Written exam for the M.Sc in Economics
Department of Economics, University of Copenhagen

Monetary Economics: Macro Aspects

Semester: Spring 2006

4-hour exam

This set contains three pages. Both questions must be answered.
In the evaluation, the two questions will be weighted equally

Any material is allowed

SUGGESTED ANSWERS AND COMPUTATIONS

QUESTION 1:

Consider an economy where output determination can be compactly described as:

$$y_t = \alpha \left(\pi_t - \mathrm{E}_{t-1}\pi_t \right) + \gamma \pi_t + \varepsilon_t, \qquad \alpha > 0, \qquad \gamma \gtreqless 0, \tag{1}$$

where y_t is log of output, π_t is the inflation rate (the monetary policy instrument), and ε_t is a mean-zero, serially uncorrelated shock with variance $\mathrm{E}_{t-1}\varepsilon_t^2 \equiv \sigma^2$. E_{t-1} is the rational expectations operator conditional on information up to and including period $t-1$.

(i) Discuss thoroughly the three components of output determination as described by (1). Explain in particular which theories can explain the various components.

A The first part states that output is increasing when inflation exceeds expected inflation. This can be explained by, e.g., imperfect information as in the Lucas Island's model, or by the presence of one-period nominal contracts. The second part states generally that inflation may or may not have effects on average output. If there is an effect it may be positive or negative. A number of explanations for this failure of superneutrality can be found in the curriculum. A positive effect can be due to the Tobin effect. A negative effect can be due to labour supply effects in models where the marginal utility of consumption is affected by real money balances (it should be noted that real money balances will decrease with higher inflation in the long run, as nominal interest rates will increase). This will affect the marginal rate of substitution between consumption and leisure. The case of $\gamma = 0$ is also found in many models (e.g., the basic CIA model), and it could be noted that empirically, $\gamma \approx 0$ is probably not unrealistic. The third part is a supply shock. This can be interpreted as stochastic variations in productive capacity, or as variations in costs of inputs beyond those captured by inflation expectations.

(ii) Discuss the output-inflation trade off of monetary policy in the short and long run implied by (1).

A There is, for *given* inflation expectations, always a short-run policy trade off between inflation and output in the sense of a conventional Phillips curve. In the long run, there may be a trade off if $\gamma \neq 0$. It is preferable if the empirical relevance of such trade offs are briefly discussed.

The objective of the economy's central bank in period t is to conduct monetary policy so as to minimize the loss function

$$L = \frac{1}{2}\left[\lambda\left(y_t - k\right)^2 + \pi_t^2\right], \qquad \lambda > 0, \qquad k > 0. \tag{2}$$

(iii) Derive the solution for discretionary monetary policy and the associated solution for output. Describe the solutions intuitively.

A To solve for discretionary policy, insert the output equation into the loss function:

$$L = \frac{1}{2}\left[\lambda\left(\alpha\left(\pi_t - \mathrm{E}_{t-1}\pi_t\right) + \gamma\pi_t + \varepsilon_t - k\right)^2 + \pi_t^2\right].$$

The relevant first-order condition is then

$$\lambda\left(\alpha + \gamma\right)\left(\alpha\left(\pi_t - \mathrm{E}_{t-1}\pi_t\right) + \gamma\pi_t + \varepsilon_t - k\right) + \pi_t = 0.$$

Under rational expectations, expected inflation is then found by taking the expected value of this expression:

$$\lambda\left(\alpha + \gamma\right)\left(\gamma\mathrm{E}_{t-1}\pi_t - k\right) + \mathrm{E}_{t-1}\pi_t = 0,$$

implying

$$\mathrm{E}_{t-1}\pi_t = \frac{\lambda\left(\alpha + \gamma\right)}{1 + \lambda\left(\alpha + \gamma\right)\gamma}k.$$

