure is optimal. For example, a relatively high variance of money demand shocks makes a money supply operating procedure unattractive as the shocks will feed into output. This will not happen under an interest rate operating procedure, which is therefore preferable (unless, on the other hand, goods demand shock variance is not too high).

- (ii) In the Fuhrer and Moore model with backward-looking output and inflation expectations, a passive Taylor-type interest rate rule (one where an increase in inflation is met by a less-than-proportional increase in the nominal rate) implies infinitely many bounded rational expectations equilibria.

FALSE. In this model, a passive Taylor rule implies that when a shock causes a deviation from steady state, explosive dynamics will result.

E.g., an expansive demand shock will increase output, then inflation and then lower the real interest rate (due to the passive interest rate rule) which will stimulate demand further and thus inflation, and so on. Hence, there is only one bounded equilibrium, and that is the steady state, which however is unstable.

- (iii) In the Barro and Gordon inflation-bias model with supply shocks, offering the central banker an appropriate linear inflation contract will eliminate the inflation bias, but distort stabilization of shocks.

FALSE. The linear inflation contract offers the optimal incentives to the central bank. It imposes a constant marginal penalty of inflation, which is exactly what is needed to counteract the constant incentive to expand output. Shock stabilization incentives are unaffected by the linear contract, and stabilization policies are not distorted.

- (iv) According to the expectations hypothesis of the term structure, an expected future reduction in the short interest rate will increase current long interest rates.

FALSE. According to the expectations hypothesis of the term structure a long interest rate is the average of current and expected future short interest rates. So if future short rates are expected to be reduced, the current long interest rate will fall immediately.

QUESTION 2:

Rogoff conservativeness in a New-Keynesian Model?

Consider a one-equation variant of a New-Keynesian model of inflation determination:

$$\pi_t = \beta E_t \pi_{t+1} + \lambda x_t + u_t, \quad (1)$$

where π_t is inflation, $0 < \beta < 1$ is a discount factor, E_t is the rational expectations operator, $\lambda > 0$ is a parameter, x_t is the output gap, and u_t is a “cost-push” shock that follows the process

$$u_t = \rho u_{t-1} + \hat{u}_t, \quad 0 < \rho < 1,$$

where \widehat{u}_t is a mean-zero i.i.d. disturbance.

- (i) Discuss the economic mechanisms leading to (1) with particular focus on why expected future inflation determines current inflation.

Equation (1) is an aggregate Phillips-type inflation adjustment mechanism that can be derived from models of staggered price setting. The essential feature is that some firms cannot freely reset their price in every period. Whenever the opportunity for a price change arises, the knowledge of this makes future aggregate prices important for optimal current price decisions. Moreover, prices are set as mark-ups over real marginal costs, and (1) subsumes a positive relationship between the output gap and marginal costs.

It is assumed that the monetary policymaker controls x_t with the aim of maximizing the discounted sum of

$$-\frac{\alpha}{2}x_t^2 - \frac{1}{2}\pi_t^2, \quad \alpha > 0. \quad (2)$$

- (ii) Show that under discretionary policymaking, optimal policy is characterized by

$$-\alpha x_t = \lambda \pi_t. \quad (3)$$

Explain the result intuitively, and describe (in words) how inflation and the output gap will respond to a positive “cost-push” shock.

Under discretion, expectations cannot be affected by policy, so maximizing

$$-\frac{1}{2}\mathbb{E}_t \sum_{i=0}^{\infty} \beta^i [\alpha x_{t+i}^2 + \pi_{t+i}^2], \quad 0 < \beta < 1$$

w.r.t. x_t subject to (2) is equivalent of maximizing

$$-\frac{\alpha}{2}x_t^2 - \frac{1}{2}\pi_t^2 + F_t$$

w.r.t. x_t subject to

$$\pi_t = \lambda x_t + f_t$$

taking as given F_t and f_t . This immediately provides the first-order condition:

$$-\alpha x_t = \lambda \pi_t$$

It describes a “leaning against the wind” policy. If inflationary pressures arise due to a positive “cost-push” shock, the policymaker should contract output ($x_t < 0$) such that the marginal cost of lower output equals the marginal gain of reducing inflation.

