Suggested answers to written exam in "Monetary Economics: Macro Aspects," June 15, 2007[1]

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QUESTION 1:

(i) **False.** The introduction of a cash-in-advance constraint increases the "price" of purchasing a given good in terms of the opportunity cost of holding the money — the nominal interest rate. Therefore many cases can be thought of where changes in the growth rate of the money supply, leading to changes in the nominal interest rate, will have real effects on agents' decisions. Superneutrality will then fail. An example could be a situation where consumption is subject to a cash-in-advance constraint, while leisure is not. Then, the choice between consumption and leisure is distorted. Another could be a case where some parts of consumption is subject to the constraint whereas other parts are not. Different inflation rates and nominal interest rates will then affect the real choice between consumption types.

(ii) **False.** The coefficients in an estimated interest rate rule does not necessarily say anything about preferences. They may as well reflect different policies for identical preferences because economic structures differ. In the example, country A and B could share identical preferences for an exclusive focus on inflation stability, but in country B, the output gap has proven to be a good indicator for inflation (in contrast with country A). Hence a positive response to x_B does not necessarily reveal a preference for output stability *per se*.

(iii) **False.** The unanticipated Home monetary expansion causes a depreciation, which raises total consumption both domestically and abroad on impact. By market clearing, global output increases as well. Home output increases by more than consumption, and this leads to a Home current account surplus, and wealth is allocated towards Home from Foreign.

[1] Note that the keyword in the title is "suggested." This means that answers pointing to other relvant facts instead of those stated here can be "awarded."

After the shock, a new steady state is reached, where consumption keeps being higher, and where the Home country is financing the higher consumption with interest payments on wealth (in runs a current account deficit). Both countries enjoy a first-order welfare increase due to the higher economic activity on impact, as this mitigates the monopolistic distortion in the economies. The welfare effects of higher long-run consumption, on the other hand, is of second-order.

(iv) **False.** In the New-Keynesian model with forward-looking inflation determination, a commitment to fight future inflation shocks *more* vigorously (i.e., being more conservative), affects current inflation expectations downward, and thus helps current stabilizing inflation. Conservatism is thus beneficial from a pure stabilization policy perspective.

QUESTION 2:

(i) Equation (1) is the Home output schedule where output is increasing in a positive inflation surprise. This could be due to the presence of one-period fixed wages, or Lucas misperceptions effects. A foreign inflation surprise has also effects on Home output. This is positive, and could be due to an associated appreciation of the Home currency causing imported input to becomes cheaper. Moreover, both countries are hit by a common supply shock. Equation (2) is the Foreign symmetric counterpart. Equation (3) is Home welfare that depends negatively on output deviations from zero as well as inflation rates different from zero. Equation (4) is the foreign counterpart.

(ii) Maximizing Home welfare taking as given foreign policy (and expectations), is equivalent of solving

$$\max_{\pi_t} U = \left[-\frac{\lambda}{2}y_t^2 - \frac{1}{2}\pi_t^2\right],$$

$$\max_{\pi_t} U = \left[-\frac{\lambda}{2}\left[\pi_t - \mathrm{E}_{t-1}[\pi_t] + a\left(\pi_t^* - \mathrm{E}_{t-1}[\pi_t^*]\right) + \varepsilon_t\right]^2 - \frac{1}{2}\pi_t^2\right].$$

The first-order condition is:

$$-\lambda\left[\pi_t - \mathrm{E}_{t-1}[\pi_t] + a\left(\pi_t^* - \mathrm{E}_{t-1}[\pi_t^*]\right) + \varepsilon_t\right] - \pi_t = 0.$$

Now use the hint of complete symmetry to get

$$-\lambda\left[\pi_t - \mathrm{E}_{t-1}[\pi_t] + a\left(\pi_t - \mathrm{E}_{t-1}[\pi_t]\right) + \varepsilon_t\right] - \pi_t = 0.$$