Actual inflation is found by inserting this expression back into the first-order condition:

$$\lambda\left(\alpha + \gamma\right)\left(\alpha\pi_t - \frac{\lambda\alpha\left(\alpha + \gamma\right)}{1 + \lambda\left(\alpha + \gamma\right)\gamma}k + \gamma\pi_t + \varepsilon_t - k\right) + \pi_t = 0$$

$$\lambda\left(\alpha + \gamma\right)\left(\left(\alpha + \gamma\right)\pi_t + \varepsilon_t - \frac{1 + \lambda\left(\alpha + \gamma\right)^2}{1 + \lambda\left(\alpha + \gamma\right)\gamma}k\right) + \pi_t = 0$$

$$\left(1+\lambda\left(\alpha+\gamma\right)^{2}\right)\pi_{t}+\lambda\left(\alpha+\gamma\right)\varepsilon_{t}-\frac{\lambda\left(\alpha+\gamma\right)\left[1+\lambda\left(\alpha+\gamma\right)^{2}\right]}{1+\lambda\left(\alpha+\gamma\right)\gamma}k=0$$

and thus

$$\pi_{t}=-\frac{\lambda\left(\alpha+\gamma\right)}{1+\lambda\left(\alpha+\gamma\right)^{2}}\varepsilon_{t}+\frac{\lambda\left(\alpha+\gamma\right)}{1+\lambda\left(\alpha+\gamma\right)\gamma}k.$$

Actual output follows as

$$\begin{aligned} y_{t} &= -\frac{\lambda\left(\alpha+\gamma\right)^{2}}{1+\lambda\left(\alpha+\gamma\right)^{2}}\varepsilon_{t}+\frac{\lambda\gamma\left(\alpha+\gamma\right)}{1+\lambda\left(\alpha+\gamma\right)\gamma}k+\varepsilon_{t} \\ &= \frac{1}{1+\lambda\left(\alpha+\gamma\right)}\varepsilon_{t}+\frac{\lambda\gamma\left(\alpha+\gamma\right)}{1+\lambda\left(\alpha+\gamma\right)\gamma}k. \end{aligned}$$

First, it should be noted that for $\gamma = 0$, the model reduces to the conventional Barro and Gordon inflation bias model. Due to the preference for high output, $k > 0$, inflation becomes inefficiently high due to the policymaker's incentive to "surprise" the private sector by higher-than-expected inflation. There will in equilibrium, however, be no effects on output as the private sector rationally foresees this incentive. Moreover, the model features stabilization of the shock, such that its effect is spread out on inflation and output. In the case of $\gamma \neq 0$ several differences occur. The central difference is that the preference for output to equal k will now have an effect on output. Indeed, the policymaker will exploit the long-run inflation output trade off to bring output closer to k at the expense of inflation (or deflation, depending upon the sign and size of γ). Moreover, the presence of a long-run policy trade off also affects stabilization policy (formally, the coefficients on the shock). For example, if γ is very large one can see that output will become more or less fully stabilized. This is because with a very large γ a large degree of output stabilization can be achieved at a small cost of inflation stabilization. Finally, it is worth mentioning that expected inflation can now be negative. If $\gamma < 0$, the policymaker will systematically engineer deflation to raise output. In the special case of $\alpha + \gamma = 0$, inflation expectations are zero, as the short-run incentive to create surprise inflation (parameterized through α) will be nullified by the long-run incentive to create deflation.

(iv) Assume now that the central banker has the ability to commit to a policy before inflation expectations are formed. Derive the solution for monetary policy and output under this assumption, and discuss any differences with the discretionary solutions found under (iii). [Hint: Assume the solution for monetary policy takes the form $\pi_t = a - b\varepsilon_t$. Determine $\mathrm{E}_{t-1}\pi_t$ and find the values of a and b that minimize $\mathrm{E}_{t-1}L$.]

A Use the hint and assume $\pi_t = a - b\varepsilon_t$. It follows then that $\mathrm{E}_{t-1}\pi_t = a$. To find the values of a and b that minimize $\mathrm{E}_{t-1}L$ the latter expression is set up with the output equation and the assumed solution substituted in:

$$\begin{aligned} \mathrm{E}_{t-1}L &= \mathrm{E}_{t-1}\frac{1}{2}\left\{\lambda\left(-\alpha b\varepsilon_t+\gamma a-\gamma b\varepsilon_t+\varepsilon_t-k\right)^2\right. \\ &\quad \left.+\left(a-b\varepsilon_t\right)^2\right\} \end{aligned}$$

First-order condition (minimizing w.r.t. a):

$$\begin{aligned} &\mathrm{E}_{t-1}\left\{\lambda\gamma\left(-\alpha b\varepsilon_t+\gamma a-\gamma b\varepsilon_t+\varepsilon_t-k\right)\right. \\ &\left.+\left(a-b\varepsilon_t\right)\right\} \\ =\ & 0 \end{aligned}$$