(iii) Assume now that the policymaker can commit to a policy of the form

$$x_t^c = -\omega u_t, \quad (4)$$

where ω is a policy-rule parameter. Find the optimal relationship between x_t^c and π_t^c . [Hint: Combine (4) with (1) to show that $\pi_t^c = [\lambda/(1 - \beta\rho)]x_t^c + [1/(1 - \beta\rho)]u_t$.]

Use the hint. Inflation follows from the Phillips curve (1), together with the policy rule (4), as:

$$\begin{aligned} \pi_t^c &= \beta \mathbf{E}_t \pi_{t+1}^c + \lambda x_t^c + u_t, \\ &= \beta \mathbf{E}_t \pi_{t+1}^c - \lambda \omega u_t + u_t. \end{aligned}$$

Solving forward:

$$\begin{aligned} \pi_t^c &= \mathbf{E}_t \sum_{i=0}^{\infty} \beta^i [-\lambda \omega u_{t+i} + u_{t+i}], \\ &= \sum_{i=0}^{\infty} \beta^i [-\lambda \omega + 1] \rho^i u_t, \\ &= \frac{1 - \lambda \omega}{1 - \beta \rho} u_t, \end{aligned}$$

or,

$$\begin{aligned} \pi_t^c &= -\frac{\lambda}{1 - \beta \rho} \omega u_t + \frac{1}{1 - \beta \rho} u_t \\ &= \frac{\lambda}{1 - \beta \rho} x_t^c + \frac{1}{1 - \beta \rho} u_t \end{aligned}$$

Then maximize

$$-\frac{1}{2} [\alpha (x_t^c)^2 + (\pi_t^c)^2]$$

w.r.t. subject x_t^c to subject to the expression for π_t^c . This gives the first-order condition:

$$\alpha x_t^c + \frac{\lambda}{1 - \beta \rho} \pi_t^c = 0.$$

Or written like in the discretionary case:

$$-\alpha x_t^c = \frac{\lambda}{1 - \beta\rho} \pi_t^c.$$

- (iv) Discuss, based on the result of (iii), whether appointing a conservative policy-maker, one characterized by $\alpha^c < \alpha$, is beneficial. Comment in particular on whether $\rho > 0$ is crucial for the answer.

One can rewrite the found relationship in (iii) as

$$-\alpha(1 - \beta\rho)x_t^c = \lambda\pi_t^c,$$

or,

$$-\alpha^c x_t^c = \lambda\pi_t^c,$$

where

$$\alpha^c \equiv \alpha(1 - \beta\rho) \leq \alpha.$$

So it is seen that as long as $\rho > 0$, the commitment solution features $\alpha^c < \alpha$, i.e., a smaller weight on output than in the social utility function. Hence, Rogoff-conservatism is optimal. It is, however, crucial that $\rho > 0$, as this implies that a current shock has implications for the future. And in this model with forward-looking expectations it is the ability to affect future expectations that is the benefit of commitment. If $\rho = 0$, the future is not affected by current shocks and there is no need to try to affect future expectations by acting conservative. But if the shock persists, being conservative implies a tougher stance on future inflation, which helps stabilize current inflation better.

QUESTION 3:

Monetary policy with a “cash-in-advance” constraint

Consider an economy formulated in discrete time and under certainty, where the utility of a representative agent is given by

$$\sum_{t=0}^{\infty} \beta^t \ln(c_t), \quad 0 < \beta < 1, \tag{1}$$

where c_t is real consumption. The agent faces the budget constraint

$$\omega_t \equiv f(k_{t-1}) + \tau_t + \frac{m_{t-1} + (1 + i_{t-1})b_{t-1}}{1 + \pi_t} = c_t + k_t + m_t + b_t, \quad (2)$$

where k_{t-1} is real capital at the end of period $t - 1$, f is a production function where $f' > 0$, $f'' < 0$, τ_t denotes real monetary transfers from the government, m_{t-1} denotes real money holdings at the end of period $t - 1$, i_{t-1} is the nominal interest rate on bonds (denoted b_{t-1} in real terms), and π_t is the rate of inflation.

