

This set contains four pages (beginning with this page)

All questions must be answered

In the evaluation, the three main questions will be weighted equally

Henrik Jensen

Department of Economics, University of Copenhagen

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QUESTION 1:

Evaluate whether the following statements are true or false. Explain your answers.

- (i) When choosing between the nominal interest rate and the money supply as the policy instrument, money demand shocks play no role in the original Poole model of optimal instrument choice.
- (ii) In the Fuhrer and Moore model with backward-looking output and inflation expectations, a passive Taylor-type interest rate rule (one where an increase in inflation is met by a less-than-proportional increase in the nominal rate) implies infinitely many bounded rational expectations equilibria.
- (iii) In the Barro and Gordon inflation-bias model with supply shocks, offering the central banker an appropriate linear inflation contract will eliminate the inflation bias, but distort stabilization of shocks.
- (iv) According to the expectations hypothesis of the term structure, an expected future reduction in the short interest rate will increase current long interest rates.

QUESTION 2:

Rogoff conservativeness in a New-Keynesian Model?

Consider a one-equation variant of a New-Keynesian model of inflation determination:

$$\pi_t = \beta E_t \pi_{t+1} + \lambda x_t + u_t, \quad (1)$$

where π_t is inflation, $0 < \beta < 1$ is a discount factor, E_t is the rational expectations operator, $\lambda > 0$ is a parameter, x_t is the output gap, and u_t is a “cost-push” shock that follows the process

$$u_t = \rho u_{t-1} + \hat{u}_t, \quad 0 < \rho < 1,$$

where \hat{u}_t is a mean-zero i.i.d. disturbance.

- (i) Discuss the economic mechanisms leading to (1) with particular focus on why expected future inflation determines current inflation.

It is assumed that the monetary policymaker controls x_t with the aim of maximizing the discounted sum of

$$-\frac{\alpha}{2} x_t^2 - \frac{1}{2} \pi_t^2, \quad \alpha > 0. \quad (2)$$

- (ii) Show that under discretionary policymaking, optimal policy is characterized by

$$-\alpha x_t = \lambda \pi_t. \quad (3)$$

Explain the result intuitively, and describe (in words) how inflation and the output gap will respond to a positive “cost-push” shock.

- (iii) Assume now that the policymaker can commit to a policy of the form

$$x_t^c = -\omega u_t, \quad (4)$$

where ω is a policy-rule parameter. Find the optimal relationship between x_t^c and π_t^c . [Hint: Combine (4) with (1) to show that $\pi_t^c = [\lambda / (1 - \beta\rho)] x_t^c + [1 / (1 - \beta\rho)] u_t$.]

- (iv) Discuss, based on the result of (iii), whether appointing a conservative policymaker, one characterized by $\alpha^c < \alpha$, is beneficial. Comment in particular on whether $\rho > 0$ is crucial for the answer.

QUESTION 3:

Monetary policy with a “cash-in-advance” constraint

Consider an economy formulated in discrete time and under certainty, where the utility of a representative agent is given by

$$\sum_{t=0}^{\infty} \beta^t \ln(c_t), \quad 0 < \beta < 1, \quad (1)$$

where c_t is real consumption. The agent faces the budget constraint

$$\omega_t \equiv f(k_{t-1}) + \tau_t + \frac{m_{t-1} + (1 + i_{t-1})b_{t-1}}{1 + \pi_t} = c_t + k_t + m_t + b_t, \quad (2)$$

where k_{t-1} is real capital at the end of period $t - 1$, f is a production function where $f' > 0$, $f'' < 0$, τ_t denotes real monetary transfers from the government, m_{t-1} denotes real money holdings at the end of period $t - 1$, i_{t-1} is the nominal interest rate on bonds (denoted b_{t-1} in real terms), and π_t is the rate of inflation.

The agent also faces the following cash-in-advance constraint on consumption:

$$c_t \leq \frac{m_{t-1}}{1 + \pi_t} + \tau_t. \quad (3)$$

- (i) Find the optimal choices of consumption, capital and real money holdings. For that purpose, show first that (2) can be rewritten as

$$\omega_{t+1} = f(k_t) + \tau_{t+1} + \frac{m_t}{1 + \pi_{t+1}} + R_t(\omega_t - c_t - k_t - m_t),$$

where $R_t \equiv (1 + i_t) / (1 + \pi_{t+1})$ is the real interest rate. Use that the agent's optimization problem can be characterized by

$$V(\omega_t, m_{t-1}) = \max \left\{ \ln(c_t) + \beta V(\omega_{t+1}, m_t) - \mu_t \left(c_t - \frac{m_{t-1}}{1 + \pi_t} - \tau_t \right) \right\},$$

where maximization is over c , k , and m , and where μ_t is the multiplier on (3). Derive and interpret the necessary optimality conditions:

$$\begin{aligned} \frac{1}{c_t} &= \beta R_t V_\omega(\omega_{t+1}, m_t) + \mu_t, \\ \beta V_\omega(\omega_{t+1}, m_t) f_k(k_t) &= \beta R_t V_\omega(\omega_{t+1}, m_t), \\ \beta \frac{1}{1 + \pi_{t+1}} V_\omega(\omega_{t+1}, m_t) + \beta V_m(\omega_{t+1}, m_t) &= \beta R_t V_\omega(\omega_{t+1}, m_t). \end{aligned}$$

(ii) Show—using the Envelope theorem—that

$$V_{\omega}(\omega_t, m_{t-1}) = \beta R_t V_{\omega}(\omega_{t+1}, m_t),$$

$$V_m(\omega_t, m_{t-1}) = \mu_t \frac{1}{1 + \pi_t}.$$

- (iii) Define $\lambda_t \equiv V_{\omega}(\omega_t, m_{t-1})$, and use the result from (ii), with the money demand relation from (i), to obtain an expression for the nominal interest rate, i_t , as a function of μ_{t+1} and λ_{t+1} . Explain this relationship with particular focus on the role of the cash-in-advance constraint.
- (iv) Has monetary policy—here different rates of nominal money growth—real effects on the steady-state value of output? Has it effects on investment or real money holdings? Explain the results.