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[Notes 2]
On the MIU model, the marginal utility of consumption and marginal value of capital

Q A question on equation (2.51). In the second lecture note (page 20) the interpretation given is, "The marginal gain of consumption (marginal utility of consumption) must equal the expected marginal loss in terms of lower capital in the next period." I have problem with this interpretation. We know that postponed consumptions do have a return which equals the marginal productivity of capital. So what I intuitively expect to have, is something like

$$u_c(c_t, m_t, l_t) = \beta \mathrm{E}_t V_k(a_{t+1}, k_t)(f_k(k_{t-1}, 1 - l_t, z_t) + 1 - \delta) \tag{2.51 conjecture}$$

The reason being that postponed consumption should have some return (which in the in competitive equilibrium equals the marginal product of capital less depreciation). So why we haven't included return on capital when agents postpone their consumption?

A To answer this, let me repeat the correct equation and explain why the conjectured (2.51 conjecture) is not correct. The correct equation reads

$$u_c(c_t, m_t, l_t) = \beta \mathrm{E}_t V_k(a_{t+1}, k_t). \tag{2.51'}$$

The intuition that increasing consumption in period t carries a loss that is dependent on the return to capital, i.e., dependent on the marginal product of capital, is correct. However, that dependence is already captured by the term $V_k(a_{t+1}, k_t)$. As $V(a_{t+1}, k_t)$ denotes the maximal lifetime utility from period $t+1$ and onwards, the term $V_k(a_{t+1}, k_t)$ indeed captures how this lifetime utility changes with a change in k_t. So $\mathrm{E}_t V_k(a_{t+1}, k_t)$ "alone" captures the relevant marginal loss from period $t+1$ and onwards of raising consumption marginally in period t. To compare it with $u_c(c_t, m_t, l_t)$, it is discounted back to period t through multiplication by β.

This, however, does *not* mean that the marginal product of capital does not matter. On the contrary! The intuition hinted at in the question has a point. To see this, go further on in the notes and note that from the Envelope theorem, we get the expression

$$V_k(a_t, k_{t-1}) = \beta \mathrm{E}_t V_k(a_{t+1}, k_t)(f_k(k_{t-1}, 1 - l_t, z_t) + 1 - \delta).$$

(See p. 21.) This is an expectational difference equation in the partial derivative of the value function. Drop for simplicity uncertainty, and one get by repeated forward substitution

$$V_k\left(a_{t,}k_{t-1}\right) = \beta^2 V_k\left(a_{t+2,}k_{t+1}\right)\beta\left(f_k\left(k_t,1-l_{t+1}\right)+1-\delta\right)\left(f_k\left(k_{t-1},1-l_t\right)+1-\delta\right)$$

$$\begin{aligned} V_k\left(a_{t,}k_{t-1}\right) &= \beta^3 V_k\left(a_{t+3,}k_{t+2}\right)\beta^2\left(f_k\left(k_{t+1},1-l_{t+2}\right)+1-\delta\right) \\ &\quad \times\beta\left(f_k\left(k_t,1-l_{t+1}\right)+1-\delta\right)\left(f_k\left(k_{t-1},1-l_t\right)+1-\delta\right) \end{aligned}$$

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$$V_k\left(a_{t,}k_{t-1}\right) = \prod_{i=0}^{\infty}\beta^i\left(f_k\left(k_{t-1+i},1-l_{t+i}\right)+1-\delta\right).$$

Hence, the marginal value of capital will be a function of all future marginal products of capital. Hence, in the first-order condition you equate the marginal utility of consumption with the marginal loss. This loss is the loss of losing the return of having not saved instead. And that loss is determined by the real return *not only tomorrow*, but the day after and the day after, and so on (you lose interest tomorrow, interest on interest the period after, and so on).

To sum up, the marginal product of capital *is* an integral part of measuring the cost of consuming more today. We just don't see it explicitly in (2.51'), as the derivative of the value function has taken it into account already. It is by nature the marginal value of capital.