The agent also faces the following cash-in-advance constraint on consumption:

$$c_t \leq \frac{m_{t-1}}{1 + \pi_t} + \tau_t. \quad (3)$$

- (i) Find the optimal choices of consumption, capital and real money holdings. For that purpose, show first that (2) can be rewritten as

$$\omega_{t+1} = f(k_t) + \tau_{t+1} + \frac{m_t}{1 + \pi_{t+1}} + R_t(\omega_t - c_t - k_t - m_t),$$

where $R_t \equiv (1 + i_t) / (1 + \pi_{t+1})$ is the real interest rate. Use that the agent's optimization problem can be characterized by

$$V(\omega_t, m_{t-1}) = \max \left\{ \ln(c_t) + \beta V(\omega_{t+1}, m_t) - \mu_t \left(c_t - \frac{m_{t-1}}{1 + \pi_t} - \tau_t \right) \right\},$$

where maximization is over c , k , and m , and where μ_t is the multiplier on (3). Derive and interpret the necessary optimality conditions:

$$\begin{aligned} \frac{1}{c_t} &= \beta R_t V_\omega(\omega_{t+1}, m_t) + \mu_t, \\ \beta V_\omega(\omega_{t+1}, m_t) f_k(k_t) &= \beta R_t V_\omega(\omega_{t+1}, m_t), \\ \beta \frac{1}{1 + \pi_{t+1}} V_\omega(\omega_{t+1}, m_t) + \beta V_m(\omega_{t+1}, m_t) &= \beta R_t V_\omega(\omega_{t+1}, m_t). \end{aligned}$$

The wealth definition in (2) can be forwarded one period:

$$\omega_{t+1} = f(k_t) + \tau_{t+1} + (1 - \delta)k_t + \frac{m_t + (1 + i_t)b_t}{1 + \pi_{t+1}}.$$

Then using that $b_t = \omega_t - c_t - k_t - m_t$, and $R_t \equiv (1 + i_t) / (1 + \pi_{t+1})$, this expression can be written as

$$\omega_{t+1} = f(k_t) + \tau_{t+1} + (1 - \delta)k_t + \frac{m_t}{1 + \pi_{t+1}} + R_t(\omega_t - c_t - k_t - m_t),$$

The agent's optimization problem is characterized by

$$V(\omega_t, m_{t-1}) = \max \left\{ \ln(c_t) + \beta V(\omega_{t+1}, m_t) - \mu_t \left(c_t - \frac{m_{t-1}}{1 + \pi_t} - \tau_t \right) \right\}.$$

The first-order condition w.r.t. c_t is

$$\frac{1}{c_t} - \beta R_t V_\omega(\omega_{t+1}, m_t) - \mu_t = 0$$

from which one readily recovers

$$\frac{1}{c_t} = \beta R_t V_\omega(\omega_{t+1}, m_t) + \mu_t.$$

This states that at the optimum the agent chooses consumption at t such that the marginal gain in terms of marginal utility equals the marginal loss, which takes the form of the utility loss arising from less wealth in the next period (multiplied by the real interest rate and discounted back to period t by β) as well as the loss accruing from the CIA constraint.

The first-order condition w.r.t. k_t is

$$\beta V_\omega(\omega_{t+1}, m_t) [f_k(k_t) + 1 - \delta] - \beta R_t V_\omega(\omega_{t+1}, m_t) = 0$$

which is readily rewritten as

$$\beta V_\omega(\omega_{t+1}, m_t) [f_k(k_t) + 1 - \delta] = \beta R_t V_\omega(\omega_{t+1}, m_t).$$

This states that capital is chosen such that the associated marginal gain in terms of more wealth in next period (multiplied by the net marginal product of capital), equals the marginal loss in terms of lower wealth in form of bonds (multiplied by the real interest rate). Note that this expression delivers the familiar relationship $R_t = f_k(k_t) + 1 - \delta$.

The first-order condition with respect to m_t is

$$\beta \frac{1}{1 + \pi_{t+1}} V_\omega(\omega_{t+1}, m_t) + \beta V_m(\omega_{t+1}, m_t) - \beta R_t V_\omega(\omega_{t+1}, m_t) = 0$$

which is readily rewritten as

$$\beta \frac{1}{1 + \pi_{t+1}} V_\omega(\omega_{t+1}, m_t) + \beta V_m(\omega_{t+1}, m_t) = \beta R_t V_\omega(\omega_{t+1}, m_t).$$

This states that real money holdings are chosen such that the marginal gains (in terms of more wealth in the next period as well as more utility of money per se due to the liquidity services it provides, if $V_m(\omega_{t+1}, m_t) > 0$, which holds when the CIA constraint binds) equal the marginal loss in terms of the utility loss of lower financial wealth.

