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[Notes 4]

On the “enforcement equation” in the repeated game version of the Barro and Gordon model

Q How is the enforcement equation (equation 8.13) on page 11 obtained?

A This equation is quantifying the net loss of cheating on the private sector’s expectation of $\bar{\pi}$ in a given period, say, period t . By the nature of the private sector’s “tit-for-tat” punishment strategy, the net loss occur in period $t + 1$ only (after that, the “good” inflation equilibrium with $\pi = \bar{\pi}$ continues until a potential cheat happens again). By the punishment strategy, the private sector punishes by expecting $\pi_{t+1}^e = \lambda a$; the inflation rate under discretion. The best policy response in that punishment period, will simply be $\pi_{t+1} = \lambda a$. This will also induce $\pi_{t+2}^e = \bar{\pi}$ by the punishment strategy, i.e., a reversal to the “good” inflation rate from period $t + 2$ onwards.

Now, the central bank’s utility of enduring the punishment in period $t + 1$ is given by

$$\lambda (a (\pi_{t+1} - \pi_{t+1}^e)) - \frac{1}{2} (\pi_{t+1})^2,$$

or, with the strategies $\pi_{t+1}^e = \lambda a$, $\pi_{t+1} = \lambda a$, inserted:

$$-\frac{1}{2} (\lambda a)^2.$$

Had the central bank not cheated from the promised inflation rate $\bar{\pi}$ in period t , the good equilibrium would have prevailed in period $t + 1$, and given the utility

$$\begin{aligned} & \lambda (a (\bar{\pi} - \bar{\pi})) - \frac{1}{2} (\bar{\pi})^2 \\ &= -\frac{1}{2} (\bar{\pi})^2. \end{aligned}$$

The net *loss* of cheating in period t is therefore

$$\frac{1}{2} (\lambda a)^2 - \frac{1}{2} (\bar{\pi})^2.$$

This measures in utility terms the loss of cheating in period t : The central bank gets a utility of $-\frac{1}{2} (\lambda a)^2$, while it by not cheating would have gotten $-\frac{1}{2} (\bar{\pi})^2$. As long as $|\bar{\pi}| < \lambda a$, the net loss is positive.

Finally, this net loss is compared to the net gain of cheating. This gain accrues in period t . To make a meaningful comparison between the net gain and net loss, one must discount the period- $t + 1$ net loss by β in order to make it comparable to period- t utility. Hence, the relevant net loss of cheating, or the *enforcement*, is given by

$$\beta \left[\frac{1}{2} (\lambda a)^2 - \frac{1}{2} (\bar{\pi})^2 \right]$$

which is equation (8.13).