Solving out expectations:

$$\lambda\gamma\left(\gamma a-k\right)+a=0$$

leading to

$$a=\frac{\lambda\gamma}{1+\lambda\gamma^2}k$$

First-order condition (minimizing w.r.t. b):

$$\begin{aligned} &\mathrm{E}_{t-1}\left\{-\lambda\left(\alpha+\gamma\right)\varepsilon_t\left(-\alpha b\varepsilon_t+\gamma a-\gamma b\varepsilon_t+\varepsilon_t-k\right)\right. \\ &\left.-\varepsilon_t\left(a-b\varepsilon_t\right)\right\} \\ =\ & 0 \end{aligned}$$

implying

$$\begin{aligned} &\mathrm{E}_{t-1}\left\{-\lambda\left(\alpha+\gamma\right)\left(1-b\left(\alpha+\gamma\right)\right)\varepsilon_t^2+b\varepsilon_t^2\right. \\ &\left.-\lambda\left(\alpha+\gamma\right)\left(\gamma a-k\right)\varepsilon_t-a\varepsilon_t\right\} \\ =\ & 0. \end{aligned}$$

Solving out expectations (using $\mathrm{E}_{t-1}\varepsilon_t^2=\sigma^2$) one gets

$$-\lambda\left(\alpha+\gamma\right)\left(1-b\left(\alpha+\gamma\right)\right)\sigma^2+b\sigma^2=0$$

leading to

$$b=\frac{\lambda\left(\alpha+\gamma\right)}{1+\lambda\left(\alpha+\gamma\right)^2}.$$

The solutions for inflation and output will thus be

$$\begin{aligned} \pi_t &= \frac{\lambda\gamma}{1+\lambda\gamma^2}k-\frac{\lambda\left(\alpha+\gamma\right)}{1+\lambda\left(\alpha+\gamma\right)^2}\varepsilon_t. \\ y_t &= \frac{\lambda\gamma^2}{1+\lambda\gamma^2}k+\frac{1}{1+\lambda\left(\alpha+\gamma\right)^2}\varepsilon_t. \end{aligned}$$

The differences with the discretionary solution is that the average values of output and inflation does not contain the α term. This is because, under commitment, inflation surprises are ruled out (as are the inefficiencies with that incentive). Expected inflation is thus always lower under commitment, irrespective of γ. To prove this (this is not required — the mention of the result is sufficient), it must be the case that

$$\frac{\lambda(\alpha+\gamma)}{1+\lambda(\alpha+\gamma)\gamma}k$$

is increasing in $\alpha > 0$. This is the case if

$$\frac{\lambda(1+\lambda(\alpha+\gamma)\gamma)-\lambda(\alpha+\gamma)\lambda\gamma}{[1+\lambda(\alpha+\gamma)\gamma]^2}k > 0,$$

or,

$$\begin{aligned}
\lambda(1+\lambda(\alpha+\gamma)\gamma)-\lambda(\alpha+\gamma)\lambda\gamma &> 0,\\
1+\lambda(\alpha+\gamma)\gamma-\lambda(\alpha+\gamma)\gamma &> 0,\\
1 &> 0
\end{aligned}$$

which is true. Hence, the model features an inefficient inflation bias of discretionary policy as in the conventional Barro and Gordon model. Output will be lower or higher depending upon the sign of γ. Hence, discretionary policymaking will for $\gamma \neq 0$ imply an output bias, in the sense that the inefficiency of the inflation bias is transmitted onto output (in a direction depending on the long-run effect of monetary policy). Finally, a comparison of the solutions show that the stabilization properties are the same under commitment and discretion. This is the same result as in the Barro and Gordon model, and is not altered by the presence of a long-run policy trade off. This long-run trade off is used optimally both under discretion and commitment, and gives no rise to time-inconsistency problems.

(v) Assume that the relevant welfare measure of the economy is indeed L. Discuss then whether it can be advantageous for the economy to appoint a "conservative" central banker with loss function

$$L = \frac{1}{2}\left[\lambda^c (y_t - k)^2 + \pi_t^2\right], \qquad \lambda > \lambda^c > 0, \tag{3}$$

if commitment is not feasible?