Find expectations from

$$-\lambda \mathrm{E}_{t-1}\left[\pi_t - \mathrm{E}_{t-1}[\pi_t] + a\left(\pi_t - \mathrm{E}_{t-1}[\pi_t]\right) + \varepsilon_t\right] - \mathrm{E}_{t-1}\pi_t = 0,$$

which yields

$$\mathrm{E}_{t-1}[\pi_t] = 0.$$

The first-order condition thus reduces to

$$-\lambda\left[\pi_t + a\pi_t + \varepsilon_t\right] - \pi_t = 0,$$

implying that Nash equilibrium policies become

$$\pi_t = \pi_t^* = -\frac{\lambda}{1+\lambda\left(1+a\right)}\varepsilon_t$$

Hence, a negative supply shock is met by expansive policy. The higher is λ, the more expansive are policy as output stability will be weighted more. The higher is a, the international spill-over of policies, the weaker is the response. One can say that each country is counting on the other country to do some of the stabilization (as spill-overs are positive).

(iii) Under cooperation, Home policy solves

$$\begin{aligned}
\max_{\pi_t} U + U^* &= -\frac{\lambda}{2}y_t^2 - \frac{1}{2}\pi_t^2 - \frac{\lambda}{2}y_t^{*2} - \frac{1}{2}\pi_t^{*2}, \\
\max_{\pi_t} U + U^* &= -\frac{\lambda}{2}\left[\pi_t - \mathrm{E}_{t-1}\left[\pi_t\right] + a\left(\pi_t^* - \mathrm{E}_{t-1}\left[\pi_t^*\right]\right) + \varepsilon_t\right]^2 - \frac{1}{2}\pi_t^2 \\
&\quad -\frac{\lambda}{2}\left[\pi_t^* - \mathrm{E}_{t-1}\left[\pi_t^*\right] + a\left(\pi_t - \mathrm{E}_{t-1}\left[\pi_t\right]\right)\right]^2 - \frac{1}{2}\pi_t^{*2}.
\end{aligned}$$

The first-order condition is:

$$\begin{aligned}
&-\lambda\left[\pi_t - \mathrm{E}_{t-1}\left[\pi_t\right] + a\left(\pi_t^* - \mathrm{E}_{t-1}\left[\pi_t^*\right]\right) + \varepsilon_t\right] - \pi_t \\
&-a\left[\pi_t^* - \mathrm{E}_{t-1}\left[\pi_t^*\right] + a\left(\pi_t - \mathrm{E}_{t-1}\left[\pi_t\right]\right)\right] \\
=\;& 0.
\end{aligned}$$

Use again the hint of complete symmetry to get

$$\begin{aligned}
&-\lambda\left[\pi_t - \mathrm{E}_{t-1}\left[\pi_t\right] + a\left(\pi_t - \mathrm{E}_{t-1}\left[\pi_t\right]\right) + \varepsilon_t\right] - \pi_t \\
&-a\left[\pi_t - \mathrm{E}_{t-1}\left[\pi_t\right] + a\left(\pi_t - \mathrm{E}_{t-1}\left[\pi_t\right]\right)\right] \\
=\;& 0.
\end{aligned}$$

and thus

$$-\lambda\left(1+a\right)\left[\pi_t - \mathrm{E}_{t-1}\left[\pi_t\right] + a\left(\pi_t - \mathrm{E}_{t-1}\left[\pi_t\right]\right) + \varepsilon_t\right] - \pi_t = 0.$$

Expectations follow from

$$-\lambda\left(1+a\right)\mathrm{E}_{t-1}\left[\pi_t - \mathrm{E}_{t-1}\left[\pi_t\right] + a\left(\pi_t - \mathrm{E}_{t-1}\left[\pi_t\right]\right) + \varepsilon_t\right] - \mathrm{E}_{t-1}\pi_t = 0,$$

and thus

$$\mathrm{E}_{t-1}\left[\pi_t\right] = 0.$$

The first-order condition therefore becomes

$$-\lambda\left(1+a\right)\left[\left(1+a\right)\pi_t + \varepsilon_t\right] - \pi_t = 0,$$

from which cooperative policies are derived as

$$\pi_t = \pi_t^* = -\frac{\lambda(1+a)}{1+\lambda(1+a)^2}\varepsilon_t.$$

Again, a negative supply shock is met by expansive policies. In comparison with non-cooperation, policies are *more* expansive (as $a > 0$). The reason is that the positive international spill-overs on the other country's output on a unilateral expansion is internalized under cooperation.

