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**[Notes 2]**
**On the basic CIA model and long-run superneutrality**

Q In the CIA model (page 32), the steady-state consumption is invariant to monetary factors. Since only consumption matters for utility [equation (3.12)], and as steady-state consumption is invariant to inflation, there is no steady-state welfare cost of inflation. On the other hand, the marginal utility of consumption is affected by nominal interest rate (the third equation on page 31). Therefore, it appears that nominal interest rates (hence inflation) should affect welfare by increasing the cost of consumption. The above two arguments appear to me contradictory. How do I compromise?

A The reason is that the marginal utility of consumption is only affected by the nominal interest rate for *given* marginal value of wealth ($\lambda_t$ in Walsh's notation). So, in the short run, there *will* be real effects of variations in the nominal interest rate, as also seen in the simulations of the stochastic CIA model. In the long run, however, this flexible price model has the standard property that output is determined by capital, and the steady-state capital is determined independently of the nominal interest rate. The reason being that capital accumulation or other production input determination are not distorted by the nominal interest rate. A higher long-run nominal interest rate, will therefore not affect steady-state capital, and thus steady-state output and consumption. This is consistent with the first-order condition, as — in the long run — a higher nominal interest rate is associated with a offsetting reduction in the marginal value of wealth as measured by $\lambda^{ss}$.

Mathematically, this can be shown as follows. Note that the steady state is determined by the following set of equations:

$$u_c\left(c^{ss}\right) = \lambda^{ss}\left(1+i^{ss}\right), \tag{1}$$

$$R^{ss} = 1/\beta, \tag{2}$$

$$R^{ss} = f_k\left(k^{ss}\right)+1-\delta, \tag{3}$$

$$R^{ss} = \frac{1+i^{ss}}{1+\theta^{ss}}, \tag{4}$$

$$c^{ss} = f\left(k^{ss}\right)-\delta k^{ss}, \tag{5}$$

$$i^{ss} = \frac{\mu^{ss}}{\lambda^{ss}}. \tag{6}$$

Equation (2) determines $R^{ss}$, then equation (3) determines $k^{ss}$, and then equation (5) determines $c^{ss}$. Equation (4) then determines, conditional on money growth, $i^{ss}$. Equation (1) then determines $\lambda^{ss}$, and finally (6) determines $\mu^{ss}$. Indeed, the solutions for the multipliers are easily found as

$$\lambda^{ss} = \frac{\beta}{1+\theta^{ss}} u_c\left(c^{ss}\right)$$

and

$$\mu^{ss} = \frac{1+\theta^{ss}-\beta}{1+\theta^{ss}} u_c\left(c^{ss}\right),$$

respectively. It thus follows that marginal utility of consumption only equals the marginal utility of wealth in the case where the Friedman rule applies, i.e., when $\theta^{ss} = \beta - 1$. In that case, $\mu^{ss} = 0$ and the CIA constraint does not bind in the steady state. For higher rates of money growth, the CIA constraint binds, as the steady-state nominal interest rate becomes positive; i.e., $\mu^{ss} > 0$. Importantly, note that a change in the steady-state rate of money growth alters the multipliers in *exact opposite directions*,

$$\frac{\mathrm{d}\lambda^{ss}}{\mathrm{d}\theta^{ss}} = -\frac{\mathrm{d}\mu^{ss}}{\mathrm{d}\theta^{ss}},$$

thus being consistent with having superneutrality and the marginal utility of consumption written as $u_c\left(c^{ss}\right) = \lambda^{ss} + \mu^{ss}$.