A A conservative central banker is always an advantage. The reason is that discretion implies an inefficient inflation bias (and an associated output bias). As seen from the commitment solution, shock stabilization is efficient under discretion. Appointing a conservative central banker will as in the case of $\gamma = 0$

thus constitute a first-order gain in terms of lower inflation, and a second-order loss in terms of distorted stabilization. Hence, some degree of conservatism is preferable. The presence of a long-run policy trade off does not alter this conclusion.

QUESTION 2:

Assume a model of a closed economy formulated in discrete time, where representative individuals have utility functions

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t), \qquad 0 < \beta < 1, \tag{1}$$

with

$$u(c_t) \equiv \frac{(c_t)^{1-\Phi} - 1}{1-\Phi}, \qquad \Phi > 0,$$

and budget constraints

$$f(k_{t-1}) + \tau_t + (1-\delta) k_{t-1} + \frac{1}{1+\pi_t} m_{t-1} = c_t + k_t + m_t, \tag{2}$$

where c_t is consumption, m_t is real money balances at the end of period t, k_{t-1} is physical capital at the end of period $t-1$, τ_t are monetary transfers, $0 < \delta < 1$ is the depreciation rate of capital, and π_t is the inflation rate. The function f is defined as

$$f(k_{t-1}) \equiv k_{t-1}^{\alpha}, \qquad 0 < \alpha < 1.$$

Purchases of consumption goods as well as investment in physical capital is subject to a cash-in-advance constraint. Formally,

$$c_t + k_t - (1-\delta) k_{t-1} \leq \tau_t + \frac{1}{1+\pi_t} m_{t-1}. \tag{3}$$

(i) Discuss briefly the model as portrayed by (1)-(3).

A The discussion should indeed be brief, with main focus on (3) and its deviation from the standard expression from the curriculum without investment.

(ii) Derive the relevant first-order conditions for optimal individual behavior, For this purpose, use the value function

$$\begin{aligned} V(k_{t-1}, m_{t-1}) &= \max \{ u(c_t) + \beta V(k_t, m_t), \\ &\quad -\mu_t [c_t + k_t - (1-\delta) k_{t-1} - \tau_t - (1/(1+\pi_t)) m_{t-1}] \} \end{aligned}$$

where μ_t is the multiplier on (3), and where the maximization is over c_t, m_t and k_t and subject to (2). [Hint: Simplify the problem by using (2) to substitute out k_t.]

A Do the substitution and one gets:

$$\begin{aligned} V(k_{t-1}, m_{t-1}) &= \max\left\{u(c_t) + \beta V\left(f(k_{t-1}) + \tau_t + (1-\delta)k_{t-1} + \frac{1}{1+\pi_t}m_{t-1} - c_t - m_t, m_t\right)\right. \\ &\quad \left. -\mu_t\left[f(k_{t-1}) - m_t\right]\right\} \end{aligned}$$

The first-order conditions are:

$$u'(c_t) = \beta V_k(k_t, m_t)$$

$$-\beta V_k(k_t, m_t) + \beta V_m(k_t, m_t) + \mu_t = 0$$

The value function derivatives are, after using the Envelope theorem:

$$V_k(k_{t-1}, m_{t-1}) = \beta V_k(k_t, m_t)\left[f'(k_{t-1}) + 1 - \delta\right] - \mu_t f'(k_{t-1})$$

$$V_m(k_{t-1}, m_{t-1}) = \beta V_k(k_t, m_t)\frac{1}{1+\pi_t}$$

(iii) Interpret the first-order conditions intuitively, and show that they can be combined (along with the expressions for the partial derivatives of the value function) into the following steady-state conditions:

$$(c^{ss})^{-\Phi} = \beta V_k(k^{ss}, m^{ss}),$$

$$V_k(k^{ss}, m^{ss}) = \beta V_k(k^{ss}, m^{ss})(1-\delta) + \beta V_m(k^{ss}, m^{ss})\,\alpha(k^{ss})^{\alpha-1},$$

$$V_m(k^{ss}, m^{ss}) = \beta V_k(k^{ss}, m^{ss})\frac{1}{1+\pi^{ss}},$$

where superscript "ss" denotes steady-state values.