(iv) If cooperation is not feasible, one can let non-cooperative policymakers be endowed with different preferences, i.e., a different λ denoted λ^b. As their policies will be

$$\pi_t = \pi_t^* = -\frac{\lambda^b}{1+\lambda^b(1+a)}\varepsilon_t,$$

they implement the cooperative policies, if λ^b is chosen such that

$$\frac{\lambda^b}{1+\lambda^b(1+a)} = \frac{\lambda(1+a)}{1+\lambda(1+a)^2},$$

since this will replicate cooperative policies. From this condition it immediately follows that

$$\lambda^b = \lambda(1+a)$$

will "do the trick." Hence, appointing policymakers that care more about output stabilization will secure implementation of cooperative policies. The intuition is that under non-cooperative policymaking, policymakers ignore the positive international output externalities. By instructing them to put more weight on output stabilization, this inefficiency can be corrected.

QUESTION 3:

(i) Using the hint, the value function can be stated as

$$V(b_{t-1}, m_{t-1}) = \max_{c_t, n_t, m_t} \left\{ \begin{array}{c} \ln c_t + \ln(1-n_t) + \beta V(b_t, m_t) \\ -\mu_t \left[c_t - \dfrac{m_{t-1}}{1+\pi_t} - \tau_t \right] \end{array} \right\},$$

where one from the budget constraint (also using the production function) has that

$$b_t = n_t^{1-\alpha} + \frac{1+i_{t-1}}{1+\pi_t} b_{t-1} + \frac{m_{t-1}}{1+\pi_t} + \tau_t - c_t - m_t.$$

Using this expression for b_t in the value function, the relevant first-order conditions follow as

$$\begin{aligned} 1/c_t - \beta V_b(b_t, m_t) - \mu_t &= 0, \\ -1/(1-n_t) + \beta V_b(b_t, m_t)(1-\alpha) n_t^{-\alpha} &= 0, \\ \beta V_m(b_t, m_t) - \beta V_b(b_t, m_t) &= 0. \end{aligned}$$

All these are interpreted as marginal gains in terms of, respectively, current consumption, leisure and money, being equal to marginal losses in terms of lost future wealth and/or current liquidity costs (of consumption due to the cash-in-advance constraint).

(ii) The value function derivatives are, by application of the envelope theorem (implying that any effect of b_{t-1} and m_{t-1} on c_t, n_t and m_t cancel out by the first-order conditions), found as

$$\begin{aligned} V_b(b_{t-1}, m_{t-1}) &= \beta V_b(b_t, m_t) \frac{1+i_{t-1}}{1+\pi_t}, \\ V_m(b_{t-1}, m_{t-1}) &= \beta V_b(b_t, m_t) \frac{1}{1+\pi_t} + \mu_t \frac{1}{1+\pi_t}. \end{aligned}$$

Use the hint and define

$$\lambda_t \equiv \beta V_b(b_t, m_t).$$

The first of the value function derivatives can therefore be written as

$$\lambda_t = \beta \lambda_{t+1} \frac{1+i_t}{1+\pi_{t+1}}.$$

The second can be rewritten as

$$\begin{aligned} V_m(b_t, m_t) &= \beta V_b(b_{t+1}, m_{t+1}) \frac{1}{1+\pi_{t+1}} + \mu_{t+1} \frac{1}{1+\pi_{t+1}}, \\ V_b(b_t, m_t) &= \beta V_b(b_{t+1}, m_{t+1}) \frac{1}{1+\pi_{t+1}} + \mu_{t+1} \frac{1}{1+\pi_{t+1}}, \\ \beta V_b(b_t, m_t) &= \beta^2 V_b(b_{t+1}, m_{t+1}) \frac{1}{1+\pi_{t+1}} + \beta \mu_{t+1} \frac{1}{1+\pi_{t+1}}, \\ \lambda_t &= \beta \frac{\lambda_{t+1} + \mu_{t+1}}{1+\pi_{t+1}}, \end{aligned}$$

where the second line uses the third of the first-order conditions. The first two first-order conditions can be rewritten as

$$\begin{gathered} 1/c_t - \lambda_t - \mu_t = 0, \\ -1/(1-n_t) + \lambda_t (1-\alpha) n_t^{-\alpha} = 0. \end{gathered}$$