(ii) Show—using the Envelope theorem—that

$$V_\omega(\omega_t, m_{t-1}) = \beta R_t V_\omega(\omega_{t+1}, m_t),$$

$$V_m(\omega_t, m_{t-1}) = \mu_t \frac{1}{1 + \pi_t}.$$

Differentiating the value function with respect to ω_t and taking into account that c_t , k_t and m_t will be optimal functions of the states (ω_t and m_{t-1}) whereby one can ignore any effects of ω_t on those variables, one immediately obtains

$$V_\omega(\omega_t, m_{t-1}) - \beta R_t V_\omega(\omega_{t+1}, m_t) = 0,$$

or

$$V_\omega(\omega_t, m_{t-1}) = \beta R_t V_\omega(\omega_{t+1}, m_t).$$

Likewise, differentiating the value function with respect to m_{t-1} holding constant c_t , k_t and m_t , one gets

$$V_m(\omega_t, m_{t-1}) - \mu_t \frac{1}{1 + \pi_t} = 0$$

or

$$V_m(\omega_t, m_{t-1}) = \mu_t \frac{1}{1 + \pi_t}.$$

Here one sees that the marginal value of money is only positive if the CIA constraint binds; i.e., in which case money provides liquidity services.

(iii) Define $\lambda_t \equiv V_\omega(\omega_t, m_{t-1})$, and use the result from (ii), with the money demand relation from (i), to obtain an expression for the nominal interest rate, i_t , as a function of μ_{t+1} and λ_{t+1} . Explain this relationship with particular focus on the role of the cash-in-advance constraint.

The money demand function, restated here,

$$\beta \frac{1}{1 + \pi_{t+1}} V_\omega(\omega_{t+1}, m_t) + \beta V_m(\omega_{t+1}, m_t) = \beta R_t V_\omega(\omega_{t+1}, m_t)$$

can now with the expression for the value function derivatives found in (iii) be expressed as

$$\frac{1}{1 + \pi_{t+1}} \lambda_{t+1} + \mu_{t+1} \frac{1}{1 + \pi_{t+1}} = R_t \lambda_{t+1}.$$

This yields

$$\begin{aligned} \lambda_{t+1} + \mu_{t+1} &= (1 + i_t) \lambda_{t+1}, \\ \frac{\lambda_{t+1} + \mu_{t+1}}{\lambda_{t+1}} &= (1 + i_t), \end{aligned}$$

and thus

$$i_t = \frac{\mu_{t+1}}{\lambda_{t+1}}.$$

It is thus seen that the nominal interest rate is only positive if the CIA binds. In that case money is necessary to purchase goods, and the price of goods is increased by the opportunity cost of holding that money, and that is indeed when the nominal interest rate is positive.

- (iv) Has monetary policy—here different rates of nominal money growth—real effects on the steady-state value of output? Has it effects on investment or real money holdings? Explain the results.

From the expression

$$V_\omega(\omega_t, m_{t-1}) = \beta R_t V_\omega(\omega_{t+1}, m_t).$$

one immediately recovers the steady-state relationship

$$R^{ss} = \frac{1}{\beta}$$

Combining this with the expression for the real interest rate, one gets

$$\frac{1}{\beta} = f_k(k^{ss}) + 1 - \delta$$

Hence, the capital stock is determined independent of monetary factors. The reason is that the capital accumulation process in the model is not distorted by the CIA constraint. The steady-state real interest rate is given by the households' subjective real interest rate ($1/\beta$), and that is exclusively given by the net marginal product of capital. From the national account identity, $c^{ss} = f(k^{ss}) - \delta k^{ss}$, it thus follows

that consumption and investment are unaffected as well. Different long-run monetary growth rates will therefore only affect monetary factors. Higher money growth will lead to higher inflation, and thus, as the real interest rate is constant, to higher nominal interest rates. This will lead to lower real money holdings.