A Juggle them around first to eliminate $\mu_t > 0$:

$$\begin{aligned} V_k(k_{t-1}, m_{t-1}) &= \beta V_k(k_t, m_t)\left[f'(k_{t-1}) + 1 - \delta\right] - \mu_t f'(k_{t-1}) \\ &= \beta V_k(k_t, m_t)\left[f'(k_{t-1}) + 1 - \delta\right] - \left[\beta V_k(k_t, m_t) - \beta V_m(k_t, m_t)\right] f'(k_{t-1}) \\ &= \beta V_k(k_t, m_t)(1-\delta) + \beta V_m(k_t, m_t) f'(k_{t-1}) \end{aligned}$$

That is, in steady state

$$u'(c) = \beta V_k$$

$$V_k = \beta V_k(1-\delta) + \beta V_m f'(k)$$

$$V_m = \beta V_k \frac{1}{1+\pi}$$

which by use of the functional forms for utility and production gives the desired expressions.

(iv) Derive the steady-state value of k, and discuss whether or not the model exhibits superneutrality. Explain the result intuitively.

A The last two steady-state equations just become:

$$V_k = \beta V_k (1-\delta) + \beta V_m f'(k)$$

$$V_m \frac{1+\pi}{\beta} = V_k$$

Thus,

$$V_k = \beta V_k (1-\delta) + \beta V_m f'(k)$$

becomes

$$V_m \frac{1+\pi}{\beta} = \beta V_m \frac{1+\pi}{\beta} (1-\delta) + \beta V_m f'(k)$$

$$\frac{1+\pi}{\beta} = (1+\pi)(1-\delta) + \beta f'(k)$$

$$\frac{(1+\pi)\left[\frac{1}{\beta} - 1 + \delta\right]}{\beta} = f'(k)$$

With the production function:

$$\frac{(1+\pi)\left[\frac{1}{\beta} - 1 + \delta\right]}{\beta} = \alpha k^{a-1}$$

$$\frac{(1+\pi)\left[\frac{1}{\beta} - 1 + \delta\right]}{\alpha\beta} = k^{a-1}$$

Hence, in steady state:

$$k^{ss} = \left[\frac{\alpha\beta}{(1+\pi)\left[\frac{1}{\beta} - 1 + \delta\right]}\right]^{\frac{1}{1-\alpha}},$$

and we see that *higher inflation reduces capital formation.* Superneutrality thus fails (in comparison with the model where only consumption is subject to a CIA constraint). The reason is that steady-state inflation raises the steady-state nominal interest rate, and thus the opportunity cost of holding money. As money must be used for investment, capital formation is depressed and steady-state capital becomes lower.

(v) In a similar economy where there is no cash-in-advance constraint, the steady-state value of k is given by

$$k^{ss} = \left[\frac{\alpha}{\frac{1}{\beta} - 1 + \delta}\right]^{\frac{1}{1-\alpha}}.$$

Derive the steady-state monetary policy (i.e., inflation rate) that supports this steady-state value of k in the cash-in-advance economy considered above. Explain the properties of this policy.

A Note that if $\mu = 0$ above, i.e., the case where the CIA constraint is not binding, we would have

$$-\beta V_k(k_t, m_t) + \beta V_m(k_t, m_t) = 0$$

and

$$V_k(k_{t-1}, m_{t-1}) = \beta V_k(k_t, m_t)\left[f'(k_{t-1}) + 1 - \delta\right]$$

In steady state we would then have

$$1 = \beta\left[f'(k) + 1 - \delta\right]$$

$$\frac{1}{\beta} - 1 + \delta = \alpha k^{\alpha-1}$$

$$k = \left[\frac{\alpha}{\frac{1}{\beta} - 1 + \delta}\right]^{\frac{1}{1-\alpha}}$$

A monetary policy relaxing the CIA constraint would therefore be one satisfying

$$\left[\frac{\alpha}{\frac{1}{\beta} - 1 + \delta}\right]^{\frac{1}{1-\alpha}} = \left[\frac{\alpha\beta}{(1+\pi)\left[\frac{1}{\beta} - 1 + \delta\right]}\right]^{\frac{1}{1-\alpha}},$$

or simply

$$\frac{\beta}{1+\pi} = 1,$$

or,

$$\beta - 1 = \pi.$$

This is the *Friedman rule*, implying a zero steady-state nominal interest rate. This would lead to a higher steady-state capital compared with the case of $\pi > \beta - 1$.