Hence, one has

$$\begin{aligned} 1/c_t - \lambda_t - \mu_t &= 0, \\ -1/(1-n_t) + \lambda_t (1-\alpha) n_t^{-\alpha} &= 0, \\ \lambda_t &= \beta \lambda_{t+1} \frac{1+i_t}{1+\pi_{t+1}}, \\ \lambda_t &= \beta \frac{\lambda_{t+1} + \mu_{t+1}}{1+\pi_{t+1}}, \end{aligned}$$

which in steady state becomes:

$$\begin{aligned} 1/c^{ss} - \lambda^{ss} - \mu^{ss} &= 0, \\ -1/(1-n^{ss}) + \lambda (1-\alpha) (n^{ss})^{-\alpha} &= 0, \\ \beta^{-1} &= \frac{1+i^{ss}}{1+\pi^{ss}}, \\ \beta^{-1} &= \frac{1+\mu^{ss}/\lambda^{ss}}{1+\pi^{ss}}. \end{aligned}$$

This is readily reformulated as

$$\begin{aligned} 1/c^{ss} &= \lambda^{ss}\left(1+i^{ss}\right), \\ 1/\left(1-n^{ss}\right) &= \lambda^{ss}\left(1-\alpha\right)\left(n^{ss}\right)^{-\alpha}, \\ \beta^{-1} &= \frac{1+i^{ss}}{1+\pi^{ss}}, \end{aligned}$$

as required.

(iii) Combining the first two steady-state requirements, on can express employment and consumption as a function of i^{ss}:

$$c^{ss}/\left(1-n^{ss}\right) = \frac{\left(1-\alpha^{ss}\right)\left(n^{ss}\right)^{-\alpha}}{1+i^{ss}}.$$

Then use the hint to express consumption as a function of employment, $c^{ss} = \left(n^{ss}\right)^{1-\alpha}$. One thus gets

$$\begin{aligned} \frac{\left(n^{ss}\right)^{1-\alpha}}{1-n} &= \frac{\left(1-\alpha\right)\left(n^{ss}\right)^{-\alpha}}{1+i^{ss}}, \\ \frac{n^{ss}}{1-n^{ss}} &= \frac{1-\alpha}{1+i^{ss}}, \end{aligned}$$

and thus

$$n^{ss} = \frac{1-\alpha}{i^{ss}+2-\alpha}.$$

One sees that employment is a decreasing function of the nominal interest rate. Monetary superneutrality thus fails in the model, as different inflation rates leads to different nominal interest rates, and thus different employment and output levels. The intuition is that consumption is "taxed" by the cash-in-advance constraint for positive nominal interest rates, while leisure is not. An increasing nominal interest rate thus makes consumption relatively more expensive than leisure, and agents substitute away from consumption and supply less labor.

(iv) The optimal monetary policy is one that alleviates the distortionary nature of the cash-in-advance constraint. Here, this will be one that implements the Friedman rule. I.e., $i^{ss} = 0$. Hence, the optimal employment level is

$$n^{ss} = \frac{1-\alpha}{2-\alpha}.$$

Technically, this can also be seen by finding the utility-maximizing employment level in steady state:

$$\max_{n^{ss}} \left\{ \ln\left[\left(n^{ss}\right)^{1-\alpha}\right] + \ln\left(1-n^{ss}\right) \right\}$$

The first-order condition is

$$\frac{(1-\alpha)}{n^{ss}} = \frac{1}{1-n^{ss}}$$

yielding

$$n^{ss} = \frac{1-\alpha}{2-\alpha}.$$

This is employment in the cash-in-advance economy only for $i^{ss} = 0$.